Algorithms for Massive, Expensive, and Otherwise Inconvenient Graphs

DAVID TENCH

UNIVERSITY OF MASSACHUSETTS AMHERST

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I’ll warn you before the tricky parts.

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8	6	1	9	3	4	5	2

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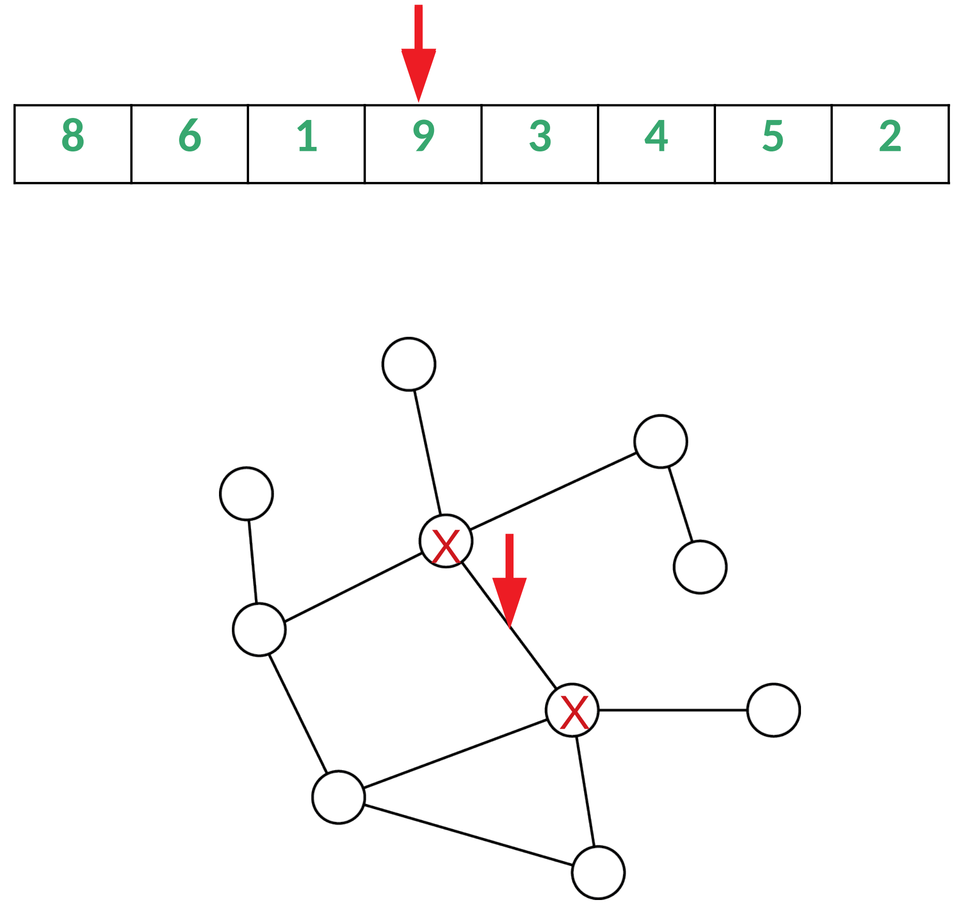
- Is there an edge between some pair of nodes in this graph?



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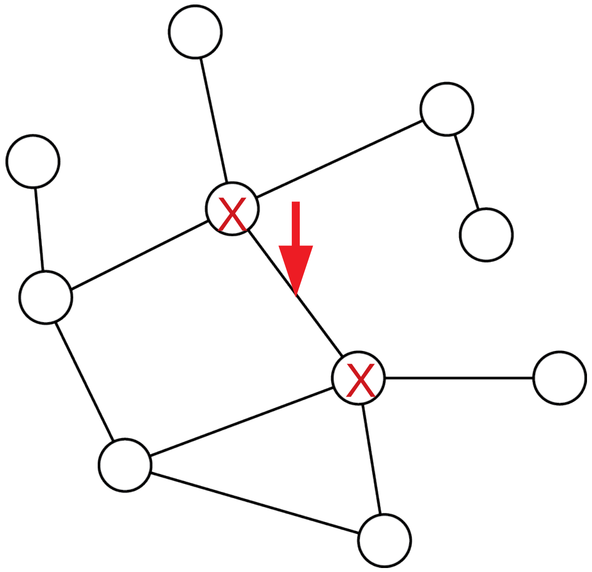

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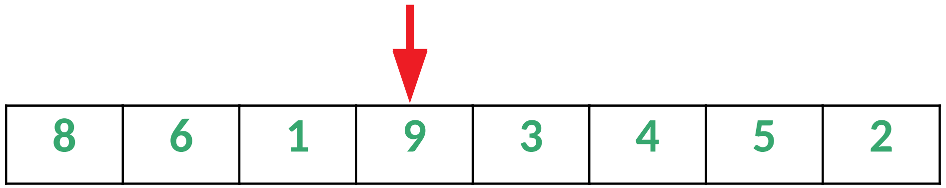

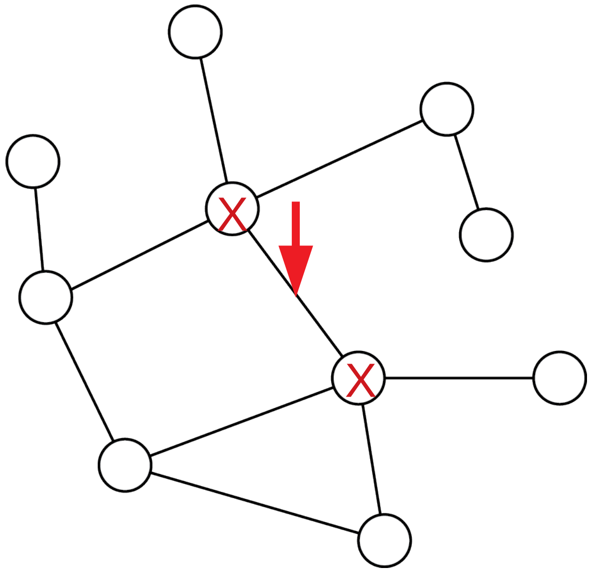

An algorithm can access **any part** of its input at **any time** at **unit cost**.

This is the *random access property*.

When Random Access Fails

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When inputs are too large
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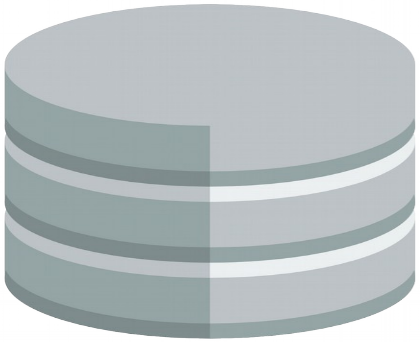

Overview of this talk

Streaming graph algorithms:
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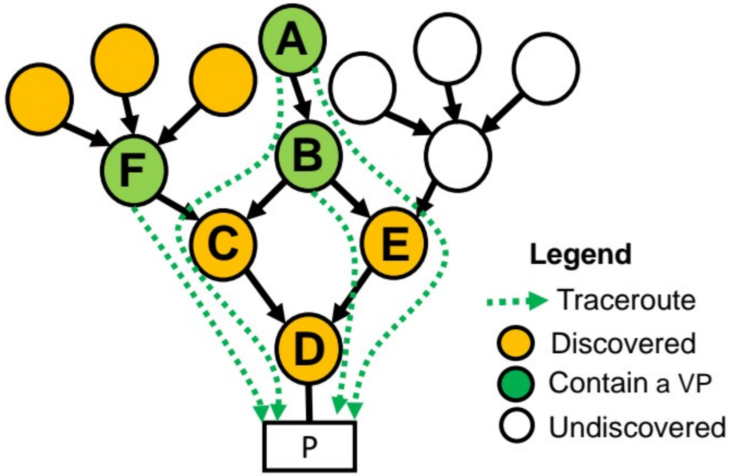

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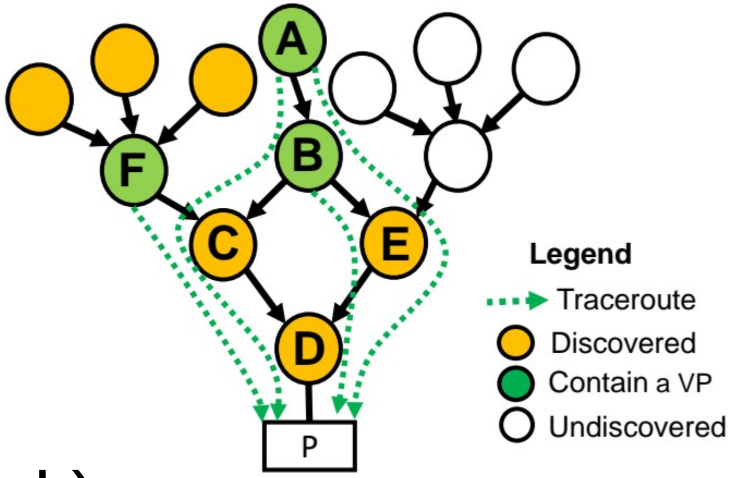

(Also future work)

Streaming

COMPUTING WITH INCREDIBLY LARGE INPUTS

When Graphs Are Too Large

Can’t store graph in memory

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Receive *stream* of edges

When Graphs Are Too Large

Can't store graph in memory

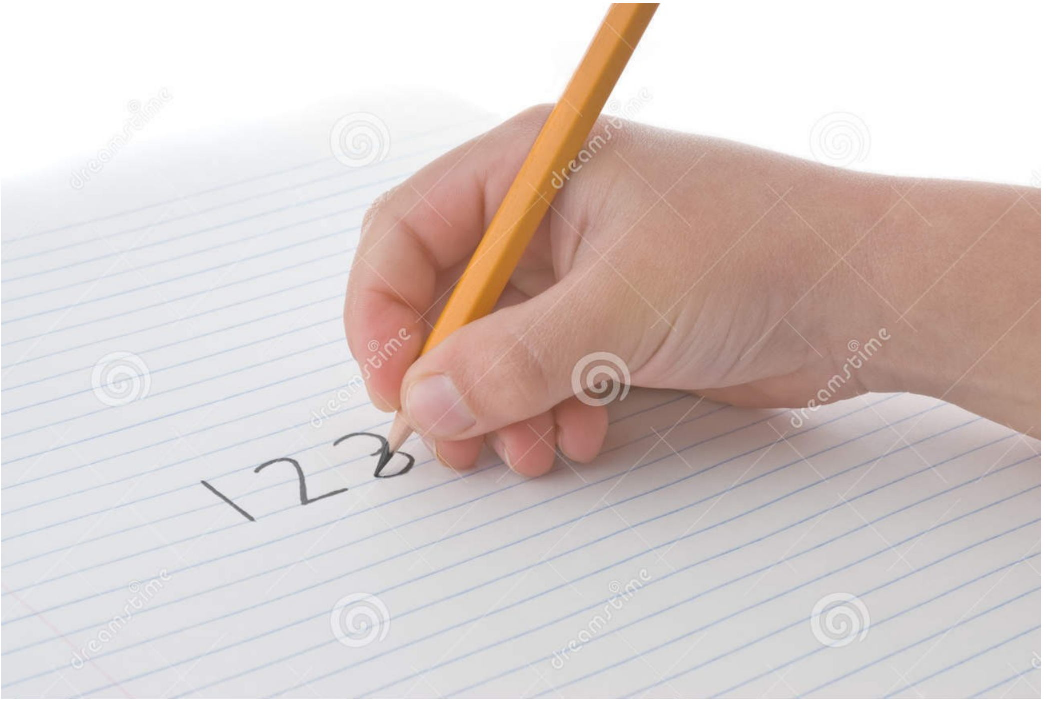
Receive *stream* of edges

- Vertex Connectivity (PODS 2015)
- Densest Subgraph (MFCS 2015)
- Unique Cover (current work)

Warm-up: Missing Number

I read you an unordered list of all the integers from 1 to 5 million – except I leave one of them out. After, I ask you which is missing.

You only have a single piece of paper and a pencil. How do you find the missing number?



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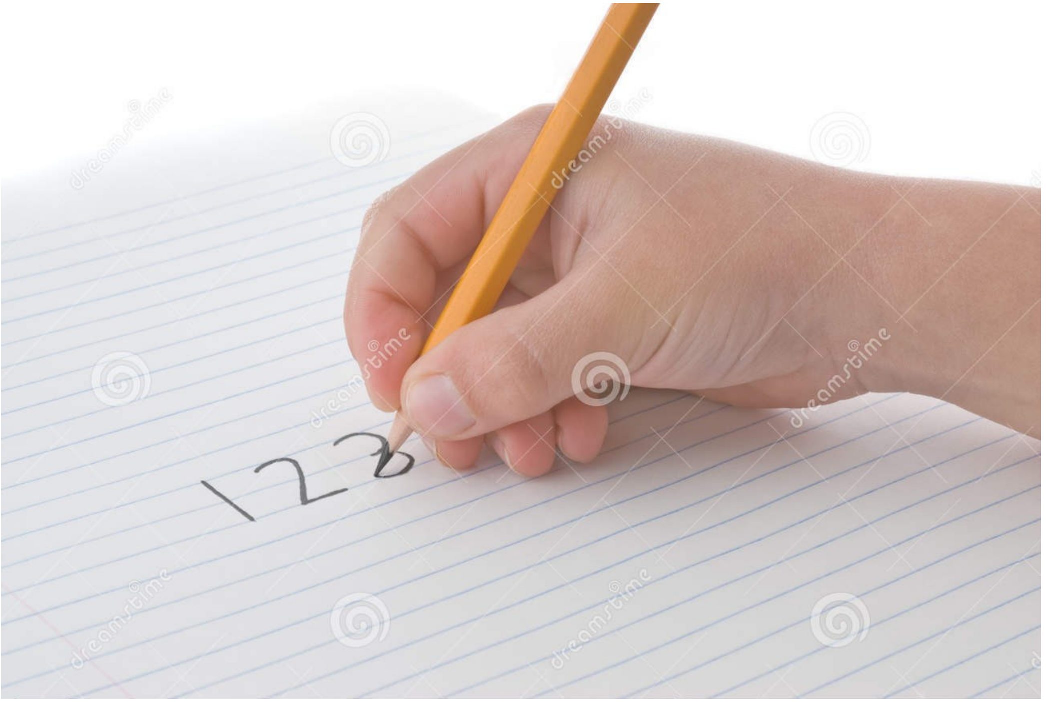
Answer: keep a running sum of all the numbers, then subtract from 1 + 2 + … + 5 million.



Graph Streaming

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Nodes = users, edges = friend relationships

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This graph could have almost $(2 \text{ billion})^2$ = 4 quintillion edges

For modern computers, storing 2 billion objects is maybe reasonable but 4 quintillion is not

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It’s possible to store some of the edges, but not all

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We have roughly n space, where n = # of nodes

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Graph edges are given as a *stream*

Defining a graph via a stream

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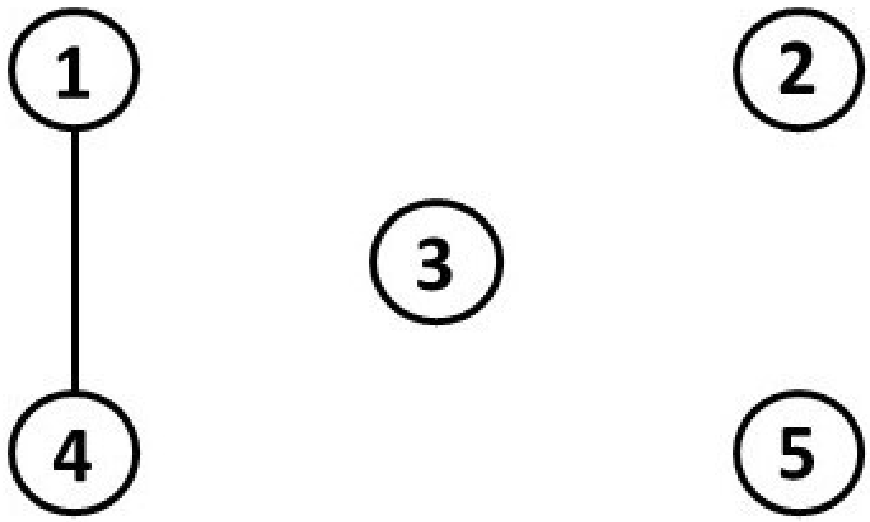
Add edge (1,4)

Defining a graph via a stream



Defining a graph via a stream

Add edge (3,4)

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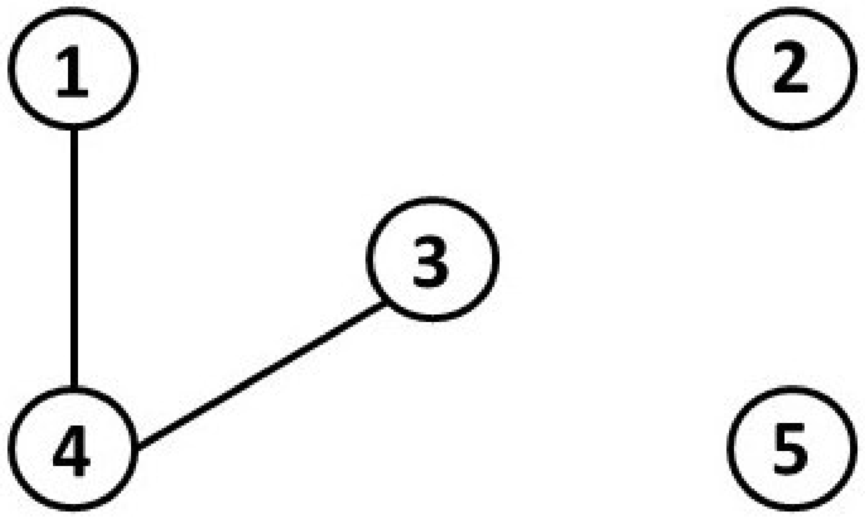

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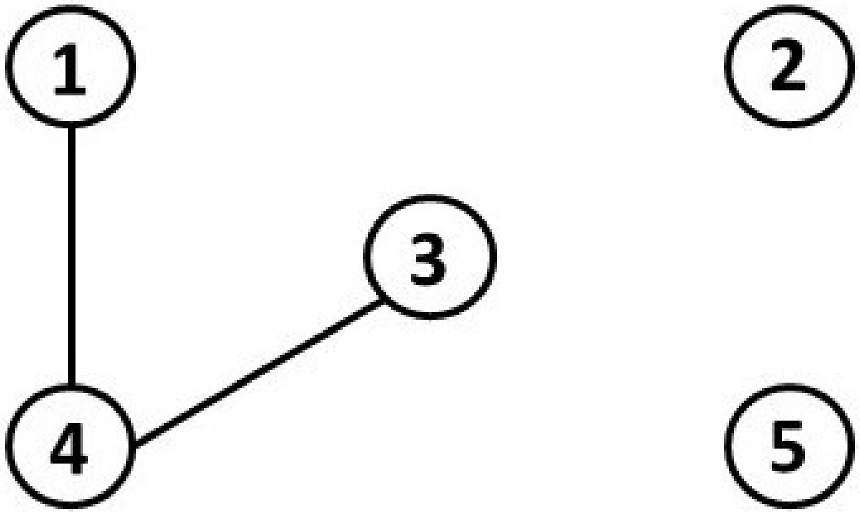
Add edge (1,3)



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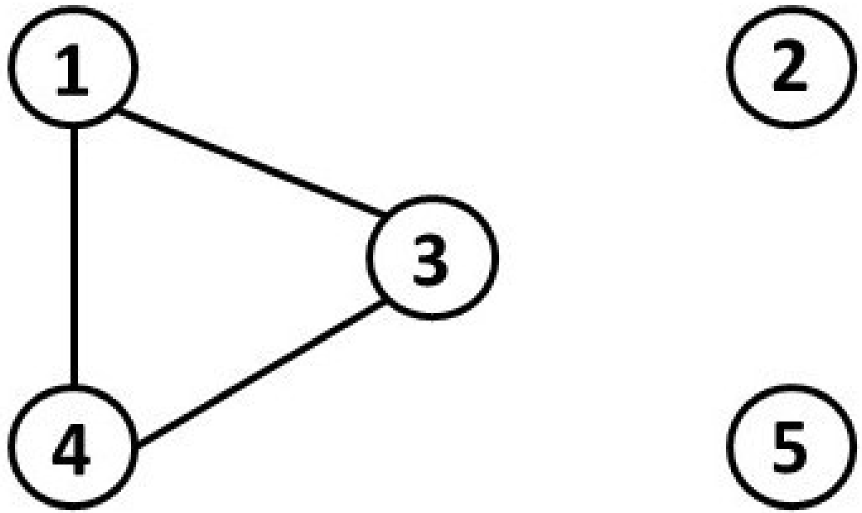
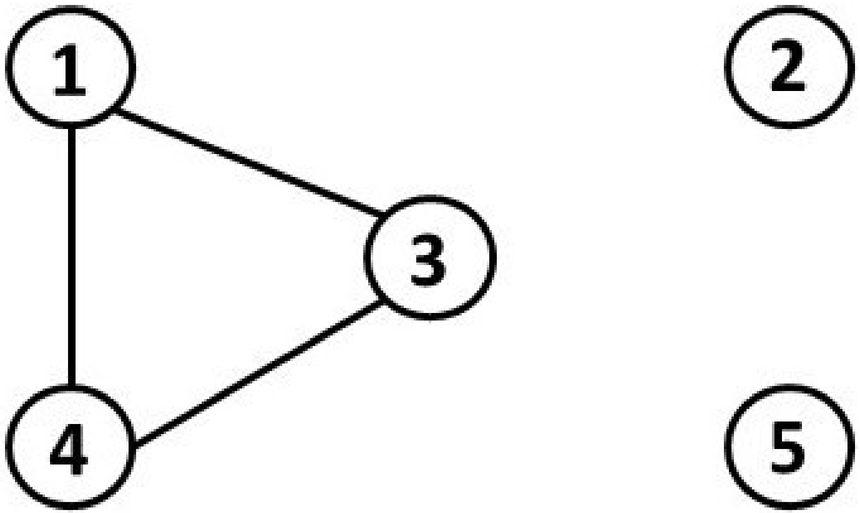


Defining a graph via a stream

Add edge (2,5)



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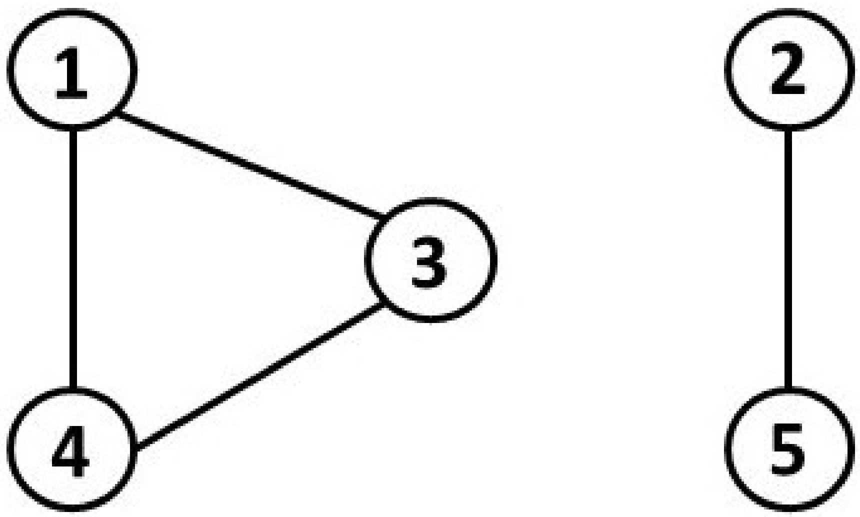

Defining a graph via a stream

Stream:
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Add edge (1,3)
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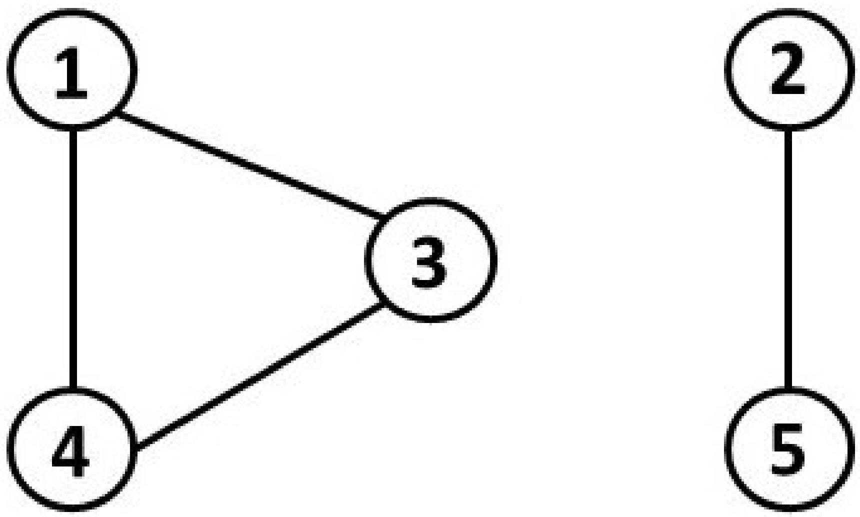
Resulting Graph:



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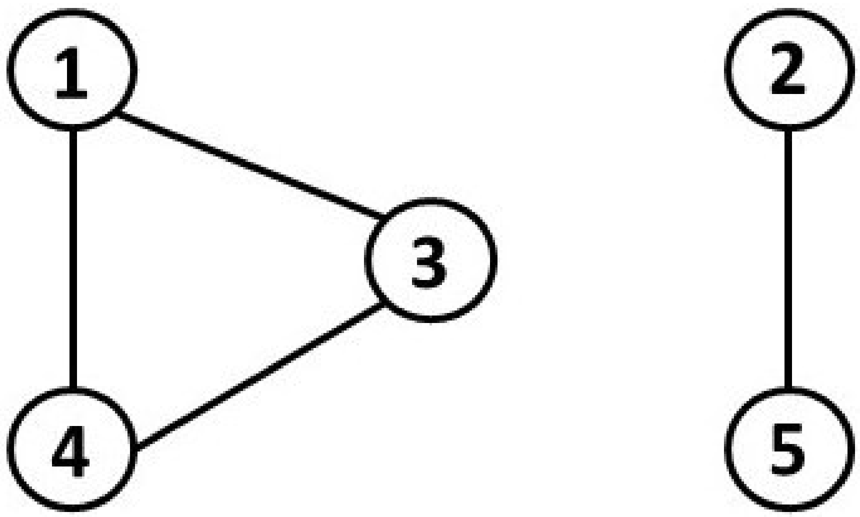
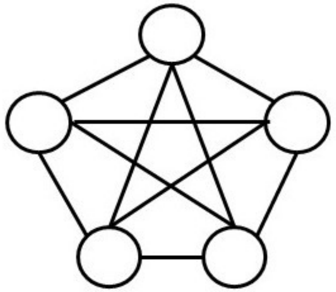
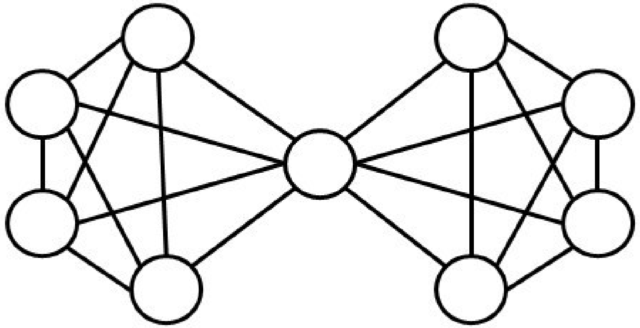


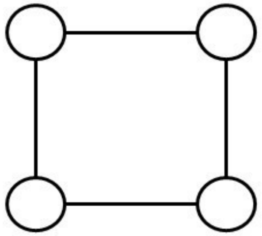
Edge deletions are also possible, but we’re ignoring them today.

