

# PHYS 122 Electromagnetism

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## # Electrical Charge and Electric Field

Quantity	Unit	Definition
Electrical field (point charge)	N/C (V/m)	$\vec{E}_s = \frac{\vec{F}_0}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$
Linear charge density	C/m	$\lambda = \frac{dQ}{dl}$
Surface charge density	C/m <sup>2</sup>	$\sigma = \frac{dQ}{dA}$
Volume charge density	C/m <sup>3</sup>	$\rho = \frac{dQ}{dV}$
Electric dipole moment (direction from - to +)	C · m	$\vec{p} = q\vec{d}$
Induced dipole moment (direction from - to +)	C · m	$\vec{p} = \alpha\vec{E}$

Description	Equations
Coulomb's law	$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$
Force on test charge by an electric field	$\vec{F}_0 = q_0 \vec{E}$
Superposition of electric forces	$\vec{F} = \sum_i \vec{F}_i$
Superposition of electric fields	$\vec{E} = \sum_i \vec{E}_i$
Torque on an electric dipole in an uniform electrical field	$\vec{\tau} = \vec{p} \times \vec{E}$
Potential energy of an electric dipole in an uniform electric field	$U = -\vec{p} \cdot \vec{E}$
Electric field of test charge on x axis caused by dipole at origin oriented in + y direction	$E_x = 0$ $E_y = -\frac{kp}{ x ^3}$
Electric field of test charge on y axis caused by dipole at origin oriented in + y direction	$E_x = 0$ $E_y = \frac{2kp}{ y ^3}$

## # Gauss's Law

Quantity	Unit	Definition
Electric flux through a surface	$\text{N} \cdot \text{m}^2/\text{C}$ $(\text{V} \cdot \text{m})$	$\Phi_E = \int \vec{E} \cdot d\vec{A}$
Description	Equations	
Electric flux of a uniform electric field	$\Phi_E = \vec{E} \cdot \vec{A}$	

Description	Equations
Electric flux of a nonuniform electric field	$\Phi_E = \int \vec{E} \cdot d\vec{A} = \int E \cos \theta \, dA$
<b>Gauss's law</b>	$\Phi_E = \oint \vec{E} \cdot d\vec{A}$
Electric flux through a closed surface	$= \oint E \cos \theta \, dA = \frac{Q}{\epsilon_0}$

## | Electric field of uniform *spherical* charge distributions

- charged = *uniformly* charged throughout (insulating)
- conducting = charge only on surface

Charge Distribution	Point in Electric Field	Electric Field Magnitude
Point charge	-	$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$
Solid conducting sphere Hollow charged sphere	Outside sphere, $r > R$	$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$
Solid conducting sphere Hollow charged sphere	Inside sphere, $r < R$	$E = 0$
Solid charged sphere	Outside sphere, $r > R$	$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$
Solid charged sphere	Inside sphere, $r < R$	$E = \frac{1}{4\pi\epsilon_0} \frac{r}{R^3} q$

## | Electric field of uniform *cylindrical* charge distributions

Charge Distribution	Point in Electric Field	Electric Field Magnitude
$\infty$ wire/rod	-	$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r} = \frac{2k\lambda}{r}$
$\infty$ solid conducting cylinder $\infty$ hollow charged cylinder	Outside cylinder, $r > R$	$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r} = \frac{2k\lambda}{r}$
$\infty$ solid conducting cylinder $\infty$ hollow charged cylinder	Inside cylinder, $r < R$	$E = 0$
$\infty$ solid charged cylinder	Outside cylinder, $r > R$	$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r} = \frac{2k\lambda}{r}$
$\infty$ solid charged cylinder	Inside cylinder, $r < R$	$E = \frac{1}{2\pi\epsilon_0} \frac{r}{R^2} \lambda = \frac{2k\lambda r}{R^2}$

## | Electric field of uniform *planar* charge distributions

Charge Distribution	Point in Electric Field	Electric Field Magnitude
$\infty$ charged sheet/plate	-	$E = \frac{\sigma}{2\epsilon_0}$
$\infty$ conducting sheet/plate	-	$E = \frac{\sigma}{\epsilon_0} = \frac{q}{2\epsilon_0 A}$ ( $q$ spreads at each surface)

Charge Distribution	Point in Electric Field	Electric Field Magnitude
Two oppositely charged conducting plates	Between plates	$E = \frac{\sigma}{\epsilon_0}$
Charged conductor	At surface	$E = \frac{\sigma}{\epsilon_0}$

