Algorithms for Massive, Expensive, and Otherwise Inconvenient Graphs

DAVID TENCH

UNIVERSITY OF MASSACHUSETTS AMHERST

Convenient Inputs

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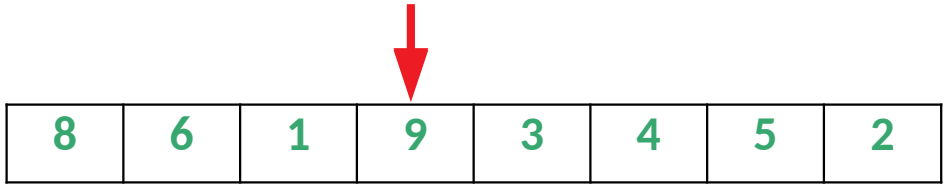
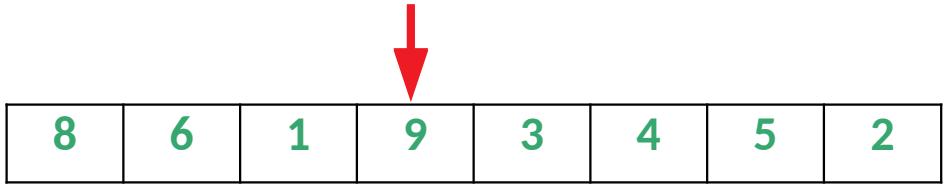
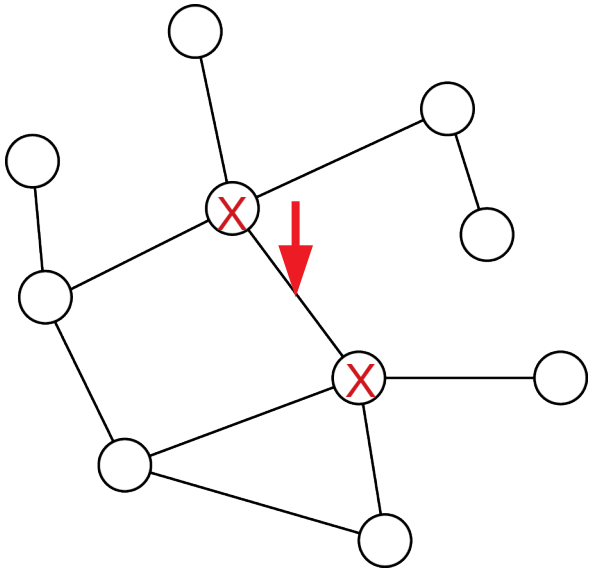
- What is the 4th element of this list?



Convenient Inputs

- What is the 4th element of this list?
- Is there an edge between some pair of nodes in this graph?





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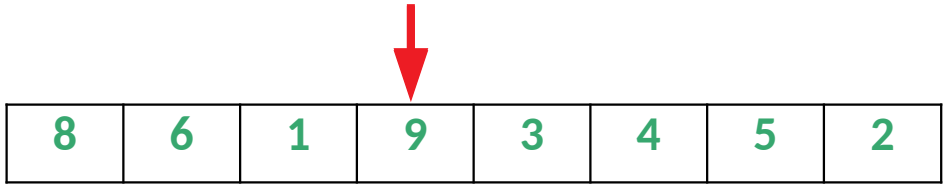

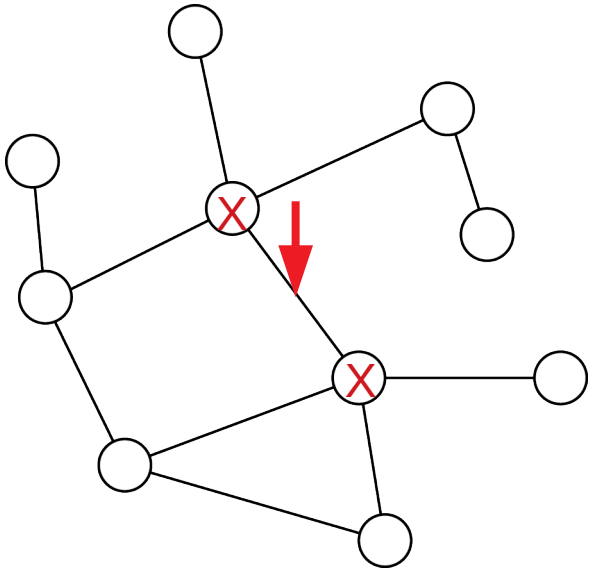

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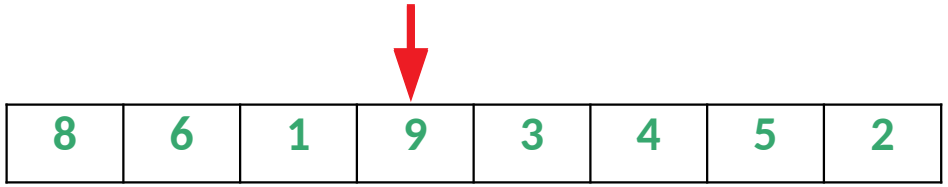

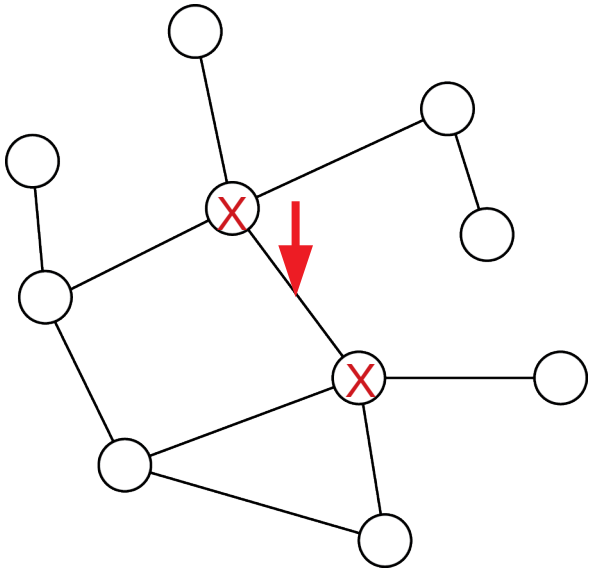

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This is the *random access property*.

When Random Access Fails

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When inputs are too large
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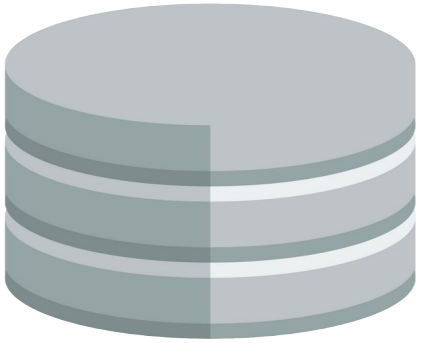

Previously Covered Work

Streaming graph algorithms: Approximating vertex connectivity in small space

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I presented algorithms that, given a stream of edges that define massive graph G, compute the smallest set of nodes whose removal would disconnect G.

(this is hard because input is **large**)

Previously Covered Work

I also presented Mesh, a system that performs memory compaction that was thought to be impossible.

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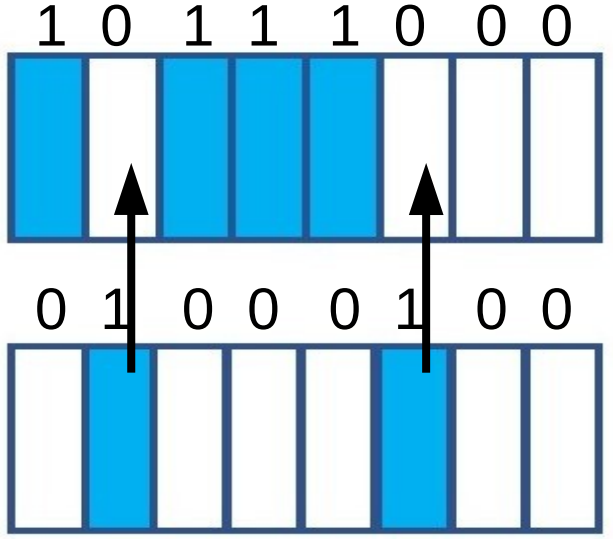
Mesh: “Impossible” C and C++ Memory Compaction via Random Graph Algorithms



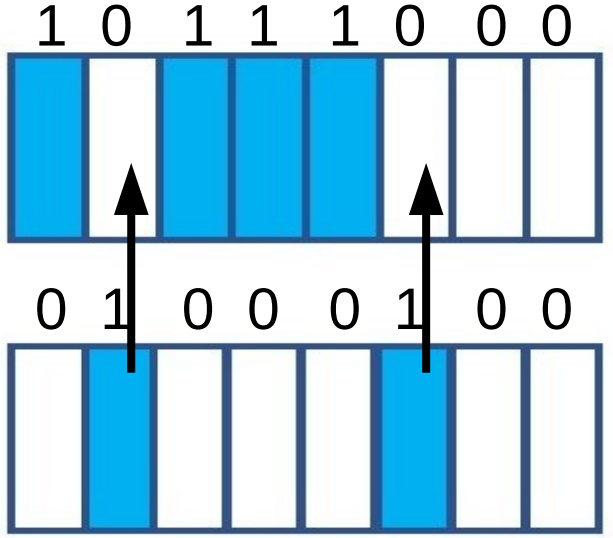
Previously Covered Work

I also presented Mesh, a system that performs memory compaction that was thought to be impossible.

Mesh does this by finding an approximately optimal maximum matching on a graph whose edges are only accessible via costly queries.

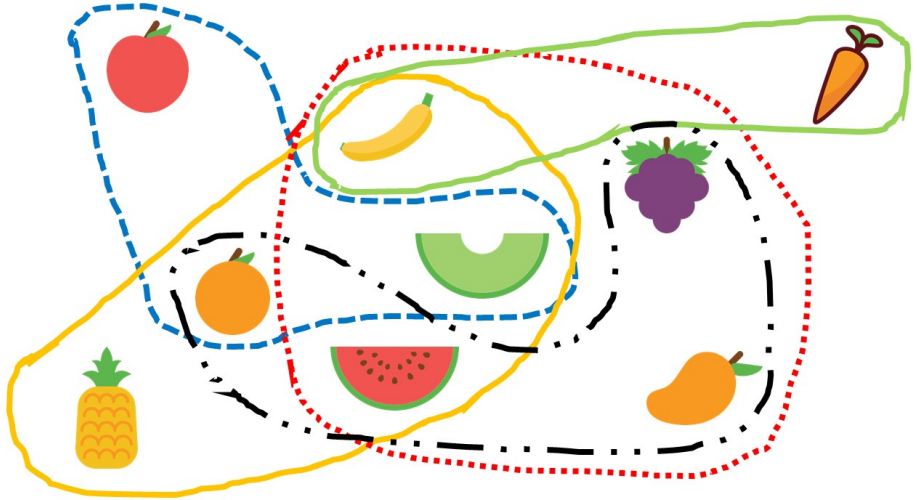
(this is hard because input is **costly**)

Mesh: “Impossible” C and C++ Memory Compaction via Random Graph Algorithms



Overview of this talk

Streaming hypergraph algorithms: Computing maximum (unique) coverage



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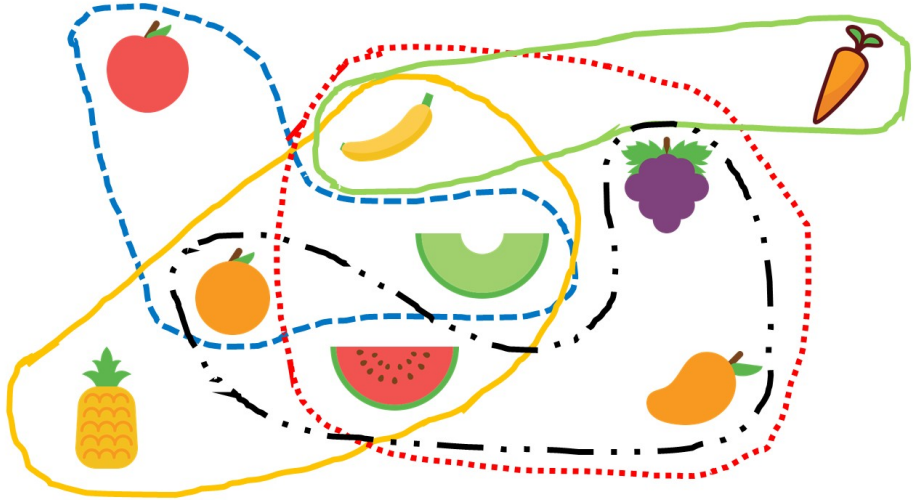

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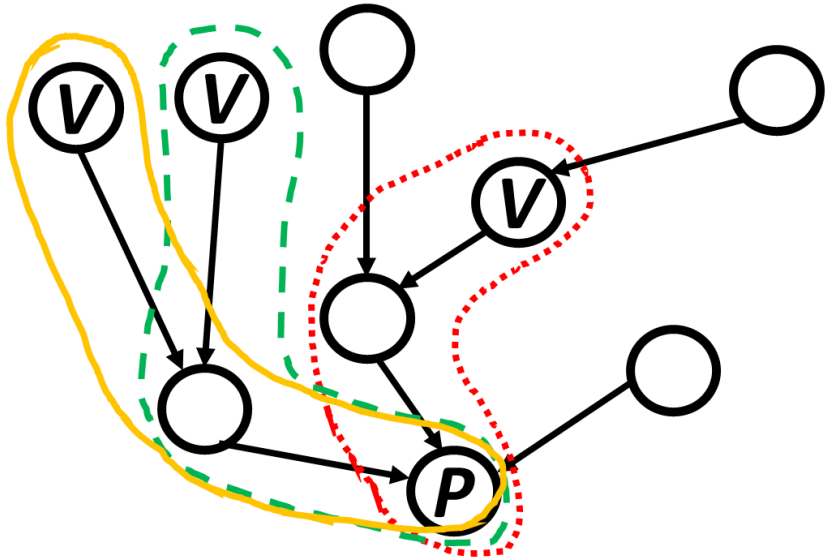
PathCache: predicting Internet paths via costly measurements



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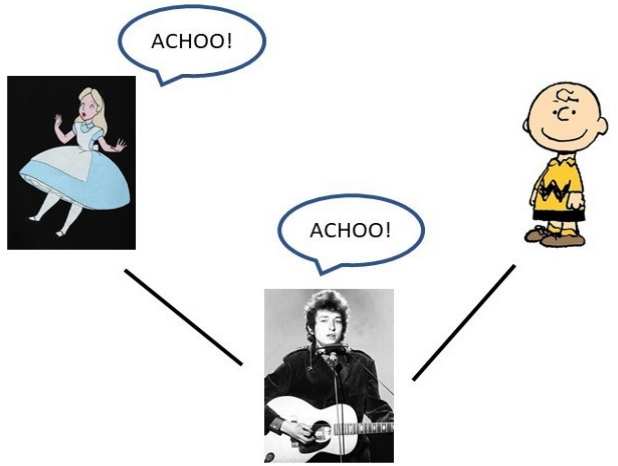
Overview of this talk

- Streaming hypergraph algorithms for coverage
- PathCache: Internet mapping

If we have time:
Temporal graph streams & applications to infection tracking



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Overview of this talk

- Streaming hypergraph algorithms for coverage
- PathCache: Internet mapping
- Temporal graph streams (?)

Future Research Directions

Overview of this talk

- Streaming hypergraph algorithms for coverage
- PathCache: Internet mapping
- Temporal graph streams (?)
- Future Research Directions

Questions before we begin?

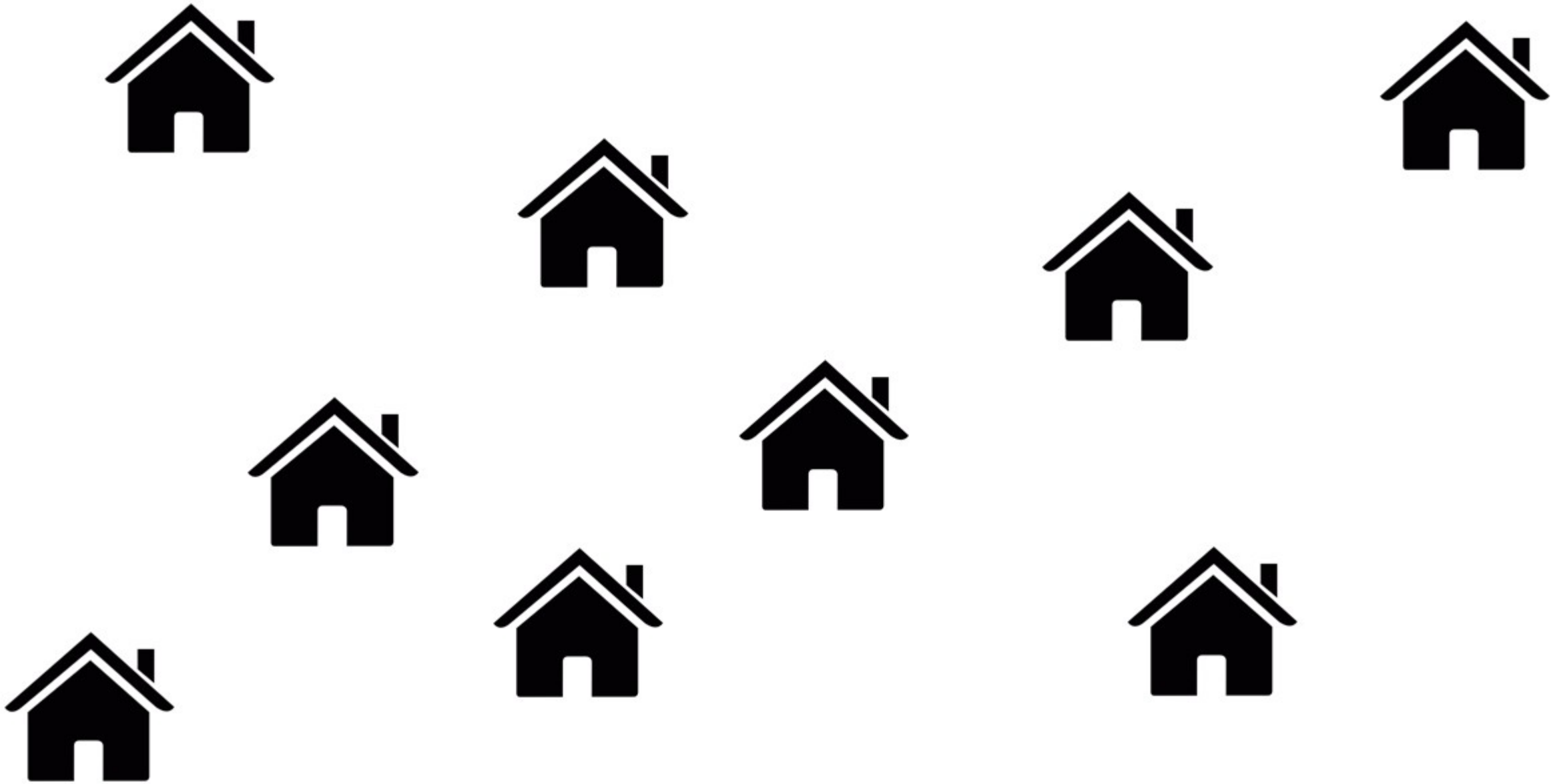
Coverage Problems in Streams

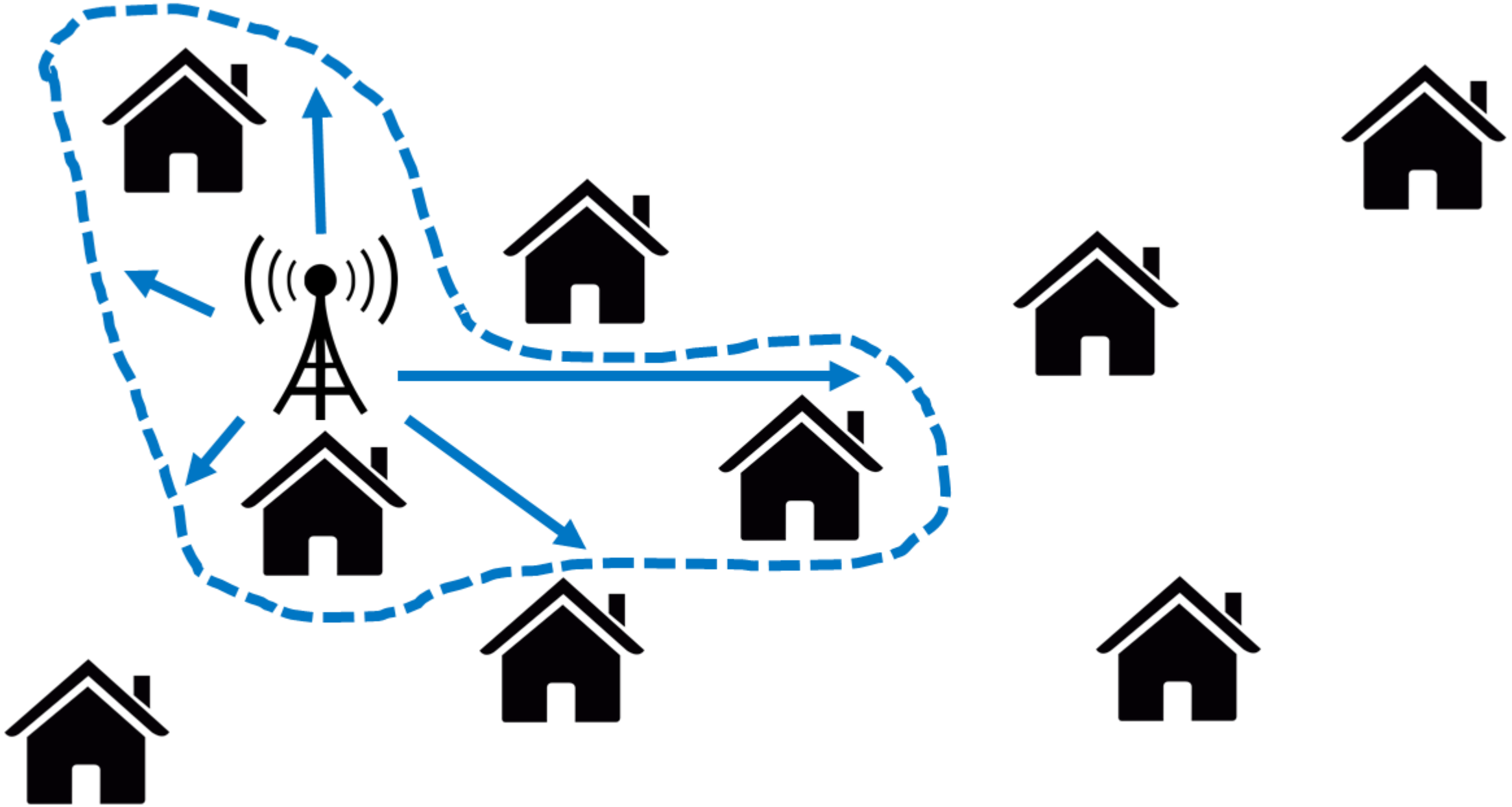
COMPUTING WITH INCREDIBLY LARGE INPUTS

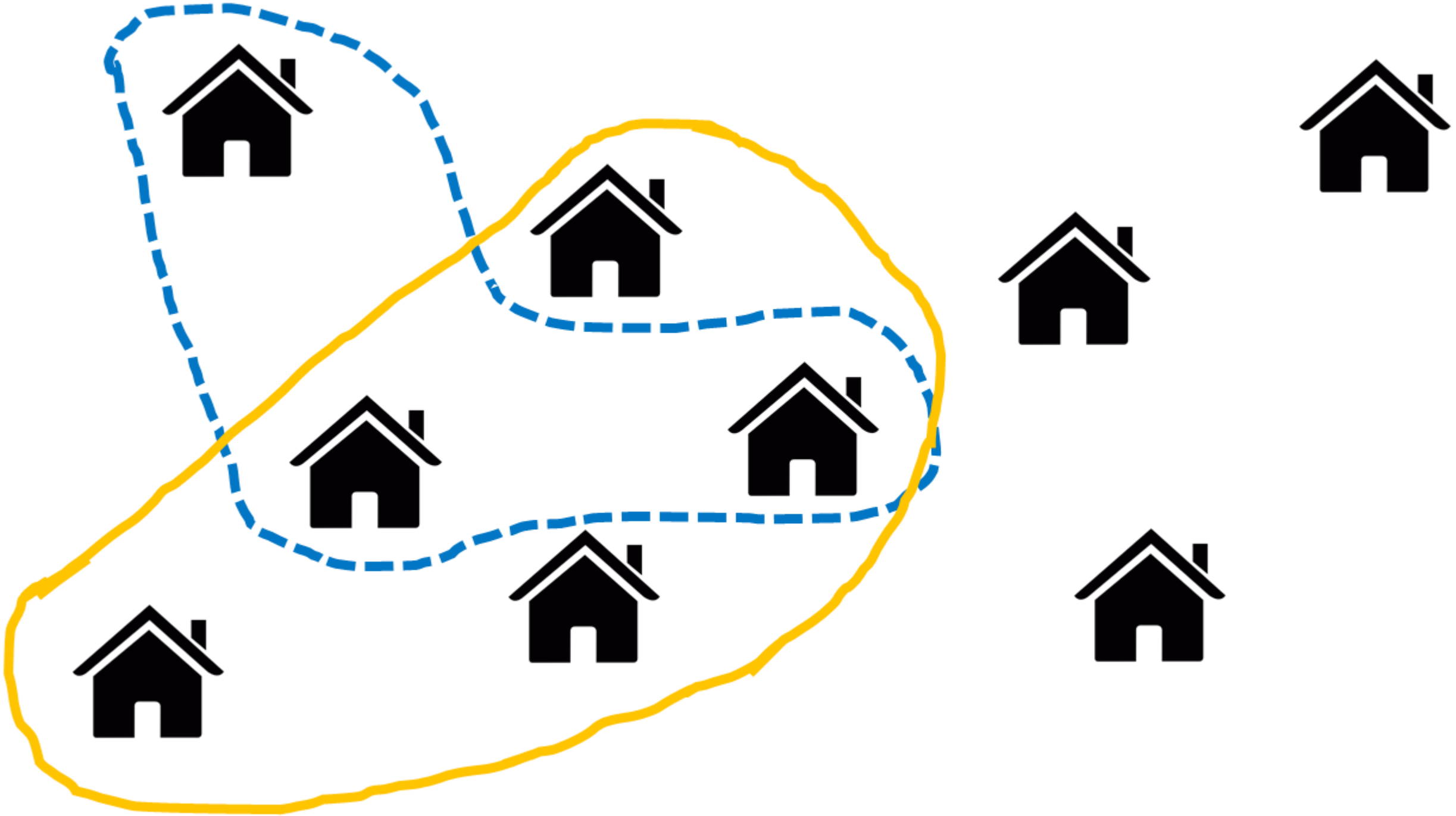
Joint work with:

