Steady-State Gain *K*

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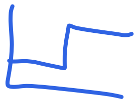
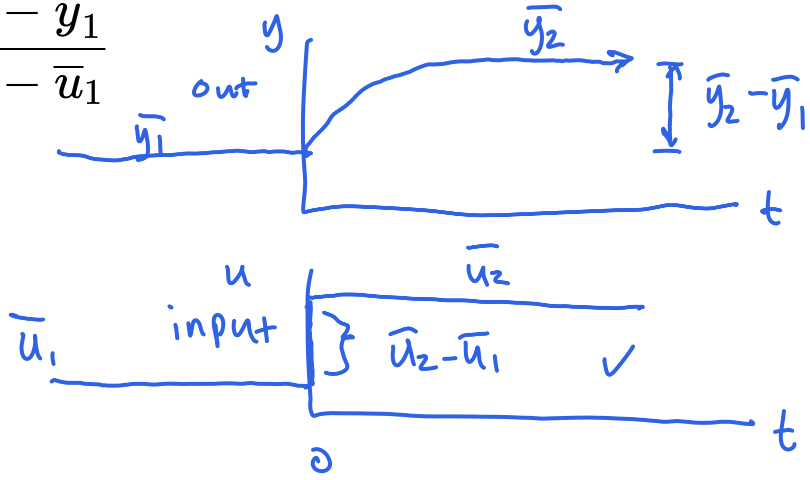
Steady-state gain quantifies how much output changes upon *unit* input change

Steady-state gain K - ratio of the output variable change to an sustained input variable change at new steady state

(step change) 

$$K = \frac{\overline{y}_2 - \overline{y}_1}{\overline{u}_2 - \overline{u}_1}$$



Steady state gain is a constant for linear processes.

Output change is normalized input change.

Steady-state gain can be evaluated from $G(0)$

Ex. Prove that steady-state gain can be evaluated from $G(s)$ by setting $s = 0$ (if the gain exists).

- Final value theorem: $\lim_{t \to \infty}[y(t)] = \lim_{s \to 0}[sY(s)]$

$$K = G(0)$$

$$K = \frac{\bar{y}_2 - \bar{y}_1}{\bar{x}_2 - \bar{x}_1} = \frac{\lim_{t\to\infty} y'}{x'} = \frac{\lim_{s\to 0}[s\,Y'(s)]}{x'}$$

$$= \frac{\lim_{s\to 0}[s\,G(s)\,X'(s)]}{x'}$$

$$= \frac{\lim_{s\to 0}\left[\cancel{s}\,G(s)\,\frac{\cancel{\bar{x}_2 - \bar{x}_1}}{\cancel{s}}\right]}{\cancel{x'}}$$

$$= \lim_{s\to 0} G(s)$$

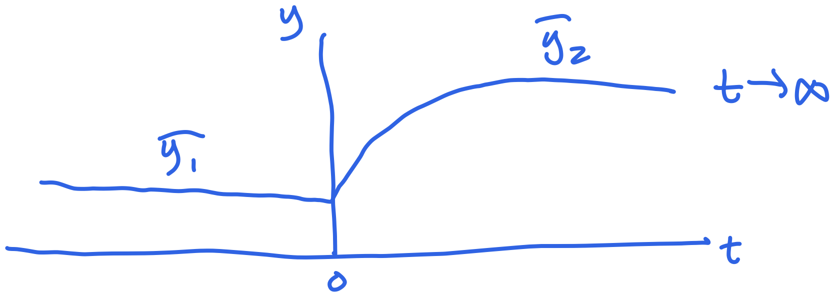
$$= G(0) \Rightarrow \boxed{K = G(0)}$$

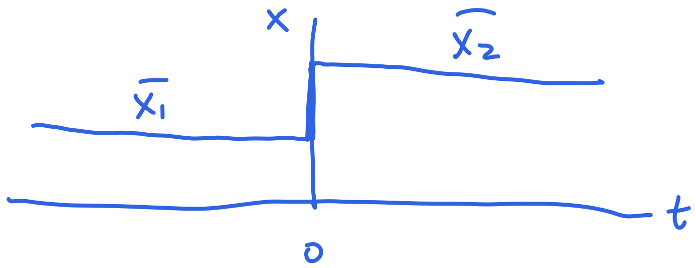
$$G(s) = \frac{Y'(s)}{X'(s)}$$

$$Y'(s) = G(s)X'(s)$$

$$x'(t) = \bar{x}_2 - \bar{x}_1$$

$$X'(s) = \frac{\bar{x}_2 - \bar{x}_1}{s}$$





Example: determining steady-state gain

Ex. Determine the steady state gain given transfer function of $G(s) = \dfrac{w_1}{\rho V s + w}$

$$K = G(0) = \frac{w_1}{\rho V(0) + w} = \boxed{\frac{w_1}{w}}$$