3 Jan 2022 L1 Intro to Heat Transfer CHEME340

<<< Intro to Heat Transfer

- thermodynamics - systems at eqm
- heat transfer - how fast system reach eqm, temperature profile formed
    - controlling reaction temperature
    - cooling material before packaging
    - heat generated in circuit
    - space travel

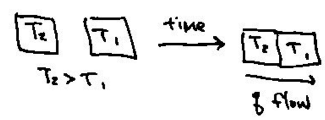
<<< Heat Transfer is derived from Thermodynamics

→ 1st law

- energy is conserved
- In - out + generation - consumption = accumulation
    - generation - rate of thermal energy generation (e.g. electric → thermal E)
    - accumulation - rate of energy stored (e.g. raising temperature)

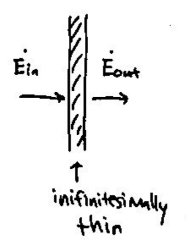
→ 2nd law

- heat flows from hot to cold unless input of work
- work can be completely converted to heat
- heat cannot be completely converted to work
- all heat transfer process are not in eqm and generate entropy in universe

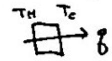


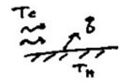
<<< Surface Energy Balance

- Boundary of system has no volume (∞ thin)
    - No generation
    - No accumulation
    - $\boxed{\dot{E}_{in} = \dot{E}_{out}}$



<<< Modes of Heat Transport

- > heat transport – thermal energy in transit due to a spatial temperature gradient / difference
  - > conduction – thermal energy in transit through a stationary substance 

  - > convection – conduction + advection 

    - > advection – transfer of heat by flow of fluid
  - > radiation – photons (EM wave) radiated and absorbed from one body to another

<<< Conduction

· Heat flux is proportional to temperature difference by Fourier's law

$$\boxed{\vec{q}\,'' = -k\nabla T}$$
Fourier's law

$$\boxed{q_x'' = -k\frac{dT}{dx}}$$
(isotropic)

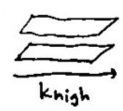
- · $\vec{q}\,''$ [W/m^2] – heat flux, amount of heat flowing thru surface
- · k [$W/(m\cdot K)$] – thermal conductivity, rate of heat flow thru material due to temp gradient
- · ∇T [K] – temperature gradient

· Heat rate is heat flux times area

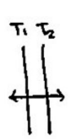
$$\boxed{\vec{q} = \int \vec{q}\,''\, dA}$$

- · $\vec{q}$ [W] – heat rate
- · $\vec{q}\,''$ [W/m^2] – heat flux

· Mechanism of conduction
- · gas – translation of molecules
- · liquid – series of collision
- · metal – movement of electron
- · solid / crystal – vibration waves (phonon)

· k can be different in different directions, needing tensor valued K
- · e.g. graphene 

EX1 Calculate heat flux between two sides of a sheet of Al ($k = 200\ W/m\cdot K$) with thickness 0.01m and temperature difference 100°C.



$k = 200 W/m\cdot K$
$\Delta x = 0.01 m$
$\Delta T = 100°C = 100K$

$$q'' = -k\frac{dT}{dx} = -k\frac{\Delta T}{\Delta x}$$
$$= -(200 W/m\cdot K)\ \frac{100K}{0.01m}$$
$$= -2\times 10^6\ \frac{W}{m^2}$$

<<< Convection

- fluid flow draws heat away faster

> Newton's law of cooling

$$\boxed{q_y'' = h(T_s - T_\infty)}$$

- q'' [W/m²] - heat flux
- h [W/m²·K] - convection heat transfer coefficient

- h is experimentally determined for diff conditions

  - free convection { gas 2-25, liquid 50-1000
  - forced convection { gas 25-250, liquid 100-20000
  - phase change : 2500-100000

- h>0 for the above equation

→ Types of Convection

> free (natural) convection - gravity causes fluid to flow, where flow removes heat
  - e.g. buoyant force cause air to flow when heated

> forced convection - flow caused by external driving force
  - e.g. pump, fan

> phase change - flow caused by phase change
  - e.g. boiling of H_2O

<<< Radiation

→ Origin

- All mass contains charged particles (e.g. protons, e-)
- When charged particles accelerate / change direction, they emit EM waves (photon)
- Any mass with finite temp has motion & change direction
  - Any mass with finite temp must emit EM radiation (photon)
  - The faster they move, the more radiation

> Planck's law - equation that relates temperature to the spectra of radiation

→ Stefan-Boltzman's law

> $$\boxed{E_b = \varepsilon \sigma T_s^4}$$

- E_b [W/m²] - emissive power, total radiation of all wavelength
- ε ($\varepsilon \in [0,1]$) - emissivity factor
- $\sigma = 5.67 \times 10^{-8}$ W/m²K⁴ - Stefan-Boltzman constant
- T_s [K] - surface temperature

> Blackbody - ideal radiator with $\varepsilon = 1$

> emissivity factor - how efficiently a surface emits energy

<<< Radiation

→ Radiation Absorption

> $\boxed{G_{abs} = \alpha G}$

- G_{abs} [W/m^2] - absorbed power, rate of radiation absorbed / unit area
- G [W/m^2] - irradiation, rate of radiation incident on surface / unit area
- $\alpha \in [0,1]$ - absorptivity

> gray surface - $\varepsilon = \alpha$

→ Net radiation heat flux

- $q''_{rad} = E_b - G_{abs}$

$$\boxed{q''_{rad} = \varepsilon \sigma T_s^4 - \alpha G}$$

- If gray surface, surrounded by T_{surr}, in vaccum

$$\boxed{q''_{rad} = \varepsilon \sigma \left(T_s^4 - T_{surr}^4\right)}$$

- Helpful to write in rate law form

$$q''_{rad} = h_{rad}(T_s - T_{surr}) \qquad \text{where} \qquad h_{rad} \equiv \varepsilon \sigma (T_s + T_{surr})(T_s^2 + T_{surr}^2)$$

so we can combine convection and radiation

$$q'' = q''_{conv} + q''_{rad}$$

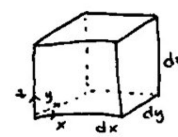
$$= (h + h_{rad})(T_s - T_{surr})$$

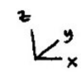
<<< Conduction Geometry

- heat flux $\vec{q}'' = -k\nabla T$
- heat rate $\vec{q} = \int \vec{q}''\, dA$

→ Cartesian Coordinate

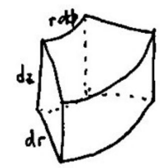
- x-dir $q_x'' = -k\frac{\partial T}{\partial x}$ $dA = dy\,dz$
- y-dir $q_y'' = -k\frac{\partial T}{\partial y}$ $dA = dx\,dz$
- z-dir $q_z'' = -k\frac{\partial T}{\partial z}$ $dA = dx\,dy$

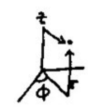




→ Cylindrical Coordinate

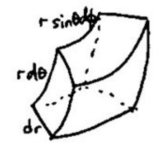
- r-dir $q_r'' = -k\frac{\partial T}{\partial r}$ $dA = r\,d\phi\,dz$
- φ-dir $q_\phi'' = -k\frac{1}{r}\frac{\partial T}{\partial \phi}$ $dA = dr\,dz$
- z-dir $q_z'' = -k\frac{\partial T}{\partial z}$ $dA = r\,dr\,d\phi$





→ Spherical Coordinate

- r-dir $q_r'' = -k\frac{\partial T}{\partial r}$ $dA = r^2\sin\theta\, d\theta\, d\phi$
- θ-dir $q_\theta'' = -k\frac{1}{r}\frac{\partial T}{\partial \theta}$ $dA = r\sin\theta\, dr\, d\phi$
- φ-dir $q_\phi'' = -k\frac{1}{\sin\theta}\frac{1}{r}\frac{\partial T}{\partial \phi}$ $dA = r\,dr\,d\theta$



$\phi \in [0, 2\pi]$
$\theta \in [0, \pi]$

$\boxed{\text{Ex1}}$ Find the surface area for r-dir transfer in cylinder

$$A = \int_0^h \int_0^{2\pi} r\, d\phi\, dz = 2\pi r h$$

$\boxed{\text{Ex2}}$ Find the surface area for r-dir transfer in sphere

$$A = \int_0^{2\pi}\int_0^{\pi} r^2 \sin\theta\, d\theta\, d\phi = 4\pi r^2$$

<<< Conduction Example

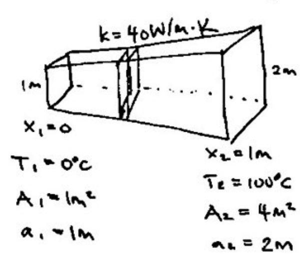
$\boxed{\text{Ex3}}$ Consider a square prism of length 1m with area $1m^2$ in one face and $4m^2$ on the other. With 0°C on small face and 100°C on other face. Calculate heat flow if $k = 40\,W/m\cdot K$.

- Shell balance suggests heat rate is constant

$$\text{in} - \text{out} + \cancel{\text{generation}}^{\,0} = \cancel{\text{accumulation}}^{\,0}$$

$$\text{in} = \text{out}$$

$$q_x = q_{x+\Delta x} = \text{const.}$$



- By Fourier's law,

$$q_x = -k\frac{dT}{dx}A \quad , \text{where } A = (x+1)^2$$

$$q_x = -k\frac{dT}{dx}(x+1)^2$$

$$\int_0^1 \frac{q_x}{(x+1)^2}dx = \int_{273}^{373} -k\,dT$$

$$q_x\left[\frac{1}{x+1}\right]_0^1 = (100K)\,k$$

$$q_x = \frac{1}{-0.5m^{-1}}(100K)(40W/m\cdot K) = -8000\,W$$

Generalized Heat Equation

- gives temperature distribution

→ Cartesian Coordinates

In − out + Generation = Accumulation

$$(q_x - q_{x+dx}) + (q_y - q_{y+dy}) + (q_z - q_{z+dz}) + \dot{q}\,dx\,dy\,dz = \rho c_p \frac{\partial T}{\partial t}\,dx\,dy\,dz$$

$$-\frac{\partial q_x}{\partial x}dx - \frac{\partial q_y}{\partial y}dy - \frac{\partial q_z}{\partial z}dz + \dot{q}\,dx\,dy\,dz = \rho c_p \frac{\partial T}{\partial t}\,dx\,dy\,dz$$

$$\frac{-\frac{\partial}{\partial x}\left(-k\frac{\partial T}{\partial x}dy\,dz\right)dx - \frac{\partial}{\partial y}\left(-k\frac{\partial T}{\partial y}dx\,dz\right)dy - \frac{\partial}{\partial z}\left(-k\frac{\partial T}{\partial z}dx\,dy\right)dz + \dot{q}\,dx\,dy\,dz}{dx\,dy\,dz} = \frac{\rho c_p \frac{\partial T}{\partial t}\,dx\,dy\,dz}{dx\,dy\,dz}$$

$$\boxed{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}}$$

$$\boxed{\nabla^2 T + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}, \quad \boxed{\alpha = \frac{k}{\rho c_p}} \text{ if } k \text{ is isotopic}$$

- From CHEME 330 L17 (28 Oct 2021), we had similar derivation using shell balance.
  - $\boxed{\nabla \cdot q'' = \dot{q} - \rho c_p \frac{\partial T}{\partial t}}$ · $\dot{q}$ is volumetric generation
  - $\boxed{\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \frac{\dot{q}}{\rho c_p}}$

→ Cylindrical Coordinates

$$\boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(k r \frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}}$$

→ Spherical Coordinates

$$\boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left(k r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(k \sin\theta \frac{\partial T}{\partial \theta}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}}$$

