Centrifugation

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Separation Processes

Sedimentation is driven by gravity and follows Stoke's law at low velocity

- Sedimentation separates solid particles dispersed in liquid
- **Stoke's law** describes flow around a sphere at low velocity (Re < 1)
    - $\boxed{v_\infty = \frac{(\rho_p - \rho_f)}{18\mu} D_p^2 g}$ for $\mathrm{Re} = \frac{D_p v_\infty \rho_f}{\mu} < 1$
      driving force
        - v_∞ - terminal velocity
        - ρ_p - particle density
        - ρ_f - fluid density
        - μ - fluid viscosity
        - D_p - particle diameter
- Sedimentation is slow... How can we speed it up?

Centrifugation is driven by centrifugal force and also follows Stoke's law

- Centrifugation separates solid particles dispersed in liquid *faster*
- **Stoke's law** describes flow around a sphere at low velocity (Re < 1)
  - $\boxed{v_\infty = \frac{(\rho_p - \rho_f)}{18\mu} D_p^2 \omega^2 r}$ for $\mathrm{Re} = \frac{D_p v_\infty \rho_f}{\mu} < 1$

    driving force
    - ω - angular velocity of centrifuge bowl
    - r - centrifuge bowl radius

- **G-force** normalizes centrifugal driving force by gravitational driving force
  - $\boxed{Z = \frac{\omega^2 r}{g}}$ $= \frac{\text{centrifgal}}{\text{gravitational}} = \qquad \times g$

Sigma factor compares performance of continuous centrifuges of the same type

- **Sigma factor** - effective area of a continuous centrifuge
  - $\boxed{\Sigma = \dfrac{\dot{V}}{2v_\infty}}$ $\quad \dfrac{\text{flow rate}}{\text{velocity}} \; [=] \; m^2$

$$1: \quad \cancel{2} v_{\infty 1} \Sigma_1 = \dot{V}_1$$
$$2: \quad \cancel{2} v_{\infty 2} \Sigma_2 = \dot{V}_2$$

ratio

$$\frac{v_{\infty 1} \Sigma_1}{v_{\infty 2} \Sigma_2} = \frac{\dot{V}_1}{\dot{V}_2}$$

- **Ex.** At small scale, cells can be centrifuged at $\Sigma_1 = 200$ and $\dot{V}_1 = 15 \text{ mL/min}$. At large scale, cells can be centrifuged at $\Sigma_2 = 9000$ and $\dot{V}_2 = 700 \text{ mL/min}$ at the same speed (ω). The densities and viscosity are unchanged. Quantify the changed physical property.

$$\frac{\overset{?}{v_{\infty 1}} \Sigma_1 \checkmark}{\underset{?}{v_{\infty 2}} \Sigma_2 \checkmark} = \frac{\dot{V}_1 \checkmark}{\dot{V}_2 \checkmark}$$

particle diameter

$\Downarrow$

$$\frac{\frac{\cancel{(\rho_p - \rho_f)}}{\cancel{18\mu}} D_{p1}^2 \cancel{\omega^2 r}}{\frac{\cancel{\rho_p - \rho_f}}{\cancel{18\mu}} D_{p2}^2 \cancel{\omega^2 r}} \frac{\Sigma_1}{\Sigma_2} = \frac{\dot{V}_1}{\dot{V}_2} \Rightarrow \frac{D_{p1}^2}{D_{p2}^2} \frac{\Sigma_1}{\Sigma_2} = \frac{\dot{V}_1}{\dot{V}_2}$$

$$\frac{D_{p1}}{D_{p2}} = \sqrt{\frac{\dot{V}_1}{\dot{V}_2} \frac{\Sigma_2}{\Sigma_1}} = \sqrt{\frac{15}{700} \frac{9000}{200}} = 0.98$$

small large

$$\boxed{D_{p1} = 0.98 \, D_{p2}}$$