Breakup of Capillary Jets

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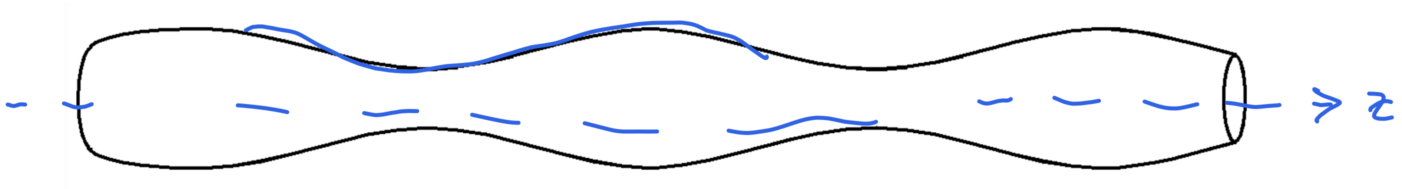
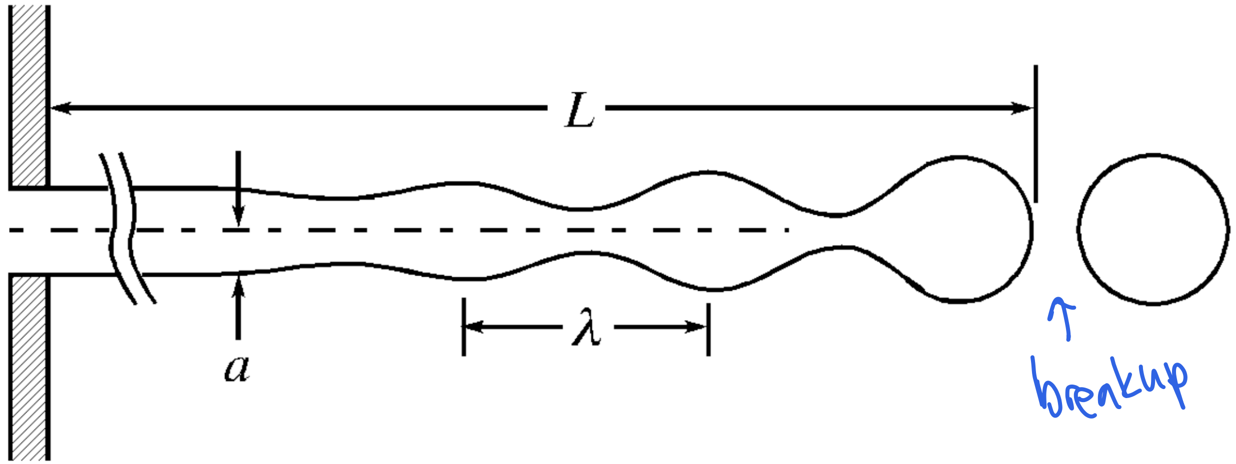
Capillary jets spontaneously breaks off to minimize system free energy



- **Capillary jet** - liquid emerge from small-diameter circular orifice into immiscible fluid at sufficient velocity
- Plateau - liquid cylinder becomes unstable when length exceeds circumference
  - Axisymmetric, sinusoidal disturbance

