CHEM E 330 Transport Processes I

-★- TRANSPORT PHENOMENA

Rate Laws for Diffusive Transport

Description	Equations
General form	flux = -(coefficient)(driving force)
Fourier's law Heat conduction	$q = -k\dfrac{dT}{dy}$
Fick's law Species diffusion	$J_A^* = -D_{AB}\dfrac{dc_a}{dy}$
Newton's law of viscosity Momentum transfer	$\tau_{yx} = -\mu\dfrac{dv_x}{dy}$

| Rate laws as concentration gradients

Description	Equations
Fourier's law	$q_y = -\alpha\dfrac{dc_H}{dy}$
Fick's law	$J_A^* = -D_{AB}\dfrac{dc_a}{dy}$
Newton's law of viscosity	$\tau_{yx} = -\nu\dfrac{dc_{px}}{dy}$
Kinematic viscosity	$\nu = \dfrac{\mu}{\rho}$
Thermal diffusivity	$\alpha = \dfrac{k}{\rho\hat{c_P}}$
Diffusivity of A in B	D_{AB}
Prandtl number	$\mathrm{Pr} = \dfrac{\nu}{\alpha} = \dfrac{\hat{C}_p\mu}{k}$
Schmidt number	$\mathrm{Sc} = \dfrac{\nu}{D_{AB}} = \dfrac{\mu}{\rho D_{AB}}$

| Heat transfer

Description	Equations
Heat flow	$\dot{Q} = \dfrac{Q}{t}$
Heat flux	$q = \dfrac{\dot{Q}}{A}$

| Mass transfer

Description	Equations
Mass (species) transport	$N_A = x_A(\sum N_i) + J_A^*$

Description	Equations
Diffusion of A through a stagnant layer of B	$N_A = -\frac{cD_{AB}}{1-x_A}\frac{dx_A}{dy}$ $N_A = -\frac{cD_{AB}}{L}\ln(1-x_A^s)$
Equimolar counter diffusion	$N_A = -cD_{AB}\frac{dx_A}{dy}$
Reaction at catalytic surface $\mathrm{A} = 2\,\mathrm{B} \implies N_B = 2N_A$	$N_A = -x_A N_A - cD_{AB}\frac{dx_A}{dy}$

| Momentum transfer

Description	Equations
Interpretation of τ_{yx}	1. viscous shear stress exerted on a y-plane in the $+x$-direction by the fluid of lesser y on that of greater y 2. flux of x-momentum across a y-plane in the $+y$-direction
Shear strain rate	$\dot{\gamma} = \frac{dv_x}{dy}$
Hooke's law ★ Hookean solid	$\tau_{yx} = -G\frac{dx}{dy} = -G\gamma$
Newton's law of viscosity ★ Newtonian fluid	$\tau_{yx} = -\mu\frac{dv_x}{dy} = -\mu\dot{\gamma}$
General Newton's law of viscosity	$\tau_{yx} = -\eta(\dot{\gamma})\dot{\gamma}$
Viscosity function of power law fluid	$\eta = m\dot{\gamma}^{n-1}$
Newton's law of viscosity ★ Power law fluid	$\tau_{yx} = -m\dot{\gamma}^n$
Carreau equation ★ Slurry	$\frac{\eta-\eta_\infty}{\eta_o-\eta_\infty} = [1+(\lambda\dot{\gamma})^2]^{(n-1)/2}$

Transport Coefficients of Fluids

| Ideal gas: Simple kinetic theory

Description	Equations
Average velocity	$\bar{u} = \sqrt{\frac{8k_BT}{\pi m}}$
Mean free path	$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$
Number density	$n = \frac{N}{V}$
Molecular flux in the y-direction	$z = \frac{1}{4}n\bar{u}$
Average distance of molecules from ref plane when they initiate their jump	$\bar{a} = \frac{2}{3}\lambda$
Viscosity of ideal gas	$\mu = \frac{1}{3}\rho\bar{u}\lambda = \frac{2}{3\pi^{3/2}}\frac{\sqrt{mk_BT}}{d^2}$
Thermal conductivity of ideal gas	$k = \frac{1}{3}\rho\hat{c_v}\bar{u}\lambda = \frac{2\hat{c_v}}{3\pi^{3/2}}\frac{\sqrt{mk_BT}}{d^2}$

Description	Equations
Diffusivity of ideal gas A in B	$D_{AB} = \frac{1}{3}\bar{u}_{AB}\lambda_{AB} = \frac{2}{\pi^{3/2}}\frac{\sqrt{k_B T^3/m_{AB}}}{d_{AB}^2 P}$
Mean mass for diffusivity	$m_{AB} = \frac{2m_A m_B}{m_A + m_B}$
Mean distance for diffusivity	$d_{AB} = \frac{1}{2}(d_A + d_B)$
Prandtl number of monoatomic ideal gas	$\mathrm{Pr}_{\mathrm{mono}} = 1$
Schmidt number of general ideal gas	$\mathrm{Sc} = 1$

