PHYS 121 Mechanics Equations

Organized by Teng-Jui Lin

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Kinematics

Quantity	Unit	Definition
Displacement	m	$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$
Instantaneous velocity	$\mathrm{m/s}$	$\vec{v} = \dfrac{d\vec{r}}{dt}$
Instantaneous acceleration	$\mathrm{m/s^2}$	$\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d^2\vec{x}}{dt^2}$

Description	Equations
Kinematics equations at constant acceleration	$x = x_0 + v_0 t + \dfrac{1}{2}at^2$ $v = v_0 + at$ $v_f^2 = v_i^2 + 2a\Delta x$
Relative velocity	$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$
Centripetal acceleration	$a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{4\pi^2 R}{T^2}$

Dynamics

Quantity	Unit	Definition
Spring force (Hooke's law)	N	$F_s = -k\Delta x$
Static friction	N	$f_s \leq (f_s)_{\max} = \mu_s F_N$ $\mu_s = \tan(\theta)$

Quantity	Unit	Definition
Kinetic friction	N	$f_k = \mu_k F_N$
Gravitational force near Earth surface	N	$\vec{F}_g = m\vec{g}$ $g = 9.81 \mathrm{m/s^2}$

Description	Equations
Newton's first law moving at constant velocity	$\sum \vec{F}_{\text{ext}} = \vec{0}$
Newton's second law	$\sum \vec{F}_{\text{ext}} = m\vec{a}$
Newton's third law	$\vec{F}_{AB} = -\vec{F}_{BA}$
Acceleration on an inclined plane	$a = g \sin\theta$

Energy

Quantity	Unit	Definition
Work	J	$W = \vec{F} \cdot \vec{x} = Fx\cos\theta$ $W = \int_{x_1}^{x_2} F\,dx$
Kinetic energy	J	$K = \frac{1}{2}mv^2$
Power	W	$P = \frac{dW}{dt} = \frac{dE}{dt}$ $P = \vec{F} \cdot \vec{v}$
Gravitational potential energy	J	$U = mgh$ $W_{\text{grav}} = -\Delta U_{\text{grav}}$
Elastic potential energy	J	$U = \frac{1}{2}kx^2$ $W_{\text{el}} = -\Delta U_{\text{el}}$
Reduced mass	kg	$\mu = \frac{m_1 m_2}{m_1 + m_2}$
Coefficient of restitution	-	$e = -\frac{v_{12,f}}{v_{12,i}}$

Description	Equations
Work-energy theorem	$W_{\text{total}} = \Delta K$
Conservation of mechanical energy	$K_i + U_i = K_f + U_f$

Description	Equations
Energy of system with external force (non-isolated system)	$K_i + U_i + W = K_f + U_f$
Conservation of energy	$\Delta K + \Delta U + \Delta U_{int} = 0$
Force as a function of potential energy	$F = -\frac{dU}{dx}\vec{F} = -\vec{\nabla}U$

Momentum

Quantity	Unit	Definition
Momentum	$\mathrm{kg \cdot m/s}$	$\vec{p} = m\vec{v}$ $\sum \vec{F}_{\text{ext}} = \frac{d\vec{p}}{dt}$
Impulse	$\mathrm{kg \cdot m/s}$	$\vec{J} = \sum \vec{F}\Delta t$ $\vec{J} = \int_{t_1}^{t_2} \sum \vec{F}\, dt$
Center of mass	m	$\vec{r}_{\text{cm}} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}$

Description	Equations
Impulse-momentum theorem	$\vec{J} = \Delta\vec{p}$
Conservation of momentum (closed system)	$\vec{p}_i = \vec{p}_f$ $\sum \vec{F}_{\text{ext}} = \frac{d\vec{p}}{dt}$
Force on extended body	$\sum \vec{F}_{\text{ext}} = m\vec{a}_{\text{cm}}$

Rotational Kinematics

Quantity	Unit	Definition
Angular displacement	rad	$\Delta\theta = \theta_f - \theta_i$
Angular velocity	$\mathrm{rad/s}$	$\omega_z = \frac{d\theta}{dt}$
Angular acceleration	$\mathrm{rad/s^2}$	$\alpha_z = \frac{d\omega_z}{dt} = \frac{d^2\theta}{dt^2}$
Rotational Inertia of particle	$\mathrm{kg \cdot m^2}$	$I = \sum_i m_i r_i^2$

Quantity	Unit	Definition
Rotational kinetic energy	J	$K = \frac{1}{2}I\omega^2$

Description	Equations
Rotational kinematics equation with constant angular acceleration	$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2$ $\omega_z = \omega_{0z} + \alpha_z t$ $\omega_{fz}^2 = \omega_{iz}^2 + 2\alpha_z\Delta\theta$
Relationship between linear kinematics and rotational kinematics	$s = r\theta$ $v = r\omega$ $a_{\tan} = r\alpha$ $a_{\mathrm{rad}} = \frac{v^2}{r} = \omega^2 r$
Parallel-axis theorem	$I_{\mathrm{parallel}} = I_{\mathrm{cm}} + md^2$

Rotational Dynamics

Quantity	Unit	Definition
Torque	$\mathrm{N \cdot m}$	$\vec{\tau} = \vec{r} \times \vec{F} = Fr\sin\theta$ $\sum\vec{\tau} = \frac{d\vec{L}}{dt}$
Angular momentum of a particle	$\mathrm{kg \cdot m^2/s}$	$\vec{L} = \vec{r} \times \vec{p} = mvr\sin\theta$
Angular momentum of rotating body	$\mathrm{kg \cdot m^2/s}$	$\vec{L} = I\vec{\omega}$

Description	Equations
Rotational Newton's second law	$\sum\tau = I\alpha_z$
Condition of mechanical equilibrium	$\sum\vec{F}_{\mathrm{ext}} = m\vec{a}$ $\sum\tau = I\alpha_z$
Total kinetic energy of rotating and translating object	$K = \frac{1}{2}mv_{\mathrm{cm}}^2 + \frac{1}{2}I_{\mathrm{cm}}\omega^2$
Rolling without slipping	$v_{\mathrm{cm}} = R\omega$
Slipping (only rolling)	$v_{\mathrm{cm}} < R\omega$
Skidding (only translating)	$v_{\mathrm{cm}} > R\omega$

Description	Equations
Rotational Work	$W = \tau_z \Delta\theta$ $W = \int_{\theta_1}^{\theta_2} \tau_z \, d\theta$ $W = \Delta K_{\text{rot}}$
Power	$P = \frac{dW}{dt}$ $P = \tau_z \omega_z$
Conservation of angular momentum (closed system)	$\vec{L}_i = \vec{L}_f$ $\sum \vec{\tau} = \frac{d\vec{L}}{dt}$

Universal Gravitation

Quantity	Unit	Definition
Gravitational force	N	$F_g = G\frac{m_1 m_2}{r^2}$
Gravitational acceleration	$\mathrm{m/s^2}$	$g = G\frac{m_E}{r^2}$
Gravitational potential energy	J	$U = -G\frac{m_E m}{r}$

Description	Equations
Escape velocity	$v_{\text{escape}} = \sqrt{\frac{2Gm_E}{R}}$
Velocity in circular orbit	$v_{\text{circ}} = \sqrt{\frac{Gm_E}{R}} = \frac{2\pi R}{T}$
Period in circular orbit	$T = \frac{2\pi R}{v} = 2\pi R\sqrt{\frac{R}{Gm_E}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$