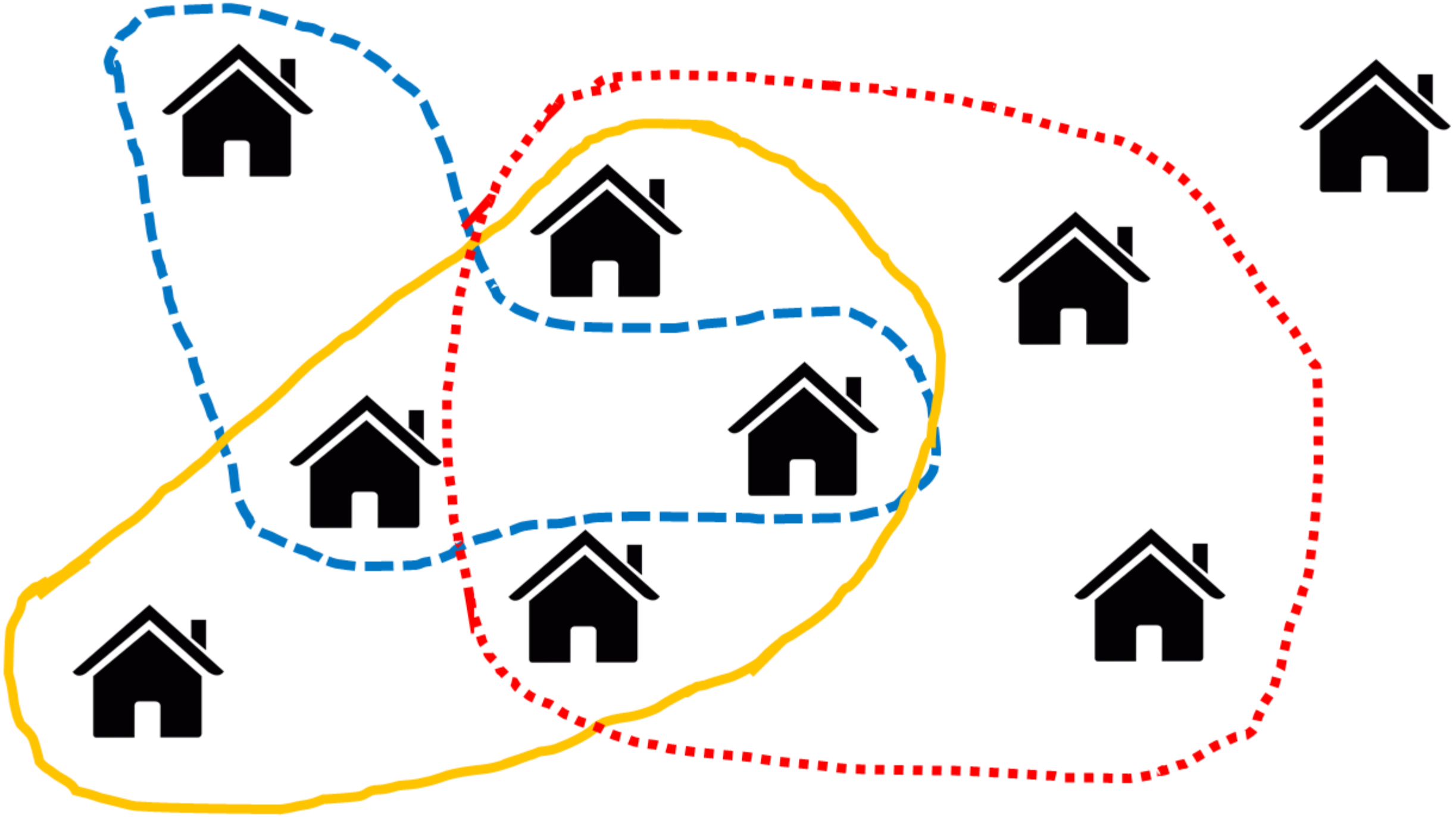
Andrew McGregor

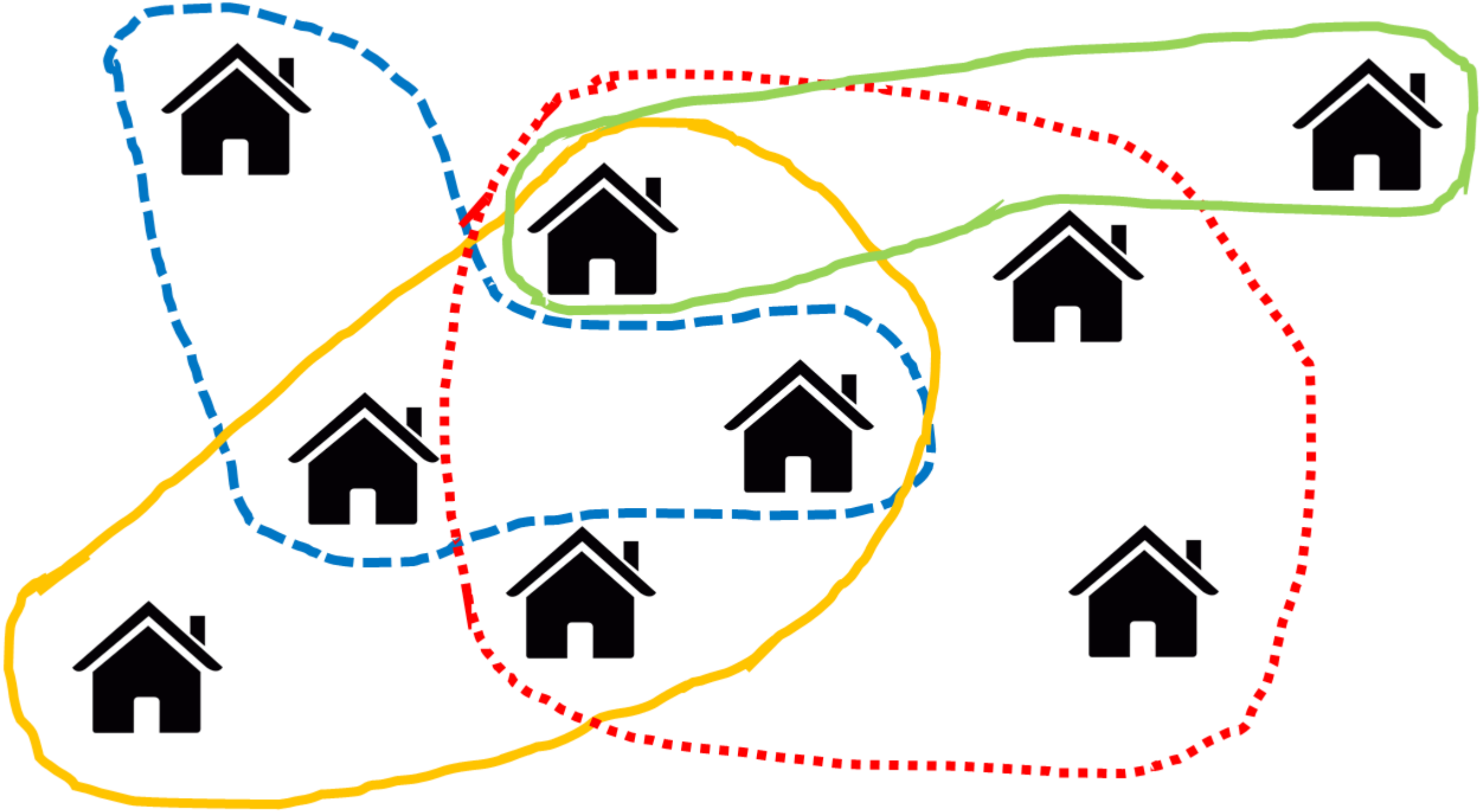
Hoa T. Vu

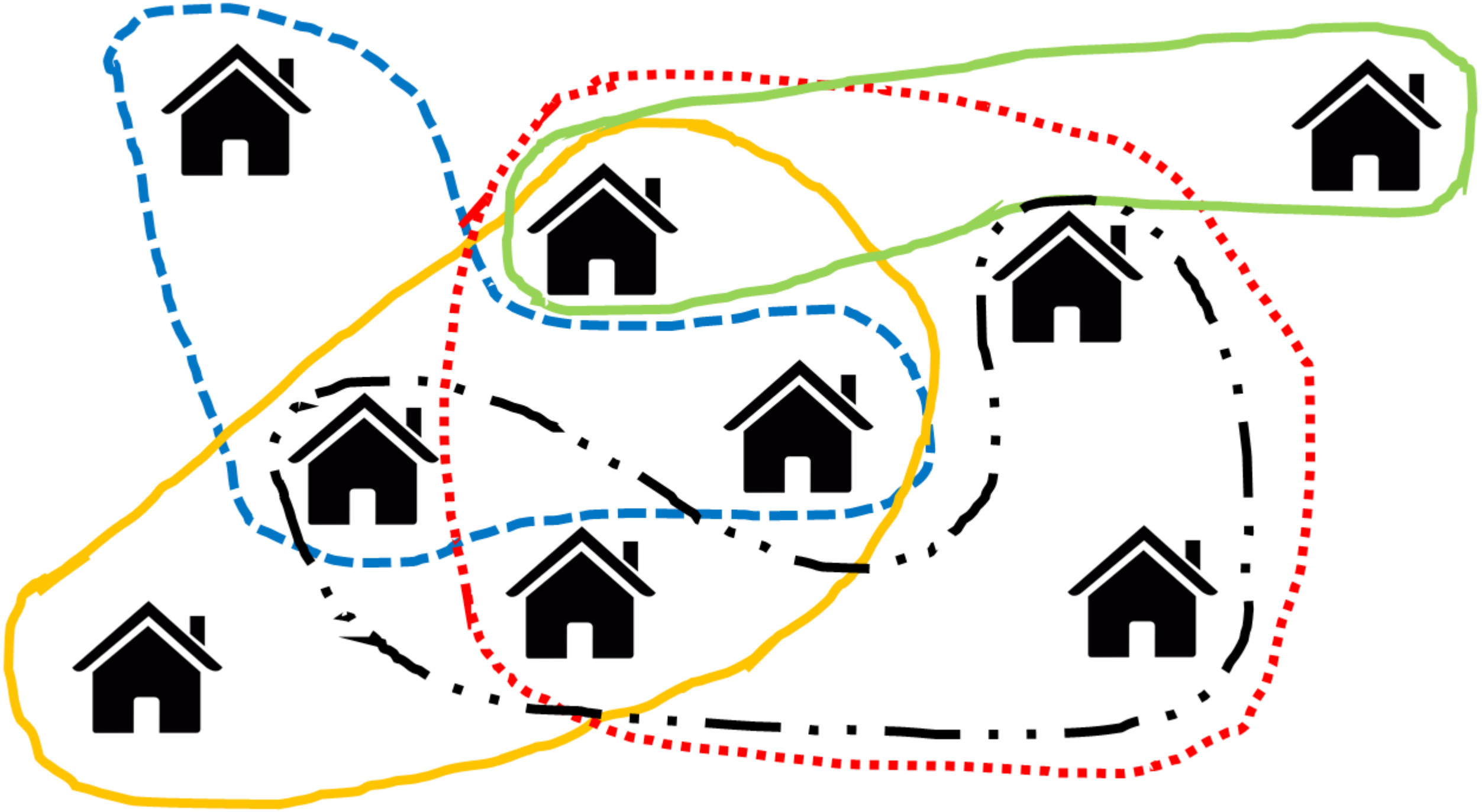


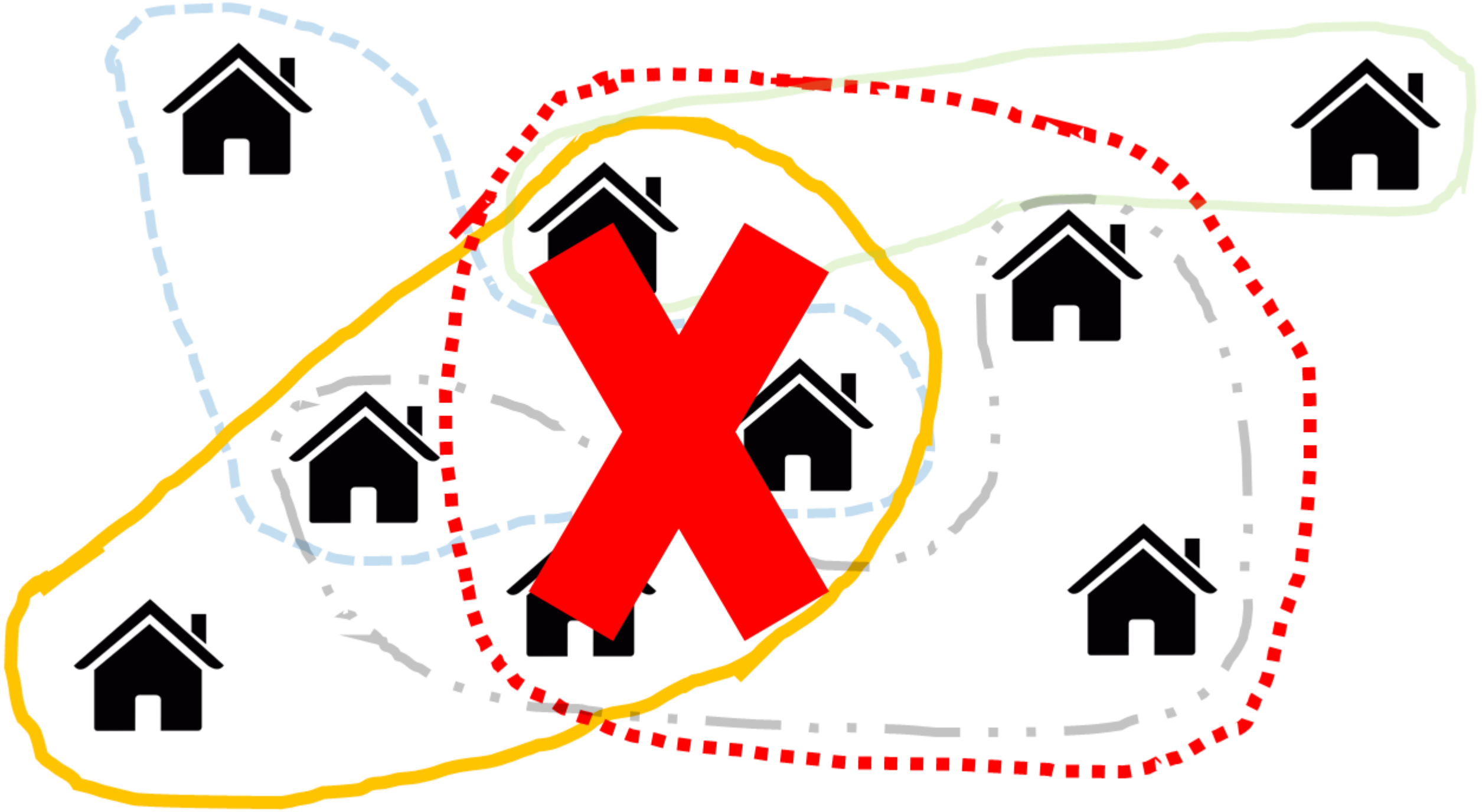


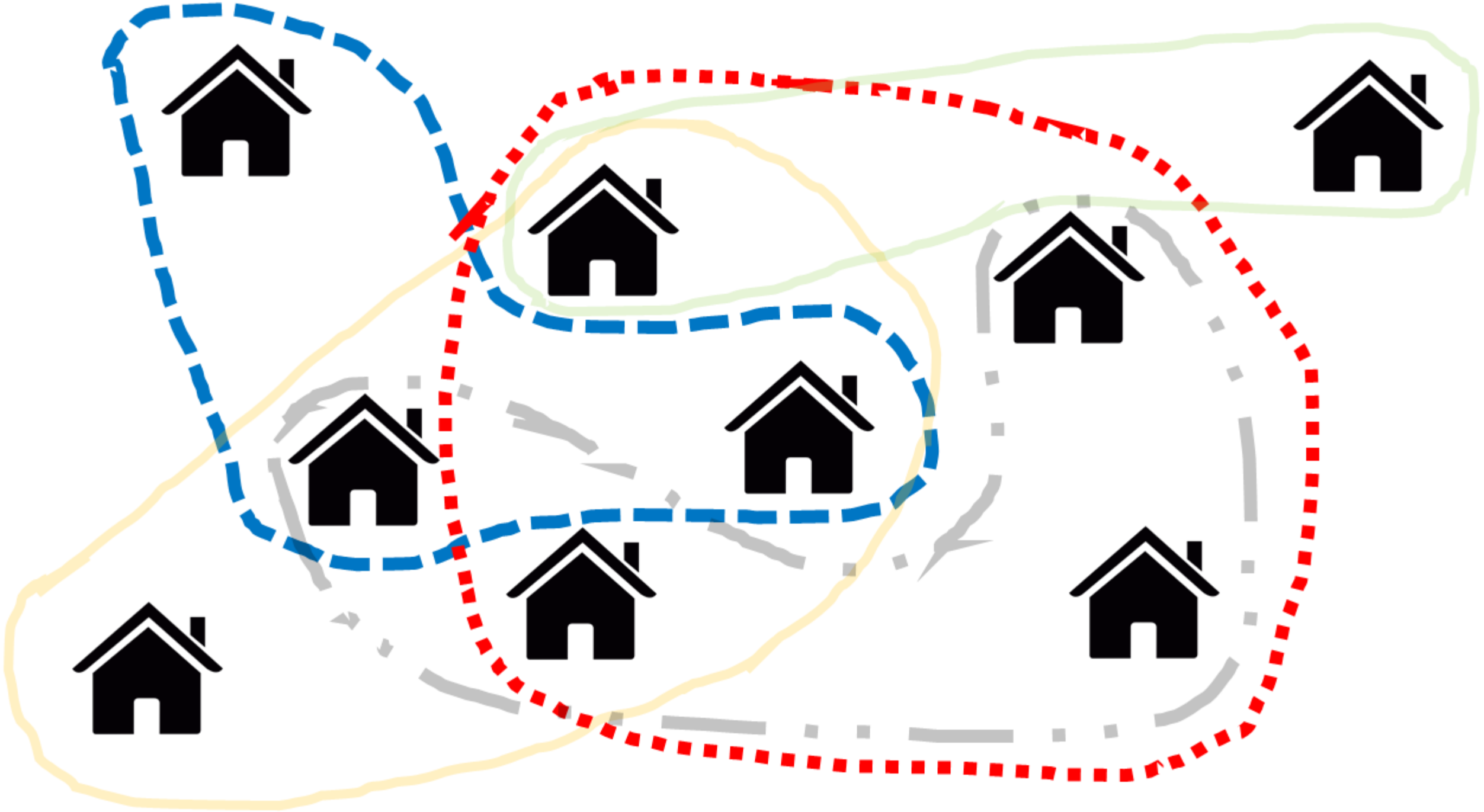


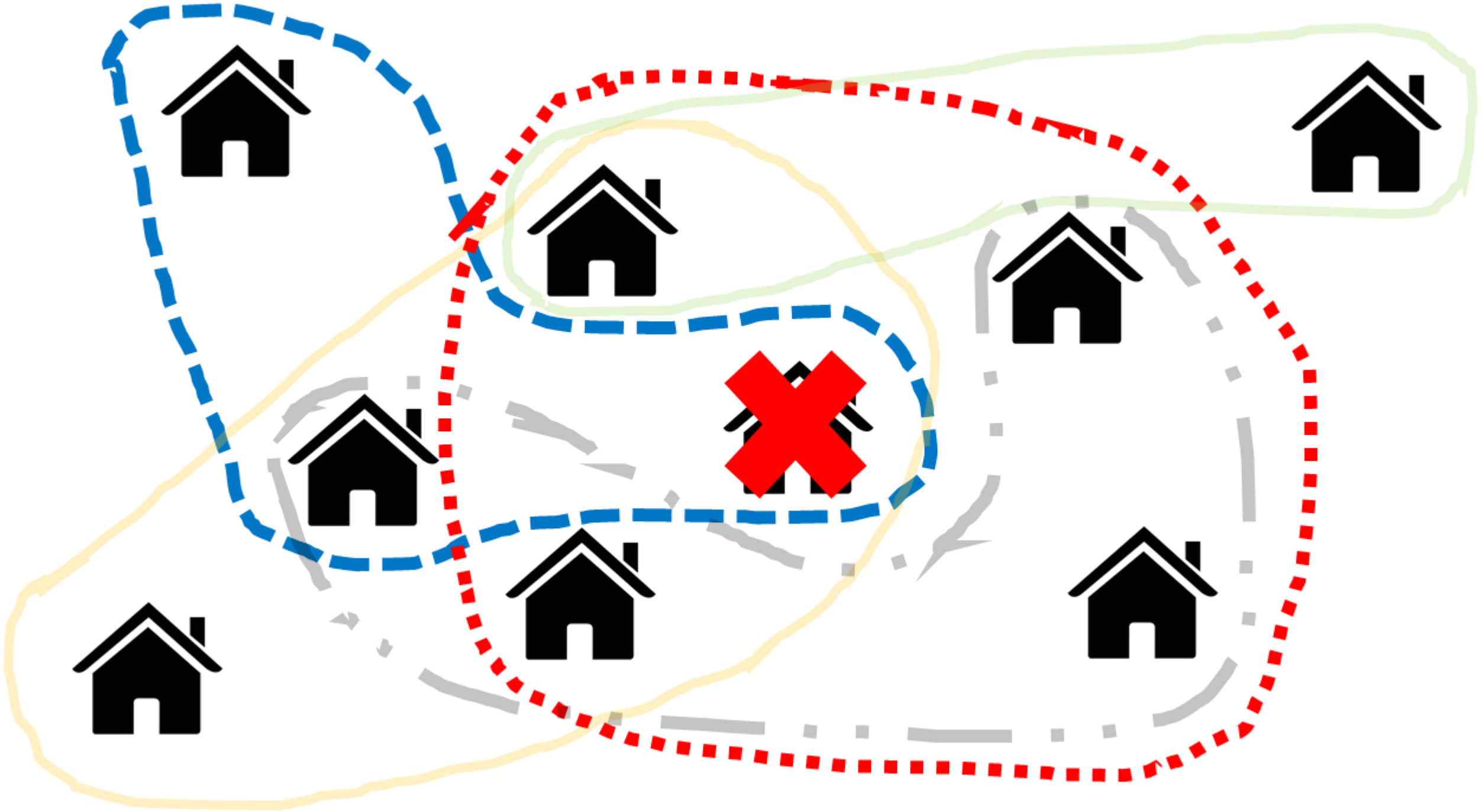






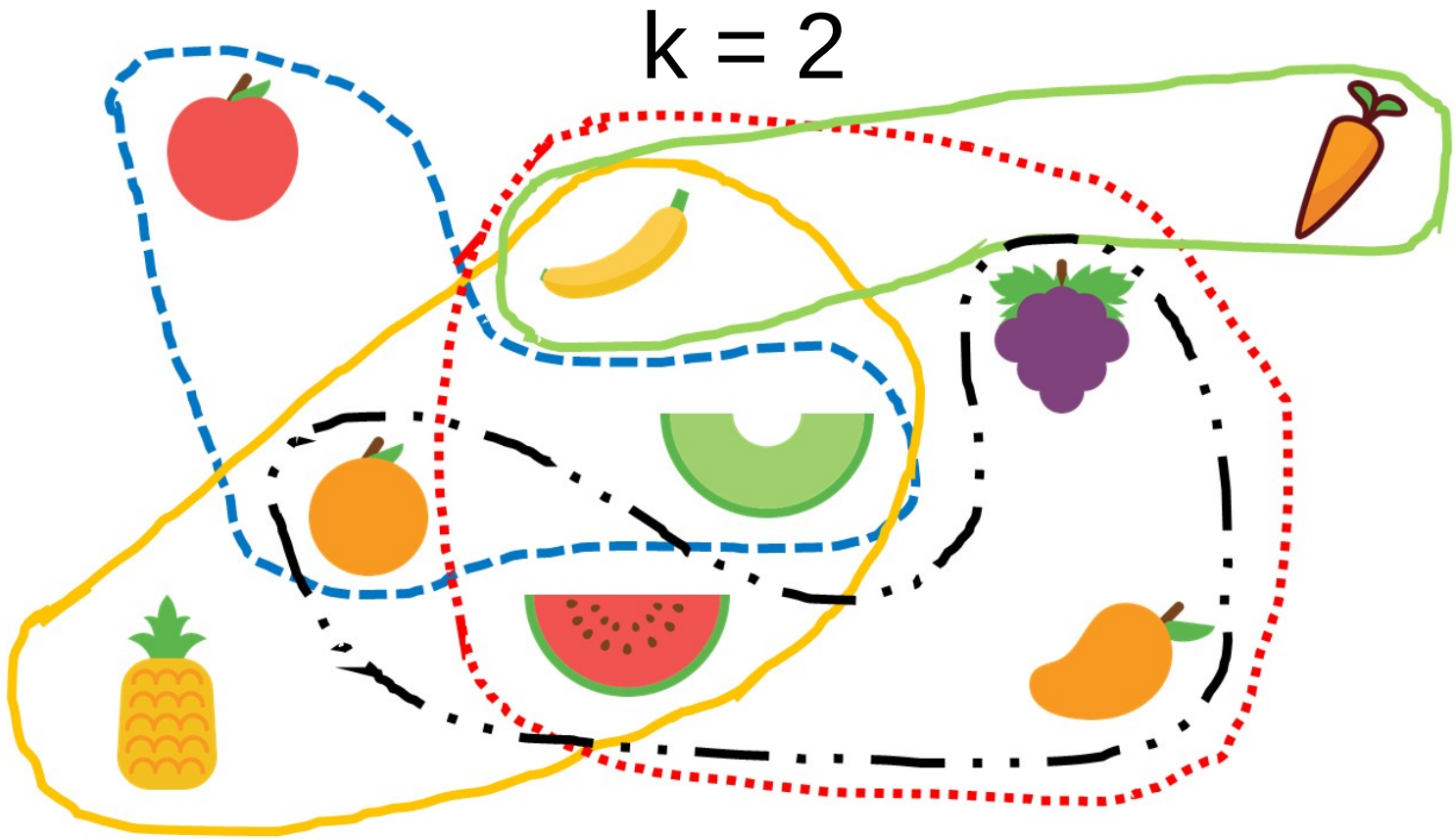






Maximum Coverage

These are examples of *coverage* problems.



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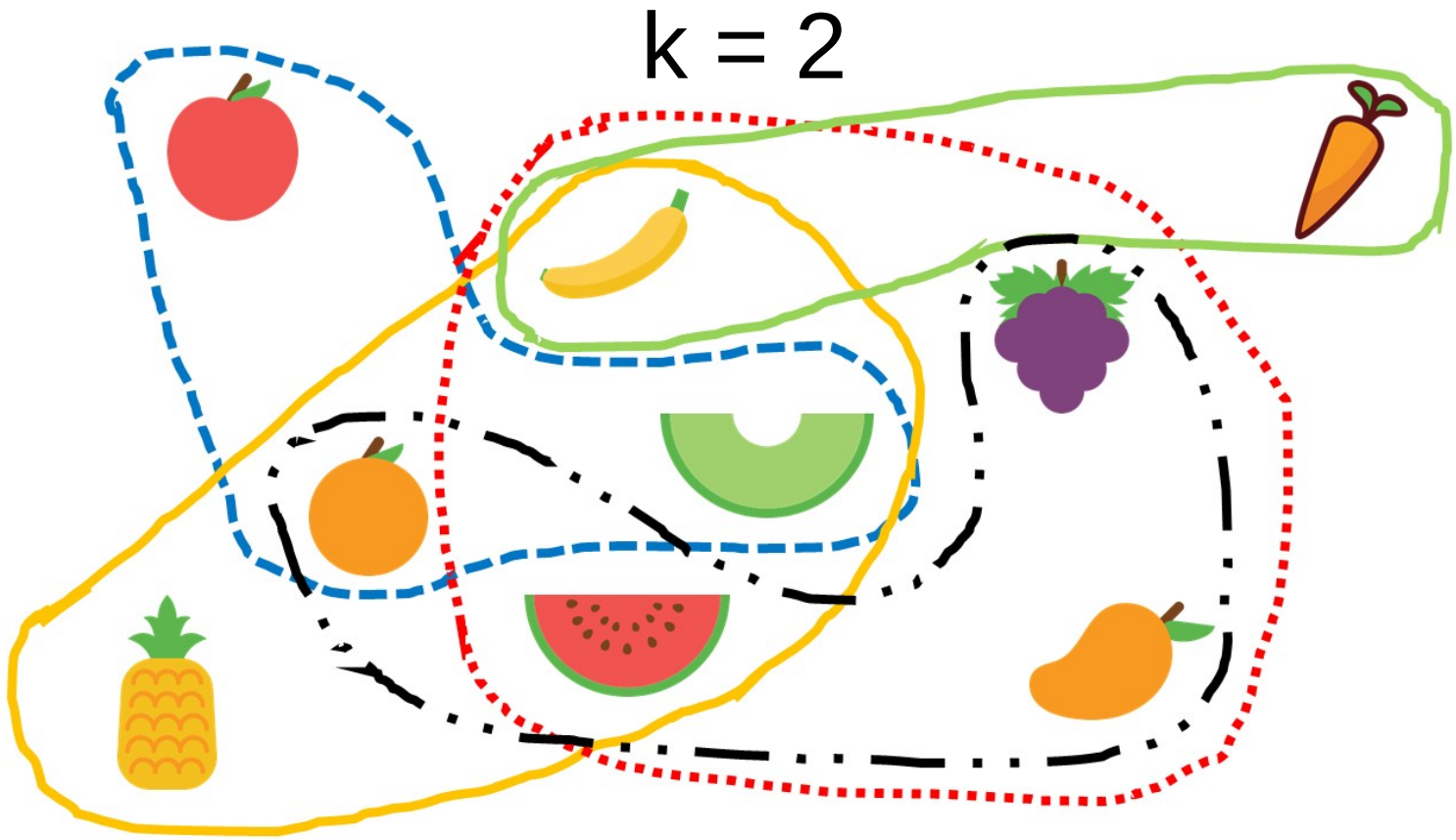
Given a universe U of n objects and m subsets of U, choose k subsets that maximize:



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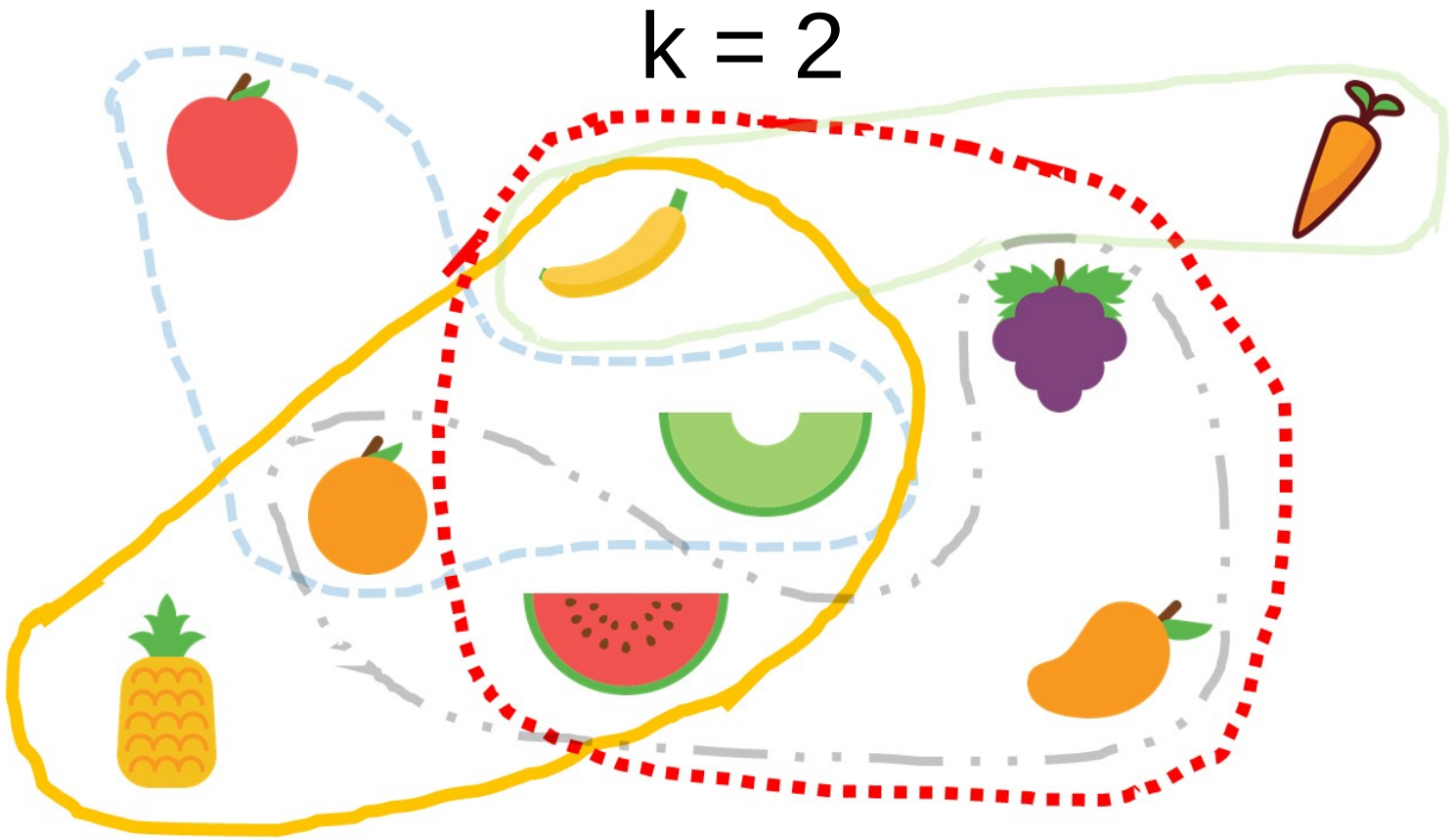

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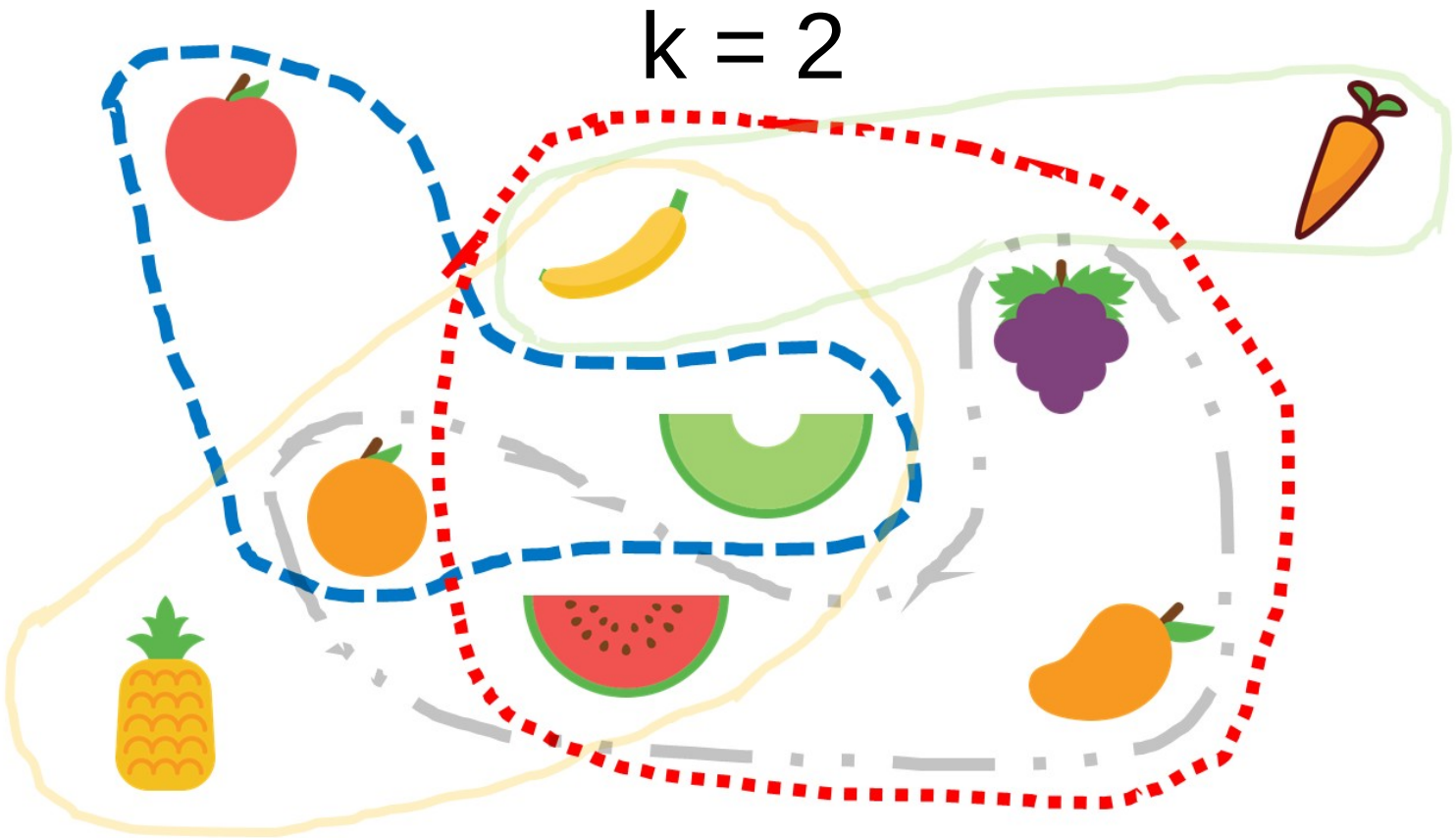
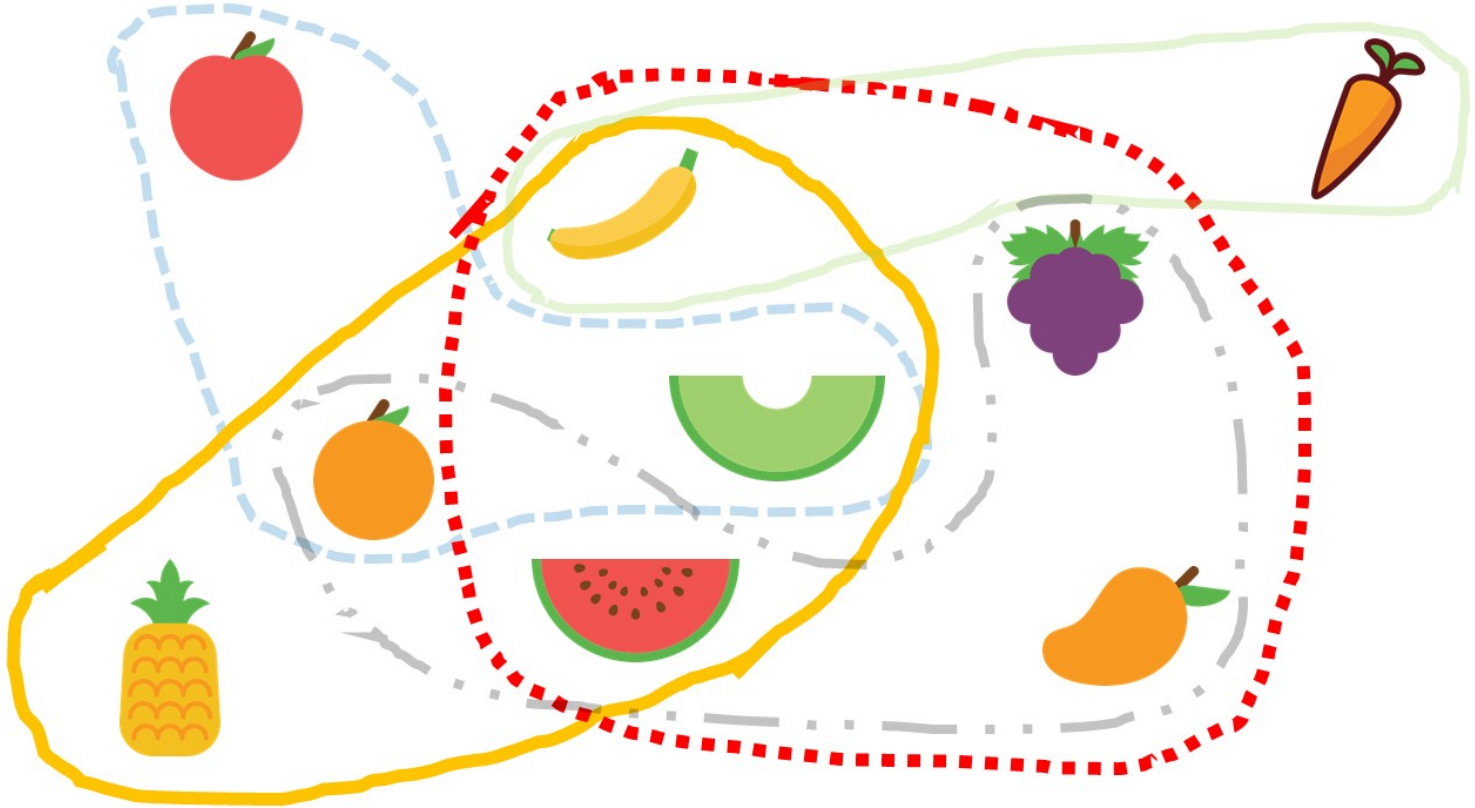
Given a universe U of n objects and m subsets of U, choose k subsets that maximize:

- # of objects covered by at least 1 subset (Max-k Coverage)
- # of objects covered by exactly 1 subset (Unique-k Coverage)

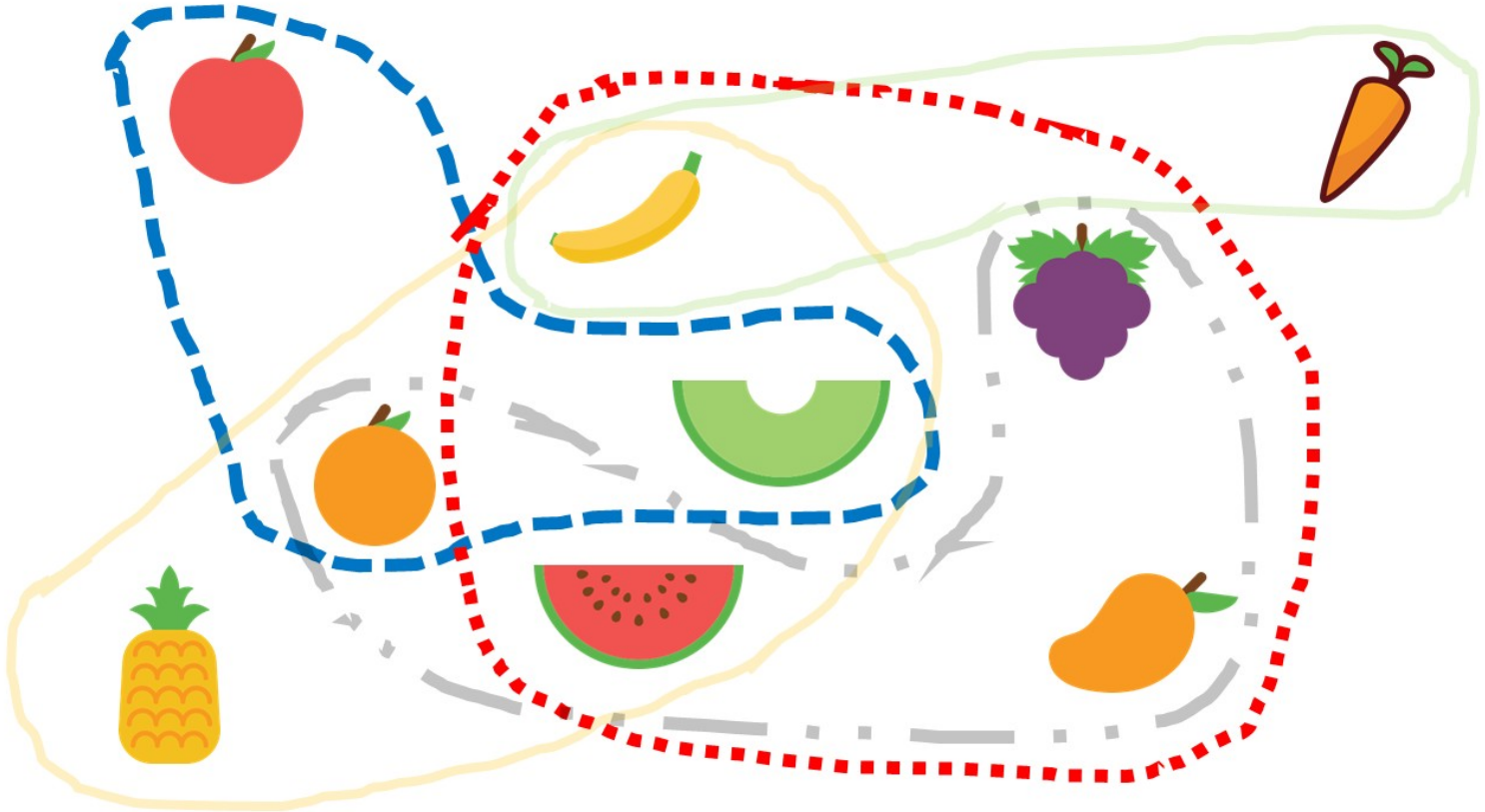


Max-k Cover



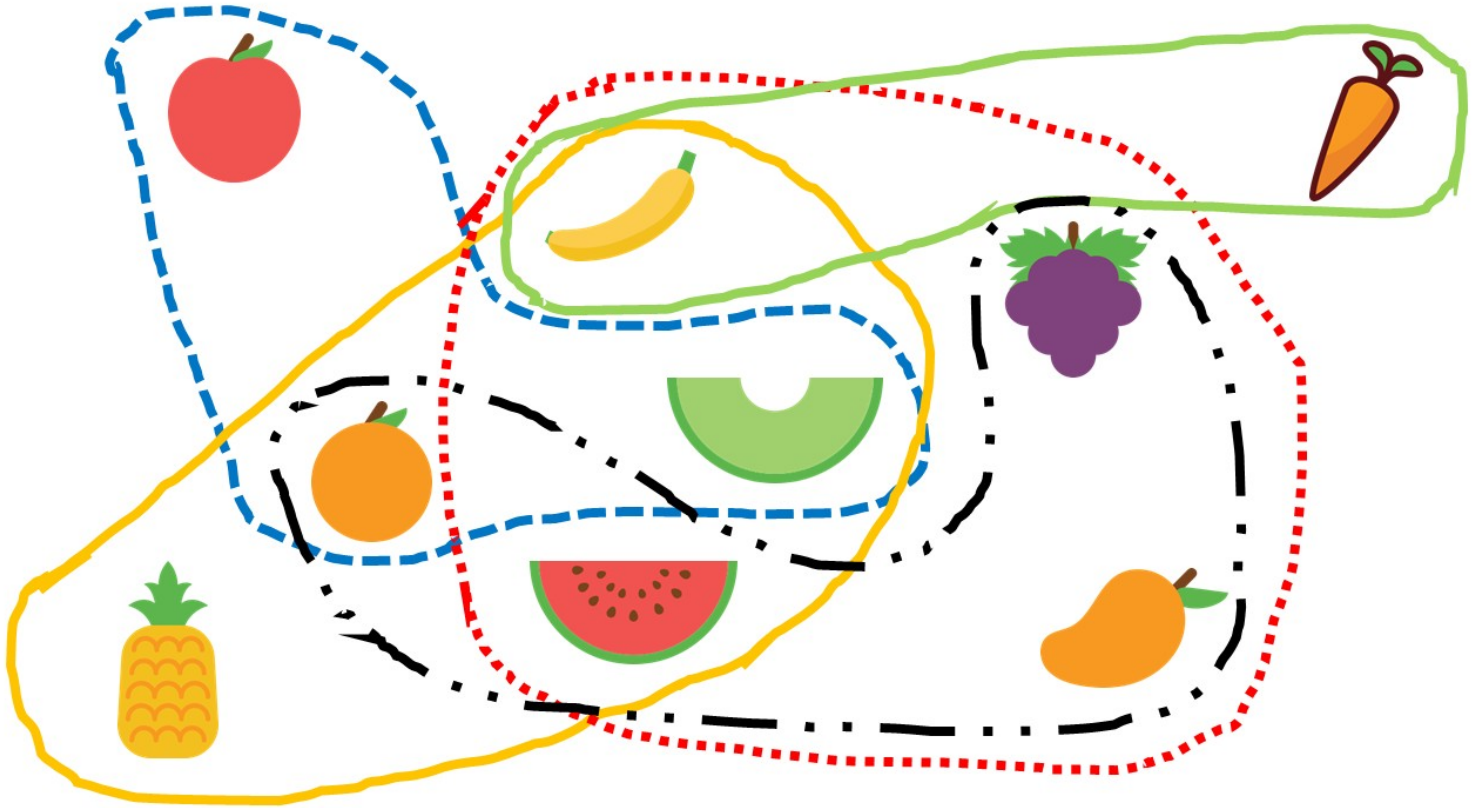
- NP-Hard
- Greedy (e/(e-1)) - approximation is the best possible

Unique-k Cover



- NP-Hard
- Probably hard to O(polylog(n))-approximate

Another Perspective



These are also hypergraphs!

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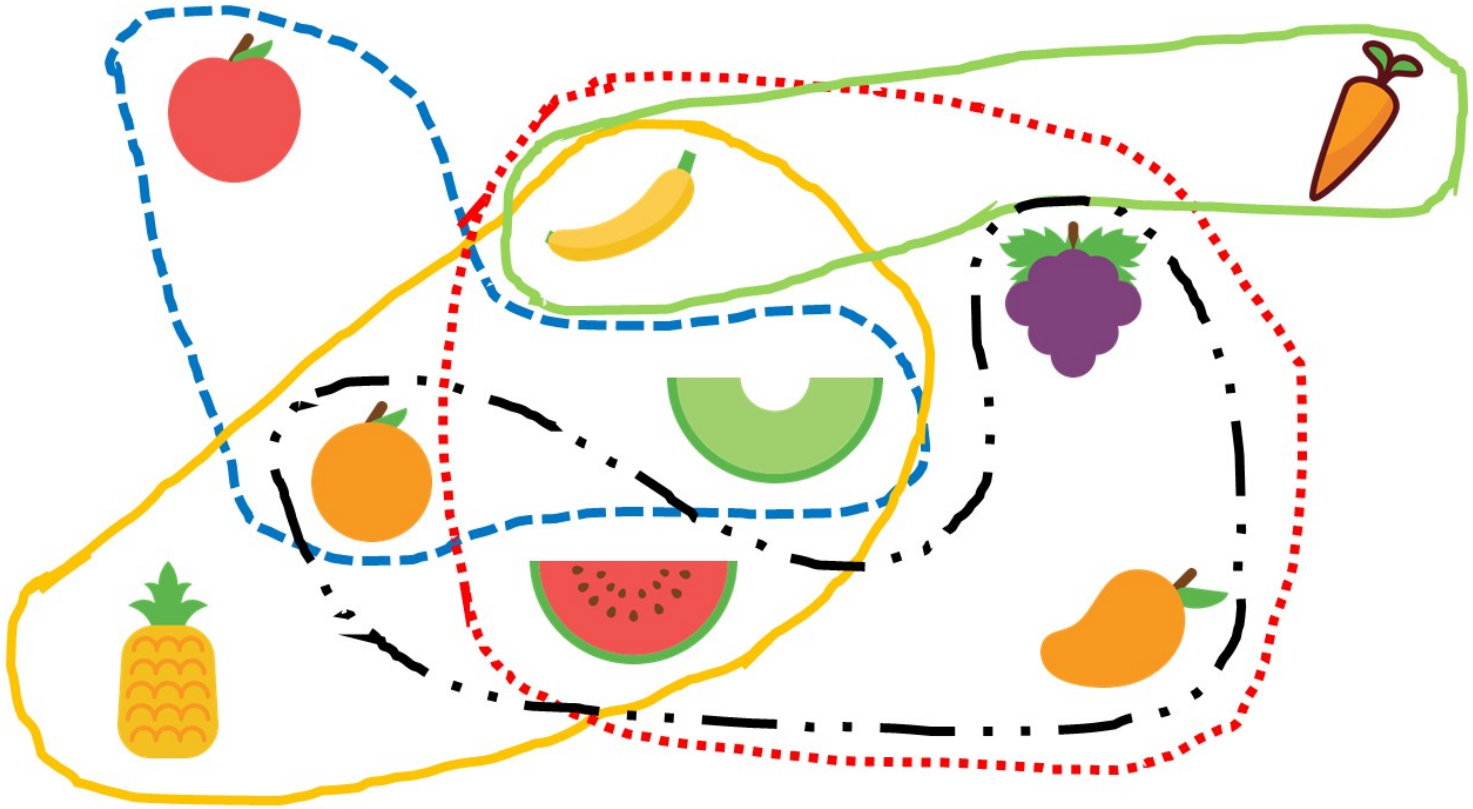

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Like graphs, but edges (colored lines) can have more than 2 endpoints.

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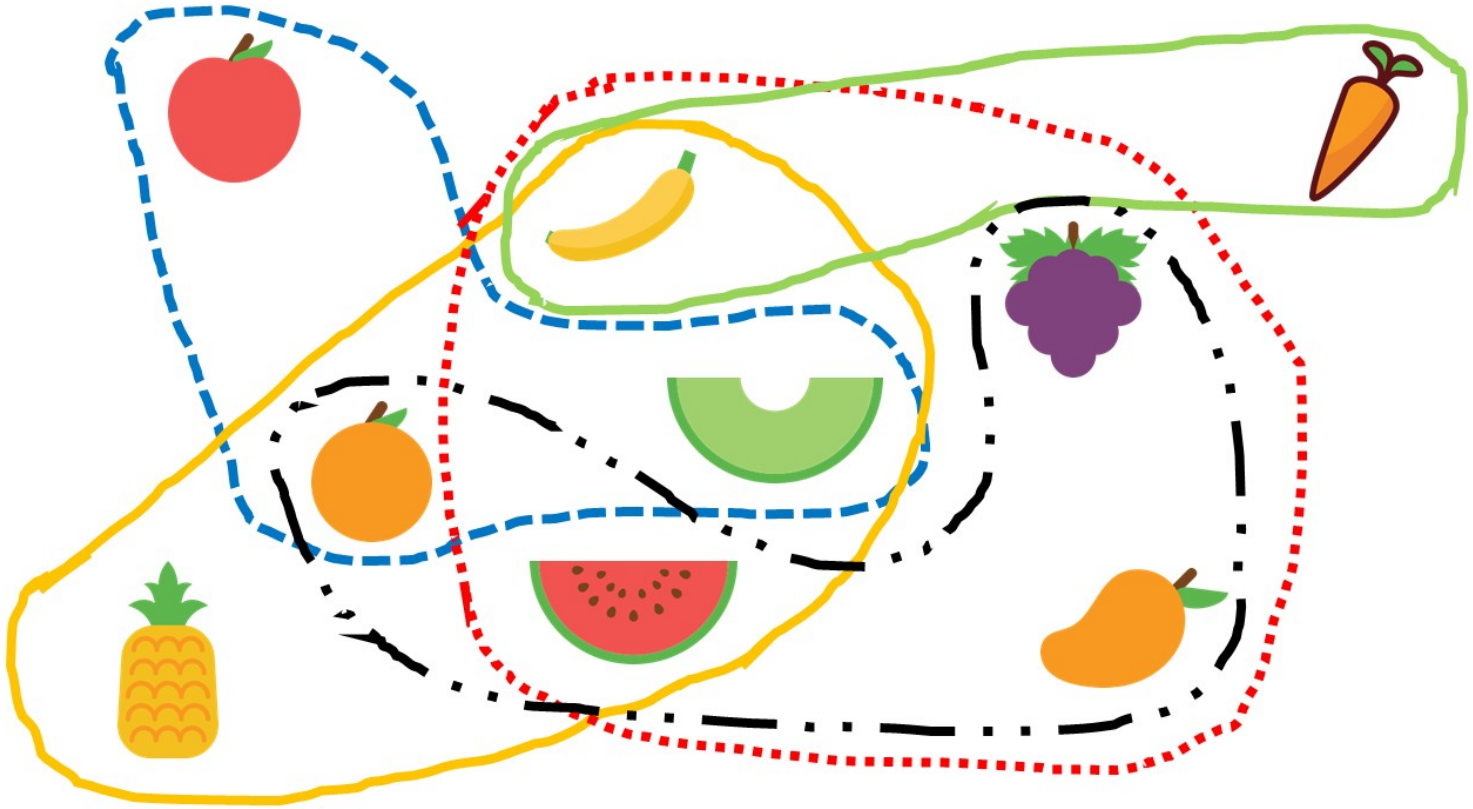

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I'll use "node"/ "object" and "set"/"edge" interchangeably.

Our version is harder!

We assume there are more subsets than we can fit into our computer's memory.