Real gas: Chapman-Enskog equations

★ Moderate pressure

Description	Equations
Lenard-Jones potential	$\varphi(r) = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$
Attractive force	$F_{\mathrm{attr}} = \frac{24\varepsilon}{r}\left[\left(\frac{\sigma}{r}\right)^{6} - 2\left(\frac{\sigma}{r}\right)^{12}\right]$
Viscosity of real gas (analytic)	$\mu = \frac{5}{16\pi}\frac{\sqrt{\pi m k_B T}}{\sigma^2\Omega_\mu}$
Thermal conductivity of real gas (analytic)	$k = \frac{25}{32\pi}\frac{\sqrt{\pi m k_B T}}{\sigma^2\Omega_k}\hat{c_v} = \frac{5}{2}\hat{c_v}\mu$
Viscosity of real gas	$\mu\left(\frac{\mathrm{g}}{\mathrm{cm \cdot s}}\right) = 2.6692\times 10^{-5}\frac{\sqrt{\mathcal{M}T}}{\sigma^2\Omega_\mu}$
Thermal conductivity of monoatomic real gas	$k_{\mathrm{mono}}\left(\frac{\mathrm{cal}}{\mathrm{cm\cdot s\cdot K}}\right) = 1.989\times 10^{-4}\frac{\sqrt{T/\mathcal{M}}}{\sigma^2\Omega_k}$
Thermal conductivity of polyatomic real gas Euken factor	$k_{\mathrm{poly}}\left(\frac{\mathrm{cal}}{\mathrm{cm\cdot s\cdot K}}\right) = \left[\hat{c_p} + \frac{5}{4}\frac{R}{\mathcal{M}}\right]\mu$
Diffusivity of real gas $T[=]\mathrm{K}$ $P[=]\mathrm{atm}$ $\sigma_{AB}[=]$ Å	$D_{AB}\left(\frac{\mathrm{cm}^2}{\mathrm{s}}\right) = 2.63\times 10^{-3}\frac{\sqrt{T^3/\mathcal{M}_{AB}}}{P\sigma_{AB}^2\Omega_D}$
Mean molar mass for diffusivity	$\mathcal{M}_{AB} = \frac{2\mathcal{M}_A\mathcal{M}_B}{\mathcal{M}_A + \mathcal{M}_B}$
Mean distance for diffusivity	$\omega_{AB} = \frac{1}{2}(\omega_A + \omega_B)$
Viscosity at different temperatures	$\mu(T_2) = \mu(T_1)\sqrt{\frac{T_2}{T_1}}\frac{\Omega_{\mu 1}}{\Omega_{\mu 2}}$
Diffusivity at different temperatures	$D_{AB}(T_2) = D_{AB}(T_1)\left(\frac{T_2}{T_1}\right)^{3/2}\frac{\Omega_{D1}}{\Omega_{D2}}$
T **and** P **dependence of transport coefficients of gases at moderate pressure**	$\mu \propto \sqrt{T}$ $k_{\mathrm{mono}} \propto \sqrt{T}$ $k_{\mathrm{poly}} = f(T, \hat{c_p}(T))$ $D_{AB} \propto T^{3/2}P^{-1}$ $D_{AB} = D_{BA}$

Ideal gas mixtures

Description	Equations
Wilke equation Viscosity of gas mixture	$\mu_{\text{mix}} = \sum_{i=1}^{N} \frac{x_i \mu_i}{\sum_{j=1}^{N} x_j \Phi_{ij}}$
Wilke equation Thermal conductivity of gas mixture	$k_{\text{mix}} = \sum_{i=1}^{N} \frac{x_i k_i}{\sum_{j=1}^{N} x_j \Phi_{ij}}$
Wilke equation parameter	$\Phi_{ij} = \frac{1}{\sqrt{8}} \left[1 + \frac{\mathcal{M}_i}{\mathcal{M}_j}\right]^{-1/2} \left[1 + \left[\frac{\mu_i}{\mu_j}\right]^{1/2} + \left[\frac{\mathcal{M}_i}{\mathcal{M}_j}\right]^{-1/4}\right]^2$
Blanc's equation Diffusivity of gas mixture	$D_{i,\text{mix}} = \left[\sum_{j \neq 1}^{N} \frac{x_j}{D_{ij}}\right]^{-1}$

Liquids

Description	Equations
Eyring model Viscosity of liquid	$\mu = \frac{N_A h}{\tilde{V}} \exp\left[0.408 \frac{\Delta U_{\text{vap}}}{RT}\right]$
Bridgeman equation Thermal conductivity of liquid	$k = 2.8 \left(\frac{N_A}{\tilde{V}}^{2/3} k_B v_s\right)$
Einstein equation	$D_{AB} \approx \frac{k_B T}{f}$
Hydrodynamic friction factor	$f = \begin{cases} 6\pi\mu_B R_A & \text{no slip} \\ 4\pi\mu_B R_A & \text{free slip} \end{cases}$
Stoke-Einstein Equation Diffusivity of dilute liquid A	$D_{AB} = \frac{k_B T}{4\pi\mu_B R_A}$
Wilke-Chang correlation Diffusivity of dilute liquid A $\tilde{V}[=]\text{cm}^3/\text{mol}$ $\mu_B[=]\text{cP}$ $T[=]\text{K}$	$D_{AB}\left(\frac{\text{cm}^2}{\text{s}}\right) = 7.4 \times 10^{-8} \frac{(\psi_B \mathcal{M}_B)^{1/2} T}{\mu \tilde{V}_A^{0.6}}$
Vigne's equation Diffusivity of liquid mixture	$D_{AB} = (D_{AB}^0)^{xB} (D_{BA}^0)^{xA}$
T dependence of transport coefficients of liquids **(no P dependence)**	$\mu = Ae^{B/T}$ $D_{AB}\mu_B \propto T$ $D_{AB} \neq D_{BA}$

Shell Balance (Bottom-Up)

Boundary conditions and shell volume

Description	Equations
Rectilinear shell volume	$\Delta V = LW\Delta y$
Cylindrical shell volume	$\Delta V = 2\pi r L \Delta r$
Spherical shell volume	$\Delta V = 4\pi r^2 \Delta r$
Newton's law of cooling	$q = h(T_{\text{solid}} - T_{\text{fluid}})$

Description	Equations
Relationship between N_A and c_A at boundary	$N_A = k_m(c_{A,\text{solid}} - c_{A,\text{fluid}})$
Reynolds number	$\text{Re} = \frac{L_\text{char} v_\text{char} \rho}{\mu}$
No slip condition	$v_1 = v_2$
Free slip condition	$-\mu_1 \left(\frac{dv_x}{dy}\right)_1 = 0$
Continuity of stress	$\tau_{y,1} = \tau_{y,2}$ $-\mu_1 \left(\frac{dv_x}{dy}\right)_1 = -\mu_2 \left(\frac{dv_x}{dy}\right)_2$

Shell balance method

1. Sketch the system with coordinate system
2. Sketch the shell that is thin in the direction of transport (change)
3. Write shell volume ΔV
4. Write shell balance OIGA of transported quantity
    - $\text{out} - \text{in} = \text{generation} - \text{accumulation}$
5. Take limit as shell thickness approach 0
    - **Differential equation of flux distribution**
6. Separate variable and integrate
    - **Flux distribution,** c_1
7. Substitute rate law
8. Separate variable and integrate
    - **Profile,** c_1, c_2
9. Evaluate c_1, c_2 using boundary conditions

Axial transport in rectilinear systems

- Rectilinear coordinates
- No generation
- No driving force
- Steady state

Description	Equations
Differential equation of flux distribution	$\frac{dq}{dy} = 0$
Temperature profile (linear)	$T(y) = T_1 - \frac{q}{k} y$
Flux distribution (inverse)	$q(y) = \frac{k(T_1 - T)}{y}$
Flux across the whole layer	$q = \frac{k(T_1 - T_2)}{H}$

