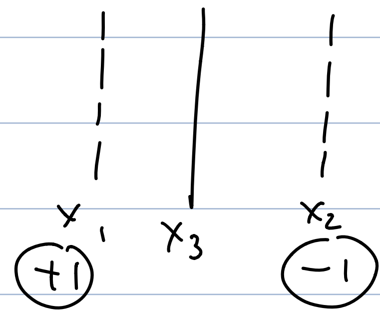
Phang Teng Fone 1003296 DL HW2

1. $x_1 = (-5, 1, 3) \quad y_1 = +1$

$x_2 = (2, 2, -3) \quad y_2 = -1$

$w x_3 + b = 0 \Rightarrow$ hyperplane



$x_3 = \frac{1}{2}(x_1 + x_2)$

$= (-1.5, 1.5, 0)$

$w = x_1 - x_3$

$= (-3.5 \ \ -0.5 \ \ -3)$

$$(-3.5 \ \ -0.5 \ \ -3) \begin{array}{c} x_3 \\ \begin{bmatrix} -1.5 \\ 1.5 \\ 0 \end{bmatrix} \end{array} + b = 0$$

$$b = -4.5$$

$\therefore x_1 w + b = [-3.5 \ \ -0.5 \ \ -3] \begin{bmatrix} -5 \\ 1 \\ 3 \end{bmatrix} - 4.5$

$= 3.5$

$\therefore x_2 w + b = [-3.5 \ \ -0.5 \ \ -3] \begin{bmatrix} 2 \\ 2 \\ -3 \end{bmatrix} - 4.5$

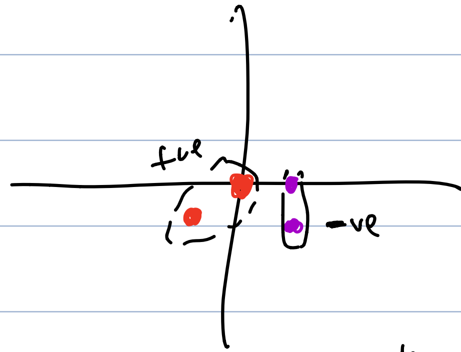
$= -3.5$

$\because$ After normalize.

$f(x) = \frac{1}{3.5} \begin{bmatrix} -3.5 \\ -0.5 \\ -3 \end{bmatrix} x_{new} - \frac{4.5}{3.5}$

2.

$$g(z) = \begin{cases} z & \text{if } |z| \leq 1 \\ \text{sign}(z) & \text{otherwise} \end{cases}$$



$z = w_1 x_1 + w_2 x_2 - w_0$

$z = w_1 x_1 + w_2 x_2 + 1$

+ve = $(0,0)$ $(0,-1)$

-ve = $(1,0)$ $(1,-1)$

By guess & check:

$z = -3x_1 + 0.5x_2 + 1$

$\therefore \underline{z = -3x_1 + \frac{1}{2}x_2 + 1}$ #

3.1 $\dfrac{\partial \log(s(a))}{\partial a} = 1 - s(a)$

LHS: $s(a) = \dfrac{\exp(a)}{1+\exp(a)}$

$= \dfrac{1}{e^{-a}+1}$

$\log(s(a)) = \log\left(\dfrac{1}{e^{-a}+1}\right)$

$= \cancel{\log(1)} - \log(e^{-a}+1)$

$= -\log(e^{-a}+1)$

$\dfrac{\partial \log(s(a))}{\partial a} = -\dfrac{1}{e^{-a}+1}(-e^{-a})$

$= \dfrac{e^{-a}}{e^{-a}+1}$

$\equiv 1 - \dfrac{1}{e^{-a}+1}$

$\therefore 1 - s(a)$ # (PROVED)

$$\log(1-s(a)) = \log\left(\frac{e^{-a}}{e^{-a}+1}\right)$$
$$= \log(e^{-a}) - \log(e^{-a}+1)$$
$$= -a - \log(e^{-a}+1)$$

$$\frac{\partial \log(1-s(a))}{\partial a} = -1 - \left[\frac{1}{e^{-a}+1} \cdot -e^{-a}\right]$$

$$= -1 + \frac{e^{-a}}{e^{-a}+1}$$

$$= -\left(\frac{e^{-a}+1}{e^{-a}+1}\right) + \frac{e^{-a}}{e^{-a}+1}$$

$$= \frac{-e^{-a} - 1 + e^{-a}}{e^{-a}+1}$$

$$= -\frac{1}{e^{-a}+1}$$

$$= -s(a) \quad \text{(PROVE)}$$

3.2 $\nabla_w L = \sum_{i=1}^{n} x_i (s(w \cdot x_i) - y_i) = \sum_{i=1}^{n} x_i (h(x_i) - y_i) \quad \because h(x) = s(w \cdot x)$