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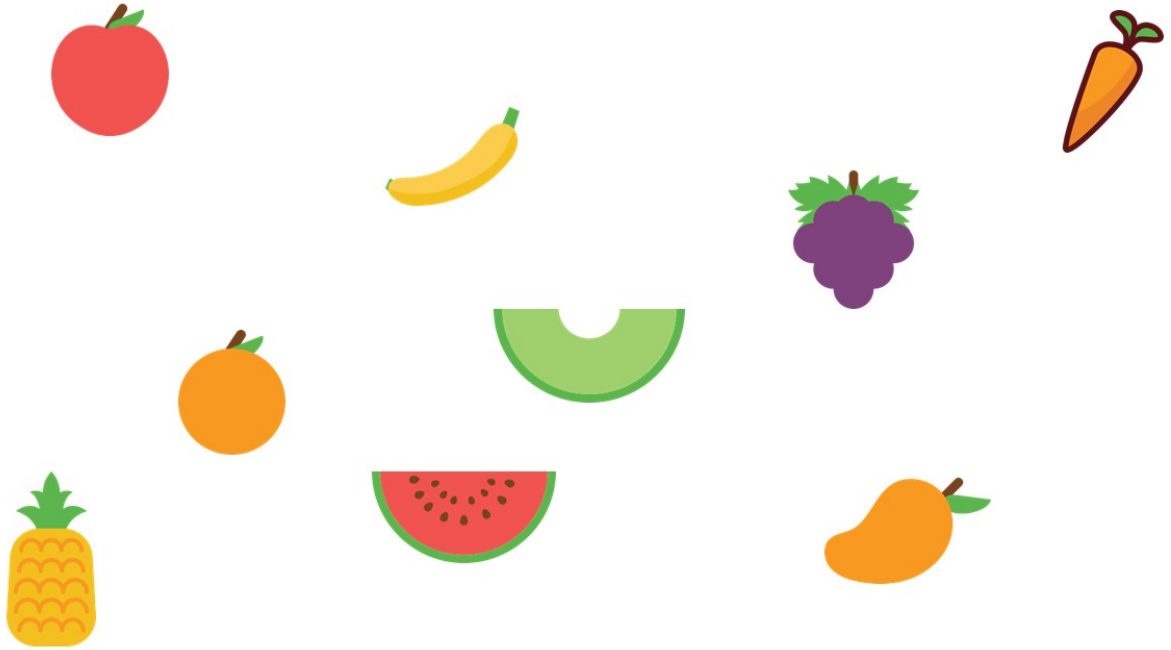

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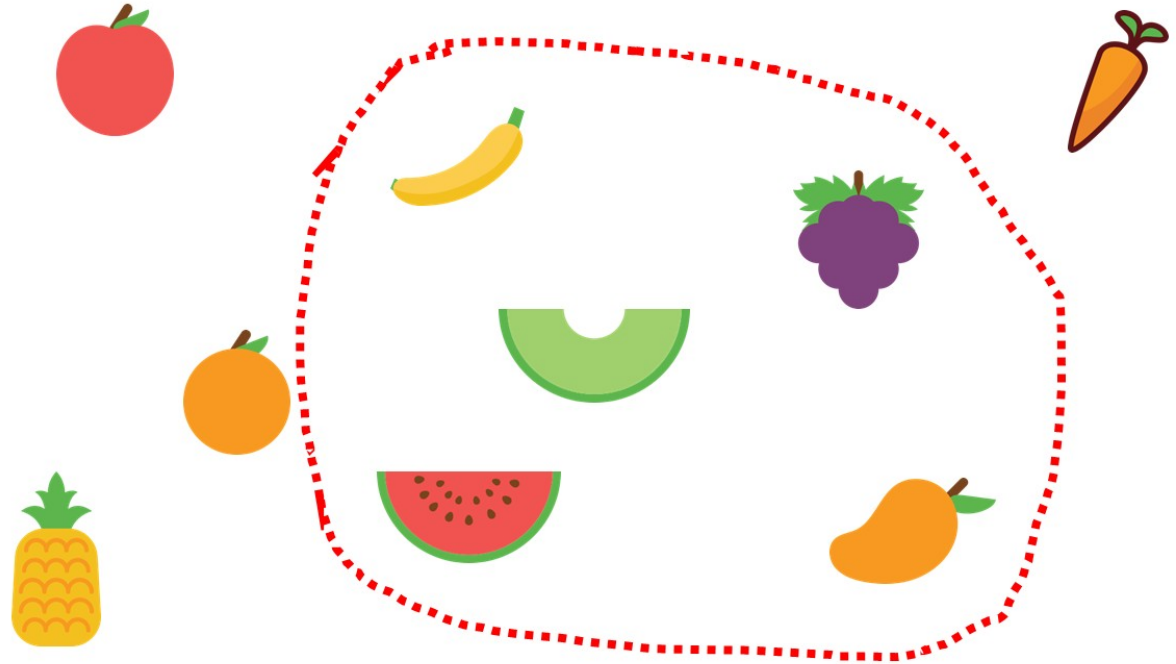

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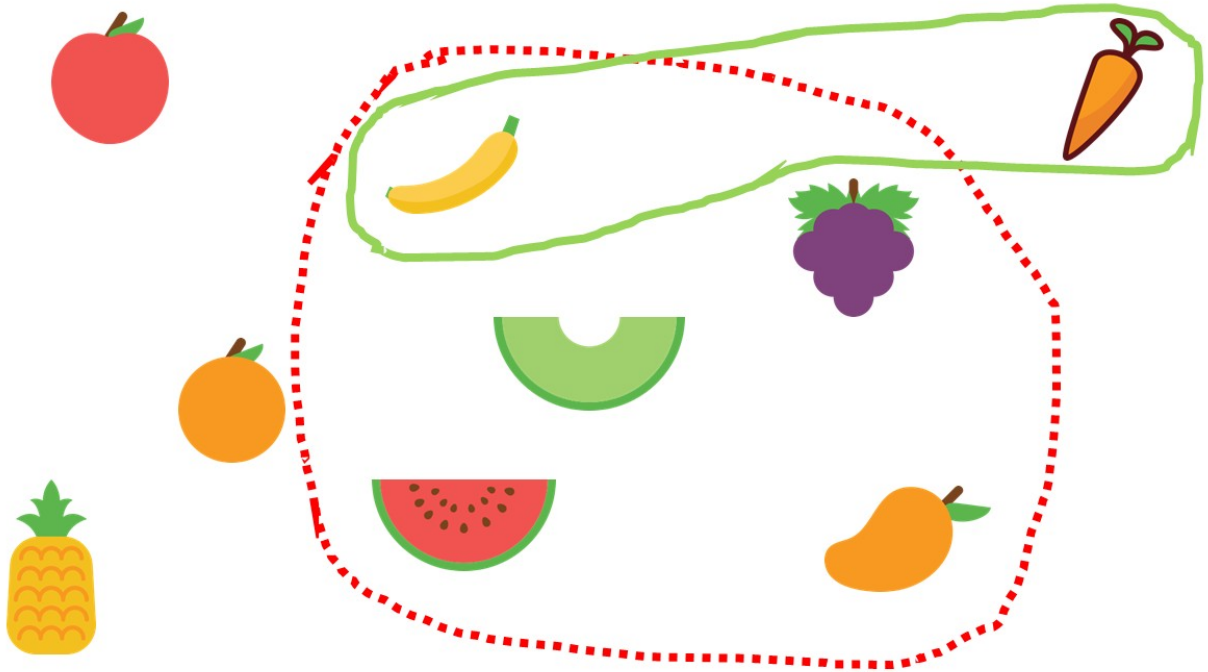

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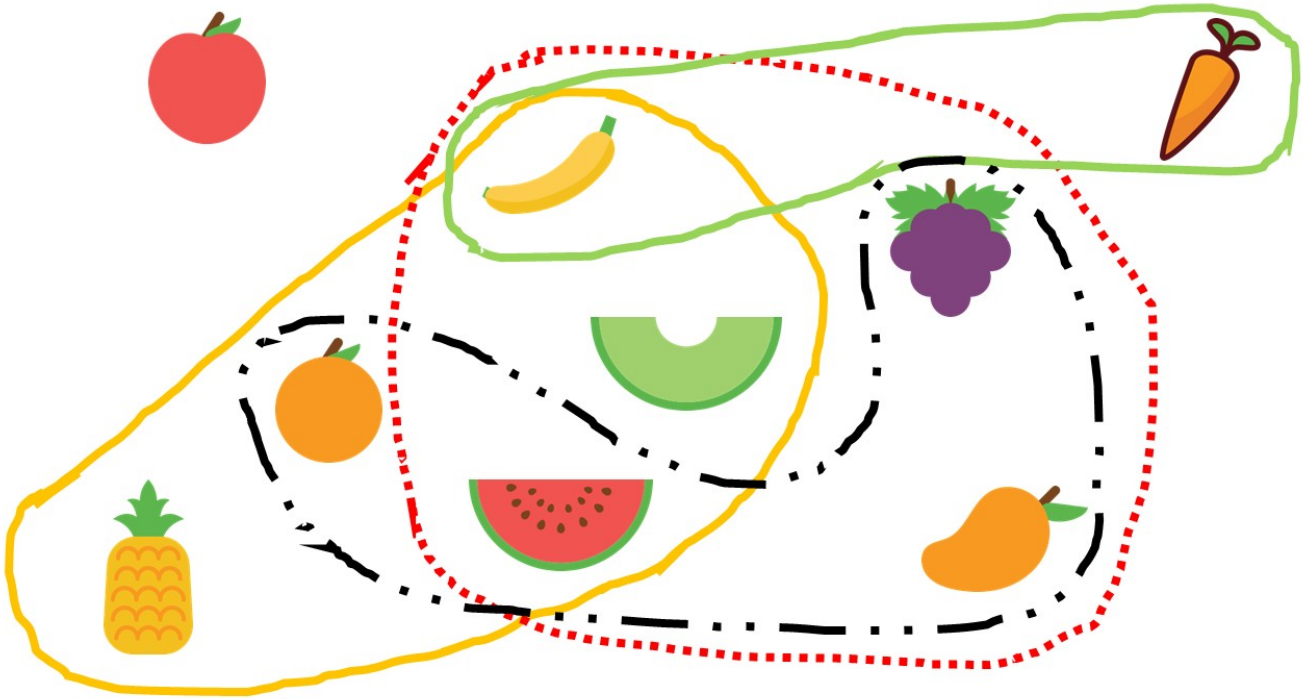

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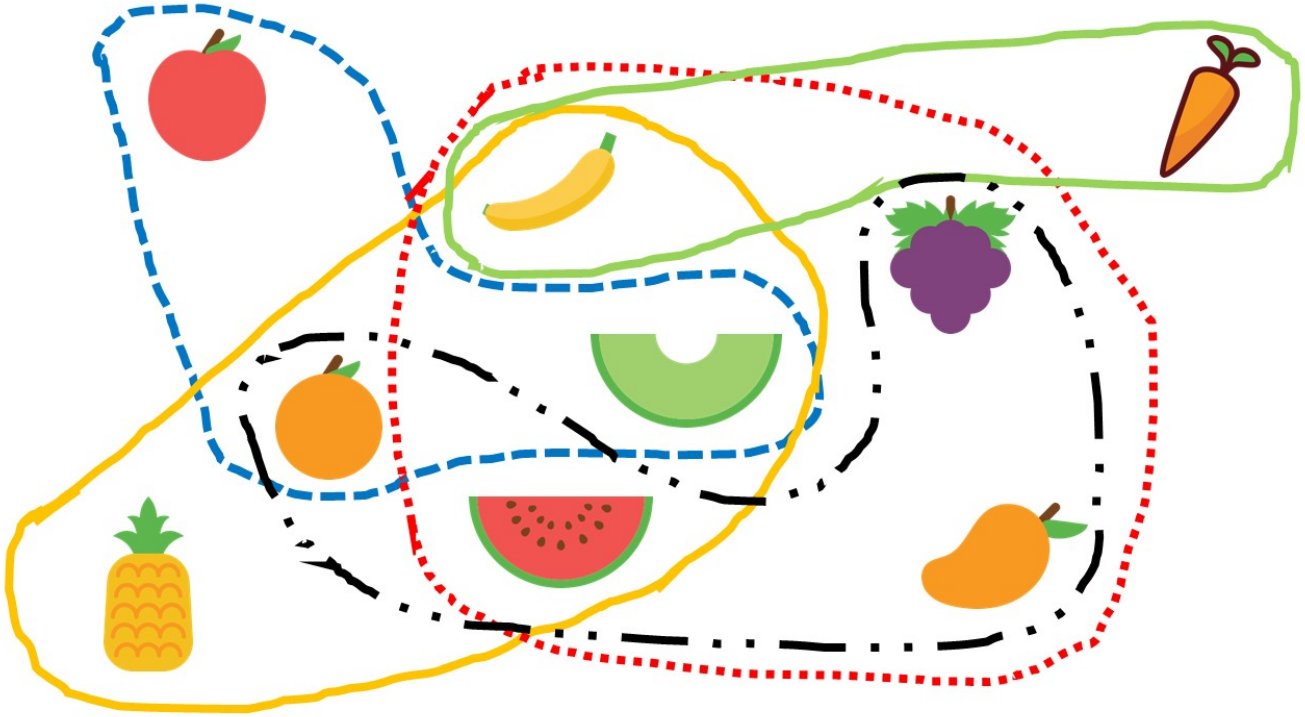

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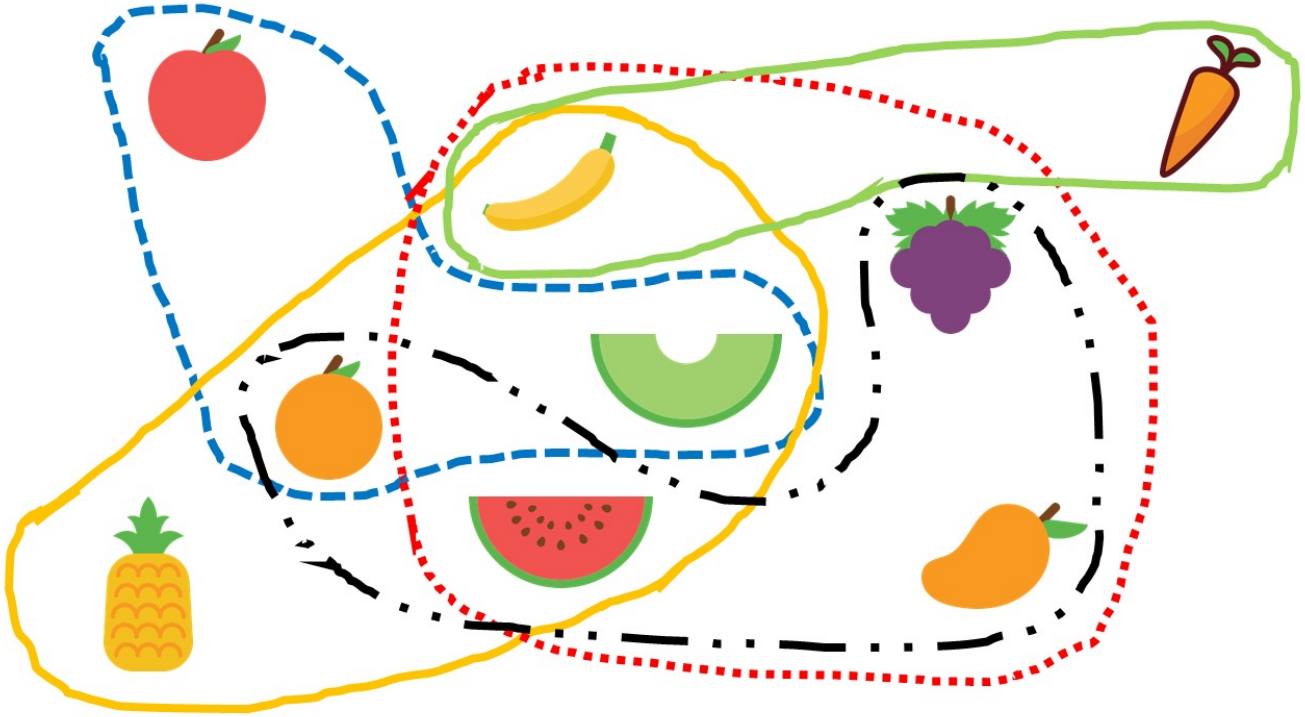
This is a generalization of the graph streaming setting.



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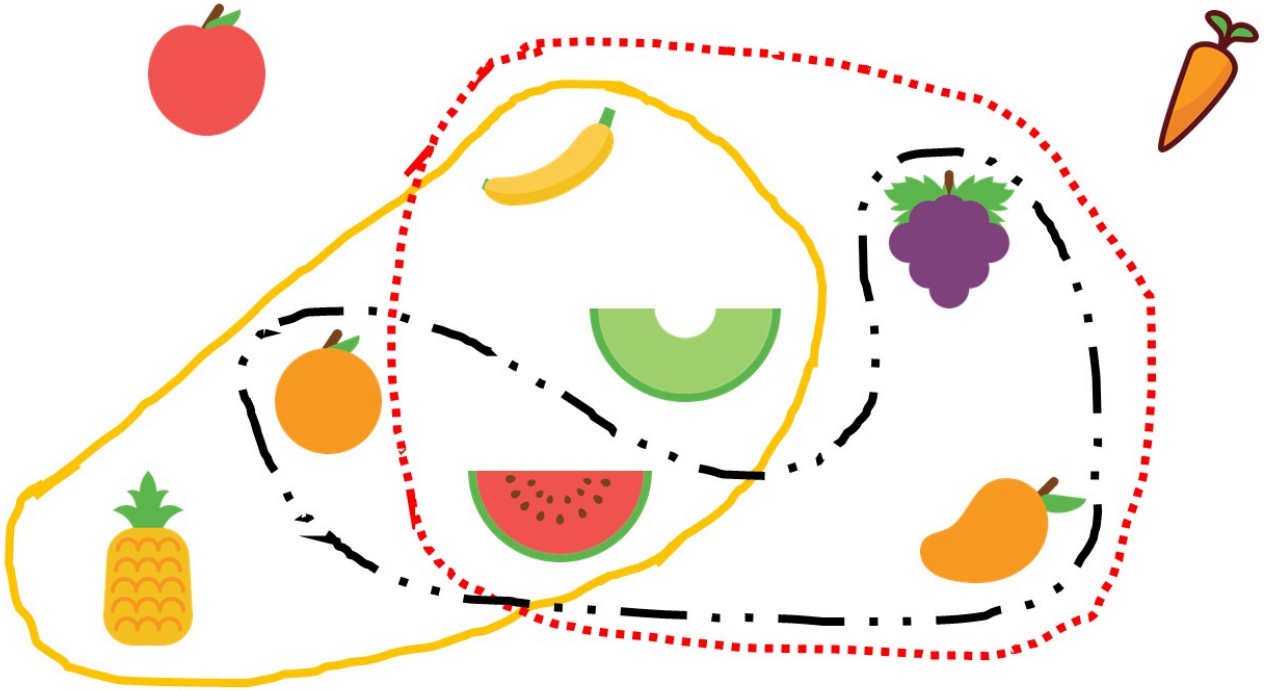
Our streaming algorithms produce *kernels*: a collection C of some (but not all) of the sets we were shown.



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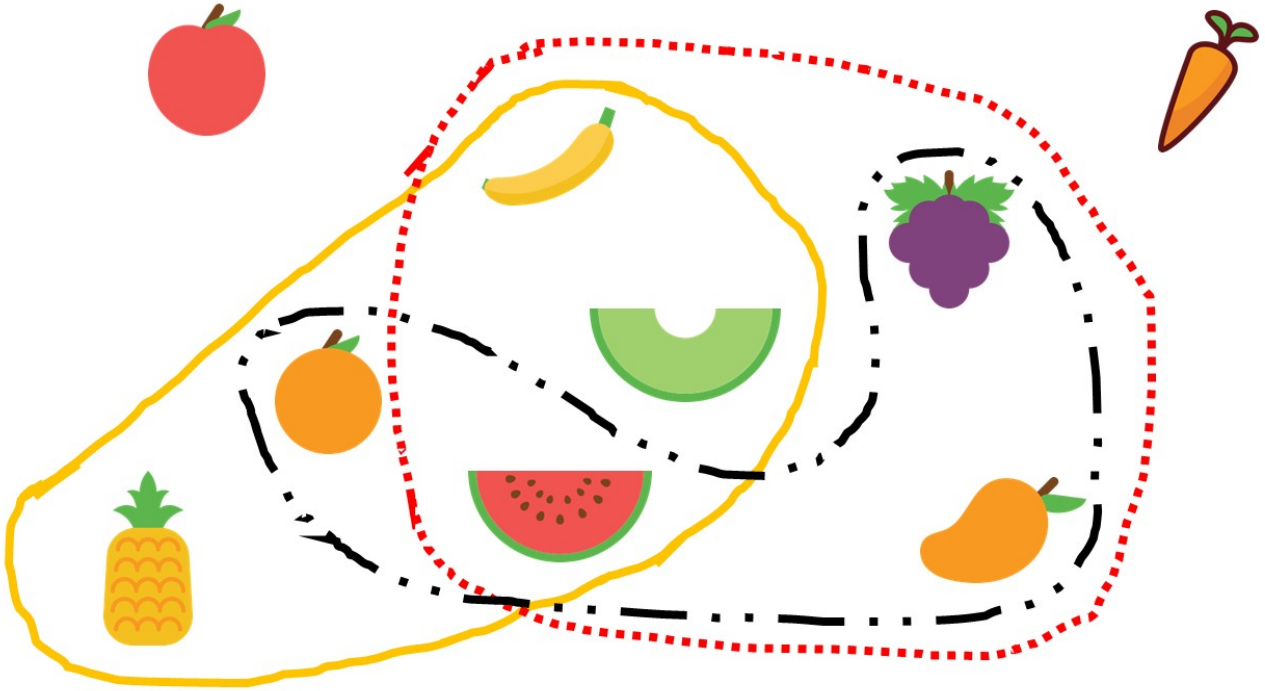
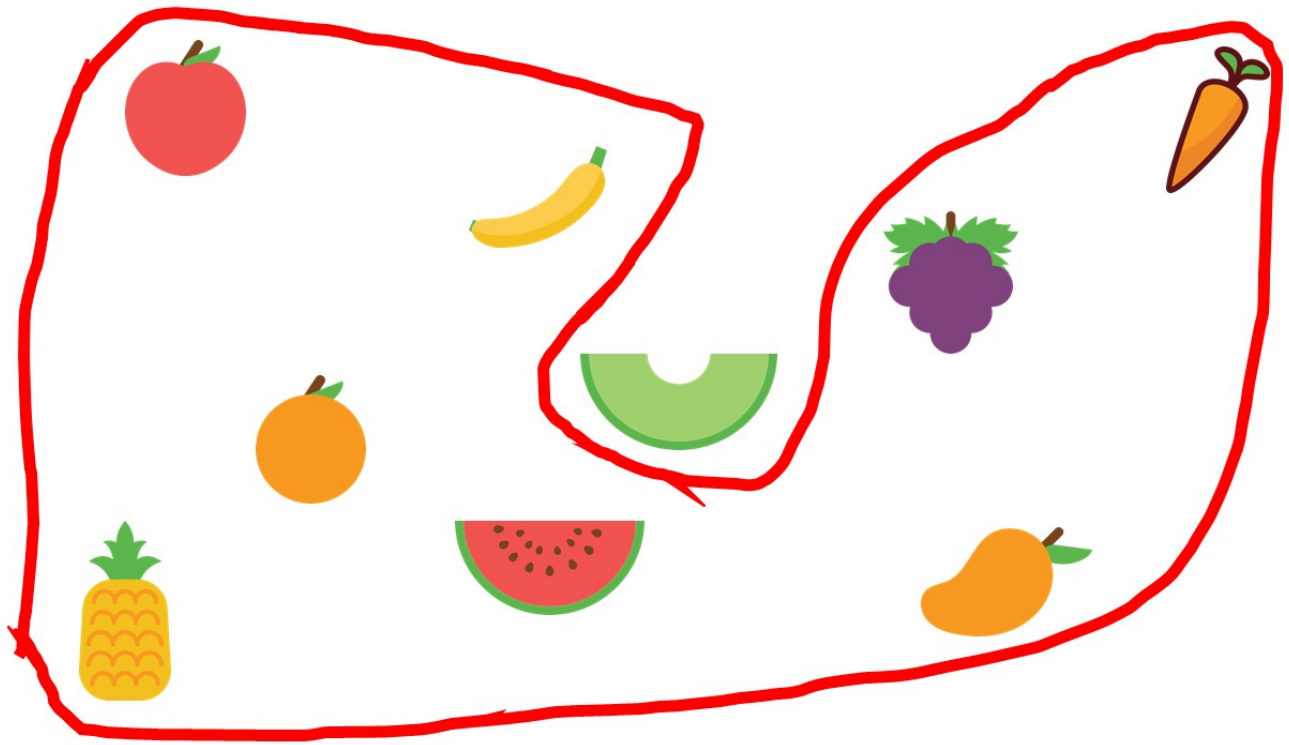
C has solutions to Max-k Cover and Unique-k Cover that are just as good (or almost as good) as the entire input.



Some structural assumptions



We sometimes assume bounded set size d

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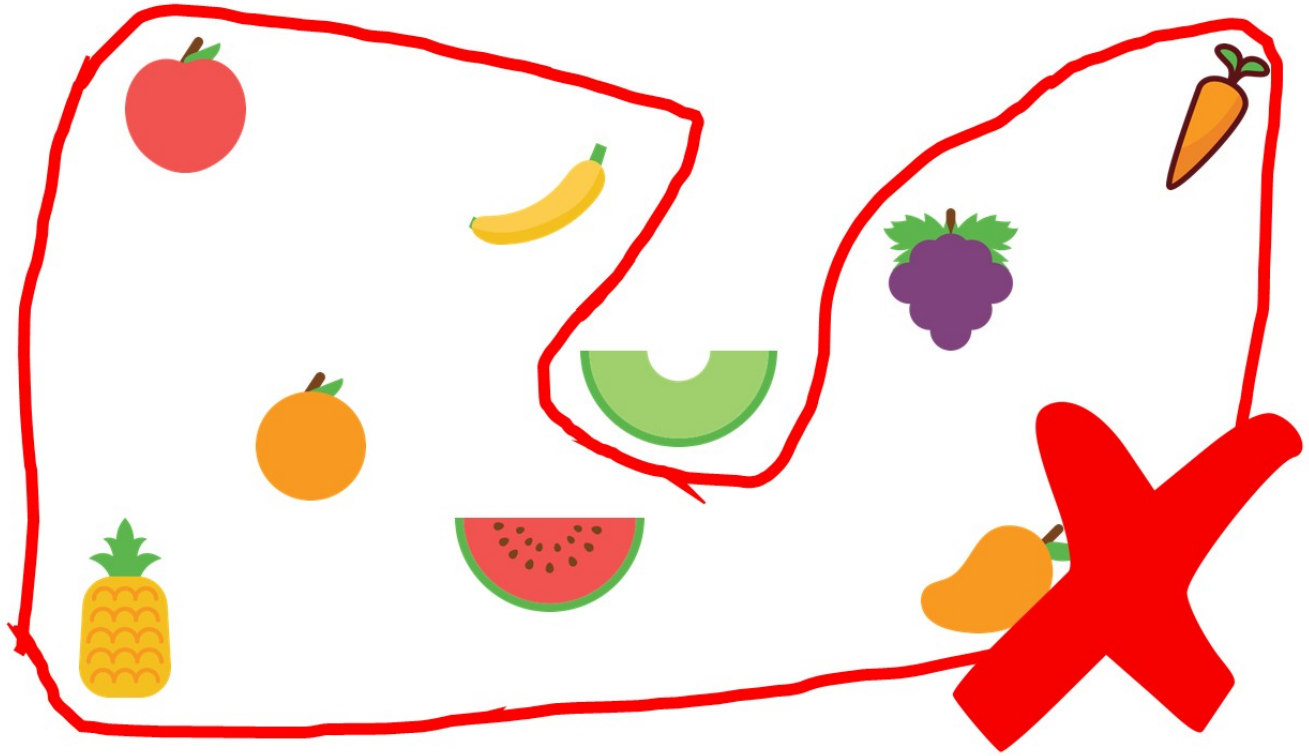

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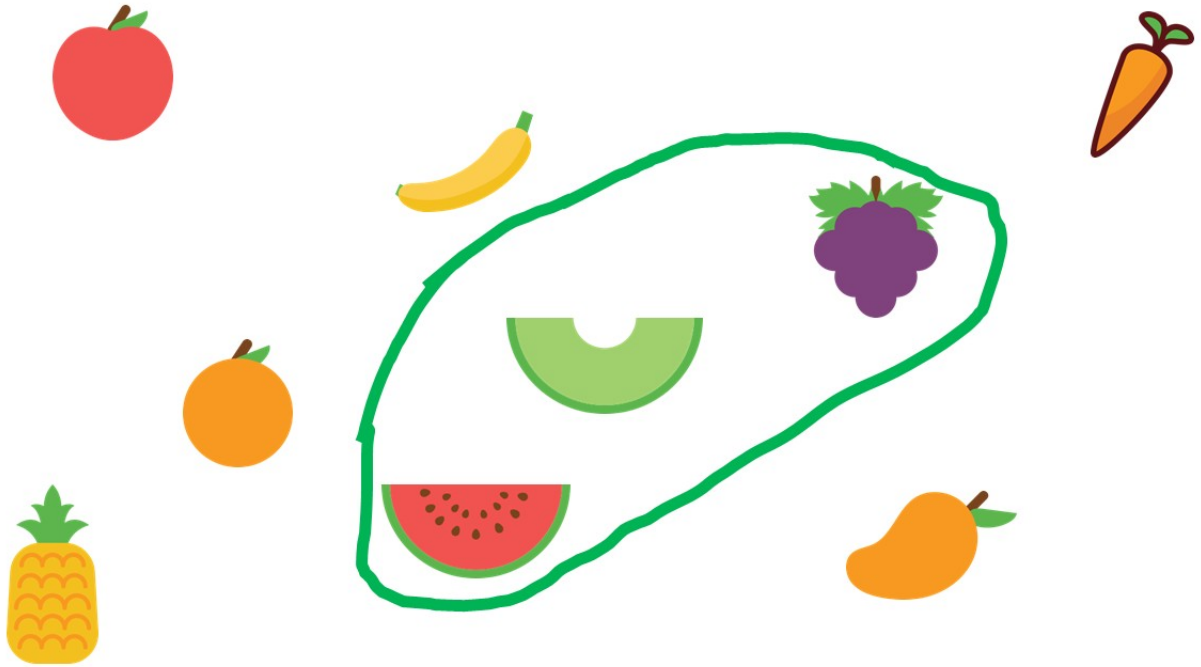

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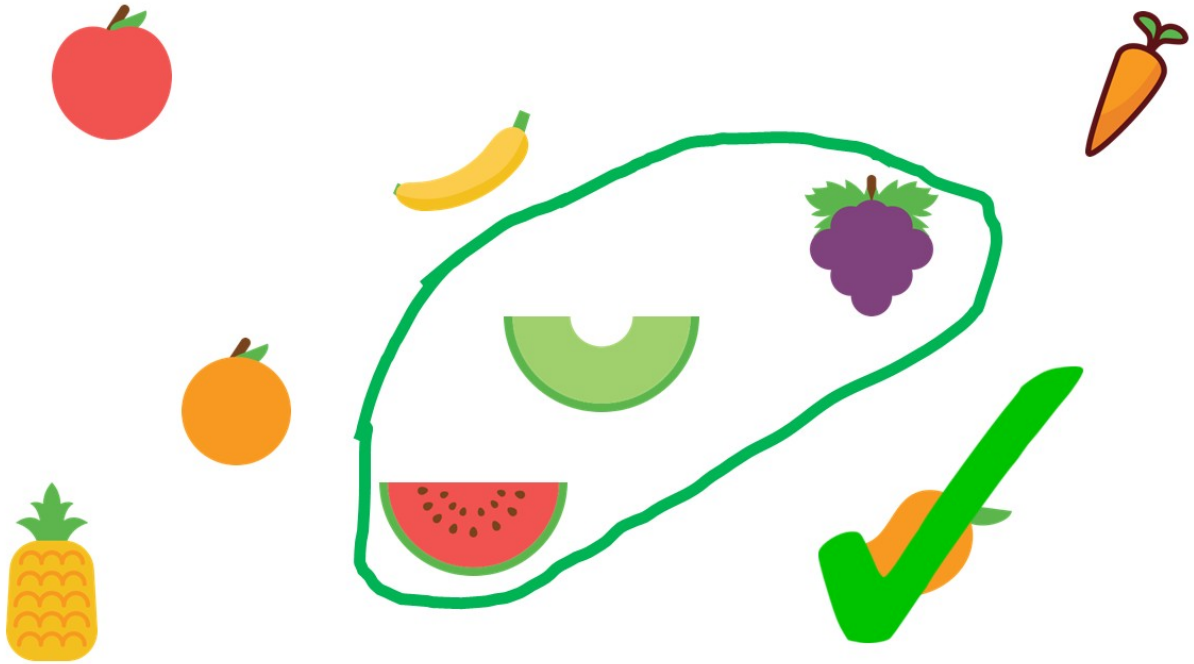

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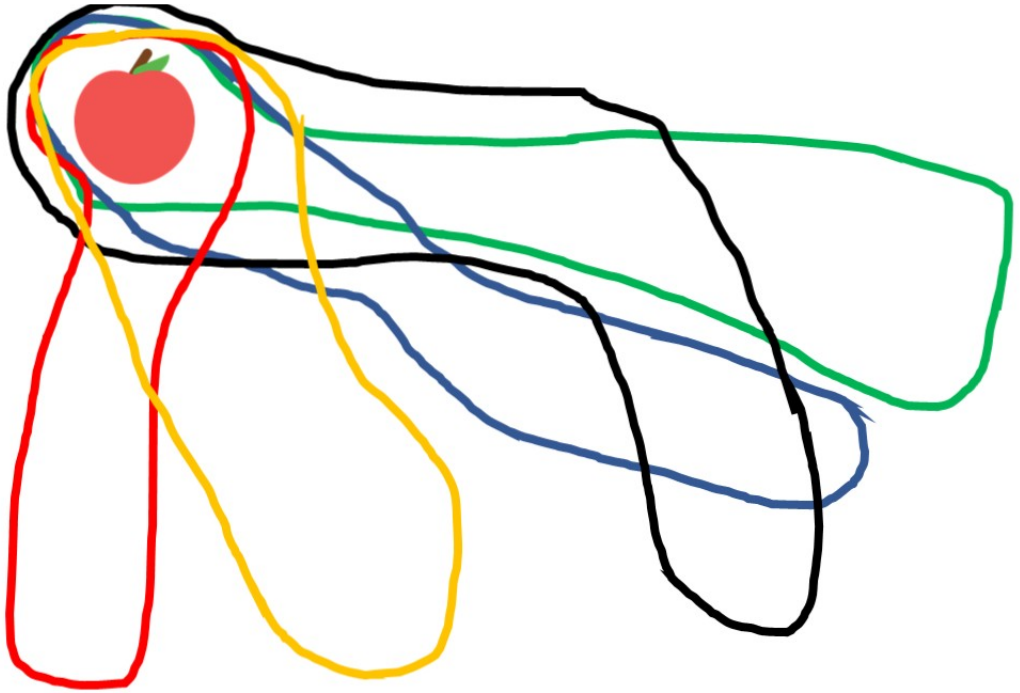

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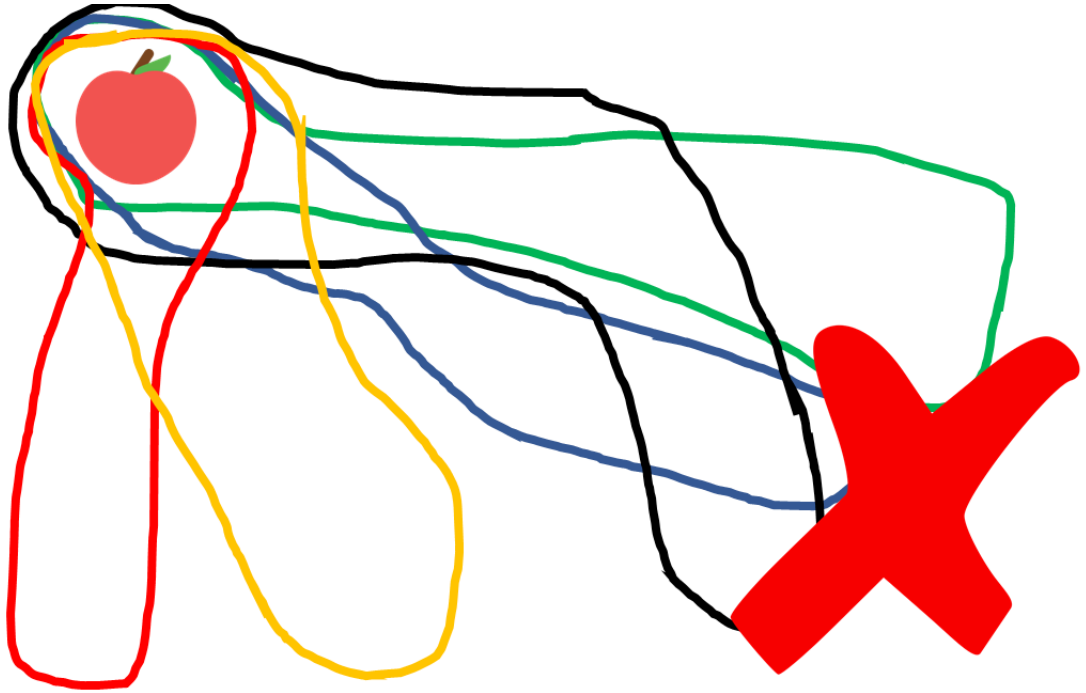

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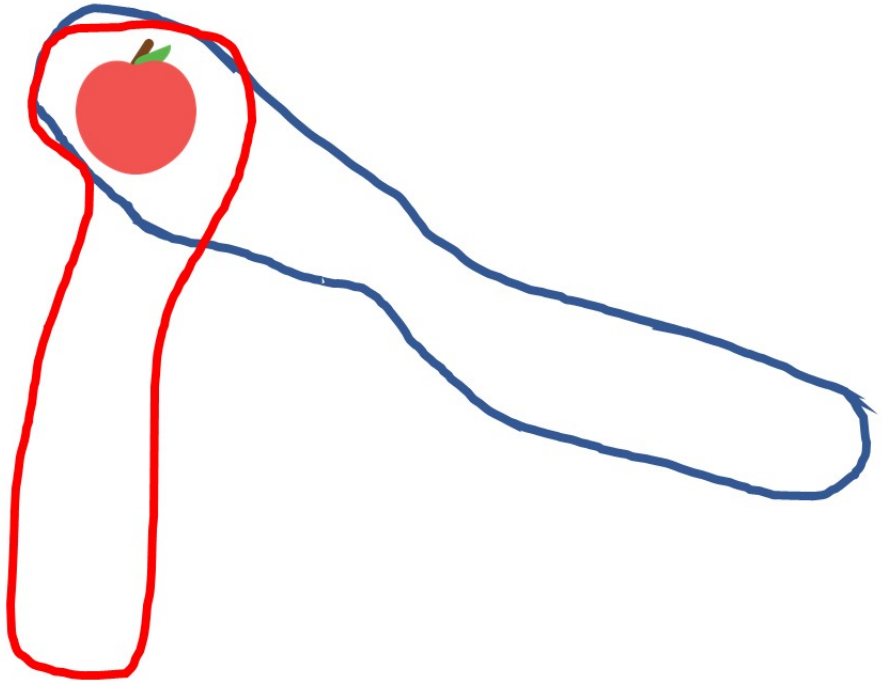

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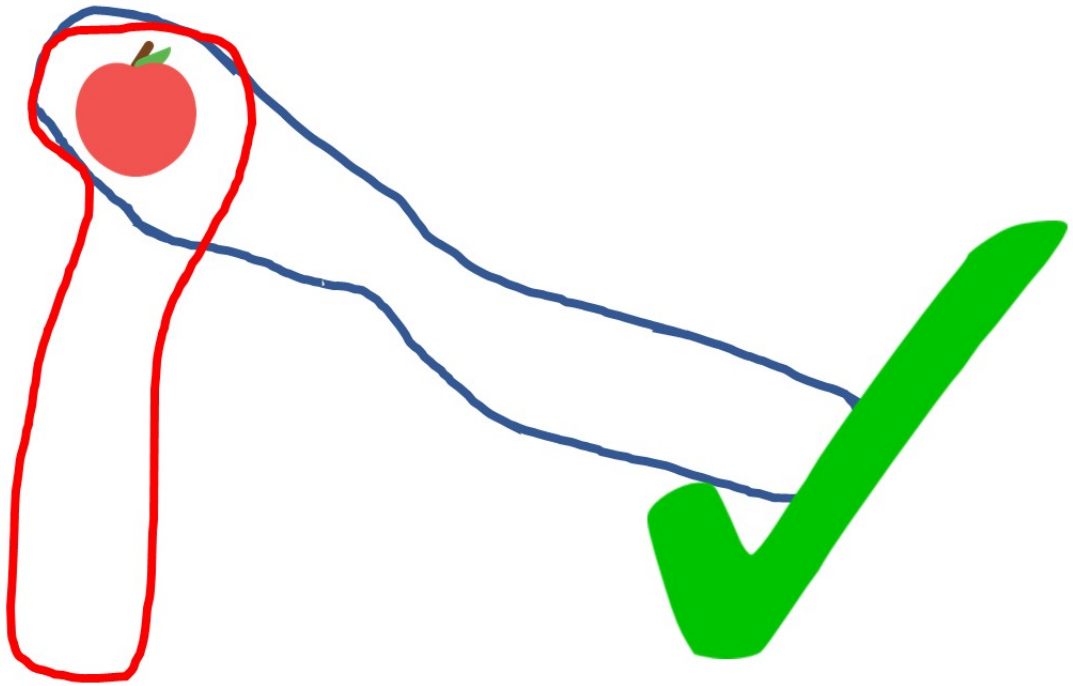

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Our Results: Algorithms

When d is bounded we:

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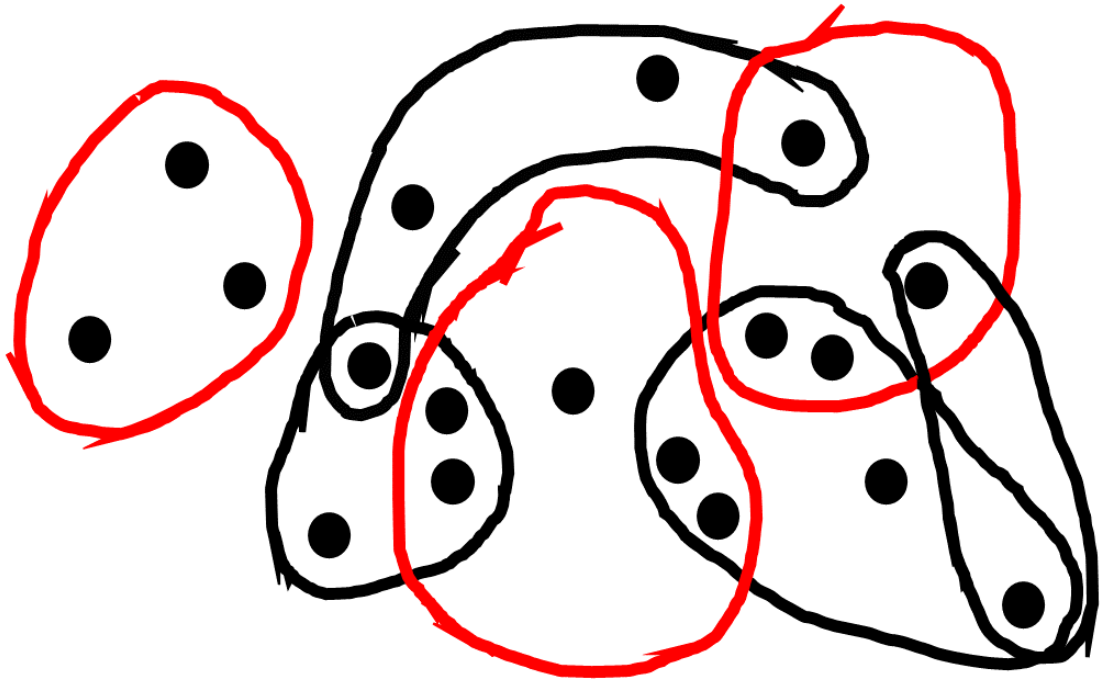
Solving Max-k Cover Exactly

Fix some maximal matching M.