Radial transport in cylindrical systems

- Cylindrical coordinates
- No generation
- No driving force

- Steady state

Description	Equations
Differential equation of flux distribution	$\frac{d(rq)}{dr} = 0$
Flux distribution (inverse)	$q(r) = \frac{k(T_i - T_0)}{r\ln(\frac{R0}{Ri})}$
Temperature profile (logarithmic)	$T(r) = T_i - \frac{T_i - T_0}{\ln(\frac{R0}{Ri})}\ln\left(\frac{r}{R_i}\right)$

Radial transport in spherical systems

- Spherical coordinates
- No generation
- No driving force
- Steady state

Description	Equations
Differential equation of flux distribution	$\frac{d(r^2 q)}{dr} = 0$
Flux distribution (inverse squared)	$q(r) = \frac{k(T_i - T_0)}{r^2(\frac{1}{Ri} - \frac{1}{R0})}$
Temperature profile (inverse)	$T(r) = T_i - \frac{T_i - T_0}{(\frac{1}{Ri} - \frac{1}{R0})}\left(\frac{1}{r} - \frac{1}{R_i}\right)$

Axial transport in rectilinear systems (with generation)

- Rectilinear coordinates
- With generation
- No driving force
- Steady state

Description	Equations
Differential equation of flux distribution	$\frac{dq}{dy} = S$
Flux distribution (linear)	$q(y) = Sy + \frac{k}{H}(T_2 - T_1) - \frac{SH}{2}$
Temperature profile (quadratic)	$T(y) = T_1 - \frac{S}{2k}y^2 + \left[\frac{SH}{2k} - \frac{T_2 - T_1}{H}\right]y$

Flow down inclined plane (falling film)

- Rectilinear coordinates
- Gravity driving force, but no pressure gradient
- Steady state

Description	Equations
Differential equation of flux distribution	$\frac{d\tau_{yx}}{dy} = \rho g \cos\beta$
Flux distribution (linear)	$\tau_{yx}(y) = -\rho g \cos\beta(\delta - y)$

Description	Equations
Velocity profile (quadratic)	$v_x(y) = \frac{g\cos\beta}{2\nu}(2\delta y - y^2)$
★ No entry length effect	$L \gg \delta$
★ No edge effect	$W \gg \delta$
★ Incompressible Newtonian fluid	$\Delta\mu = 0, \Delta\rho = 0$
★ No end effect, no ripple	$\mathrm{Re}_{\text{rippling}} \lesssim 20$
Reynolds number for falling film	$\mathrm{Re} = \frac{4\delta\langle v_x\rangle\rho}{\mu}$

Flow descriptors

Description	Equations
Skin friction	$\tau^0 = \rho g\cos(\beta)\delta$
Free surface velocity	$v_x^{\text{surf}} = \frac{g\cos\beta}{2\nu}\delta^2$
Volumetric flow rate	$Q = \int v_\perp \, dA$
Volumetric flow rate per unit area	$\frac{Q}{W} = \frac{g\cos(\beta)\delta^3}{3\nu}$
Average velocity	$\langle v_x\rangle = \frac{g\cos(\beta)\delta^2}{3\nu}$
Mass flow rate	$\dot{m} = \rho Q$
Mass flow rate per unit width	$\Gamma = \frac{\rho Q}{W} = \frac{\rho g\cos(\beta)\delta^3}{3\nu}$
Film thickness given Γ	$\delta = \sqrt[3]{\frac{3\nu\Gamma}{\rho g\cos\beta}}$

Flow in round tube (Hagen-Poiseuille flow)

- Cylindrical coordinates
- Pressure-gravity driving force
- Steady state
- No tube bents, constant cross section
- Negligible P dependence with r

Description	Equations
Modified pressure	$\mathcal{P} = P + \rho g h$
Pressure-gravity driving force	$-\frac{dP}{dz} + \rho g\cos\beta = \frac{\mathcal{P}_1 - \mathcal{P}_2}{L}$
Differential equation of flux distribution	$\frac{d(r\tau_{rz})}{dr} = \left(\frac{\mathcal{P}_1 - \mathcal{P}_2}{L}\right) r$
Flux distribution (linear)	$\tau_{rz}(r) = \frac{1}{2}\left(\frac{\mathcal{P}_1 - \mathcal{P}_2}{L}\right) r$
Velocity profile (quadratic)	$v_z(r) = \frac{R^2}{4\mu}\left(\frac{\mathcal{P}_1 - \mathcal{P}_2}{L}\right)\left[1 - \left(\frac{r}{R}\right)^2\right]$
★ Incompressible Newtonian fluid	$\Delta\mu = 0, \Delta\rho = 0$
★ Laminar flow	$\mathrm{Re}_{\text{laminar}} \leq 2100$

Description	Equations
★ Fully developed flow (no entry length effect)	$L_e \approxeq 0.035 D\mathrm{Re}$
Reynolds number for pipe flow	$\mathrm{Re}_{\text{pipe}} = \dfrac{D\langle v_z\rangle\rho}{\mu}$

Flow descriptors

Description	Equations
Skin friction	$\tau_{rz}^0 = \dfrac{1}{2}\left(\dfrac{\mathcal{P}_1 - \mathcal{P}_2}{L}\right) R$
Volumetric flow	$Q = \dfrac{R^4\pi}{8\mu}\left(\dfrac{\mathcal{P}_1 - \mathcal{P}_2}{L}\right)$
Average velocity	$\langle v_z\rangle = \dfrac{R^2}{8\mu}\left(\dfrac{\mathcal{P}_1 - \mathcal{P}_2}{L}\right)$
Mass flow rate	$\dot{m} = \dfrac{R^4\pi\rho}{8\mu}\left(\dfrac{\mathcal{P}_1 - \mathcal{P}_2}{L}\right)$

Laminar flow through porous media

Description	Equations
Darcy's law - average velocity κ - bed permeability	$\langle v\rangle = \dfrac{\kappa}{\mu L}(\mathcal{P}_1 - \mathcal{P}_2)$
Darcy's law - volumetric flow rate A - empty bed cross section ε - porosity, void fraction	$Q = \dfrac{\kappa A\varepsilon}{\mu L}(\mathcal{P}_1 - \mathcal{P}_2)$
Blake-Kozeny model Bed permeability	$\kappa = \dfrac{D_p^2}{150}\left(\dfrac{\varepsilon}{1-\varepsilon}\right)^2$
Effective packing particle diameter	$D_p = \dfrac{6}{a_v} = \dfrac{6V}{A}$ $D_{p,\text{spheres}} = D$
Bed Reynolds number	$\mathrm{Re}_{\text{bed}} = \dfrac{D_p Q\rho}{\mu A(1-\varepsilon)}$
★ Laminar flow	$\mathrm{Re}_{\text{laminar}} < 10$

