

# **Transfer Functions of Multivariable ODEs**

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**Process Dynamics and Control**

## Transfer functions of multivariable ODEs show linearity

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**Seborg Ex. 4.4a** Given constant liquid density  $\rho$ , volume  $V$ , mass flow rates  $w_1$ ,  $w_2$ , and  $w$ , the governing equation for a continuous blending process is

$$\rho V \frac{dx}{dt} = w_1 x_1 + w_2 x_2 - wx$$

where  $x$  are compositions. Determine the transfer function for output  $x$  and varying input  $x_1$  while  $x_2$  is held constant.

# Transfer functions of multivariable ODEs show linearity

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**Seborg Ex. 4.4a (cont.)**

# Transfer functions of multivariable ODEs show linearity

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**Seborg Ex. 4.4a (cont.)**

## Transfer functions of multivariable ODEs show linearity

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**Seborg Ex. 4.4b** Given constant liquid density  $\rho$ , volume  $V$ , mass flow rates  $w_1$ ,  $w_2$ , and  $w$ , the governing equation for a continuous blending process is

$$\rho V \frac{dx}{dt} = w_1 x_1 + w_2 x_2 - w x$$

where  $x$  are compositions. Determine the transfer function for output  $x$  and varying inputs  $x_1$  and  $x_2$ .

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**Seborg Ex. 4.4b** (cont.)

# Transfer functions of multivariable ODEs show linearity

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**Seborg Ex. 4.4b** (cont.)

## Linearity and input-output relationship

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$$X'(s) = G_1(s)X'_1(s) + G_2(s)X'_2(s)$$