## # Electric Potential

Quantity	Unit	Definition
Electric potential energy (point charge) (choose $U = 0$ at $\infty$ )	J	$U = \frac{1}{4\pi\epsilon_0} \frac{q_s q_0}{r}$
Electric potential (point charge) (choose $V = 0$ at $\infty$ )	V (J/C)	$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q_s}{r}$

Description	Equations
Electric potential energy of a test charge due to many source charges	$U = \frac{q_0}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$
Total electric potential energy of all source charges	$U = \frac{1}{4\pi\epsilon_0} \sum_{i < j} \frac{q_i q_j}{r_{ij}}$
Electric potential due to many source charges	$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$
Electric potential due to continuous distribution of charges	$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$
Electric potential and potential energy of point charges	$U = q_2 V_1$
Work by electric force and electric field	$W_{a \rightarrow b} = \int_a^b \vec{F} \cdot d\vec{l} = q \int_a^b \vec{E} \cdot d\vec{l}$
Work by electric force on a closed path	$W_{a \rightarrow b \rightarrow a} = q \oint \vec{E} \cdot d\vec{l} = 0$
Work by electric force and change in potential energy	$W_{a \rightarrow b} = -\Delta U$
Potential difference	$V_{ab} = V_b - V_a$
Potential difference between terminals of battery	$V_{\text{batt}} = V_{-+} = V_+ - V_-$
Potential difference and work, potential energy difference	$V_{ab} = \frac{\Delta U}{q_0} = -\frac{W_{a \rightarrow b}}{q_0}$
Potential difference and electric field	$V_{ab} = -\int_a^b \vec{E} \cdot d\vec{l} = -\int E \cos \theta \, dl$
Electric field and potential gradient	$\vec{E} = -\vec{\nabla} V$ $= \left\langle -\frac{\partial V}{\partial x}, -\frac{\partial V}{\partial y}, -\frac{\partial V}{\partial z} \right\rangle$

## # Capacitance and Dielectrics

Quantity	Unit	Definition
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Quantity	Unit	Definition
Capacitance (in vacuum)	F (C/V = C <sup>2</sup> /J)	$C = \frac{Q}{V_{-+}} = \frac{Q}{V_{+} - V_{-}}$
Electric energy density (in vacuum)	J/m <sup>3</sup>	$u = \frac{U}{Ad} = \frac{1}{2}\epsilon_0 E^2$

Description	Equations
Capacitance of a parallel-plate capacitor in vacuum	$C = \frac{Q}{V_{-+}} = \epsilon_0 \frac{A}{d}$
Potential energy stored in a charged capacitor (define $U_{\text{uncharged}} \equiv 0$ )	$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$
Electric energy density in vacuum	$u = \frac{U}{Ad} = \frac{1}{2}\epsilon_0 E^2$
Dielectric constant	$\kappa = \frac{C}{C_0} = \frac{V_0}{V} = \frac{E_0}{E}$
Induced surface charge density on a dielectric in an isolated capacitor	$\sigma_{\text{induced}} = \sigma_{\text{bound}}$ $\sigma_0 = \sigma_{\text{free}}$ $\sigma_{\text{induced}} = \sigma_0 \left(1 - \frac{1}{\kappa}\right)$
Permittivity of a dielectric	$\epsilon = \kappa\epsilon_0$
Capacitance of a parallel-plate capacitor with dielectric between plates	$C = \kappa C_0 = \kappa\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$
Electric energy density in a dielectric	$u = \frac{1}{2}\kappa\epsilon_0 E^2 = \frac{1}{2}\epsilon E^2$
Gauss's law in dielectrics	$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{free,enc}}}{\kappa\epsilon_0}$

## # Current, Resistance, and emf

Quantity	Unit	Definition
Current	A (C/s)	$I = \frac{dQ}{dt}$
Current density (per unit cross-section area)	A/m <sup>2</sup>	$\vec{J} = nq\vec{v}_d$ $J = \frac{I}{A} = n q v_d$
Conductivity (intrinsic to a material)	(Ω · m) <sup>−1</sup> A/(V · m)	$\sigma = \frac{J}{E}$
Resistivity (intrinsic to a material)	Ω · m	$\rho = \frac{E}{J}$
Resistance	Ω	$R = \frac{V}{I} = \frac{\rho L}{A} = \frac{L}{\sigma A}$
	Description	Equations
	Drift velocity of charge carrier	$\vec{v}_d = -\frac{q\vec{E}}{m}\tau$
	Current and conductor properties	$I = \frac{dQ}{dt} = n q v_dA = JA$