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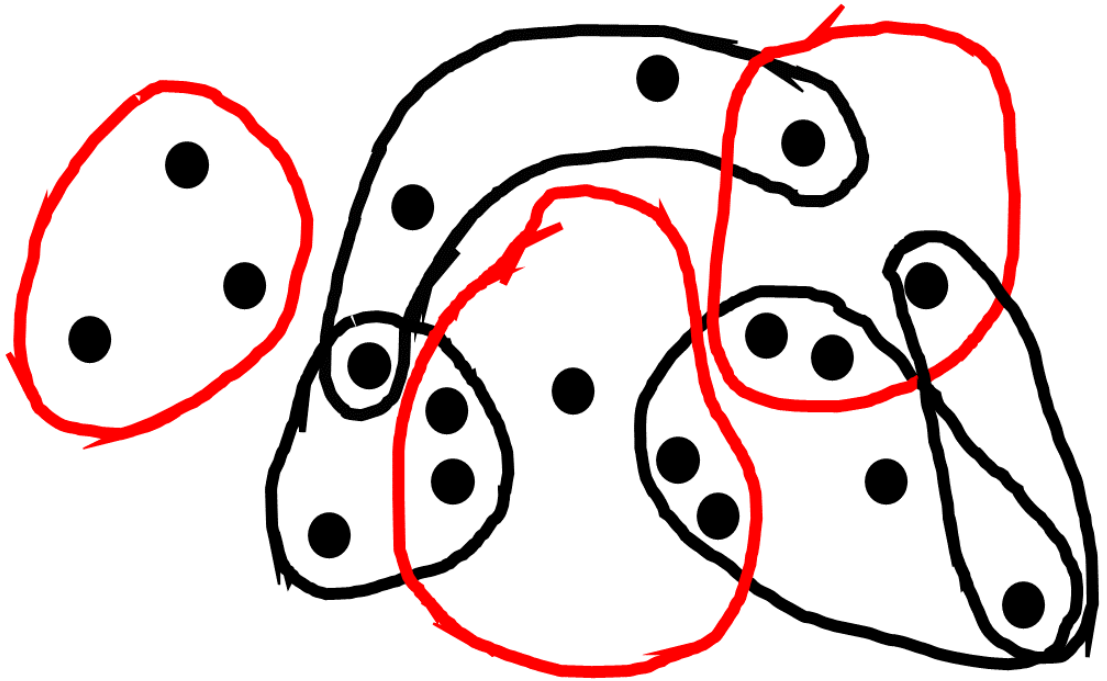

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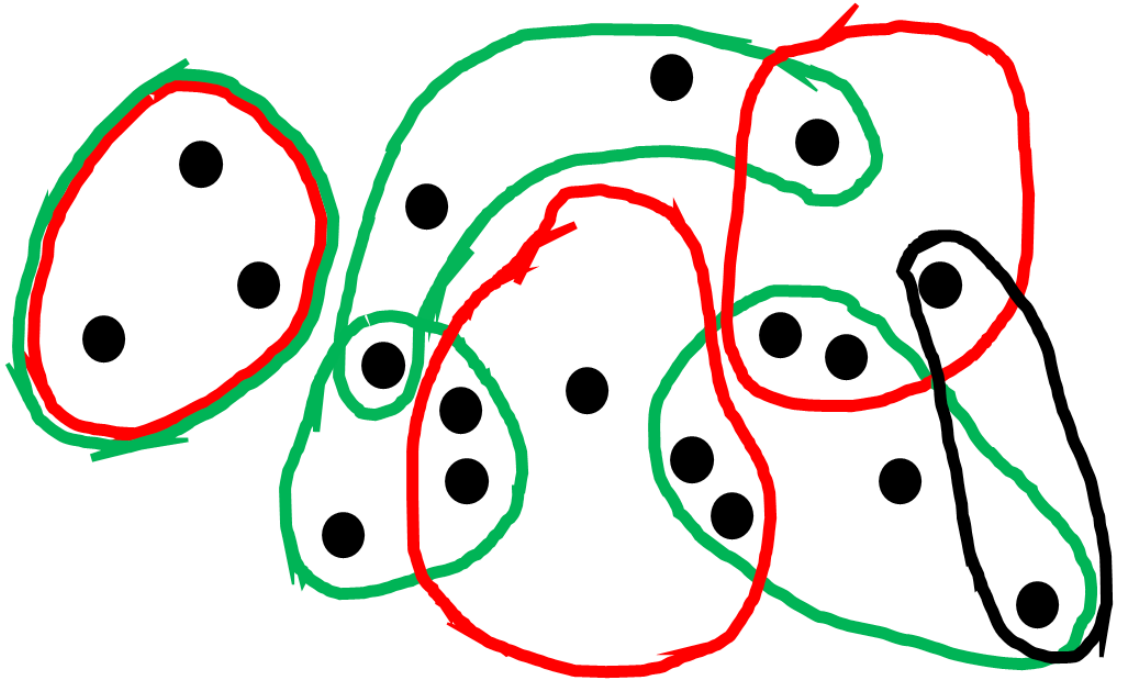

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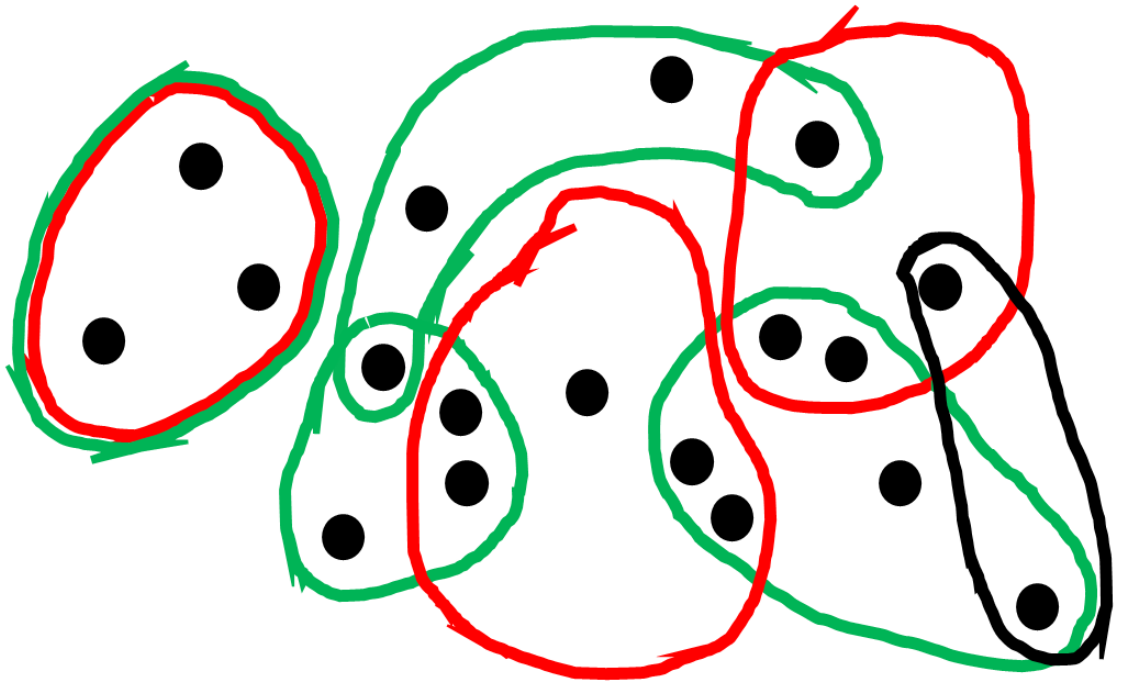
If we build M from the stream, and keep everything which overlaps M, we preserve OPT.



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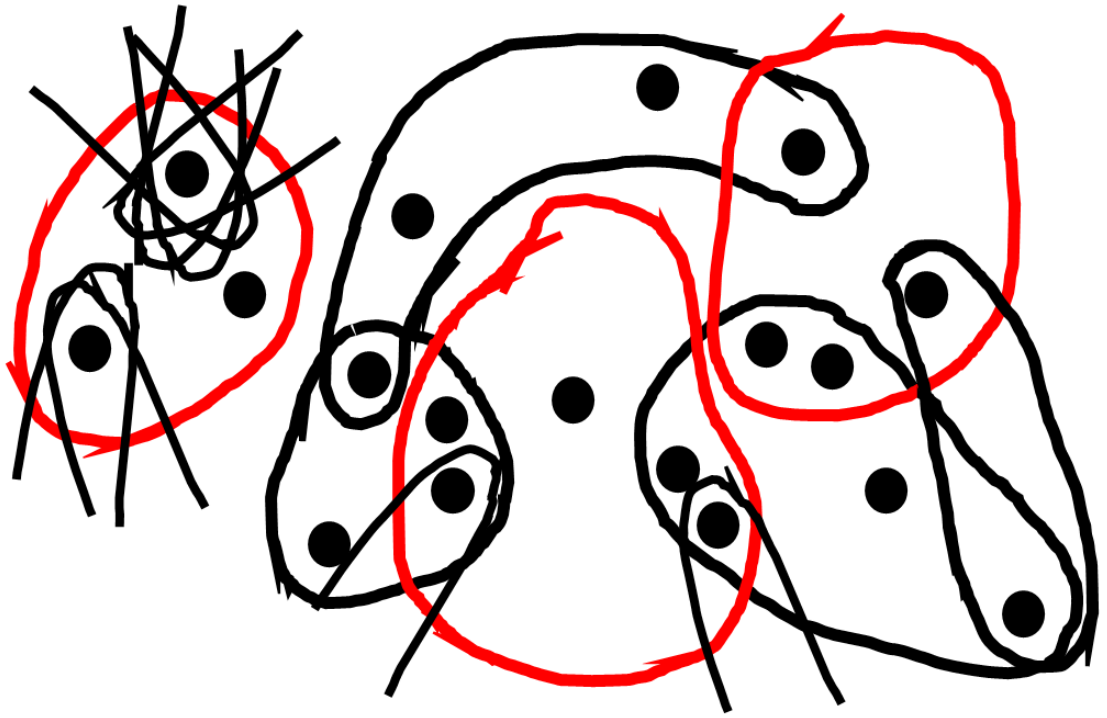
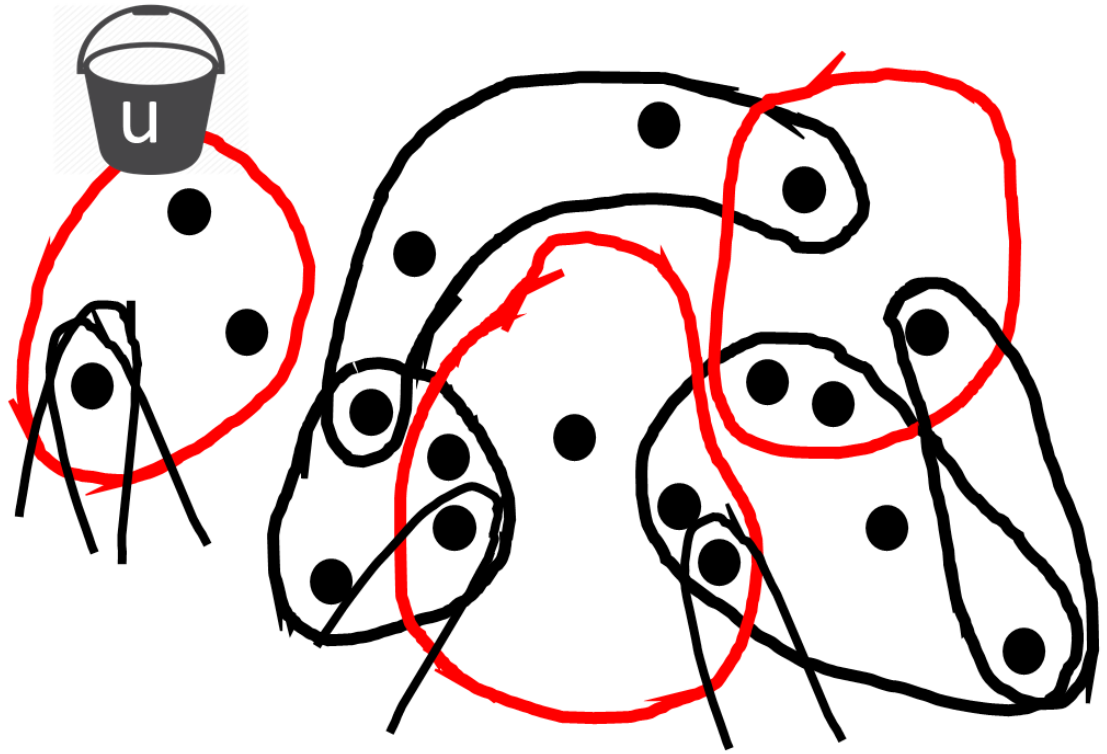
But since we assume no bound on r, Ω(m) sets can intersect with M. Too many to store!



Solving Max-k Cover Exactly

M + intersecting sets too big.

Solution: store M and a "bucket" b_u for each object u in M. When a set intersects with M at node u, we keep it unless it is too similar to the contents of b_u.



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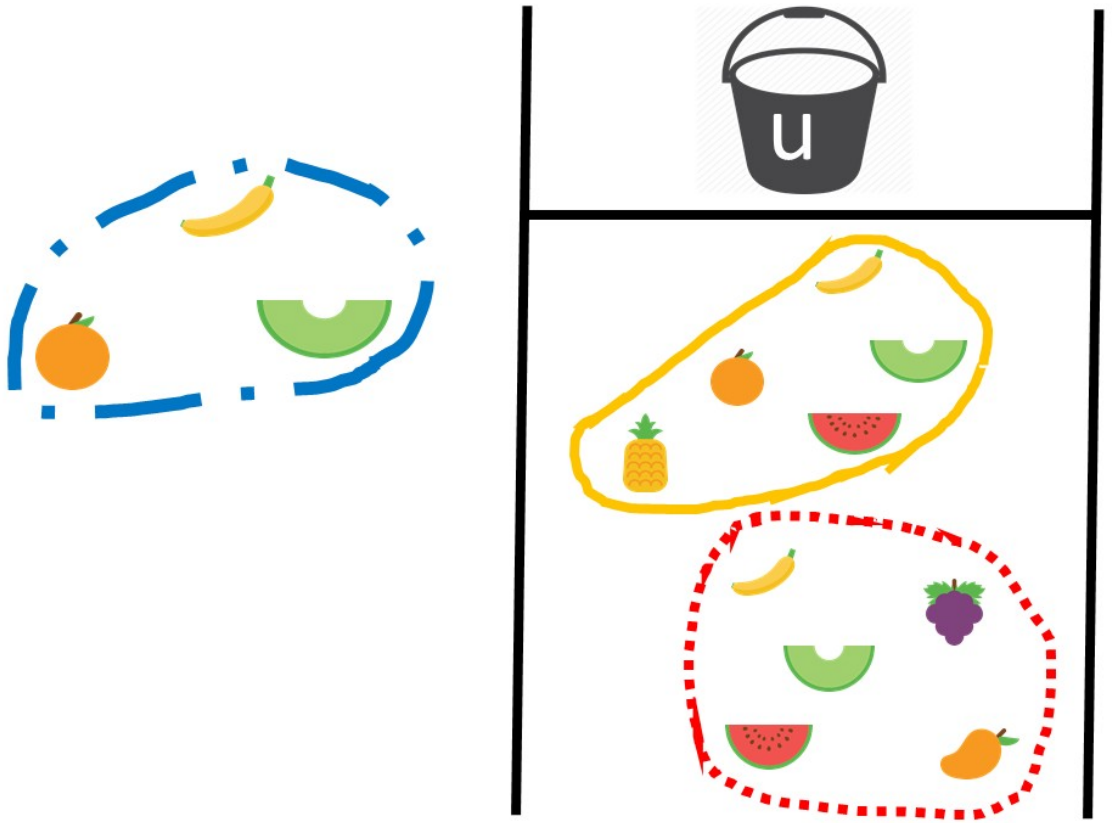

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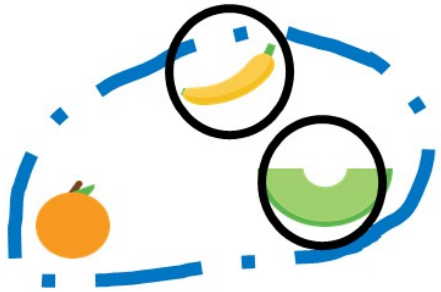

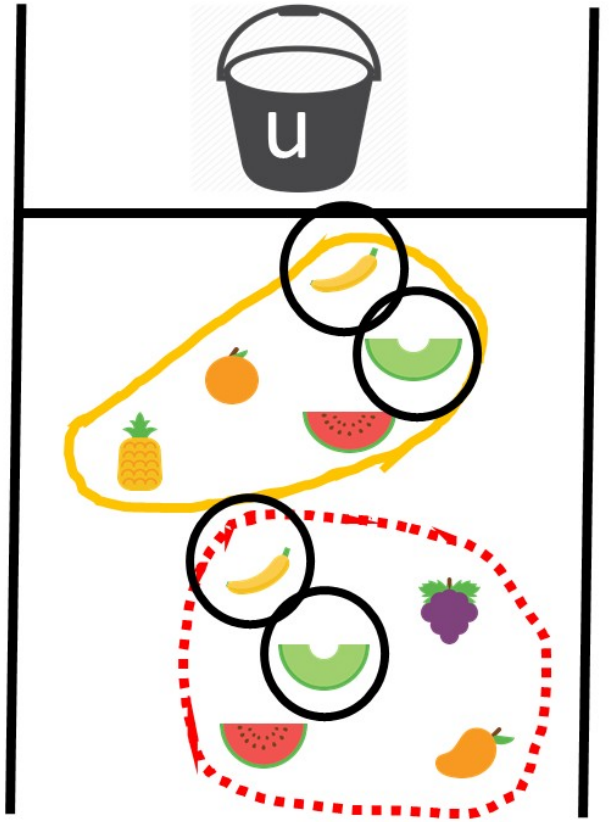

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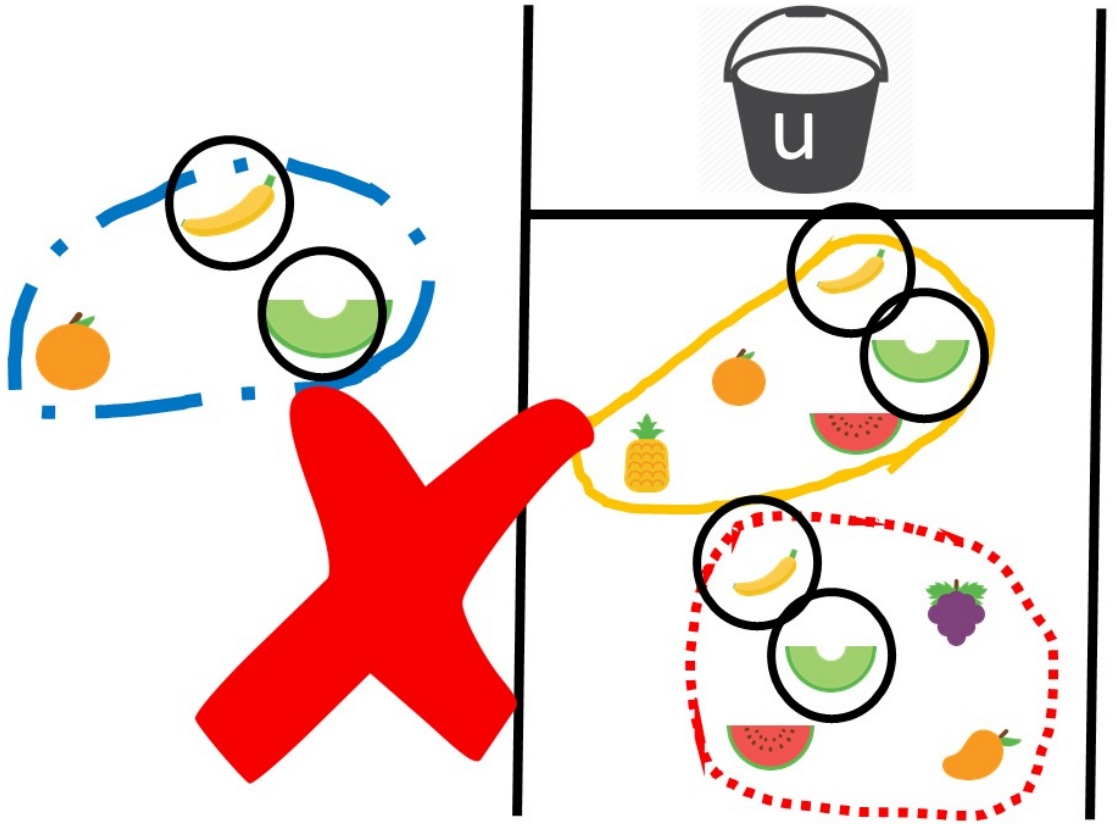

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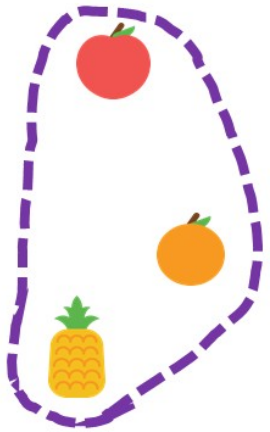

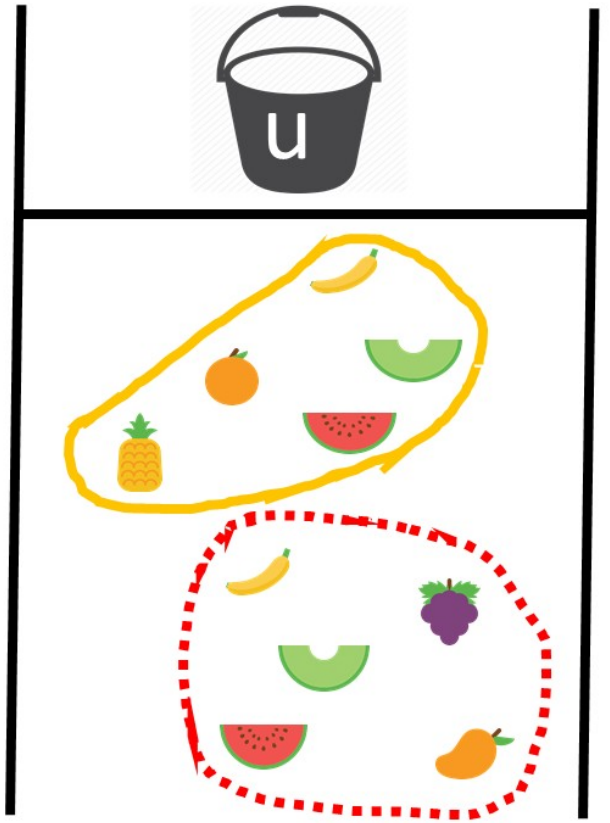

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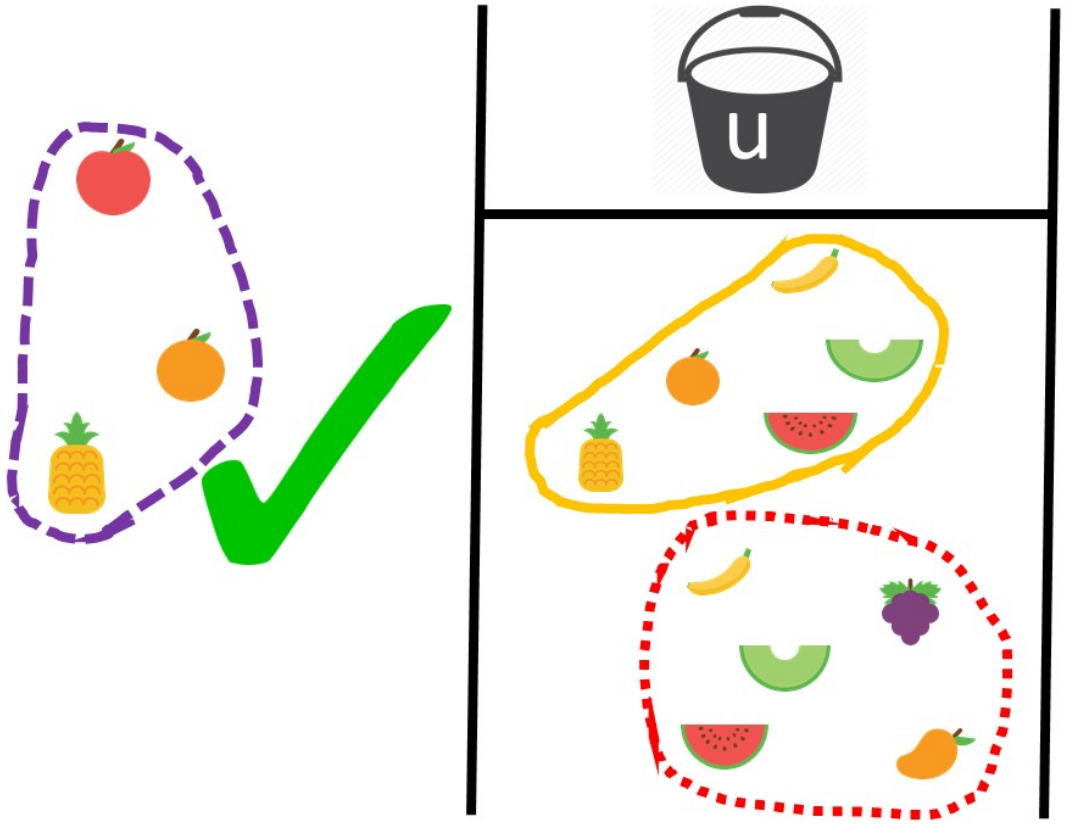
Intuition: we only discard sets when there's a similar set in the bucket that can substitute for it.



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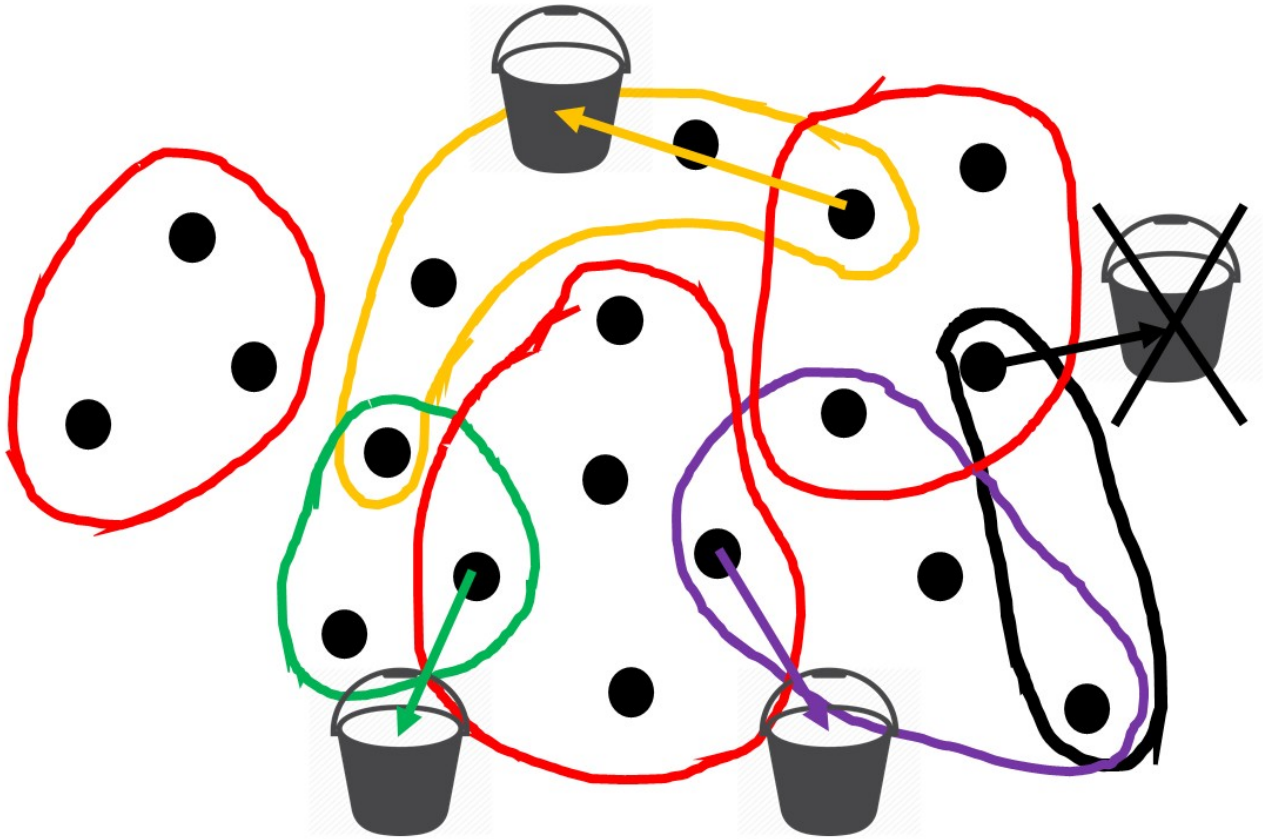

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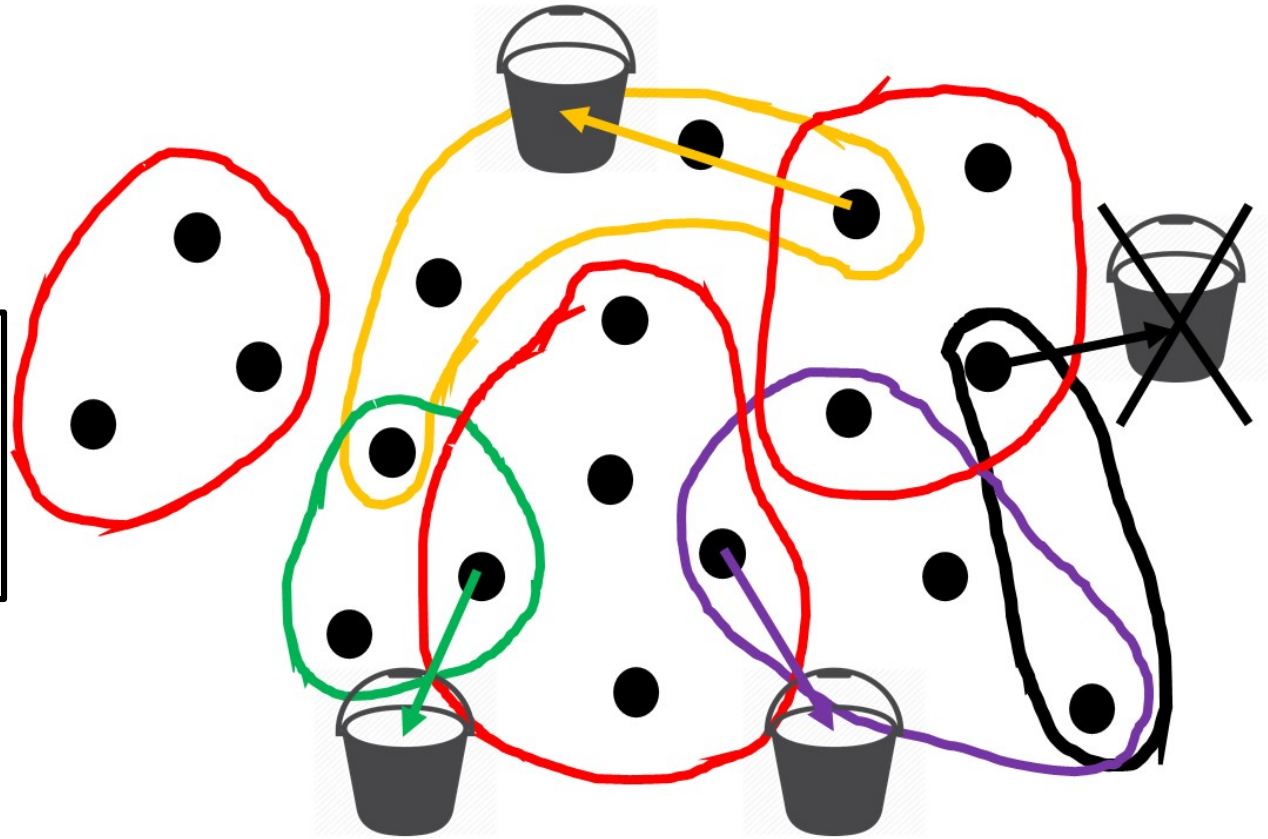

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Note: assume that all sets have size exactly d.

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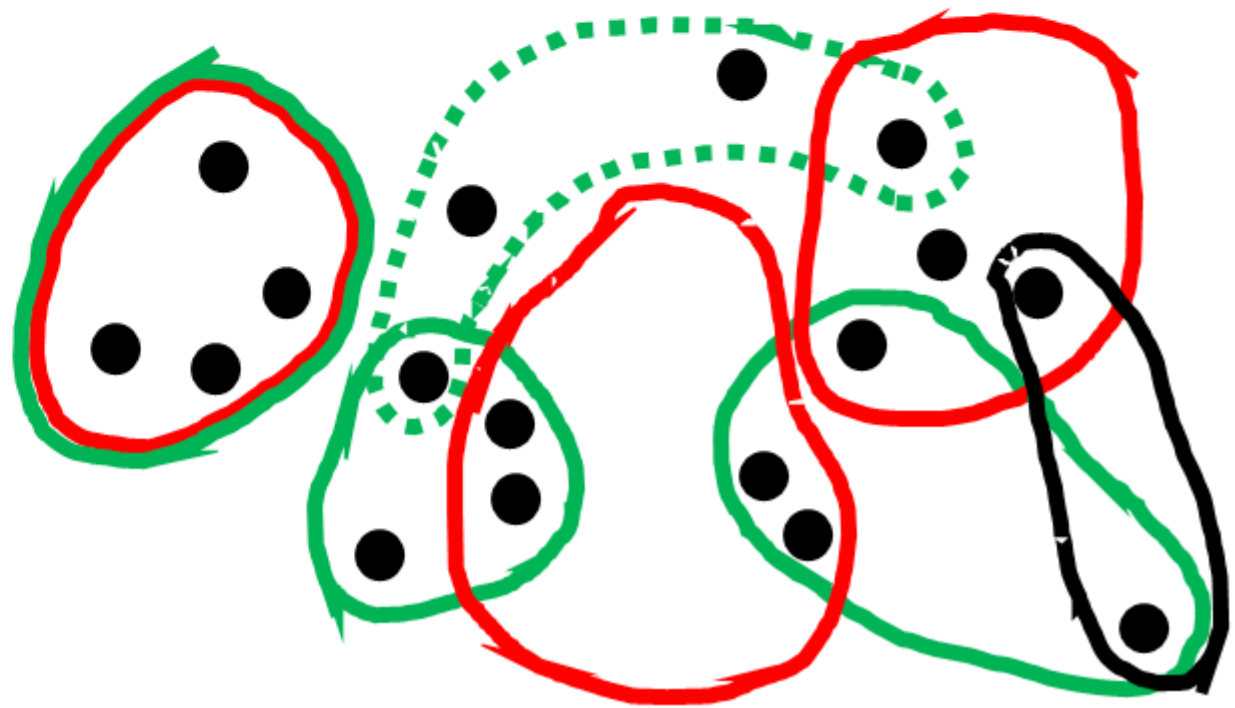

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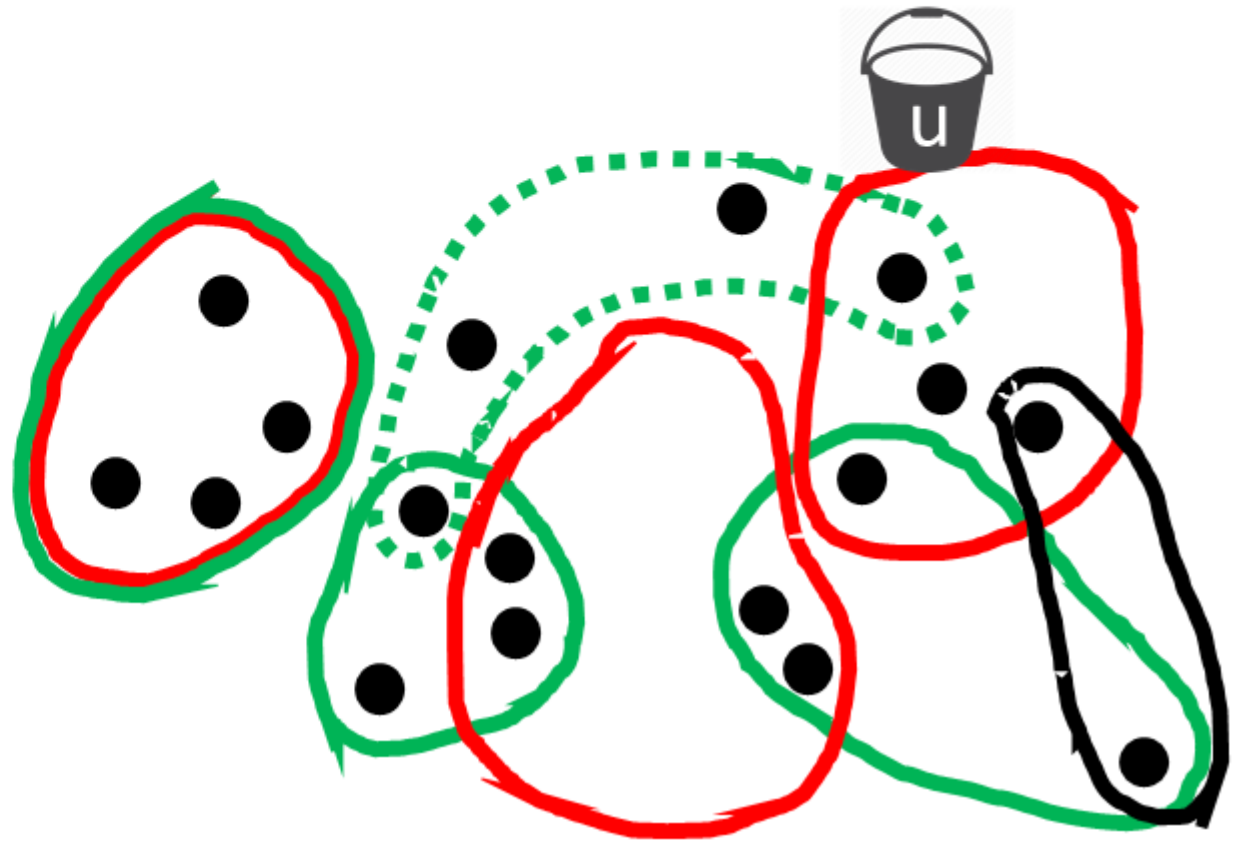

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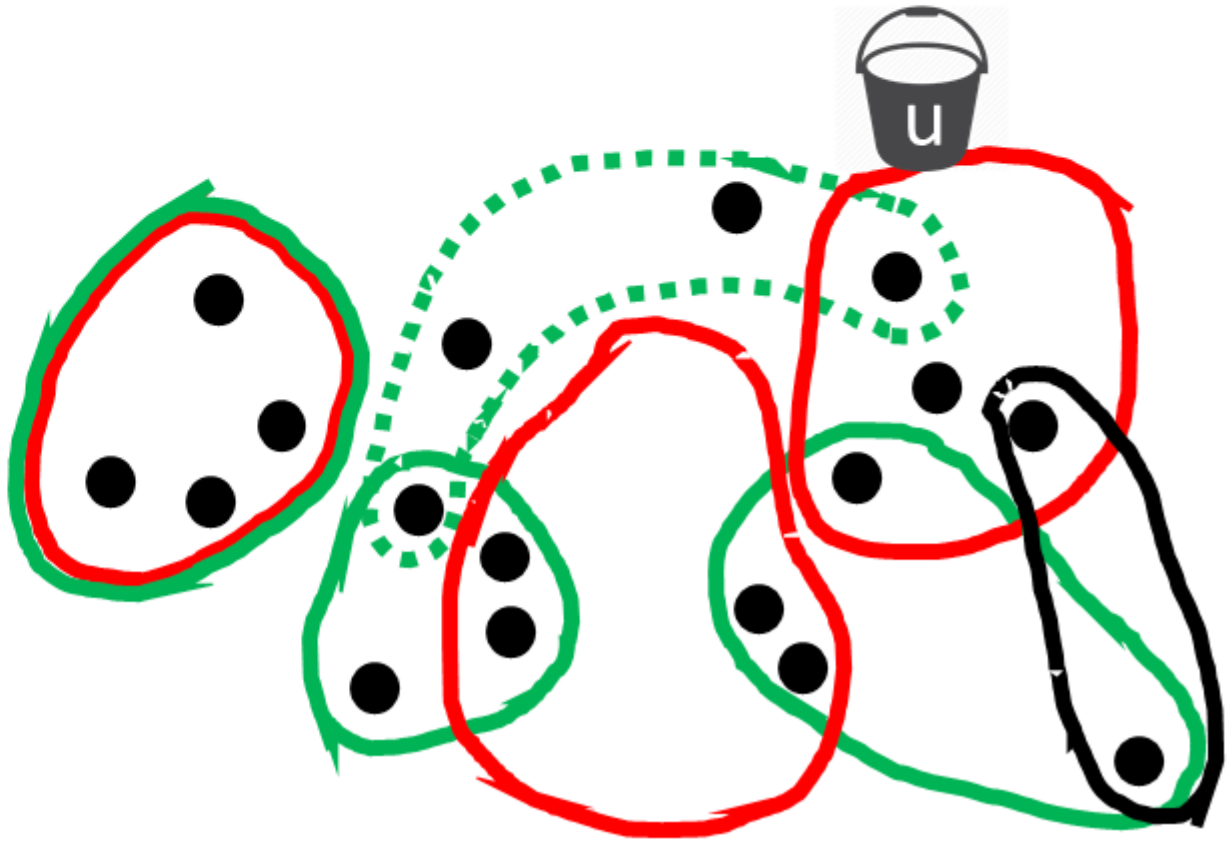

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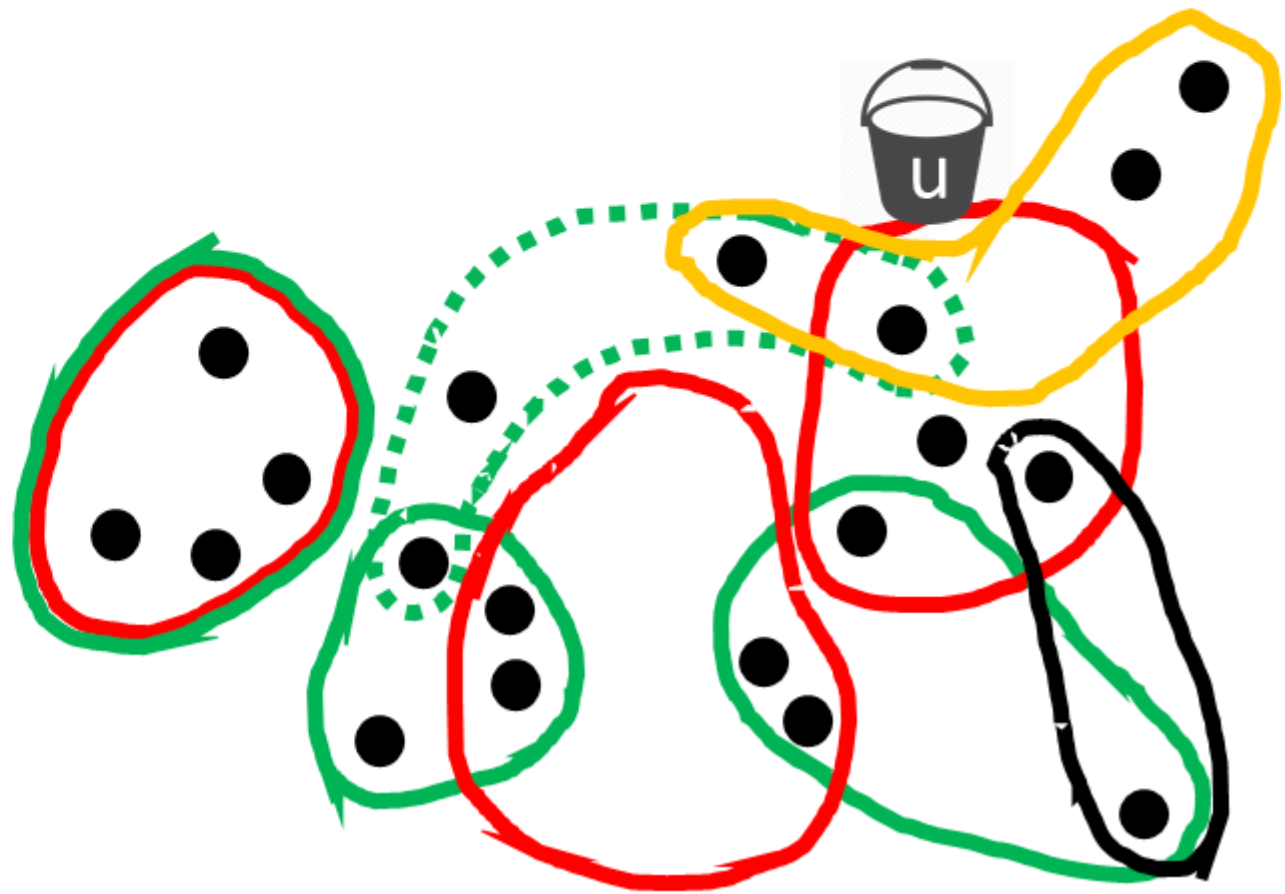
Lemma: $X = b_u, B = OPT \backslash\{S_i\}$, then b_u has replacement S': $|S' \cap B| = 0$.