Problem: Vertex Connectivity





What is the minimum number of nodes we can remove to disconnect a graph G?



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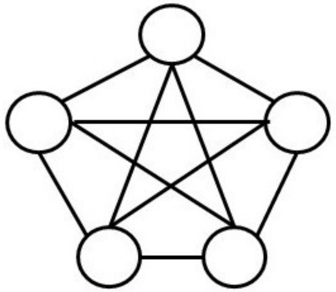
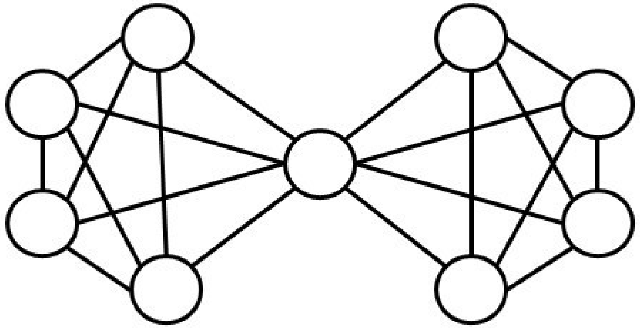
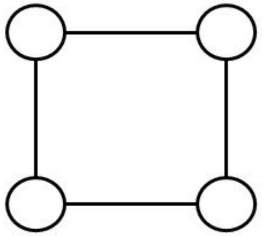




1



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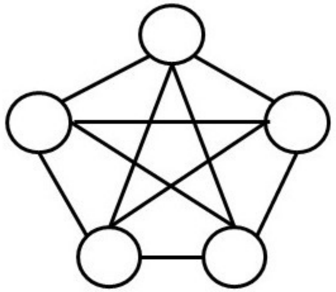

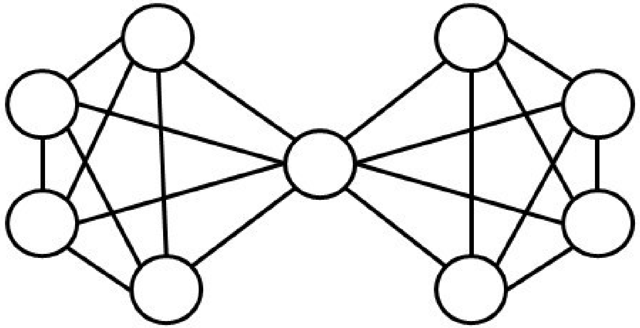

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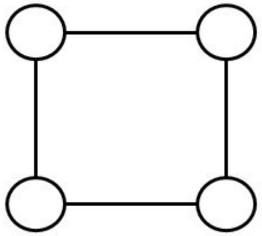

2

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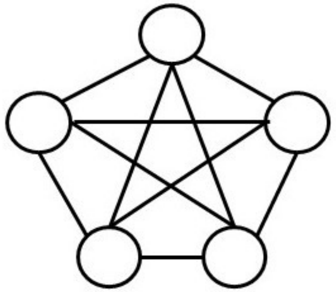

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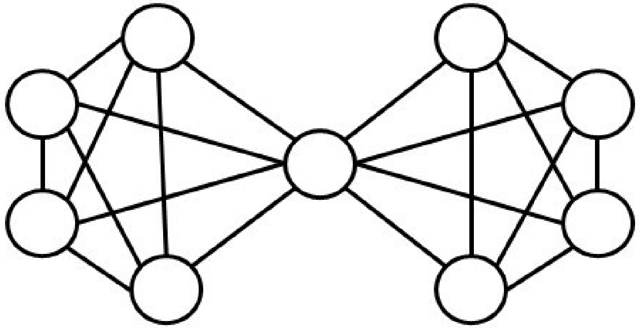

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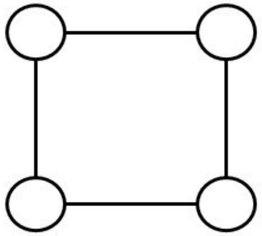

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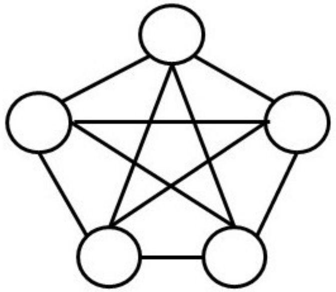

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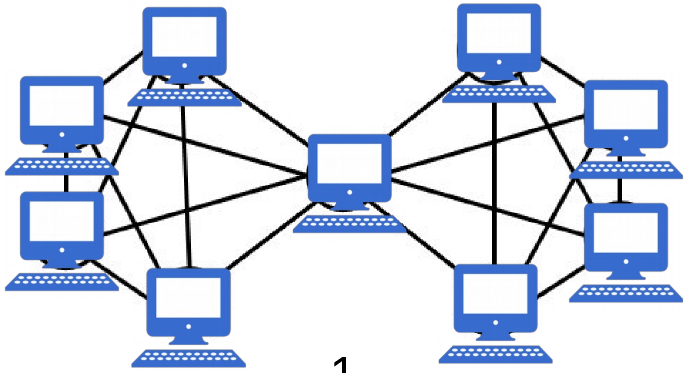

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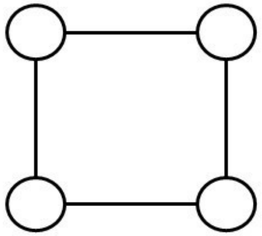

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How many computers in a network can fail before the remaining network is disconnected?

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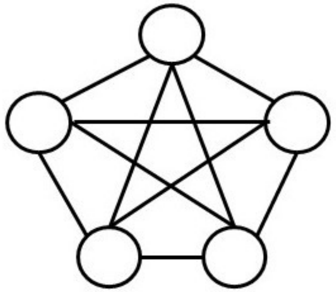

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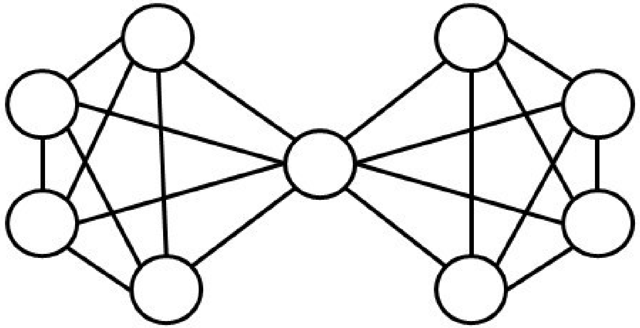

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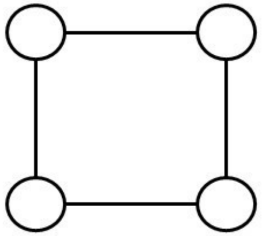

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Easily solvable using max flow algorithm, but this requires random access to the graph.

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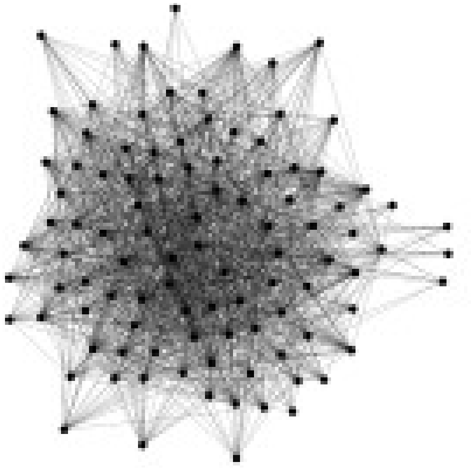
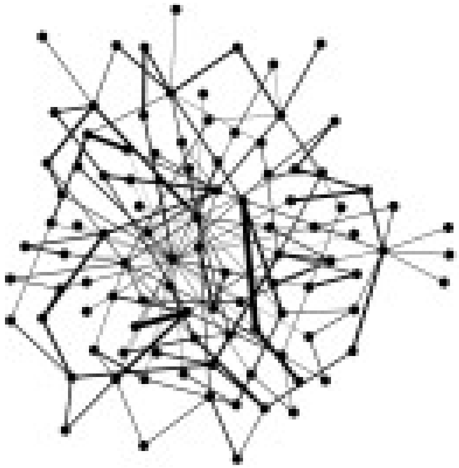


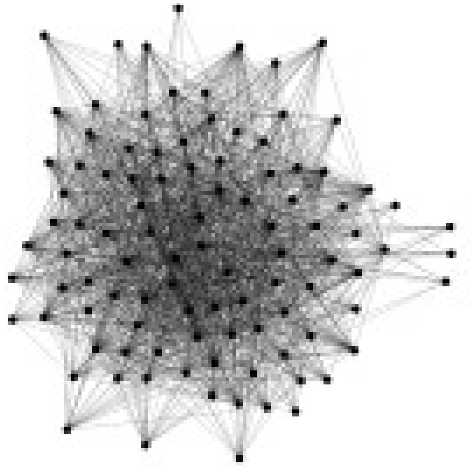
G



H

We show how to create a *certificate* graph H that matches G’s vertex connectivity up to constant k, but has only roughly kn edges.

Problem: Vertex Connectivity



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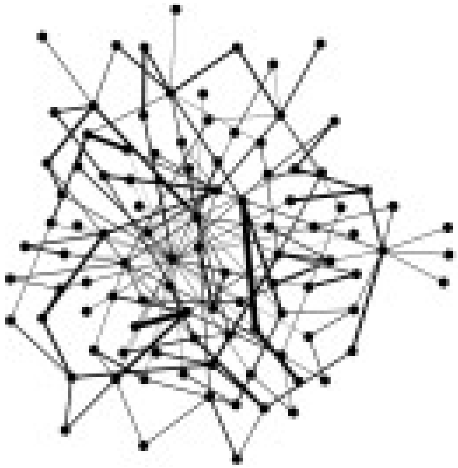

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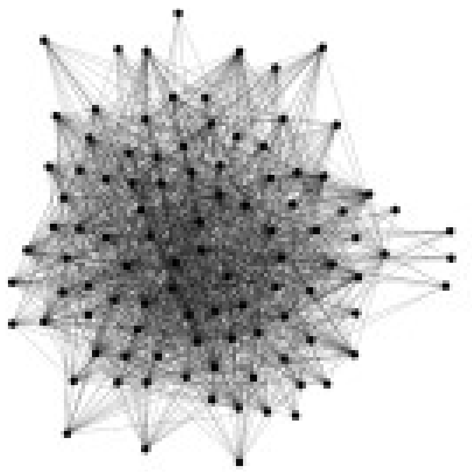

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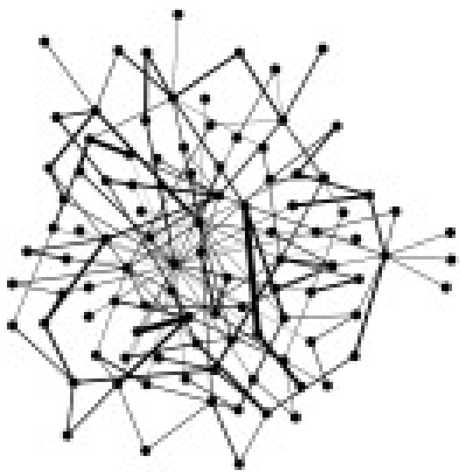

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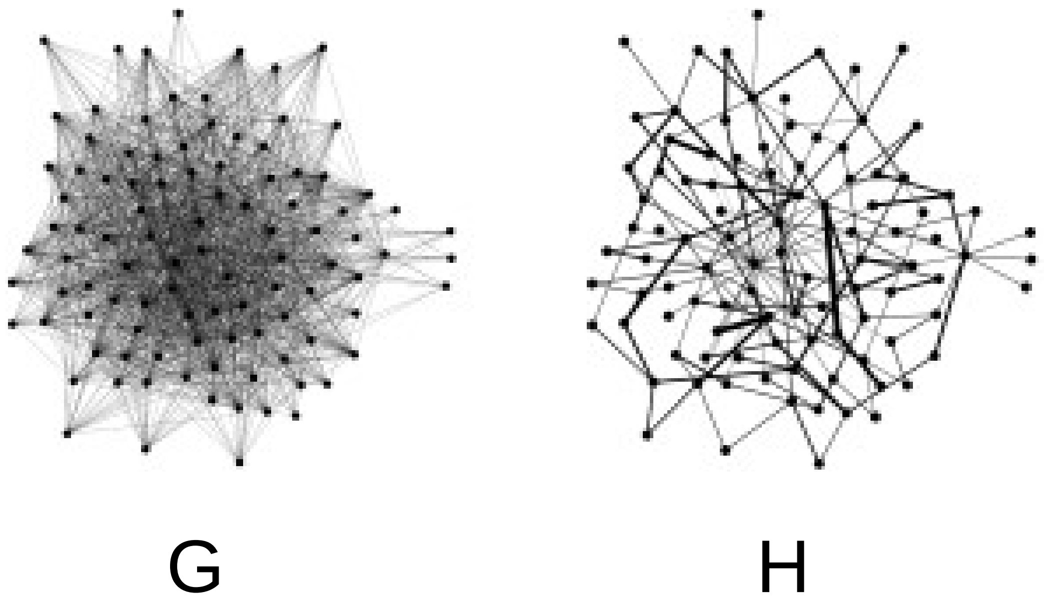

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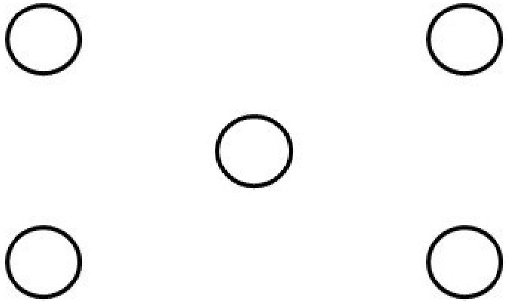
Warm-up: Is the Graph Connected?

Original graph



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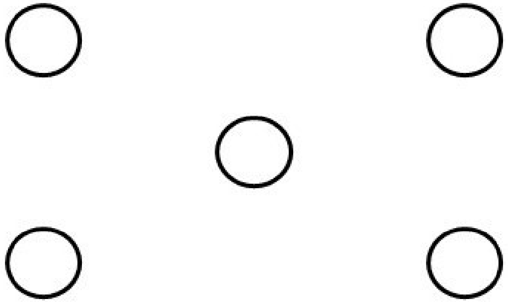

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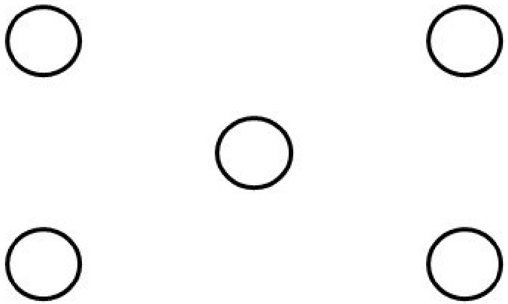

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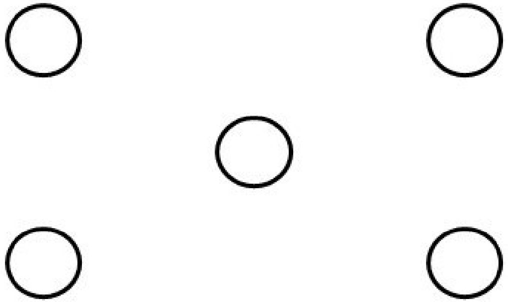

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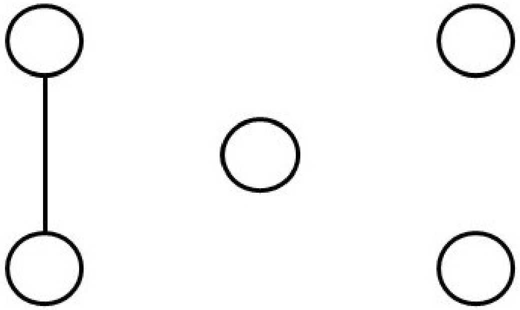

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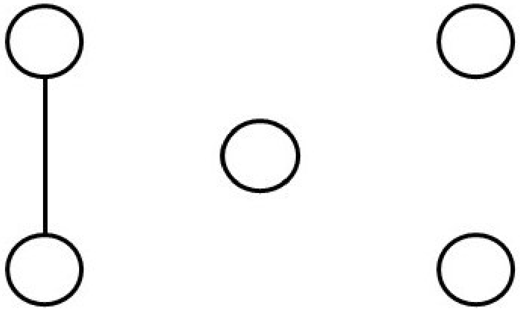

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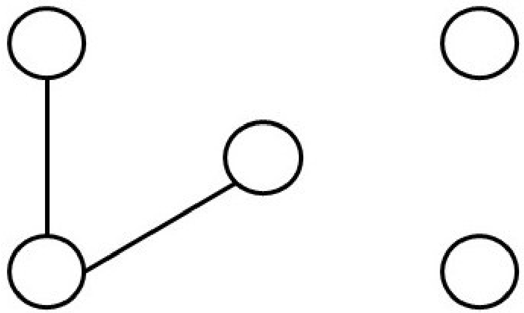

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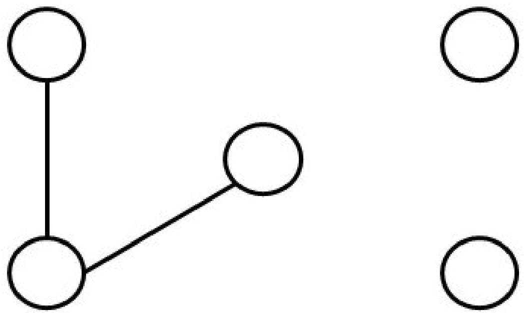

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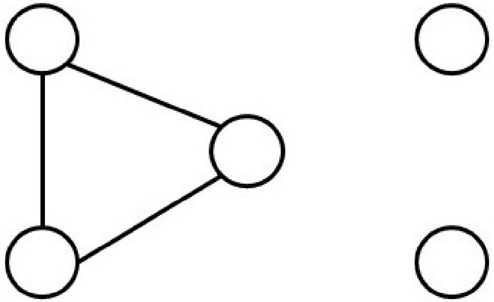

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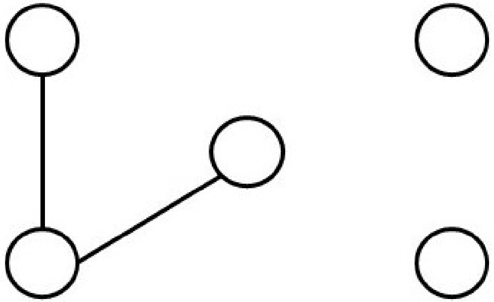

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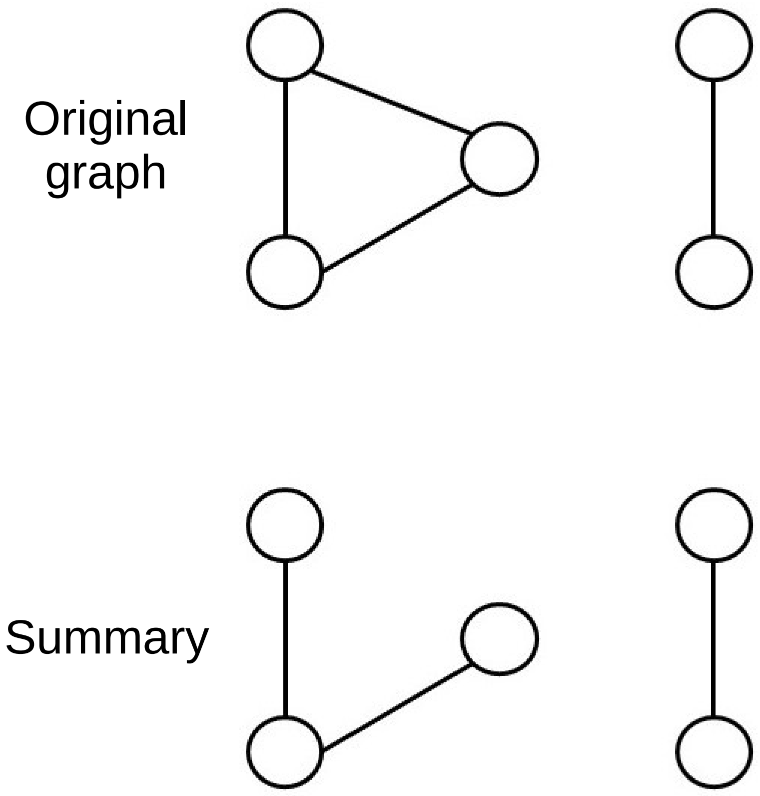

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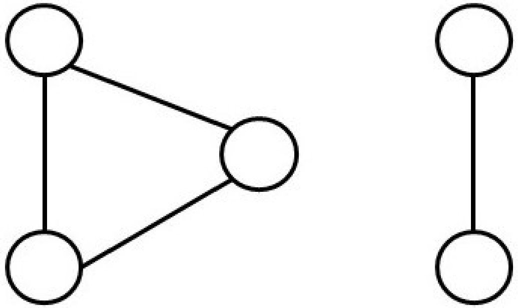
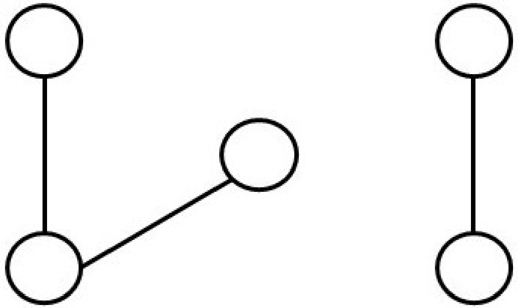


Spanning Forest



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This is a **spanning forest** and has at most n-1 edges.

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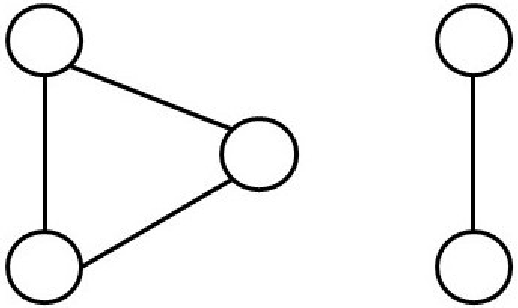
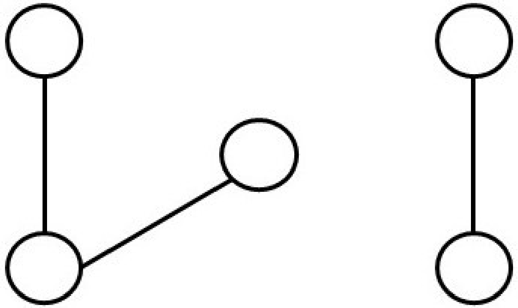


Spanning Forest

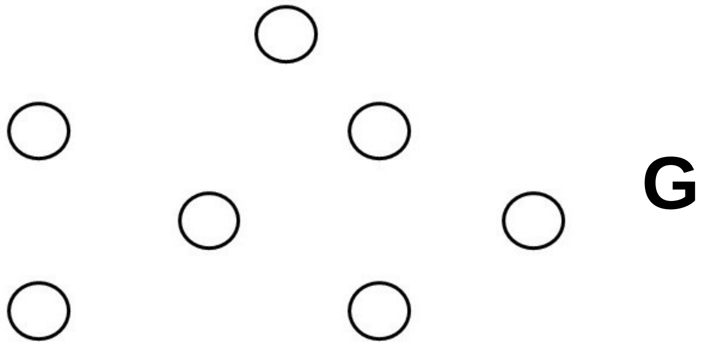


As each edge arrives in the stream, we keep it only if its endpoints were not already connected.

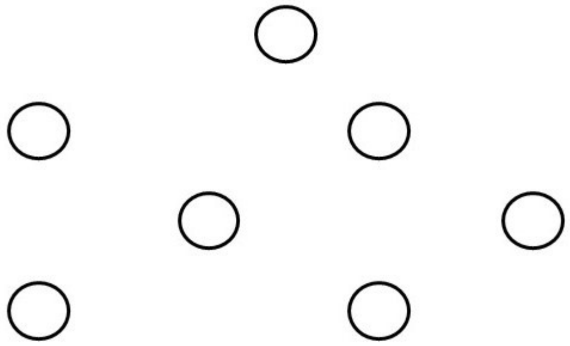
This is a **spanning forest** and has at most n-1 edges.

If the spanning forest is connected, we know the original graph was as well.

Back to Vertex Connectivity



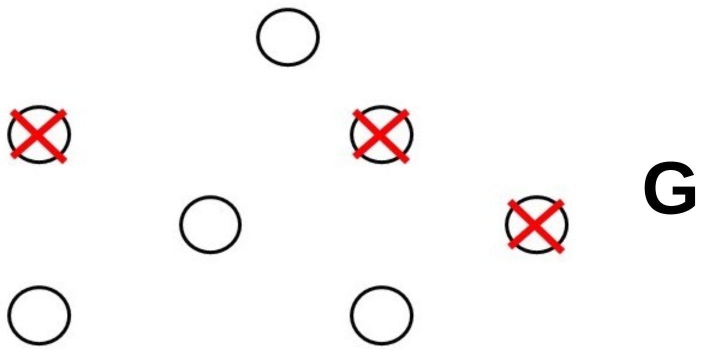
Back to Vertex Connectivity



G

First we delete each node in G with probability 1-1/k.

