Wicking Flow in Porous Media

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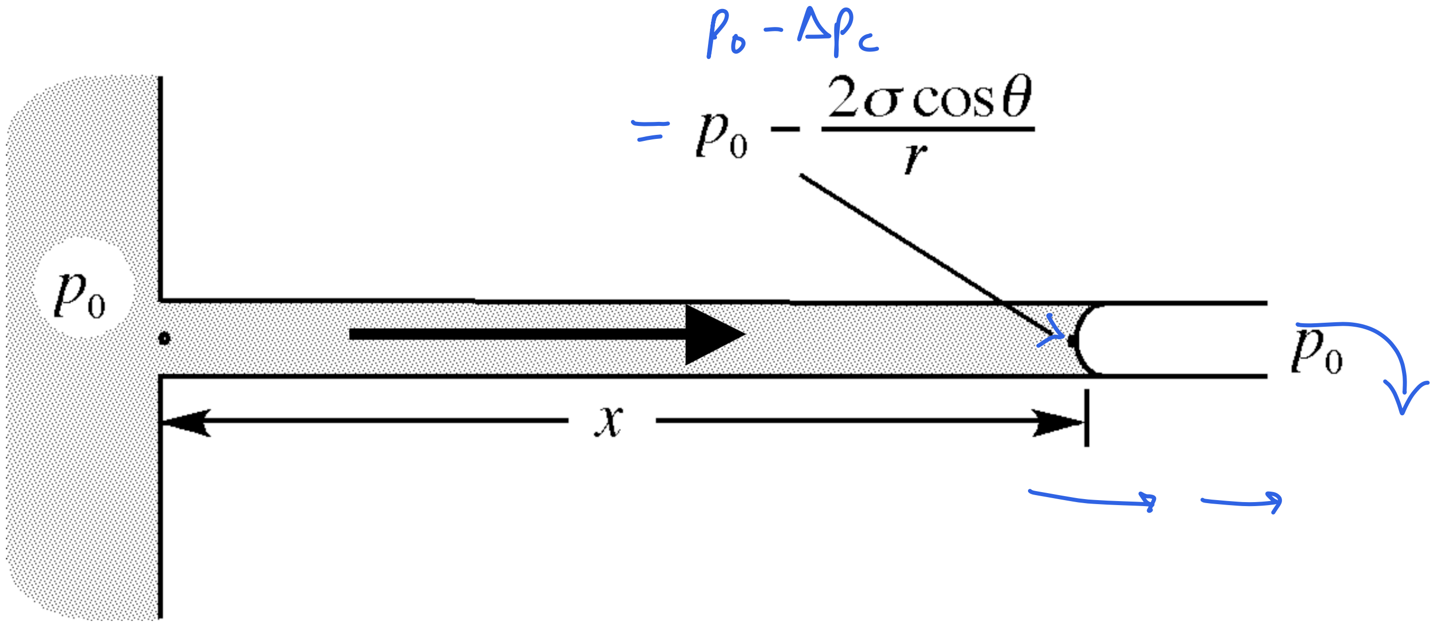
Surface and Colloid Science

Wicking flow in a horizontal tube is driven by Young-Laplace pressure gradient



- Young-Laplace equation

$$\boxed{\Delta p_c = \frac{2\sigma}{R_m}} = \frac{2\sigma}{\frac{r}{\cos\theta}} = \frac{2\sigma\cos\theta}{r}$$

  - Part-of-sphere approx

$$R_m = \frac{r}{\cos\theta}$$

Washburn equation is derived from Hagen-Poiseuille equation

- Hagen-Poiseuille equation

$$\frac{dx}{dt} = \boxed{v = \frac{r^2}{8\mu}\frac{dp}{dx}} = \frac{r^2}{8\mu}\frac{\Delta p_c}{x} = \frac{r^2}{8\mu}\frac{2\sigma\cos\theta}{r x} = \frac{r\sigma\cos\theta}{4\mu x}$$

$$\int dt = \int \frac{4\mu x}{r\sigma\cos\theta}\,dx$$

$$t = \frac{4\mu}{r\sigma\cos\theta}\frac{1}{2}x^2 = \frac{2\mu x^2}{\cos\theta\, r\sigma} \Rightarrow x = \sqrt{\frac{t r\sigma\cos\theta}{2\mu}} = \sqrt{\frac{r\sigma\cos\theta}{2\mu}t}$$