Description	Equations
Current density (per unit cross-section area)	$\vec{J} = nq\vec{v}_d$ $J = \frac{I}{A} = n q v_d = \frac{nq^2\tau}{m_q}E$
Conductivity (intrinsic to a material)	$\sigma = \frac{J}{E} = \frac{nq^2\tau}{m_q}$
Temperature dependence of resistivity	$\rho(T) = \rho_0(1 + \alpha(T - T_0))$
Temperature dependence of resistance	$R(T) = R_0(1 + \alpha(T - T_0))$

## # Direct-Current (DC) Circuits

### | Circuit analysis

Description	Equations
Circuit elements in series $ Q , I$ - Equal $V, R$ - Add $C$ - Reciprocal	$ Q  =  Q_1  = \dots =  Q_i $ $I = I_1 = \dots = I_i$ $V = \sum_i V_i$ $R = \sum_i R_i$ $\frac{1}{C} = \sum_i \frac{1}{C_i}$
Circuit elements in parallel $V$ - Equal $Q, I, C$ - Add $R$ - Reciprocal	$V = V_1 = \dots = V_i$ $Q = \sum_i Q_i$ $I = \sum_i I_i$ $C = \sum_i C_i$ $\frac{1}{R} = \sum_i \frac{1}{R_i}$
Algebra of reciprocal values of two elements	$\frac{1}{A} = \frac{1}{A_1} + \frac{1}{A_2} \Rightarrow A = \frac{A_1 A_2}{A_1 + A_2}$
Kirchhoff's junction rule (conservation of charge)	$\sum I = 0$
Kirchhoff's loop rule (conservation of energy)	$\sum V = 0$
Battery ( $- \rightarrow +$ )	$+\mathcal{E}$
Resistor (along reference direction)	$-IR$
Capacitor ( $- \rightarrow +$ )	$+\frac{q(t)}{C}$

### | Ohm's law and power

Description	Equations
Ohm's law	$V = IR$
Potential difference of source with internal resistance	$V_{-+} = \mathcal{E} - Ir = IR$
Current of source with internal resistance	$I = \frac{\mathcal{E}}{R + r}$

Description	Equations
Power delivered to or extracted from a circuit element	$P = IV$
Power delivered to a resistor (Note: both $I$ and $V$ depend on $R$ )	$P = IV = I^2 R = \frac{V^2}{R}$
Power output of a source	$P = I\mathcal{E} = IV + I^2 r = I^2(R + r)$

## | R-C circuit

Description	Equations
Time constant	$\tau = RC$
Charge when charging capacitors	$q(t) = C\mathcal{E}(1 - e^{-t/RC})$ $= Q_f(1 - e^{-t/RC})$
Current when charging capacitors	$i(t) = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC}$
Charge when discharging capacitors	$q(t) = Q_0 e^{-t/RC}$
Current when discharging capacitors	$i(t) = -\frac{Q_0}{RC}e^{-t/RC} = I_0 e^{-t/RC}$
Power of battery in R-C circuit	$P = i\mathcal{E} = i^2 R + \frac{iq}{C}$
Total energy stored in capacitor	$U = \frac{1}{2}QV = \frac{1}{2}Q_f\mathcal{E}$

## # Magnetic Force and Motion

Quantity	Unit	Definition
Magnetic force	N	$\vec{F} = q\vec{v} \times \vec{B}$ $=  q vB \sin \theta$
Magnetic flux through a surface	Wb (T · m <sup>2</sup> )	$\Phi_B = \int \vec{B} \cdot d\vec{A}$
Magnetic dipole moment (direction from S to N)	A · m <sup>2</sup> J/T	$\vec{\mu} = I\vec{A}$

## | Magnetic interactions of charged particles

Description	Equations
Magnetic force on a charged particle	$\vec{F} = q\vec{v} \times \vec{B} =  q vB \sin \theta$
Radius of a circular orbit in a magnetic field (charge where $v \perp B$ )	$R = \frac{mv}{ q B}$
Angular speed (frequency) of circular motion	$\omega = 2\pi f = \frac{2\pi}{T} = \frac{ q B}{m}$
Frequency of circular motion	$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{ q B}{2\pi m}$
Period of circular motion	$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi m}{ q B}$