Fluid pressure, hydrostatic, manometer

Description	Equations
Equation of hydrostatic	$P_1 - P_2 = \rho g(h_2 - h_1)$
Manometer equation	$P_1 - P_2 = (\rho_m - \rho)gH + \rho g(h_2 - h_1)$
Manometer equation	$\mathcal{P}_1 - \mathcal{P}_2 = (\rho_m - \rho)gH$

Unsteady state transport

Description	Equations

Description	Equations
Unsteady state conduction in rectilinear system	$\left(\frac{\partial T}{\partial t}\right)_y = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{S}{\rho \hat{c_p}}$
Unsteady state diffusion in rectilinear system	$\left(\frac{\partial c_A}{\partial t}\right)_y = D_{AB} \frac{\partial^2 c_A}{\partial y^2} + R_A$
Unsteady state Couette flow (1D rectilinear shear flow)	$\left(\frac{\partial v_x}{\partial t}\right)_y = \nu \left(\frac{\partial^2 v_x}{\partial y^2}\right)_t$
Unsteady state flow in cylindrical system	$\left(\frac{\partial v_z}{\partial t}\right)_r = \nu \left[\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r}\right] + \frac{1}{\rho}\left[\frac{\mathcal{P}_1 - \mathcal{P}_2}{L}\right]$

Rate Laws in 3D

Description	Equations
Fourier's law in 3D	$\underset{\sim}{q} = -k\nabla T$
Fick's law in 3D	$\underset{\sim}{J}_A^* = -D_{AB}\nabla c_A$
Newton's law of viscosity in 3D	$\underset{\approx}{\tau} = -\mu(\underset{\approx}{\Delta} + \underset{\approx}{\Delta}^\dagger)$
Viscous stress tensor	$\underset{\approx}{\tau} = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix}$
Rate of strain tensor	$\underset{\approx}{\Delta} = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} & \frac{\partial v_x}{\partial z} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} & \frac{\partial v_y}{\partial z} \\ \frac{\partial v_z}{\partial x} & \frac{\partial v_z}{\partial y} & \frac{\partial v_z}{\partial z} \end{bmatrix}$

Conservation Laws in 3D

Description	Equations
Conservation of thermal energy	$\nabla \cdot \underset{\sim}{q} = S - \rho \hat{c_p} \frac{\partial T}{\partial t}$
Conduction equation ★ No convection	$\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \frac{S}{\rho \hat{c_p}}$
Molecular diffusion equation ★ No convection	$\frac{\partial c_A}{\partial t} = D_{AB}\nabla^2 c_A + R_A$

-★- FLUID MECHANICS

Navier-Stokes Equation

Description	Equations
Continuity equation	$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underset{\sim}{v}) = 0$
Continuity equation of incompressible liquid ★ Constant ρ	$\nabla \cdot \underset{\sim}{v} = 0$

Description	Equations
Equation of motion (v-form)	$\rho\frac{D\underset{\sim}{v}}{Dt} = -\nabla p + \mu\nabla^2\underset{\sim}{v} + \rho g$
Equation of motion (τ-form)	$\rho\frac{D\underset{\sim}{v}}{Dt} = -\nabla p - \nabla\cdot\underset{\approx}{\tau} + \rho g$
Equation of motion (x-component)	$\rho\left[\frac{\partial v_x}{\partial t} + \underset{\sim}{v}\cdot\nabla v_x\right] = -\frac{\partial p}{\partial x} - \left[\frac{\partial\tau_{xx}}{\partial x} + \frac{\partial\tau_{yx}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z}\right] + \rho g_x$

| Operators

Description	Equations
Gradient operator ∇	Operates on scalar to give a vector, whose magnitude is the maximum rate of change of the scalar with position, and whose direction points in the direction of that change
Divergence operator $(\nabla\cdot)$	Operates on a vector to give a scalar
Divergence of a flux vector $(\nabla\cdot\underset{\sim}{f})$	Rate of efflux (outflow) of the transported quantity per unit volume
Laplacian operator	$\nabla^2 = \nabla\cdot\nabla$
Substantial derivative operator	$\frac{D}{Dt} = \frac{\partial}{\partial t} + \underset{\sim}{v}\cdot\nabla$

| Generalization to convection

Description	Equations
Thermal energy equation	$\frac{DT}{Dt} = \alpha\nabla^2 T + \frac{S}{\rho\hat{c}_p}$
Convective diffusion equation	$\frac{Dc_A}{Dt} = D_{AB}\nabla^2 c_A + R_A$

| Flow in conduit

Description	Equations
Mach number	$\mathrm{Ma} = \frac{v_{\text{char}}}{v_{\text{sound}}}$
Conduit flow	$\dot{m}_1 = \dot{m}_2$ $\rho_1 Q_1 = \rho_2 Q_2$
Incompressible conduit flow ★ Constant ρ	$Q_1 = Q_2$ $A_1\langle v\rangle_1 = A_2\langle v\rangle_2$

Apply N-S Equations (Top-Down)

| Flow between parallel plates

Assumptions	Equations

Assumptions	Equations
Rectilinear coordinates	$f(x, y, z)$
Constant ρ, μ	$\frac{\partial \rho}{\partial t} = 0, \frac{\partial \mu}{\partial t} = 0$
Laminar flow	$\mathrm{Re} < \mathrm{Re}_{\mathrm{cr}}$
Steady state	$\frac{\partial}{\partial t} = 0$
v_x component only	$v_y = v_z = 0$
No edge effect	$\frac{\partial}{\partial z} = 0$
No end effect	$\frac{\partial v_x}{\partial x} = 0$
No hydrostatic pressure diff between plates	$b \ll W, L \implies -\frac{\partial p}{\partial y} + \rho g_y = 0$

Description	Equations
x-momentum equation	$\frac{\mathcal{P}_0 - \mathcal{P}_L}{L} + \mu \frac{\partial^2 v_x}{\partial y^2} = 0$
Velocity profile (quadratic)	$v_x(y) = \frac{1}{2\mu} \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{L} \right) (-y^2 + by)$
Average velocity	$\langle v_x \rangle = \frac{b^2}{12\mu} \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{L} \right)$
Skin friction at bottom plate	$\tau^0 = \frac{b}{2} \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{L} \right)$