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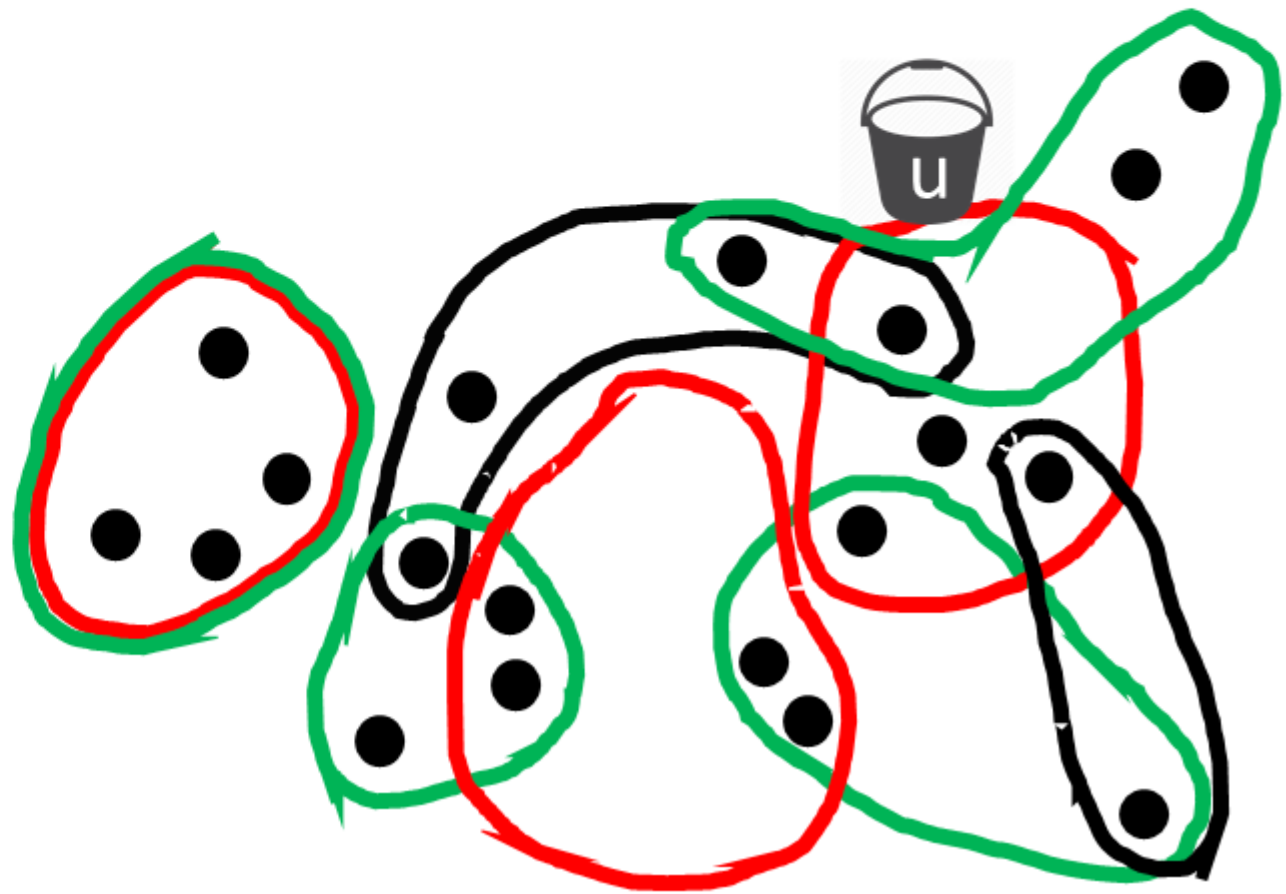
Since $|S' \cap B| = 0$, we can replace OPT with $(OPT\backslash\{S_i\}) \cup \{S'\}$. This new solution is just as good.



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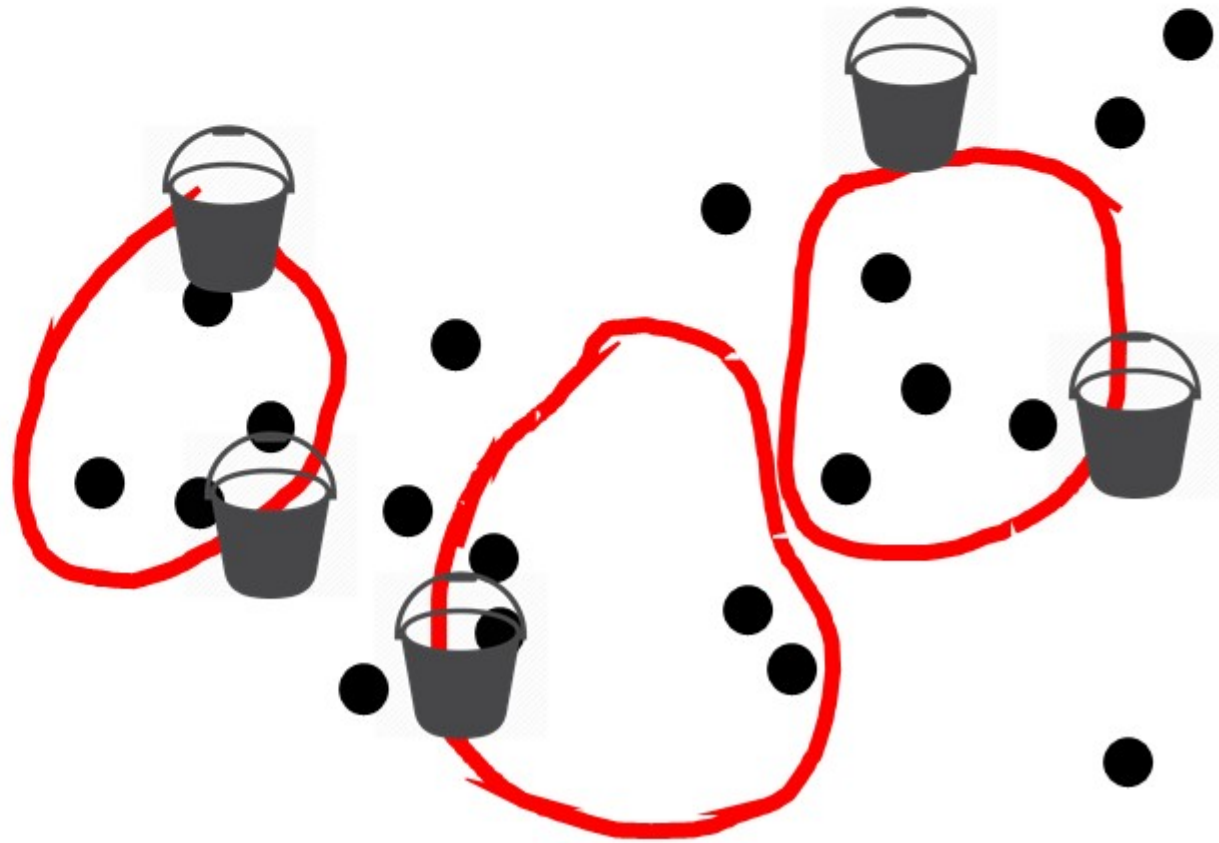
If there is T $\subset S\backslash\{u\}$ that already appears in ℓ_T sets in b_u, discard S. b_u has similar sets already.

We kept matching M and some number of buckets. How many edges in total?



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$$\ell_T := (d(k-1)+1)^{d-1-|T|}$$

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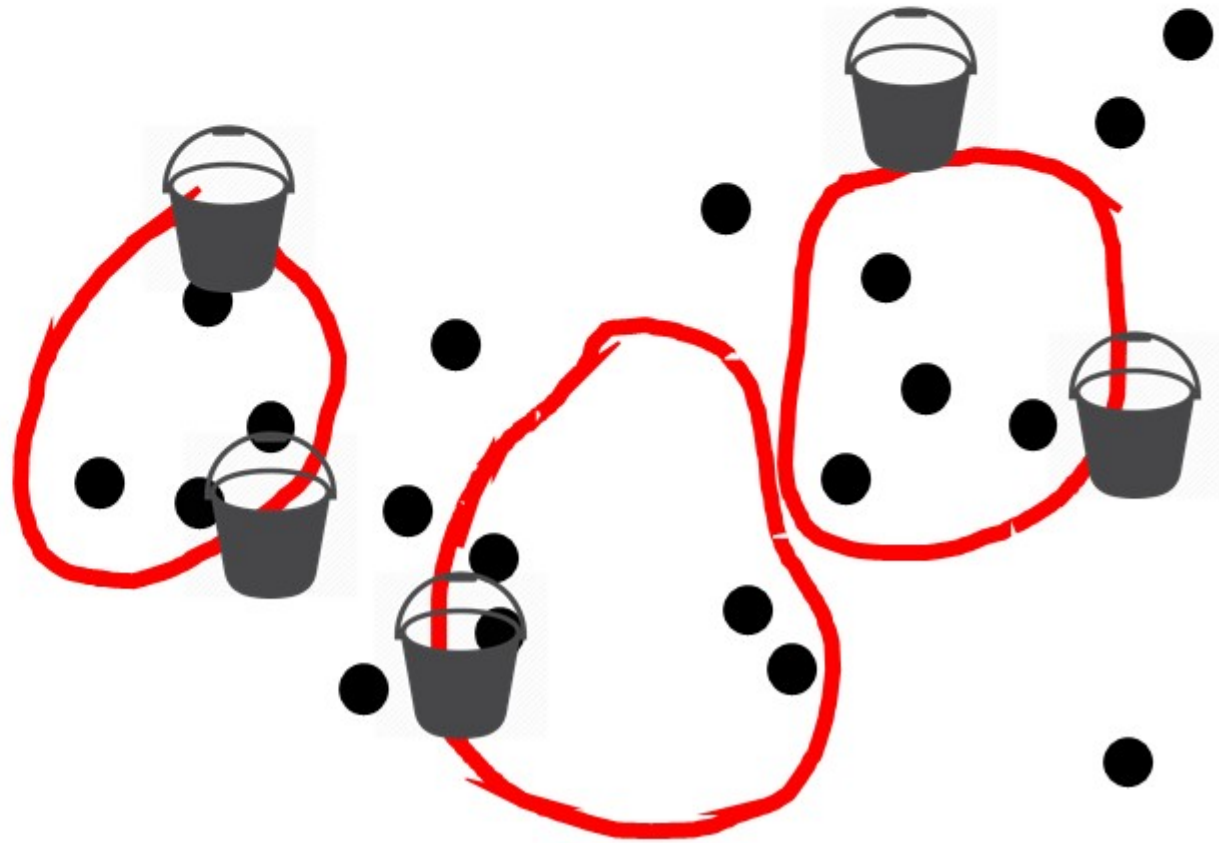

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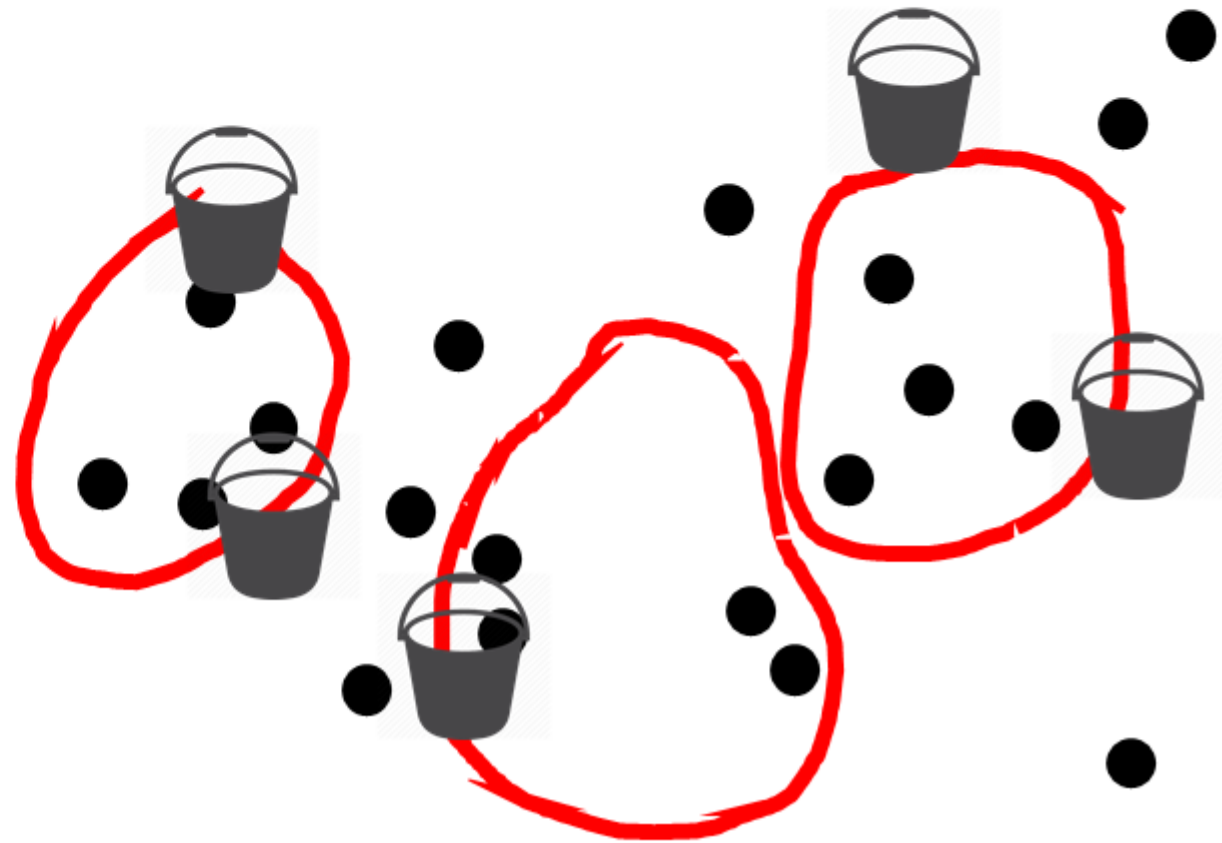
Each bucket can have at most

$\ell_0 = (d(k-1)+1)^{d-1-0}$

$= O((dk)^{d-1})$ edges because every edge has the subset {}.

We kept matching M and some number of buckets. How many edges in total?



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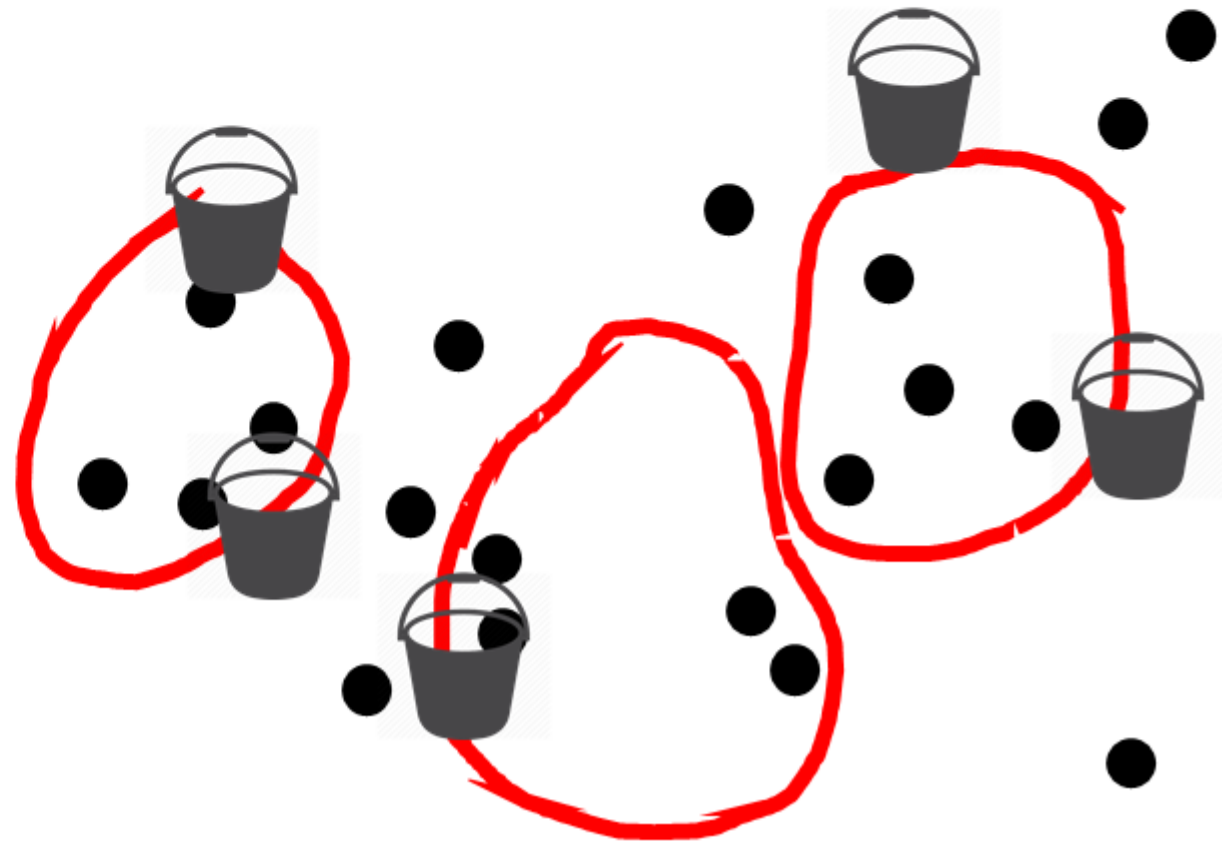

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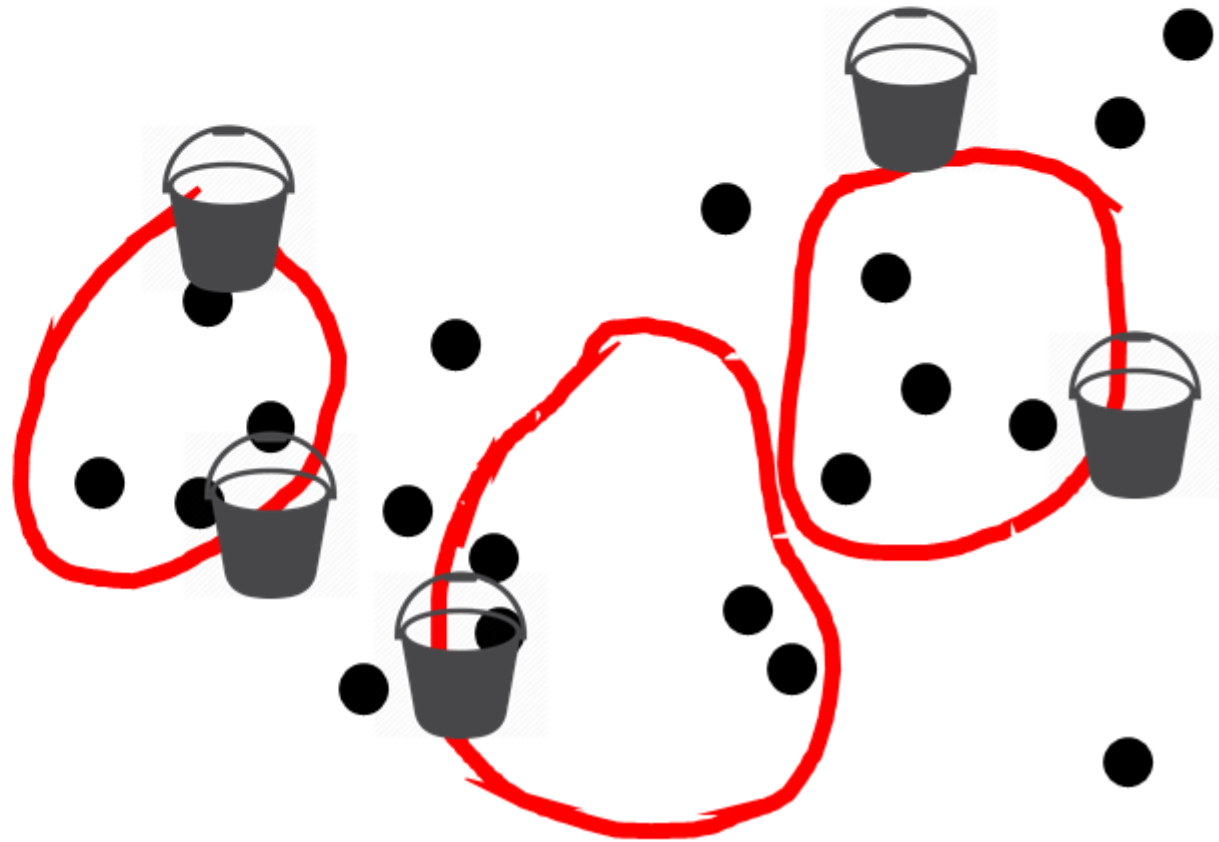
So we keep

$|M| + dk\, O\left((dk)^{d-1}\right) = O((dk)^d)$

edges.

d nodes per edge means total space is $O(d^{d+1}k^d)$.

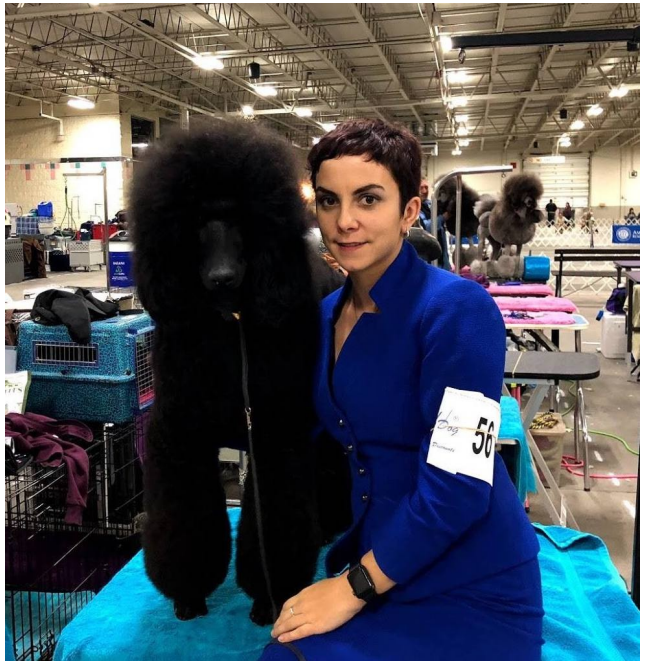


PathCache

EFFICIENTLY MAPPING THE INTERNET

Joint work with:

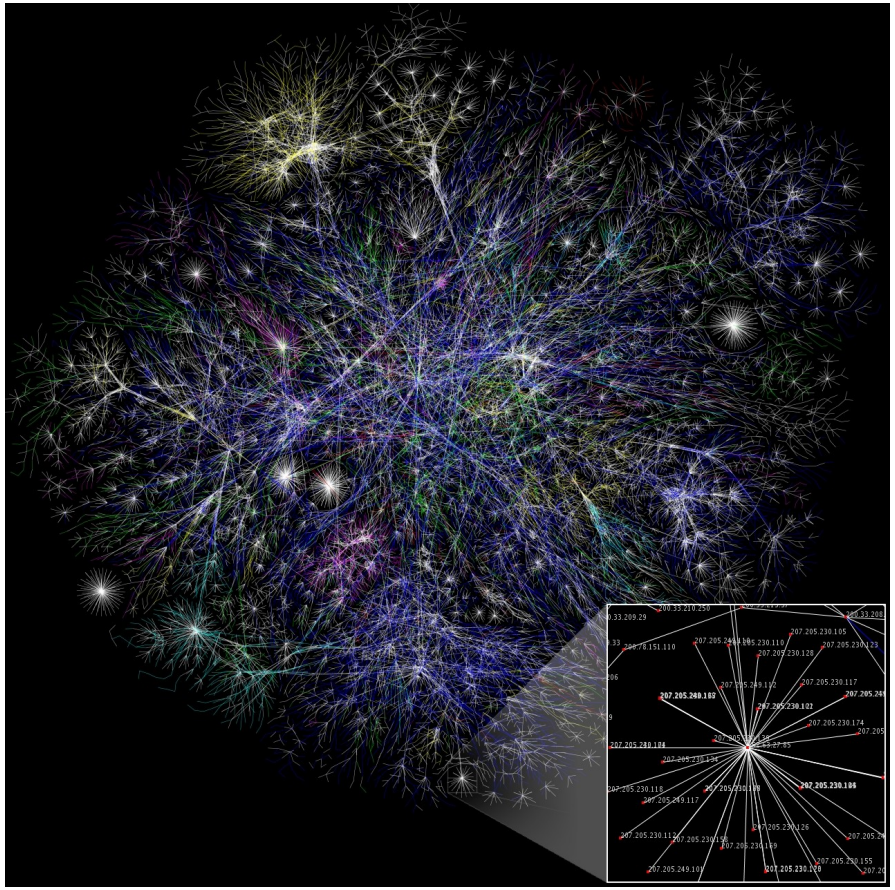
Andrew McGregor



Phillipa Gill

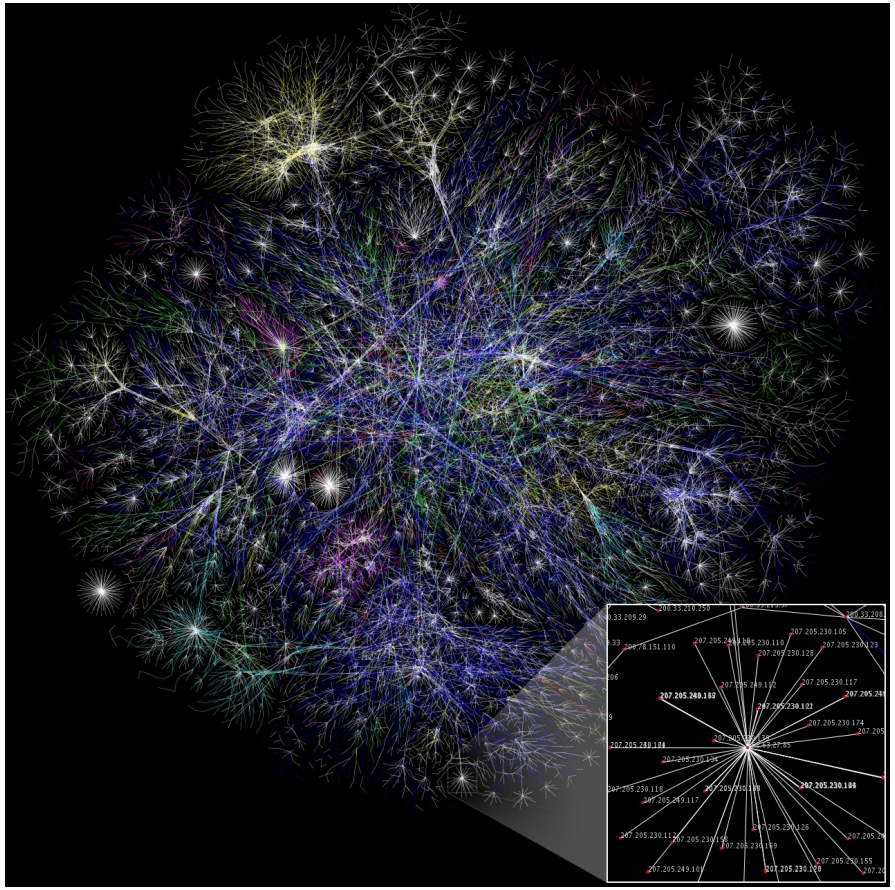
Rachee Singh

Understanding Internet Paths



Want to map the Internet to predict paths. Useful for:

Understanding Internet Paths



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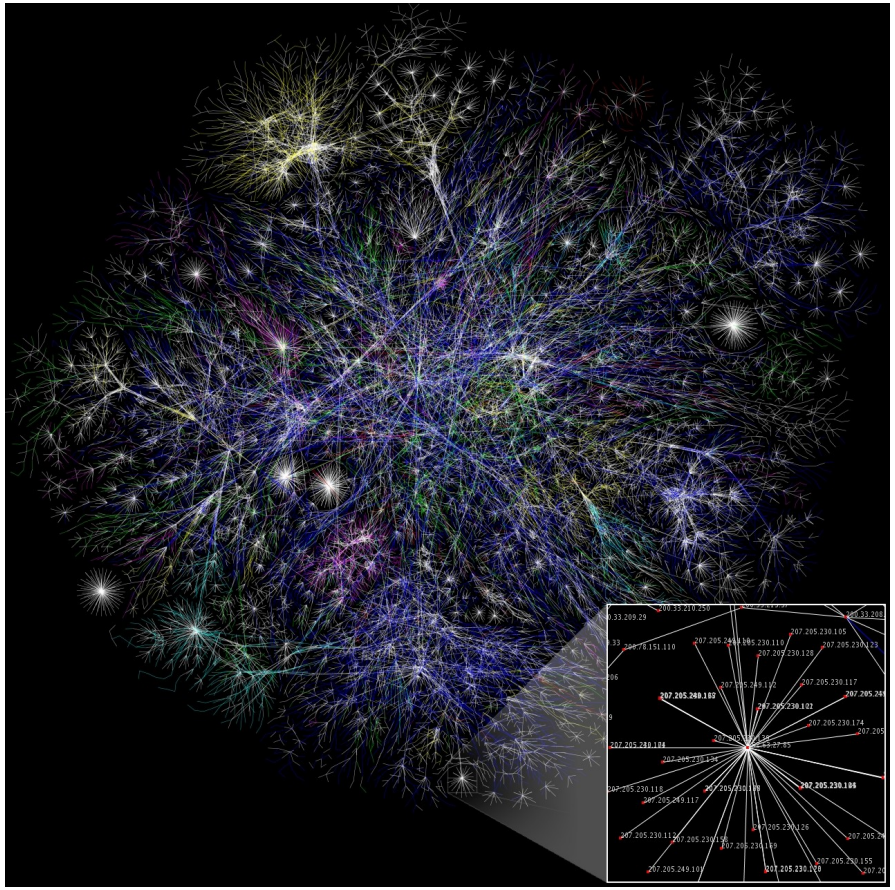

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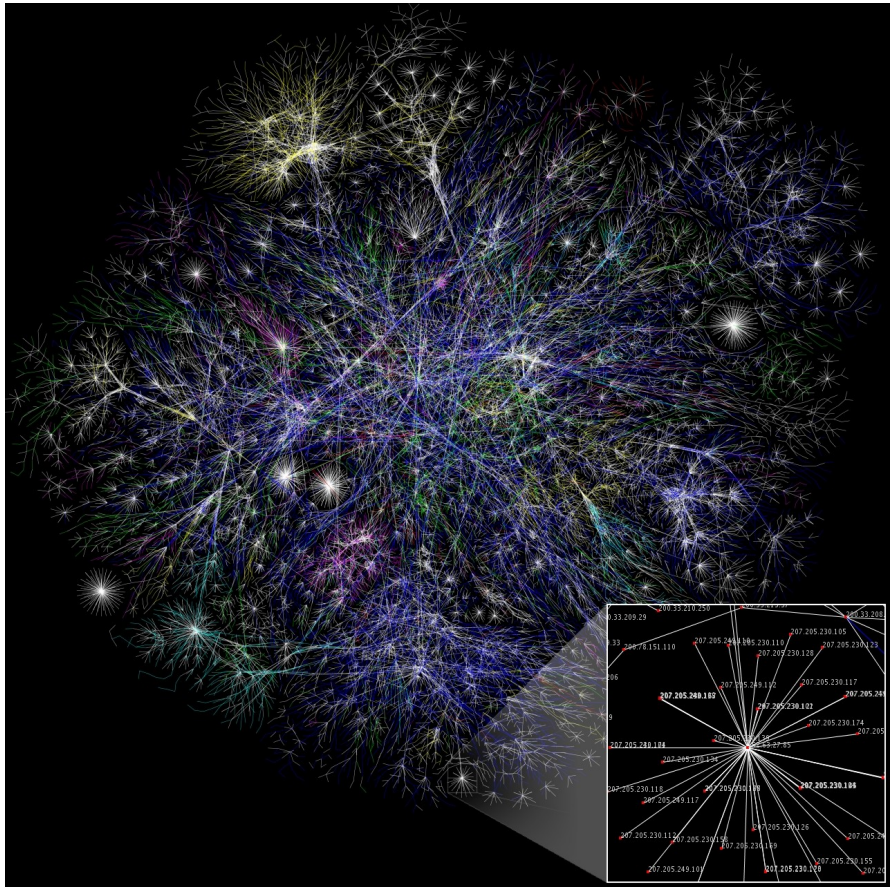

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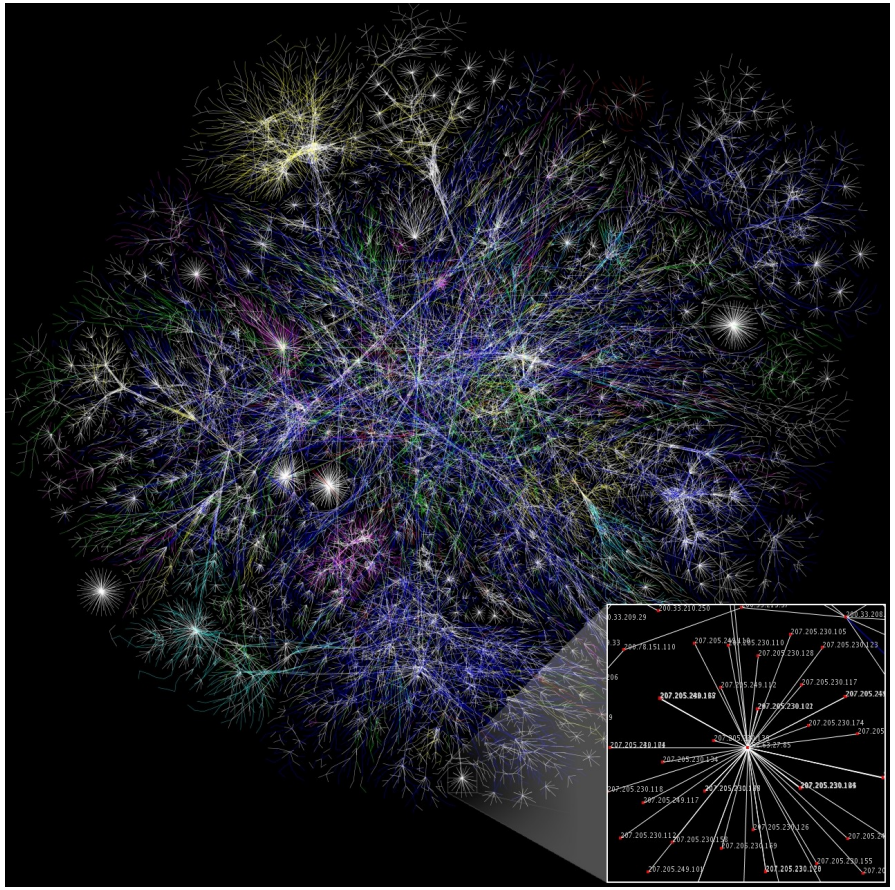

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Introducing PathCache, a system for predicting Internet paths accurately & efficiently.

Internet Basics

172.16.254.1

IP address: numerical label assigned to connected device

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172.16.254.1

172.16.254.0/24

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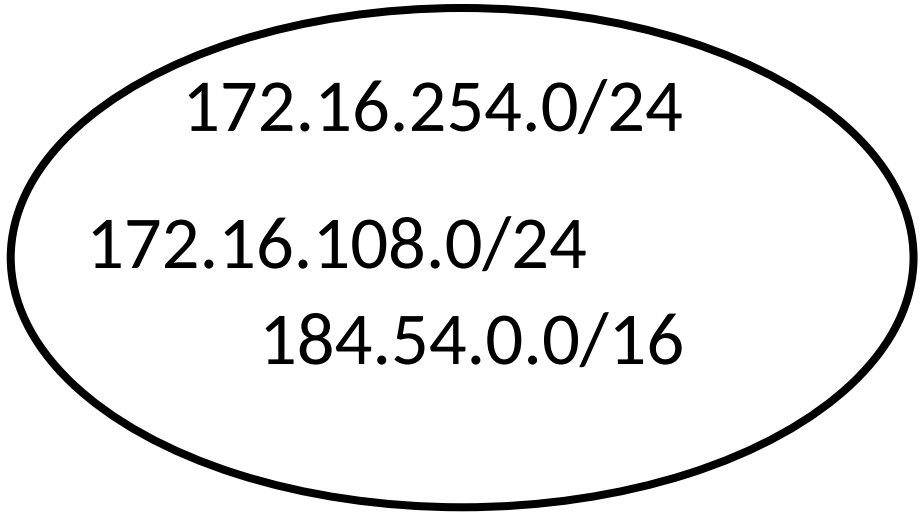


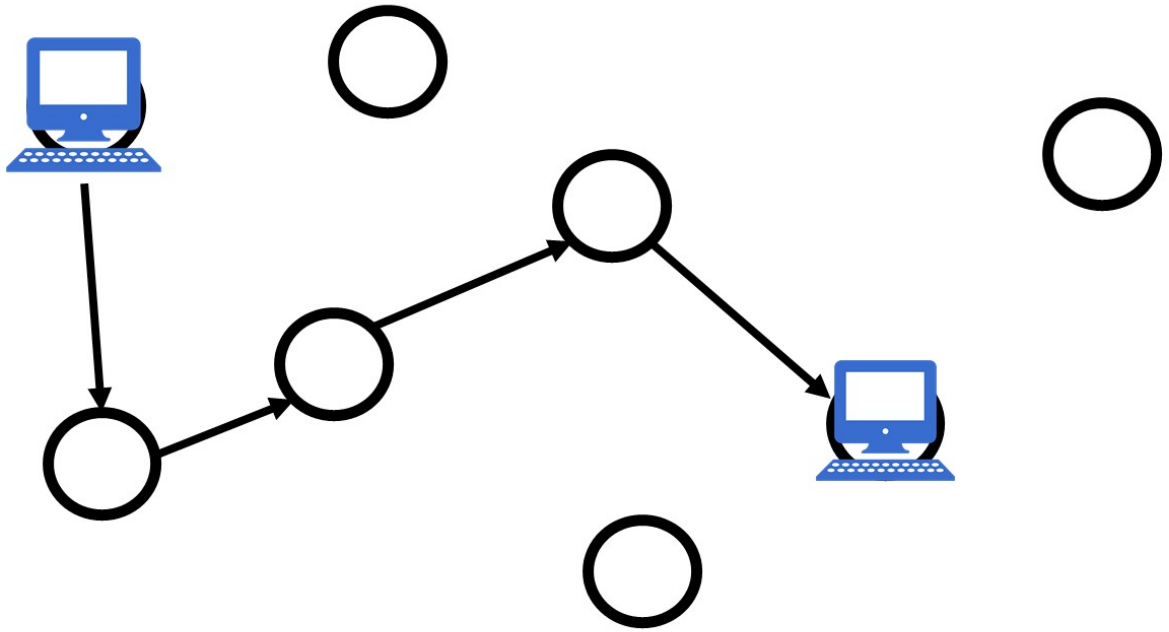
IP address: numerical label assigned to connected device

IP prefix: range of sequential IP addresses with same prefix

Autonomous System (AS): collection of IP prefixes owned by network w/ 1 routing policy

At a high level, the Internet is made up of ASes. Routing handled by BGP.

Internet Basics



Traceroute: path measurement between host and other device

You need to control a host to issue a traceroute from it.

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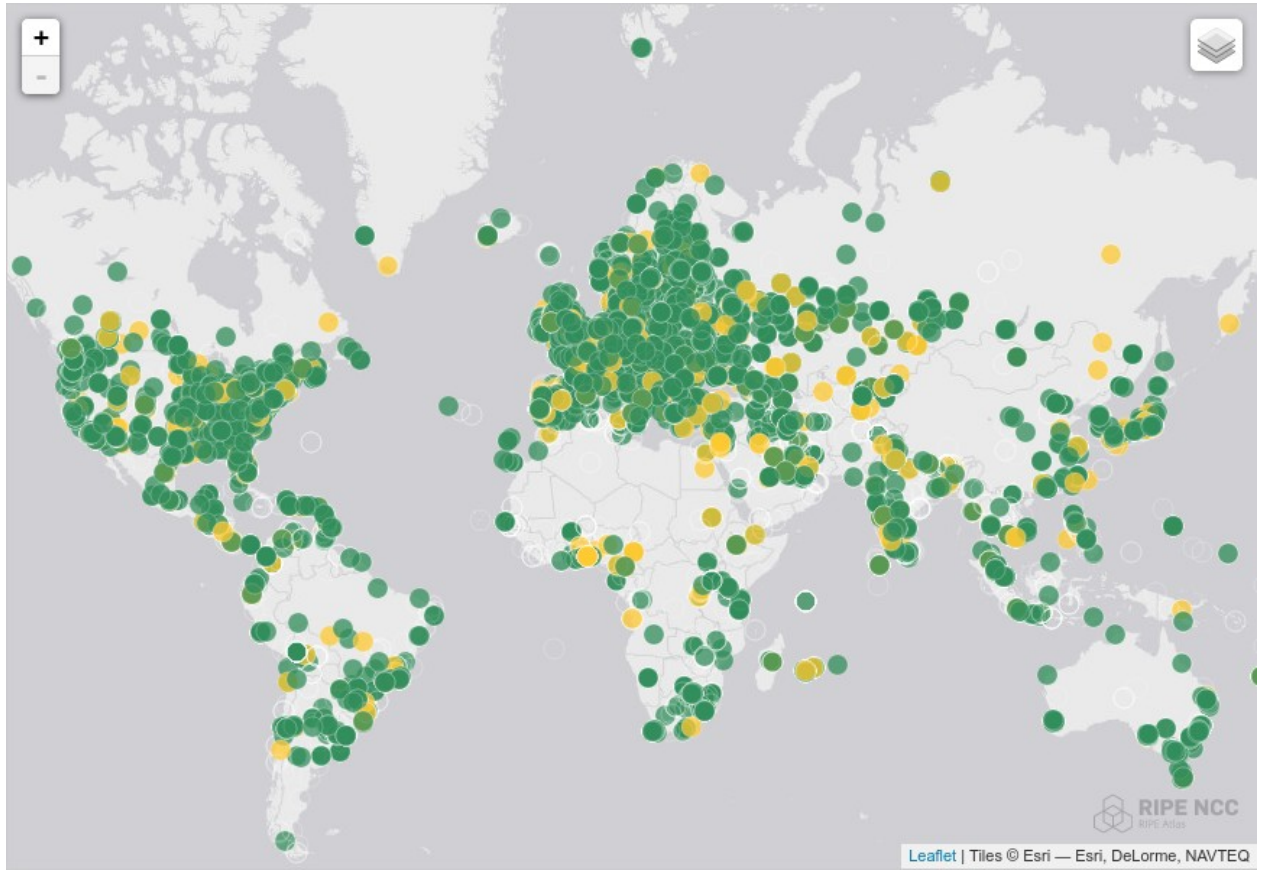

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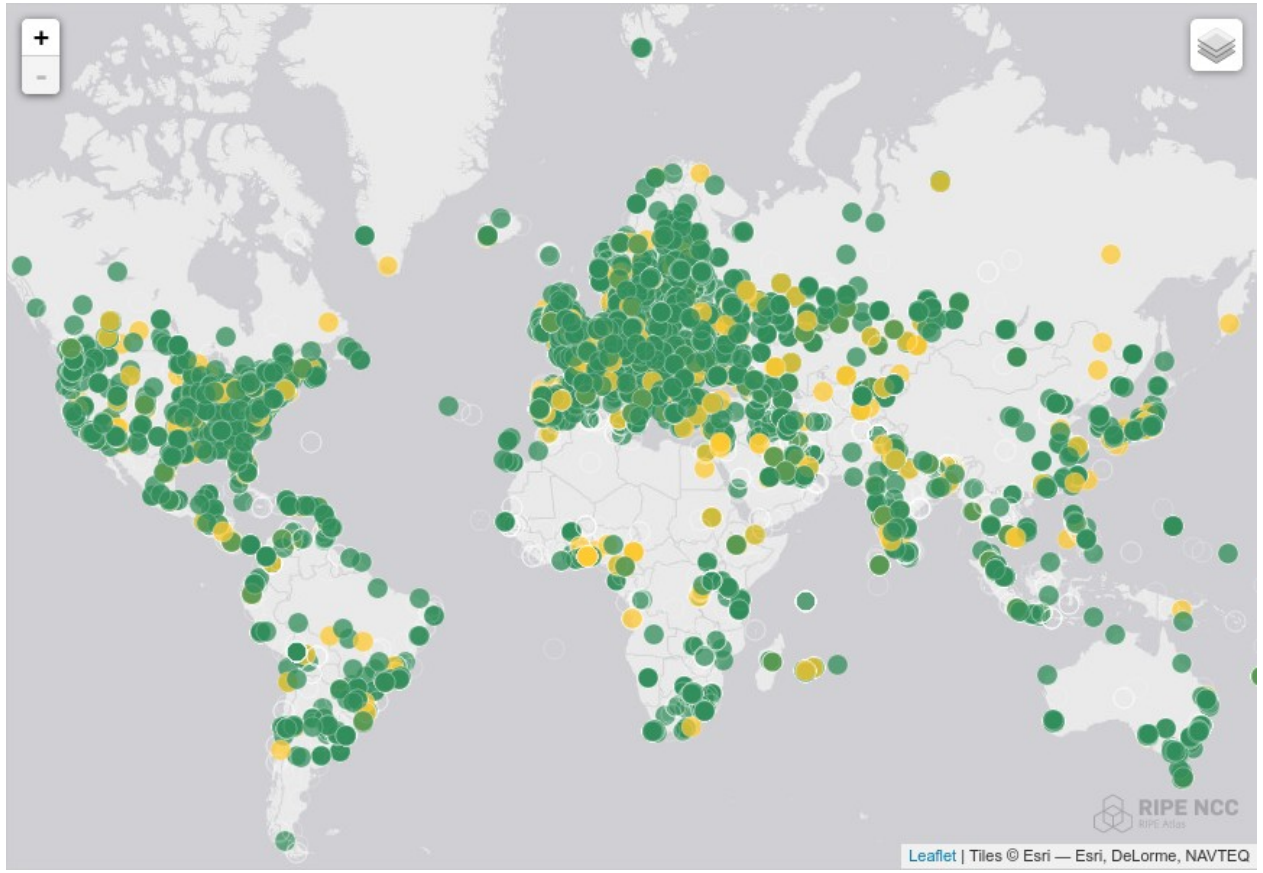

Traceroute: path measurement between host and other device

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Vantage points: devices which do traceroutes for researchers. VPs can't be used too often.