Description	Equations
Velocity selector	$v = \frac{E}{B}$
Thompson's experiment	$v = \sqrt{\frac{2qV}{m}}$ $\frac{q}{m} = \frac{E^2}{2VB^2}$
Mass spectrometers	$m = \frac{ q B^2R}{E}$

### | Magnetic interactions of current-carrying conductor

Description	Equations
Magnetic force on a straight wire segment	$\vec{F} = I\vec{l} \times \vec{B}$
Magnetic force on an infinitesimal wire segment	$d\vec{F} = I d\vec{l} \times \vec{B}$
Magnetic dipole moment	$\vec{\mu} = I\vec{A}$
Magnetic torque on a current loop	$\vec{\tau} = \vec{\mu} \times \vec{B} = IAB \sin \theta$
Magnetic torque on a solenoid	$\vec{\tau} = N\vec{\mu} \times \vec{B} = NIAB \sin \theta$
Potential energy for a magnetic dipole in B field	$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \theta$

### | Magnetic flux and other effects

Description	Equations
Magnetic flux through a surface	$\Phi_B = \int \vec{B} \cdot d\vec{A}$
Gauss's law for magnetism	$\oint \vec{B} \cdot d\vec{A} = 0$
Electromagnetic (Lorentz) force	$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$
Hall effect	$nq = \frac{-J_x B_y}{E_z}$

### # Magnetic Field

Quantity	Unit	Definition
Magnetic field	T N/(A · m) 1G = 10 <sup>-4</sup> T	$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{Id\vec{l} \times \hat{r}}{r^2}$

  

Description	Equations
Ampere's law	$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$
Magnetic field of a point charge	$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$
Biot-Savart law	$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{l} \times \hat{r}}{r^2}$
Magnetic field of infinitesimal length of wire	

Description	Equations
Force on two $\infty$ parallel wires per unit length	$\vec{F} = q\vec{v} \times \vec{B}$ $\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$
Force on two moving charges	$\vec{F} = I\vec{l} \times \vec{B}$ $\vec{F}_{1 \rightarrow 2} = \frac{\mu_0}{4\pi} \frac{q_1 q_2}{r} \vec{v}_2 \times \vec{v}_1 \times \hat{r}$

### | Magnetic field of *linear* conductors

Conductor Form	Magnetic Field Magnitude
$\infty$ straight wire	$B = \frac{\mu_0 I}{2\pi r}$
$\infty$ current-conducting plane	$B = \frac{1}{2}\mu_0 K$

### | Magnetic field of *circular* conductors

Conductor Form	Magnetic Field Magnitude
On the axis of circular wire loop	$B_x = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$
On the axis of N circular wire loops	$B_x = \frac{N\mu_0 I R^2}{2(x^2 + R^2)^{3/2}} = \frac{\mu_0 \mu}{2\pi(x^2 + R^2)^{3/2}}$
At the center of N circular wire loops	$B_x = \frac{N\mu_0 I}{2a}$
At the center of a circular arc	$B = \frac{\mu_0 I \theta}{4\pi r}$
Inside cylindrical conductor	$B = \frac{\mu_0 I}{2\pi} \frac{r}{R^2} \quad (r < R)$
Outside cylindrical conductor	$B = \frac{\mu_0 I}{2\pi r} \quad (r > R)$
Inside $\infty$ solenoid	$B = N\mu_0 I$
Inside finite length solenoid	$B = \frac{N\mu_0 I}{l}$
Inside toroid	$B = \frac{N\mu_0 I}{2\pi r}$

### # Changing Magnetic Field (Induction)

Quantity	Unit	Definition
Inductance	H V · s/A	$L = \frac{\Phi_B}{i}$

Description	Equations
Faraday's law	$\mathcal{E} = -\frac{d\Phi_B}{dt}$
Motional emf	$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$ $\mathcal{E} = vBl$



Description	Equations
Faraday's law for stationary integration path (Induced electric field and magnetic flux)	$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$
Inductance of a solenoid	$L = \frac{\mu_0 N^2 A}{l}$
Inductance as amount of change in current associated with change in magnetic flux	$\mathcal{E} = -L \frac{di}{dt}$ $\frac{d\Phi_B}{dt} = L \frac{di}{dt}$
Magnetic potential energy	$U = \frac{1}{2} L I^2$
Magnetic energy density	$u = \frac{1}{2} \frac{B^2}{\mu_0}$

