- **Continuous:** $E(X) = \mu_X = \int_{-\infty}^{\infty} xf(x)\,dx$
- **Remark**: The expected value exists provided the sum/integral exists

2.6.2 Expectation of a function of an R.V.

$\forall\, g(X)$ with p.f. $f_X(x)$

- **Discrete:** $E[g(X)] = \sum_x g(x) f_X(x)$
- **Continuous:** $E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x)\,dx$
- Provided the sum/integral exists.

2.6.3 Variance ($\sigma_X^2 = V(X)$)

- $g(x) = (x - \mu_X)^2$, Let X be an R.V. with p.f. $f(x)$
- $\sigma_X^2 = V(X) = E[(X - \mu_X)^2]$
- $E[(X-\mu_X)^2] = \begin{cases} \sum_x (x-\mu_X)^2 f_X(x) & \text{if } X \text{ is discrete} \\ \int_{-\infty}^{\infty} (x-\mu_X)^2 f_X(X)\,dx & \text{if } X \text{ is continuous} \end{cases}$
- $V(X) \geq 0$, $V(X) = E(X^2) - [E(X)]^2$
- **Standard deviation** = $\sigma_X = \sqrt{V(X)}$

2.6.4 K-th moment of X

- **Definition:** $E(X^k)$, use $g(x) = x^k$ in expectation of a fn

2.6.5 Properties of Expectation

1. $E(aX + b) = aE(X) + b$
2. $V(X) = E(X^2) - [E(X)]^2$
3. $V(aX + b) = a^2 V(X)$

2.7 Chebyshev's Inequality

- Let X be an R.V. with $E(X) = \mu$, $V(X) = \sigma^2$
- $\forall\, k > 0$, $\Pr(|X - \mu| > k\sigma) \leq \frac{1}{k^2}$
- Alternatively, $\Pr(|X - \mu| \leq k\sigma) \geq 1 - \frac{1}{k^2}$
- Holds for **all** distributions with finite mean and variance
- Gives a **lower bound** but not exact probability.

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3.1 Taylor Series of f at a

$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$

3.2 MacLaurin Series

Taylor series of f at 0, i.e. $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

3.3 List of common MacLaurin Series

- $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, -1 < x < 1, R = 1$
- $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n, -1 < x < 1, R = 1$
- $\frac{1}{1+x^2} = \sum_{n=0}^{\infty} x^2 n, -1 < x < 1, R = 1$
- $ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}, -1 < x < 1, R = 1$
- $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, -\infty < x < \infty, R = \infty$
- $\cos x = \sum_{n=1}^{\infty} \frac{(-1)^n x^2 n}{(2n)!}, -\infty < x < \infty, R = \infty$
- $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, -\infty < x < \infty, R = \infty$
- $\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} x^{2n+1}, -1 \leq x \leq 1, R = 1$
- $\frac{1}{(1-x)^2} = \sum_{n=1}^{\infty} nx^{n-1}, -1 < x < 1, R = 1$
- $\frac{1}{(1-x)^3} = \frac{1}{2} \sum_{n=2}^{\infty} n(n-1)x^{n-2}, -1 < x < 1, R = 1$
- $(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n, -1 < x < 1, R = 1$
- $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ..., -1 < x < 1, R = 1$

3.4 Indefinite Integral

Denoted by $\int f(x)dx = F(x) + C$

3.5 Rules of Indefinite Integration

1. $\int kf(x)dx = k\int f(x)dx$
2. $\int -f(x)dx = -\int f(x)dx$
3. $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$