Couette flow between concentric rotating cylinders

Assumptions	Equations
Cylindrical coordinates	$f(r, \theta, z)$
Constant ρ, μ	$\frac{\partial \rho}{\partial t} = 0, \frac{\partial \mu}{\partial t} = 0$
Laminar flow	$\mathrm{Re} < \mathrm{Re}_{\mathrm{cr}}$
Steady state	$\frac{\partial}{\partial t} = 0$
v_θ component only	$v_r = v_z = 0$
Axial symmetry	$\frac{\partial}{\partial \theta} = 0$
No end effect	$\frac{\partial v \theta}{\partial z} = 0$
Vertical orientation	$g_z = -g, g_\theta = g_r = 0$

Description	Equations
r-momentum equation	$-\rho \frac{v_\theta^2}{r} = -\frac{\partial p}{\partial r}$
θ-momentum equation	$\mu \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r v_\theta) \right) = 0$
z-momentum equation	$-\frac{\partial p}{\partial z} - \rho g = 0$
Velocity profile (general form)	$v_\theta(r) = c_1 \frac{r}{2} + \frac{c_2}{r}$
Velocity profile	$v_\theta(r) = \frac{\Omega_0}{1 - \kappa^2} \left[r - \frac{(\kappa R)^2}{r} \right]$

Description	Equations
Pressure profile	$P - P_{\kappa R} = \frac{1}{2}\rho\left(\frac{\Omega_0 \kappa R}{1-\kappa^2}\right)^2\left[\left(\frac{r}{\kappa R}\right)^2 - \left(\frac{\kappa R}{r}\right)^2 - 4\ln\left(\frac{r}{\kappa R}\right)\right]$
Shear stress distribution	$\tau_{r\theta} = -2\mu\kappa^2\left(\frac{\Omega_0}{1-\kappa^2}\right)\left(\frac{R}{r}\right)^2$
Torque	$\mathcal{T} = 4\pi\mu L\Omega_0 R^2\frac{\kappa^2}{1-\kappa^2}$
Couette viscometer	$\mu = \frac{\mathcal{T}}{4\pi L\Omega_0 R^2}\frac{1-\kappa^2}{\kappa^2}$

Stoke’s law: Flow around a sphere

Assumptions	Equations
Spherical coordinates	$f(r,\theta,\phi)$
Constant ρ, μ	$\frac{\partial\rho}{\partial t} = 0, \frac{\partial\mu}{\partial t} = 0$
Laminar flow	$\mathrm{Re} < \mathrm{Re}_{\mathrm{cr}}$
Steady state	$\frac{\partial}{\partial t} = 0$
Axial symmetry	$\frac{\partial}{\partial\phi} = 0$
No spinning	$v_\phi = 0$
Vertical orientation	$g_r = -g\cos\theta, g_\theta = g\sin\theta, g_\phi = 0$
v_θ component only	$v_r = v_z = 0$

Description	Equations
r velocity profile	$v_r = v_\infty\left[1 - \frac{3}{2}\left(\frac{R}{r}\right) + \frac{1}{2}\left(\frac{R}{r}\right)^2\right]\cos\theta$
θ velocity profile	$v_\theta = -v_\infty\left[1 - \frac{3}{4}\left(\frac{R}{r}\right) - \frac{1}{4}\left(\frac{R}{r}\right)^3\right]\sin\theta$
Pressure profile	$p = p_0 - \rho g z - \frac{3}{2}\frac{\mu v_\infty}{R}\left(\frac{R}{r}\right)^2\cos\theta$
Viscous drag	$4\pi\mu v_\infty R$
Pressure force (buoyancy + form frag)	$\frac{4}{3}\pi R^3\rho g + 2\pi R\mu v_\infty$
Stoke’s law	$v_\infty = \frac{2R^2(\rho_s - \rho)g}{9\mu}$
Falling ball viscometer	$\mu = \frac{2R^2(\rho_s - \rho)g}{9v_\infty}$

Centrifuge viscometer

Description	Equations
Terminal velocity	$v_\infty = \frac{2R^2(\rho_s - \rho)\omega^2 r}{9\mu}$

Description	Equations
Centrifuge viscometer	$\mu = \frac{2R^2(\rho_s - \rho)\omega^2}{9\ln\left(\frac{R2}{R1}\right)}\Delta t$

Turbulence

Transition to turbulence

Geometry	Reynolds Number	Critical Reynolds Number
Circular tube flow	$\mathrm{Re} = \frac{D\langle v\rangle\rho}{\mu}$	$\mathrm{Re}_c \approx 2100$
Falling film	$\mathrm{Re} = \frac{4\delta\langle v\rangle\rho}{\mu}$	$\mathrm{Re}_c \approx 1500$
Flow between parallel plates	$\mathrm{Re} = \frac{2b\langle v\rangle\rho}{\mu}$	$\mathrm{Re}_c \approx 1780$
Tangential flow in an annulus (Couette flow between rotating cylinders)	$\mathrm{Re} = \frac{\Omega_0 R^2\langle v\rangle\rho}{\mu}$	$\mathrm{Re}_c \approx 50000$

Laminar vs. turbulent

Property	Laminar Flow $(\mathrm{Re} < 2100)$	Turbulent Flow $(\mathrm{Re} \in [10^4, 10^5])$
Velocity profile	$\frac{v_z}{v_{z,\max}} = 1 - \left(\frac{r}{R}\right)^2$	$\frac{v_z}{v_{z,\max}} \approx \left(1 - \frac{r}{R}\right)^{1/7}$
Average velocity	$\langle v_z\rangle = \frac{1}{2}v_{z,\max}$	$\langle v_z\rangle \approx \frac{4}{5}\bar{v}_{z,\max}$
Volumetric flow rate	$Q = \frac{\pi R^4}{8\mu}\left(\frac{\mathcal{P}_0 - \mathcal{P}_1}{L}\right)$	$Q \propto \left(\frac{\mathcal{P}_0 - \mathcal{P}_1}{L}\right)^{4/7}$
Entry length	$L_e = 0.035D\mathrm{Re}$	$L_e \approx 40D$
Derivation	From theory	From experiment

Description	Equations
Velocity decomposition	$v_z = \bar{v}_z + v_z'$
Velocity profile in turbulent flow	$\bar{v}_z = \bar{v}_{z,\max}\left(1 - \frac{r}{R}\right)^{1/n}$ $n = \begin{cases} 6 & \mathrm{Re} \in [2\times 10^3, 10^4] \\ 7 & \mathrm{Re} \in [10^4, 10^5] \\ 8 & \mathrm{Re} \in [10^5, 10^6] \end{cases}$

