# **CHEM E 467 Biochemical Engineering**

# # Enzyme Kinetics

Description	Equations
Arrhenius equation	$k = A \exp \left(rac{E_A}{RT} ight)$
Power law	$r_P = k \prod C_i^{ u_i}$
Linearized 1st order rate law	$\ln\left(rac{[A]}{[A]_0} ight) - kt$

#### | Michaelis-Menten kinetics

Description	Equations
Overall reaction	$E + S \longrightarrow E + P$
Reaction mechanism	$S + E \xrightarrow[k_{-1}]{k_{1}} ES$ $ES \xrightarrow{k_{2}} P + E$
Enzyme balance	$egin{aligned} [\mathbf{E}_{\mathrm{T}}] &= [\mathbf{E}] + [\mathbf{ES}] \ [\mathbf{E}] &= [\mathbf{E}_{\mathrm{T}}] - [\mathbf{ES}] \end{aligned}$
Pseudo-steady-state approximation	$egin{aligned} r_{ ext{ES}} &= k_1[ ext{S}][ ext{E}] - k_2[ ext{ES}] = 0 \ r_{ ext{ES}} &= k_1[ ext{S}]([ ext{E}_{ ext{T}}] - [ ext{ES}]) - k_{-1}[ ext{ES}] - k_2[ ext{ES}] = 0 \ [ ext{ES}] &= rac{k_1[ ext{S}][ ext{E}_{ ext{T}}]}{k_1[ ext{S}] + k_{-1} + k_2} = rac{[ ext{E}_{ ext{T}}][ ext{S}]}{K_M + [ ext{S}]} \end{aligned}$
Turnover number (# substrates converted to product per unit time on one enzyme at saturation)	$k_{ m cat}=k_2$
Michaelis-Menten constant (attraction of enzyme of its substrate, [Substrate] which rate of rxn is 1/2 max)	$K_M = rac{k_{\mathrm{cat}} + k_{-1}}{k_1}$
Maximum rate	$V_{ m max} = k_{ m cat} [{ m E_T}]$
<b>Michaelis-Menten equation</b> Rate of reaction	$egin{aligned} r_{ m P} &= k_2 [{ m ES}] \ &= rac{k_1 k_2 [{ m S}] [{ m E_T}]}{k_1 [{ m S}] + k_{-1} + k_2} \ &= rac{k_{ m cat} [{ m S}] [{ m E_T}]}{K_M + [{ m S}]} \ &= rac{V_{ m max} [{ m S}]}{K_M + [{ m S}]} \end{aligned}$
Lineweaver-Burk equation	$rac{1}{r_{ ext{P}}} = rac{K_M}{V_{ ext{max}}}rac{1}{[ ext{S}]} + rac{1}{V_{ ext{max}}}$
Eadie-Hofstee equation	$r_{ m P} = V_{ m max} - K_M rac{r_{ m P}}{[{ m S}]}$
Hanes-Woolf equation	$rac{[\mathrm{S}]}{r_{\mathrm{P}}} = rac{K_M}{V_{\mathrm{max}}} + rac{1}{V_{\mathrm{max}}} [\mathrm{S}]$

#### | Enzymatic inhibition

#### | Competitive inhibition

Description	Equations
Reaction mechanism	$E + S \Longrightarrow ES$ $ES \longrightarrow E + P$ $E + I \Longrightarrow EI \text{ (inactive)}$
Reaction rate	$r_{ ext{P}} = rac{V_{ ext{max}}[ ext{S}]}{[ ext{S}] + K_M \left[1 + rac{[ ext{I}]}{K_I} ight]}$
Lineweaver-Burk form $\uparrow K_I, \uparrow  ext{slope}$	$rac{1}{r_{ m P}} = rac{1}{[{ m S}]} \left[rac{K_M}{V_{ m max}} \left[1 + rac{[{ m I}]}{K_I} ight] ight] + rac{1}{V_{ m max}}$

# | Uncompetitive inhibition

Description	Equations
Reaction mechanism	$E + S \Longrightarrow ES$ $ES \longrightarrow E + P$ $ES + I \Longrightarrow ESI \text{ (inactive)}$
Reaction rate	$r_{ ext{P}} = rac{V_{ ext{max}}[ ext{S}]}{K_M + [ ext{S}] \left[1 + rac{[ ext{I}]}{K_I} ight]}$
Lineweaver-Burk form $\uparrow K_I, \uparrow  ext{intercept}$	$rac{1}{r_{ ext{P}}} = rac{1}{ ext{[S]}} rac{K_M}{V_{ ext{max}}} + rac{1}{V_{ ext{max}}} \left[ 1 + rac{ ext{[I]}}{K_I}  ight]$

