Motivating Transfer Functions

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Laplace-transformed output in original variables are hard to find

Ex. Find the Laplace-transformed output $Y(s)$ of the ODE

$$\frac{dy}{dt} = x - y$$

where x is the input and y is the output.

Deviation variables have vanishing initial condition at steady-state in $\mathcal{L}$ space

Ex. Find the Laplace-transformed output deviation $Y'(s)$ of the ODE

$$\frac{dy}{dt} = x - y$$

where x is the input and y is the output, by defining deviation variable of $y' = y - \overline{y}$.

- Derive governing ODE
- Identify variables
    - Input variable (cause)
    - Output variable (effect)
- Derive steady-state expression
- Derive ODE in deviation variables
- Get transfer function in $\mathcal{L}$ space

Deviation variables have vanishing initial condition at steady-state in $\mathcal{L}$ space

Ex. Find the Laplace-transformed output deviation $Y'(s)$ of the ODE

$$\frac{dy}{dt} = x - y$$

where x is the input and y is the output, by defining deviation variable of $y' = y - \overline{y}$.

Transfer function maps input-output relationship in the $\mathcal{L}$ space

- **Transfer function** - characterizes dynamic relationship of input and output variables in the Laplace space

$$\boxed{G(s) = \frac{\mathcal{L}[\text{output}]}{\mathcal{L}[\text{input}]}}$$

- **Deviation variable** - deviation from a nominal steady-state
    - Have vanishing initial condition at steady-state in $\mathcal{L}$ space

Laplace transform of multivariable ODE show linearity of transfer function

Ex. Find the Laplace-transformed output deviation $Z'(s)$ of the ODE

$$\frac{dz}{dt} = x - y - z$$

where x and y are the inputs and z is the output.

Transfer function can predict step change

Ex. Determine the response $y(t)$ to a step change in x of magnitude M at $t = 0$ using the transfer function

$$G(s) = \frac{1}{s+1}$$

- Rearrange for $Y'(s)$
- Determine $X'(s)$ from $x'(t)$
- Get deviation in time space
- Get response in time space

Obtaining transfer function from ODE

Seborg Ex. 4.1a Find the transfer function model between liquid level h and inlet flow rate q_i in deviation variables with governing ODE of

$$A\frac{dh}{dt} = q_i - \frac{1}{R_v}h$$

- Derive steady-state expression

- Derive ODE in deviation variables

Obtaining transfer function from ODE

Seborg Ex. 4.1a Find the transfer function model between liquid level h and inlet flow rate q_i in deviation variables with governing ODE of

$$A\frac{dh}{dt} = q_i - \frac{1}{R_v}h$$

- Know ODE in deviation variables

$$A\frac{dh'}{dt} = q_i' - \frac{1}{R_v}h'$$

- Get transfer function in $\mathcal{L}$ space
  - Initial condition

Use transfer function to predict step change

Seborg Ex. 4.1b Determine the response $h(t)$ to a step change in q_i of magnitude M at $t = 0$ using the transfer function

$$G(s) = \frac{H'(s)}{Q_i'(s)} = \frac{R_v}{AR_v s + 1}$$

- Rearrange for $H'(s)$
- Determine $Q_i'(s)$ from $q_i'(t)$
- Get deviation response in time space
- Get response in time space