CHEM E 467 Biochemical Engineering

Enzyme Kinetics

Description	Equations
Arrhenius equation	$k = A\exp\left(\frac{E_A}{RT}\right)$
Power law	$r_P = k\prod C_i^{\nu_i}$
Linearized 1st order rate law	$\ln\left(\frac{[A]}{[A]_0}\right) - kt$

Michaelis-Menten kinetics

Description	Equations
Overall reaction	$\mathrm{E} + \mathrm{S} \longrightarrow \mathrm{E} + \mathrm{P}$
Reaction mechanism	$\mathrm{S} + \mathrm{E} \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} \mathrm{ES}$ $\mathrm{ES} \xrightarrow{k_2} \mathrm{P} + \mathrm{E}$
Enzyme balance	$[\mathrm{E_T}] = [\mathrm{E}] + [\mathrm{ES}]$ $[\mathrm{E}] = [\mathrm{E_T}] - [\mathrm{ES}]$
Pseudo-steady-state approximation	$r_{\mathrm{ES}} = k_1[\mathrm{S}][\mathrm{E}] - k_{-1}[\mathrm{ES}] - k_2[\mathrm{ES}] = 0$ $r_{\mathrm{ES}} = k_1[\mathrm{S}]([\mathrm{E_T}] - [\mathrm{ES}]) - k_{-1}[\mathrm{ES}] - k_2[\mathrm{ES}] = 0$ $[\mathrm{ES}] = \frac{k_1[\mathrm{S}][\mathrm{E_T}]}{k_1[\mathrm{S}] + k_{-1} + k_2} = \frac{[\mathrm{E_T}][\mathrm{S}]}{K_M + [\mathrm{S}]}$
Turnover number (# substrates converted to product per unit time on one enzyme at saturation)	$k_{\mathrm{cat}} = k_2$
Michaelis-Menten constant (attraction of enzyme of its substrate, [Substrate] which rate of rxn is 1/2 max)	$K_M = \frac{k_{\mathrm{cat}} + k_{-1}}{k_1}$
Maximum rate	$V_{\max} = k_{\mathrm{cat}}[\mathrm{E_T}]$
Michaelis-Menten equation Rate of reaction	$r_{\mathrm{P}} = k_2[\mathrm{ES}]$ $= \frac{k_1k_2[\mathrm{S}][\mathrm{E_T}]}{k_1[\mathrm{S}] + k_{-1} + k_2}$ $= \frac{k_{\mathrm{cat}}[\mathrm{S}][\mathrm{E_T}]}{K_M + [\mathrm{S}]}$ $= \frac{V_{\max}[\mathrm{S}]}{K_M + [\mathrm{S}]}$
Lineweaver-Burk equation	$\frac{1}{r_{\mathrm{P}}} = \frac{K_M}{V_{\max}}\frac{1}{[\mathrm{S}]} + \frac{1}{V_{\max}}$
Eadie-Hofstee equation	$r_{\mathrm{P}} = V_{\max} - K_M\frac{r_{\mathrm{P}}}{[\mathrm{S}]}$
Hanes-Woolf equation	$\frac{[\mathrm{S}]}{r_{\mathrm{P}}} = \frac{K_M}{V_{\max}} + \frac{1}{V_{\max}}[\mathrm{S}]$

Enzymatic inhibition

Competitive inhibition

Description	Equations
Reaction mechanism	$\mathrm{E} + \mathrm{S} \rightleftharpoons \mathrm{ES}$ $\mathrm{ES} \longrightarrow \mathrm{E} + \mathrm{P}$ $\mathrm{E} + \mathrm{I} \rightleftharpoons \mathrm{EI}\ (\text{inactive})$
Reaction rate	$r_\mathrm{P} = \dfrac{V_\text{max}[\mathrm{S}]}{[\mathrm{S}] + K_M\left[1 + \dfrac{[\mathrm{I}]}{K_I}\right]}$
Lineweaver-Burk form $\uparrow K_I, \uparrow \text{slope}$	$\dfrac{1}{r_\mathrm{P}} = \dfrac{1}{[\mathrm{S}]}\left[\dfrac{K_M}{V_\text{max}}\left[1 + \dfrac{[\mathrm{I}]}{K_I}\right]\right] + \dfrac{1}{V_\text{max}}$

Uncompetitive inhibition

Description	Equations
Reaction mechanism	$\mathrm{E} + \mathrm{S} \rightleftharpoons \mathrm{ES}$ $\mathrm{ES} \longrightarrow \mathrm{E} + \mathrm{P}$ $\mathrm{ES} + \mathrm{I} \rightleftharpoons \mathrm{ESI}\ (\text{inactive})$
Reaction rate	$r_\mathrm{P} = \dfrac{V_\text{max}[\mathrm{S}]}{K_M + [\mathrm{S}]\left[1 + \dfrac{[\mathrm{I}]}{K_I}\right]}$
Lineweaver-Burk form $\uparrow K_I, \uparrow \text{intercept}$	$\dfrac{1}{r_\mathrm{P}} = \dfrac{1}{[\mathrm{S}]}\dfrac{K_M}{V_\text{max}} + \dfrac{1}{V_\text{max}}\left[1 + \dfrac{[\mathrm{I}]}{K_I}\right]$

