Annuities and Discount Factors

Teng-Jui Lin
Department of Chemical Engineering, University of Washington
Process Design

Annuities give same amount of cash flow for each consecutive year

Ex. Verify that the future value F_n of annuities with each transaction value of A for n consecutive years is

$$\boxed{F_n = A\left[\frac{(1+i)^n - 1}{i}\right]}$$

where i is the annual interest rate.

- **Annuities** - A series of uniform value cash transactions in consecutive years
- Hint: nth partial sum of geometric series: $S_n = a_1 \dfrac{1 - r^n}{1 - r}$

Discount factor converts money value between the present, future, and annuities

- Discount factor for converting A to F

$$F_n = A \left[\frac{(1+i)^n - 1}{i} \right]$$

- Notation

Discount factors are derived from known discount factors

Ex. Derive the discount factor for converting A to P.

Common discount factors are tabulated

Turton Table 9.1

Conversion	Symbol	Common Name	Formula
P to F	F/P	Single payment compound amount factor	$(1+i)^n$
F to P	P/F	Single payment present worth factor	$\frac{1}{(1+i)^n}$
A to F	F/A	Uniform series compound amount factor; Future worth of annuity	$\frac{(1+i)^n-1}{i}$
F to A	A/F	Sinking fund factor	$\frac{1}{(1+i)^n-1}$
P to A	A/P	Capital recovery factor	$\frac{i(1+i)^n}{(1+i)^n-1}$
A to P	P/A	Uniform series present worth factor; Present worth of annuity	$\frac{(1+i)^n-1}{i(1+i)^n}$

Example of using discount factor as a conversion factor

Turton Ex. 9.14 A lottery winner will receive $200,000/year for the next 20 years.
(**a**) What is the equivalent present value of the winnings if there is a secure investment opportunity providing 7.5% p.a.?
(**b**) What rate of return in part (a) would be needed for a present value of $2.5 million?