  ↓ surface area, ↓ system free energy, spontaneous

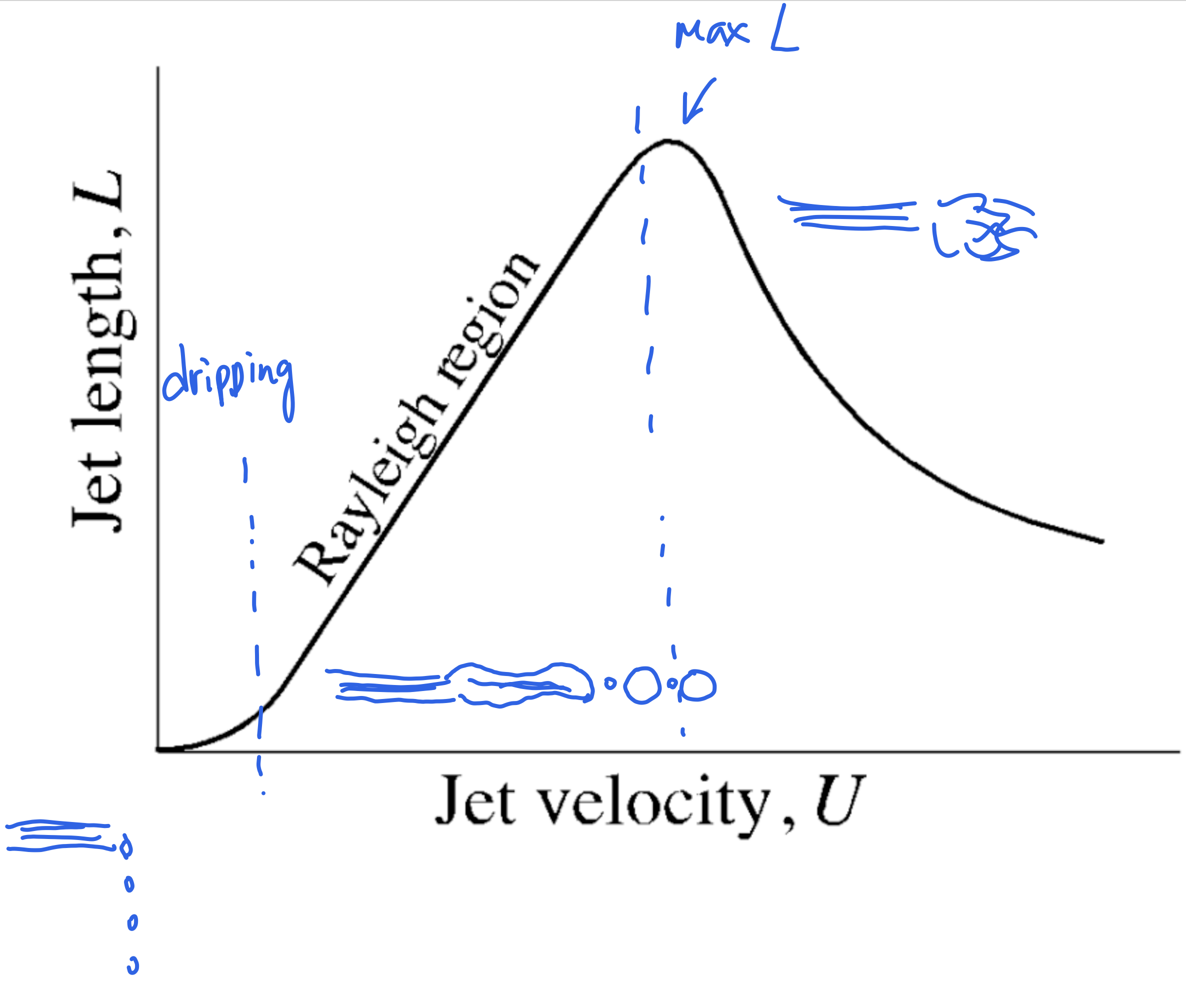


Jet length depends on jet velocity for laminar flow

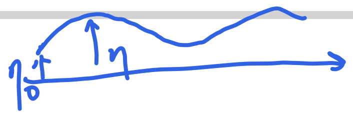
- Dripping regime
  - Low flow rate, not jet formation
- Rayleigh region
  - Laminar jet formation

    Jet length $\propto$ jet velocity

  - Regular jet break off with uniform drop size and spacing (could form satellite drops)
- After maximum jet length
  - Irregular jet breakup and spacing
- High jet velocity
  - Atomization - spray of droplets
  - Hard to define jet length