Back to Vertex Connectivity



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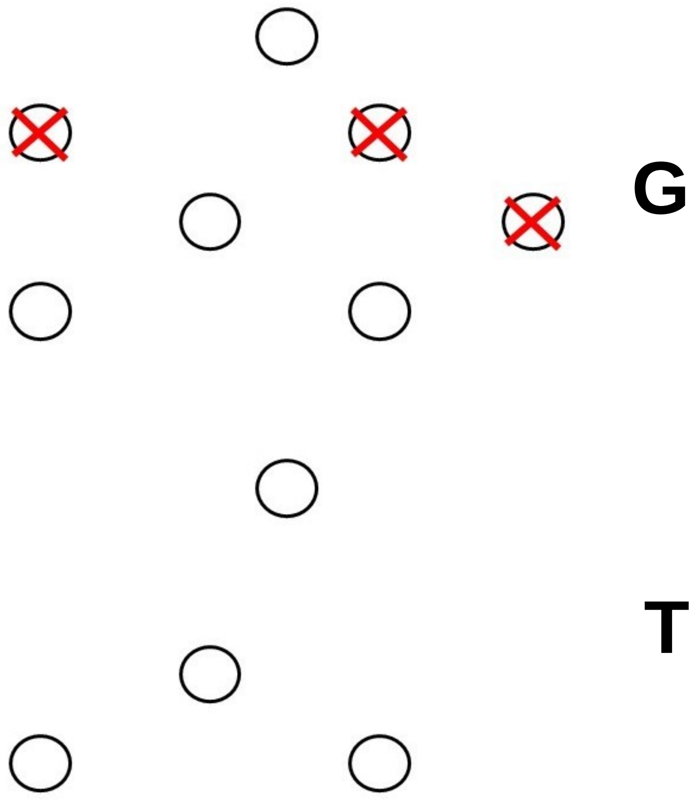
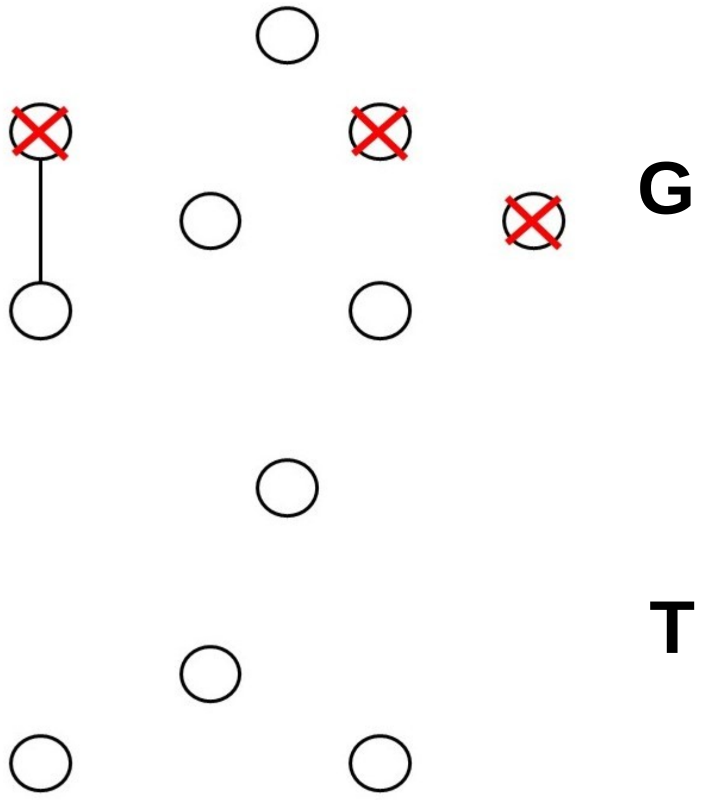


First we delete each node in G with probability 1-1/k.

During the stream we maintain a spanning forest T on the remaining nodes.

Back to Vertex Connectivity



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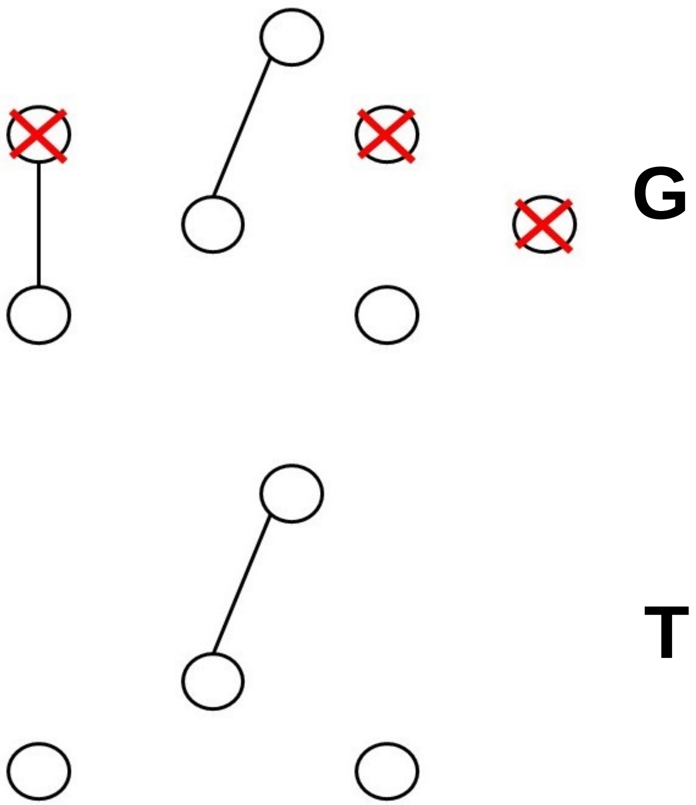

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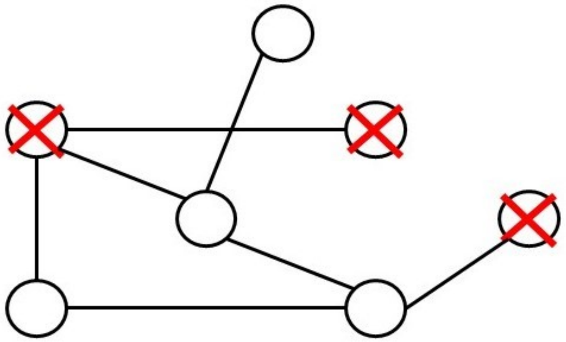

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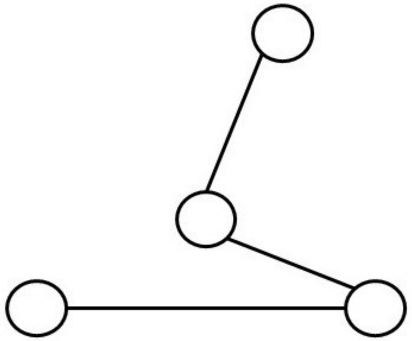

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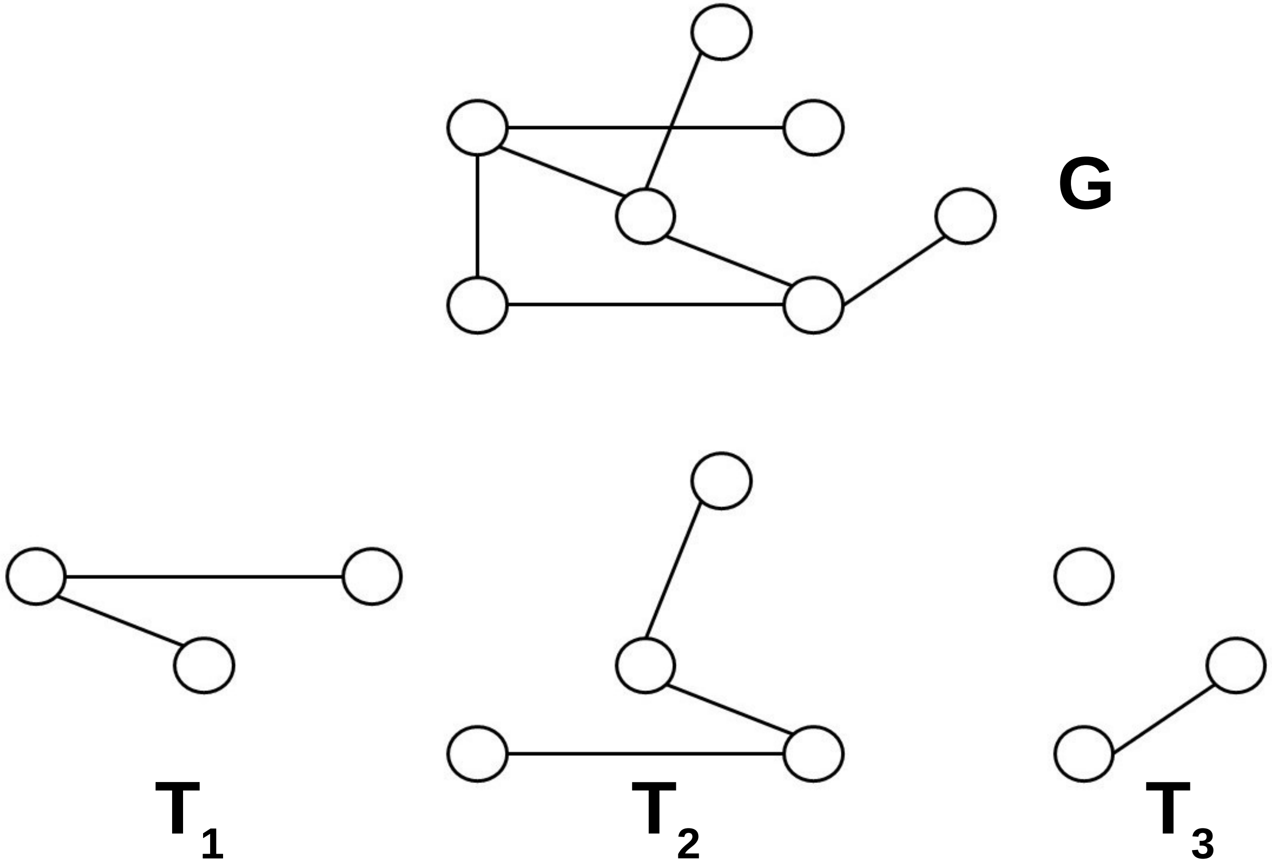
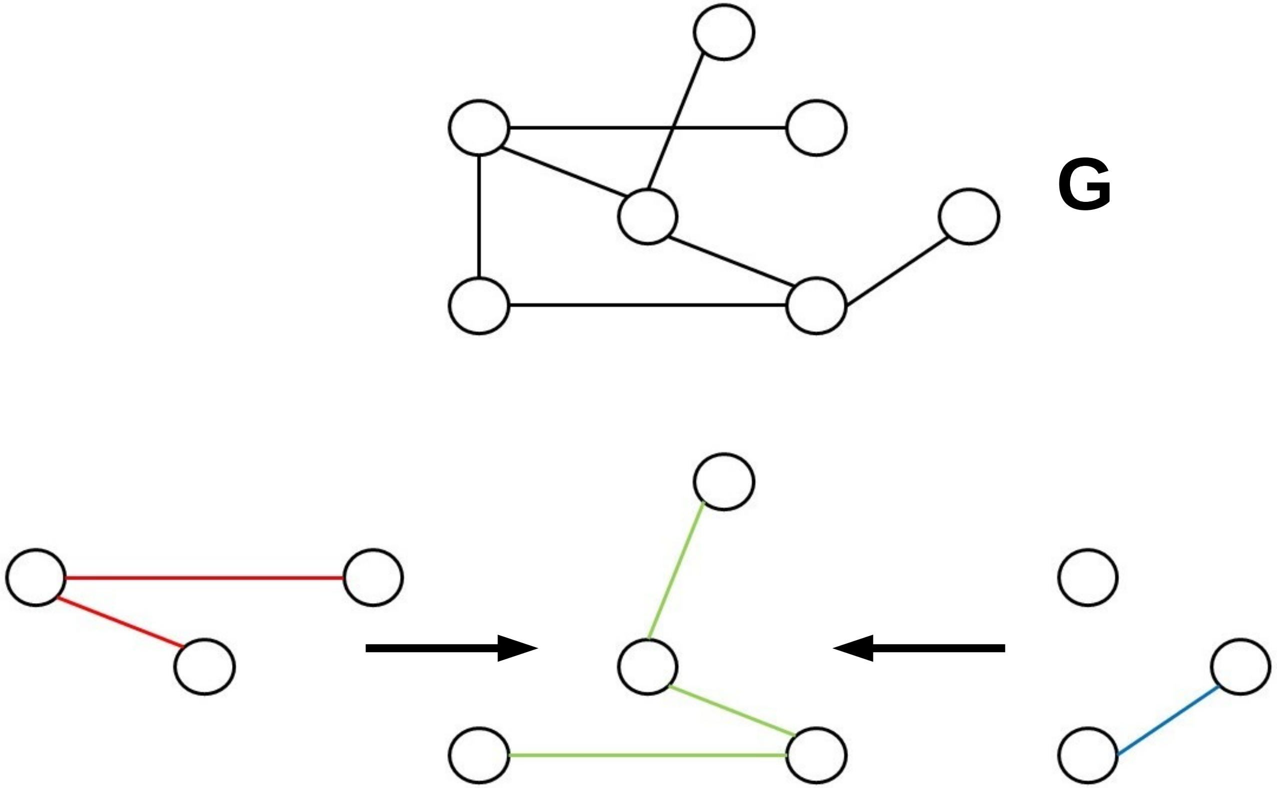


First we delete each node in G with probability 1-1/k.

During the stream we maintain a spanning forest T on the remaining nodes.

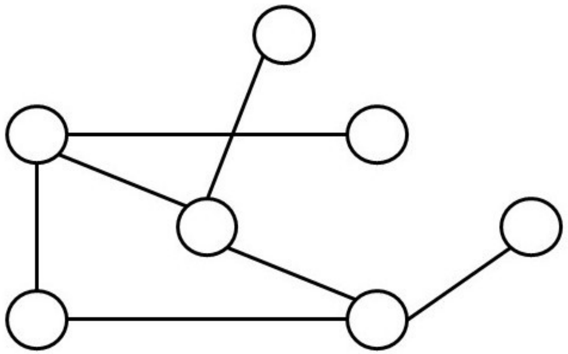
Repeat roughly k^2 times in parallel* to make T_1, T_2, …

Back to Vertex Connectivity



After the stream, merge all T_i to form H, our vertex connectivity certificate.

Back to Vertex Connectivity



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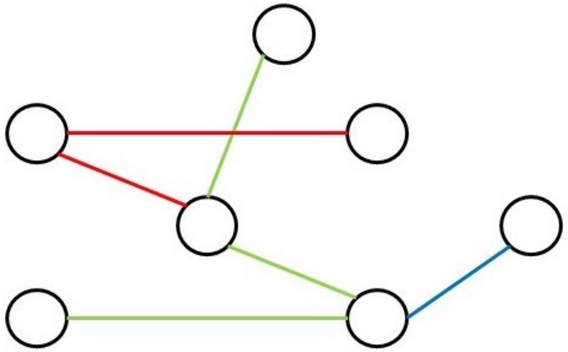

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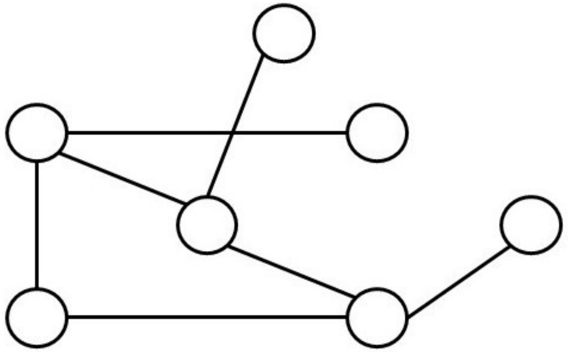

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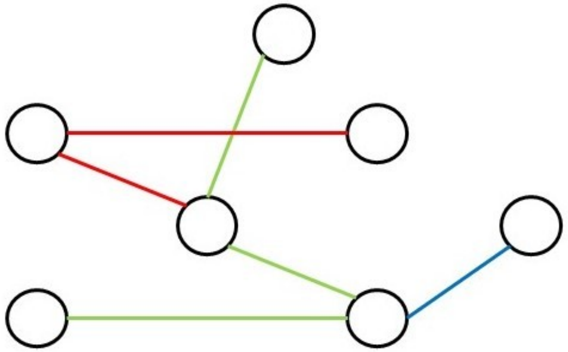

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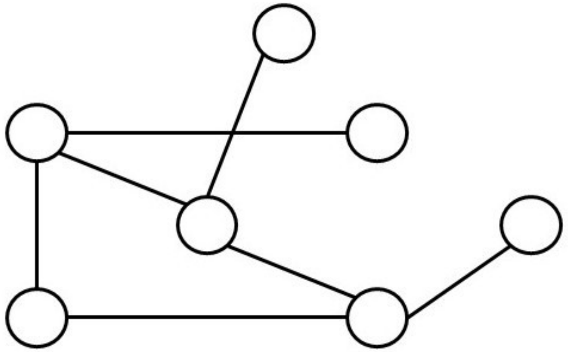
After the stream, merge all T_i to form H, our vertex connectivity certificate.

Each spanning forest has n/k edges in expectation and there are k^2 of them so H has kn edges in expectation.

Back to Vertex Connectivity



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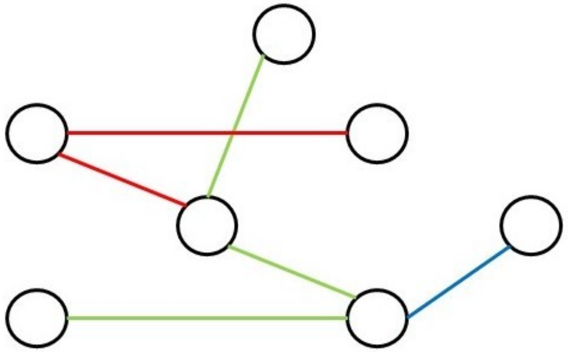

H

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Theorem: For some arbitrary set S of at most k nodes, G\S is disconnected iff H\S is disconnected.

Proof Sketch of Theorem

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To prove: When G\S is disconnected, H\S must be disconnected

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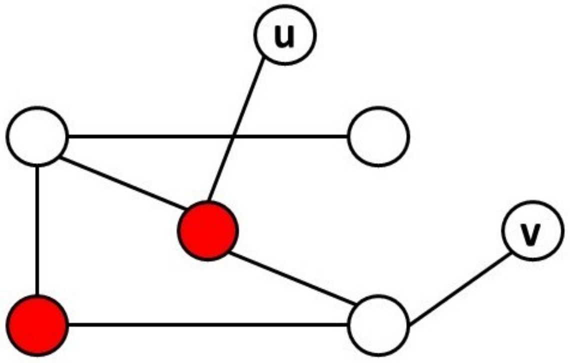

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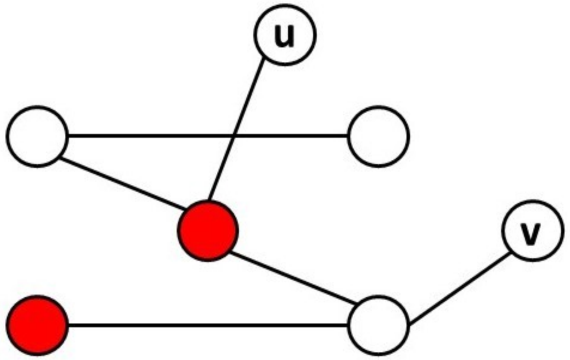
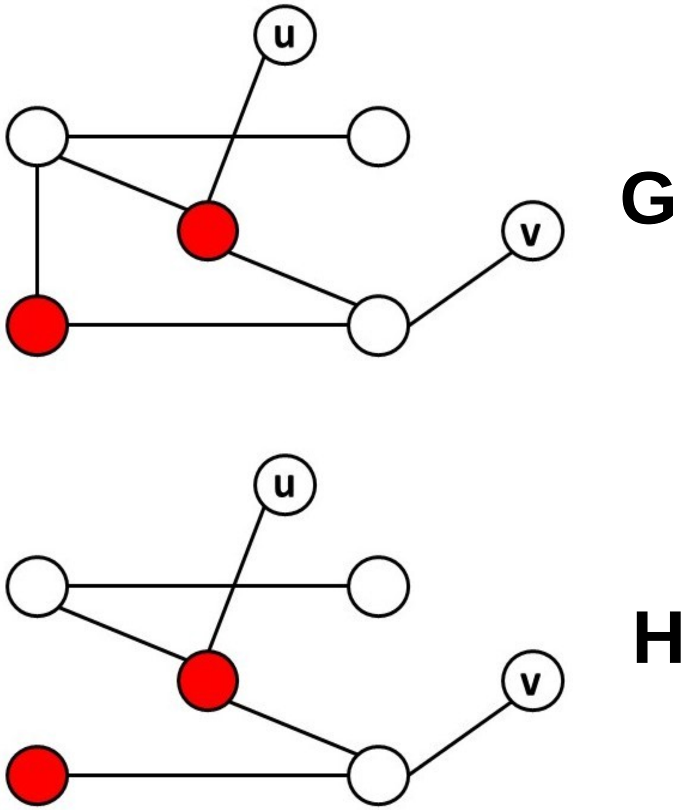


H

Proof Sketch of Theorem

To prove: When G\S is disconnected, H\S must be disconnected

H is a subgraph of G, so if G\S lacks a path between nodes u and v, so does H\S.



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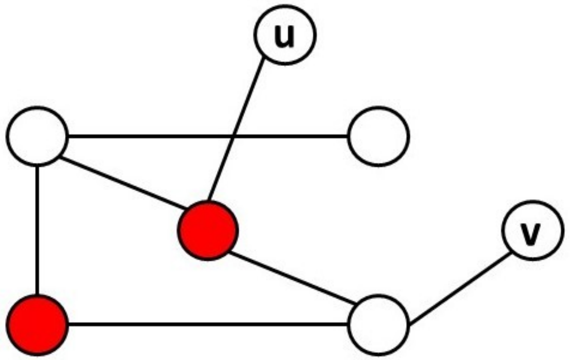
With high probability, H has all of G's nodes. //



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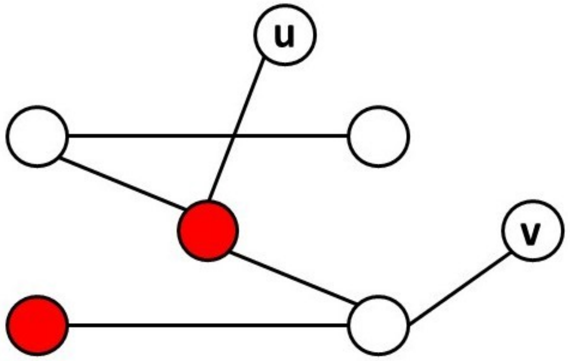


H

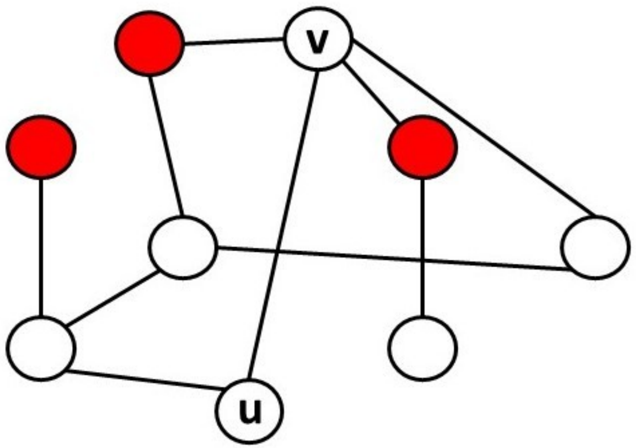
Proof Sketch of Theorem

To prove: When G\S is connected, H\S must be connected

Proof Sketch of Theorem

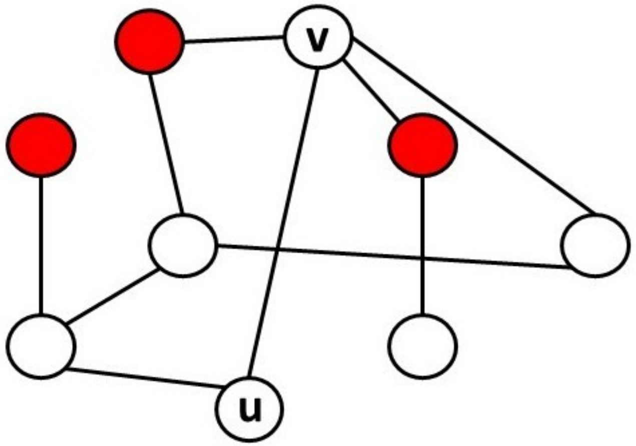
To prove: ~~When G\S is connected, H\S must be connected~~
To prove: If some edge (u,v) exists in G\S, then there is a path between u and v in H\S.

Proof Sketch of Theorem



To prove: If edge (u,v) exists in G\S, u and v are connected in H\S.

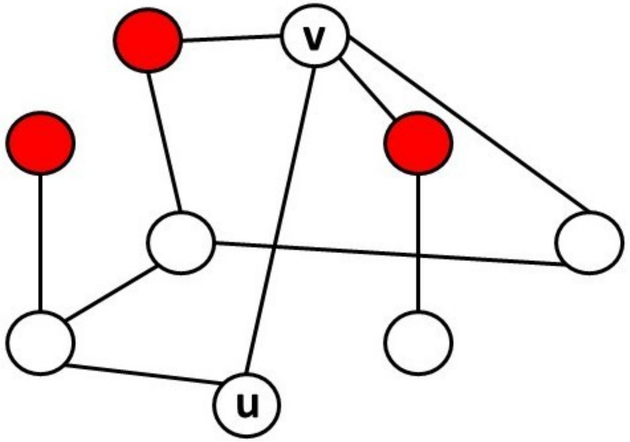
Proof Sketch of Theorem



To prove: If edge (u,v) exists in G\S, u and v are connected in H\S.

If there's some T_i that contains both u and v...

Proof Sketch of Theorem



G

$\mathbf{T_i}$

To prove: If edge (u,v) exists in G\S, u and v are connected in H\S.

