9 Concepts and modes of analysis

9.1 Simple Interest

Simple Interest is the interest paid only on the principal amount borrowed. No interest is paid on the interest accrued during the term of the loan

$$SI = Pnr \; ; \; A = P(1 + nr)$$

$P = ₹10,000$, $n = 8$ months $r = 10\%$

$$SI = 10000 \times \frac{8}{12} \times \frac{10}{100} = Rs\ 1,250$$

9.2 Compound Interest

Compound interest means that the interest calculated on interest. The interest accrued on a principal amount is added back to the principal sum and the whole amount is then treated as new principal, for the calculation of the interest for the next period

$$CA = P(1 + i)^n$$

$P = ₹100.00$; $i = 7.5\%$ compounded quaterly $n = 5$

$$CA = 10000\left(1 + \frac{7.5}{100 \times 4}\right)^{5 \times 4} = Rs\ 14,499.48$$

Comparison with SI

Amount for SI = $P(1+rn)$

$= 10000(1+0.075(5))$

$= ₹\ 13,750$

CI gives ₹ 749.48 more

Power of compounding

Higher rate of return & longer duration
→ exponential growth

Rate of return

5% vs 20% over 25 years shows huge growth due to compounding

9.3 Time value of Money (TVM)

⇒ Definition - Rupee today is worth more than rupee in the future

⇒ Reason - Can earn interest, inflation reduces future purchasing power

⇒ Present value (PV) - Current discounted value of future cash flow

⇒ Future value (FV) - Compounded value of present amount

⇒ Discount factor - PV of ₹ in future

⇒ Compounding factor - FV of ₹1 now

9.3.2 How is time value of money computed?

1. Future value of a single cash flow

=> Discrete intervals

Future value (FV) = Present Value (PV) $(1+r)^t$

$$FV = PV(1+r)^t$$

=> Continuous compounding

$$FV = PV * e^{rt}$$

value of exponential (e) = 2.7183

Eg: Calculate the value of a deposit of ₹2000 made today, 3 years hence if the interest rate is 10%.

By discrete compounding

$$FV = 2,000 * (1+0.10)^3 = ₹ 2,662$$

By continuous compounding

$$FV = 2000 * e^{(0.10*3)} = ₹ 2699.72$$

2) Future value of an annuity

$$FVA = CF * (1+r)^{t-1} + CF(1+r)^{t-2} + \ldots + CF(1+r)^1 + CF$$

Eg: you deposit 3000 annually in a bank for 5yr and your deposit earn a compound interest rate of 10%, what will be value of this series of deposits (an annuity at the end of 5 years) Assume that each deposit occurs at the end of the year

FV of this annuity

$= ₹3000 * (1.10)^4 + ₹3000 * (1.10)^3 + ₹3000 * (1.10)^2$

$+ ₹3000 * (1.10) + ₹3000$

$= Rs\ 18315.30$

$$FVA = CF * \frac{(1+r)^t - 1}{r}$$

$$= ₹3000 * \frac{(1.10)^5 - 1}{0.1}$$

$= ₹3000 * 6.1051$

$= ₹18,315.30$

3 Present value of a single cash flow

$PV = FV / (1+r)^t$ → discrete discounting

Eg: What is the present value of ₹5000 hence if the interest rate is 10% pa

$PV = 5000 / 1.10^3 = ₹3756.57$

$PV = FV * e^{-rt}$ → continuous discounting

$PV = 10000 / e^{(0.1*2)} = ₹8187.297$

4 Present value of an annuity

Discrete discounting

$$PVA = FV \frac{(1+r)^t - 1}{r*(1+r)^t}$$

Continuous discounting

$$PVA = FV * \frac{(1 - e^{-rt})}{r}$$

Eg What is the present value of ₹2000 received at the end of each year for 3 continuous years

$$PVa = 2000 * [1/1.10] + 2000 * [1/1.10]^2 + 2000 * [1/1.10]^3$$

$$= ₹ 4973.704$$

Using discrete discounting formula

$$PVa = FV \frac{(1+r)^t - 1}{r*(1+r)^t}$$

$$= 2000 \times \frac{1.10^3 - 1}{0.1 * 1.10^3}$$

$$= 2000 \times 2.4868$$

$$= ₹ 4973.704$$

1. If the question gives cash flows like this
   ⇒ ₹2000 received at the end of each year
   ⇒ ₹5000 at the end of year 1, year 2, year 3
   ⇒ PV of ₹10000 received after 5 years
   ⇒ Use 10% discount rate
   → Discrete discounting

2. Continuous discounting
   ⇒ Continuous compounded rate
   ⇒ discounted continuously
   ⇒ Use continuous compounding
   ⇒ use e^{-rt}

3. If question mentions "per year" or "at the end of each year" → it is DISCRETE discounting.

9.3.3 Effective Annual Return (EAR)

Actual annual return accounting for intra year compounding

10% pa with quaterly compounding

EAR = 10.38%