Rayleigh analysis predicts jet length and drop size



- General form
  - Surface disturbance amplitude

    init ← growth rate

    $$\eta = \eta_0 e^{\beta t}\cos(kz)$$

    time, wavenumber $k = \frac{2\pi}{\lambda}$

  - Jet length

    $$L = \frac{U}{\beta^*}\ln\left(\frac{a}{\eta_0}\right)$$

    $\eta = \eta_0 e^{\beta t} \cancel{\cos(kz)}^{\,1}$

    $a = \eta_0 e^{\beta^* t}$

    $\frac{1}{\beta^*}\ln\left(\frac{a}{\eta_0}\right) = t \approx \frac{L}{U}$ ← velocity

    $L = \frac{U}{\beta^*}\ln\left(\frac{a}{\eta_0}\right)$

  - Drop size

    $$V = \pi a^2 \lambda^*$$

- Rayleigh analysis assumptions
  - Axisymmetric disturbance ✓
  - Inviscid (zero viscosity) liquid jet H_2O
  - Inviscid (zero viscosity), zero density medium air
  - No gravity

- Rayleigh analysis
  - Wave number that maximizes β

    $$k^* \approx \frac{0.697}{a}$$

  - Wavelength that maximizes β

    $$\lambda^* = 2\pi k^* \approx 9.02a$$

  - Maximum growth constant

    $$\beta^* = \sqrt{0.12\frac{\sigma}{\rho a^3}}$$

    σ ← surface tension; a ← jet radius; ρ ← density

  - Jet length

    ✓ $$\boxed{L = 8.33\ln\left(\frac{a}{\eta_0}\right)U\left(\frac{\rho a^3}{\sigma}\right)^{1/2}}$$

  - Drop size

    ✓ $$\boxed{V = 28.3a^3}$$

Weber analysis relaxes some assumptions for Rayleigh analysis

- Weber's number

velocity

density of external medium

$$\frac{\text{drag force}}{\text{s.f force}} = \mathrm{We} = \frac{U^2 \rho_e 2a}{\sigma} \begin{cases} \leq 0.1 & \text{Rayleigh analysis} \\ > 0.1 & \text{Weber analysis} \end{cases}$$

surface tension

radius

- Weber analysis assumptions
  - Finite jet liquid viscosity
  - Finite density for medium
  - Asymmetrical disturbance
- Weber analysis
  - Maximum growth constant

$$\beta^* = \left[\left(\frac{8\rho a^3}{\sigma} \right)^{1/2} + \left(\frac{6\mu a}{\sigma} \right) \right]^{-1} \checkmark$$

  - Wave number that maximizes β

$$k^* = \left[2a^2 + \left(\frac{9\mu^2 a}{\rho\sigma} \right) \right]^{-1/2} \checkmark$$

- Weber analysis (derived quantities)
  - Wavelength that maximizes β

$$\checkmark \quad \lambda^* = 2\pi k^* = 2\pi \left[2a^2 + \left(\frac{9\mu^2 a}{\rho\sigma} \right) \right]^{-1/2}$$

  - Jet length

$$\boxed{L = U \left[\left(\frac{8\rho a^3}{\sigma} \right)^{1/2} + \left(\frac{6\mu a}{\sigma} \right) \right] \ln \left(\frac{a}{\eta_0} \right)}$$

  - Drop size

$$\boxed{V = 2\pi^2 a^2 \left[2a^2 + \left(\frac{9\mu^2 a}{\rho\sigma} \right) \right]^{-1/2}}$$

Experimental setup of capillary jet breakup

- Rayleigh analysis
  - Jet length

$$\boxed{L = \underbrace{8.33 \ln\left(\frac{a}{\eta_0}\right)}_{\text{const}} U \underbrace{\left(\frac{\rho a^3}{\sigma}\right)^{1/2}}_{\text{const}}}$$

  - Drop size

$$\boxed{V = 28.3a^3}$$

- Measurement by image analysis
  - ✓ L - Jet length
  - ✓ r - Drop radius $\Rightarrow$ V - Drop size
  - ✓ λ - Breakup wavelength
  - ✓ a - Undisturbed jet radius
- Measurement by bucket and stopwatch

  $\dot{V}$ - Volumetric flow rate $\Rightarrow$ U - Jet velocity

$$\dot{V} = \frac{V}{t} = \frac{m/\rho}{t}$$