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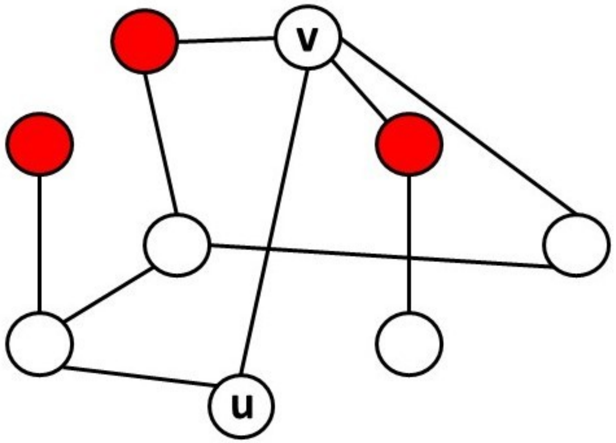
Either edge (u,v) is in T_i or

Proof Sketch of Theorem



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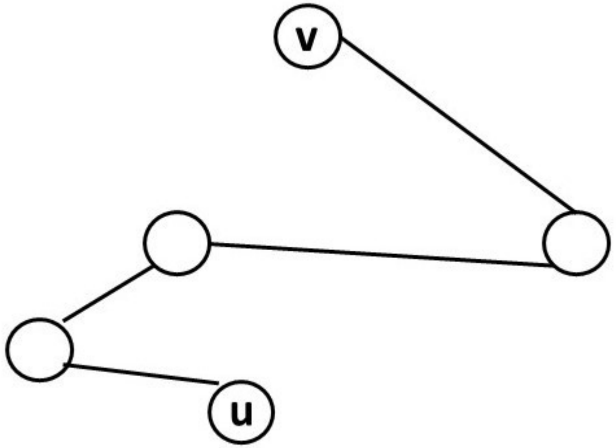


T_i

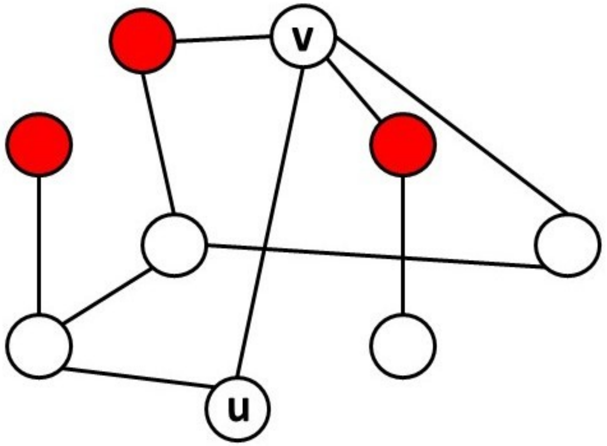
To prove: If edge (u,v) exists in G\S, u and v are connected in H\S.

If there's some T_i that contains both u and v…

Either edge (u,v) is in T_i or

Some other path between u and v is in T_i

Proof Sketch of Theorem



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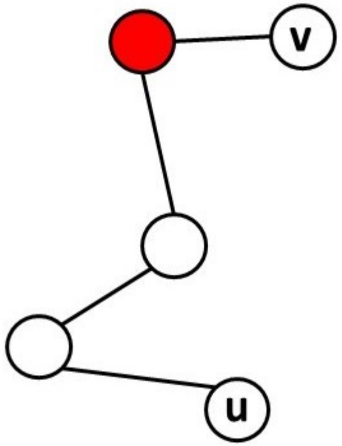


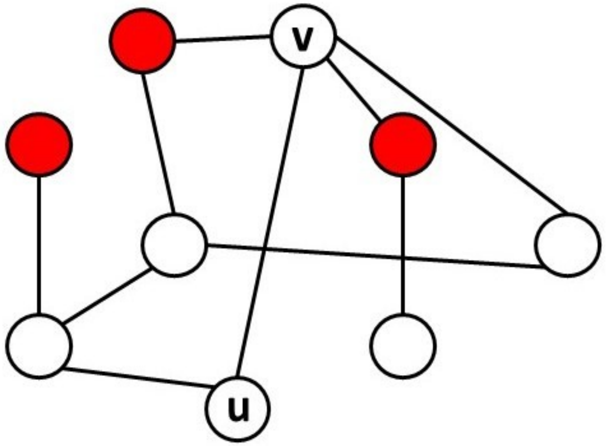
T_i

To prove: If edge (u,v) exists in G\S, u and v are connected in H\S.

If there’s some T_i that contains both u and v…

The only potential problem is that this alternate path may contain a node that’s in S (red node).

Proof Sketch of Theorem



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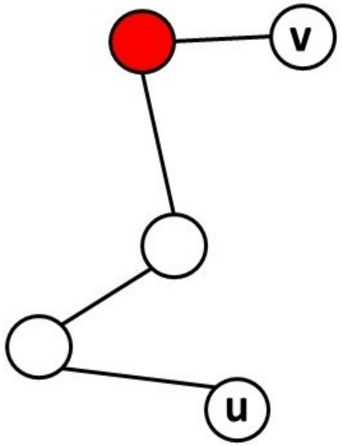

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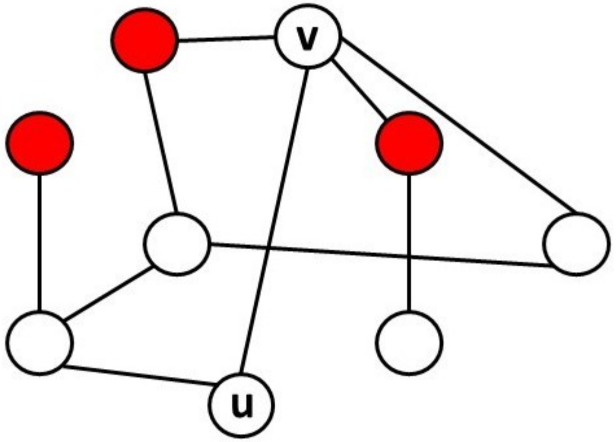
So we need T_i to not contain any nodes in S.

Proof Sketch of Theorem



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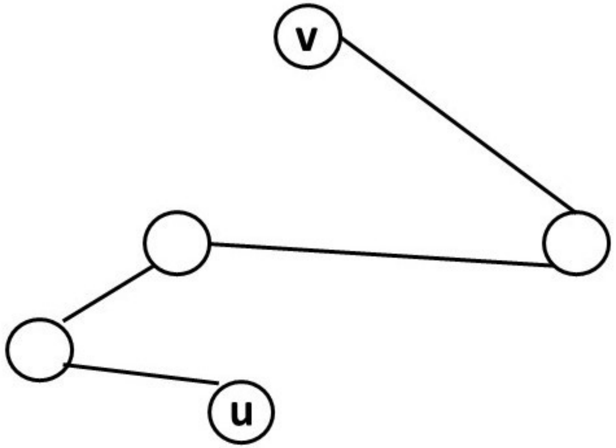

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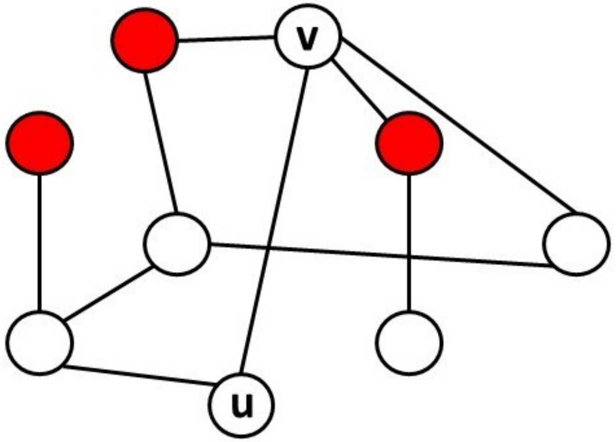
To prove: If edge (u,v) exists in G\S, u and v are connected in H\S.

We need T_i to contain u and v, and no red S nodes.

$$P(u \text{ and } v \text{ connected in } T_i \backslash S) = \frac{1}{k^2}\left(1 - \frac{1}{k}\right)^k$$

Proof Sketch of Theorem



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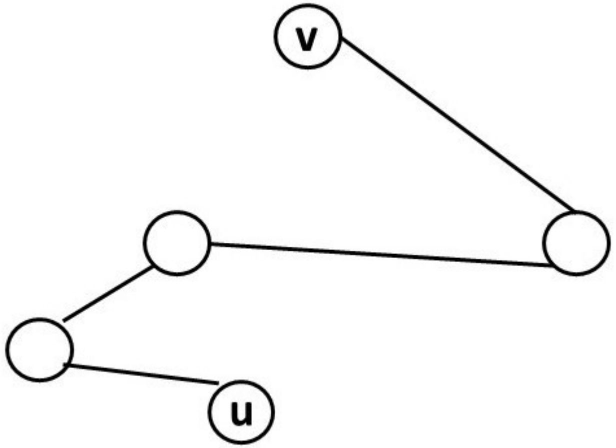

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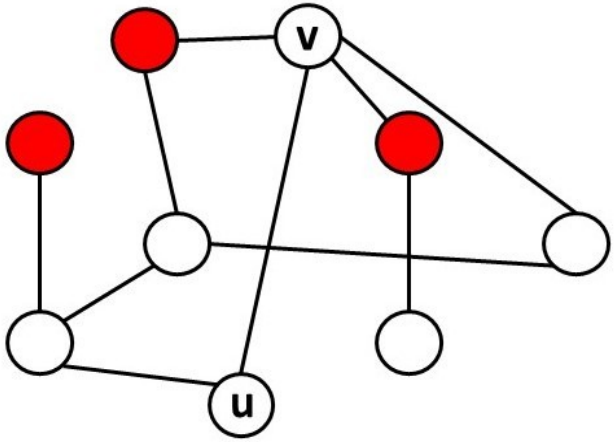

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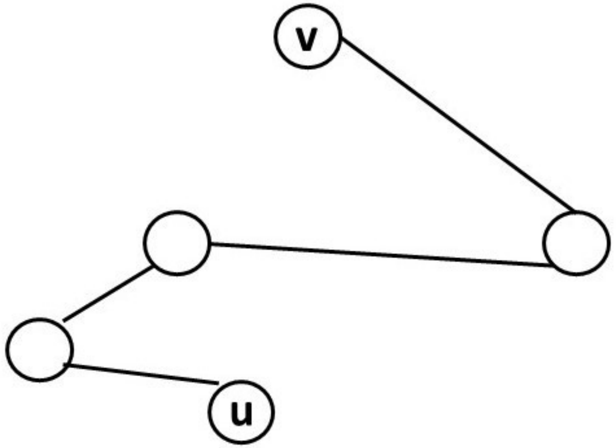
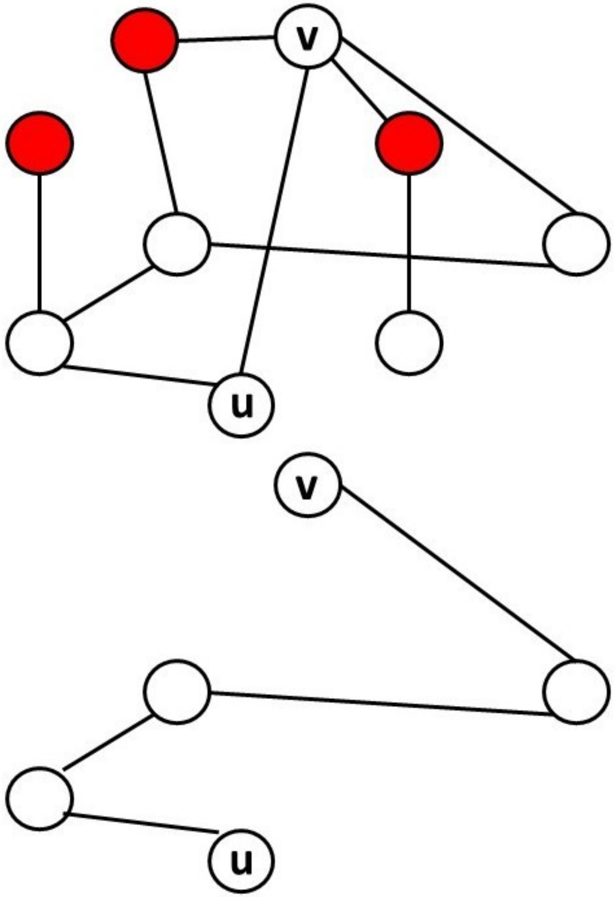


T_i

$$P(u \text{ and } v \text{ disconnected in } H \backslash S) = \left(1 - \frac{1}{k^2}\left(1 - \frac{1}{k}\right)^k\right)^{O(k^2 log(n))} \leq \frac{1}{n^4}$$

Proof Sketch of Theorem



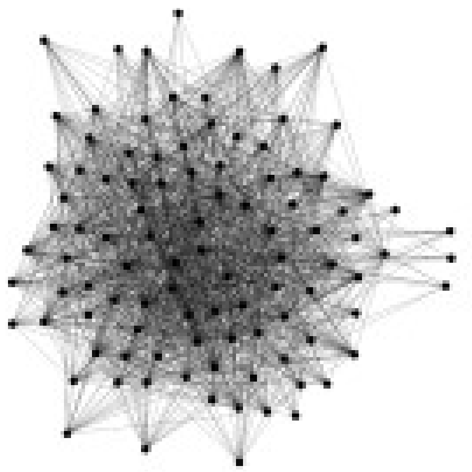
To prove: If edge (u,v) exists in G\S, u and v are connected in H\S.

We need T_i to contain u and v, and no red S nodes.

The probability that no T_i meets this requirement is at most $1/n^4$.

So u and v are connected in H\S with high probability. //

We did it!



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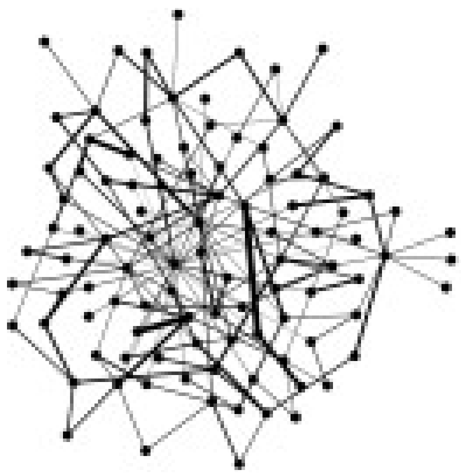
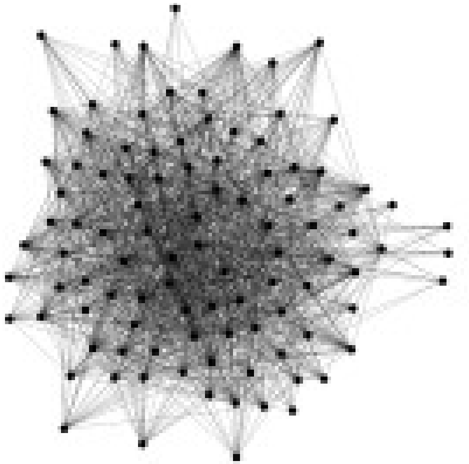


H

We showed how to create a *certificate* graph H that matches G’s vertex connectivity up to constant k, but has only roughly kn edges.

We did it!



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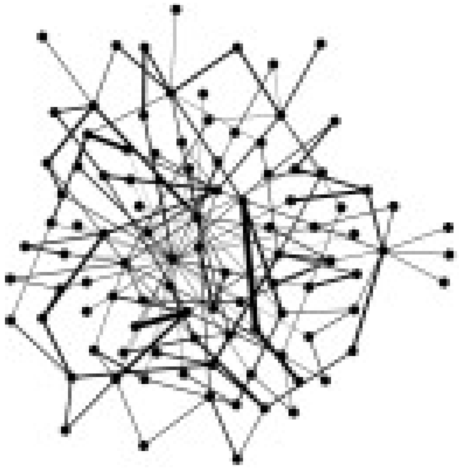

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We showed how to create a *certificate* graph H that matches G’s vertex connectivity up to constant k, but has only roughly kn edges.

You can process a massive stream of the edges in G to create H, which is much smaller. Then you can run a traditional vertex connectivity algorithm on H to get your answer.

Query-Based Algorithms

WHEN DISCOVERING GRAPH EDGES IS COSTLY

When Graph Edges Are Costly

Can check existence of any edge at any time, but pay a significant cost

Want to minimize # of queries

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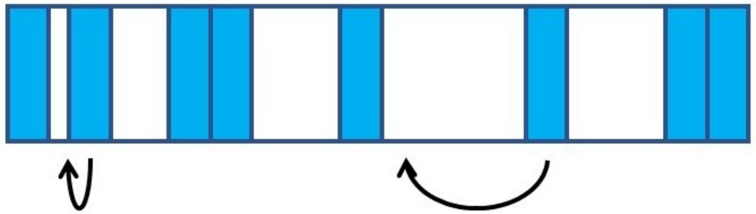
Mesh memory manager (PLDI 2019)



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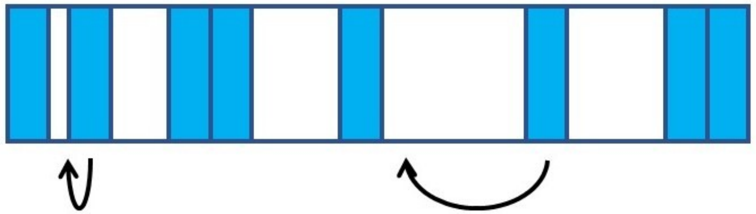
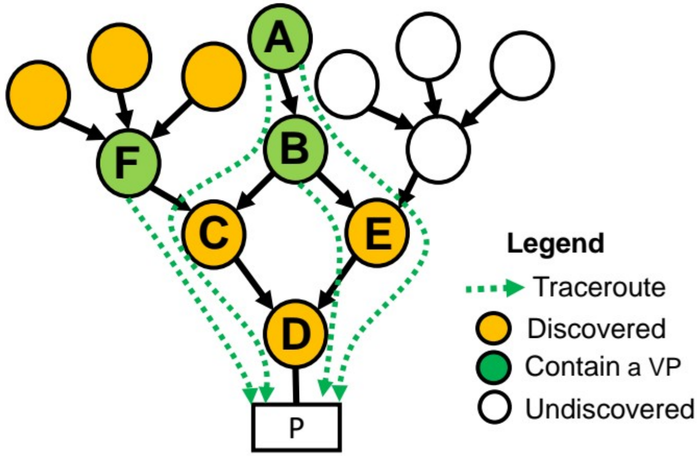


PathCache network path predictor
(to be submitted SIGCOMM 2020)



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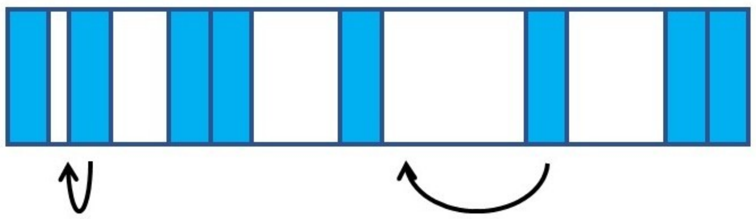

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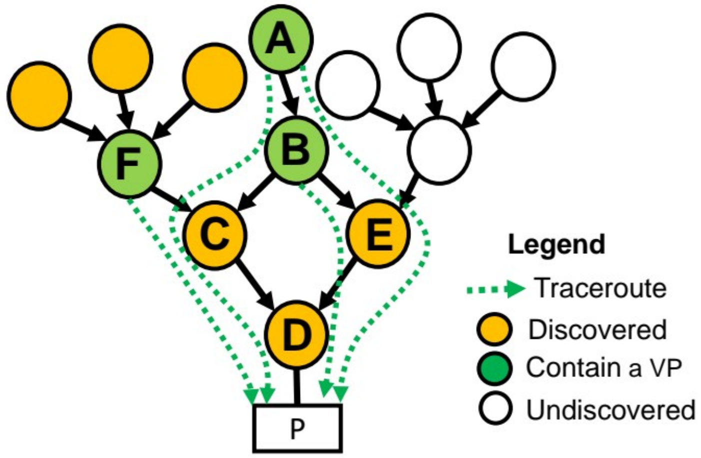

Memory Fragmentation

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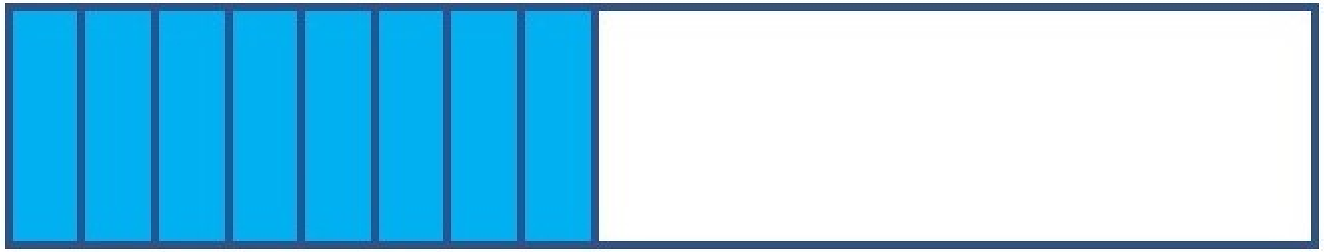
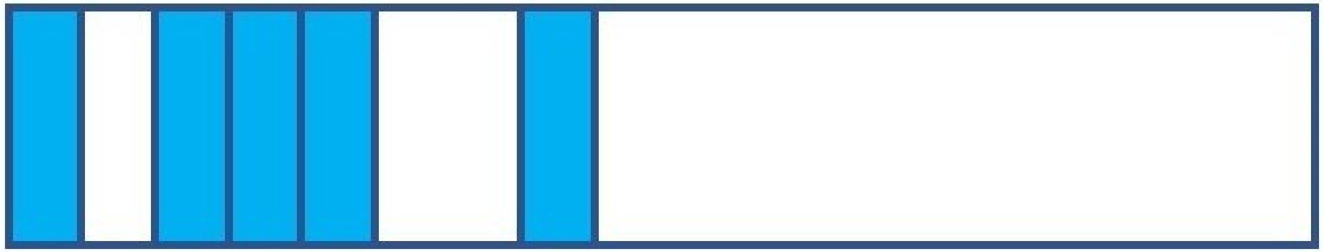


Memory Fragmentation



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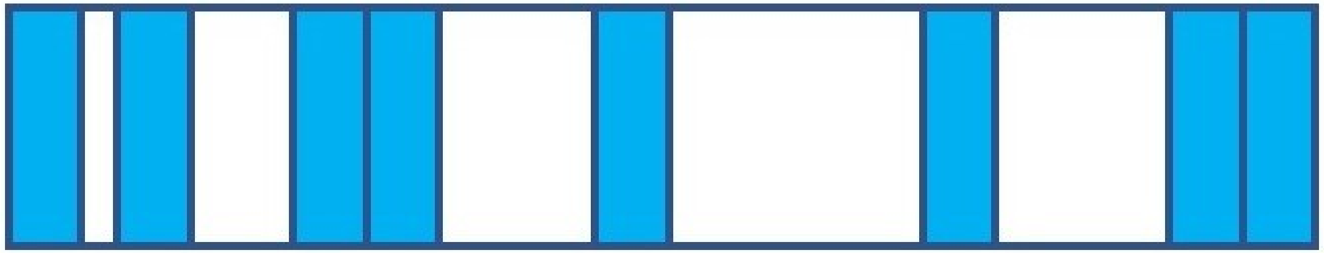

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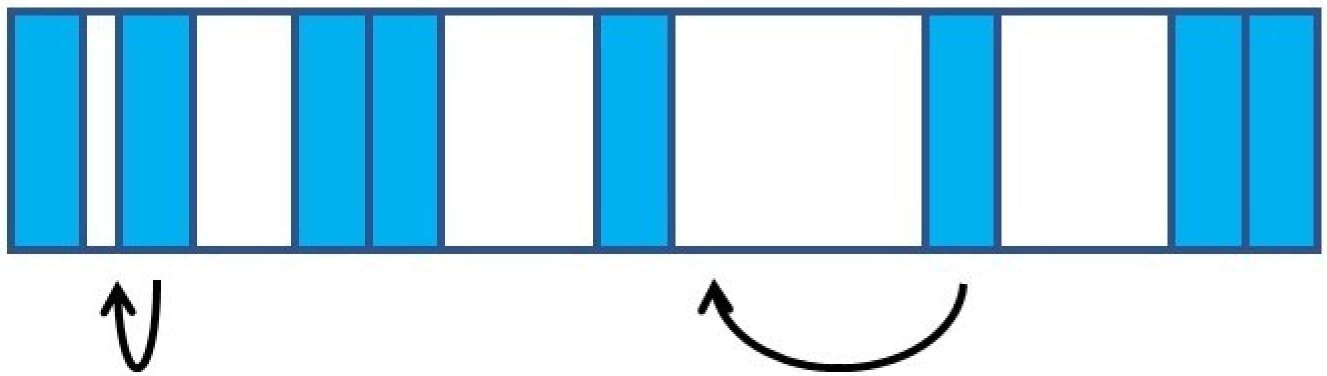
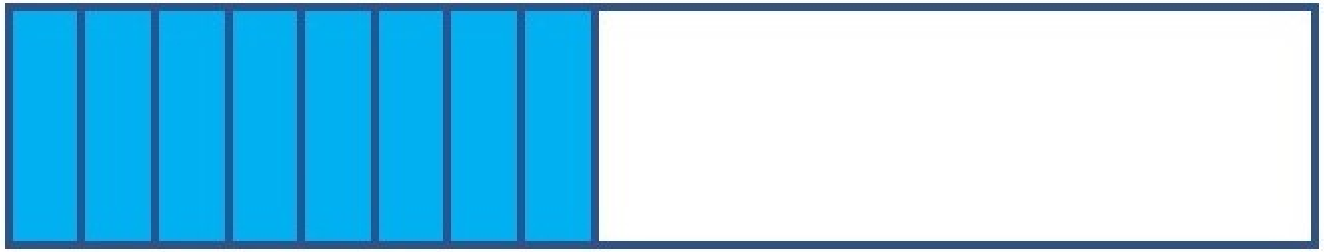


We want to reorganize or “compact” the allocated regions of memory to be contiguous.