- Washburn equation

$$\boxed{x = k_W\sqrt{t}} \qquad x \propto \sqrt{t}$$

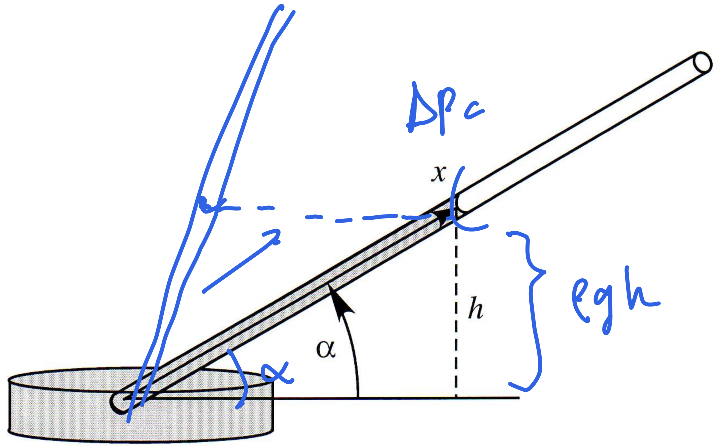
  - Washburn constant

radius of cap. surface tension contact angle

$$k_W = \sqrt{\frac{r\sigma\cos\theta}{2\mu}}$$

viscosity

Wicking flow in an inclined tube is affected by gravity



$\sin\alpha = \frac{h}{x}$

$h = x\sin\alpha$

- Pressure drop

$$\Delta p = \Delta p_c - \rho g h = 0 = \frac{2\sigma\cos\theta}{r} - \rho g x \sin\alpha$$

- Rise height

$$\boxed{H = \frac{2\sigma\cos\theta}{\rho g r}} \neq f(\alpha)$$

$$\frac{2\sigma\cos\theta}{r\rho g \sin\alpha} = x$$

- Wicking distance

$$X = \frac{H}{\sin\alpha} = \frac{2\sigma\cos\theta}{\rho g r \sin\alpha}$$

Wicking distance with respect to time in an inclined tube

- Hagen-Poiseuille equation

quasi-steady
laminar

$$\frac{dx}{dt} \approx \boxed{v = \frac{r^2}{8\mu}\frac{dp}{dx}} = \frac{r^2}{8\mu}\frac{\Delta p}{x} = \frac{r^2}{8\mu}\frac{1}{x}\left[\frac{2\sigma\cos\theta}{r} - \rho g x\sin\alpha\right]$$

$$\frac{dx}{dt} = \frac{r^2}{8\mu}\left[\frac{2\sigma\cos\theta}{rx} - \rho g\sin\alpha\right]$$

- Integrate

$$t = \frac{8\mu X}{\rho g r^2}\frac{1}{\sin\alpha}\left[\underbrace{-\ln\left(1-\frac{x}{X}\right)} - \frac{x}{X}\right]$$

wicking eqn for inclined capillary

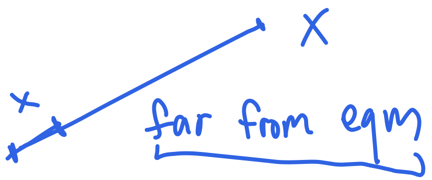
$$\underline{X} = \frac{2\sigma\cos\theta}{\rho g r\sin\alpha}$$

- Taylor series approximation

$$t \approx \frac{\cancel{8}\mu\cancel{X}}{\rho g r^2}\frac{1}{\sin\alpha}\left[\frac{1}{\cancel{2}}\left(\frac{x}{X}\right)^{\cancel{2}} + \cancelto{0}{\mathcal{O}\left(\frac{x}{X}\right)^3}\right]$$

Reduces to Washburn equation when $\boxed{x/X \text{ is small}}$

< 0.3



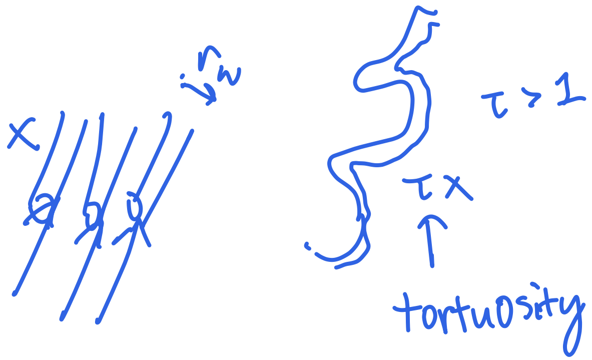
wicking eqn for inclined cap

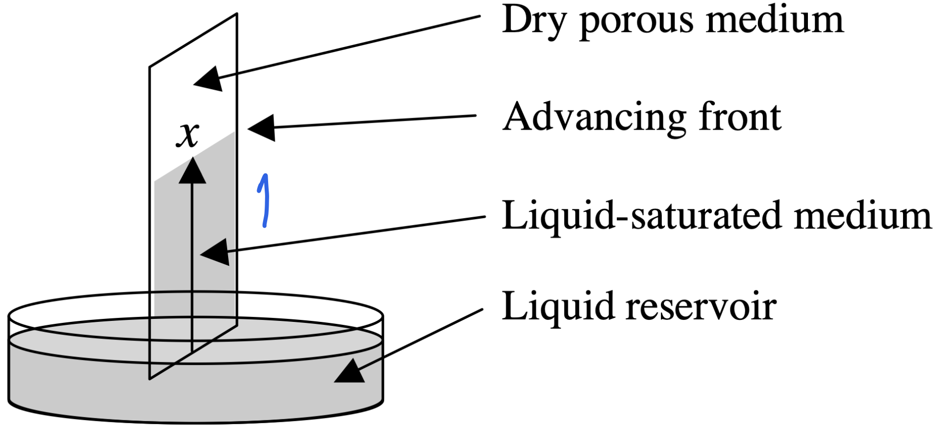
negligible g effect

$$t \approx \frac{4\mu}{\underline{X}}\frac{x^2}{\rho g r^2\sin\alpha} = \frac{\cancel{4}^{2}\mu x^2}{\cancel{\rho g}r^{\cancel{2}}\cancel{\sin\alpha}}\frac{\cancel{\rho g r}\cancel{\sin\alpha}}{\cancel{2}\sigma\cos\theta} = \frac{2\mu x^2}{r\sigma\cos\theta} \Rightarrow x = \sqrt{\frac{t r\sigma\cos\theta}{2\mu}}$$

Washburn eqn for horiz. cap.