EX1 Find the temperature profile and q_r leaving a cylinder (steady-state, no generation)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k r \frac{\partial T}{\partial r}\right) + \underbrace{\cancel{\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)}}_{\text{no } \phi\text{-dir}} + \underbrace{\cancel{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}}_{\text{no } z\text{-dir}} + \underbrace{\cancel{\dot{q}}}_{\text{no generation}} = \underbrace{\cancel{\rho c_p \frac{\partial T}{\partial t}}}_{\text{steady-state}}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k r \frac{\partial T}{\partial r}\right) = 0$$

$$k r \frac{\partial T}{\partial r} = c_1$$

$$\int \partial T = \int \frac{c_1}{k}\frac{1}{r}\partial r$$

$$T = \frac{c_1}{k}\ln(r) + c_2 \qquad \text{b.c.} \begin{cases} T(R_i) = T_i \\ T(R_o) = T_o \end{cases}$$

$$\boxed{T(r) = \frac{T_i - T_o}{\ln\left(\frac{R_i}{R_o}\right)}\ln\left(\frac{r}{R_o}\right) + T_o}$$

$$q_r' = -k\frac{\partial T}{\partial r}A \qquad A = 2\pi r L$$

$$\boxed{q_r = \frac{2\pi L k (T_i - T_o)}{\ln\left(\frac{R_o}{R_i}\right)}} \text{ is a constant, but } q'' \text{ is not.}$$

<<< Thermal Resistance

- Thermal resistance and equivalent thermal circuit can solve 1D heat transport thru many materials
- Write heat rate in terms of driving force, solve for resistance R
  - $q = -k \frac{\Delta T}{\Delta x} A = -\frac{\Delta T}{R} = \frac{\text{driving force}}{\text{resistance}}$

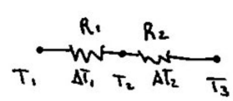
    $R = \frac{\Delta x}{kA}$
  - $\boxed{q = -\frac{\Delta T}{R} = \frac{T_1 - T_2}{R}}$

Mode of Transport	Heat rate	Heat rate (resistance)	Resistance
Conduction (Cartesian)	$q = \frac{kA}{L}(T_1 - T_2)$	$q = \frac{T_1 - T_2}{R}$	$R = \frac{L}{kA}$
Conduction (Cylindrical)	$q = \frac{2\pi L k}{\ln(R_o/R_i)}(T_i - T_o)$	$q = \frac{T_i - T_o}{R}$	$R = \frac{\ln(R_o/R_i)}{2\pi L k}$
Conduction (Spherical)	$q = \frac{4\pi k}{\frac{1}{R_i} - \frac{1}{R_o}}(T_i - T_o)$	$q = \frac{T_i - T_o}{R}$	$R = \frac{\frac{1}{R_i} - \frac{1}{R_o}}{4\pi k}$
Convection	$q = h(T_s - T_\infty)A$	$q = \frac{T_s - T_\infty}{R}$	$R = \frac{1}{hA}$
Radiation	$q = h(T_s - T_{surr})A$ $h = \varepsilon\sigma(T_s + T_{surr})(T_s^2 + T_{surr}^2)$	$q = \frac{T_s - T_{surr}}{R}$	$R = \frac{1}{hA}$

<<< Equivalent Thermal Circuit

- Assume no generation, no accumulation
  - $E_{in} = E_{out}$
- Assume 1D transport

→ Series

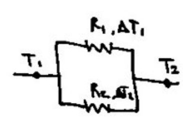


(I): $q = q_1 = q_2$

(V): $\Delta T = \Delta T_1 + \Delta T_2$.

(R): $R = R_1 + R_2$

→ Parallel



(I): $q = q_1 + q_2$

(V): $\Delta T = \Delta T_1 = \Delta T_2$

(R): $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$R = \frac{R_1 R_2}{R_1 + R_2}$

<< Total Heat Transfer

- $\boxed{q = UA(T_1 - T_2)}$

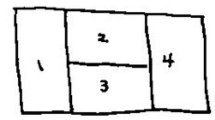
U - overall heat transfer coefficient

$T_1 - T_2$ - overall temp diff

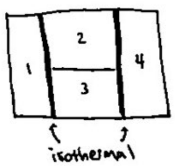
A - cross-sectional area

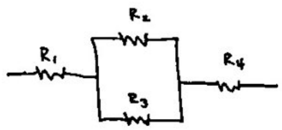
$$U = \frac{1}{R_{tot}A}$$

Ex1 Convert the system with 4 diff materials into equivalent thermal circuit.

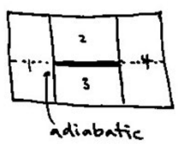


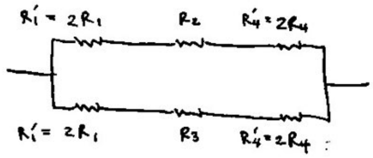
(1) Assume isothermal surface for normal to x-dir





(2) Assume adiabatic surface for parallel to x-dir

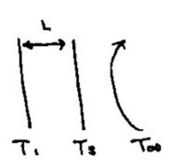


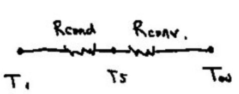


- Note that
  - $\frac{1}{R_1} = \frac{1}{R_1'} + \frac{1}{R_1'} = \frac{2}{R_1'} \Rightarrow R_1' = 2R_1$
  - $R_1 = \frac{L}{kA}$
  - $R_1' = \frac{L}{k(\frac{A}{2})}$

  $\Big\} \rightarrow R_1' = 2R_1$

Ex2 Given T_1, T_∞, solve for T_s.





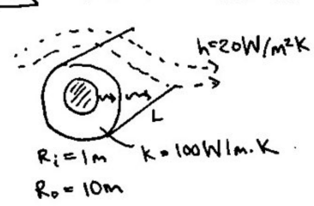
$$\left.\begin{aligned} R_{cond} &= \frac{L}{kA} \\ R_{conv} &= \frac{1}{hA} \end{aligned}\right\} R_{tot} = R_{cond} + R_{conv} = \frac{L}{kA} + \frac{1}{hA}$$

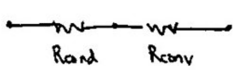
$$q = q_{cond} = q_{conv}$$

$$\frac{1}{R_{tot}}(T_1 - T_\infty) = \frac{1}{R_{cond}}(T_1 - T_s)$$

$$T_s = -\frac{R_{cond}}{R_{tot}}(T_1 - T_\infty) + T_1$$

Ex3 Calculate resistance between fluid inside & outside the pipe.





$$R_{tot} = R_{cond} + R_{conv}$$

$$= \frac{\ln(R_o/R_i)}{2\pi Lk} + \frac{1}{h(2\pi R_o L)}$$

$$= \frac{\ln(10)}{200\pi L} + \frac{1}{400\pi L}$$

$q_{cond} = q_{conv.}$

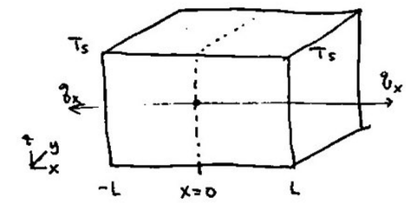
<<< Heat Generation in Cartesian Coordinates

- no accumulation
- 1D transport
- Cartesian coordinates
- constant generation

* Heat generation is due to energy conversion from other types of energy (e.g. chem rxn, current)

* Heat generation $\neq$ Heat accumulation (storage)

$$k\nabla^2 T + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$



$$k\left(\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}}^{\,0} + \cancel{\frac{\partial^2 T}{\partial z^2}}^{\,0}\right) + \dot{q} = \cancel{\rho c_p \frac{\partial T}{\partial t}}^{\,0}$$

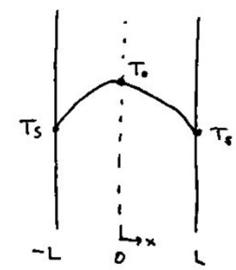
$$k\frac{\partial^2 T}{\partial x^2} + \dot{q} = 0$$

$$\frac{\partial T}{\partial x} = \frac{\dot{q}}{k}x + c_1$$

$$T = \frac{\dot{q}}{2k}x^2 + c_1 x + c_2 \quad \rightarrow \text{b.c.} \begin{cases} x=-L, \ T=T_s \\ x=L, \ T=T_s \end{cases}$$

$$T(x) = \frac{\dot{q}L^2}{2k}\left(1-\frac{x^2}{L^2}\right) + T_s$$

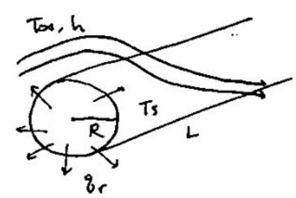
$$T(0) = \frac{\dot{q}L^2}{2k} + T_s$$



- Both q_x and q_x'' are NOT constant $\because$ generation.

<<< Heat Generation in Cylindrical Coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)}^{\,0} + \cancel{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}^{\,0} + \dot{q} = \cancel{\rho c_p \frac{\partial T}{\partial t}}^{\,0}$$



$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \dot{q} = 0$$

$$r\frac{\partial T}{\partial r} = -\frac{\dot{q}}{2k}r^2 + c_1$$

$$T = -\frac{\dot{q}}{4k}r^2 + c_1\ln(r) + c_2 \qquad \text{b.c.} \begin{cases} r=0, & \frac{\partial T}{\partial r}=0 \\ r=R, & T=T_s \end{cases} \Rightarrow c_1 = 0$$

$$T(r) = \frac{\dot{q}R^2}{4k}\left(1-\frac{r^2}{R^2}\right) + T_s$$

To relate T_s and T_∞, use overall energy balance,

$$q_{gen} = q_{conv}$$

$$\dot{q}V = h(T_s - T_\infty)A$$

$$\dot{q}(\pi R^2 L) = h(T_s - T_\infty)2\pi R L$$

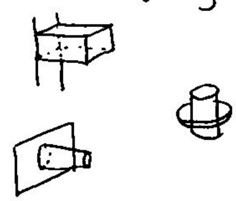
$$T_s = T_\infty + \frac{\dot{q}R}{2h}$$

So the T profile in terms of T_∞ is

$$T(r) = \frac{\dot{q}R^2}{4k}\left(1-\frac{r^2}{R}\right) + \frac{\dot{q}R}{2h} + T_\infty$$

<< Heat Transfer from Extended Surfaces

- fin - extended surface used to enhance heat trasfer between solid and adjoining fluid
  - straight fin - attached to a plane wall
  - annular fin - circumferentially attached to a cylinder
  - pin fin (spine) - has circular cross sections

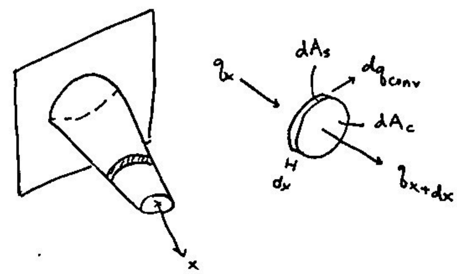


- fins increase surface area for heat transfer
  - heat sinks in microprocessors
  - radiators in car engine
  - air conditioner

<<< General Fin Equation

→ Assumptions

- 1D heat flow
- T(x only)
- steady state $\frac{\partial T}{\partial t} = 0$
- constant k
- constant h
- no generation $\dot{q} = 0$
- neglegible radiation $h_{rad} \ll h_{cond}$



→ Derivation

$$\text{In} - \text{out} + \cancel{\text{generation}}^{0} = \cancel{\text{accumulation}}^{0}$$

$$q_x - q_{x+dx} - dq_{conv} = 0$$

$$-\frac{q_{x+dx} - q_x}{dx}\,dx - dq_{conv} = 0$$

$$-\frac{dq}{dx}\,dx - h\,dA_s(T - T_\infty) = 0$$

$$-\frac{d}{dx}\left(-k\frac{dT}{dx}A_c\right)dx - h\,dA_s(T - T_\infty) = 0$$

$$\frac{d}{dx}\left(\frac{dT}{dx}A_c\right) - \frac{h}{k}\frac{dA_s}{dx}(T - T_\infty) = 0$$

$$\frac{d^2T}{dx^2}A_c + \frac{dT}{dx}\frac{dA_c}{dx} - \frac{h}{k}\frac{dA_s}{dx}(T - T_\infty) = 0$$

$$\boxed{\frac{d^2T}{dx^2} + \frac{1}{A_c}\frac{dA_c}{dx}\frac{dT}{dx} - \frac{h}{kA_c}\frac{dA_s}{dx}(T - T_\infty) = 0}$$

note $\frac{dA_s}{dx}$ = Perimeter = P

<<< Fins of Uniform Cross Sectional Area

→ Additional Assumptions

- uniform A_c $\frac{dA_c}{dx} = 0$
- $\frac{dA_s}{dx}$ = Perimeter $\equiv P$

→ Fin Equation for constant A_c

- $\boxed{\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0}$

  > excess temperature - $\boxed{\theta(x) = T(x) - T_\infty}$

  - how cold/hot the fin is at a point compared to surrounding
  - $\frac{d\theta}{dx} = \frac{dT}{dx}$, $\frac{d^2\theta}{dx^2} = \frac{d^2T}{dx^2}$