## # Changing Electric Field

Description	Equations
Conduction current	$i_C = \frac{dq}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt}$
Displacement current	$i_D = \varepsilon_0 \frac{d\Phi_E}{dt}$
<b>Maxwell-Ampere's law</b>	$\oint \vec{B} \cdot d\vec{l} = \mu_0 (i_C + i_D)$ $= \mu_0 i_C + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$
Magnetic field inside a circular capacitor	$B = \frac{\mu_0 I r}{2\pi R^2} \quad (r < R)$ $B = \frac{\mu_0 I}{2\pi R} \quad (r \geq R)$
<b>Maxwell's Equations</b>	$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$ $\oint \vec{B} \cdot d\vec{A} = 0$ $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ $\oint \vec{B} \cdot d\vec{l} = \mu_0 \left( i_C + \varepsilon_0 \frac{d\Phi_E}{dt} \right)$
Maxwell's equation in empty free space	$\oint \vec{E} \cdot d\vec{A} = 0$ $\oint \vec{B} \cdot d\vec{A} = 0$ $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ $\oint \vec{B} \cdot d\vec{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

## # Alternating-Current (AC) Circuits

Quantity	Unit	Definition
Capacitive reactance	$\Omega$	$X_C = \frac{1}{\omega C}$
Inductive reactance	$\Omega$	$X_L = \omega L$
Impedance	$\Omega$	$Z = \frac{\mathcal{E}_{\max}}{I}$

Description	Equations
AC source in AC circuit	$\mathcal{E} = \mathcal{E}_{\max} \sin(\omega t)$
Angular frequency of oscillation	$\omega = 2\pi f$
Resistor in AC circuit (i and v in phase)	$v_R = \mathcal{E}_{\max} \sin(\omega t)$ $i = I \sin(\omega t)$ $V_R = IR$
Capacitor in AC circuit (i leads v by 90 deg)	$v_C = \mathcal{E}_{\max} \sin(\omega t)$ $i = I \sin(\omega t + 90^\circ)$ $V_C = IX_C = \frac{I}{\omega C}$
Inductor in AC circuit (i lags v by 90 deg)	$v_L = \mathcal{E}_{\max} \sin(\omega t)$ $i = I \sin(\omega t - 90^\circ)$ $V_L = IX_L = I\omega L$
RC series AC circuit	$Z_{RC} = \sqrt{R^2 + 1/(\omega C)^2}$ $\tan \phi = -\frac{V_C}{V_R} = -\frac{1}{\omega RC}$
RLC series AC circuit	$Z_{RLC} = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$ $\tan \phi = \frac{V_L - V_C}{V_R} = \frac{\omega L - 1/\omega C}{R}$
RC filters	$V_C = V_R$ $\omega_{\text{cutoff}} = \frac{1}{RC}$ High pass measures R Low pass measures C
RL filters	$V_R = V_L$ $\omega_{\text{cutoff}} = \frac{R}{L}$ High pass measures L Low pass measures R
Trigonometric identities	$\sin \theta = \cos(\frac{\pi}{2} - \theta)$ $\cos \theta = \sin(\frac{\pi}{2} - \theta)$ $\sin(-\theta) = -\sin \theta$ $\cos(-\theta) = \cos \theta$

## # Special Relativity

Description	Equations
Lorentz factor	$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$
Time dilation	$\Delta t_v = \gamma \Delta t_{\text{proper}}$
Length contraction	$l_v = \frac{l_{\text{proper}}}{\gamma}$
Space-time interval	$s^2 = (c\Delta t)^2 - (\Delta x)^2$
Lorentz transformation	$x' = \gamma(x - ut)$ $y' = y$ $z' = z$ $t' = \gamma(t - ux/c^2)$

Description	Equations
	$v'_x = \frac{v_x - u}{1 - uv_x/c^2}$
Relativistic inertia	$m_v = \gamma m$
Relativistic momentum	$p = \gamma mv$
Relativistic kinetic energy	$K = (\gamma - 1)mc^2$
Internal (rest) energy	$E_{\text{int}} = mc^2$
Total energy	$E = K + E_{\text{int}}$ $E^2 = (mc^2)^2 + (pc)^2$

## # Appendix: List of Constants

Quantity	Value
Coulomb constant	$k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$
Electric constant	$\varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$
Magnetic constant	$\mu_0 = 1.26 \times 10^{-6} \text{ H/m}$
Elementary charge	$q_e = 1.60 \times 10^{-19} \text{ C}$
Mass of electron	$m_e = 9.11 \times 10^{-31} \text{ kg}$
Speed of light in vacuum	$c = 3.00 \times 10^8 \text{ m/s}$