Time-smoothed N-S equation

Description	Equations
Time-smoothed continuity equation	$\nabla \cdot \underset{\sim}{\bar{v}} = 0$ $\nabla \cdot \underset{\sim}{v'} = 0$
Time-smoothed equation of motion (τ-form)	$\rho\frac{D\underset{\sim}{\bar{v}}}{Dt} = -\nabla\bar{p} - \nabla \cdot \underset{\approx}{\bar{\tau}}^{\text{total}} + \rho g$

Description	Equations
Time-smoothed equation of motion (x-component)	$\rho\left[\frac{\partial\bar{v}_x}{\partial t}+\underset{\sim}{\bar{v}}\cdot\nabla\bar{v}_x\right] = -\frac{\partial\bar{p}}{\partial x}-\left[\frac{\partial\bar{\tau}_{xx}^{\text{total}}}{\partial x}+\frac{\partial\bar{\tau}_{yx}^{\text{total}}}{\partial y}+\frac{\partial\bar{\tau}_{zx}^{\text{total}}}{\partial z}\right]+\rho g_x$
Total shear stress (viscous + turbulent)	$\bar{\tau}_{yx}^{\text{total}} = \bar{\tau}_{yx}^{(v)}+\bar{\tau}_{yx}^{(t)} = \bar{\tau}_{yx}+\rho\overline{v_y' v_x'}$

| Shear stress distribution

Description	Equations
Shear stress distribution in round tube	$\tau_{r\theta}=\frac{1}{2}\left[\frac{\mathcal{P}_0-\mathcal{P}_1}{L}\right]r$
Shear stress distribution in general conduit	$\tau_{r\theta}=\left[\frac{\mathcal{P}_0-\mathcal{P}_1}{L}\right]R_H$
Hydraulic radius	$R_H=\frac{\text{cross sectional area}}{\text{wetted perimeter}}$
Characteristic length	$l_{\text{char}}=4R_H$
Characteristic velocity	$v_{\text{char}}=\langle v_z\rangle$

| Universal velocity profile

Layer	Normalized velocity	Normalized length range
Laminar sublayer	$v^+=y^+$	$y^+\in(0,5)$
Buffer layer	$v^+=5\ln(y^++0.205)-3.27$	$y^+\in(5,30)$
Turbulent core	$v^+=2.5\ln(y^+)+5.5$	$y^+\in(30,\infty)$

Description	Equations
Characteristic length	$y_*=\frac{\mu}{v_*\rho}$
Characteristic velocity	$v_*=\sqrt{\frac{\tau^0}{\rho}}$
Normalized length	$y^+=\frac{y}{y_*}$
Normalized velocity	$v^+=\frac{v}{v_*}$
Eddie viscosity	$\mu^{(t)}=-\frac{\bar{\tau}_{yz}^{\text{total}}}{\left(\frac{dvz}{dy}\right)}-\mu=-\frac{\left[\frac{\mathcal{P}0-\mathcal{P}1}{L}\right]\frac{r}{2}}{\left(\frac{dvz}{dy}\right)}-\mu$

Dynamic Similarity and Dimensional Analysis

| Flow around a sphere outside of Stoke's law

Description	Equations
★ Non-Stoke's law condition	$\text{Re}\geq 0.1$

Description	Equations
Nondimensionalized continuity equation	$\breve{\nabla} \cdot \underset{\sim}{\breve{v}} = 0$
x-component of momentum equation	$\frac{D\breve{v}_x}{D\breve{t}} = -\frac{\partial \breve{p}}{\partial \breve{x}} + \frac{1}{\mathrm{Re}}\breve{\nabla}^2\breve{v}_x + \frac{1}{\mathrm{Fr}}\breve{g}_x$
Drag coefficient Friction factor	$c_D = f = \frac{F_D}{\frac{1}{2}\rho v_\infty^2 A_{\text{approach}}}$
Drag coefficient in Stoke's law region	$c_D = \frac{24}{\mathrm{Re}}$
Drag coefficient in non-Stoke's law region	$c_D = \left(\sqrt{\frac{24}{\mathrm{Re}}} + 0.5407\right)^2$

| Dimensionless groups

Description	Equations
Reynolds number	$\mathrm{Re} = \frac{l_0 v_0 \rho}{\mu} = \frac{\text{inertial forces}}{\text{viscous forces}}$
Froude number	$\mathrm{Fr} = \frac{v_0^2}{g l_0} = \frac{\text{inertial forces}}{\text{gravitational forces}}$
Capillary number	$\mathrm{Ca} = \frac{\mu v_0}{\sigma} = \frac{\text{viscous forces}}{\text{surface tension forces}}$
Weber number	$\mathrm{Fr} = \frac{l_0 \rho v_0^2}{\sigma} = \frac{\text{inertial forces}}{\text{surface tension forces}}$
Euler's number	$\mathrm{Eu} = \frac{(\Delta p) D^4}{\rho Q^2}$

| Dimensional analysis

- *Buckingham π theorem* - A function $f(X_1, X_2, \ldots, X_k)$ with dimensional variables X_i can be rewritten in a function $\Phi(\Pi_1, \Pi_2, \ldots, \Pi_{k-n})$ with dimensionless variables Π_j by enforcing dimensional consistency using n fundamental dimensions.
  - Define fundamental dimensions
  - Choose stand-in variables for fundamental dimensions
  - Rewrite other variables in terms of stand-in variables to get dimensionless groups

Bernoulli Analysis and Applications

| N-S equation for steady flow in stream tubes

Assumptions	Equations
Constant density fluid	$\Delta\rho = 0$
1D flow in z direction	$v_r = v_\theta = 0$
Plug flow - uniform velocity across cross section	$\langle v \rangle = v = \text{constant}$ $v_z = v_z(z)$
Inviscid flow	$\mu \approx 0, \mathrm{Re} \geq 10000$
No sharp bends	Straight stream lines

Description	Equations

Description	Equations
Continuity equation	$Q_1 = Q_2$ $A_1\langle v\rangle_1 = A_2\langle v\rangle_2$
Equation of motion	$\rho v\frac{dv}{dz} = -\frac{dp}{dz} - \rho g\frac{dh}{dz}$