Wicking flow in porous media can be approximated by Washburn analysis





- Δp_c varies point to point, but Washburn analysis is good approx

$$k_W = \sqrt{\frac{r_W \sigma \cos\theta}{2\mu}}$$

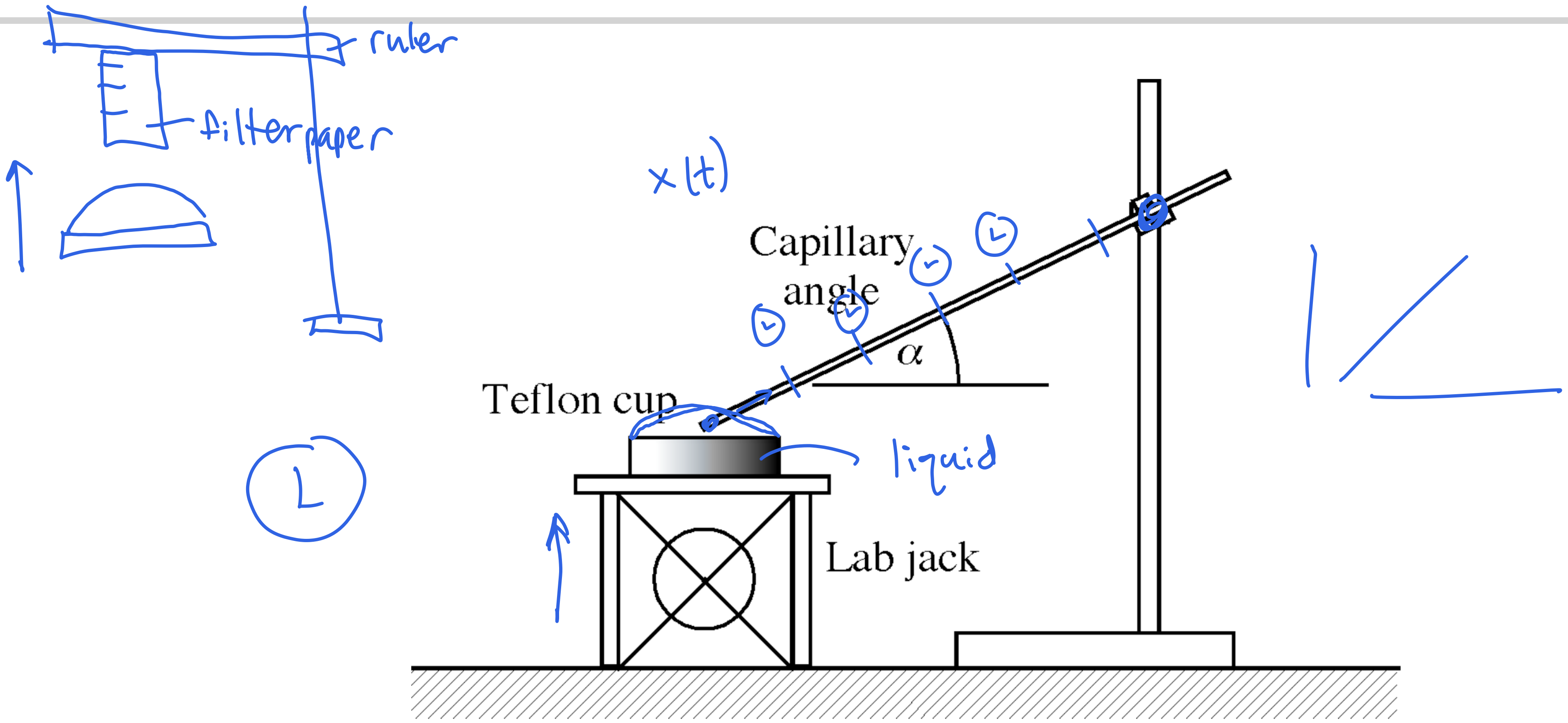
  - r_W - Wicking equivalent radius, effective cylindrical pore radius for Washburn analysis
    - One order of magnitude smaller than actual pore radius
    - Tortuosity correction - replace x as τx

$$x = \sqrt{\frac{r \quad \sigma\cos\theta}{2\tau\mu}t} = \sqrt{\frac{r_W \sigma\cos\theta}{2\mu}t} \implies r_W = \frac{r}{\tau}$$

actual

- Gravity effect negligible due to small pore radius

Experimental setup



- **Wear safety goggles at all times!**
- Variables: liquid, capillary radius, tilt angle