Memory Fragmentation



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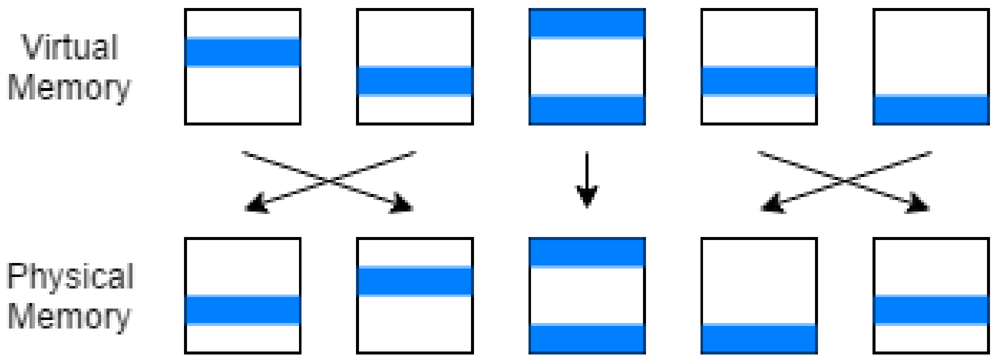
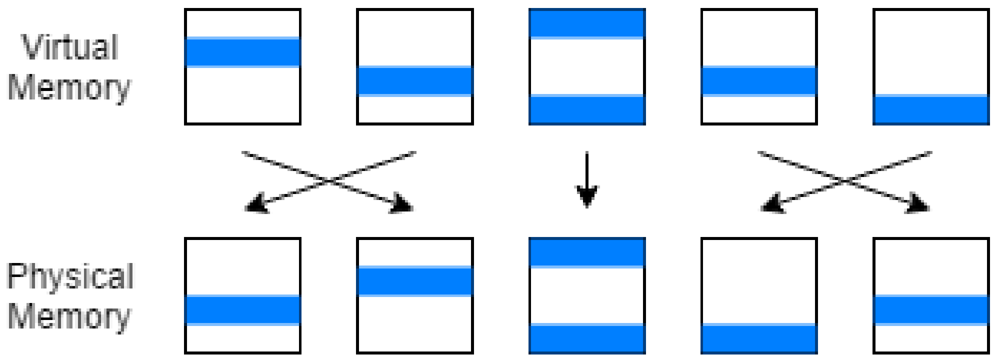
Virtual Memory – A Quick Primer

Modern operating systems maintain a mapping between virtual and physical memory.



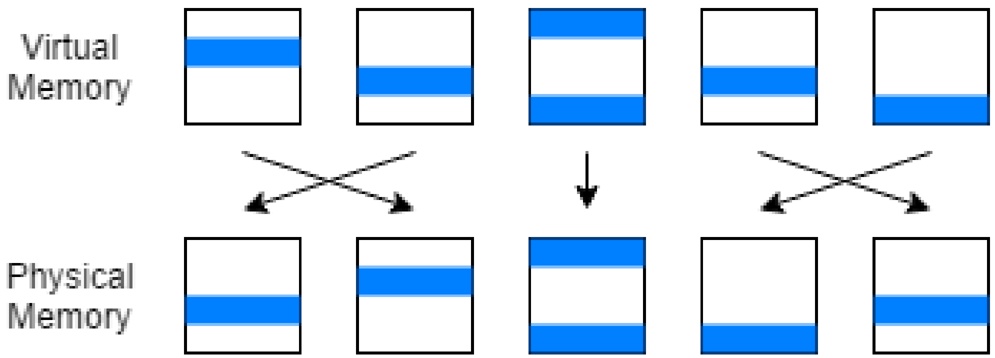
Virtual Memory – A Quick Primer

If we relocate objects in physical memory, we have to update their virtual addresses as well.



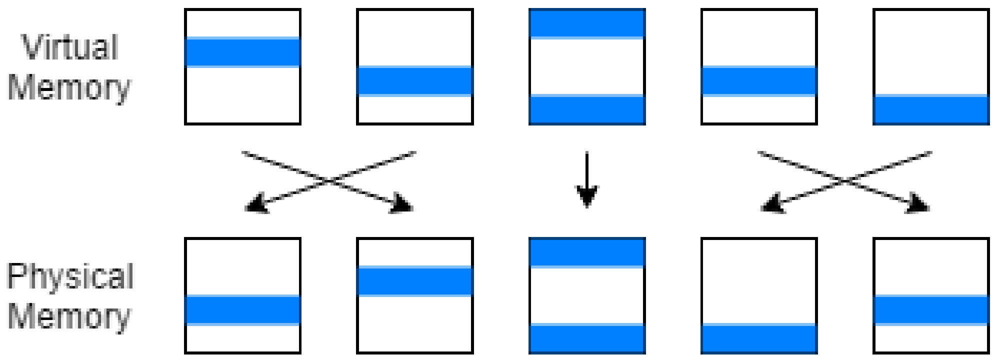
Virtual Memory – A Quick Primer

But in C and C++, we can’t alter virtual addresses safely.



Virtual Memory – A Quick Primer

How can we relocate objects without changing their virtual addresses?



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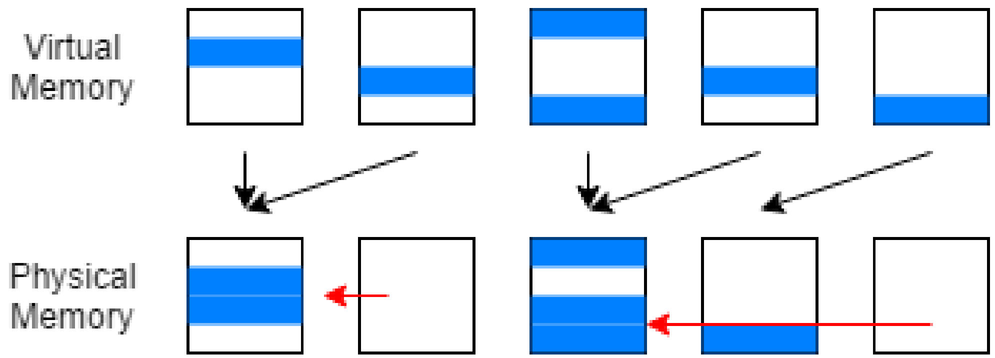
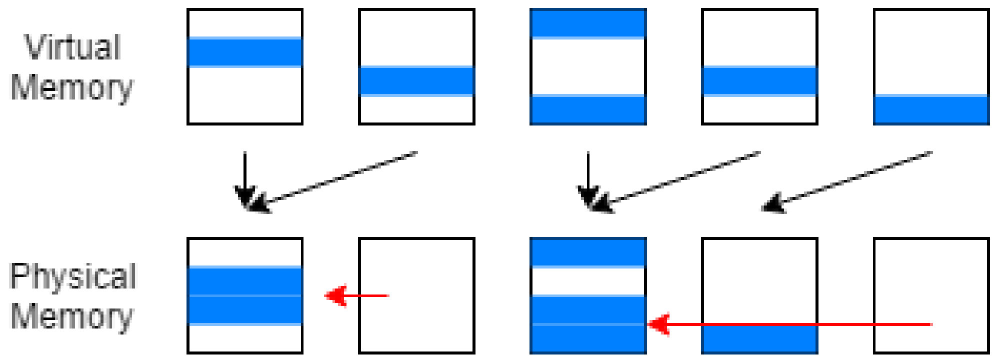
We can remap two virtual pages onto the same physical page in memory*, and discard one of the physical pages.

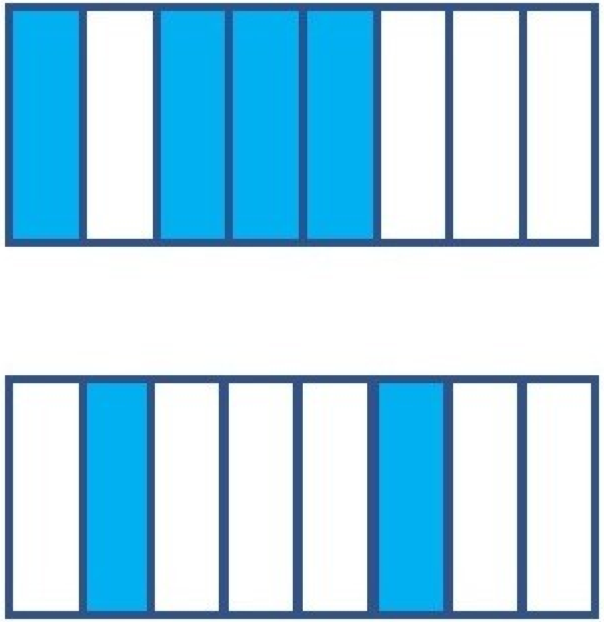


Virtual Memory – A Quick Primer

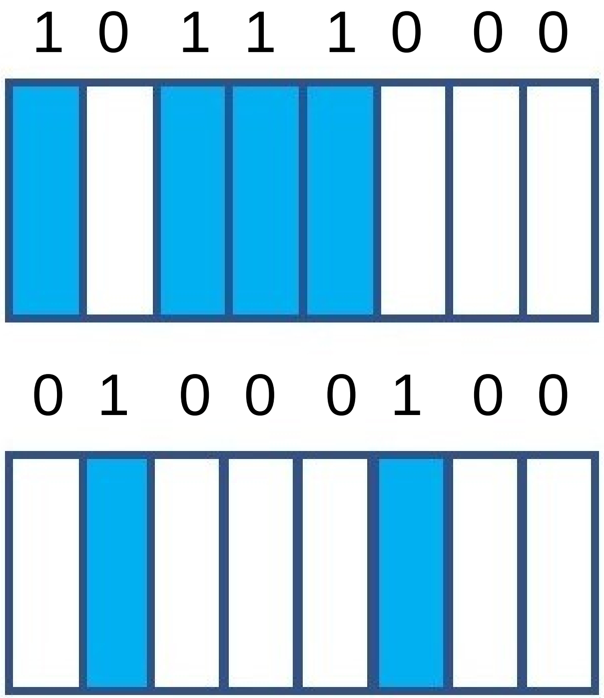
We can remap two virtual pages onto the same physical page in memory*, and discard one of the physical pages.

* provided there are no collisions between objects on the two pages.



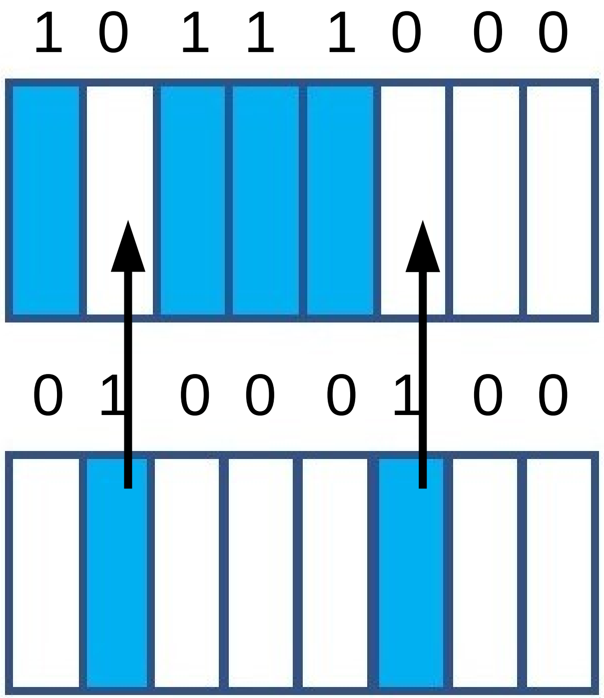


- Memory organized into pages
- Each page holds same # of objects (8)
- Objects are placed on page uniformly at random



We can represent each page as a bitstring, where 0 indicates a free slot and 1 indicates an occupied slot.

We can *mesh* two pages together if they don't have 1s in the same position.



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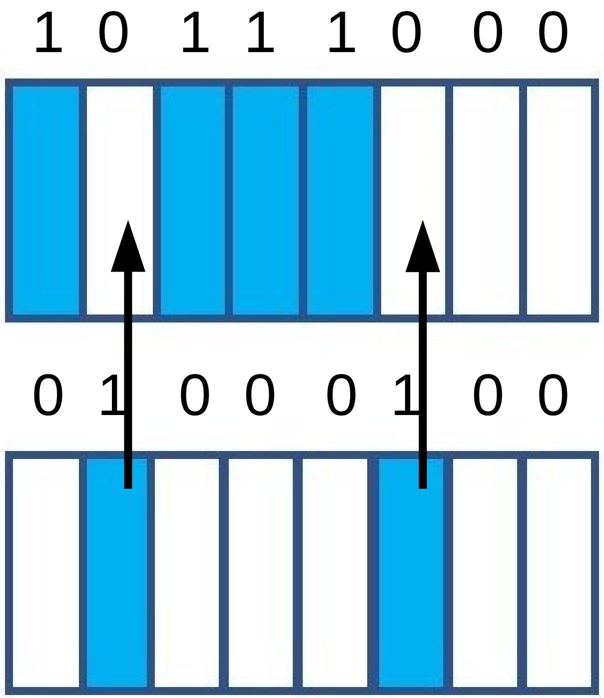
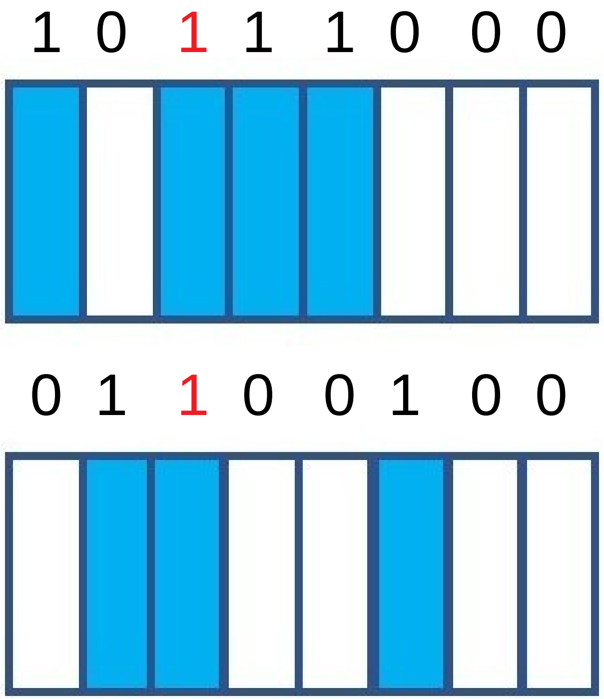
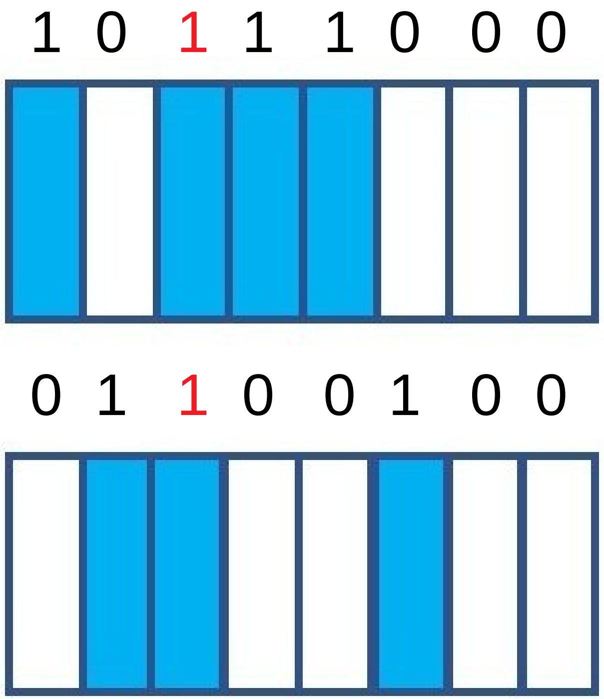


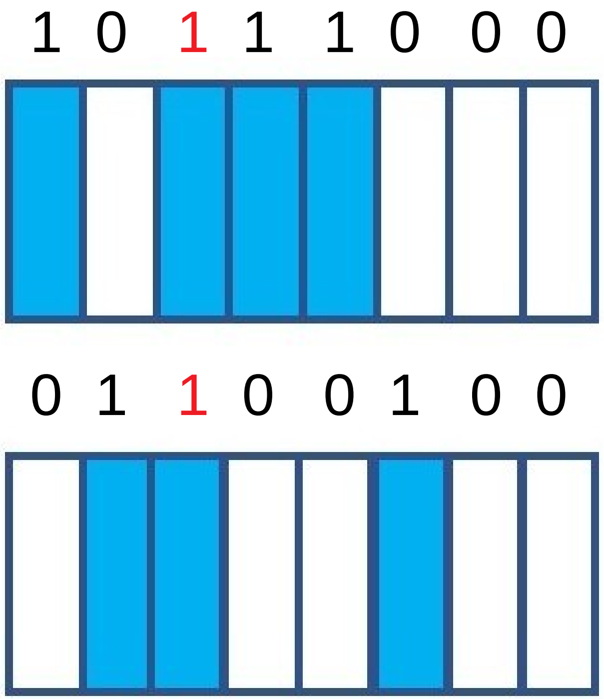
THEY MESH!



If both bitstrings have a 1 in some position, we can’t mesh the strings together.



NO MESH



Now we can forget about the details of memory, and think about our problem in terms of finding meshable pairs of bitstrings.

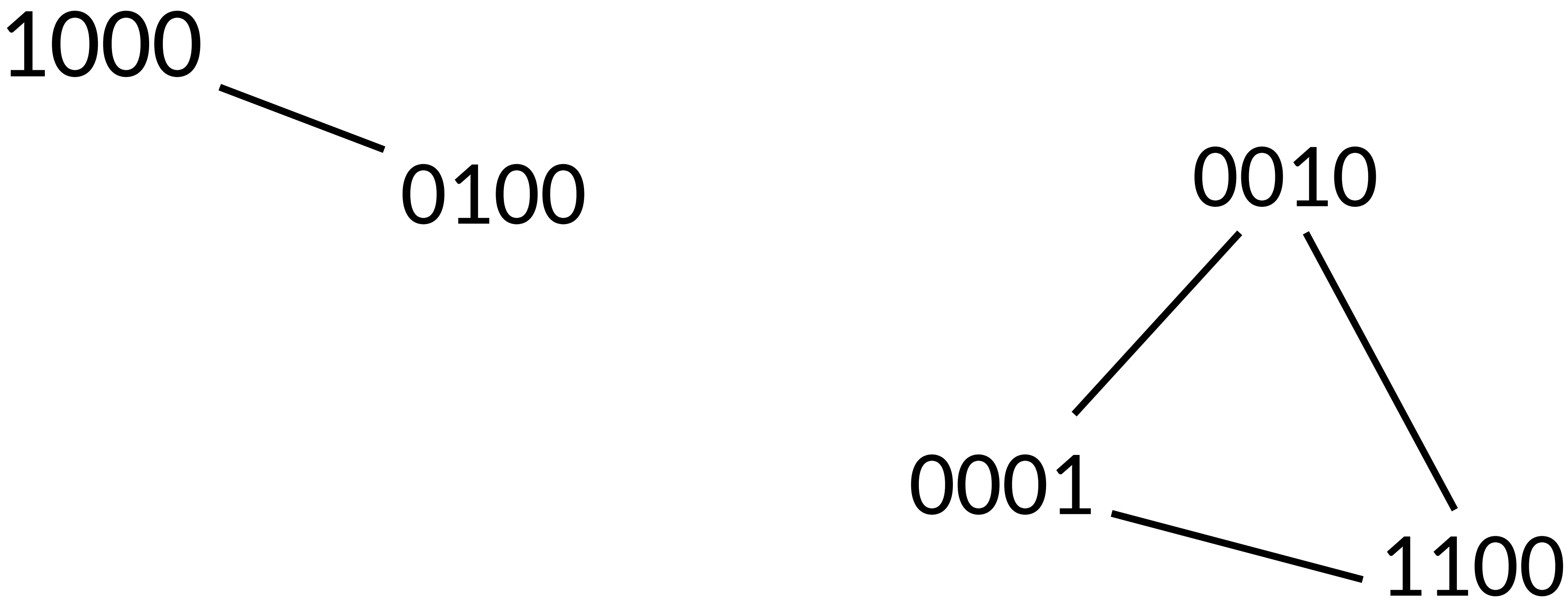
We want to mesh as many pairs of strings as possible.

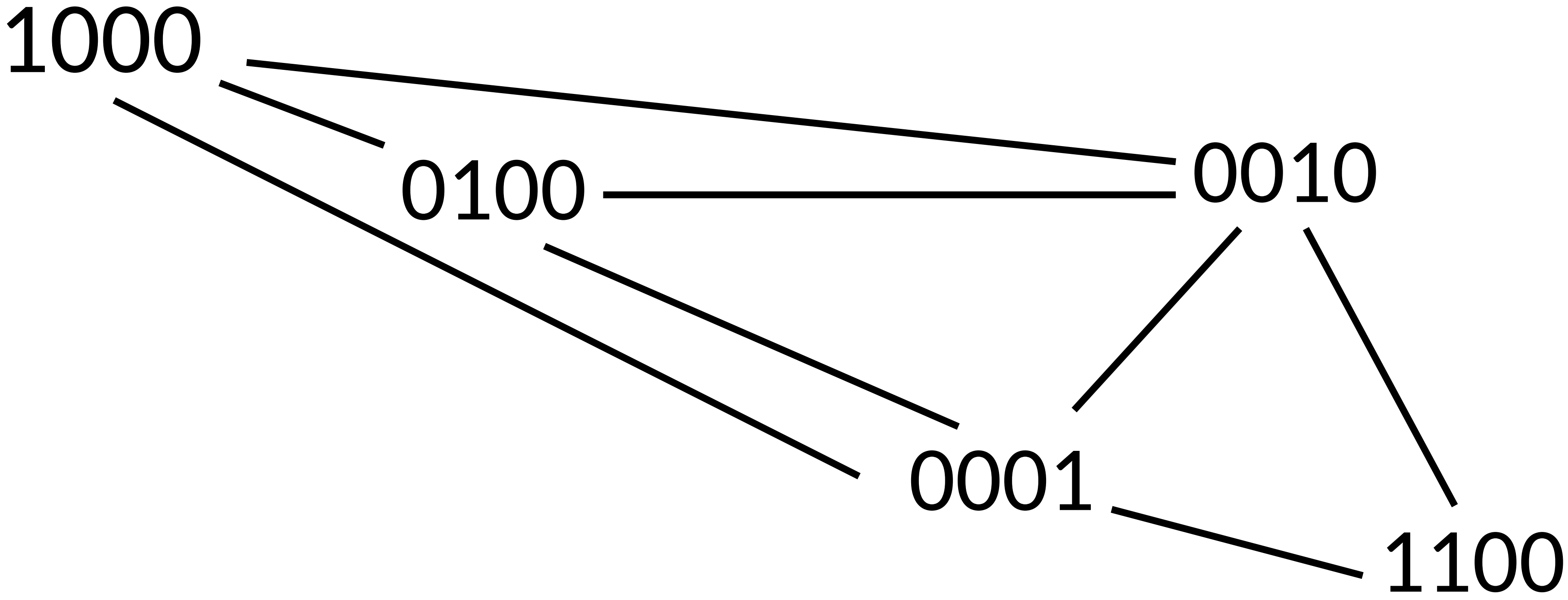
1000

0100

1000

0100


1000
0100
0010
0001
1100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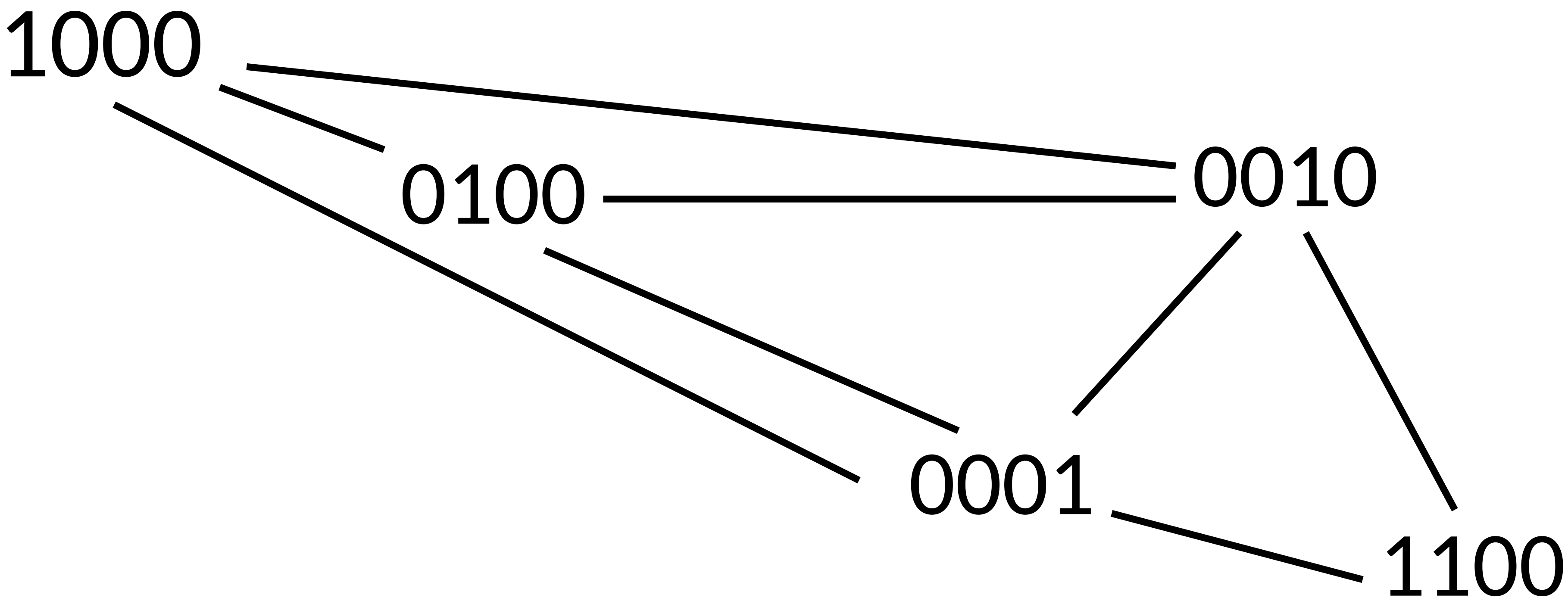
We can think of this as a graph problem!