  - constant $\boxed{m^2 = \frac{hP}{kA_c}}$

- $\boxed{\frac{d^2\theta}{dx^2} - m^2\theta = 0}$

  - General soln : $\theta = C_1 e^{mx} + C_2 e^{-mx}$

→ Temperature distribution, Heat Rate, Boundary Conditions

- b.c. 1 : $T(0) = T_b \Leftrightarrow \theta(0) = T_b - T_\infty \equiv \theta_b$
- b.c. 2 : tip condition at $x = L$

Case	Use case	Tip Condition ($x=L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	convection at tip	$q_{conv} = q_{cond}$ $h\theta(L) = -k\frac{d\theta}{dx}\Big]_{x=L}$	$\frac{\cosh[m(L-x)] + \frac{h}{mk}\sinh[m(L-x)]}{\cosh(mL) + \frac{h}{mk}\sinh(mL)}$	$M\frac{\sinh(mL) + \frac{h}{mk}\cosh(mL)}{\cosh(mL) + \frac{h}{mk}\sinh(mL)}$
B	Convection neglegible, small tip area	$\frac{d\theta}{dx}\Big]_{x=L} = 0$	$\frac{\cosh[m(L-x)]}{\cosh(mL)}$	$M\tanh(mL)$
C	given tip temperature	$\theta(L) = \theta_L$	$\frac{\frac{\theta_L}{\theta_b}\sinh(mx) + \sinh[m(L-x)]}{\sinh(mL)}$	$M\frac{\cosh(mL) - \theta_L/\theta_b}{\sinh(mL)}$
D	Infinitely long fin ($L \to \infty$), temp equal to surrounding	$\theta(L) = 0$	e^{-mx}	M

$\theta \equiv T - T_\infty$ $m^2 = \frac{hP}{kA_c}$

$\theta_b = \theta(0) = T_b - T_\infty$ $M = \theta_b\sqrt{hPkA_c}$

Fin Performance Parameters

→ Fin Heat Transfer Rate q_f

> amount of heat transferred from the entire fin

- q_f tabulated for various boundary conditions
- Calculated by heat balance at base
  - $q_f = q_b = -k \frac{dT}{dx}\Big]_{x=0} A_c = -k \frac{d\theta}{dx}\Big]_{x=0} A_c$

→ Fin Effectiveness ε_f

> Ratio of fin heat transfer rate to heat transfer rate without a fin

- $\boxed{\varepsilon_f = \dfrac{q_f}{q_{no\,fin}} = \dfrac{q_f}{h A_{c,b} \theta_b}}$
- larger = better
- $\varepsilon_f \gtrsim 2$ to use a fin

→ Fin Efficiency η_f

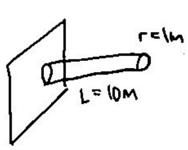
> Ratio of fin heat transfer rate to max possible heat transfer rate if entire fin were at T_b

- $\boxed{\eta_f = \dfrac{q_f}{q_{max}} = \dfrac{q_f}{h A_f \theta_b}}$
- $A_f = A_c + A_s$ (total surface area of fin)

Non-uniform Cross-sections

- See Table 3.5 for various geometry's fin efficiency η_f
- Approach to fin heat transfer problem
  - Identify boundary condition & geometry
  - Calculate T profile $\frac{\theta}{\theta_b}$ to find T
  - Solve for unknowns

$\boxed{EX1}$ Calculate fin effectiveness



$\varepsilon = ?$

$h = 1\,W/m^2K$
$k = 1\,W/mK$
$L = 10m$
$T_b = 400K$
$T_\infty = 300K$

For ∞ long fin, $q_f = M$, so

$$\varepsilon = \frac{q_f}{hA_{c,b}\theta_b} = \frac{M}{hA_c\theta_b} = \frac{\cancel{\theta_b}\sqrt{hPkA_c}}{hA_c\cancel{\theta_b}}$$

$$= \frac{\sqrt{(1)2\pi(1)(1)\pi(1^2)}}{(1)\pi(1^2)} = \sqrt{2}$$

<<< Heat Conduction in 2D

- Analytic soln - separation of variables
- Emperical soln - pre-solved soln (shape factors)
- Numerical soln - computer soln

<<< Shape Factor

> shape factor S - $\boxed{q = -Sk\Delta T} = Sk(T_1 - T_2)$

> 2D conduction resistance $\boxed{R_t = \frac{1}{Sk}}$

} Table 4.1a

<<< Dimensionless Conduction Heat Rate

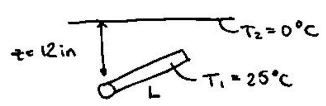
> Dimensionless cond. heat rate q^*_{ss} - $\boxed{q = \frac{-q^*_{ss}\, k A_s \Delta T}{L_c}}$

> characteristic length - $\boxed{L_c = \sqrt{\frac{A_s}{4\pi}}}$

} Table 4.1b infinite medium

- constant object Temperature
- no heat generation in medium

$\boxed{\text{Ex1}}$ A 1" diameter pipe is buried 1' underground in sand ($k = 1 W/mK$) carrying water at 25°C. It is 0°C outside. Calculate heat rate per length.



shape factor case 2 ($L \gg D$, $z > \frac{3D}{2}$):

$$S = \frac{2\pi L}{\ln(4z/D)}$$

So heat rate is

$$q = -Sk\Delta T$$

$$q = -\frac{2\pi L}{\ln(4z/D)} k (T_2 - T_1)$$

$$\frac{q}{L} = \frac{2\pi k}{\ln(4z/D)} (T_1 - T_2)$$

$$\frac{q}{L} = \frac{2\pi(1 W/mK)}{\ln\left(\frac{4(12in)}{1in}\right)} (25°C - 0°C)$$

$$\frac{q}{L} = 41 W/m$$

<<< Finite Difference Method

> node (nodal point) - reference point for calculating T

> nodal network (grid, mesh) - collection of nodes

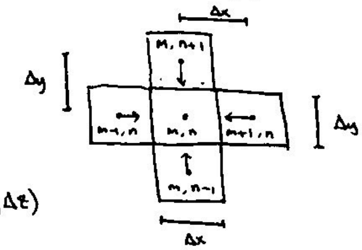
> finite difference method - numerical method calculating T at nodes

- Assume T at node is the average in $\Delta x \Delta y$ area.
- Use energy balance to obtain an eqn at each point
- Solve the system of eqn to get T at node

→ Interior point

- Assume steady state
- Assume all heat flow into the control volume (E_out taken account with sign)
- Energy balance

$$\dot{E}_{in} + \dot{E}_{gen} = 0$$



$$\sum_{i=1}^{4} q_{(i)\to(m,n)} + \dot{q}\,\Delta x \Delta y \Delta z = 0$$

Assume $\Delta z = 1$, $\Delta x = \Delta y$

$$q_{(m-1,n)\to(m,n)} = -k\,\frac{T_{m,n}-T_{m-1,n}}{\Delta x}(\Delta y \Delta z)$$

$$q_{(m+1,n)\to(m,n)} = -k\,\frac{T_{m,n}-T_{m+1,n}}{\Delta x}(\Delta y \Delta z)$$

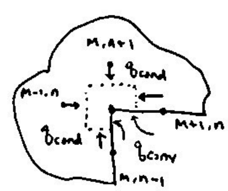
$$q_{(m,n-1)\to(m,n)} = -k\,\frac{T_{m,n}-T_{m,n-1}}{\Delta y}(\Delta x \Delta z)$$

$$q_{(m,n+1)\to(m,n)} = -k\,\frac{T_{m,n}-T_{m,n+1}}{\Delta y}(\Delta x \Delta z)$$

$$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} + \frac{\dot{q}(\Delta x)^2}{k} - 4T_{m,n} = 0$$

- Write the eqn for every point
- Solve system of eqn to get T at every point

→ Alternative Geometry



$$\dot{E}_{in} + \dot{E}_{gen} = 0$$

Assume $\Delta z = 1$, $\Delta x = \Delta y$, $\dot{E}_{gen} = 0$, steady state, 2D transport

$$q_{(m,n+1)\to(m,n)} = -k\,\frac{T_{m,n}-T_{m,n+1}}{\Delta y}(\Delta x \Delta z)$$

$$q_{(m,n-1)\to(m,n)} = -k\,\frac{T_{m,n}-T_{m,n-1}}{\Delta y}\left(\frac{\Delta x}{2}\Delta z\right)$$

$$q_{(m+1,n)\to(m,n)} = -k\,\frac{T_{m,n}-T_{m+1,n}}{\Delta x}\left(\frac{\Delta y}{2}\Delta z\right)$$

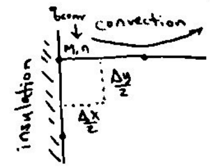
$$q_{(m-1,n)\to(m,n)} = -k\,\frac{T_{m,n}-T_{m-1,n}}{\Delta x}(\Delta y \Delta z)$$

$$q_{\infty\to(m,n)} = h(T_\infty - T_{m,n})\left[\frac{\Delta x}{2}\Delta z + \frac{\Delta y}{2}\Delta z\right]$$

$$T_{m-1,n} + T_{m,n+1} + \frac{1}{2}(T_{m+1,n} + T_{m,n-1}) + \frac{h\Delta x}{k}T_\infty - \left(3 + \frac{h\Delta x}{k}\right)T_{m,n} = 0$$

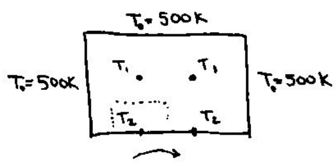
Examples of Finite Difference Method

$\boxed{\text{Ex1}}$ Write heat flow into (m,n) due to convection.



$$q_{conv} = h(T_\infty - T_{m,n})\left(\frac{\Delta x}{2}\right)$$

$\boxed{\text{Ex2}}$ Solve for T_1 and T_2 given isothermal surfaces and convection. No generation



$T_\infty = 300K$

$h = 10 W/m^2 \cdot K$

$k = 1 W/m \cdot K$

$\Delta x = \Delta y = 0.25 m$

$\Delta z = 1 m$

Point 1 : Interior point

$$T_{m,n+1} + T_{m,n-1} + T_{m-1,n} + T_{m+1,n} - 4T_{m,n} + \cancel{\frac{\dot{q}(\Delta x)^2}{k}}^{0} = 0$$

$$500 + T_2 + 500 + T_1 - 4T_1 + 0 = 0$$

$$-3T_1 + T_2 + 1000 = 0$$

Point 2 : Edge

$$q_{top} = -k\frac{T_2 - T_1}{\cancel{\Delta y}}(\cancel{\Delta x}\cancel{\Delta z})^1 = k(T_1 - T_2)$$

$$q_{bottom} = h(T_\infty - T_2)(\Delta x \cancel{\Delta z})^1 = h(T_\infty - T_2)\Delta x$$

$$q_{left} = -k\frac{T_2 - T_0}{\cancel{\Delta x}}\left(\frac{\cancel{\Delta y}}{2}\cancel{\Delta z}\right)^1 = \frac{k}{2}(T_0 - T_2)$$

$$q_{right} = -k\frac{\cancel{T_2 - T_2}^{0}}{\Delta x}\left(\frac{\Delta y}{2}\Delta z\right) = 0$$

$$\Sigma q = 0$$

$$k(T_1 - T_2) + h(T_\infty - T_2)\Delta x + \frac{k}{2}(T_0 - T_2) = 0$$

$$(1)(T_1 - T_2) + 10(300 - T_2)(0.25) + \frac{(1)}{2}(500 - T_2) = 0$$

$$T_1 - 4T_2 = -1000$$

System

$$\begin{cases} -3T_1 + T_2 = -1000 \\ T_1 - 4T_2 = -1000 \end{cases}$$

$$\begin{bmatrix} -3 & 1 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} -1000 \\ -1000 \end{bmatrix}$$