Goal: make few measurements & predict paths from results.

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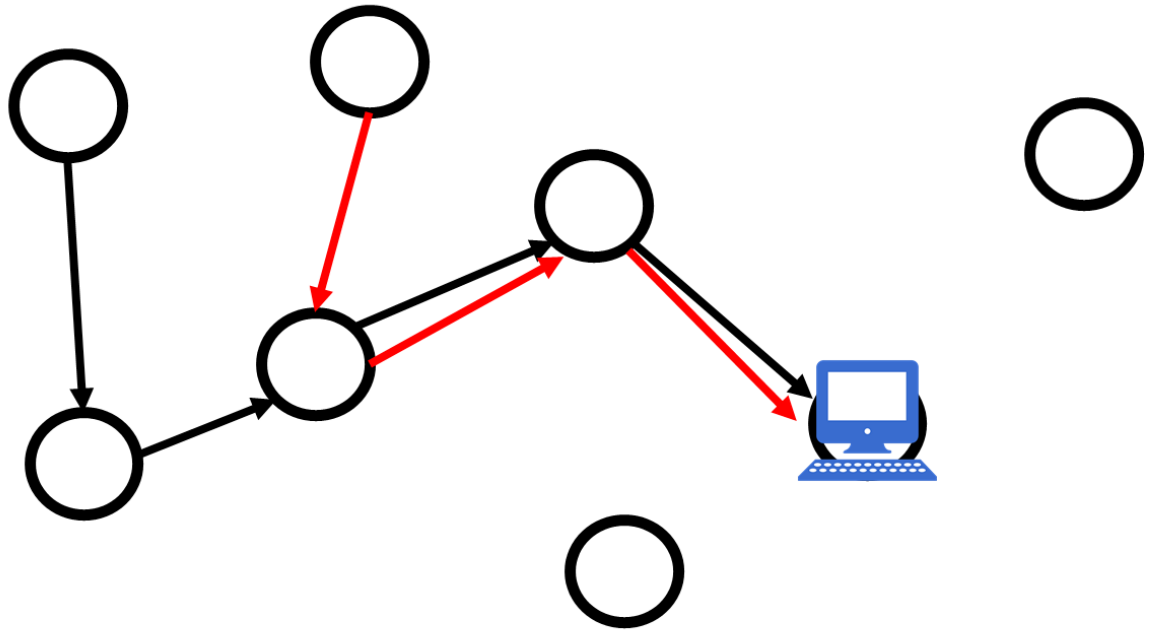

<u>Destination-based routing (DBR):</u>
Messages to same destination take consistent paths thru AS graph.

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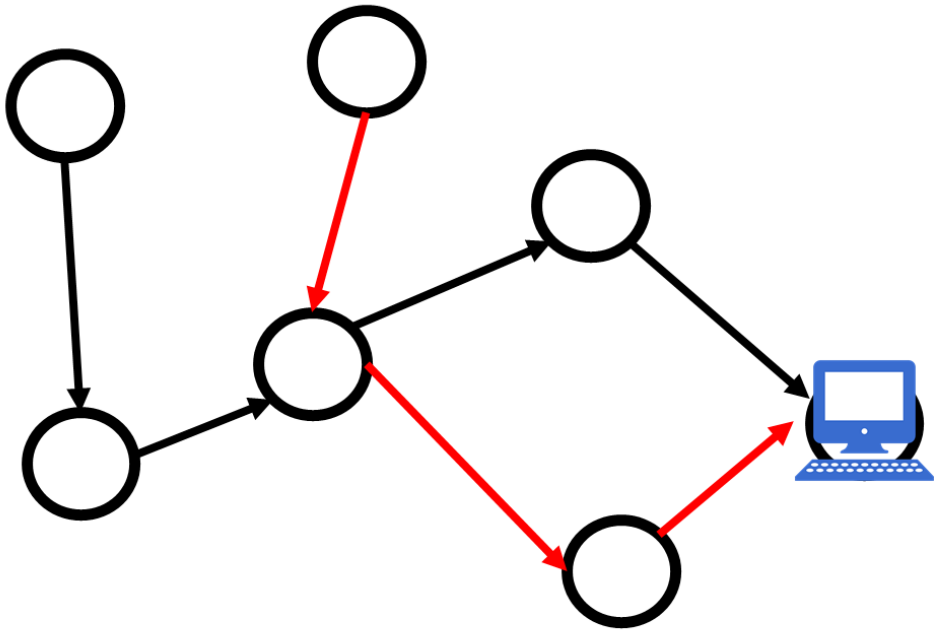

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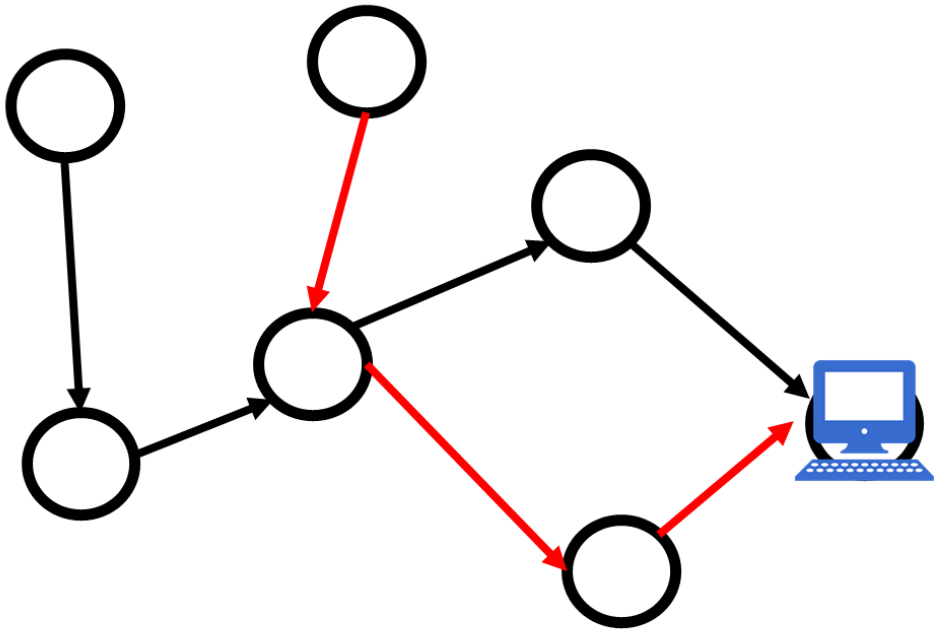

<u>Destination-based routing (DBR):</u>
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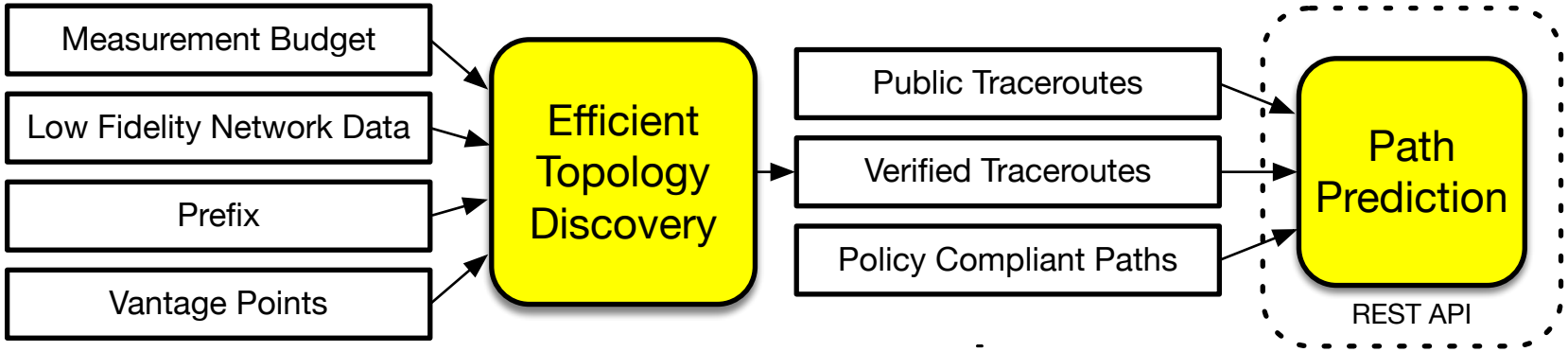
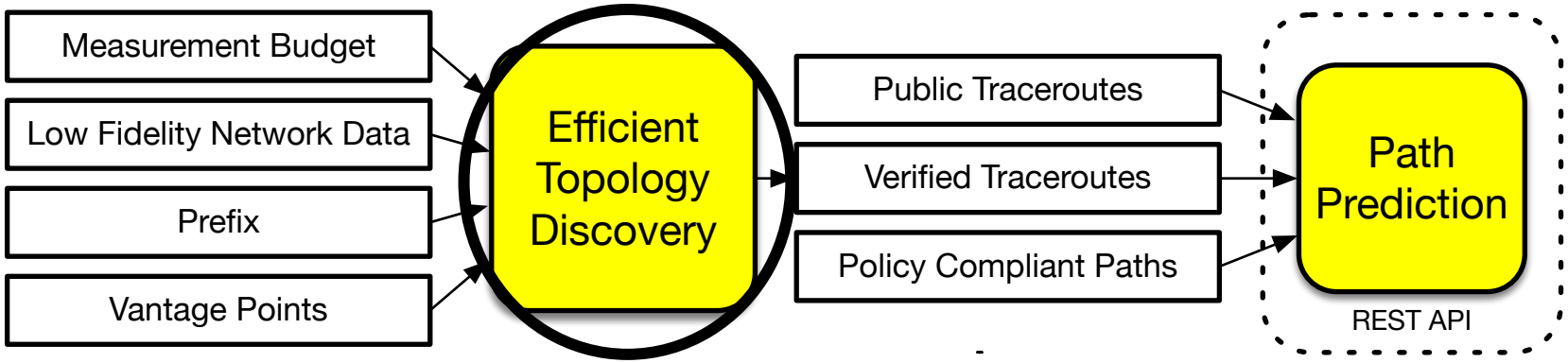
PathCache insight: DBR consistency allows organizing measurements & path predictions by destination

PathCache System overview



PathCache starts with untrusted routing data towards a prefix *P*. It:

PathCache System overview



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1. Uses this data to make traceroutes to *P* from VPs via a cover algorithm.

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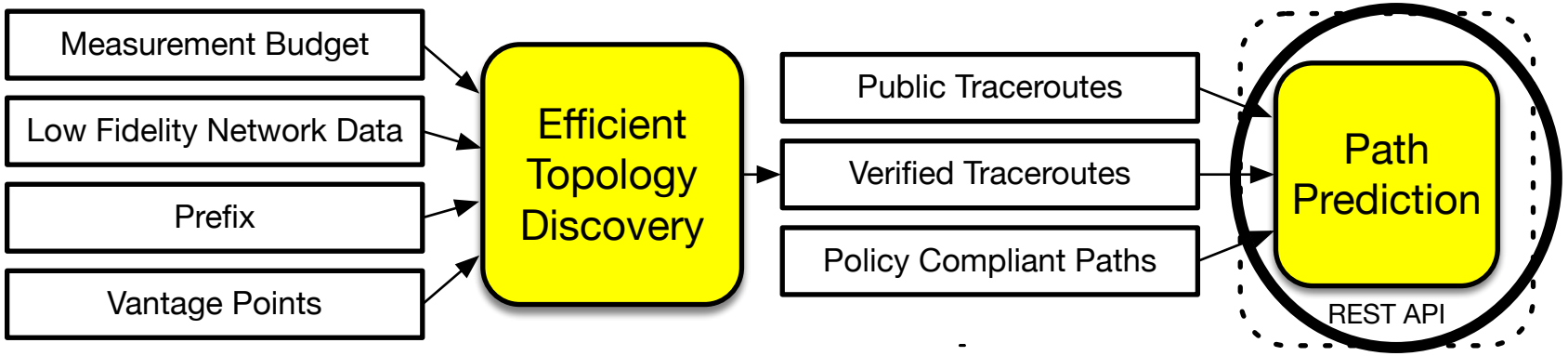
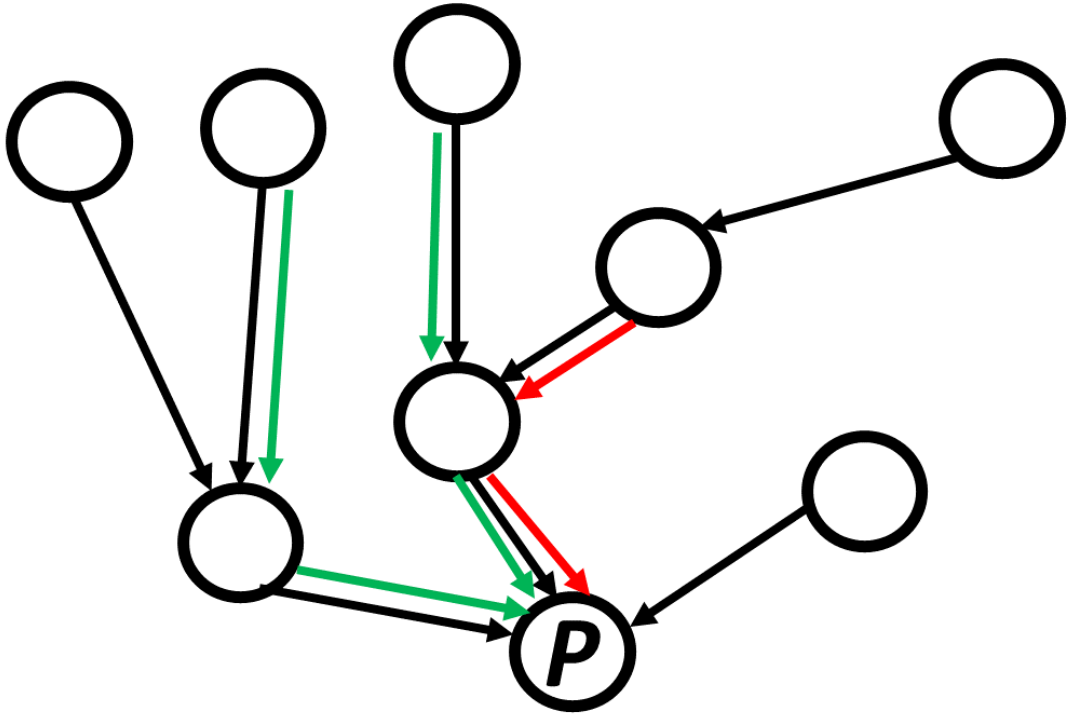


PathCache starts with untrusted routing data towards a prefix *P*. It:

1. Uses this data to make traceroutes to *P* from VPs via a cover algorithm.

2. Combines traceroute results to make a directed graph which it uses to predict path from <u>any</u> starting location to prefix *P*.

Efficient Topology Discovery



Input is untrusted path data

- Stale traceroutes (out of date?)
- BGP routing info (maybe wrong)

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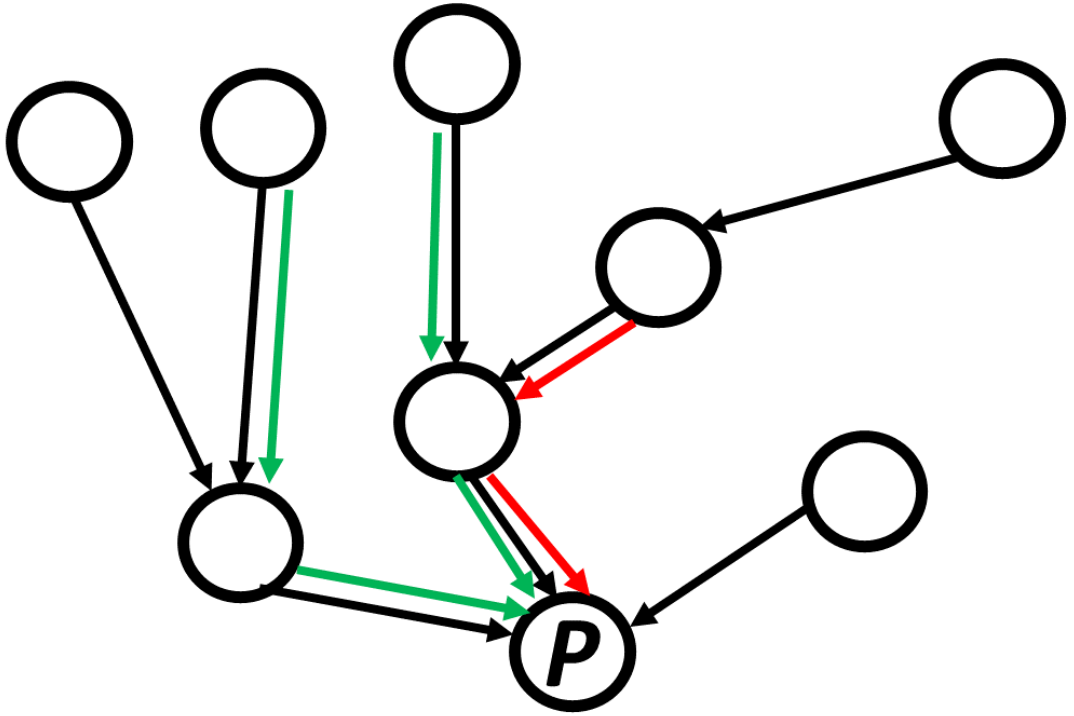

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We expect the paths in this data to form a directed tree rooted at *P*.

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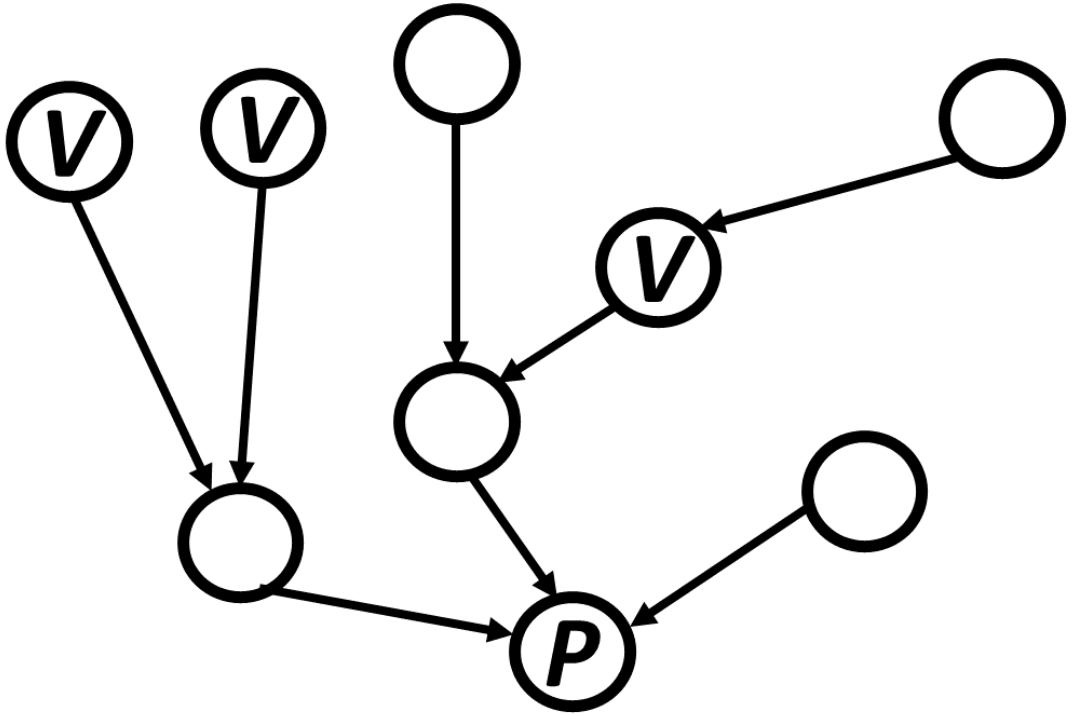

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Output: k VPs whose traceroutes will maximize edges discovered.

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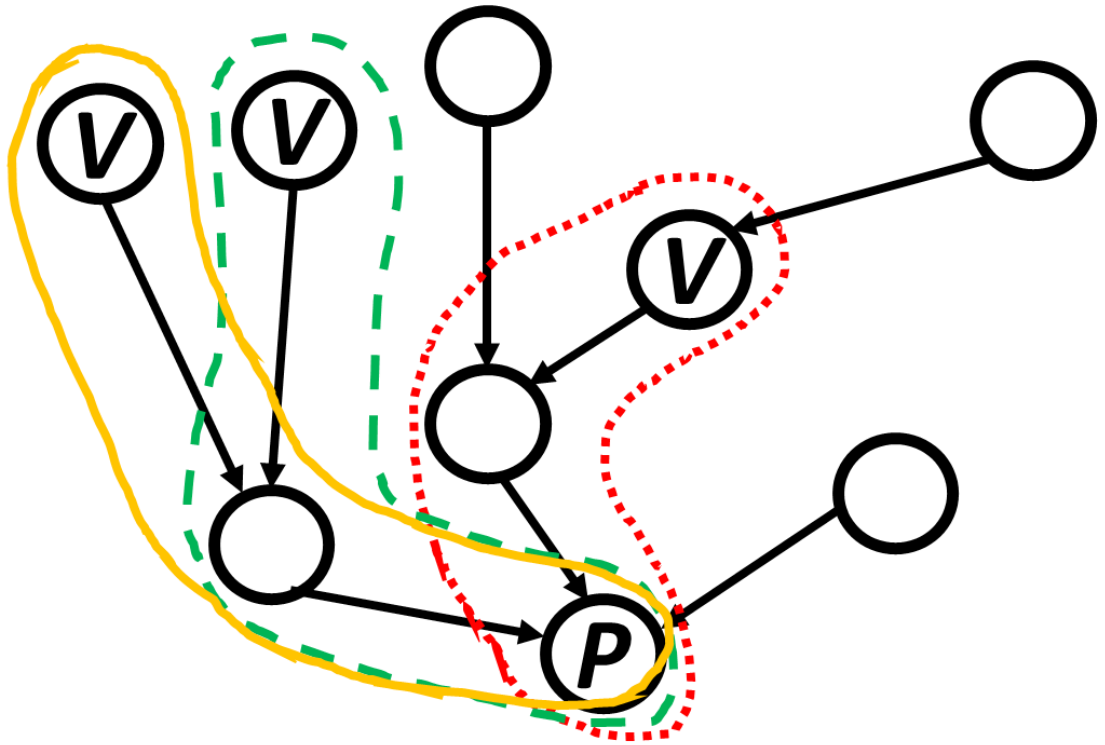

We can think of each VP as a set: the path from VP to prefix *P*. Pick k sets that maximize edge coverage. Sound familiar?

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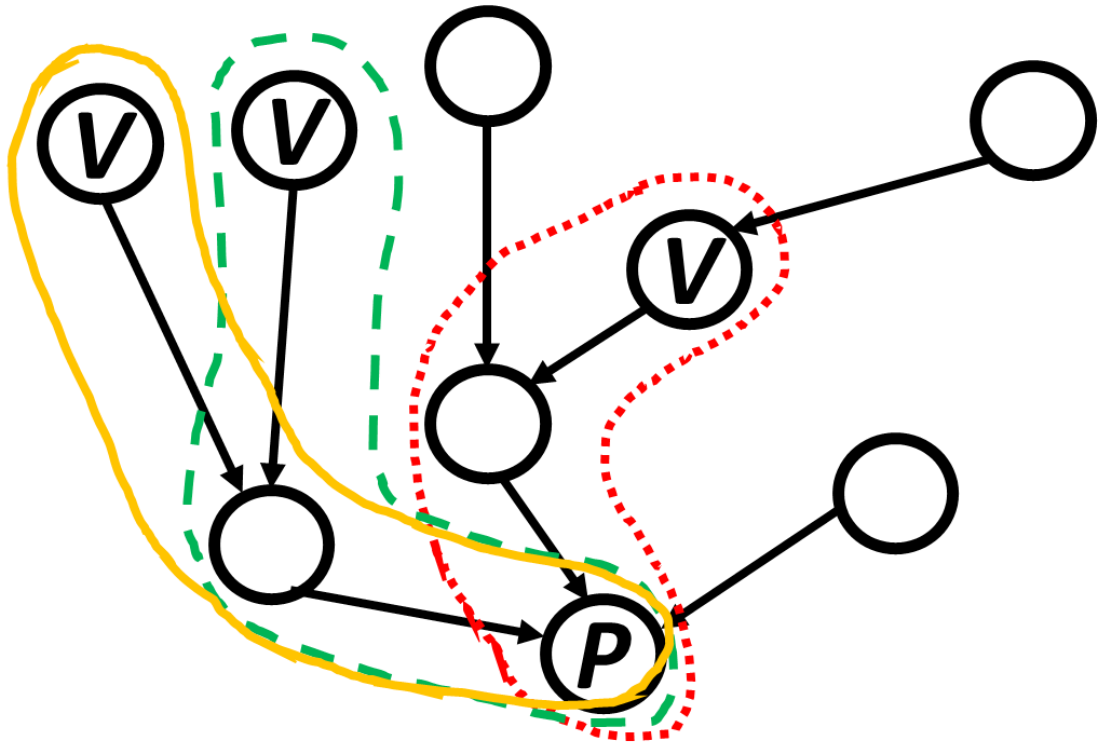

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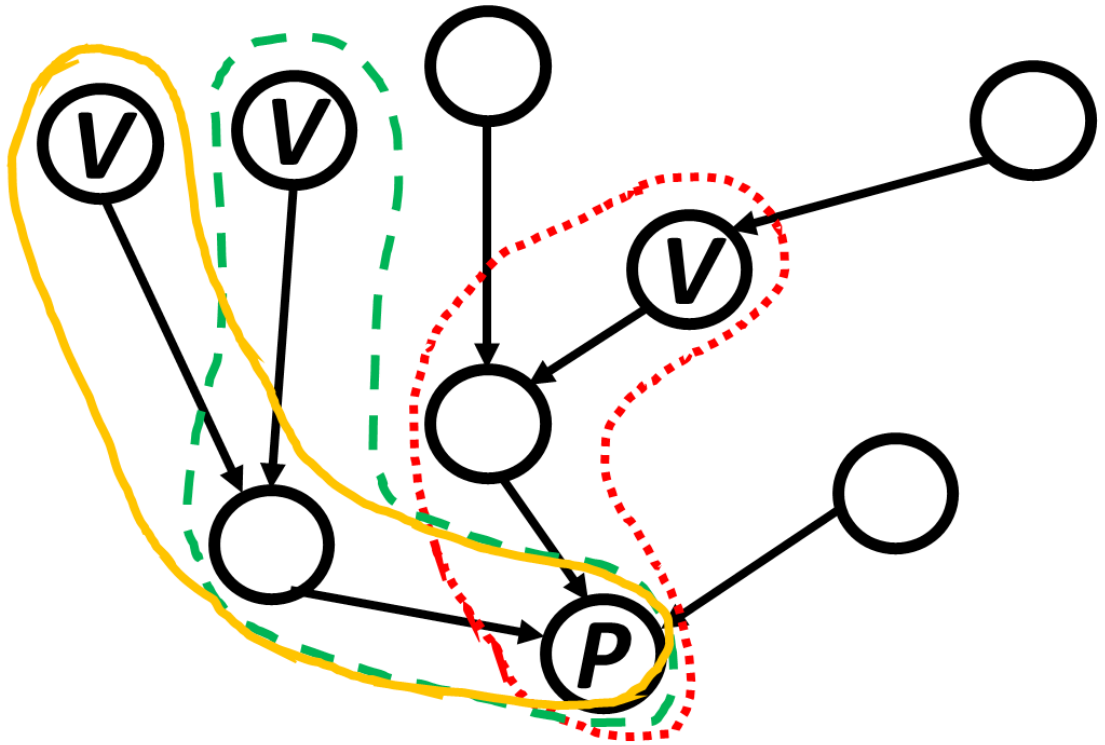

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Recall: NP-Hard, greedy algorithm gives (1-1/e)-approximation

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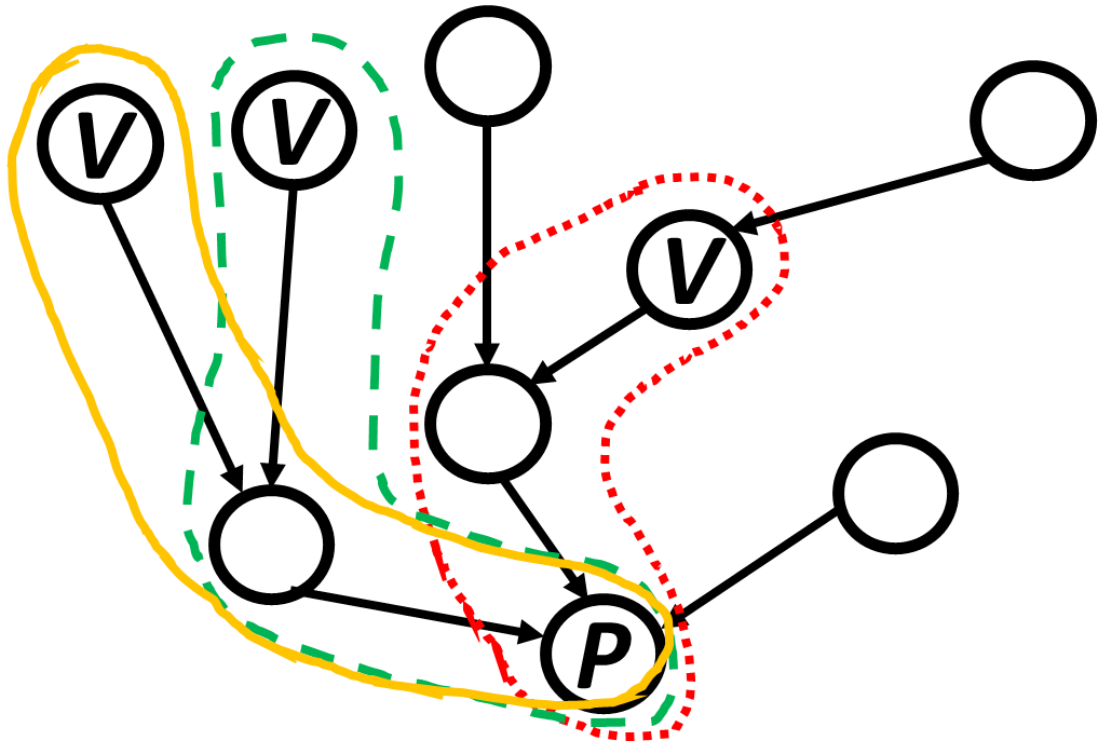

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In this special case: greedy algorithm gives optimal coverage.

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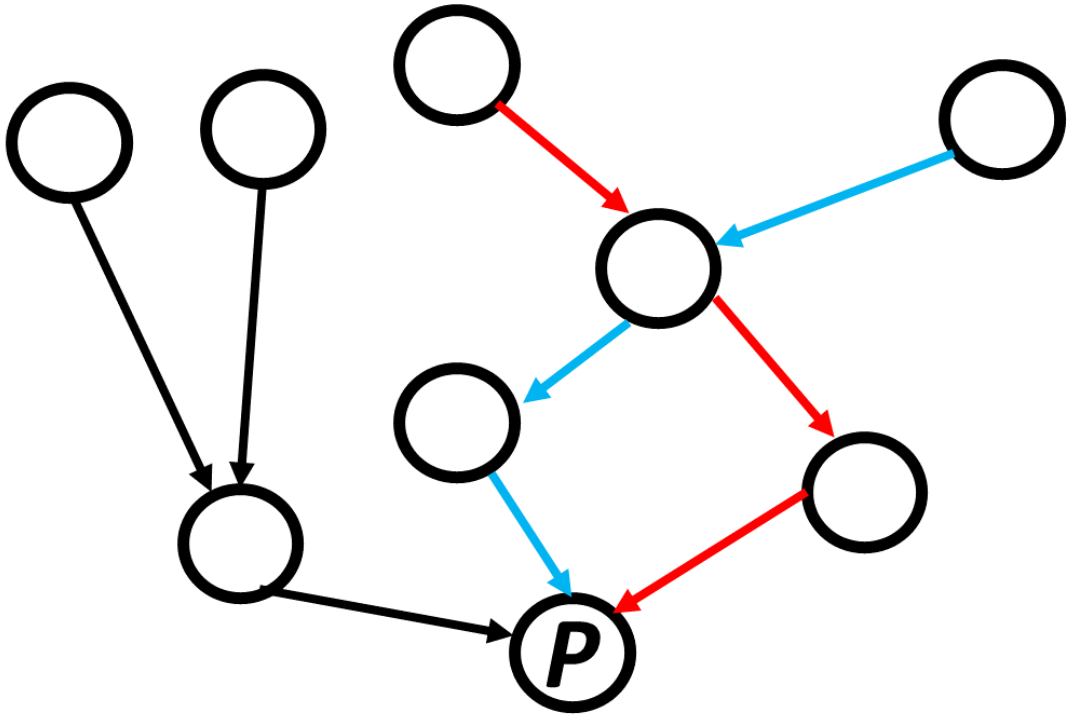

Sometimes our untrusted input data violates DBR, and isn't a tree.

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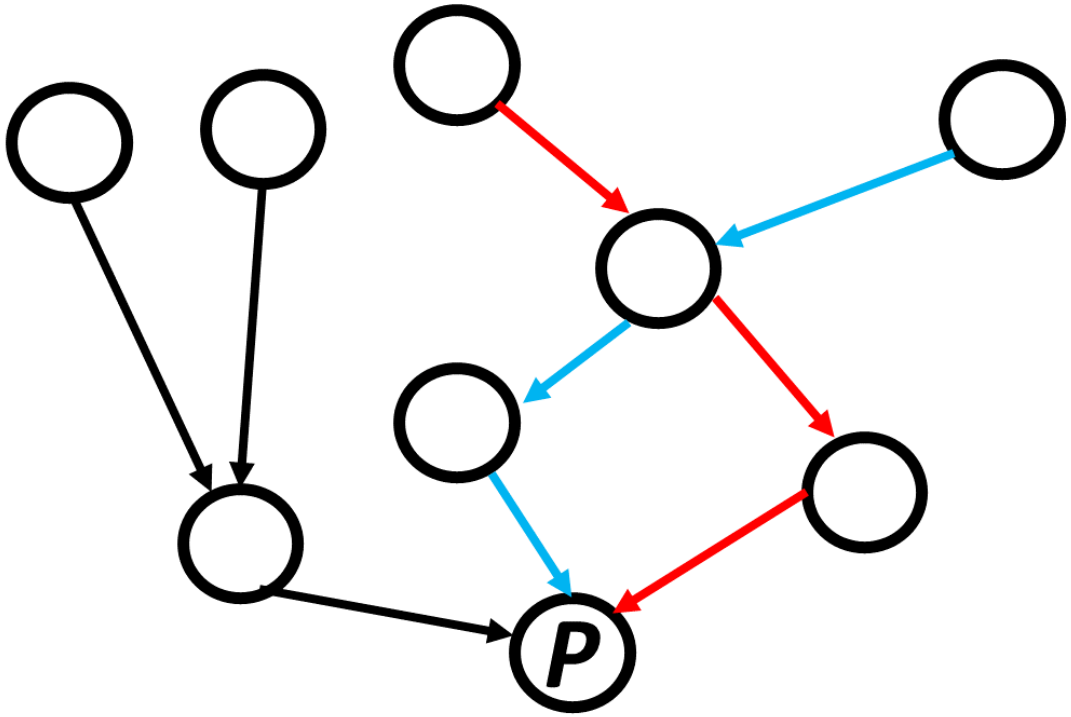

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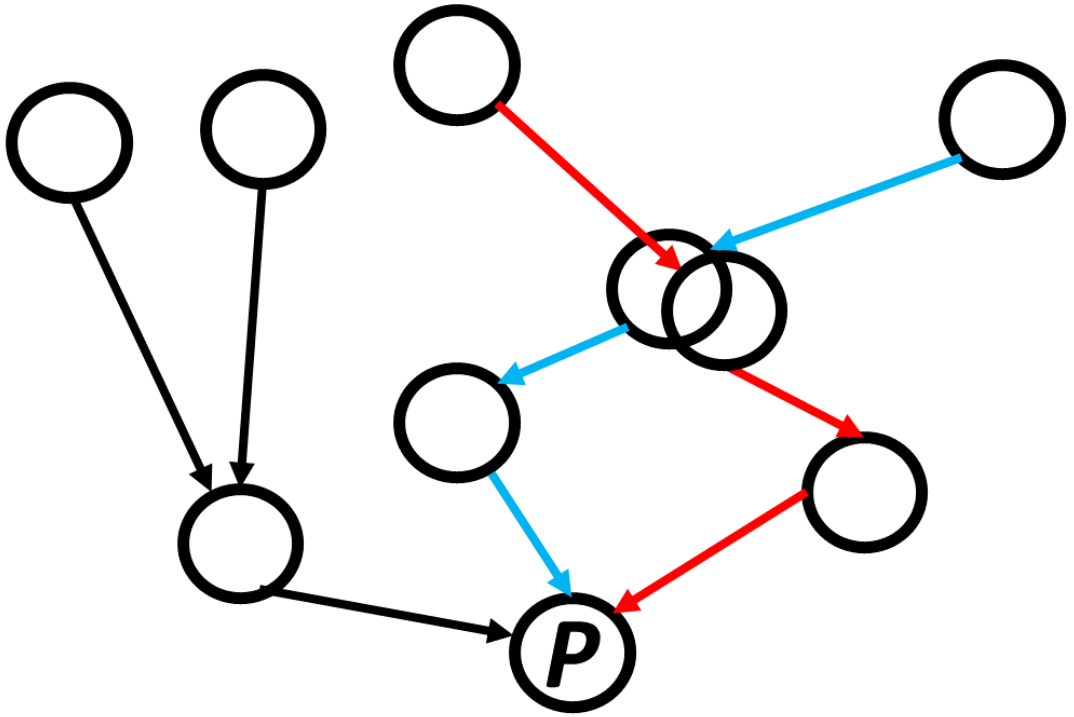

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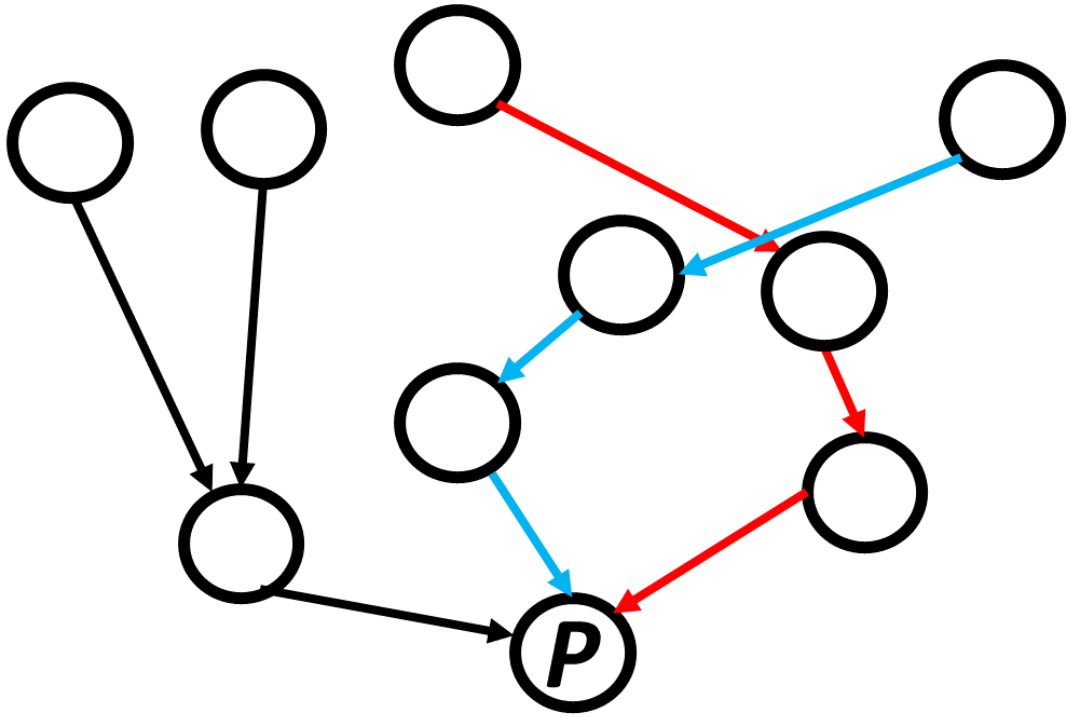

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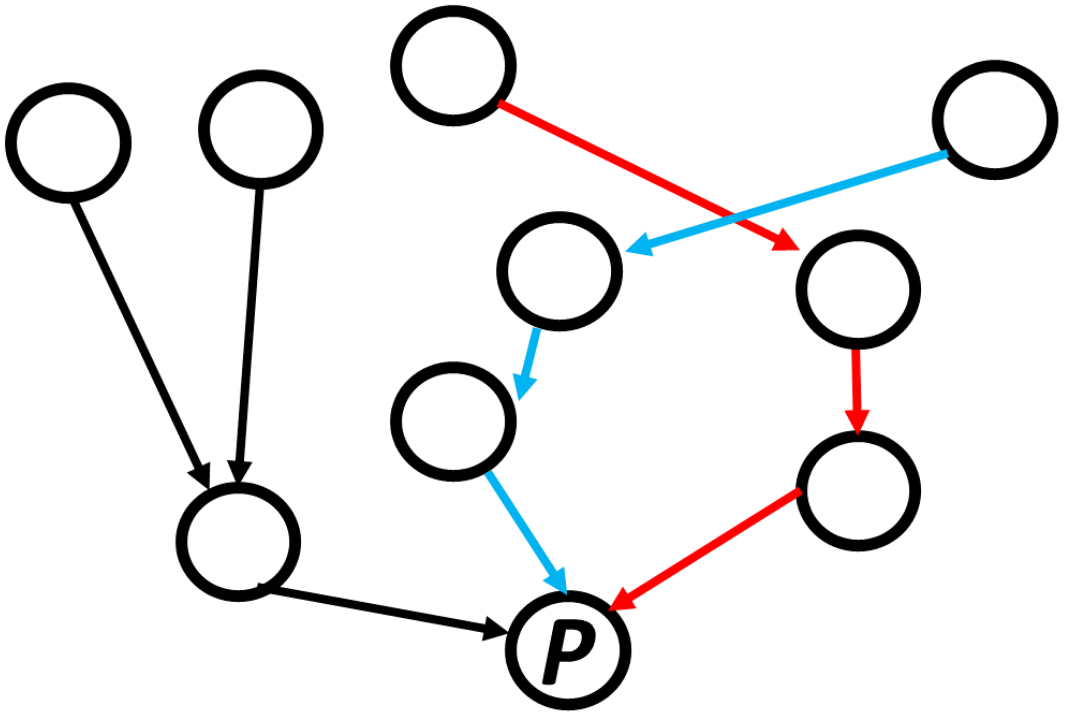