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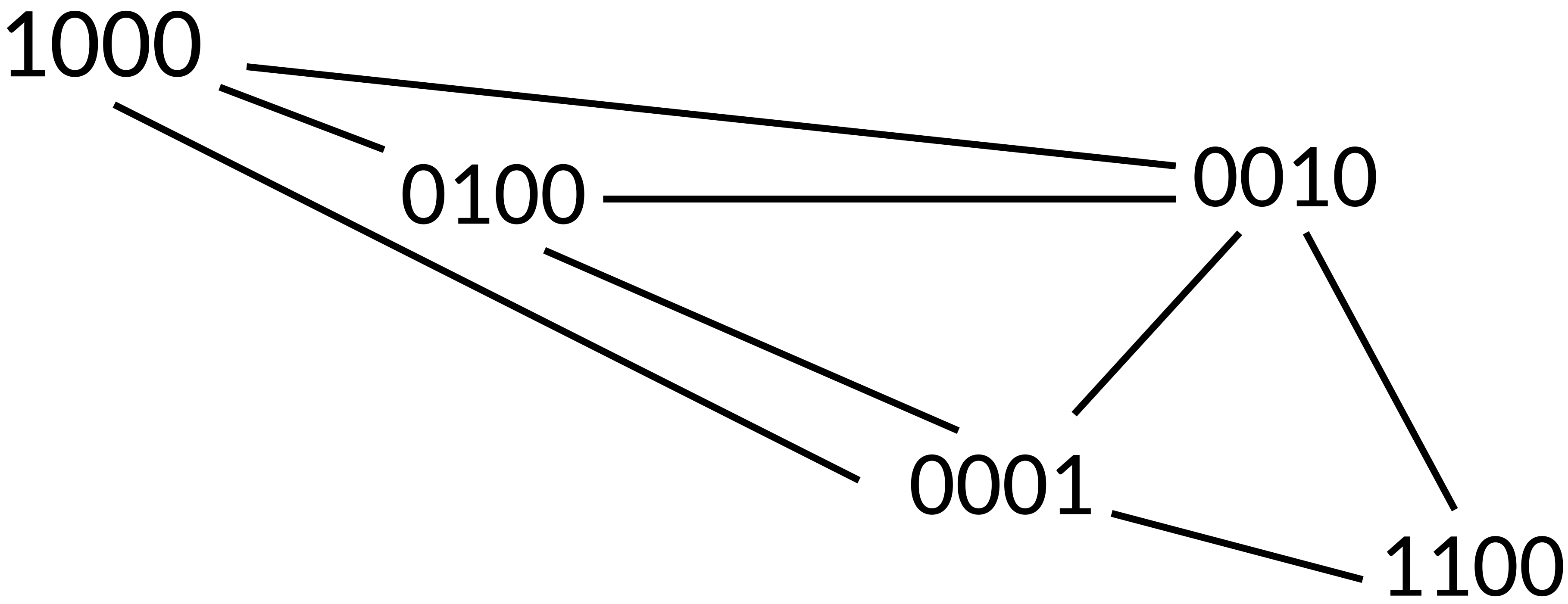
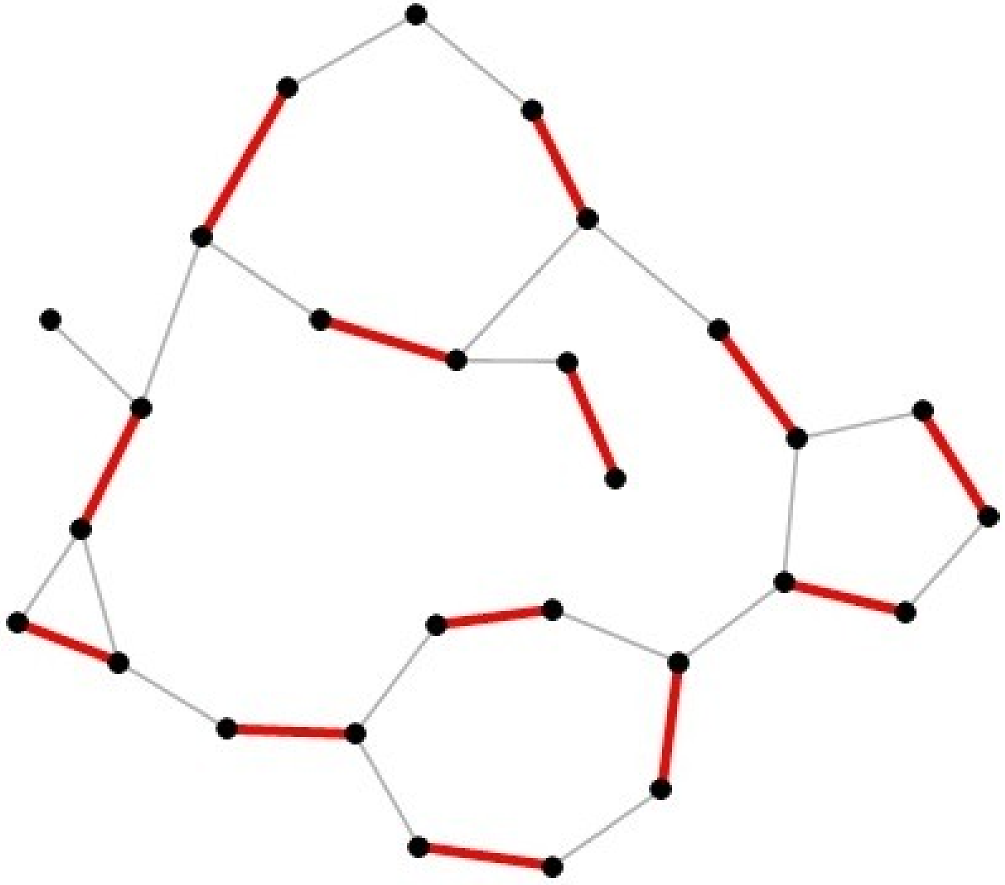
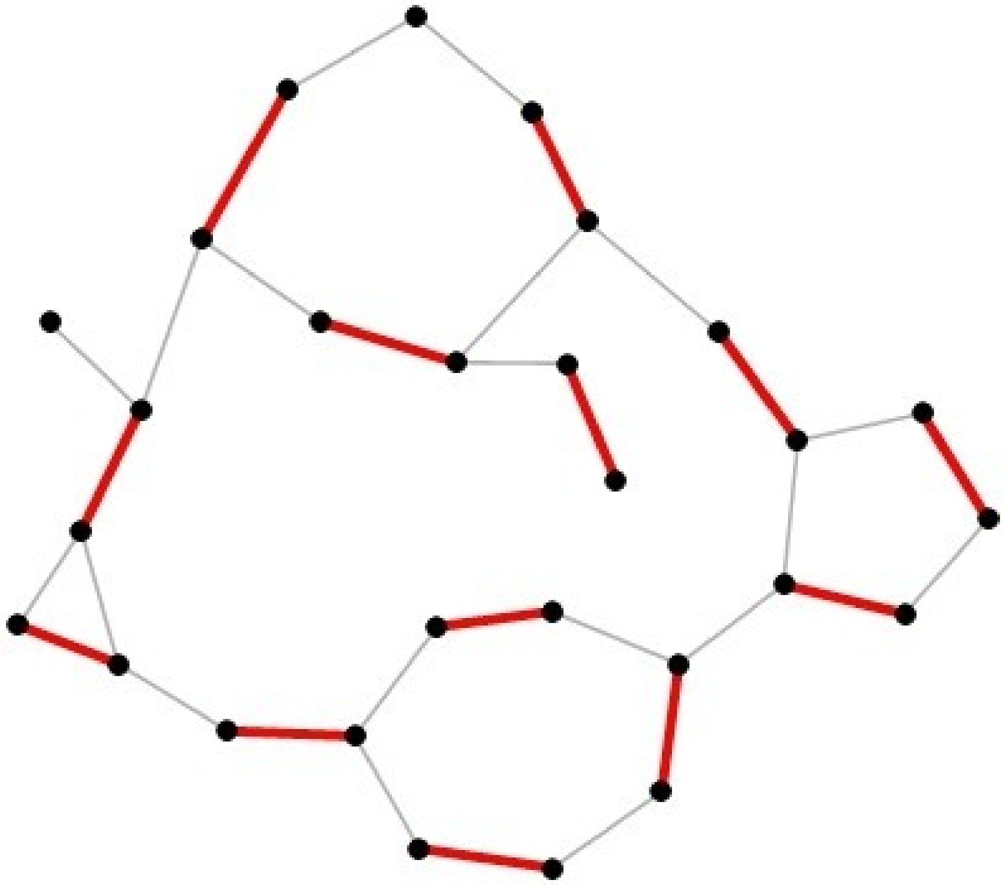


Maximum Matching



Maximum Matching

Well-known polynomial time algorithm



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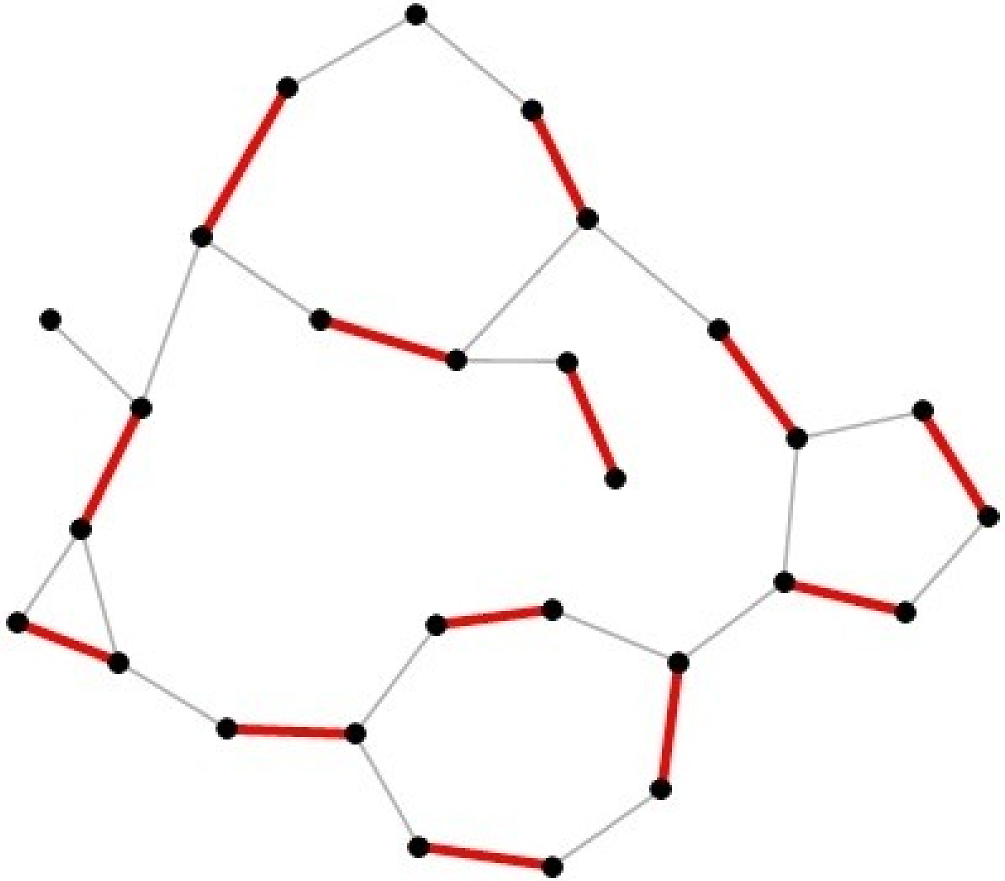
Requires random access to graph



Maximum Matching

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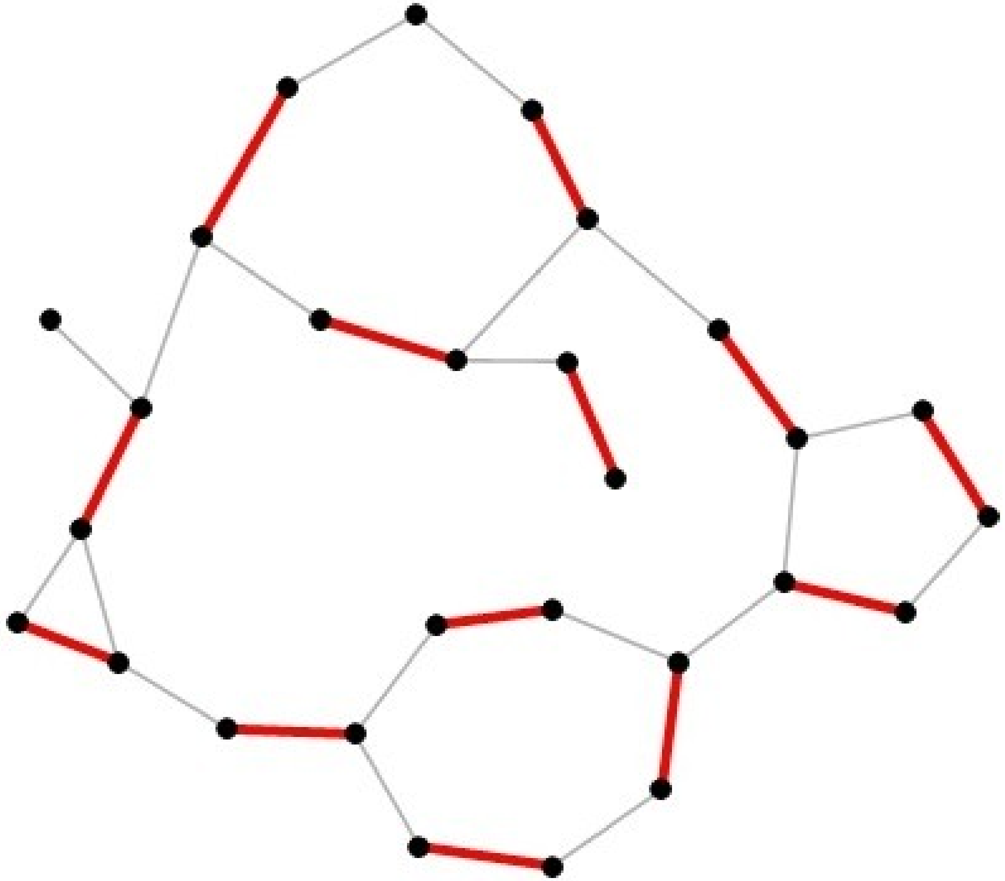
Do we have random access?
<u>No</u>.



Not Enough Time!

We mesh **during program execution.**

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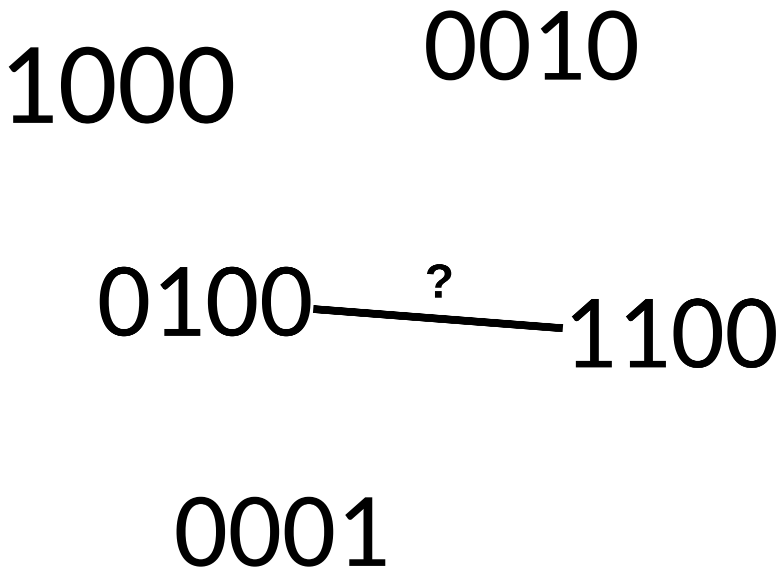

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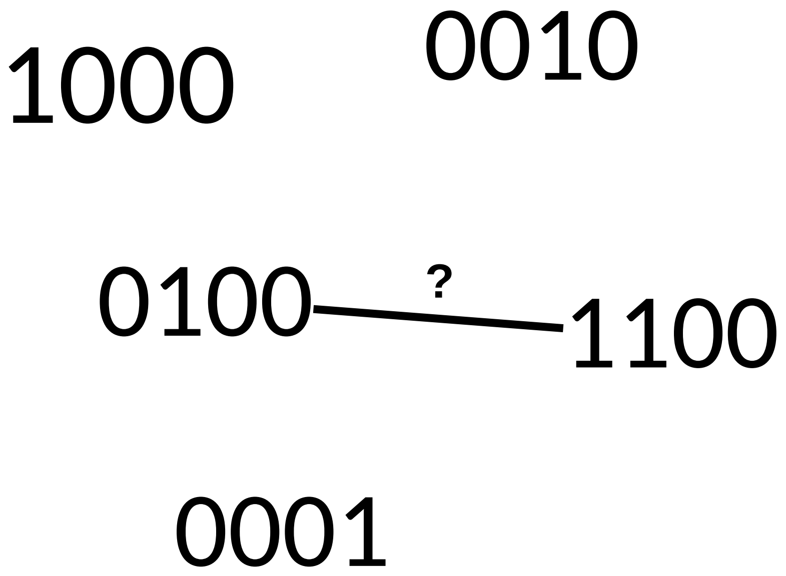
Checking an edge is a costly query.



Random Graphs

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That's not the case for meshing graphs!

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Edge Dependence Example

Say we know all of our bitstrings are half full.

???? (50% full)

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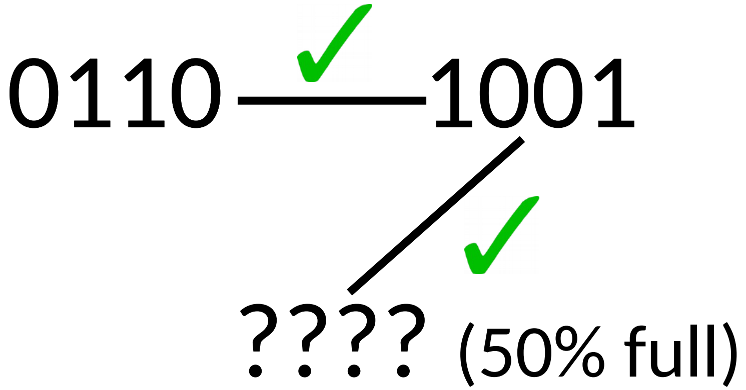

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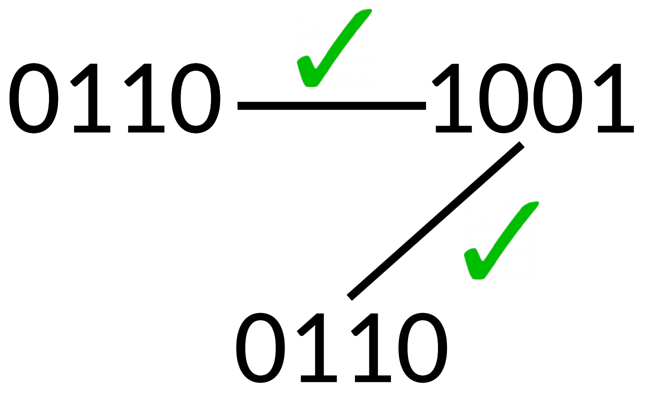

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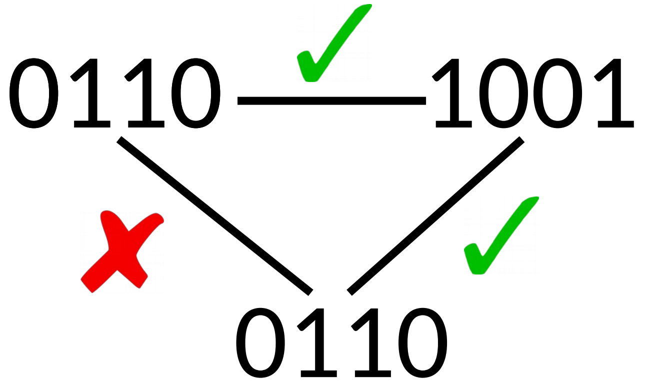

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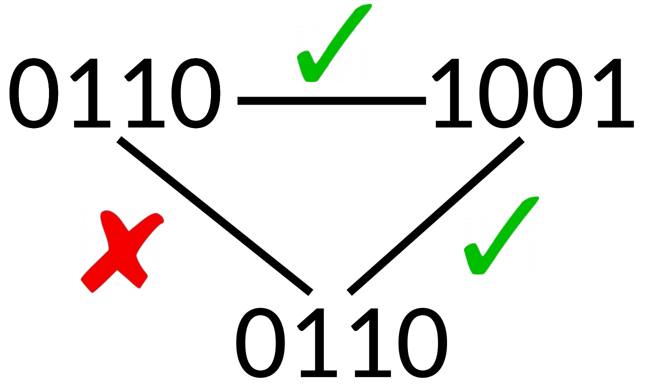
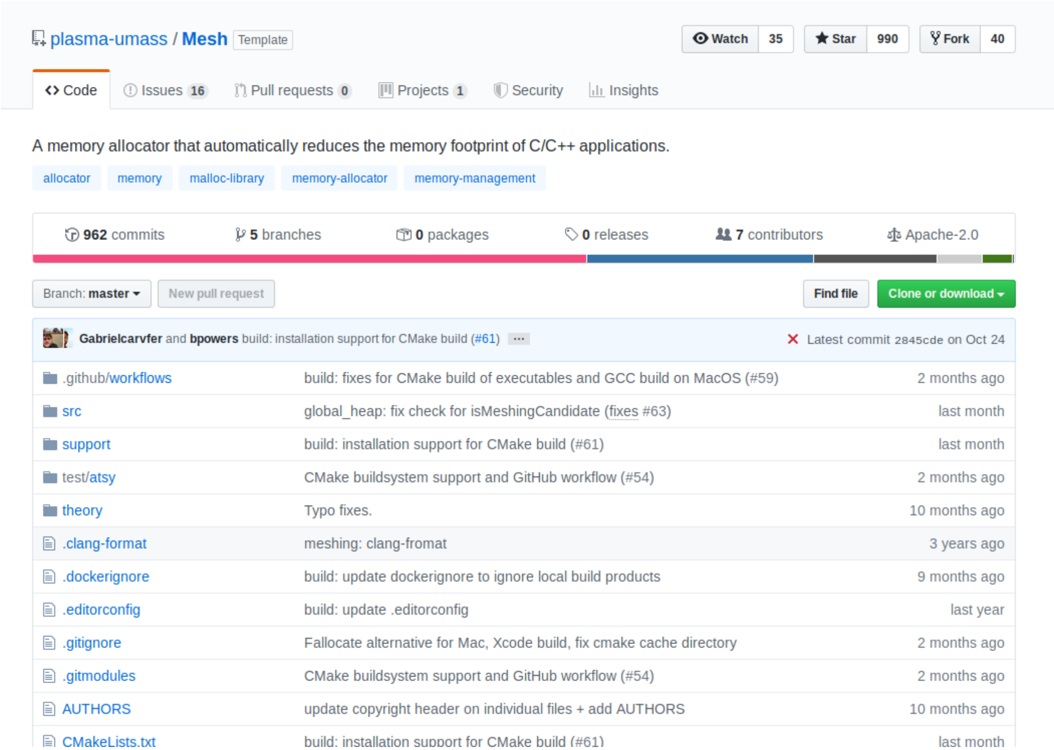


This is a novel and interesting mathematical structure!

Mesh

We built Mesh, a memory manager powered by a query-limited matching algorithm that can perform memory compaction in C and C++.



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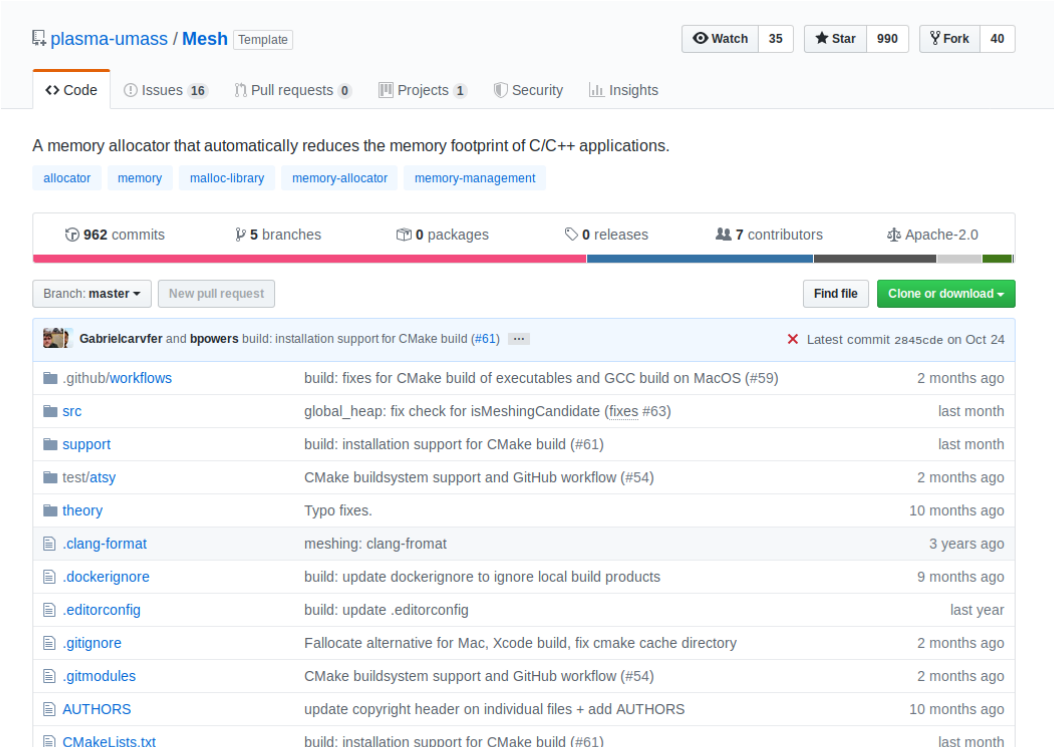

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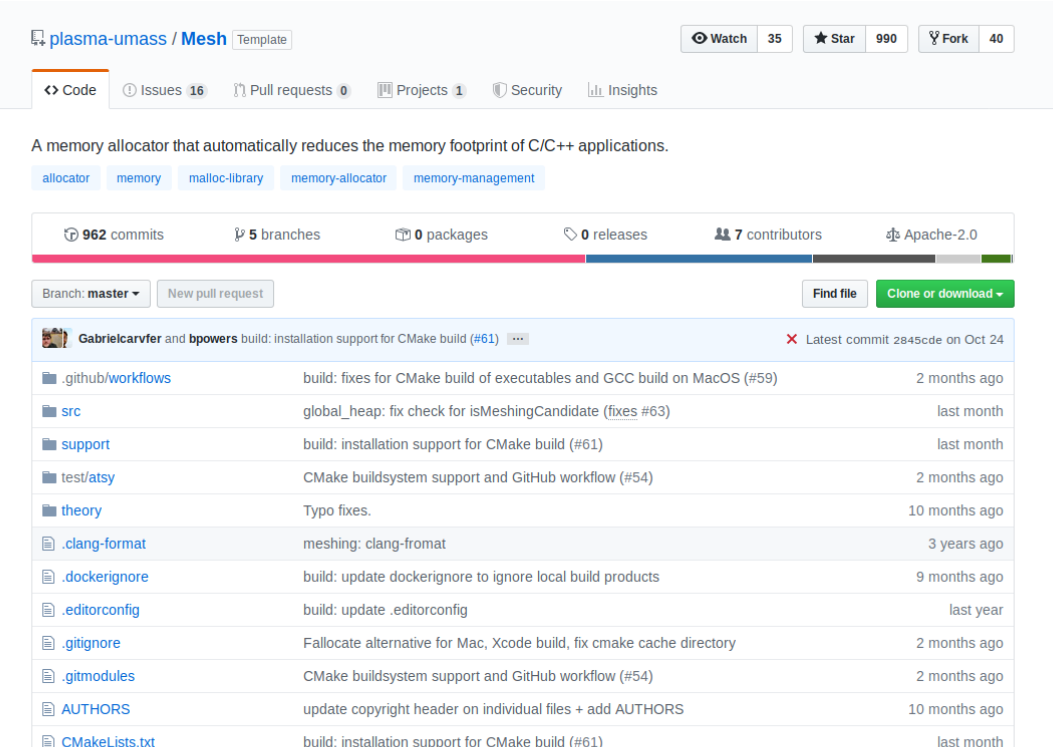
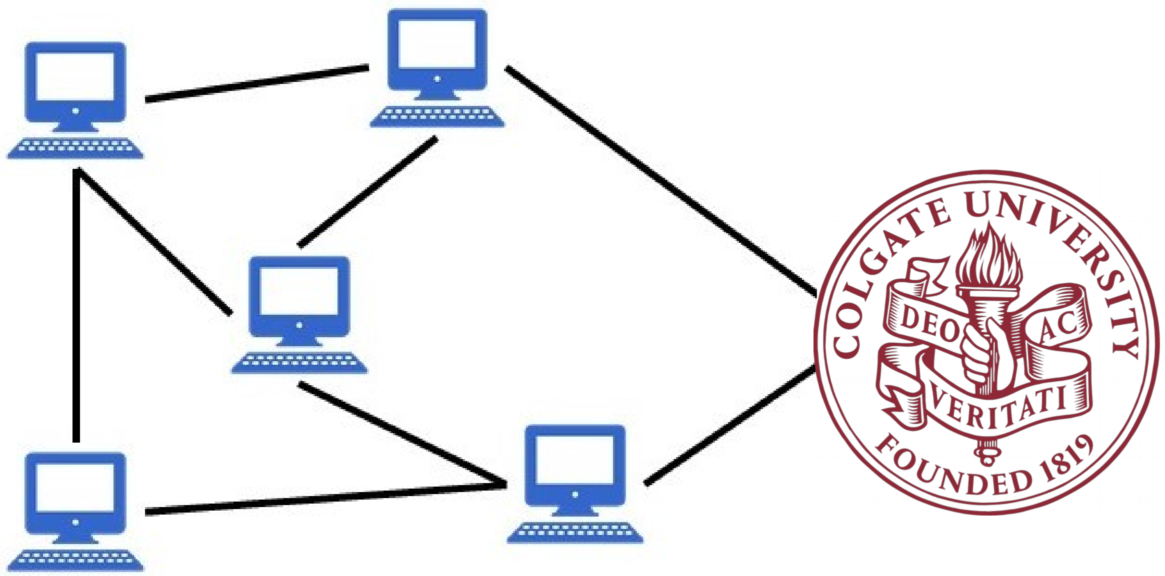


Finding the Shape of the Internet

I want to visit www.colgate.edu. What path through the internet will my HTTP request take?