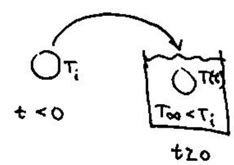
$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} 455 \\ 364 \end{bmatrix} \text{ Kelvin}$$

31 Jan 2022 L13 Transient Heat Conduction & Lumped Capacitance CHEME340

<<< Lumped Capacitance Method

- transient heat conduction occurs
  - $\frac{\partial T}{\partial t} \neq 0$
  - have accumulation
- Assume T is spatially constant (uniform)
  - $T \neq f(x,y,z)$
  - $T = f(t)$

$\boxed{\text{Ex1}}$ Derive temperature as function of time for hot object immerse in cold fluid.



$$\cancelto{0}{\text{In}} - \text{out} + \cancelto{0}{\text{generation}} = \text{accumulation}$$

$$-h(T_s - T_\infty)A_s = \rho c_p V \frac{\partial T}{\partial t}$$

$$-h\theta A_s = \rho c_p V \frac{\partial \theta}{\partial t} \qquad \text{let } \boxed{\theta \equiv T - T_\infty}, \text{ so } \frac{\partial \theta}{\partial t} = \frac{\partial T}{\partial t}$$

$$\int_0^t -dt = \int_{\theta_i}^{\theta} \frac{\rho c_p V}{h A_s} \frac{\partial \theta}{\theta}$$

$$t = \frac{\rho c_p V}{h A_s} \ln\left(\frac{\theta_i}{\theta}\right)$$

- temperature as function of time $\boxed{\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{hA_s}{\rho V c_p} t\right)}$ $\quad \frac{\theta}{\theta_i} = \exp\left(-\frac{t}{\tau}\right)$

> lumped thermal conductance - $\boxed{C \equiv \rho V c_p}$ $\quad C$ [=] J/K

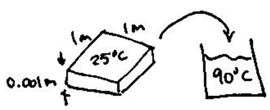
> thermal time constant - $\boxed{\tau = RC = \frac{\rho V c_p}{h A_s}}$ $\quad \tau$ [=] s

- total energy transferred - $Q = \int_0^t q \, dt$

$$= h A_s \int_0^t \theta \, dt$$

$$\boxed{Q = \rho V c_p \theta_i \left[1 - \exp\left(-\frac{t}{\tau}\right)\right]}$$

$\boxed{\text{Ex2}}$ Calculate time for solid to reach 80°C.



$h = 500\ W/m^2K$

$\rho = 2.7\ g/cm^3$

$c_p = 0.89\ J/g°C$

$\theta = 80°C - 90°C = -10°C$

$\theta_i = 25°C - 90°C = -65°C$

$$t = \frac{\rho c_p V}{h A_s} \ln\left(\frac{\theta_i}{\theta}\right)$$

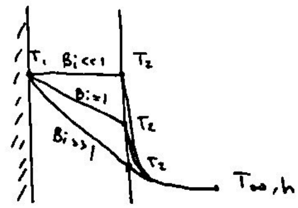
$$= \frac{(2.7\ g/cm^3)(0.89\ J/g°C)(0.1cm)(100cm)(100cm)}{(500\ W/m^2K)(2m^2)} \ln\left(\frac{-65°C}{-10°C}\right)$$

$$= 4.5\ s$$

- Note $T \rightarrow T_\infty$ as $t \rightarrow \infty$

<<< Validity of Lumped Capacitance Method

- Assumed T is spatially uniform within solid
  - need criteria to quantify
- Assume steady-state, energy balance gives



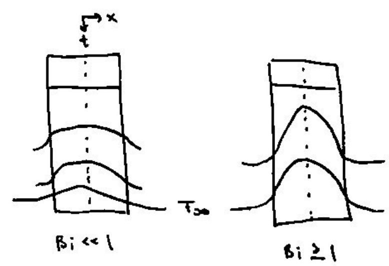
$$q_{cond} = q_{conv}$$

$$\frac{kA}{L}(T_1 - T_2) = hA(T_2 - T_\infty)$$

$$\frac{T_1 - T_2}{T_2 - T_\infty} = \frac{hA}{kA/L} = \frac{R_{cond}}{R_{conv}} = \frac{hL}{k} = Bi$$

> Biot number - $\boxed{Bi \equiv \frac{hL_c}{k}}$

- $\boxed{Bi = \frac{R_{cond}}{R_{conv}}}$
- ratio of thermal resistance due to conduction vs. convection.
- lump capacitance method valid when conduction << convection
  - $\boxed{Bi < 0.1}$
- characteristic length $L_c = \frac{V}{A_s}$
  - plane wall - $L_c = \frac{1}{2}L$
  - cylinder - $L_c = R_o/2$
  - sphere - $L_c = R_o/3$



<< Dimensionless Time

- $$\frac{t}{\tau} = \frac{hA_s t}{\rho V c_p} = \frac{ht}{\rho c_p L_c}\frac{L_c}{L_c}\frac{k}{k} = \underbrace{\frac{hL_c}{k}}_{Bi}\underbrace{\frac{k}{c_p \rho}}_{\alpha}\frac{t}{L_c^2} = Bi\frac{\alpha t}{L_c^2} = Bi \cdot F_o$$

> Fourier number - $\boxed{F_o \equiv \frac{\alpha t}{L_c^2}}$

- dimensionless time

- temp as function of time of Lumped Capacitance method - $\boxed{\frac{\theta}{\theta_i} = \exp(-Bi \cdot F_o)}$

<<< Spatial Effect with Nondimensionalization

- Assume no generation, 1D transport
- When $Bi > 0.1$, spatial effect is not neglegible

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$T = T(x, t, T_i, T_\infty, L, k, \alpha, h)$$

> dimless time - $\boxed{t^* = F_o = \frac{\alpha t}{L^2}}$

> dimless temperature - $\boxed{\theta^* = \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty}}$

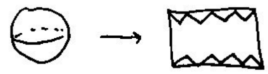
> dimless spatial coord - $\boxed{x^* = \frac{x}{L}}$

$$\frac{\partial^2 \theta^*}{\partial x^{*2}} = \frac{\partial \theta^*}{\partial F_o}$$

$$\boxed{\theta^* = \theta^*(x^*, F_o, Bi)}$$

<<< Example of Transient Heat Transfer

$\boxed{\text{Ex1}}$ Calculate time for sphere at T_i to reach T_f in T_∞ at its center.



$T_i = 25°C$ $\quad$ $T_\infty = 180°C$

$m = 2.8\,kg$ $\quad$ $T_f = 90°C$

$k = 0.65\,W/m°C$ $\quad$ $h = 15.5\,W/m^2°C$

$\rho = 940\,kg/m^3$

$C_p = 3850\,J/kg°C$

$\boxed{1}$ Calculate Bi to test validity of lumped capacitance method

$$V = \frac{4}{3}\pi R^3 = \frac{m}{\rho}$$

$$R = \sqrt[3]{\frac{m}{\rho\pi}\frac{3}{4}} = 0.0893\,m = r_o$$

$$Bi = \frac{hL_c}{k} = \frac{hR}{k\,3} = 0.709 > 0.1$$

- Lumped capacitance method is not valid.
- Use correlation instead.

$\boxed{2}$ Use correlation for sphere to calculate time.

- Assume $Fo > 0.2$, at center $r^* = \frac{r}{r_o} = 0$,

$$\theta_o^* = C_1 \exp(-\zeta_1^2 Fo)$$

$$\ln\left(\frac{\theta_o^*}{C_1}\right) = -\zeta_1^2 \frac{\alpha}{r_o^2} t$$

$$t = -\ln\left(\frac{\theta_o^*}{C_1}\right)\frac{r_o^2}{\zeta_1^2 \alpha}$$

$$t = -\ln\left(\frac{T_f - T_\infty}{T_i - T_\infty}\frac{1}{C_1}\right)\frac{r_o^2}{\zeta_1^2}\frac{C_p\rho}{k}$$

$Bi = \frac{hR}{k} = 2.13$ (corrected for correlation)

Bi	ζ_1	C_1	
2.0	2.0288	1.4793	
3.0	2.2889	1.6227	
2.13	2.063	1.498	(interpolate)

$$t = 9881\,s = 2.7\,h$$

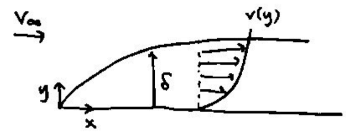
- Verify that $F_o = \frac{\alpha t}{r_o^2} = \frac{kt}{C_p\rho r_o^2} = 0.22 > 0.2$ ✓

7 Feb 2022 — L16 Boundary Layers, Convection, Turbulence — CHEME 340

<<< Boundary Layers

→ Velocity boundary layer

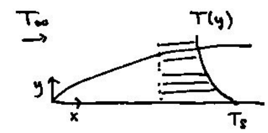
- fluid velocity gradient exist near surface due to no slip condition and shear stress
- > Newton's law of viscocity – $\boxed{\tau_{yx} = \mu \frac{\partial v_x}{\partial y}}$



- > boundary layer thickness δ – y at which $v = 0.99\, v_\infty$

→ Thermal boundary layer

- temperature gradient exist near surface
- > boundary layer thickness δ_t – y at which $\frac{T_s - T}{T_s - T_\infty} = 0.99$



- > Fourier's law – $\boxed{q_s'' = -k \frac{dT}{dy}}$
- > Newton's law of cooling – $\boxed{q_s'' = h(T_s - T_\infty)}$
- thermal boundary layer affects convection coefficient
  - h is not constant
  - $h = \frac{-k \frac{dT}{dy}\big|_{y=0}}{T_s - T_\infty}$
    - $\frac{dT}{dy}\big|_{y=0}$ decreases with x
    - h decreases with x

<<< Average Convection Coefficient

- Convection coefficient h is not constant, being function of x : h(x)
- $q = \int_{A_s} q'' \, dA_s$

  $= (T_s - T_\infty) \int_{A_s} h \, dA_s$
- $\boxed{q = \bar{h}\,(T_s - T_\infty)\, A_s}$

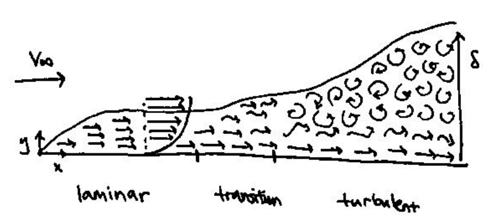
> Average convection coefficient – $\boxed{\bar{h} = \frac{1}{A_s} \int_{A_s} h \, dA_s}$

- 1D case – $\bar{h} = \frac{1}{L} \int_0^L h \, dx$

<<< Laminar & Turbulent Flow

> Reynolds number – $\boxed{Re = \frac{\rho V_\infty L_c}{\mu}} = \frac{\text{inertial force}}{\text{viscous force}}$

- determines transition point from laminar to turbulent
- depends on geometry
  - flat plate $\begin{cases} L_c = x \\ Re_c = 5 \times 10^5 \end{cases}$



<<< Methods of Solving Convection Problem

- Analytic soln (rare)
- Empirical soln (correlations)
- Numerical soln (COMSOL)

<<< Dynamic Similarity & Nondimensionalization

→ Dimensionless parameters

> Reynolds number - $\boxed{Re = \frac{\rho v L_c}{\mu}} = \frac{\text{inertial force}}{\text{viscous force}}$

> Prandtl number - $\boxed{Pr = \frac{\nu}{\alpha}} = \frac{\mu}{\rho}\frac{\rho C_p}{k} = \frac{\mu C_p}{k} = \frac{\text{momentum diffusion}}{\text{thermal diffusion}}$

> Peclet number - $\boxed{Pe = \frac{vL}{\alpha}} = Re\,Pr = \frac{\text{advection}}{\text{convection}}$

> Nusselt number - $\boxed{Nu = \frac{hL}{k_{fluid}}} = \frac{\text{convection}}{\text{conduction}} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$

> Biot number - $\boxed{Bi = \frac{hL}{k_{solid}}} = \frac{\text{internal thermal resistance of solid}}{\text{boundary layer thermal resistance}}$

- $Nu \neq Bi$
- L in Nu is length along slab
- L in Bi is thickness

→ Nondimensionalizing variables

- $x^* = \frac{x}{L}$
- $y^* = \frac{y}{L}$
- $V_x^* = \frac{V_x}{V_\infty}$
- $V_y^* = \frac{V_y}{V_\infty}$
- $T^* = \frac{T-T_s}{T_\infty - T_s}$

<<< External Flow Example

$\boxed{EX1}$ Calculate heat rate q from the surface.