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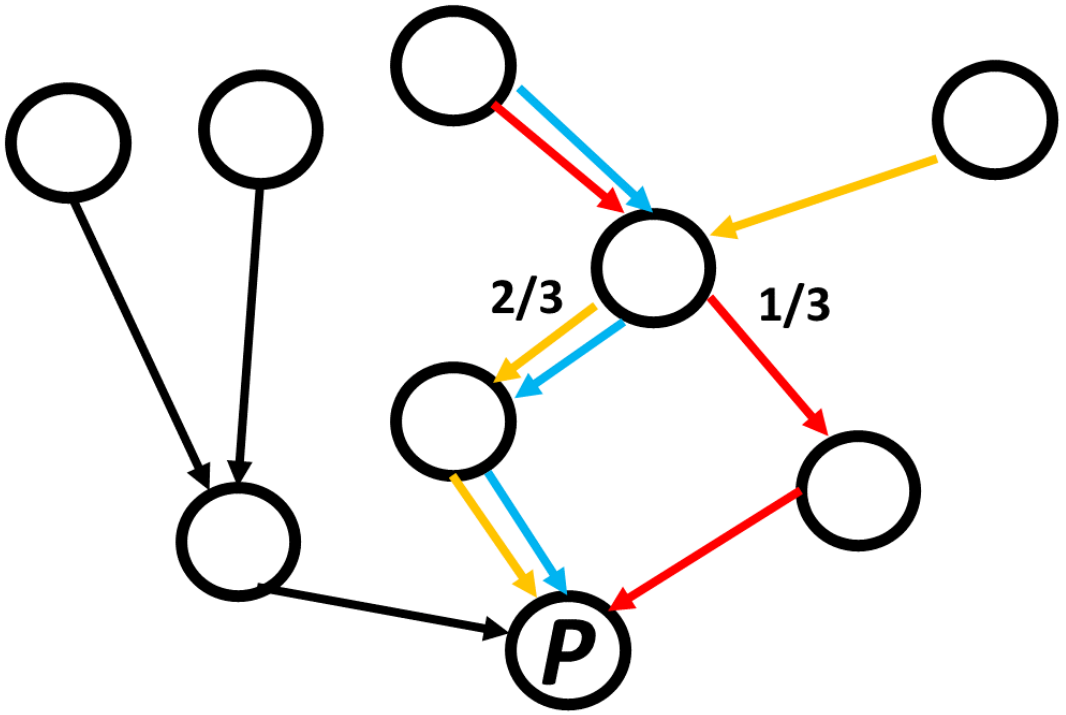

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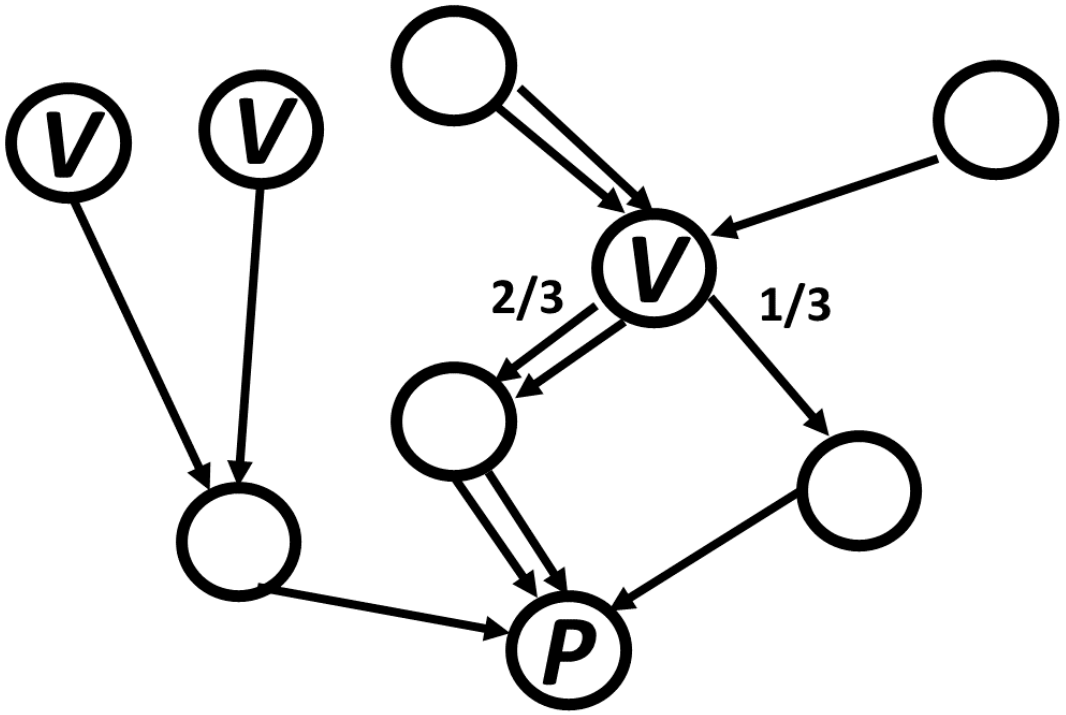

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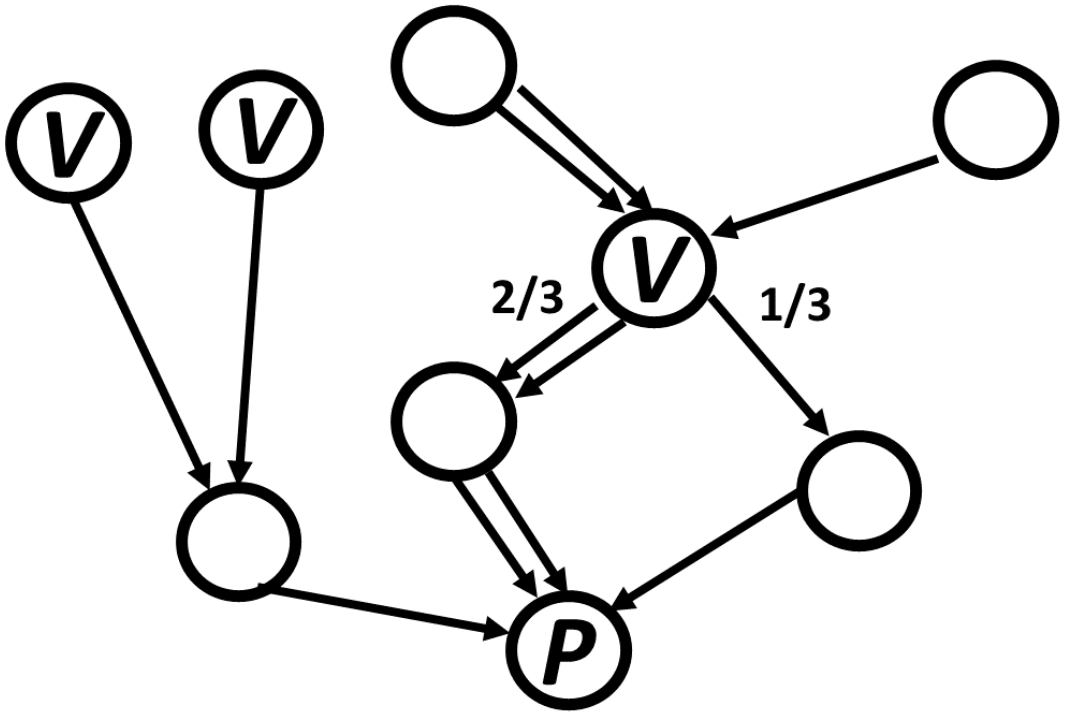

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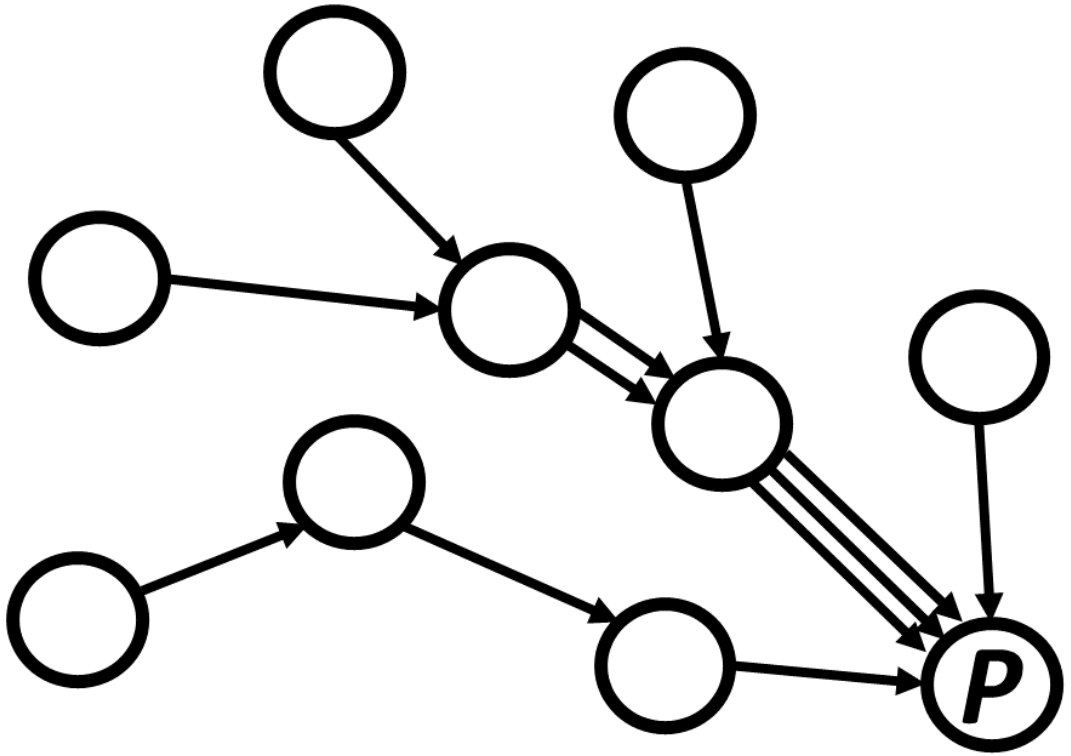
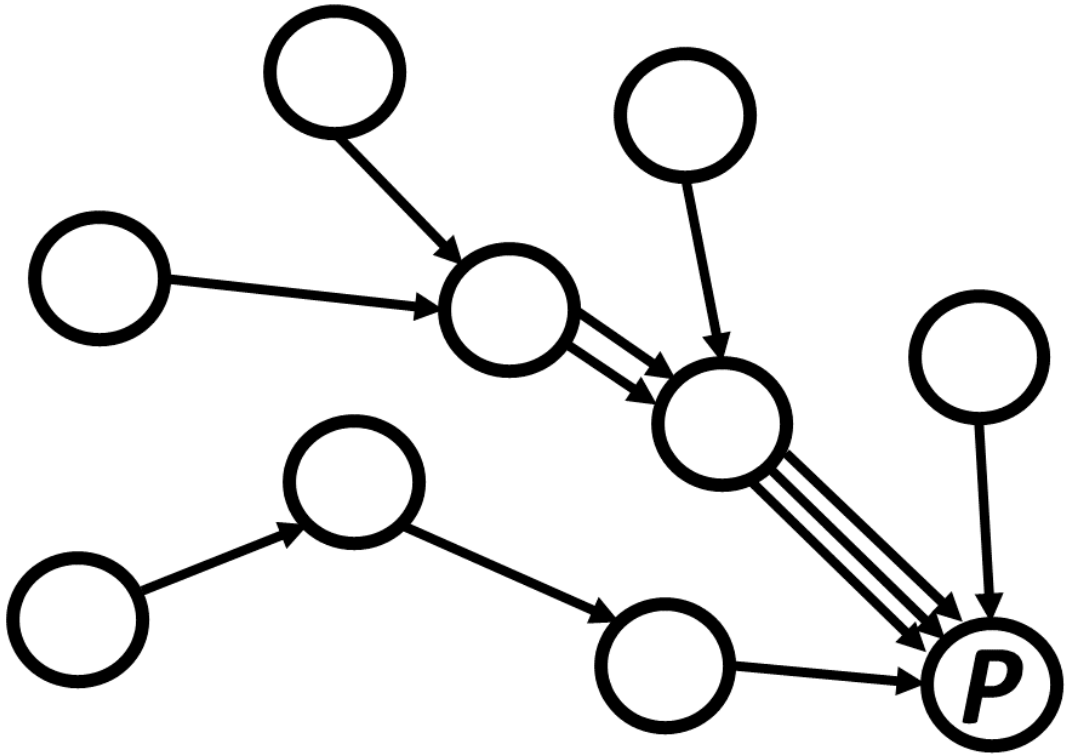
In this random routing case, a generalized greedy algorithm chooses VP with max expected coverage and gets $(e/(e-1))^2$ appx.

Path Prediction



We run traceroutes from each VP in our chosen set to *P*. Now we have a big collection of paths.

Path Prediction



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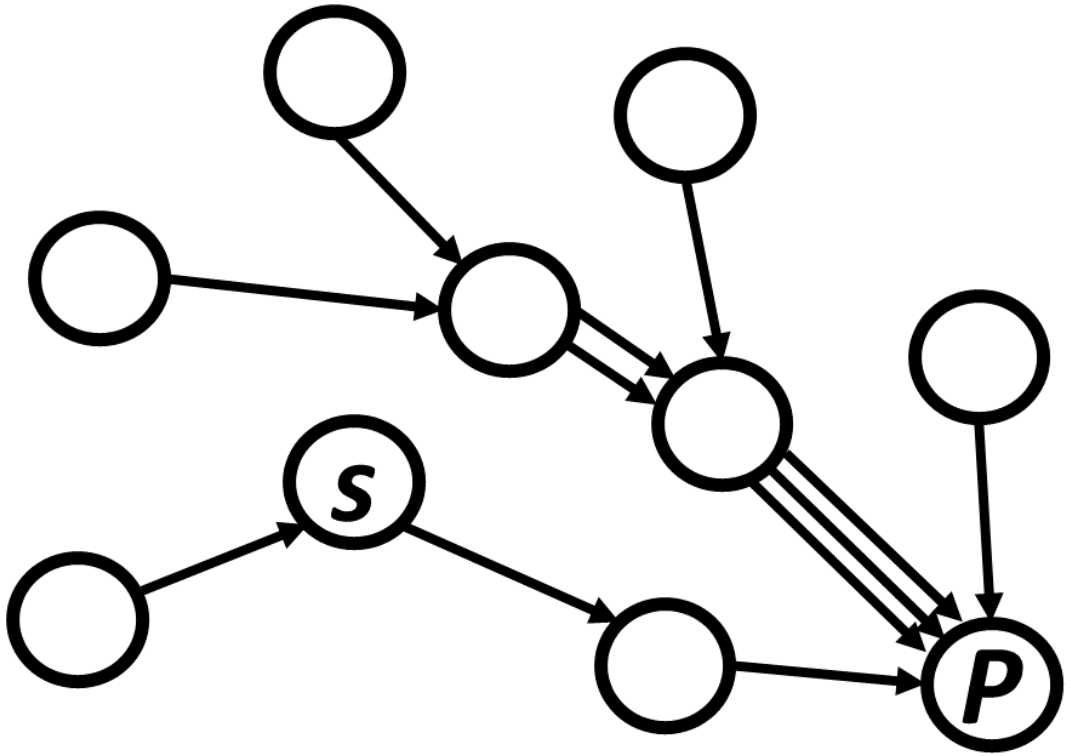

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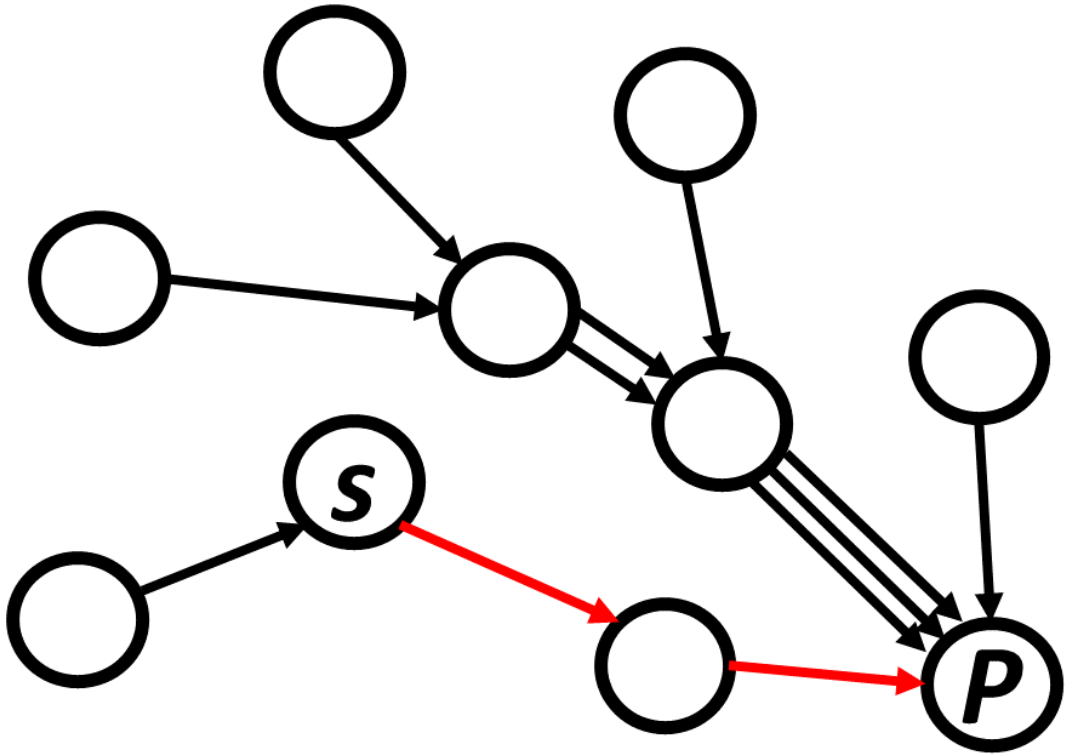

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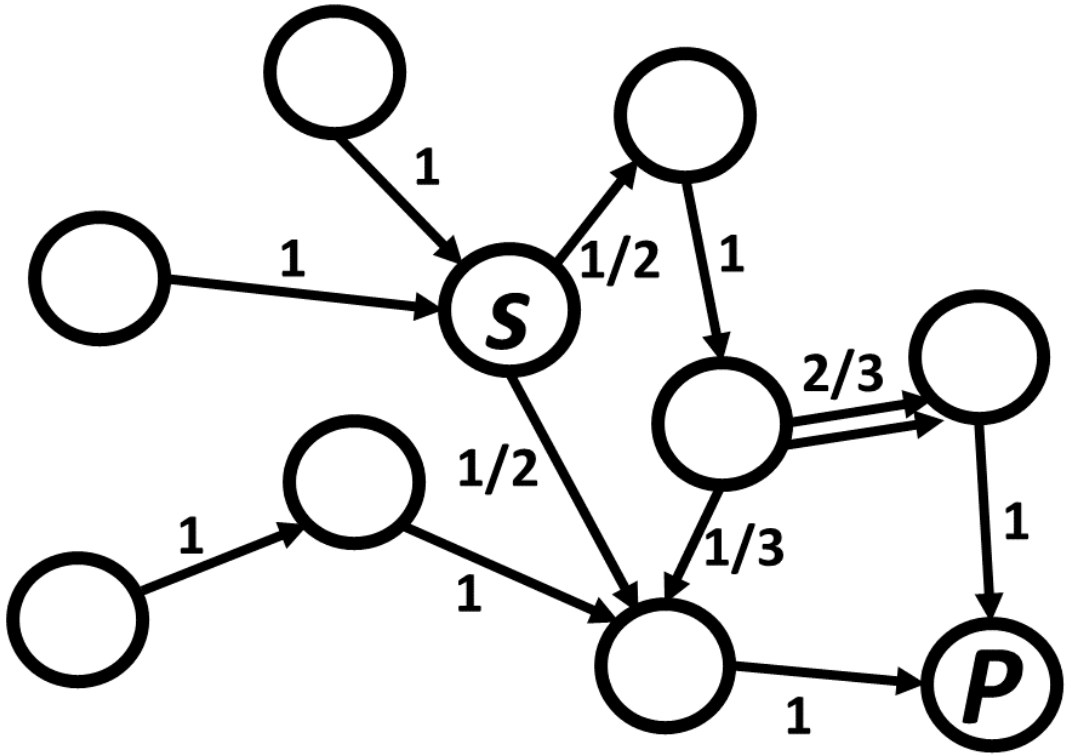
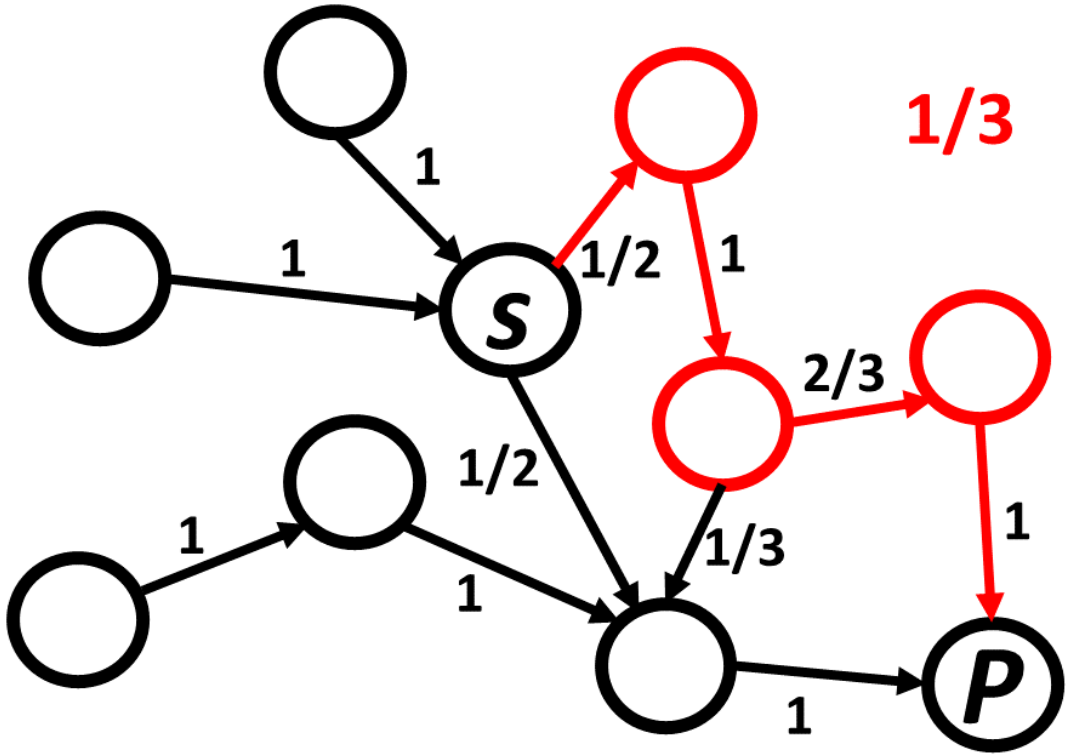


But *D* is basically never an in-tree, it turns out. So we model each node as routing randomly like before. *D* encodes a Markov chain.

Path Prediction



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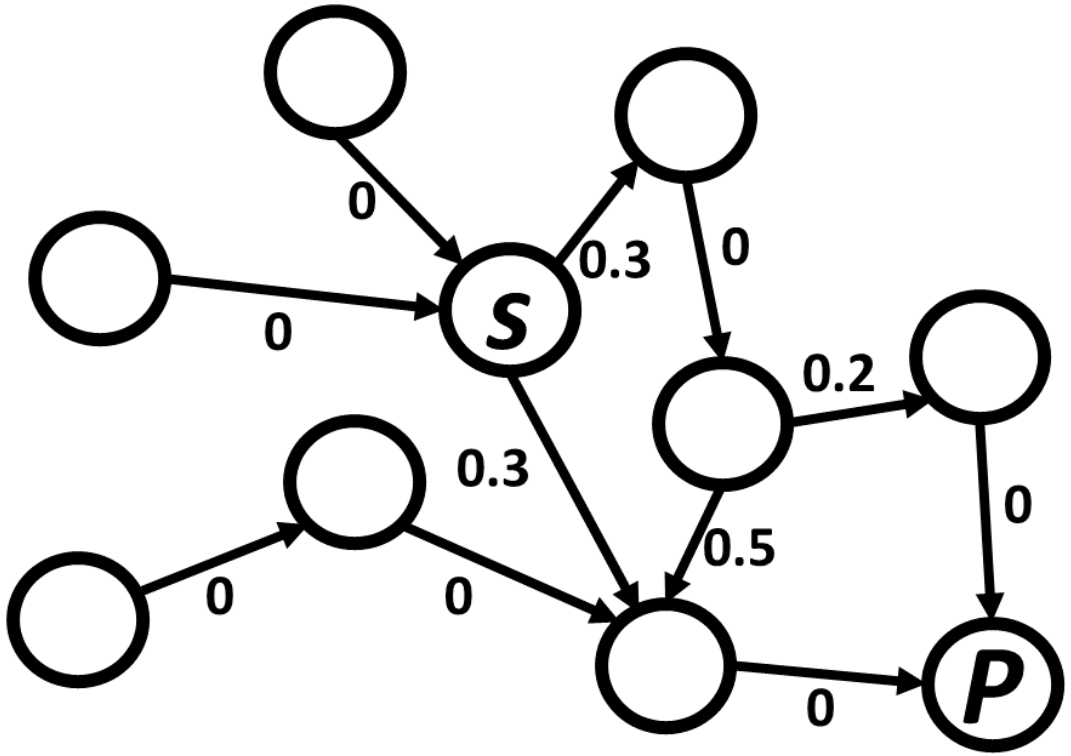

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Given a query source *s*, we can efficiently predict the most likely p paths from *s* to *P* by taking the –log of each edge probability and using Yen's p-shortest paths algorithm.

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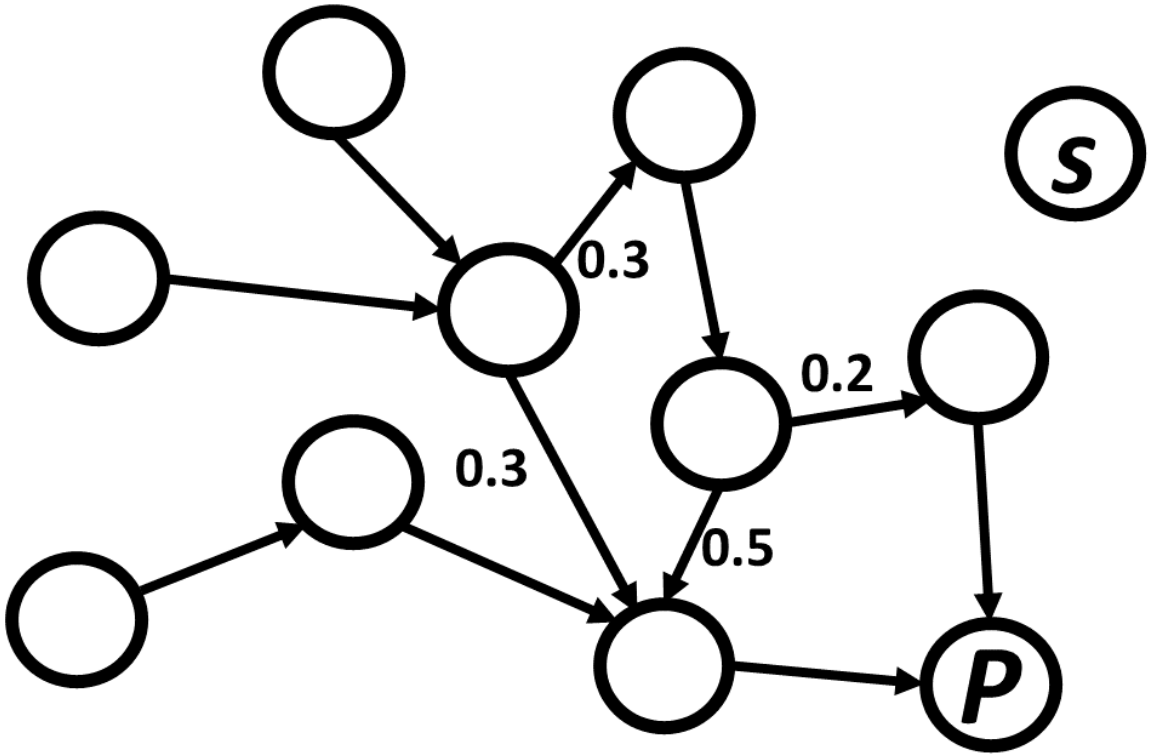

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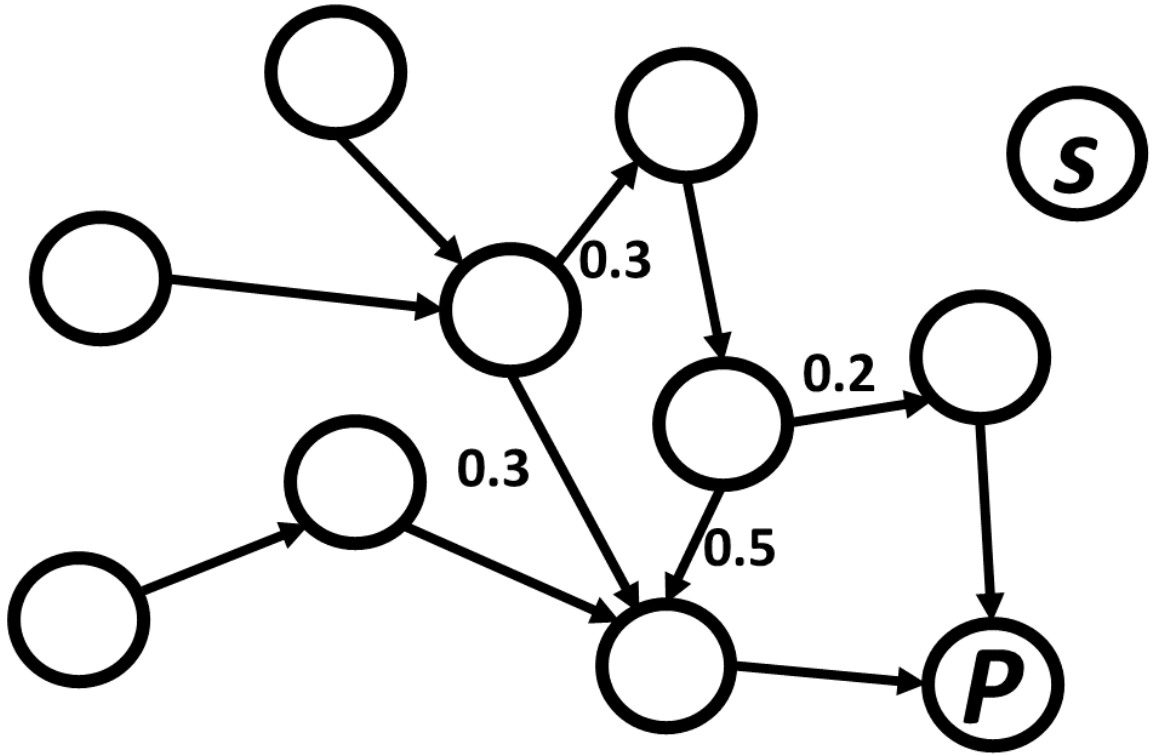

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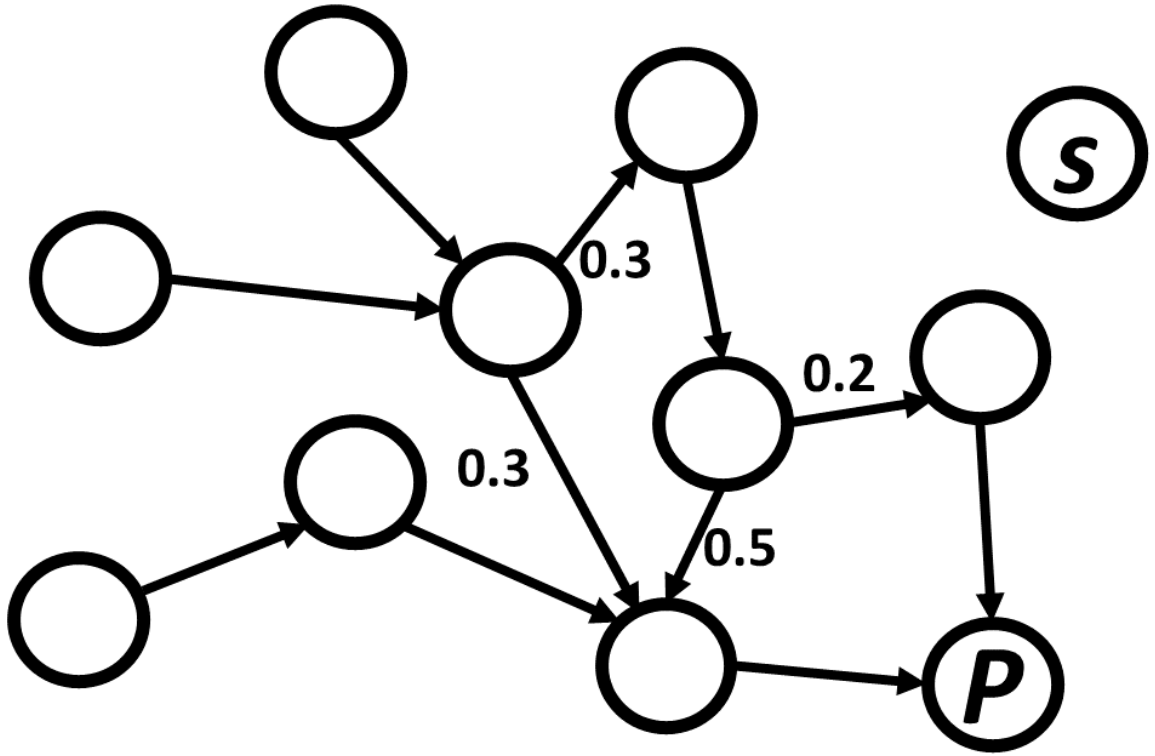

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How can we predict paths from *s*?

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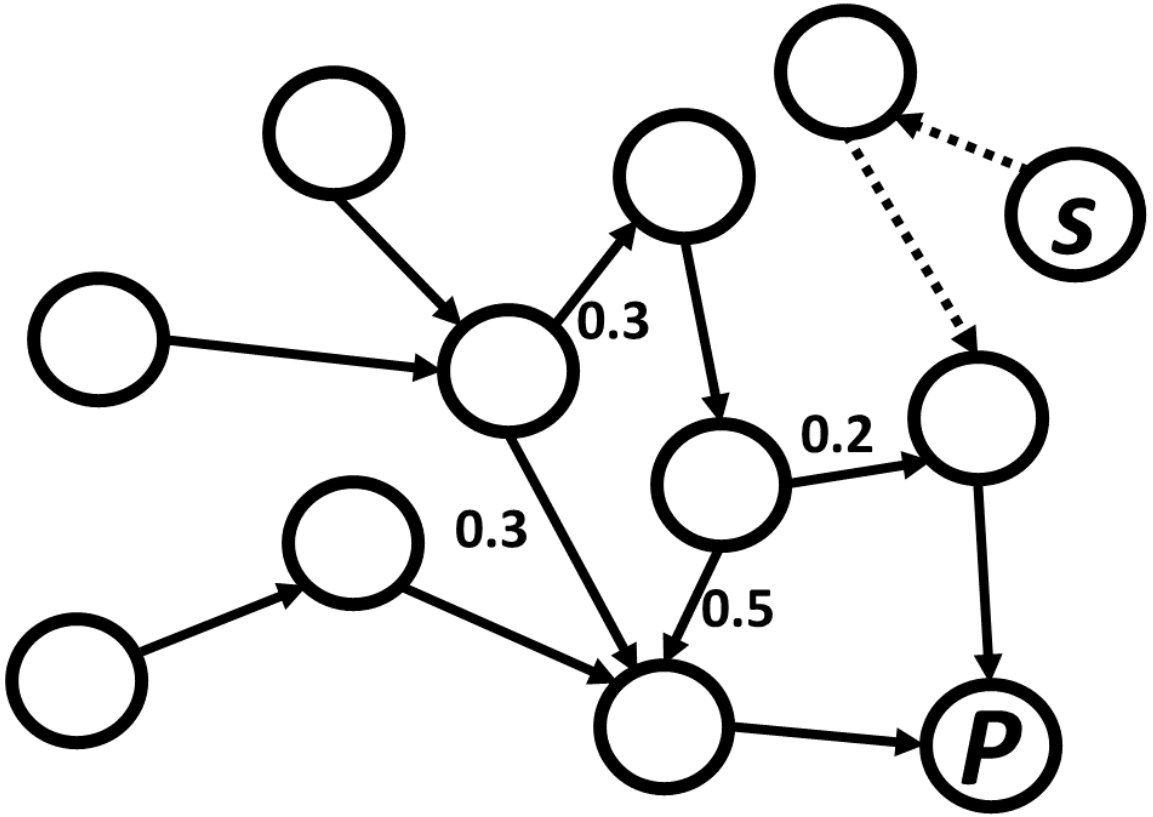

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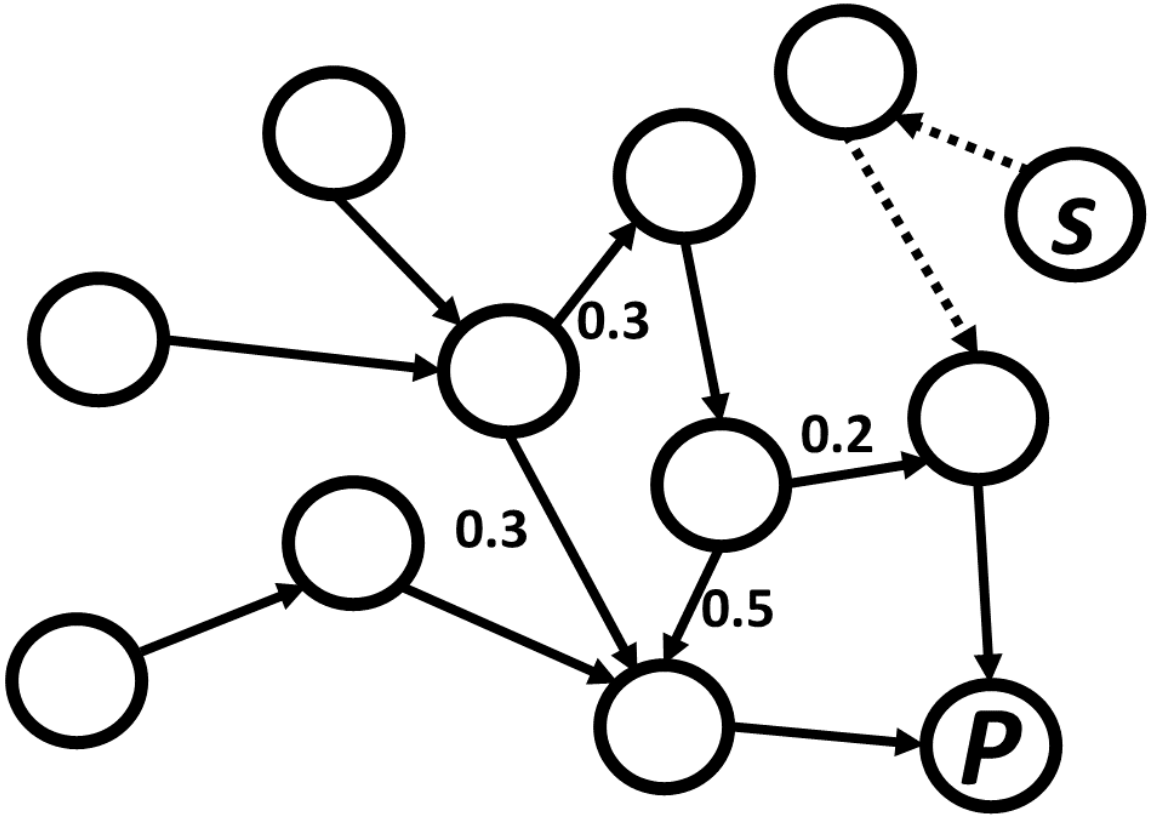

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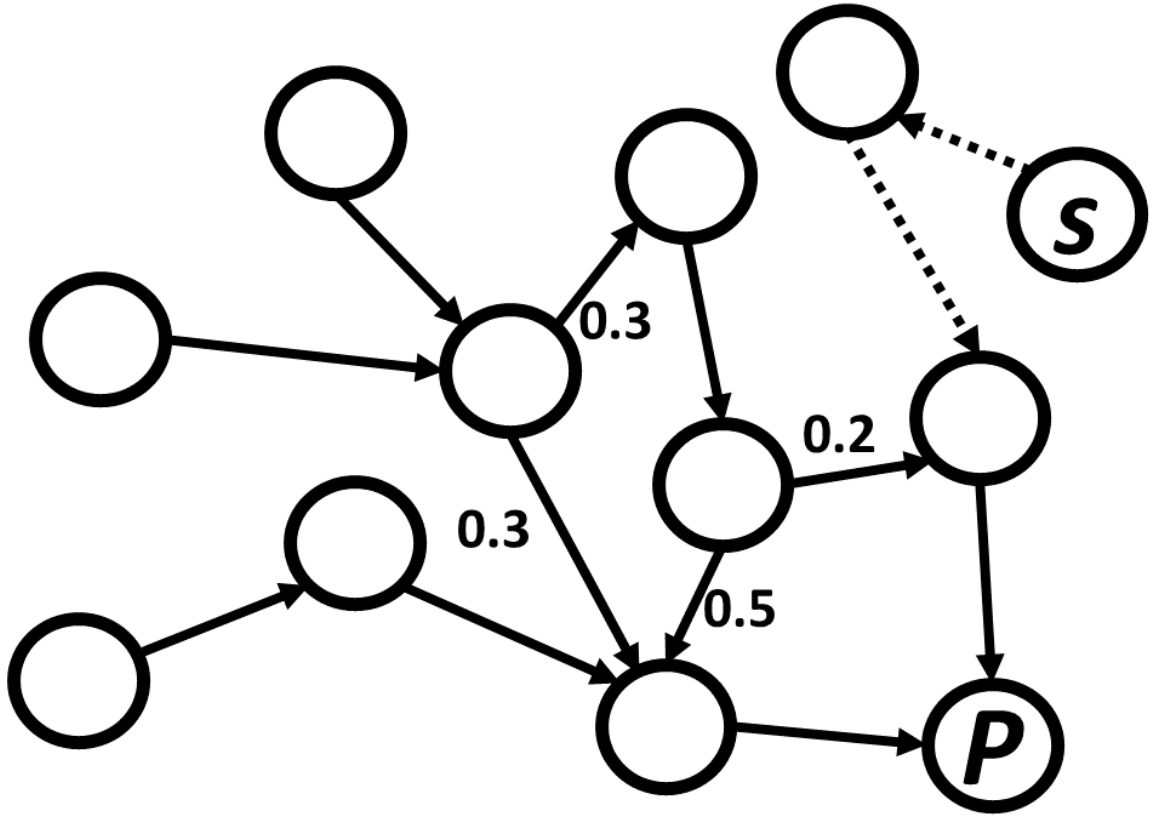

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BGPSim always returns a path, but it might not be totally accurate.