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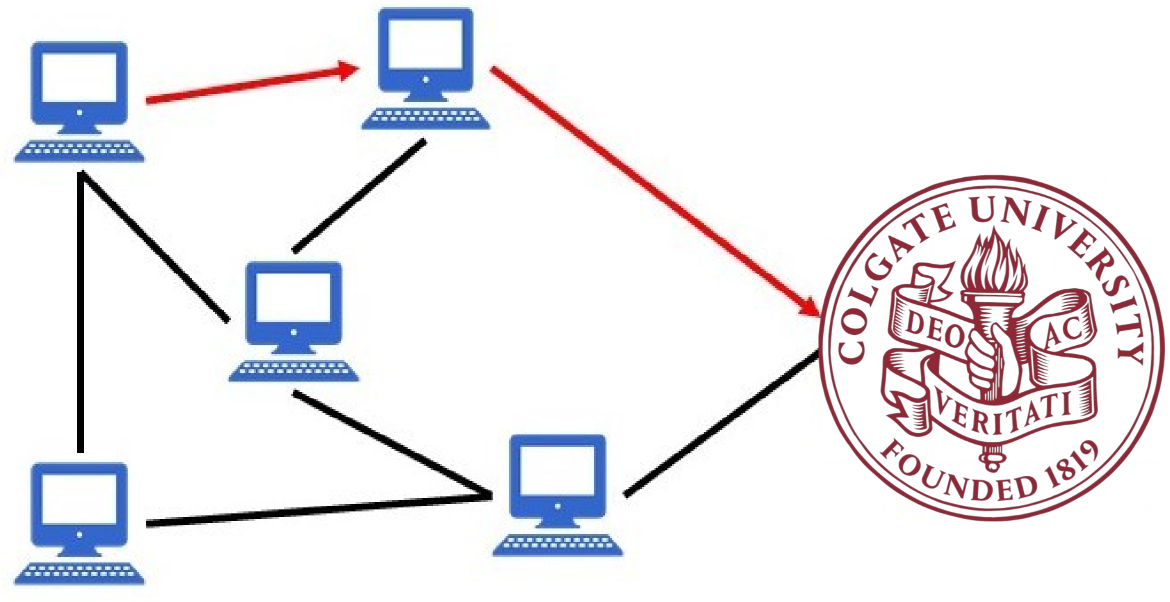

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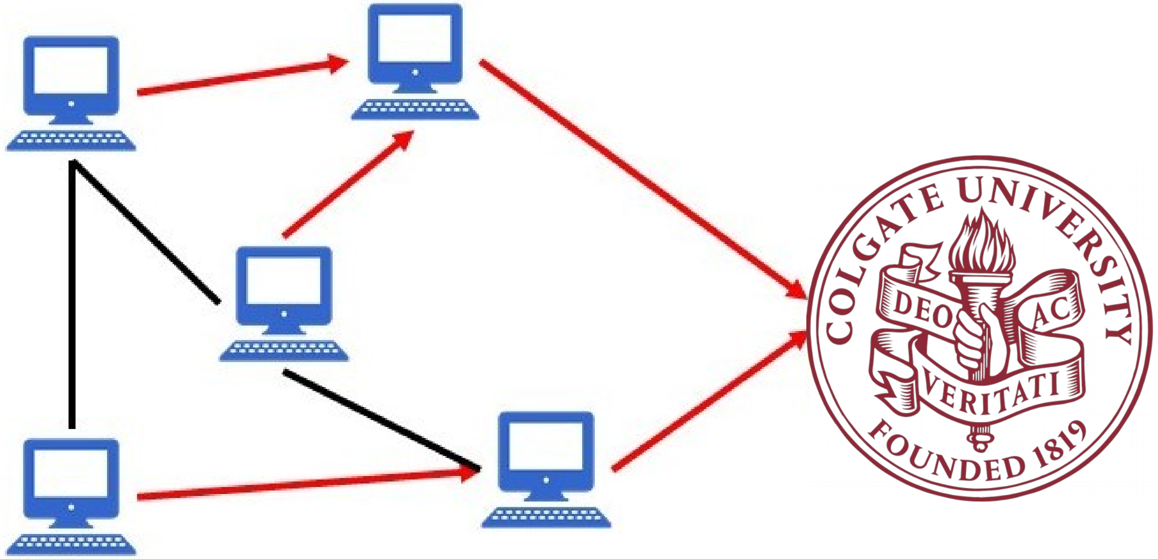

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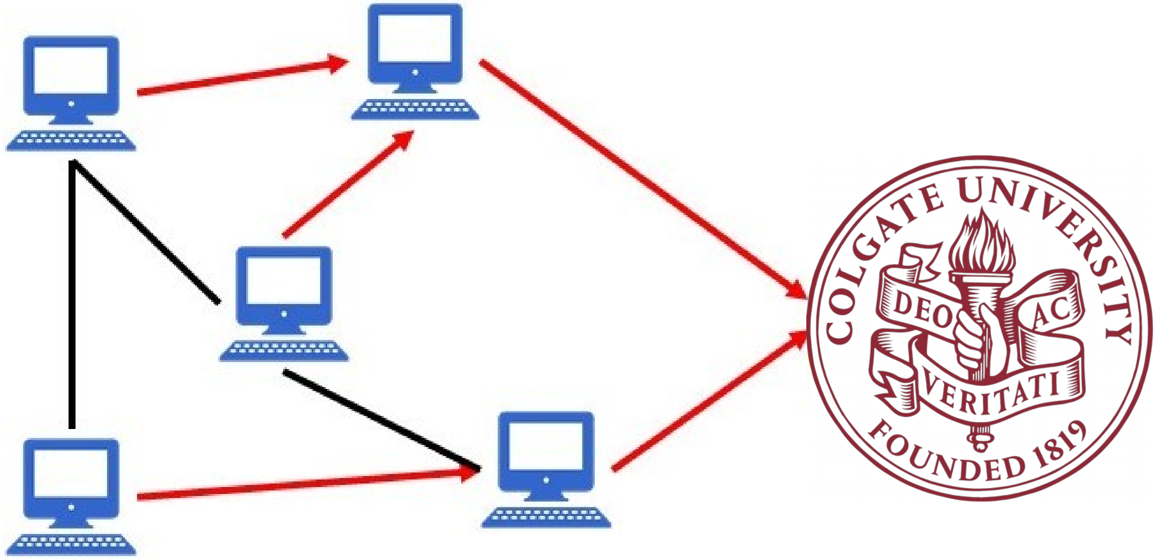

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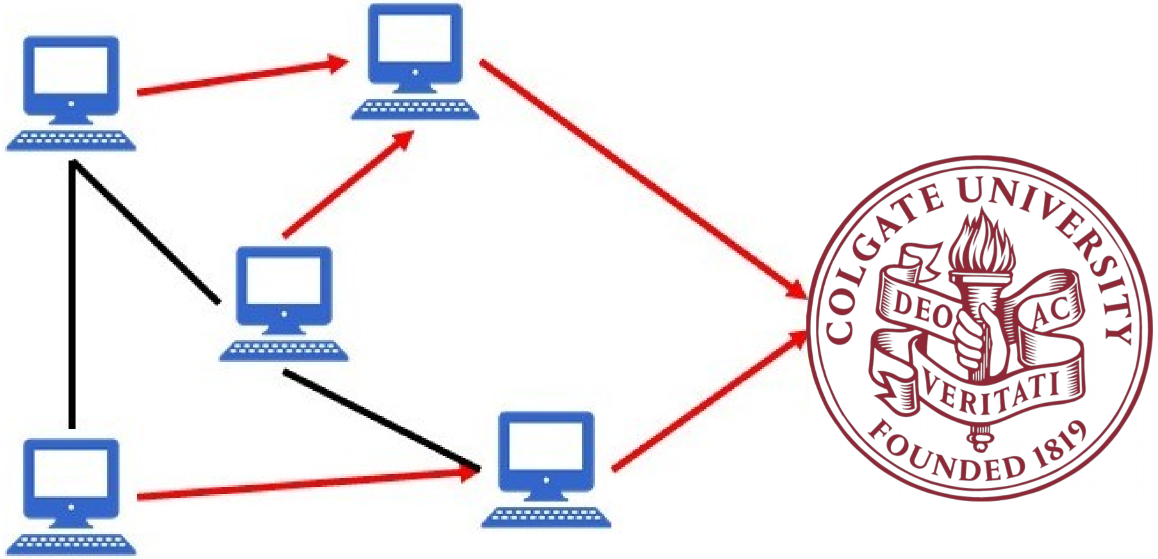

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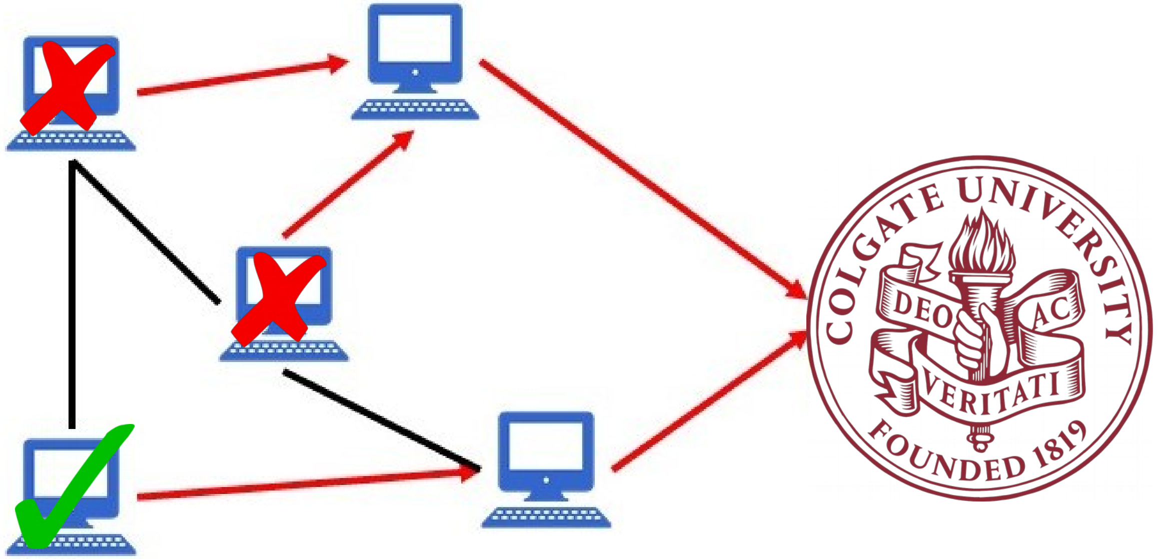

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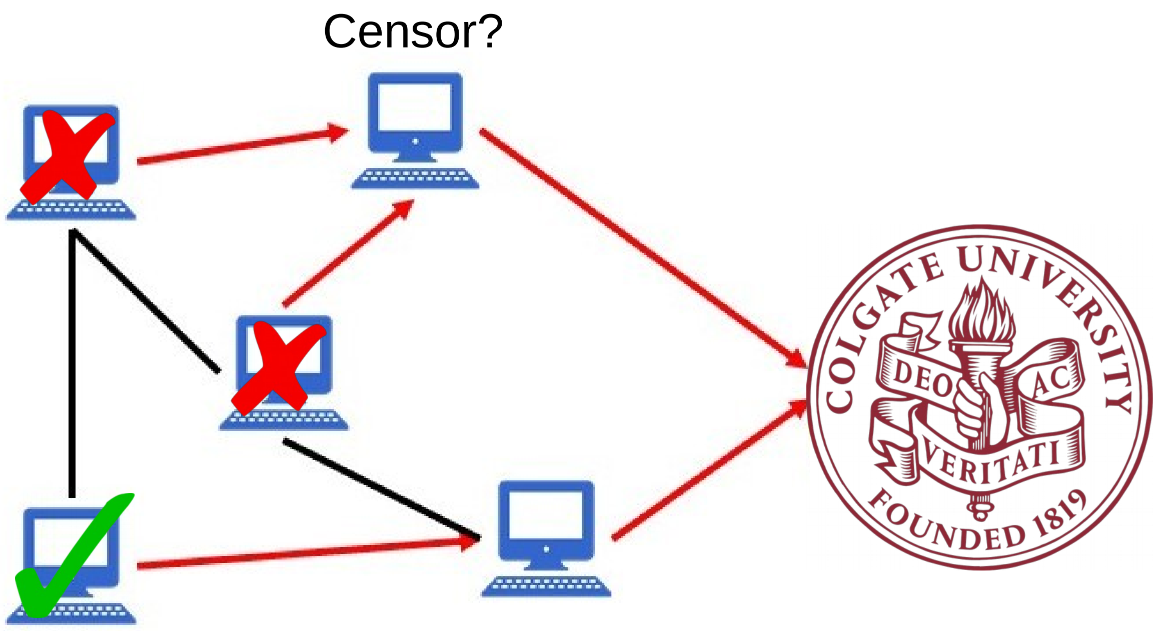

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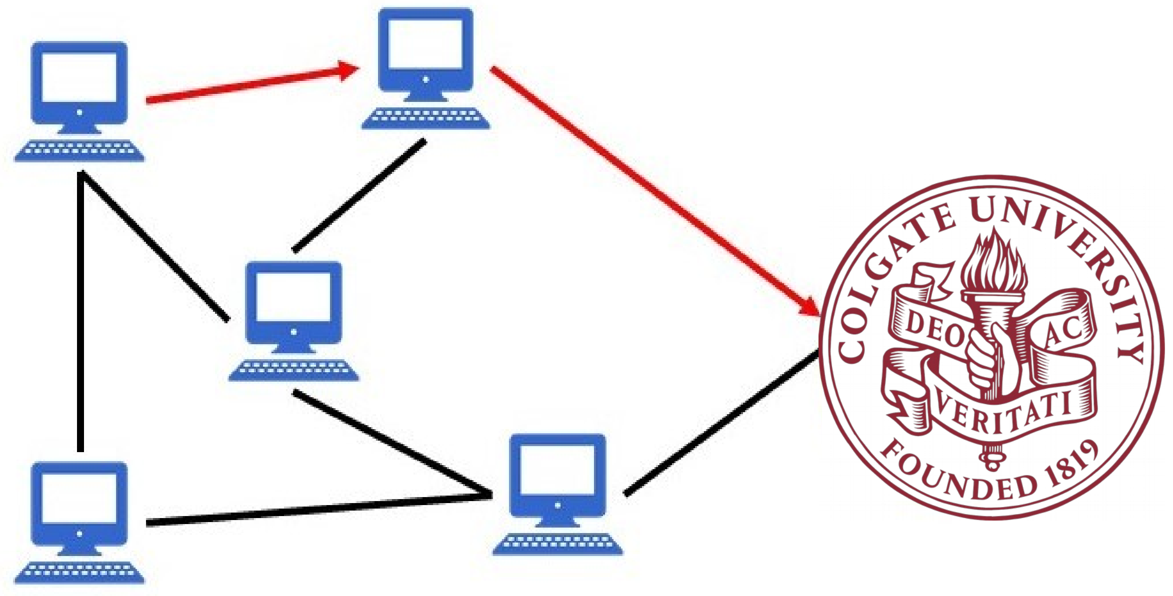
Measuring a path is a costly query called a *traceroute*. We want to discover as much of the Internet as possible using minimal traceroutes.

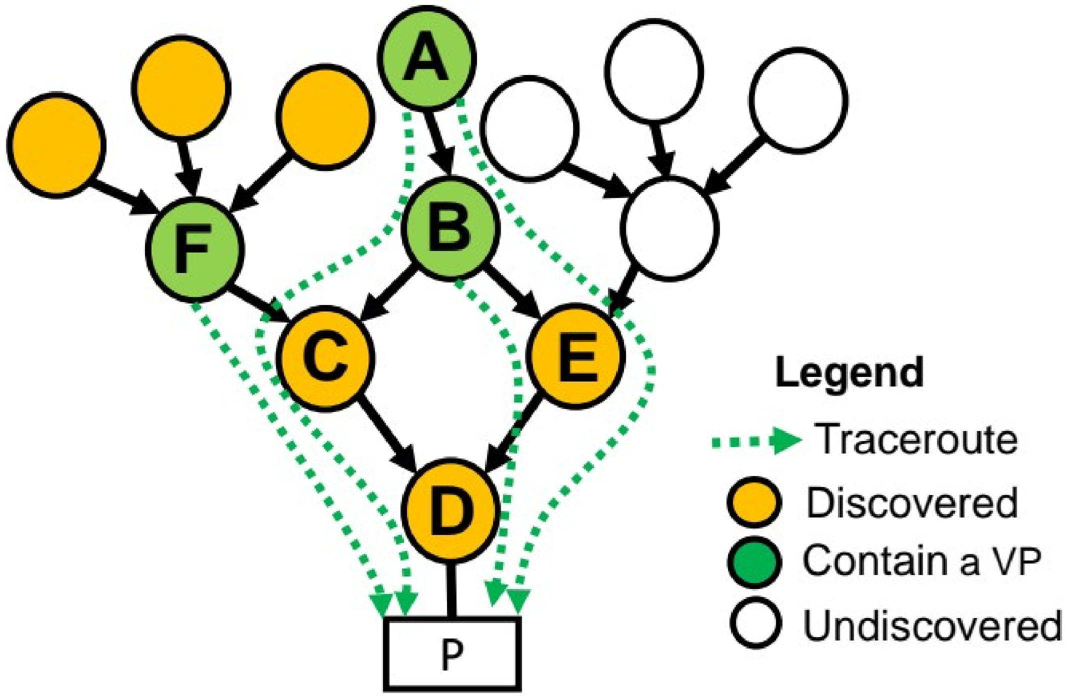


PathCache

A system for efficiently using limited VP measurements to predict paths towards Internet destinations.

Current version discovers 4 times more connections than pre-existing measurement strategies with comparable traceroute budget.

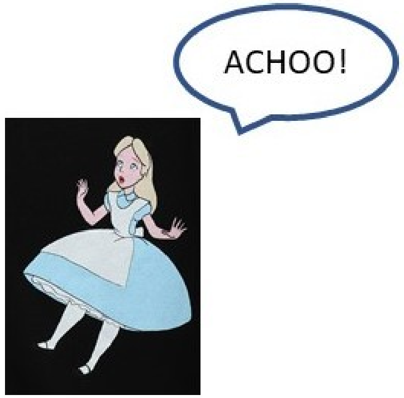
Predicts correct or nearly-correct paths 75% of the time.



Future Work

GRAPHS THAT ARE EVEN MORE INCONVENIENT

Time-Dependent Graph Streams

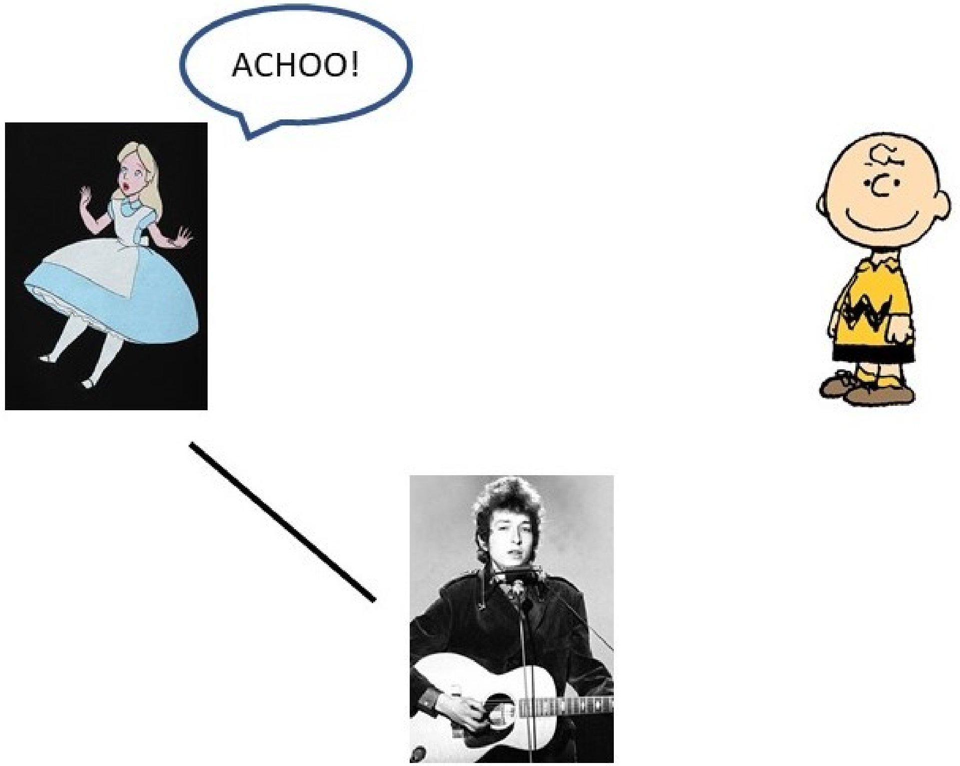


In the typical streaming model, graph is the same **regardless of the order edges appear in the stream.**

What if the order mattered?