# | Noncompetitive (mixed) inhibition

Description	Equations
Reaction mechanism	$E + S \Longrightarrow ES$ $ES \longrightarrow E + P$ $E + I \Longrightarrow EI \text{ (inactive)}$ $ES + I \Longrightarrow ESI \text{ (inactive)}$ $S + EI \Longrightarrow ESI \text{ (inactive)}$
Reaction rate	$r_{ ext{P}} = rac{V_{ ext{max}}[ ext{S}]}{([ ext{S}] + K_M)\left[1 + rac{[ ext{I}]}{K_I} ight]}$
Lineweaver-Burk form $\uparrow K_I, \uparrow  ext{slope}, \uparrow  ext{intercept}$	$rac{1}{r_{ m P}} = rac{1}{\left[ m S ight]}rac{K_M}{V_{ m max}}\left[1 + rac{\left[ m I ight]}{K_I} ight] + rac{1}{V_{ m max}}\left[1 + rac{\left[ m I ight]}{K_I} ight]$

#### **# Cell Growth Kinetics**

Description	Equations
Nomenclature	S = substrate mass $P$ = product mass $X$ = cell mass
Mass balance	$\Delta S + \Delta P = \Delta X$
Yield coefficient (mass basis)	$Y_{X/S} = -rac{\Delta X}{\Delta S} = rac{[X] - [X]_0}{[S]_0 - [S]}$

Description	Equations
Exponential growth	$[X] = [X]_0 \exp(\mu_{\max} t)$
Doubling time	$t_d = rac{\ln 2}{\mu}$
Exponential death	$[X] = [X]_0 \exp(-k_d t)$
Optical density (absorbance)	$\mathrm{OD}_{\lambda} = A_{\lambda} = \log rac{I}{I_0}$

# # Cell Reaction Stoichiometry

# | Elemental (atomic) balance

Description	Equations
Cell reaction	$egin{aligned} &\operatorname{C}_w \operatorname{H}_x \operatorname{O}_y \operatorname{N}_z + a \operatorname{O}_2 + b \operatorname{NH}_3 \ &\longrightarrow c \operatorname{CH}_lpha \operatorname{O}_eta \operatorname{N}_\delta \ + d \operatorname{CO}_2 + e \operatorname{H}_2 \operatorname{O} + f \operatorname{C}_j \operatorname{H}_k \operatorname{O}_l \operatorname{N}_m \end{aligned}$
Respiratory quotient	$ ext{RQ} = rac{ ext{mol CO}_2  ext{ produced}}{ ext{mol O}_2  ext{ consumed}} = rac{d}{a}$
Biomass yield (mass basis)	$Y_{X/S} = rac{\Delta X}{\Delta S} = rac{ ext{g cell}}{ ext{g substrate}}$
Product yield (mass basis)	$Y_{P/S} = rac{\Delta P}{\Delta S} = rac{ ext{g product}}{ ext{g substrate}}$
Substrate utilization rate (SUR)	$ ext{SUR} = rac{ ext{mol substrate}}{ ext{time}}$
Oxygen utilization rate (OUR)	$\mathrm{OUR} \equiv A = a\mathrm{SUR}$
Carbon dioxide evolution rate (CER)	$\mathrm{CER} \equiv D = d\mathrm{SUR}$

#### | Electron balance

Description	Equations
Cell reaction	$egin{aligned} &\operatorname{C}_w \operatorname{H}_x \operatorname{O}_y \operatorname{N}_z + a \operatorname{O}_2 + b \operatorname{NH}_3 \ &\longrightarrow c \operatorname{CH}_{lpha} \operatorname{O}_{eta} \operatorname{N}_{\delta} \ + d \operatorname{CO}_2 + e \operatorname{H}_2 \operatorname{O} + f \operatorname{C}_j \operatorname{H}_k \operatorname{O}_l \operatorname{N}_m \end{aligned}$
Degree of reduction	$\gamma_i = rac{ ext{(Valance \# of element } i)(\#  ext{ atom in element } i)}{\#  ext{C atom}}$
Stoichiometric coefficient of oxygen	$a=rac{1}{4}(w\gamma_S-c\gamma_X-fj\gamma_P)$