Noncompetitive (mixed) inhibition

Description	Equations
Reaction mechanism	$\mathrm{E} + \mathrm{S} \rightleftharpoons \mathrm{ES}$ $\mathrm{ES} \longrightarrow \mathrm{E} + \mathrm{P}$ $\mathrm{E} + \mathrm{I} \rightleftharpoons \mathrm{EI}\ (\text{inactive})$ $\mathrm{ES} + \mathrm{I} \rightleftharpoons \mathrm{ESI}\ (\text{inactive})$ $\mathrm{S} + \mathrm{EI} \rightleftharpoons \mathrm{ESI}\ (\text{inactive})$
Reaction rate	$r_\mathrm{P} = \dfrac{V_\text{max}[\mathrm{S}]}{([\mathrm{S}] + K_M)\left[1 + \dfrac{[\mathrm{I}]}{K_I}\right]}$
Lineweaver-Burk form $\uparrow K_I, \uparrow \text{slope}, \uparrow \text{intercept}$	$\dfrac{1}{r_\mathrm{P}} = \dfrac{1}{[\mathrm{S}]}\dfrac{K_M}{V_\text{max}}\left[1 + \dfrac{[\mathrm{I}]}{K_I}\right] + \dfrac{1}{V_\text{max}}\left[1 + \dfrac{[\mathrm{I}]}{K_I}\right]$

Cell Growth Kinetics

Description	Equations
Nomenclature	S = substrate mass P = product mass X = cell mass
Mass balance	$\Delta S + \Delta P = \Delta X$
Yield coefficient (mass basis)	$Y_{X/S} = -\dfrac{\Delta X}{\Delta S} = \dfrac{[X] - [X]_0}{[S]_0 - [S]}$

Description	Equations
Exponential growth	$[X] = [X]_0 \exp(\mu_{\max} t)$
Doubling time	$t_d = \dfrac{\ln 2}{\mu}$
Exponential death	$[X] = [X]_0 \exp(-k_d t)$
Optical density (absorbance)	$\mathrm{OD}_\lambda = A_\lambda = \log \dfrac{I}{I_0}$

Cell Reaction Stoichiometry

| Elemental (atomic) balance

Description	Equations
Cell reaction	$\mathrm{C}_w\mathrm{H}_x\mathrm{O}_y\mathrm{N}_z + a\,\mathrm{O}_2 + b\,\mathrm{NH}_3$ $\longrightarrow c\,\mathrm{CH}_\alpha\mathrm{O}_\beta\mathrm{N}_\delta\ + d\,\mathrm{CO}_2 + e\,\mathrm{H}_2\mathrm{O} + f\,\mathrm{C}_j\mathrm{H}_k\mathrm{O}_l\mathrm{N}_m$
Respiratory quotient	$\mathrm{RQ} = \dfrac{\text{mol CO}_2\text{ produced}}{\text{mol O}_2\text{ consumed}} = \dfrac{d}{a}$
Biomass yield (mass basis)	$Y_{X/S} = \dfrac{\Delta X}{\Delta S} = \dfrac{\text{g cell}}{\text{g substrate}}$
Product yield (mass basis)	$Y_{P/S} = \dfrac{\Delta P}{\Delta S} = \dfrac{\text{g product}}{\text{g substrate}}$
Substrate utilization rate (SUR)	$\mathrm{SUR} = \dfrac{\text{mol substrate}}{\text{time}}$
Oxygen utilization rate (OUR)	$\mathrm{OUR} \equiv A = a\,\mathrm{SUR}$
Carbon dioxide evolution rate (CER)	$\mathrm{CER} \equiv D = d\,\mathrm{SUR}$

| Electron balance

Description	Equations
Cell reaction	$\mathrm{C}_w\mathrm{H}_x\mathrm{O}_y\mathrm{N}_z + a\,\mathrm{O}_2 + b\,\mathrm{NH}_3$ $\longrightarrow c\,\mathrm{CH}_\alpha\mathrm{O}_\beta\mathrm{N}_\delta\ + d\,\mathrm{CO}_2 + e\,\mathrm{H}_2\mathrm{O} + f\,\mathrm{C}_j\mathrm{H}_k\mathrm{O}_l\mathrm{N}_m$
Degree of reduction	$\gamma_i = \dfrac{(\text{Valance \# of element } i)(\text{\# atom in element } i)}{\text{\#C atom}}$
Stoichiometric coefficient of oxygen	$a = \frac{1}{4}(w\gamma_S - c\gamma_X - fj\gamma_P)$