Bernoulli equation

Description	Equations
Bernoulli equation (energy form)	$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$
Bernoulli equation (head form)	$\frac{v_1^2}{2g} + \frac{p_1}{\rho g} + h_1 = \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + h_2$
Bernoulli head	$\mathcal{B} = \frac{v^2}{2g} + \frac{p}{\rho g} + h = \text{constant}$
Drag coefficient	$c_D = \frac{F_D}{\frac{1}{2}\rho v_\infty^2 A_{\text{approach}}}$
Lift coefficient	$c_L = \frac{F_L}{\frac{1}{2}\rho v_\infty^2 A_{\text{planform}}}$
Pressure change in contracting conduit $\Delta p \equiv p_1 - p_2$	$\Delta p = \frac{8\rho Q^2}{\pi^2 D_1^4}\left[\left(\frac{D_1}{D_2}\right)^4 - 1\right] + \rho g(h_2 - h_1)$
Torricelli's law	$\langle v\rangle = \sqrt{2g\Delta h}$
Pressure at stagnation point	$p = p_{\text{static}} + p_{\text{dynamic}}$ $= p_{\text{static}} + \frac{1}{2}\rho v_\infty^2$

Flow-metering devices

Description	Equations
Manometer equation	$\Delta p = (\rho_{\text{m}} - \rho)gH$
Local velocity **Pitot tube**	$v = \sqrt{\frac{2\Delta p}{\rho}}$
Volumetric flow rate **Venturi meter** $c_0 \in [0.96, 0.98]$ **Orfice meter** $c_0 \in [0.40, 0.80]$ **Nozzle meter** $c_0 \in [0.96, 0.98]$	$Q = c_0\pi D_0^2\sqrt{\frac{\Delta p}{8\rho[1 - (\frac{D_0}{D})^4]}}$
Rotameter	Calibrated specifically to the fluid with falling sphere

Full Bernoulli analysis

Description	Equations
Full Bernoulli equation	$\frac{v_1^2}{2g} + \frac{p_1}{\rho g} + h_1 = \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + h_2 + H_{L12}$
Head loss	$H_{L12} = H_{L12f} + H_{L12c}$
Skin friction loss H_{L12f}	Viscous work done per unit weight by fluid on walls of conduit in moving from 1 to 2

Description	Equations
Skin friction loss (general)	$H_{L12f} = \dfrac{\tau^0 L}{\rho g R_H}$
Skin friction loss for circular tube	$H_{L12f} = \dfrac{4\tau^0 L}{\rho g D}$
Fanning friction factor	$f = \dfrac{\tau^0}{\frac{1}{2}\rho\langle v\rangle^2}$
Skin friction loss for circular tube	$H_{L12f} = \dfrac{2\langle v\rangle^2 L}{gD} f = \dfrac{32Q^2 L}{\pi^2 D^5 g} f$
Skin friction loss for non-circular tube	$H_{L12f} = \dfrac{\langle v\rangle^2 L}{2gR_H} f = \dfrac{Q^2 L}{2gA_c^2 R_H} f$
Reynolds number for noncircular pipes	$\mathrm{Re} = \dfrac{4R_H\langle v\rangle\rho}{\mu}$
Configurational loss of one fitting in circular tube	$H_{Lc} = e_v \dfrac{\langle v\rangle^2_{\text{downstream}}}{2g}$
Configurational loss of all fittings in circular tube	$H_{L12c} = \dfrac{\langle v\rangle^2_{\text{down}}}{2g}(\sum_i e_{v,i}) = \dfrac{8Q^2}{\pi^2 D^4 g}(\sum_i e_{v,i})$
Total head loss for circular tube	$H_{L12} = \begin{cases} \dfrac{2\langle v\rangle^2}{Dg}[(\sum_i L_i)f + \frac{D}{4}(\sum_i e_{v,i})] \\ \dfrac{32Q^2}{\pi^2 D^5 g}[(\sum_i L_i)f + \frac{D}{4}(\sum_i e_{v,i})] \end{cases}$
Kinetic head correction factor	$\alpha = \dfrac{\langle v^3\rangle}{\langle v\rangle^3}$
Brake horse power	$\mathrm{bhp} = \dfrac{P}{\eta} = \dfrac{H_p \rho g Q}{\eta}$

Fanning friction factor correlations

Description	Equations	Conditions
Hydraulically smooth pipes (Blasius)	$f = \dfrac{0.0791}{\mathrm{Re}^1/4}$	$\mathrm{Re} \in [2100, 10^5]$
Hydraulically smooth pipes (Koo)	$f = 0.0014 + \dfrac{0.125}{\mathrm{Re}^{0.32}}$	$\mathrm{Re} \in [10^4, 10^7]$
Pipes of general roughness (Haaland)	$\dfrac{1}{\sqrt{f}} = -3.6\log_{10}\left[\dfrac{6.9}{\mathrm{Re}} + \left(\dfrac{k/D}{3.7}\right)^{10/9}\right]$	$\mathrm{Re} \in [4\times 10^4, 10^7]$ $k/D < 0.05$
Commercial standard piping (Drew)	$f = 0.0014 + \dfrac{0.090}{\mathrm{Re}^{0.27}}$	$\mathrm{Re} \in [10^4, 10^7]$ $k/D \approx 0.00015$
Full rough conduit	$\dfrac{1}{\sqrt{f}} = 2.28 - 4.0\log_{10}\left(\dfrac{k}{D}\right)$	$\mathrm{Re} > 10^4$ $k/D > 0.01$

Kinetic head correction factor

Re	n	α
$2\times 10^3 \sim 10^4$	6	1.08
$10^4 \sim 10^5$	7	1.06
$10^5 \sim 10^7$	8	1.05