$T_\infty = 22°C$

$V_\infty = 0.65 m/s$

$\rho = 1.0134\ kg/m^3$

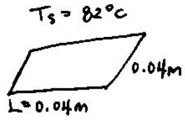
$C_p = 1.0065\ kJ/kg\ K$

$\mu = 1.985 \times 10^{-5}\ kg/m \cdot s$

$k = 0.02822\ W/K \cdot m$

Air

$T_s = 82°C$



$A_s = L^2 = 0.0016 m^2$

$Re = \frac{\rho V_\infty x}{\mu} = \frac{\rho V_\infty L}{\mu} = 1350 < 500000 \rightarrow$ laminar

$T = T_f = \frac{T_s + T_\infty}{2} = 325 K$

$\nu = \frac{\mu}{\rho} = 1.825 \times 10^{-5} m^2/s$

$\alpha = \frac{k}{\rho C_p} = 2.59 \times 10^{-5} m^2/s$

$Pr = \frac{\nu}{\alpha} = 0.701$

Correlation: $(Pr \geq 0.6)$

$\bar{h} = 0.664 (Pr)^{\frac{1}{3}} (Re)^{\frac{1}{2}} \frac{k}{x} = 15.3\ W/m^2 \cdot K$

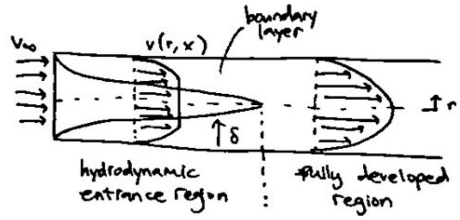
$(x = L = 0.04 m)$

$q = \bar{h} (T_s - T_\infty) A_s = 1.47 W$

<<< Hydrodynamic considerations

→ Flow conditions

> Reynolds number for circular pipe - $\boxed{Re = \frac{\rho \bar{v} D}{\mu}}$



. critical Re - $Re_c = 2300$

→ Mean velocity

> mass flow rate - $\boxed{\dot{m} = \rho \bar{v} A_c}$

· $\bar{v} = \frac{\dot{m}}{\rho A_c} = \frac{1}{\rho A_c} \int_{A_c} \rho v \, dA_c$

$\bar{v} = \frac{2}{r_o^2} \int_0^{r_o} v \, r \, dr$ (incompressible fluid, circular tube)

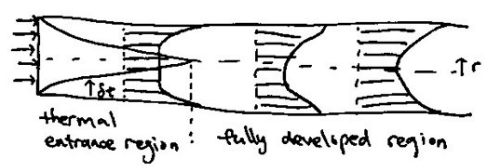
<<< Thermal considerations

→ Thermal entry length

· At fully developed region,

$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right] = 0$$

· Relative shape of dimensionless temp is unchanged



→ Mean temperature

· $T_m = \frac{1}{\bar{v}} \int_{A_c} T v \, dA_c$ (constant ρ, c_p)

· $T_m = \frac{2}{\bar{v} r_o^2} \int_0^{r_o} T v \, r \, dr$ (circular pipe)

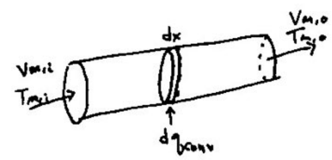
→ Axial temperature profile in a tube

· Overall system balance : $q_{conv} = \dot{m} c_p (T_{m,o} - T_{m,i})$

$d q_{conv} = \dot{m} c_p \, dT_m$

· Shell balance : $dq_{conv} = q_s'' \, dA_s$

$dq_{conv} = q_s'' \, P \, dx$



· Equate : $\dot{m} c_p \, dT_m = q_s'' \, P \, dx$

$$\boxed{\frac{dT_m}{dx} = \frac{q_s'' P}{\dot{m} c_p} = \frac{P}{\dot{m} c_p} h (T_s - T_m)}$$

→ Correlations

· Uniform surface flux

· Uniform surface temperature

· Uniform external fluid temperature

EX1 Calculate average convection coefficient $\bar{h}$.

D = 0.05m

L = 6m

Ts = 100°C

$\dot{m}$ = 0.25 kg/s water

c_p = 4.18 kJ/g°C

$$\frac{T_s - T_{m,x}}{T_s - T_{m,i}} = \exp\left[-\frac{PL}{\dot{m} c_p} \bar{h}\right]$$

$$\bar{h} = -\ln\left(\frac{T_s - T_{m,x}}{T_s - T_{m,i}}\right) \frac{\dot{m} c_p}{PL}$$

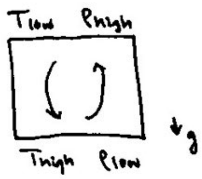
$$= -\ln\left(\frac{100-57}{100-15}\right) \frac{(0.25)(4.18)}{\pi(0.05)(6)}$$

$$= 0.76 \ W/m^2K$$

<<< Free Convection

> free convection - fluid motion is caused by temperature gradients which drive density gradient that creates buoyancy in the presence of gravity

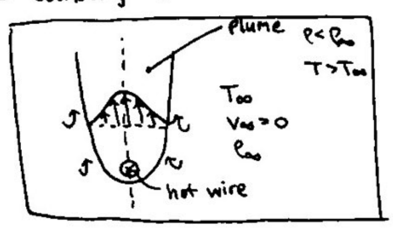
- prescence of temperature or density gradient does not guaruntee free convection.



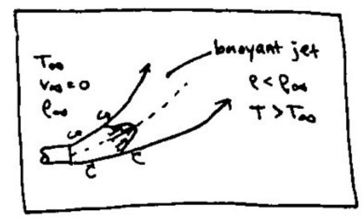
free convection (unstable)

stable (no fluid motion)

- Examples of free convection
  - free boundary flow

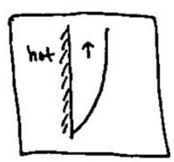


Plume



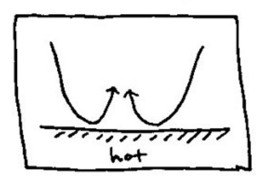
Buoyant jet

  - flows with bound surface



verticle plate

inclined plane



horizontal plate

<<< Governing Equation

> volumetric thermal expansion coefficient - $\boxed{\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P}$ $[=]\ K^{-1}$

$$\beta = -\frac{1}{\rho}\frac{\Delta \rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T}$$

> Boussinesq approximation - $\boxed{\rho_\infty - \rho = \rho \beta (T - T_\infty)}$

- Navier-Stokes eqn gives $v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_y}{\partial y} = g\beta(T - T_\infty) + \nu \frac{\partial^2 v_x}{\partial y^2}$

<<< Dynamic Similarity & Nondimensionalization

- $x^* = \frac{x}{L}$
- $y^* = \frac{y}{L}$
- $V_x^* = \frac{V_x}{V_{x,0}}$, where $V_{x,0} = \sqrt{g\beta(T_s - T_\infty)L_c}$
- $V_y^* = \frac{V_y}{V_{y,0}}$
- $T^* = \frac{T - T_\infty}{T_s - T_\infty}$

> $\boxed{Re = \sqrt{\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}}}$

> Grashof number - $\boxed{Gr = Re^2 = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}} = \frac{\text{buoyancy force}}{\text{viscous force}}$

- Gr characterizes free convection
- Re characterizes forced convection

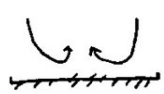
> Rayleigh number - $\boxed{Ra = Gr\,Pr = \frac{g\beta(T_s - T_\infty)x^3}{\nu\alpha}}$

- $Ra_c = 10^9$
- determines laminar vs turbulent

- problem solving approach
  - $Nu \Leftrightarrow h \Leftrightarrow q$

<<< Example of using Correlations

$\boxed{\text{Ex1}}$ Calculate the heat transfer rate from the plate by free convection.



$T_\infty = 22°C$

$T_s = 82°C$

$L = 0.04\,m$

$\rho = 1.0134\ kg/m^3$

$C_p = 1.0065\ kJ/kg\,K$

$\mu = 1.985 \times 10^{-5}\ kg/m\,s$

$k = 0.02822\ W/m\,K$

$\beta = 1.7 \times 10^{-3}\ K^{-1}$

- Use horizontal plate correlation
- $T_f = \frac{T_s + T_\infty}{2} = 325\ K$
- $\nu = \frac{\mu}{\rho} = 1.825 \times 10^{-5}\ m^2/s$
- $\alpha = \frac{k}{\rho C_p} = 2.59 \times 10^{-5}\ m^2/s$
- $Pr = \frac{\nu}{\alpha} = 0.701$
- $L_c = \frac{A_s}{P} = \frac{(0.04)^2}{4(0.04)} = 0.01\,m$
- $Ra = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu\alpha} = 33800 \ll 10^9 \rightarrow$ laminar
- $\overline{Nu} = \frac{\bar{h}L}{k} = 0.27(Ra)^{1/4} = 3.66$

  $\bar{h} = \frac{\overline{Nu}\,k}{L} = 10.3\ W/m^2 \cdot K$
- $q = \bar{h} A_s (T_s - T_\infty) = 0.984\,W$

<<< COMSOL Multiphysics

- uses finite element differential equation solver

→ Method

1. Define coordinate system
2. Specify physics needed for simulation
3. Specify time dependence of problem
4. Construct geometry
   - construct shapes with properties of concern
     - e.g. fin problem construct fin, but not fluid
     - e.g. fluid in pipe problem construct fluid, not pipe
   - use operations like "union", "intersection" to get desired shape
5. Select material
6. Assign areas to material and physics
7. Assign boundary conditions
8. Build mesh
9. Simulate and calculate properties

<<< Boiling Modes

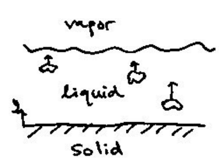
> boiling - evaporation at solid-liquid interface

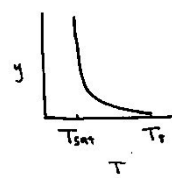
- heat transfer from solid to liquid
- $q_s'' = h(T_s - T_{sat})$
  $= h\,\Delta T_e$
- $\Delta T_e \equiv T_s - T_{sat}$ excess temperature

- boiling modes
  > pool boiling - liquid quiescent, motion near surface due to free convection and mixing induced by bubble growth and detachment
  > forced convection boiling - fluid motion caused by external means, in addition to free convection and bubble-induced mixing
  > subcooled boiling - $T_{liq} < T_{sat}$, bubble formed at surface condense to liquid
  > saturated boiling - $T_{liq} \geq T_{sat}$, bubble formed at surface propelled thru liquid by buoyancy force, escaping from free surface

<<< Pool Boiling

→ The Boiling Curve





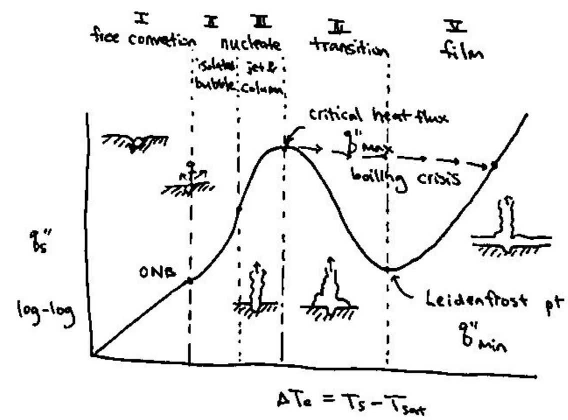
- Temperature changes rapidly near solid surface
  - $T_{sat} < T_{liq} < T_{solid}$

→ Modes of Pool Boiling

$\boxed{\text{I}}$ Free convection
- no bubbles forming
- fluid motion due to free convection
> onset of nucleate boiling (ONB) - T above which bubbles begin nucleating at defect

$\boxed{\text{II}}$ Isolated bubbles (Nucleate boiling)
- bubbles nucleate at defects
- most heat transfer from fluid motion, high mixing

$\boxed{\text{III}}$ Jet & Column (Nucleate boiling)
- bubbles coalesce into jet & column
> critical heat flux q''_{max} - further increase in ΔT_e are balanced by decrease in $\bar{h}$ because liquid no longer fully wets surface
  - vapor is less thermally conductive than liquid



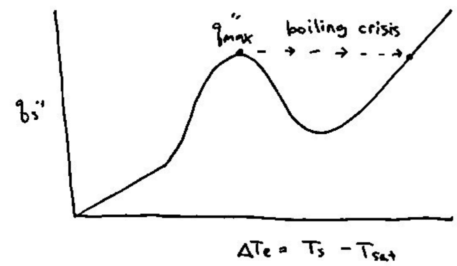
$\boxed{\text{IV}}$ Transition Boiling
- a vapor film starts to form on surface
- $\bar{h}$ decreases with increasing ΔT_e
> Leidenfrost point q''_{min} - surface is coated in vapor blanket

$\boxed{\text{V}}$ Film boiling
- radiation thru the film which dominates q''
- q'' increases with increasing ΔT_e

<<< Pool Boiling (cont.)