We follow the BGPSim path until we get to *D*, then predict from *D*.

PathCache’s Performance

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Temporal Graph Streaming

AND APPLICATIONS TO DISEASE TRACKING

Joint work with:

Andrew McGregor

Cameron Musco

Time-Dependent Graph Streams

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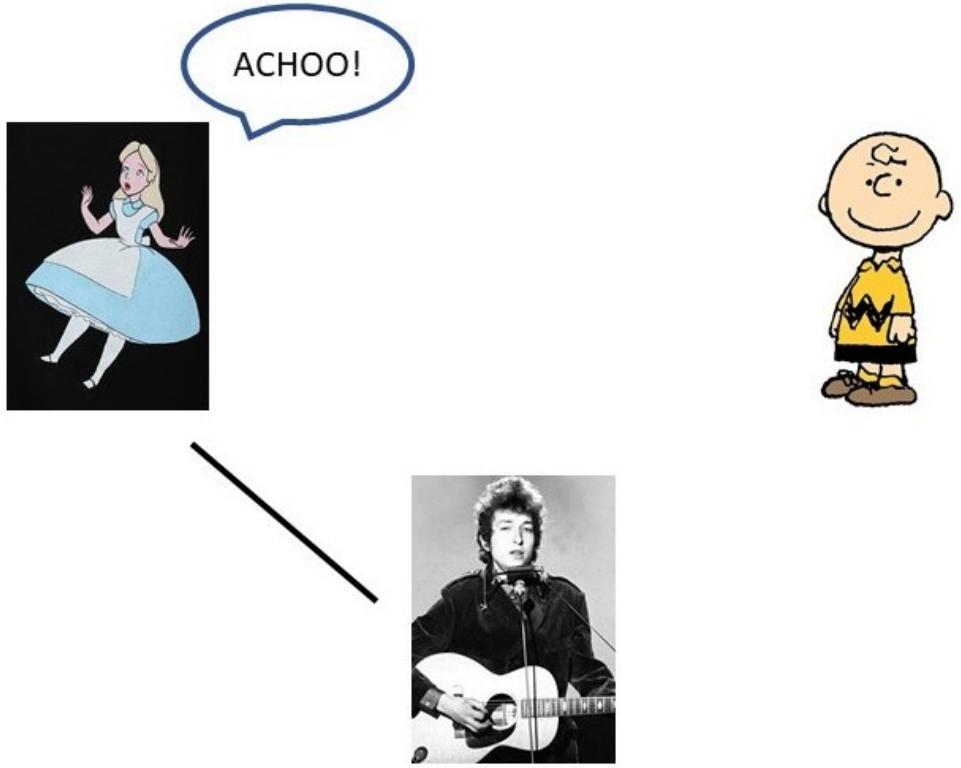

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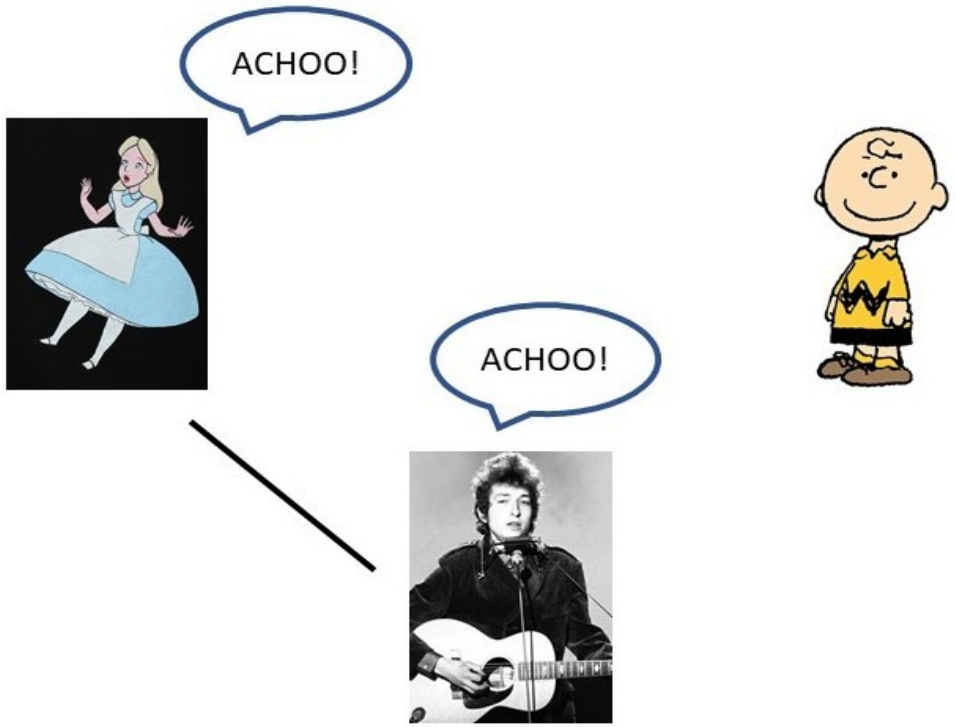

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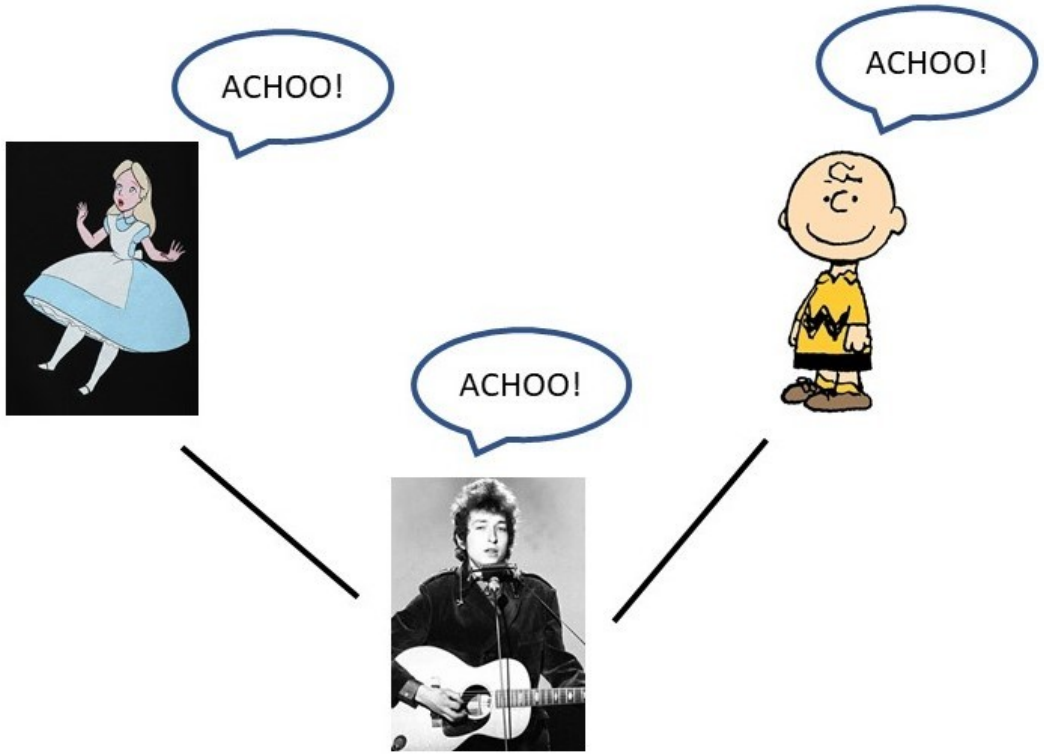

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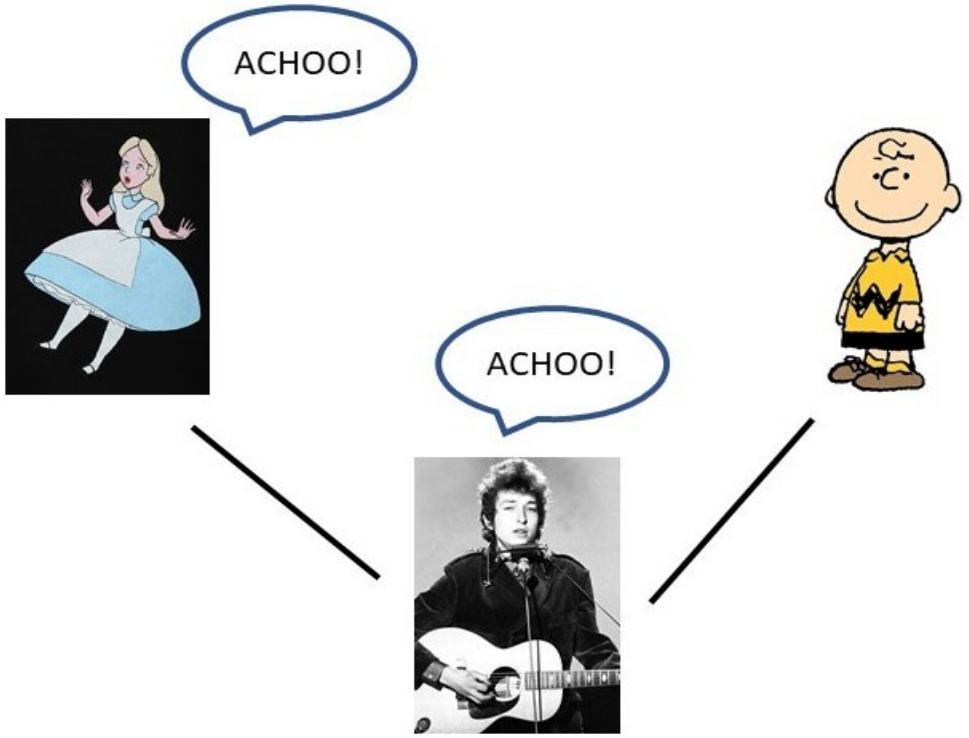

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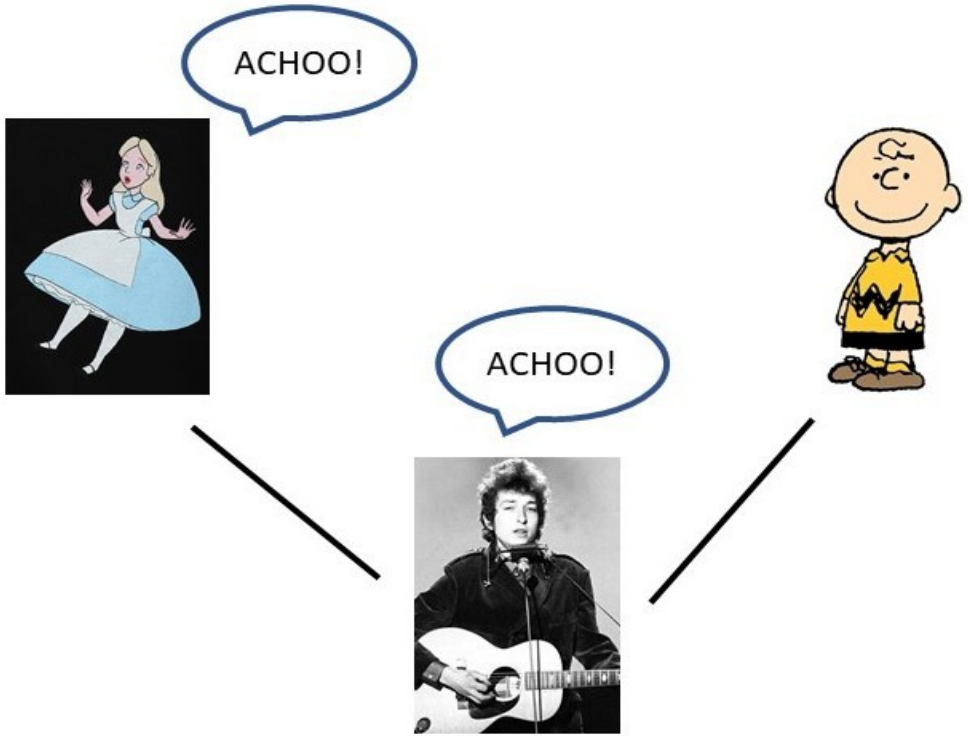

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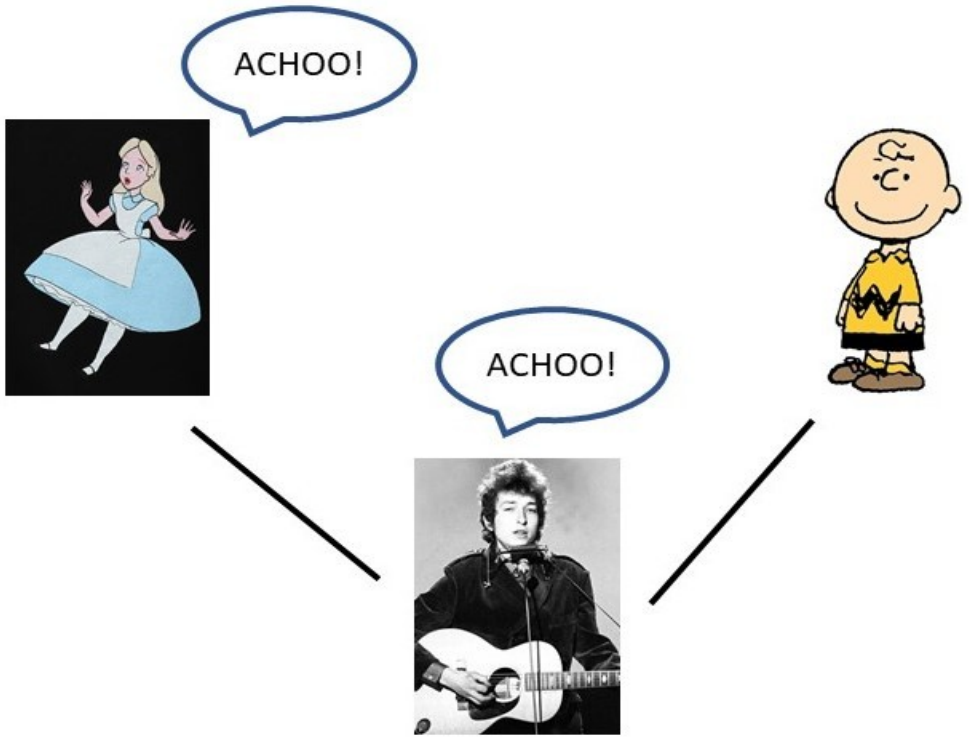

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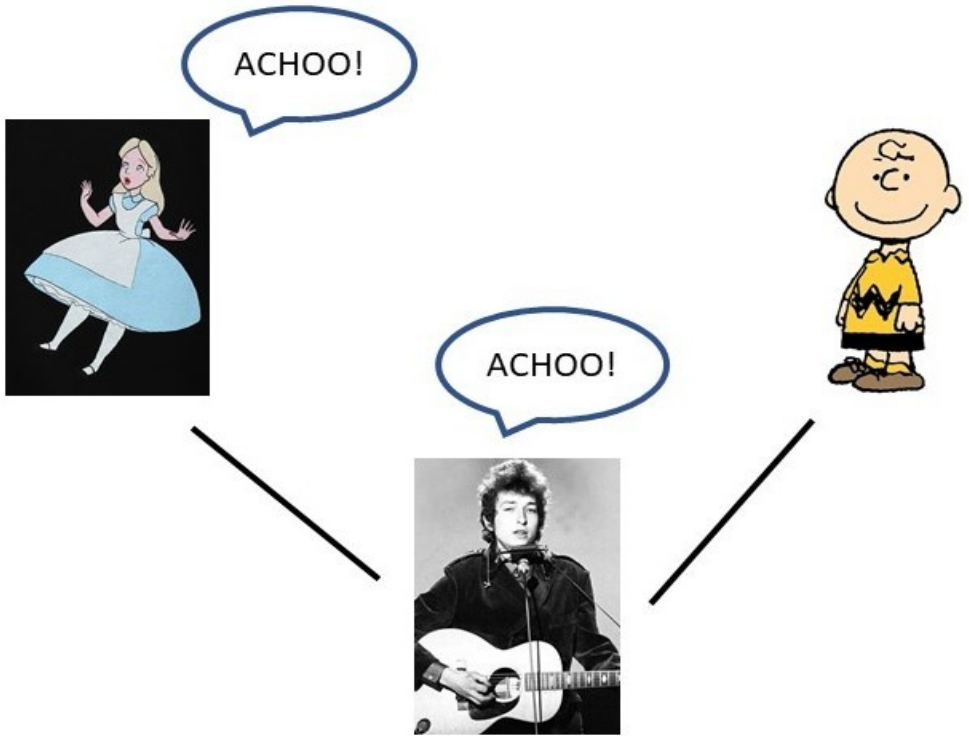

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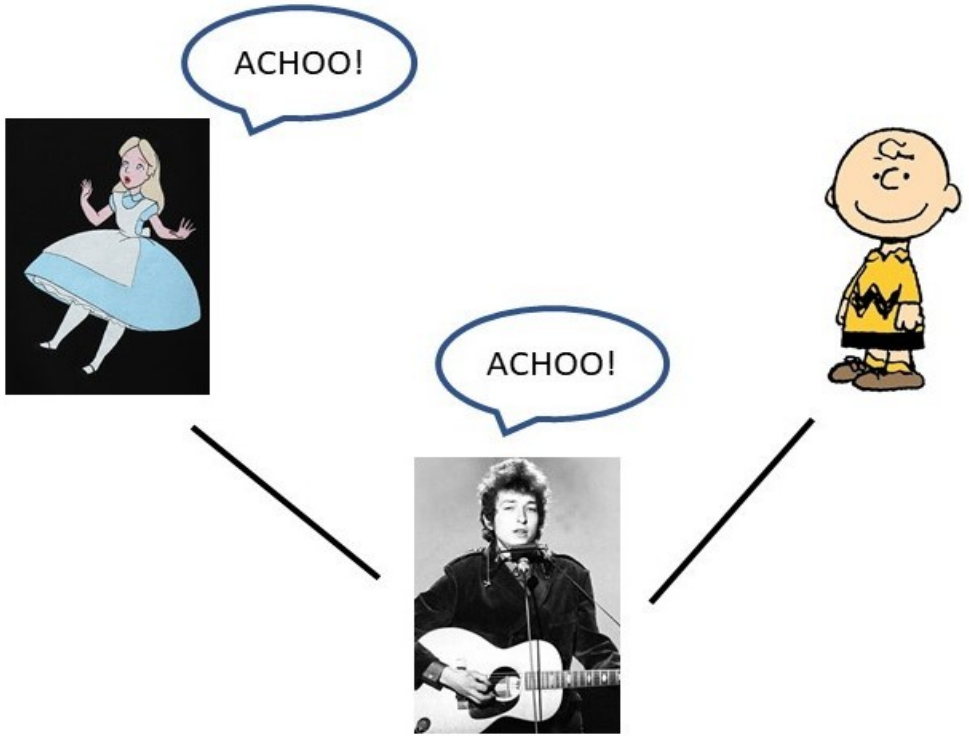

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However, if the edges appear in order we can solve some similar problems.

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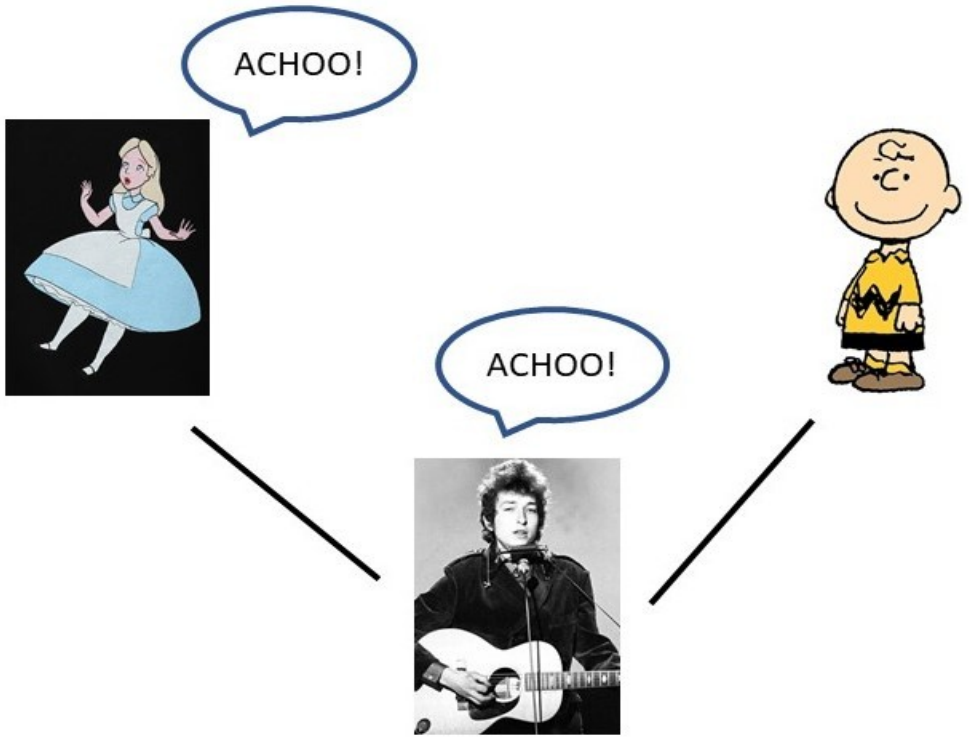

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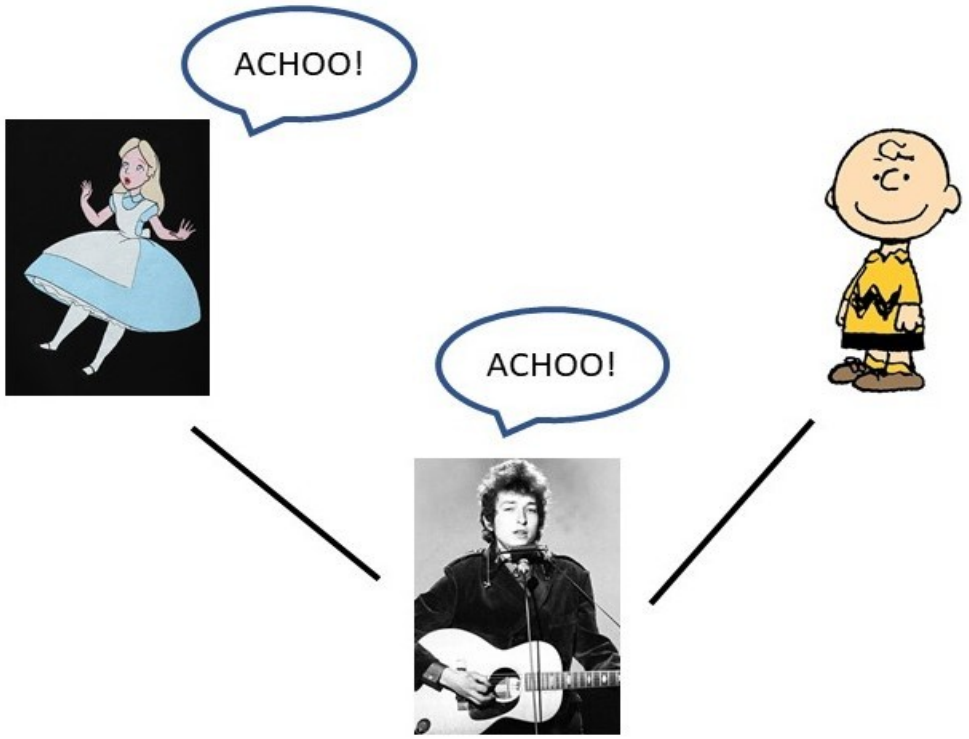

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We can also randomly sample from the set of potential infectors.

Future Research Directions

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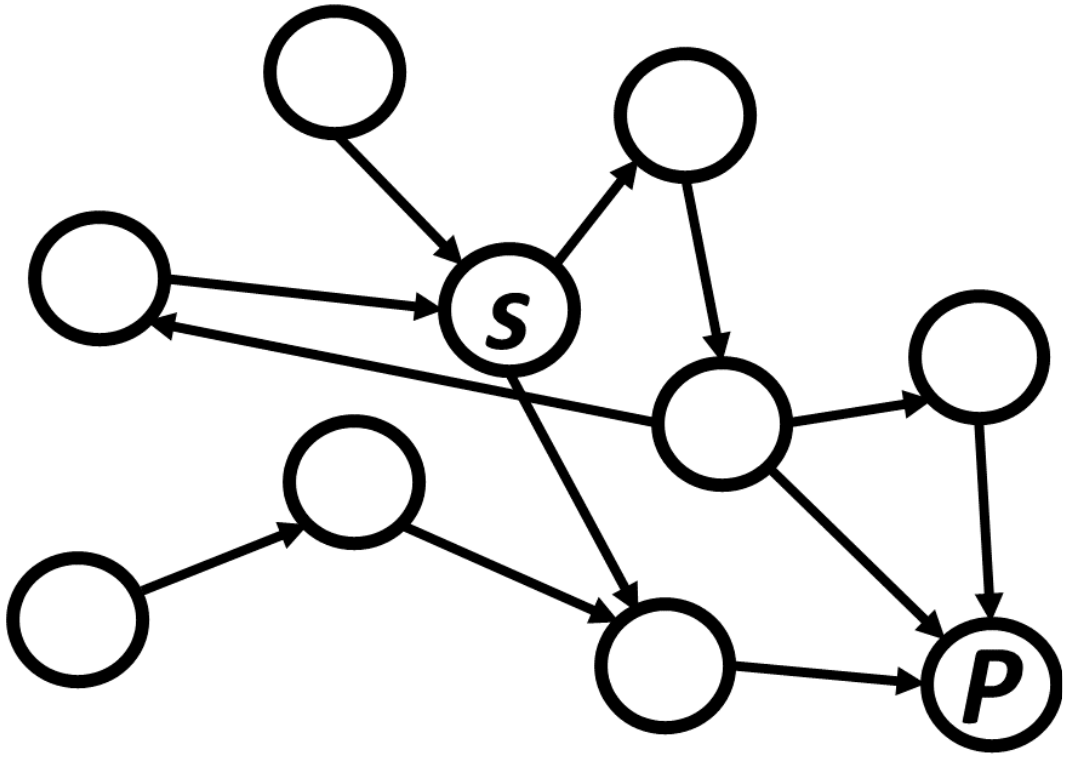
- Can techniques from streaming help utilize large sequentially-accessible working memory to design new data structures?

Cycles in PathCache “DAGs”



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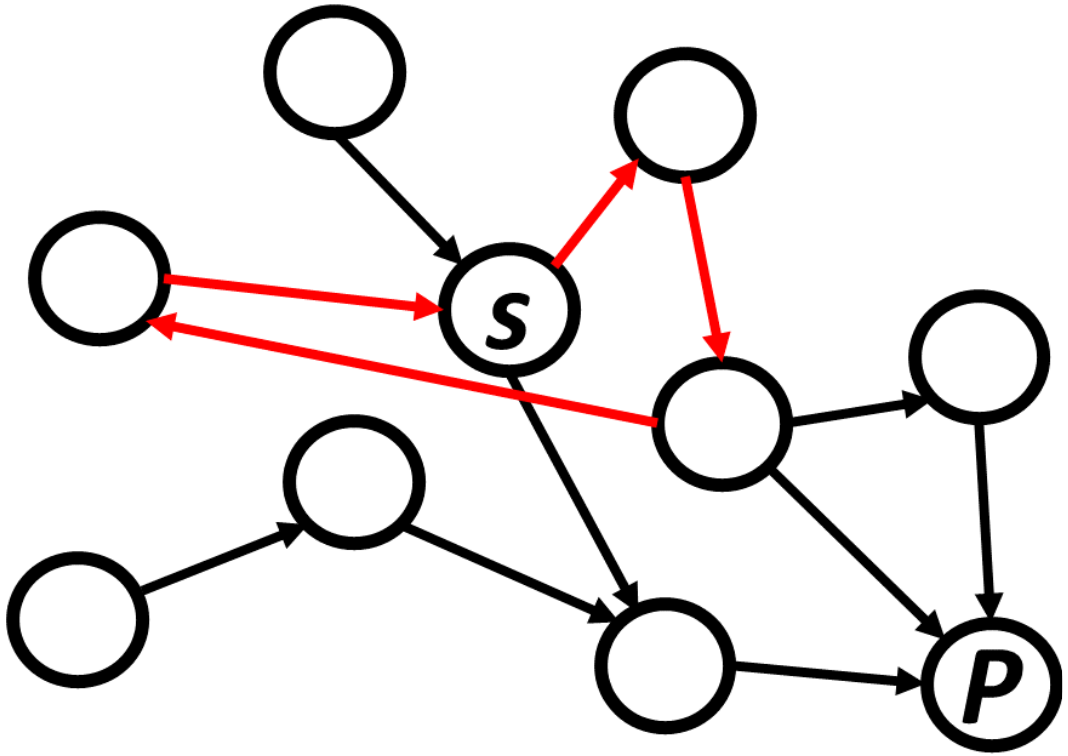

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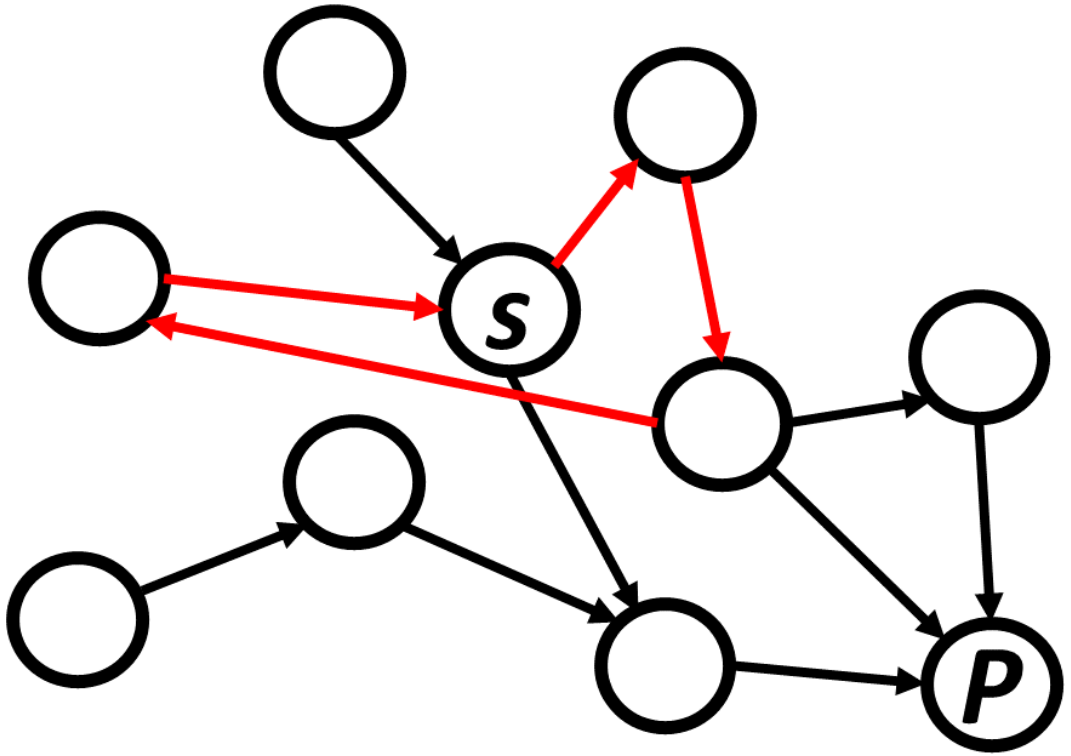

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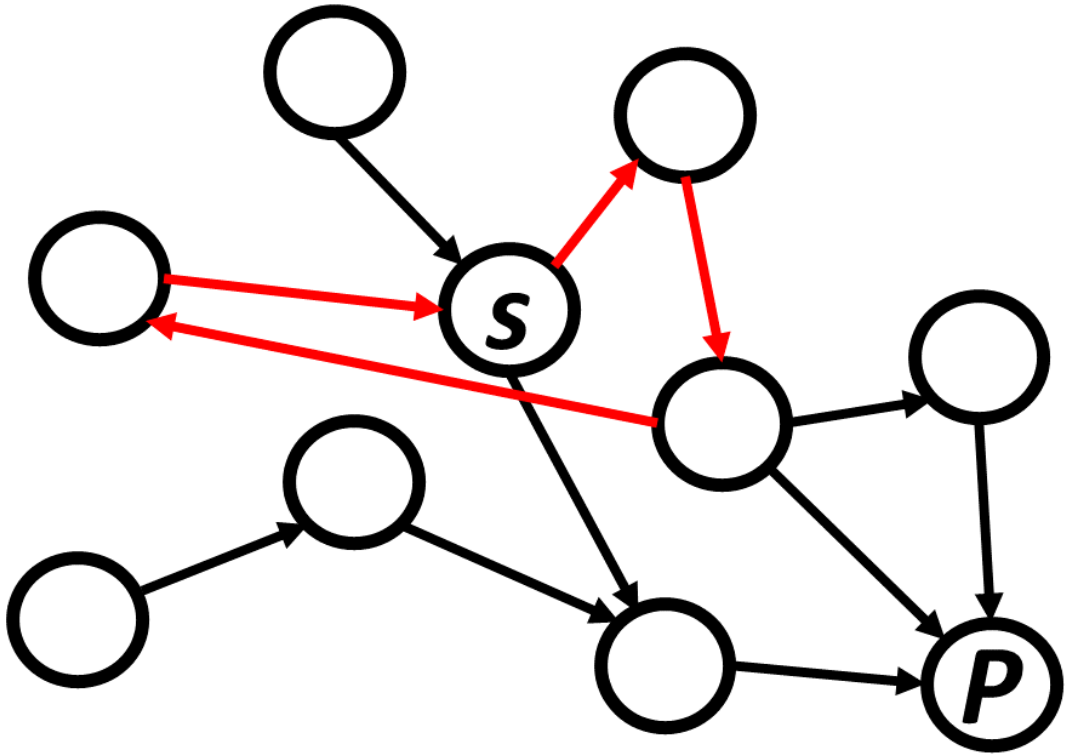

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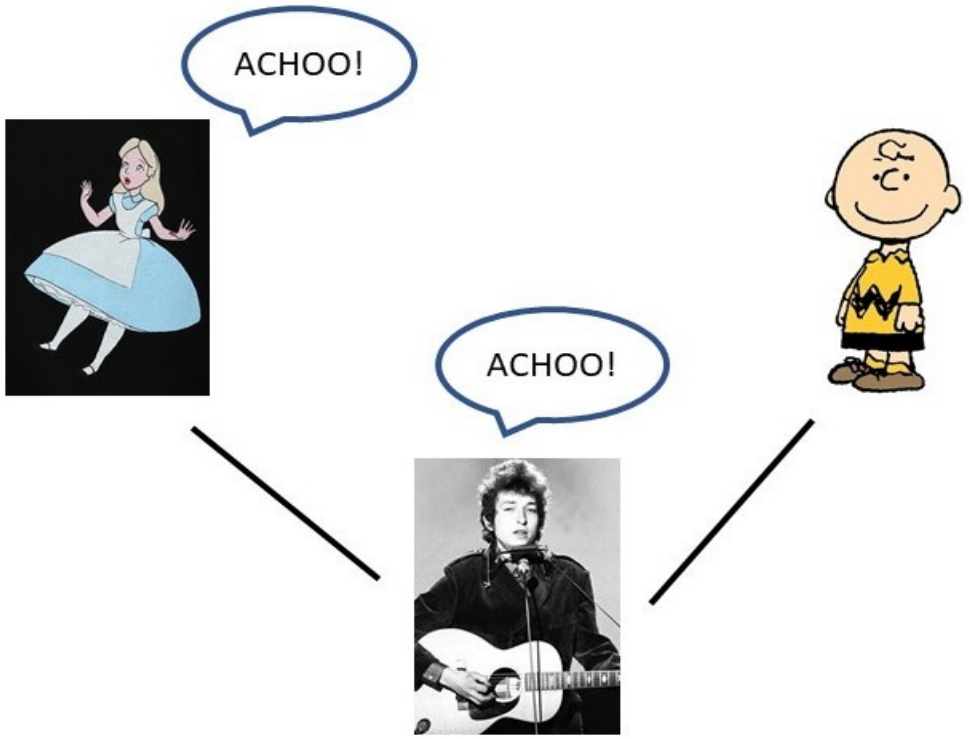
Studying this could reveal new perspectives on Internet routing!

Temporal Graph Streaming



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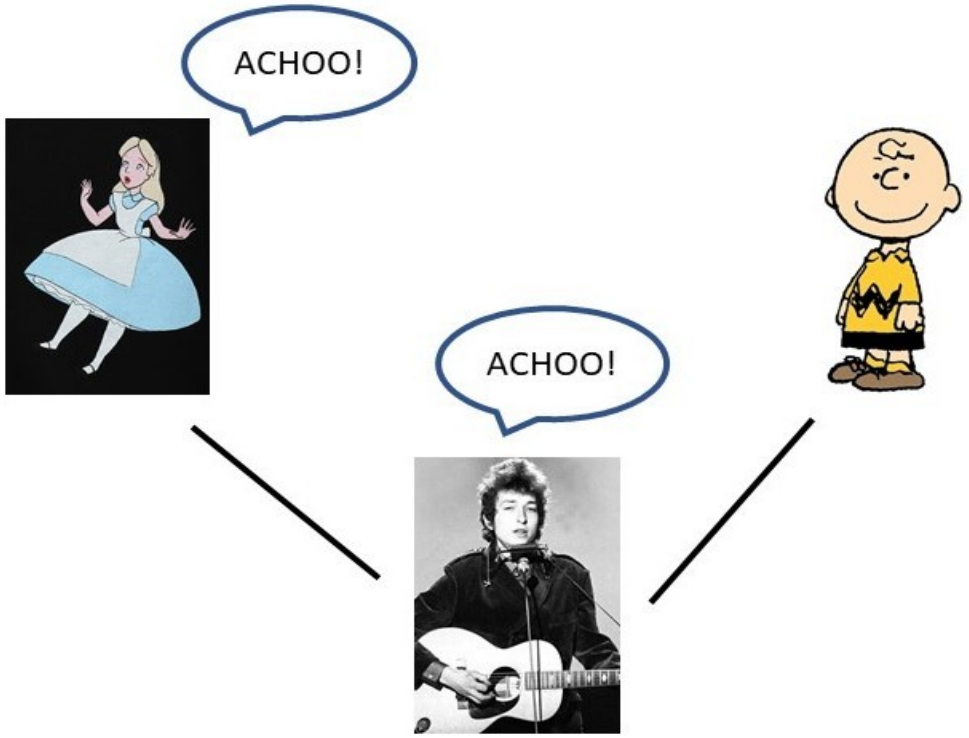

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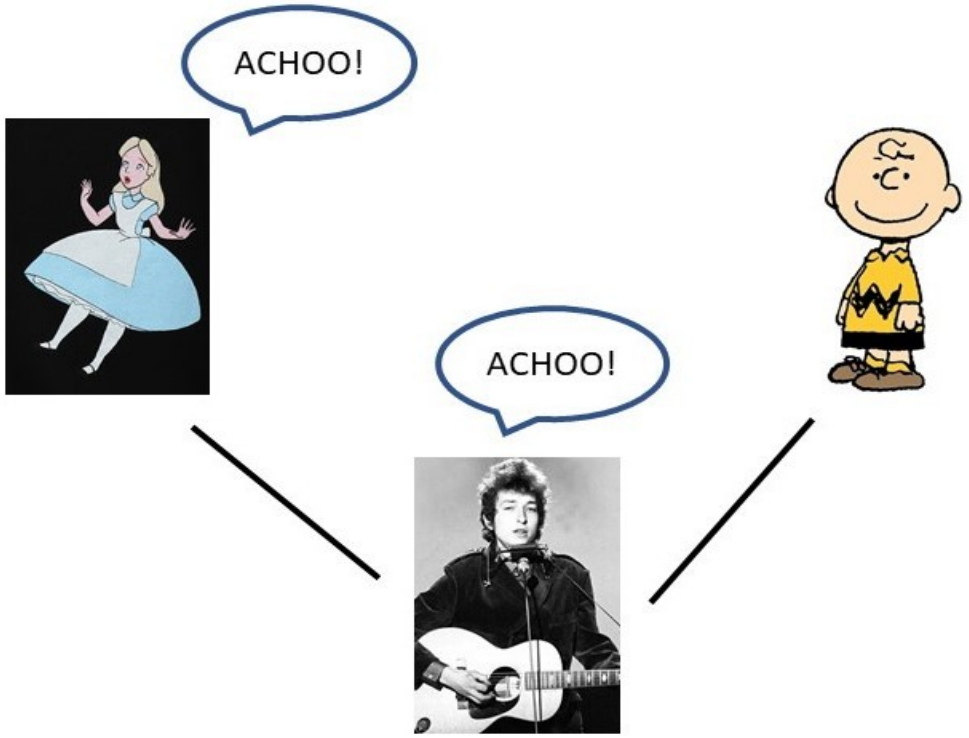

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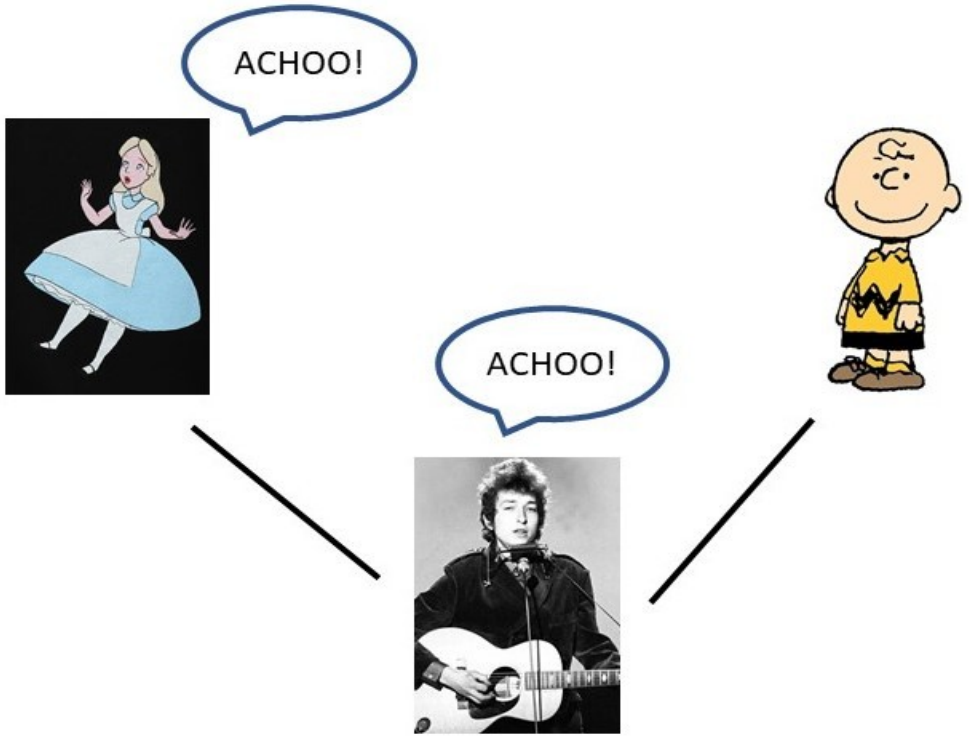

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Can we sparsify temporal graphs while retaining reachability?

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Thanks for listening!