LHS: $L = (-1) \cdot \sum_{i=1}^{n} y_i \log[h(x_i)) + (1-y_i)\log(1-h(x_i)]$

Sub $h(x_i)$ into $s(a)$ from 3.1

$L = (-1) \sum_{i=1}^{n} y_i \log[s(w \cdot x_i) + (1-y_i)\log(1-s(w \cdot x_i))]$

$$\nabla_w L = (-1)\sum_{i=1}^{n} y_i \left[(1 - S(w \cdot x_i))(x_i) + (1-y_i)(-S(w_i x_i))x_i \right]$$

$$= \sum_{i=1}^{n} -x_i \left(y_i - \cancel{y_i S(w \cdot x_i)} - S(w \cdot x_i) + \cancel{y_i S(w \cdot x_i)} \right)$$

$$= \sum_{i=1}^{n} -x_i \left(y_i - S(w \cdot x_i) \right)$$

$$= \sum_{i=1}^{n} x_i \left(S(w \cdot x_i) - y_i \right)$$

$$= \sum_{i=1}^{n} x_i \left(h(x_i) - y_i \right) \text{ ※ (PROVE)}$$

4. $$\frac{\partial L}{\partial n_4} = \frac{\partial L}{\partial n_2} \cdot \frac{\partial n_2}{\partial n_4} + \frac{\partial L}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_4}$$

$$\frac{\partial L}{\partial n_5} = \frac{\partial L}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_5} = \frac{\partial L}{\partial n_2} \cdot \frac{\partial n_2}{\partial n_4} \frac{\partial n_4}{\partial n_5} + \frac{\partial L}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_5}$$

$$\frac{\partial L}{\partial n_6} = \frac{\partial L}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_6} + \frac{\partial L}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_6} = \frac{\partial L}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_6} + \frac{\partial L}{\partial n_2} \cdot \frac{\partial n_2}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_6} + \frac{\partial L}{\partial n_3} \frac{\partial n_3}{\partial n_4} \frac{\partial n_4}{\partial n_6}$$

$$\frac{\partial L}{\partial n_7} = \frac{\partial L}{\partial n_5} \cdot \frac{\partial n_5}{\partial n_7} + \frac{\partial L}{\partial n_6} \cdot \frac{\partial n_6}{\partial n_7}$$

$$= \frac{\partial L}{\partial n_2} \cdot \frac{\partial n_2}{\partial n_4} \frac{\partial n_4}{\partial n_5} \frac{\partial n_5}{\partial n_7} + \frac{\partial L}{\partial n_3} \cdot \frac{\partial n_3}{\partial n_4} \cdot \frac{\partial n_4}{\partial n_5} \frac{\partial n_5}{\partial n_7} +$$

$$\frac{\partial L}{\partial n_3}\cdot\frac{\partial n_3}{\partial n_6}\frac{\partial 6}{\partial n_7}+\frac{\partial L}{\partial n_2}\cdot\frac{\partial n_2}{\partial n_4}\cdot\frac{\partial n_4}{\partial n_6}\frac{\partial n_6}{\partial n_7}+\frac{\partial L}{\partial n_3}\frac{\partial n_3}{\partial n_4}\frac{\partial n_4}{\partial n_6}\frac{\partial n_6}{\partial n_7}$$

$$\frac{\partial L}{\partial x}=\frac{\partial L}{\partial n_7}\cdot\frac{\partial n_7}{\partial x}$$

$$=\frac{\partial L}{\partial n_2}\cdot\frac{\partial n_2}{\partial n_4}\frac{\partial n_4}{\partial n_5}\frac{\partial n_5}{\partial n_7}\frac{\partial n_7}{\partial n_x}+\frac{\partial L}{\partial n_3}\cdot\frac{\partial n_3}{\partial n_4}\cdot\frac{\partial n_4}{\partial n_5}\frac{\partial n_5}{\partial n_7}\frac{\partial n_7}{\partial n_x}+$$

$$\frac{\partial L}{\partial n_3}\cdot\frac{\partial n_3}{\partial n_6}\frac{\partial 6}{\partial n_7}\frac{\partial n_7}{\partial n_x}+\frac{\partial L}{\partial n_2}\cdot\frac{\partial n_2}{\partial n_4}\cdot\frac{\partial n_4}{\partial n_6}\frac{\partial n_6}{\partial n_7}\frac{\partial n_7}{\partial n_x}$$

$$+\frac{\partial L}{\partial n_3}\frac{\partial n_3}{\partial n_4}\frac{\partial n_4}{\partial n_6}\frac{\partial n_6}{\partial n_7}\frac{\partial n_7}{\partial n_x}$$

5

```python
#Q5.1
q1a = torch.randn(3,2,5)
q1b = torch.randn(3)
torch.einsum('ijk,i->jk',[q1a,q1b])
```

```
tensor([[-1.2061, -1.1289,  3.2449,  2.0738, -0.5003],
        [-1.0436, -0.5560, -0.4786,  2.1590, -2.8435]])
```

```python
#Q5.2
q2 = torch.randn(3,2,5,3)
torch.einsum('ijkl->ik',[q2])
```

```
tensor([[-6.7280, -0.6500, -0.7285,  2.2631, -1.4154],
        [ 2.4351,  1.3366, -4.5421, -0.2333,  4.3568],
        [ 1.2702,  1.9210,  0.1328, -1.2590,  0.4741]])
```

```python
#Q5.3
q3 = torch.randn(3,2,5,3)
torch.einsum('ijkl->ki',[q3])
```

```
tensor([[ 0.0547,  3.6758, -0.9661],
        [-2.4279,  0.6336, -5.0624],
        [-4.4450,  0.0596,  1.5150],
        [ 2.8499,  1.4035,  1.6870],
        [-2.2888, -0.8119, -4.2149]])
```

```python
#Q5.4
q4 = torch.randn(3,2,5)
torch.einsum('ijk,ijk->i',[q4,q4])
```

```
tensor([19.4891, 15.3892,  7.2393])
```

```python
#Q5.5
q5a = torch.arange(6).reshape(2, 3)
q5g = torch.arange(6).reshape(2, 3)
q5gt = q5g.t()
q5b = torch.arange(6).reshape(2, 3)
torch.einsum('de,ef,fl->dl',[q5a,q5gt,q5b])
```

```
tensor([[ 42,  61,  80],
        [150, 214, 278]])
```

2-tensor order