→ Modes of pool boiling (cont.)

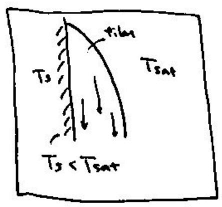
- boiling crisis
  - Instruments usually control q'', but not ΔT_e
  - If $q'' > q''_{max}$, there will be a sharpe increase of ΔT_e
    - q'' decreases in transition, So by energy balance, T_s increases (more accumulation)



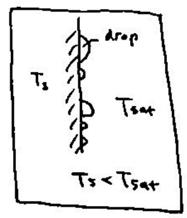
→ Correlations

- correlations avaliable in different regions

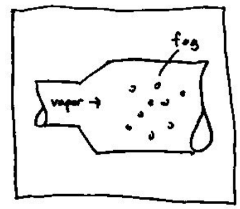
<<< Modes of Condensation



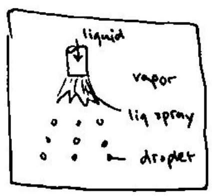
Film condensation



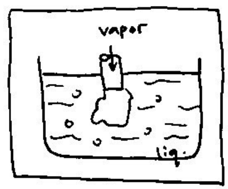
Dropwise condensation



Homogeneous condensation



Direct contact condensation (liq in vap)



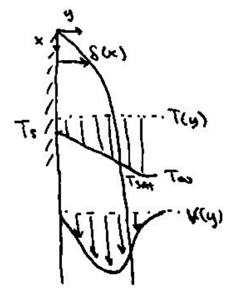
Direct contact Condensation (Vap in liq)

- Heat transfer is faster in dropwise condensation compared to film condensation
  - Shorter distance between T_s, T_{sat}
  - less thermal resistance

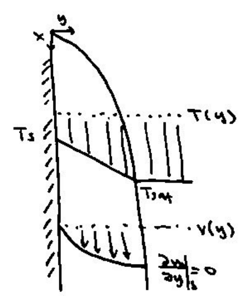
<<< Laminar Film Condensation on Vertical Plate

→ Assumptions

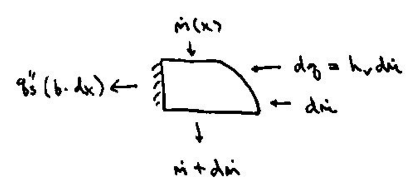
- laminar flow $Re = \frac{4 \rho_l \bar{v} \delta}{\mu}$, $Re_{,c} = 1800$
- constant film properties
- uniform vapor temp $T(\delta) = T_{sat}$
- no temp gradient in vapor $\left.\frac{\partial T}{\partial y}\right|_\delta = 0$
- heat transfer by condensation only at interface
- shear stress at vapor-liquid interface negligible $\left.\frac{\partial v_x}{\partial y}\right|_\delta = 0$
- no slip condition $v_x(0) = 0$
- low fluid velocity - conduction only
  - no advection
  - linear T profile



Reality



Assumption



→ Method

- Objective: find film thickness δ
  - velocity boundary consideration
  - thermal boundary consideration

→ Velocity Boundary Condition

$$\rho\left(\cancel{v_x}^{\,0}\frac{\partial v_x}{\partial x}+\cancel{v_y}^{\,0}\frac{\partial v_y}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\cancel{\frac{\partial^2 v_x}{\partial x^2}}^{\,0}+\frac{\partial^2 v_x}{\partial y^2}\right)+\rho g \qquad \text{Navier-Stokes eqn}$$

$$0=-\rho g+\mu\frac{\partial^2 v_x}{\partial y^2}+\rho_v g$$

$$\frac{\partial^2 v_x}{\partial y^2}=-\frac{g}{\mu}(\rho_\ell-\rho_v)$$

· velocity profile

$$v_x(y)=\frac{g(\rho_\ell-\rho_v)}{\mu_\ell}\left[\frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^2\right] \qquad \begin{cases}\text{b.c. 1} & y=0 \quad v_x=0\\ \text{b.c. 2} & y=\delta \quad \frac{\partial v_x}{\partial y}=0\end{cases}$$

· mass flow rate per unit width

$$\Gamma(x)=\frac{\dot m(x)}{b}=\int_0^{\delta(x)}\rho_\ell v_x(y)\,dy$$

$$\Gamma(x)=\frac{g\rho_\ell(\rho_\ell-\rho_v)\delta^3}{3\mu_\ell} \quad \rightarrow \quad \frac{d\Gamma}{dx}=\frac{g\rho_\ell(\rho_\ell-\rho_v)\delta^2}{\mu_\ell}\frac{d\delta}{dx}$$

→ Thermal Boundary Conditions

· mass flow rate per unit width

$$\Gamma(x)=\frac{\dot m(x)}{b}$$

$dq=q_s''(b\cdot dx)\leftarrow$  $\leftarrow dq=h_v\,d\dot m$

$$\frac{d\Gamma}{dx}=\frac{1}{b}\frac{d\dot m}{dx} \qquad dq=h_v\,d\dot m$$

$$\frac{d\Gamma}{dx}=\frac{1}{b}\frac{dq}{dx}\frac{1}{h_v} \qquad dq=q_s''(b\cdot dx)$$

$$\frac{d\Gamma}{dx}=\frac{1}{bh_v}q''b$$

$$\frac{d\Gamma}{dx}=\frac{q''}{h_v} \qquad q''=-k\left.\frac{dT}{dy}\right|_0=+k\frac{T_{sat}-T_s}{\delta}$$

$$\frac{d\Gamma}{dx}=\frac{k_\ell(T_{sat}-T_s)}{\delta h_v}$$

→ Solve for film thickness δ

· Equate

$$\frac{d\Gamma}{dx}=\frac{g\rho_\ell(\rho_\ell-\rho_v)\delta^2}{\mu_\ell}\frac{d\delta}{dx}=\frac{k_\ell(T_{sat}-T_s)}{\delta h_v}$$

$$\delta^3\,d\delta=\frac{k_\ell\mu_\ell(T_{sat}-T_s)}{g\rho_\ell(\rho_\ell-\rho_v)h_v}dx$$

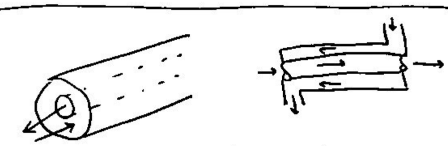
$$\boxed{\delta(x)=\left[\frac{4k_\ell\mu_\ell(T_{sat}-T_s)}{g\rho_\ell(\rho_\ell-\rho_v)h_v}\right]^{\frac{1}{4}}}$$

<< Types of Heat Exchangers

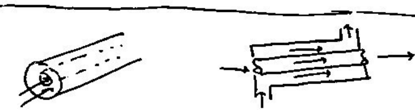
> heat exchangers – device to exchange heat between two fluids at diff T and separated by solid wall

- important to make process more efficient

- Classify by flow arrangement
  - parallel flow $\rightrightarrows$
  - counter flow $\rightleftarrows$
  - cross flow

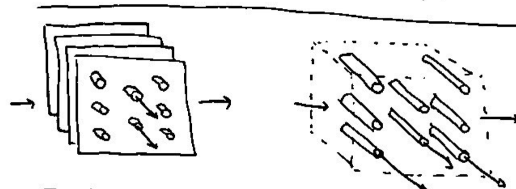


Counterflow double pipe



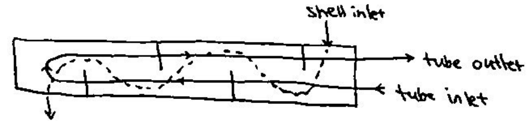
Parallel flow double pipe

- Classify by configuration
  - Concentric tube (Double pipe) heat exchanger
  - Finned cross flow heat exchanger
  - Unfinned cross flow heat exchanger
  - Shell & tube heat exchanger
  - Compact heat exchanger
    - circular / flat tubes
    - circular / plate fins
    - single / multipass



Finned cross flow (unMixed flow) Unfinned cross flow (mixed flow)

- Note in cross flow heat exchanger
  - finned – unmixed flow
    - T gradient along x, y
  - unfinned – mixed flow
    - Fluid mix in all directions
    - T gradient along main flow direction x



(one shell, two tube pass)
shell & tube heat exchanger

<< Overall Heat Transfer Coefficient

- For wall separating two fluids

$$R_{tot} = R_{conv,h} + R_{foul,h} + R_{wall} + R_{foul,c} + R_{conv,c}$$ h = hot, cold = c

$$\frac{1}{UA} = \left(\frac{1}{hA} + \frac{R''_{foul}}{A}\right)_h + R_{wall} + \left(\frac{R''_{foul}}{A} + \frac{1}{hA}\right)_c$$ unfinned surface

$$\frac{1}{UA} = \left(\frac{1}{\eta_o hA} + \frac{R''_{foul}}{\eta_o A}\right)_h + R_{wall} + \left(\frac{R''_{foul}}{\eta_o A} + \frac{1}{\eta_o hA}\right)_c$$ finned surface

> overall surface efficiency – $\eta_o = \frac{q_{tot}}{q_{max}} = \frac{q_t}{hA_t\theta_b}$

q_t – total heat rate from finned surface
A_t – total area of finned surface
- $A_t = NA_f + A_b$
  - N – # of fins
  - A_f – area of one fin
  - A_b – area of exposed unfinned surface

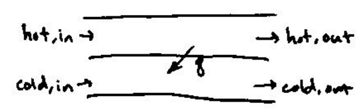
$$\eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f)$$

- total heat rate – $q_t = N(\eta_f hA_f\theta_b) + hA_b\theta_b$

- resistance of fin array – $R_t = \frac{T_b - T_\infty}{q_t} = \frac{1}{\eta_o hA_t}$

<<< General Analysis

- $q = \dot{m}_h (i_{h,i} - i_{h,o}) \xrightarrow[\text{no phase change}]{\text{constant } c_p} q = \dot{m}_h c_{p,h} (T_{h,i} - T_{h,o})$
- $q = \dot{m}_c (i_{c,o} - i_{c,i}) \qquad q = \dot{m}_c c_{p,c} (T_{c,o} - T_{c,i})$

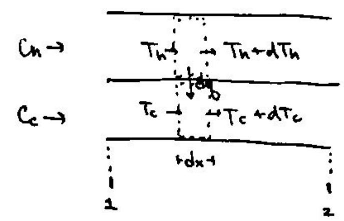


- $q = UA\Delta T$. $\qquad$ i = fluid enthalpy
  - $\Delta T = T_h(x) - T_c(x)$
- $dq = U \Delta T \, dA$

→ Parallel Flow Heat Exchanger

→ Assumptions

- insulated from surrounding . heat exchange only between fluid
- neglegible axial heat conduction
- neglegible ΔPE, ΔKE
- constant fluid specific heat c_p
- constant overall heat transfer coefficient U



h = hot $\quad$ i = in $\quad$ 1 = left
c = cold $\quad$ o = out $\quad$ 2 = right