Ex: disease spreading

Time-Dependent Graph Streams



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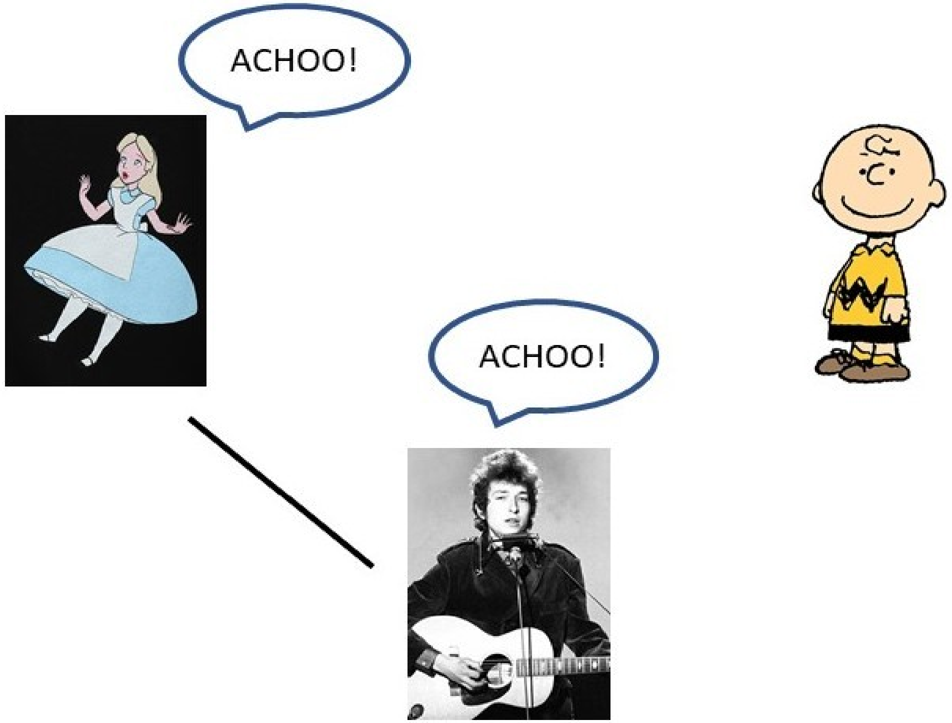

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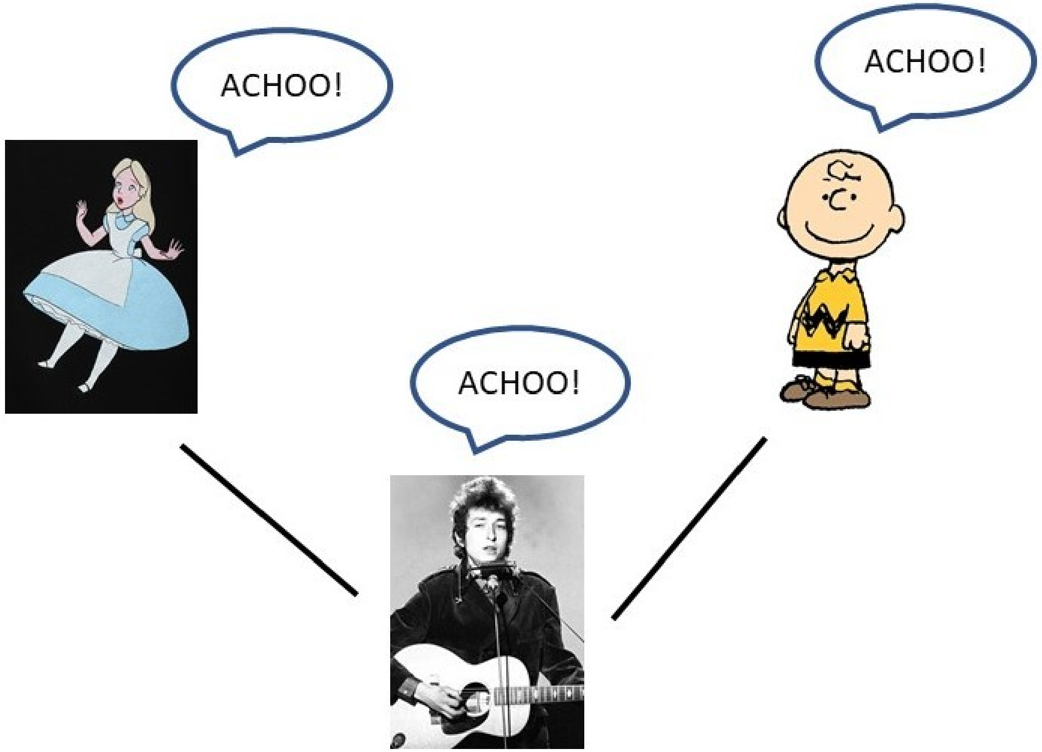

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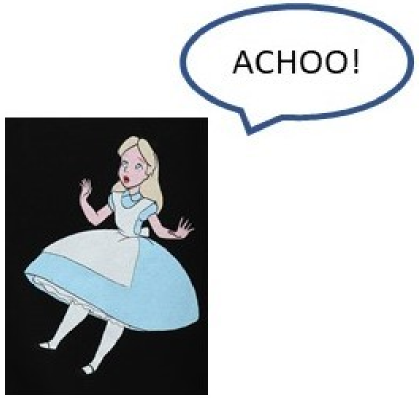

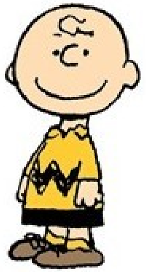

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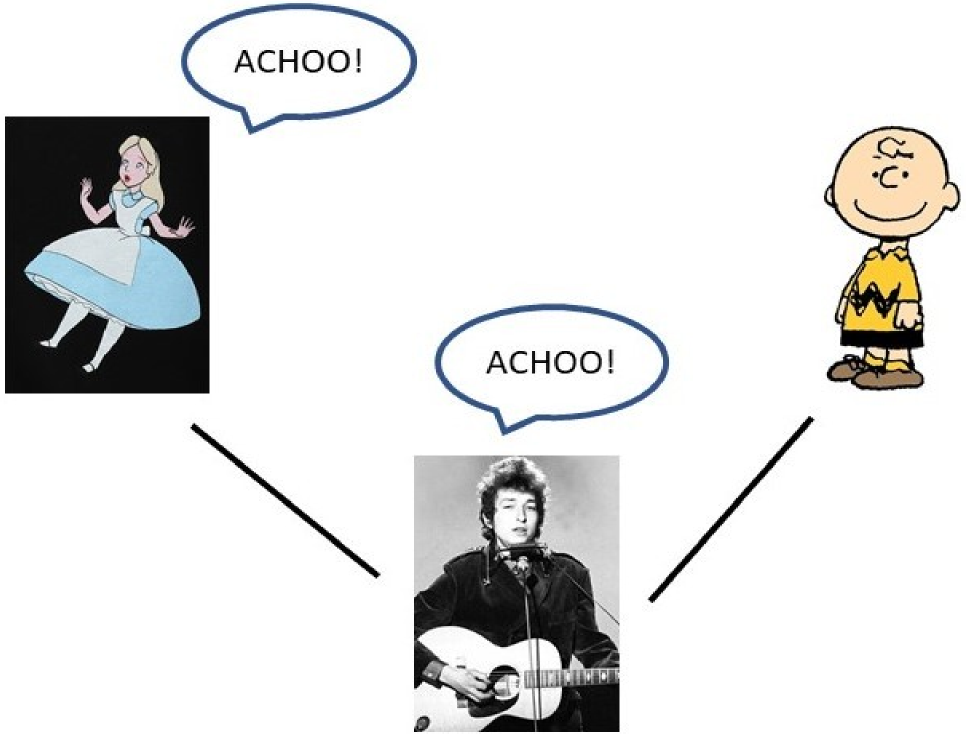

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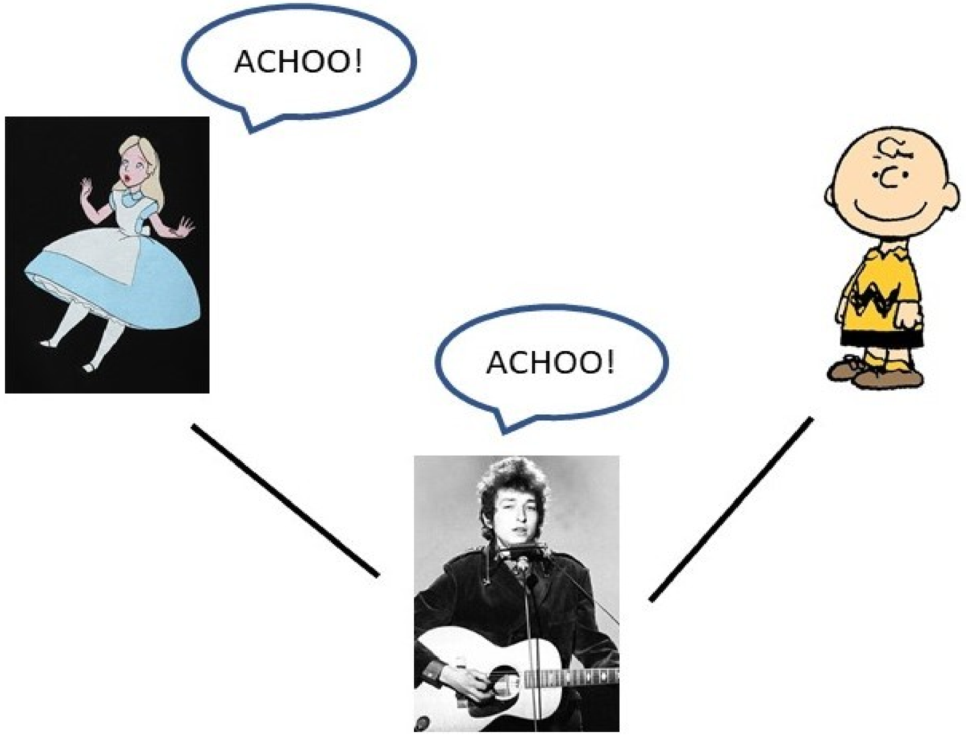
Temporal graphs are just beginning to be investigated. No streaming work.

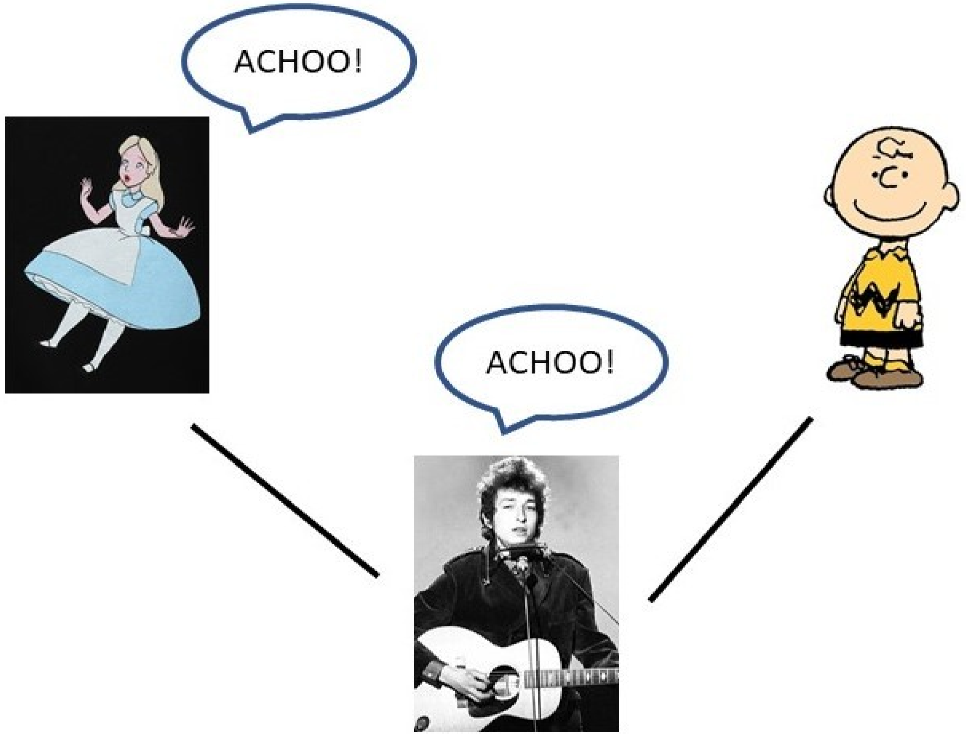
Time-Dependent Graph Streams



Imagine we receive a massive stream of handshakes between many people. Later, we learn one of those people was sick.

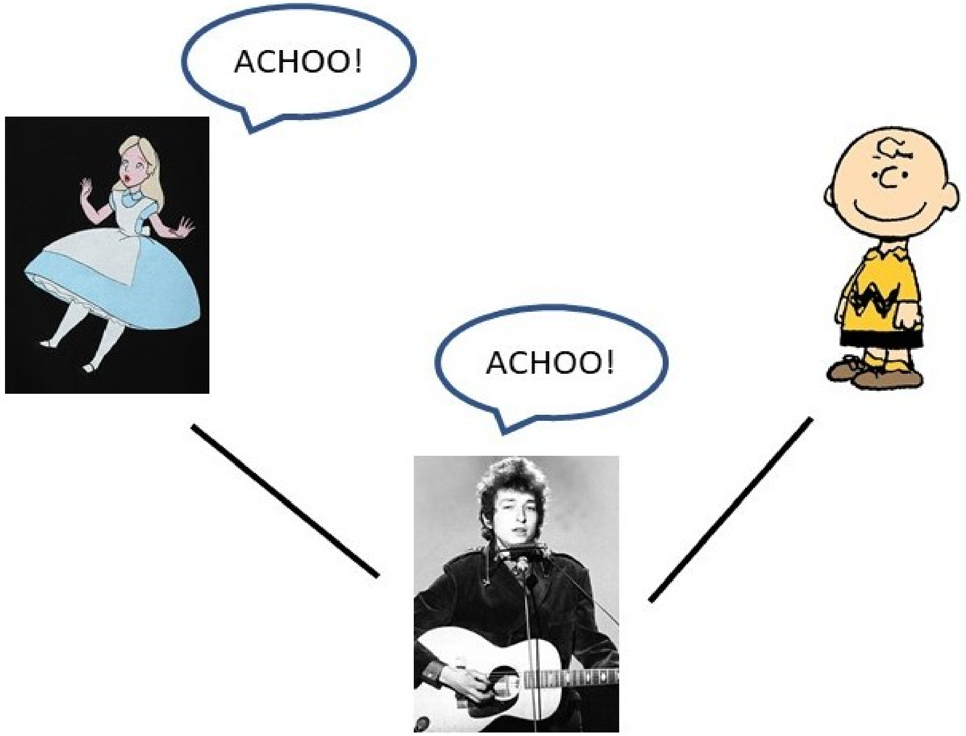
Can we determine who is infected without storing the entire stream?

Time-Dependent Graph Streams



This problem is about connectivity or reachability on temporal graphs.

Time-Dependent Graph Streams



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Other potential problems:

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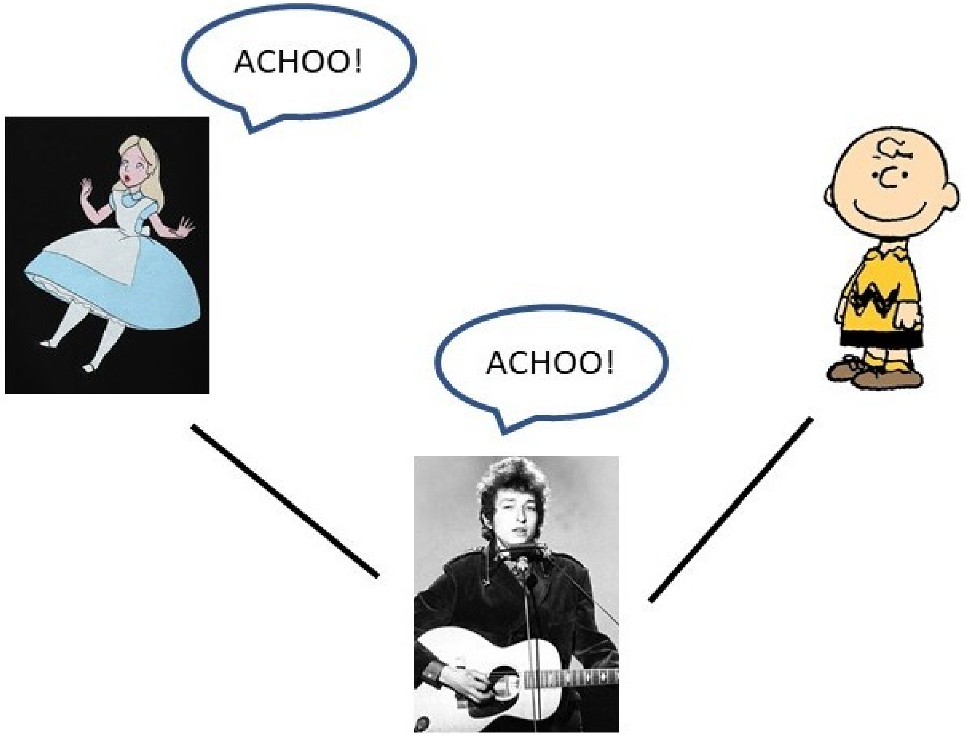

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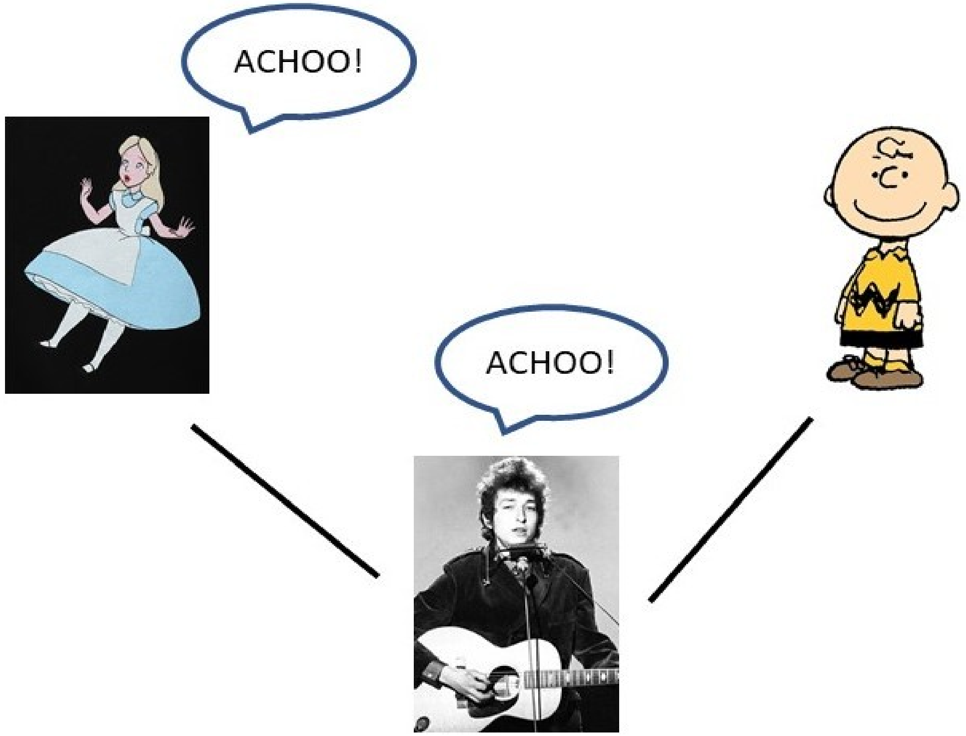

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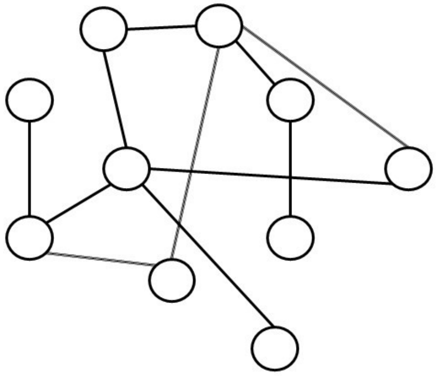
Other potential problems:

Determine how long it takes for a disease (or information, or goods) to reach every part of a network.

Estimate how many different Patient Zeros could infect a particular person.

Reconstructing Graphs

We want to reconstruct a graph G.
We can make a query which returns a random, unlabeled induced subgraph of G. How many queries are needed?



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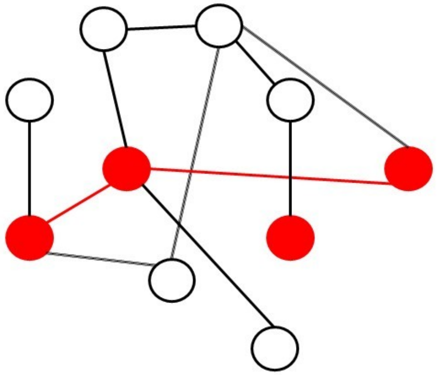

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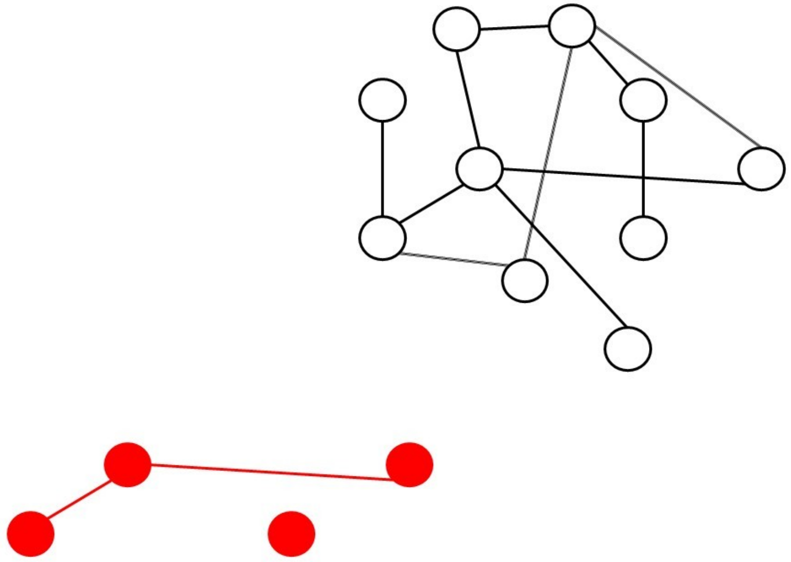

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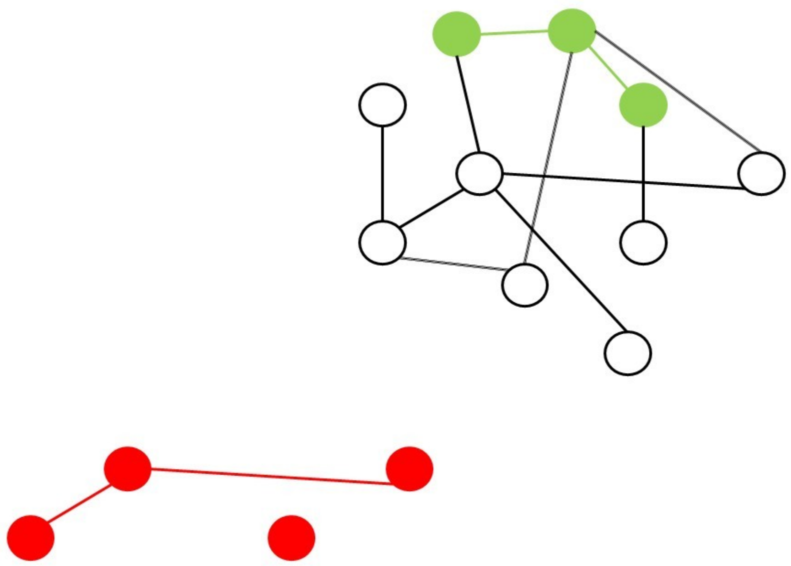

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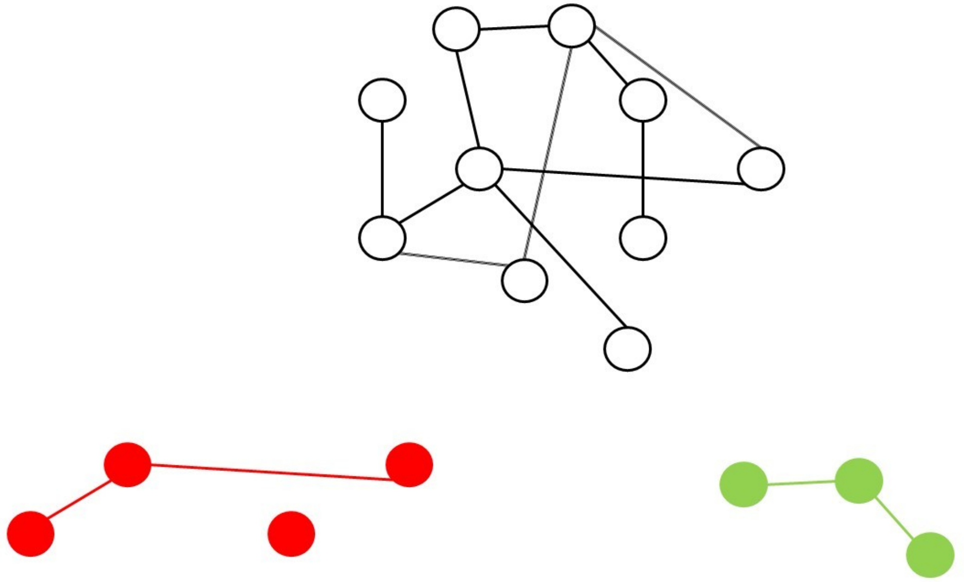

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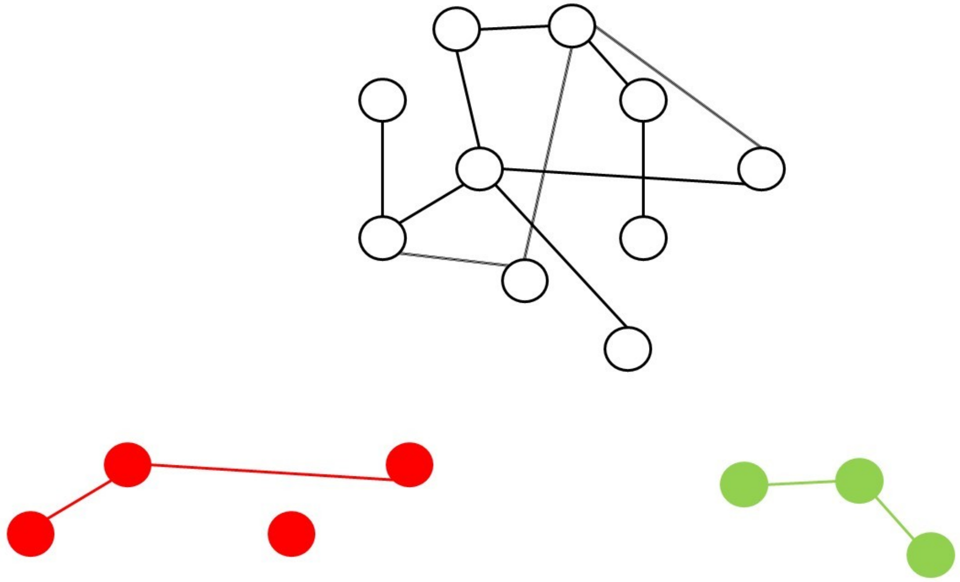
More generally, we must reconstruct a matrix M and can make queries which return a submatrix where rows and columns are deleted randomly. How many queries are needed?



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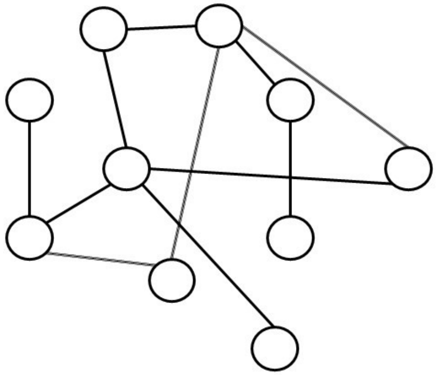

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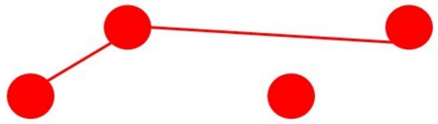

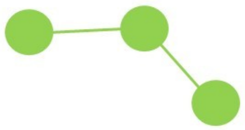

Reconstruction from other queries?

Thanks for listening!