# **PHYS 121 Mechanics Equations**

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#### **Kinematics**

Quantity	Unit	Definition	
Displacement	m	$\Delta ec{r} = ec{r}_f - ec{r}_i$	
Instantaneous velocity	$\mathrm{m/s}$	$ec{v}=rac{dec{r}}{dt}$	
Instantaneous acceleration	$ m m/s^2$	$ec{a}=rac{dec{v}}{dt}=rac{d^2ec{x}}{dt^2}$	

Description	Equations
Kinematics equations at constant acceleration	$x = x_0 + v_0 t + rac{1}{2} a t^2 \ v = v_0 + a t \ v_f^2 = v_i^2 + 2 a \Delta x$
Relative velocity	$ec{v}_{P/A} = ec{v}_{P/B} + ec{v}_{B/A}$
Centripetal acceleration	$a_{ m rad}=rac{v^2}{R}=rac{4\pi^2R}{T^2}$

## **Dynamics**

Quantity	Unit	Definition
Spring force (Hooke's law)	N	$F_s = -k\Delta x$
Static friction	N	$f_s \leq (f_s)_{ ext{max}} = \mu_s F_N \ \mu_s =  an( heta)$

Quantity	Unit	Definition	
Kinetic friction	N	$f_k = \mu_k F_N$	
Gravitational force near Earth surface	N	$ec{F}_g = m ec{g} \ g = 9.81 \mathrm{m/s^2}$	

Description	Equations
Newton's first law moving at constant velocity	$\sum {ec F}_{ m ext} = ec 0$
Newton's second law	$\sum {ec F}_{ m ext} = m ec a$
Newton's third law	$ec{F}_{AB} = -ec{F}_{BA}$
Acceleration on an inclined plane	$a=g\sin heta$

## **Energy**

Quantity	Unit	Definition
Work	J	$W=ec{F}\cdotec{x}=Fx\cos heta \ W=\int_{x_1}^{x_2}F\ dx$
Kinetic energy	J	$K=\frac{1}{2}mv^2$
Power	W	$P = rac{dW}{dt} = rac{dE}{dt} \ P = ec{F} \cdot ec{v}$
Gravitational potential energy	J	$egin{aligned} U = mgh \ W_{ ext{grav}} = -\Delta U_{ ext{grav}} \end{aligned}$
Elastic potential energy	J	$egin{aligned} U &= rac{1}{2} k x^2 \ W_{ ext{el}} &= -\Delta U_{ ext{el}} \end{aligned}$
Reduced mass	kg	$\mu=\frac{m_1m_2}{m_1+m_2}$
Coefficient of restitution	-	$e = -\frac{v_{12,f}}{v_{12,i}}$

Description	Equations
Work-energy theorem	$W_{ m total} = \Delta K$
Conservation of mechanical energy	$K_i + U_i = K_f + U_f$

Description	Equations
Energy of system with external force (non- isolated system)	$K_i + U_i + W = K_f + U_f$
Conservation of energy	$\Delta K + \Delta U + \Delta U_{int} = 0$
Force as a function of potential energy	$F=-rac{dU}{dx}ec{F}=-ec{ abla}U$

### **Momentum**

Quantity	Unit	Definition
Momentum	${ m kg\cdot m/s}$	$ec{p}=mec{v} \ \sum {ec{F}}_{ m ext} = rac{dec{p}}{dt}$
Impulse	${ m kg\cdot m/s}$	$ec{J} = \sum_{t_1} ec{F} \Delta t \ ec{J} = \int_{t_1}^{t_2} \sum_{} ec{F} \; dt$
Center of mass	m	$ec{r}_{ m cm} = rac{\sum\limits_{i} m_i ec{r}_i}{\sum\limits_{i} m_i}$

Description	Equations
Impulse-momentum theorem	<del>-</del>
Conservation of momentum (closed system)	$ec{p_i} = ec{p_f} \ \sum ec{F}_{ m ext} = rac{dec{p}}{dt}$
Force on extended body	$\sum ec{F}_{ m ext} = m ec{a}_{ m cm}$

### **Rotational Kinematics**

Quantity	Unit	Definition
Angular displacement	$\operatorname{rad}$	$\Delta  heta =  heta_f -  heta_i$
Angular velocity	m rad/s	$\omega_z=rac{d heta}{dt}$
Angular acceleration	$ m rad/s^2$	$lpha_z=rac{d\omega_z}{dt}=rac{d^2 heta}{dt^2}$
Rotational Inertia of particle	${ m kg\cdot m^2}$	$I=\sum_i m_i r_i^2$

Quantity	Unit	Definition	
Rotational kinetic energy	J	$K=rac{1}{2}I\omega^2$	

Description	Equations
Rotational kinematics equation with constant angular acceleration	$ heta =  heta_0 + \omega_{0z} t + rac{1}{2} lpha_z t^2$
	$egin{aligned} \omega_z &= \omega_{0z} + lpha_z t \ \omega_{fz}^2 &= \omega_{iz}^2 + 2lpha_z \Delta  heta \end{aligned}$
Relationship between linear kinematics and rotational kinematics	s=r heta
	$v=r\omega$
	$a_{ an}=rlpha$
	$a_{ m rad}=rac{v^2}{r}=\omega^2 r$
Parallel-axis theorem	$I_{ m parallel} = I_{ m cm} + m d^2$

# **Rotational Dynamics**

Quantity	Unit	Definition
Torque	${ m N}\cdot{ m m}$	$ec{ au}=ec{r} imesec{F}=Fr\sin heta \ \sum ec{ au}=rac{dec{L}}{dt}$
Angular momentum of a particle	${ m kg\cdot m^2/s}$	$ec{L}=ec{r} imesec{p}=mvr\sin heta$
Angular momentum of rotating body	${ m kg\cdot m^2/s}$	$ec{L}=Iec{\omega}$

Description	Equations
Rotational Newton's second law	$\sum  au = I lpha_z$
Condition of mechanical equilibrium	$\sum ec{F}_{ m ext} = m ec{a} \ \sum  au = I lpha_z$
Total kinetic energy of rotating and translating object	$K=rac{1}{2}mv_{ m cm}^2+rac{1}{2}I_{ m cm}\omega^2$
Rolling without slipping	$v_{ m cm}=R\omega$
Slipping (only rolling)	$v_{ m cm} < R \omega$
Skidding (only translating)	$v_{ m cm} > R \omega$

Description	Equations
Rotational Work	$egin{aligned} W &=  au_z \Delta  heta \ W &= \int_{ heta_1}^{ heta_2}  au_z \; d heta \ W &= \Delta K_{ ext{rot}} \end{aligned}$
Power	$egin{aligned} P &= rac{dW}{dt} \ P &=  au_z \omega_z \end{aligned}$
Conservation of angular momentum (closed system)	$ec{L}_i = ec{L}_f \ \sum ec{ au} = rac{dec{L}}{dt}$

## **Universal Gravitation**

Quantity	Unit	Definition
Gravitational force	N	$F_g=Grac{m_1m_2}{r^2}$
Gravitational acceleration	$ m m/s^2$	$g=Grac{m_E}{r^2}$
Gravitational potential energy	J	$U=-Grac{m_E m}{r}$

Description	Equations
Escape velocity	$v_{ m escape} = \sqrt{rac{2Gm_E}{R}}$
Velocity in circular orbit	$v_{ m circ} = \sqrt{rac{Gm_E}{R}} = rac{2\pi R}{T}$
Period in circular orbit	$T=rac{2\pi R}{v}=2\pi R\sqrt{rac{R}{Gm_E}}=rac{2\pi r^{3/2}}{\sqrt{Gm_E}}$