→ Analysis

> heat capacity rate - $\boxed{C_p = \dot{m} c_p}$

- $dq = -\dot{m}_h c_{p,h} dT_h = -C_h dT_h$
- $dq = \dot{m}_c c_{p,c} dT_c = -C_c dT_c$
- $d(\Delta T) = dT_h - dT_c$

$$d(\Delta T) = -dq \left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

$$d(\Delta T) = -U \Delta T \, dA \left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

$$\int_1^2 \frac{d(\Delta T)}{\Delta T} = -U\left(\frac{1}{C_h} - \frac{1}{C_c}\right) \int_1^2 dA$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left(\frac{1}{C_h} - \frac{1}{C_c}\right)$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -UA\left[\frac{T_{h,i} - T_{h,o}}{q} + \frac{T_{c,o} - T_{c,i}}{q}\right]$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -\frac{UA}{q}\left((T_{h,i} - T_{c,i}) - (T_{h,o} - T_{c,o})\right)$$

$$\left.\begin{array}{l} \Delta T_1 \equiv T_{h,i} - T_{c,i} \\ \Delta T_2 \equiv T_{h,o} - T_{c,o} \end{array}\right\} \text{parallel flow heat exchanger}$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = -\frac{UA}{q}(\Delta T_1 - \Delta T_2)$$

$$q = UA \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

$$\boxed{q = UA \Delta T_{lm}} \qquad \boxed{\Delta T_{lm} \equiv \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)}}$$

delta T log mean, log mean temp diff

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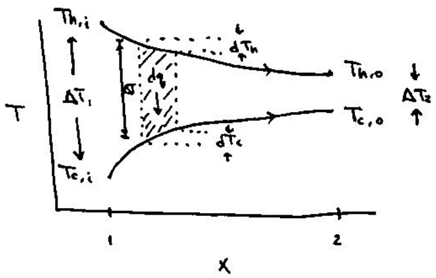
··· Log Mean Temp Diff ΔT_{lm}

› $\boxed{q = UA\,\Delta T_{lm}}$

› $\boxed{\Delta T_{lm} = \dfrac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}}$

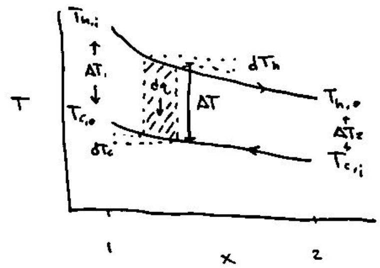
→ Parallel Flow



$\Delta T_1 \equiv T_{h,1} - T_{c,1} = T_{h,i} - T_{c,i}$

$\Delta T_2 \equiv T_{h,2} - T_{c,2} = T_{h,o} - T_{c,o}$

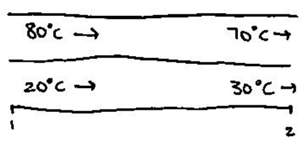
→ Counter Flow



$\Delta T_1 \equiv T_{h,1} - T_{c,1} = T_{h,i} - T_{c,o}$

$\Delta T_2 \equiv T_{h,2} - T_{c,2} = T_{h,o} - T_{c,i}$

$\boxed{\text{EX1}}$

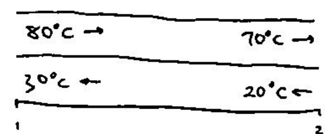


$\Delta T_1 = 80°C - 20°C = 60°C$

$\Delta T_2 = 70°C - 30°C = 40°C$

$\Delta T_{lm} = \dfrac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} = \dfrac{60-40}{\ln(60/40)} = 49.3°C$

$\boxed{\text{EX2}}$



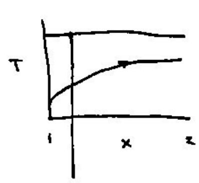
$\Delta T_1 = 80°C - 30°C = 50°C$

$\Delta T_2 = 70°C - 20°C = 50°C$

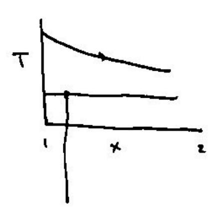
$\Delta T_{lm} = \Delta T_1 = \Delta T_2 = 50°C$

- ΔT_{lm} greater for counter flow at same inlet and outlet T
  - counterflow needs less surface area
- $T_{c,o}$ can exceed $T_{h,o}$ for counterflow, but not parallel flow

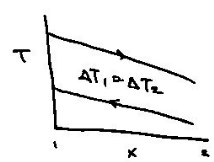
→ Special Operating Conditions



- $C_h \gg C_c$
- condensing vapor keep T_h const.



- $C_h \ll C_c$
- evaporating liquid keep T_c const

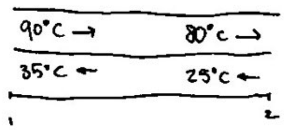


- $C_c = C_h$

<<< Summary of Heat Analysis Problems

- Design problems
  - Given : $\dot{m}$, $T_{h,i}$, $T_{h,o}$, $T_{c,i}$, $T_{c,o}$
  - Calculate : surface area, choose heat exchanger
- Performance problems
  - Given : heat exchanger surface area, $T_{h,i}$, $T_{c,i}$, $\dot{m}$
  - Calculate : q, $T_{h,o}$, $T_{c,o}$

$\boxed{Ex1}$ Find the area of heat exchanger. $C = 1 J/K$ for both fluid. $U = 1 W/m^2K$



$\Delta T_1 = 90°C - 35°C = 55°C$

$\Delta T_2 = 80°C - 25°C = 55°C$

If $\Delta T_1 = \Delta T_2$, $\Delta T_{lm} = \Delta T_1 = \Delta T_2 = 55°C$

$q = UA\Delta T_{lm} = C\Delta T$

$$A = \frac{C\Delta T}{U\Delta T_{lm}} = \frac{(1\ J/K)(90°C - 80°C)}{(1\ W/m^2K)(55°C)} = 0.18\ m^2$$

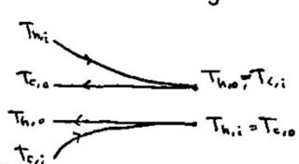
<<< Effectiveness - NTU Method

- correlations needed for more complex heat exchangers

> effectiveness of heat exchanger - actual heat transfer divided by theoretical maximum

> $\boxed{\varepsilon = \frac{q}{q_{max}}}$

- max efficiency achieved by ∞ length counterflow



- If $C_c > C_h$, $q_{max} = C_h(T_{h,i} - T_{c,i})$ hot stream has max q
- If $C_c < C_h$, $q_{max} = C_c(T_{h,i} - T_{c,i})$ cold stream has max q
- In general, $\boxed{q_{max} = C_{min}(T_{h,i} - T_{c,i})}$

- $\varepsilon = \frac{C_h(T_{h,i} - T_{h,o})}{C_{min}(T_{h,i} - T_{c,i})}$
- $\varepsilon = \frac{C_c(T_{c,i} - T_{c,o})}{C_{min}(T_{h,i} - T_{c,i})}$
- $\varepsilon = f\left(NTU, \frac{C_{min}}{C_{max}}\right)$

  > Number of transfer unit - $\boxed{NTU = \frac{UA}{C_{min}}}$

- $\boxed{q = \varepsilon C_{min}(T_{h,i} - T_{c,i})}$

> heat capacity ratio - $\boxed{C_r = \frac{C_{min}}{C_{max}}}$

<<< Effectiveness - NTU Method Example

- Design problem - Calculate C_r, ε, use correlation for NTU, then solve for area A
- Performance problem - Calculate C_r, NTU, use correlation for ε, solve for T_f with $q = \varepsilon q_{max}$

$\boxed{EX1}$ Hot gas enters finned, cross flow heat exchanger at 300°C and leaves at 100°C. Cold water of 1kg/s goes from 35°C to 125°C. Find surface area at hot side A_h.

Given: $U_h = 100\ W/m^2 \cdot K$

$c_{p,c} = c_{p,H_2O} = 4197\ J/K\,kg$

$c_{p,h} = c_{p,gas} = 1000\ J/K\,kg$

- $C_c = \dot{m}_c c_{p,c} = (1kg/s)(4197 J/K\,kg) = 4197\ W/K$
- $q = C_c \Delta T_c = (4197 W/K)(125-35)°C = 3.78\times10^5\ W$
- $C_h = \dot{m}_h c_{p,h} = \frac{q}{\Delta T_h} = \frac{3.78\times10^5 W}{(300-100)°C} = 1889\ W/K$ ← C_{min}
- $q_{max} = C_{min}(T_{h,i} - T_{c,i}) = (1889 W/K)(300°C - 35°C) = 5.00\times10^5\ W$
- $\varepsilon = \frac{q}{q_{max}} = \frac{3.78\times10^5 W}{5.00\times10^5 W} = 0.755$
- $C_r = \frac{C_{min}}{C_{max}} = \frac{1889\ W/K}{4197 W/K} = 0.45$
- From unmixed flow (finned) correlation (pg 666. Fig 11.14), NTU ≈ 2.0
  - $NTU = \frac{U_h A_h}{C_h} \Rightarrow A_h = \frac{C_h\, NTU}{U_h} = \frac{(2.0)(1889 W/K)}{100 W/m^2\cdot K} = 37.8\,m^2$

<<< Intro to Radiation

- All matter of finite temperature emits electromagnetic radiation in all directions.
    - The radiation can be absorbed by other matter
- For gas & transparent solids, radiation is volumetric phenomena
    - don't absorb radiation back
- For liquids & solids, radiation is surface phenomena
    - absorbs radiation back

→ Radiation Heat Fluxes (overall wavelengths and all directions)

> emissive power E - rate at which radiation is emitted from a surface per unit area

> irradiation G - rate at which radiation is incident upon a surface per unit area

- reflection - reflectivity ρ - fraction of irradiation reflected
- absorption - absorptivity α - fraction of irradiation absorbed
- transmission - transmitivity τ - fraction of irradiation transmitted
- $\rho + \alpha + \tau = 1$

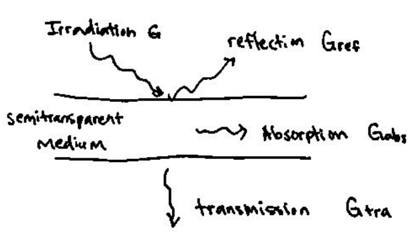
> opaque - $\tau = 0$

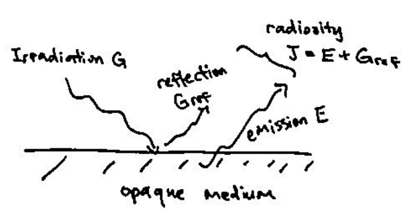
> radiosity J - rate at which radiation leaves a surface per unit area

- $J = E + \rho G$ (opaque)

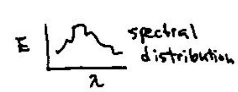
> net radiative flux q''_{rad} - net rate of radiation leaving a surface per unit area

- $q''_{rad} = \varepsilon \sigma T^4 - \alpha G$ (opaque)

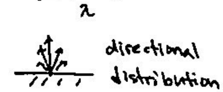




- radiation has dependency on wavelength (spectral)



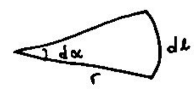
- radiation has dependency on direction (directionality)



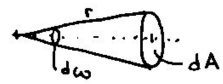
<<< Spectral Intensity

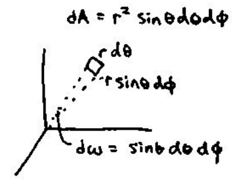
→ Math Review

> differential plane angle - $\boxed{d\alpha = \frac{dl}{r}}$



> differential solid angle - $\boxed{d\omega = \frac{dA}{r^2}}$





· unit - steradian (sr)

· $\boxed{d\omega = \sin\theta \, d\theta \, d\phi}$

→ Radiation Intensity & Emission

> spectral intensity $I_{\lambda,e}$ - rate at which radiant energy is emitted at wavelength λ in the direction θ,ϕ, per unit area of emitting surface normal to this direction, per unit solid angle about this direction, per unit wavelength interval $d\lambda$ about λ.