Flow through packed bed

Description	Equations
Specific area of packing element	$a_v = \dfrac{\text{area of packing element}}{\text{volume of packing element}}$
Effective diameter of packing element (particle)	$D_p = \dfrac{6}{a_v}$
Darcy's law $\star\ \mathrm{Re}_{\mathrm{bed}} \lesssim 10$	$\langle v \rangle = \dfrac{\kappa}{\mu}\left[\dfrac{\mathcal{P}_0 - \mathcal{P}_L}{L}\right]$
Volumetric flow rate	$Q = \langle v \rangle \varepsilon A = v_0 A$
Superficial velocity	$v_0 = \langle v \rangle \varepsilon$
Bed Reynolds number	$\begin{aligned}\mathrm{Re}_{\mathrm{bed}} &= \dfrac{D_p v_0 \rho}{\mu}\dfrac{1}{1-\varepsilon} \\ &= \dfrac{D_p \langle v \rangle \rho}{\mu}\dfrac{\varepsilon}{1-\varepsilon} \\ &= \dfrac{D_p Q \rho}{\mu A}\dfrac{1}{1-\varepsilon}\end{aligned}$
Tube Reynolds number	$\mathrm{Re}_{\mathrm{tube}} = \dfrac{2}{3}\mathrm{Re}_{\mathrm{bed}}$
Hydrolic radius	$R_H = \dfrac{D_p \varepsilon}{6(1-\varepsilon)}$
Friction factor of tube $\star\ \mathrm{Re}_{\mathrm{bed}} \leq 10$	$f_{\mathrm{tube}} = \dfrac{24(1-\varepsilon)\mu}{D_p v_0 \rho}$
Friction factor of tube $\star\ \mathrm{Re}_{\mathrm{bed}} > 1000$	$f_{\mathrm{tube}} = \dfrac{7}{12}$
Bed permeability	$\kappa = \dfrac{D_p^2}{150}\left(\dfrac{\varepsilon}{1-\varepsilon}\right)^2$
Blake-Kozeny equation $\star\ \mathrm{Re}_{\mathrm{bed}} \leq 10$	$\left[\dfrac{\mathcal{P}_0 - \mathcal{P}_L}{L}\right] = 150\dfrac{\mu v_0}{D_p^2}\dfrac{(1-\varepsilon)^2}{\varepsilon^3}$
Burke-Plummer equation $\star\ \mathrm{Re}_{\mathrm{bed}} > 1000$	$\left[\dfrac{\mathcal{P}_0 - \mathcal{P}_L}{L}\right] = \dfrac{7}{4}\dfrac{\rho v_0^2}{D_p}\dfrac{1-\varepsilon}{\varepsilon^2}$
Superficial mass flux	$G_0 = \rho v_0 = \dfrac{\dot{m}}{A}$
Ergun equation $\star\ \mathrm{Re}_{\mathrm{bed}} \in [10, 1000]$	$\begin{aligned}&\left[\dfrac{(\mathcal{P}_0 - \mathcal{P}_L)\rho}{G_0^2}\right]\dfrac{D_p}{L}\dfrac{\varepsilon^3}{1-\varepsilon} = 150\left[\dfrac{1-\varepsilon}{\frac{DpG0}{\mu}}\right] + \dfrac{7}{4} \\ &\left[\dfrac{(\mathcal{P}_0 - \mathcal{P}_L)\rho}{G_0^2}\right]\dfrac{D_p}{L}\dfrac{\varepsilon^3}{1-\varepsilon} = 150\dfrac{1}{\mathrm{Re}_{\mathrm{bed}}} + \dfrac{7}{4}\end{aligned}$

Cavitation and vortex motion

Description	Equations
Cavitation number	$\sigma = \dfrac{p_A - p_C}{\frac{1}{2}\rho v_\infty^2}$

Forced vortex flow in rotating cylinder

Description	Equations

Description	Equations
Velocity profile	$v_\theta = r\Omega$
Pressure difference ★ 1 defined arbitrarily, 2 defined at center	$p_2 - p_1 = \frac{1}{2}\rho\Omega^2(r_2^2 - r_1^2) + \rho g(z_1 - z_2)$
Height	$h = \frac{\Omega^2}{2g}r^2$

| Free vortex flow during drainage

Description	Equations
Pressure difference ★ 1 defined arbitrarily, 2 defined at $r \to \infty$	$p_2 - p_1 = \frac{1}{2}\rho C^2\left(\frac{1}{r_1^2} - \frac{1}{r_2^2}\right) + \rho g(z_1 - z_2)$
Depth	$h = \frac{C^2}{2g}\frac{1}{r^2}$

Microfluidics*

| Validity of continuum description

Description	Equations
Mean free path	$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$ $\lambda(\mu\mathrm{m}) \approx 3.1 \times 10^{-3}\frac{T(\mathrm{K})}{\sigma^2(\mathrm{\mathring{A}}^2)p(\mathrm{atm})}$
Knudsen number	$\mathrm{Kn} = \frac{\lambda}{L_c}$

Characteristics	Range
Molecular flow	$\mathrm{Kn} \in (10, \infty)$
Transition flow	$\mathrm{Kn} \in (0.1, 10)$
N-S equations hold, but no-slip condition fails	$\mathrm{Kn} \in (0.001, 0.1)$
N-S equations hold, and no-slip condition holds	$\mathrm{Kn} \in (0, 0.001)$

| Forces in microfluidic flows

- Viscous force dominate over inertial forces and gravity forces
    - Driving force
        - Pressure
        - Capillary (surface tension) forces
        - Electro-kinetic forces
        - Magnetic forces
    - Resisting forces: viscous force, dominated by wall effects

Description	Equations
Reynolds number ★ Creeping flow	$\mathrm{Re} = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{Lv\rho}{\mu} \to 0$

Description	Equations
Froude number	$\mathrm{Fr} = \frac{\text{inertial forces}}{\text{gravity forces}} = \frac{v^2}{gL}$
Viscous force dominates gravity force	$\frac{\mathrm{Re}}{\mathrm{Fr}} = \frac{\text{gravity forces}}{\text{viscous forces}} = \frac{gL^2}{\mu v} \to 0$

Generalized Hagen-Poiseuille flow

Description	Equations
Differential equation of generalized H-P flow	$0 = \frac{\Delta p}{L} + \mu \left(\frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} \right)$
No-slip condition $F(x, y)$ is equation of conduit perimeter	$v_z(x, y) = 0 for F(x, y) = 0$
Velocity profile	$v_z(x, y) = \frac{\Delta p}{\mu L} F(x, y)$
Volumetric flow rate	$Q = \frac{\Delta p}{\mu L} \iint F(x, y)\, dy\, dx$

Hydraulic resistance in micro-channels

Description	Equations
Flow equation	$\Delta p = \mathcal{R}_{\text{hyd}} Q$
Volumetric flow rate	$Q = \frac{\Delta p}{\mathcal{R}_{\text{hyd}}}$

Capillary driving force and wicking phenomena

Description	Equations
Pressure difference	$\Delta p = \sigma \kappa = \frac{2\sigma}{R}$
Wicking velocity	$v = \frac{r^2}{8\mu} \frac{\Delta P}{x} = \frac{r \sigma \cos\theta}{4 \mu x}$
Washburn equation	$x = \sqrt{\frac{r \sigma \cos\theta}{2\mu} t} \propto \sqrt{t}$
Wicking into porous media	$h = \sqrt{\frac{r_e \sigma \cos\theta}{2\mu} t} \propto \sqrt{t}$