$I_{\lambda,e}$ [=] $W/m^2 \cdot sr \cdot \mu m$

> $\boxed{I_{\lambda,e}(\lambda,\theta,\phi) = \frac{dq}{dA_1 \cos\theta \, d\omega \, d\lambda}}$

$\cos\theta$ accounts for projection

· $dq = I_{\lambda,e}(\lambda,\theta,\phi) \, dA_1 \cos\theta \, d\omega \, d\lambda$

$dq_\lambda = I_{\lambda,e}(\lambda,\theta,\phi) \, dA_1 \cos\theta \, d\omega$ $\boxed{dq_\lambda \equiv \frac{dq}{d\lambda}}$

$dq''_\lambda = I_{\lambda,e}(\lambda,\theta,\phi) \cos\theta \, d\omega$

$dq''_\lambda = I_{\lambda,e}(\lambda,\theta,\phi) \cos\theta \sin\theta \, d\theta \, d\phi$

> spectral hemispherical emissive power E_λ - flux of λ, radiation from surface in all directions

· $\boxed{E_\lambda(\lambda) = q''_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} I_{\lambda,e}(\lambda,\theta,\phi) \cos\theta \sin\theta \, d\theta \, d\phi}$

> total hemispherical emissive power E - flux of all radiation from surface in all directions

· $\boxed{E = \int_0^\infty E_\lambda(\lambda) \, d\lambda}$

> diffusive matter - spectral intensity I is independent of direction $I_{\lambda,e}(\lambda,\theta,\phi) = I_{\lambda,e}(\lambda)$

· $E_\lambda(\lambda) = \pi I_{\lambda,e}(\lambda)$

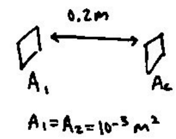
· $E = \pi I_e$

<<< Spectral Intensity

→ Emission (cont.)

$\boxed{EX1}$ Both surfaces has total intensity associated with emission $I = 5000\ W/m^2\ sr$

Calculate net flux q_{rad} emitted from surface 1 to 2?



- A_1, A_2 are small and faraway, so treat as differential surfaces dA_2 and a point at A_1.

$$dq = I\ dA_1 \cos\theta\ d\omega$$

$$q = I\ A_1 \cos\theta\ \omega \qquad \text{(differential surface)}$$

$$q = I\ A_1 \cos\theta\ \frac{A_2}{r^2}$$

$$q = (5000\ W/m^2\ sr)(10^{-3}m^2)\cos(0)\ \frac{(10^{-3}m^2)}{(0.2m)^2} = 0.125\ W$$

→ Irradiation

> spectral intensity $I_{\lambda,i}(\lambda,\theta,\phi)$ – rate at which radiant energy of wavelength λ is incident from θ,ϕ direction, per unit area of intercepting surface normal to this direction, per unit solid angle about this direction, and per unit wavelength interval $d\lambda$ about λ.

- practically, $I_{\lambda,i}$ is all fluxes reversed of $I_{\lambda,e}$

> spectral irradiation – flux of λ radiation incident on a surface

- $$\boxed{G_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} I_{\lambda,i}(\lambda,\theta,\phi)\cos\theta\sin\theta\ d\theta\ d\phi}$$

> total hemispherical irradiation – flux of all radiation incident on a surface

- $$\boxed{G = \int_0^\infty G_\lambda(\lambda)\ d\lambda}$$

- For diffuse irradiation,

- $$\boxed{G_\lambda = \pi I_{\lambda,i}}$$
- $$\boxed{G = \pi I_i}$$

<<< Spectral Intensity

→ Radiosity

> spectral radiosity J_λ - flux of radiation leaving surface at wavelength λ

· intensity associated with emission and reflection $I_{\lambda,e+r}(\lambda,\theta,\phi)$

· $\boxed{J_\lambda(\lambda) = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} J_\lambda\,l\,(\lambda,\theta,\phi)\cos\theta\sin\theta\,d\theta\,d\phi}$

> total radiosity J - flux of radiation leaving surface at all wavelengths

· $\boxed{J = \int_0^\infty J_\lambda(\lambda)\,d\lambda}$

· For diffusive reflector and diffusive emitter,

· $J_\lambda = \pi I_{\lambda,e+r}$

· $J = \pi I_{e+r}$

<<< Blackbody Radiation

> blackbody
1. absorbs all incident radiation, regardless of wavelength and direction
2. emits maximum energy, compared to other object at same temp & wavelength
3. is a diffusive emitter (radiation emitted not dependent on direction)

· best approximated by isothermal cavity

· blackbody radiation exists within the cavity irrespective whether the cavity surface is highly reflective or absortive.

→ Planck Distribution

> blackbody spectral intensity $\boxed{I_{\lambda,b}(\lambda,T) = \dfrac{2hc_0^2}{\lambda^5 \exp\left[\left(\frac{hc_0}{\lambda k_B T}\right)-1\right]}}$

> blackbody spectral emissive power $\boxed{E_{\lambda,b}(\lambda,T) = \pi I_{\lambda,b}(\lambda,T)}$

→ Wien's Displacement Law

> $\boxed{\lambda_{max} T = 2898\ \mu m \cdot K}$

→ Stefan - Boltzmann Law

> blackbody total emissive power - $E_b = \int_0^\infty E_{\lambda,b}(\lambda,T)\,d\lambda$

$= \int_0^\infty \pi I_{\lambda,b}(\lambda,T)\,d\lambda$

$\boxed{E_b = \sigma T^4}$

> blackbody total intensity - $\boxed{I_b = \frac{E_b}{\pi} = \frac{\sigma}{\pi}T^4}$

<<< Emission from Real Surfaces

> spectral directional emissivity - ratio of intensity of radiation emitted at λ in θ, ϕ to the intensity of radiation emitted by a blackbody at same T, λ.

$$\boxed{\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T) = \frac{I_{\lambda,e}(\lambda,\theta,\phi,T)}{I_{\lambda,b}(\lambda,T)}}$$

> total directional emissivity - $\boxed{\varepsilon_\theta(\theta,\phi,T) = \frac{I_e(\theta,\phi,T)}{I_b(T)}}$

> spectral hemispherical emissivity - $\boxed{\varepsilon_\lambda(\lambda,T) = \frac{E_\lambda(\lambda,T)}{E_{\lambda,b}(\lambda,T)}}$

> total hemispherical emissivity - $\boxed{\varepsilon(T) = \frac{\int_0^\infty \varepsilon_\lambda(\lambda,T)\, E_{\lambda,b}(\lambda,T)\, d\lambda}{E_b(T)}}$

· $\boxed{E = \varepsilon E_b = \varepsilon \sigma T^4}$

<< Irradiation on a Real Surface

→ Absorption

> spectral directional absorptivity - fraction of spectral intensity incident in direction θ, ϕ that is absorbed

$$\boxed{\alpha_{\lambda,\theta}(\lambda,\theta,\phi) = \frac{I_{\lambda,i,abs}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}}$$

> spectral hemispherical absorptivity - $\boxed{\alpha_\lambda(\lambda) = \frac{G_{\lambda,abs}(\lambda)}{G_\lambda(\lambda)}}$

> total hemispherical absorptivity - $\boxed{\alpha = \frac{G_{abs}}{G}}$

→ Reflection

> spectral directional reflectivity - $\boxed{\rho_{\lambda,\theta}(\lambda,\theta,\phi) = \frac{I_{\lambda,i,ref}(\lambda,\theta,\phi)}{I_{\lambda,i}(\lambda,\theta,\phi)}}$

> spectral hemispherical reflectivity - $\boxed{\rho_\lambda(\lambda) = \frac{G_{\lambda,ref}(\lambda)}{G_\lambda(\lambda)}}$

> total hemispherical reflectivity - $\boxed{\rho = \frac{G_{ref}}{G}}$

→ Transmition

> spectral hemispherical transmissivity - $\boxed{\tau_\lambda = \frac{G_{\lambda,tr}(\lambda)}{G_\lambda(\lambda)}}$

> total hemispherical transmissivity - $\boxed{\tau = \frac{G_{tr}}{G}}$

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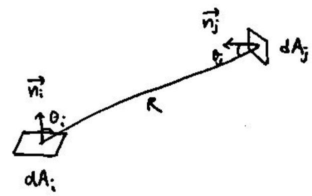
<<< View Factor

> view factor – fraction of radiation leaving surface i that is intercepted by surface j

- $\boxed{F_{ij} = \dfrac{q_{i\to j}}{A_i J_i}} \Rightarrow \boxed{q_{i\to j} = F_{ij} A_i J_i}$



- $dq_{i\to j} = I_{e+r,i} \cos\theta_i \, dA_i \, d\omega_{ji}$

$$dq_{i\to j} = I_{e+r,i} \cos\theta_i \, dA_i \frac{\cos\theta_j \, dA_j}{R^2}$$

$$dq_{i\to j} = \frac{J_i}{\pi} \frac{\cos\theta_i \cos\theta_j}{R^2} dA_i \, dA_j$$

$$q_{i\to j} = J_i \int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_j \, dA_i$$

- $$\boxed{F_{ij} = \frac{1}{A_i}\int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_j \, dA_i}$$

$$F_{ji} = \frac{1}{A_j}\int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} dA_j \, dA_i$$

→ View Factor Relations

- reciprocity relation – $\boxed{A_i F_{ij} = A_j F_{ji}}$
- summation rule for enclosure – $\boxed{\sum_{j=1}^{N} F_{ij} = 1}$

<<< Blackbody Radiation Exchange

- net radiative exchange

$$q_{ij} = q_{i\to j} - q_{j\to i}$$

$$= F_{ij} A_i J_i - F_{ji} A_j J_j$$

$$= F_{ij} A_i E_{bi} - F_{ji} A_j E_{bj}$$ no reflection. $J_i = E_{bi}$

$$= F_{ij} A_i (E_{bi} - E_{bj})$$ reciprocity rule

$$\boxed{q_{ij} = F_{ij} A_i \sigma (T_i^4 - T_j^4)}$$ net rate of radiation leaving i by interacting with j

$$q_{ji} = F_{ji} A_j \sigma (T_i^4 - T_j^4)$$

<<< Radiation Exchange Between Opaque, Diffuse, Grey Surfaces in Enclosure

→ Net radiation exchange at surface

- $\boxed{q_i = A_i (J_i - G_i)}$

  $q_i = A_i (E_i + \rho_i G_i - G_i)$

- $\boxed{q_i = A_i (E_i - \alpha_i G_i)}$ $\alpha_i = 1 - \rho_i$ for opaque surface $(\tau = 0)$

  - $J_i = E_i + \rho_i G_i$

    $J_i = \varepsilon_i E_{bi} + (1 - \varepsilon_i) G_i$ $\alpha_i = \varepsilon_i$ for gray surface

    $G_i = \dfrac{J_i - \varepsilon_i E_{bi}}{1 - \varepsilon_i}$

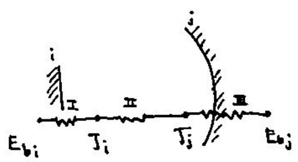
- $q_i = A_i \left(E_i - \alpha_i \dfrac{J_i - \varepsilon_i E_{bi}}{1 - \varepsilon_i}\right)$

  $\boxed{q_i = \dfrac{E_{bi} - J_i}{(1 - \varepsilon_i)/\varepsilon_i A_i}}$

  > surface radiative resistance – $\boxed{R_i = \dfrac{1 - \varepsilon_i}{\varepsilon_i A_i}}$

  > space resistance – $\boxed{R_i = \dfrac{1}{A_i F_{ij}}}$

EX1 Derive expression for blackbody radiative heat transfer rate for enclosure.



$$\left.\begin{aligned} R_I &= \frac{1-\varepsilon_i}{\varepsilon_i A_i} \\ R_{II} &= \frac{1}{A_i F_{ij}} \\ R_{III} &= \frac{1-\varepsilon_j}{\varepsilon_j A_j} \end{aligned}\right\} \quad R = \sum R_i = \frac{1-\varepsilon_i}{\varepsilon_i A_i} + \frac{1}{A_i F_{ij}} + \frac{1-\varepsilon_j}{\varepsilon_j A_j}$$

- $q_{ij} = \dfrac{1}{R} (E_{bi} - E_{bj})$

$$= \frac{\sigma (T_i^4 - T_j^4)}{\frac{1-\varepsilon_i}{\varepsilon_i A_i} + \frac{1}{A_i F_{ij}} + \frac{1-\varepsilon_j}{\varepsilon_j A_j}} \qquad \text{blackbody } E = \sigma T^4$$

$$= A_i \sigma (T_i^4 - T_j^4) \qquad \text{blackbody } \varepsilon_i = \varepsilon_j = 1$$

$F_{ij} = 1$ (everything absorbed)