

Laplace Transform of Derivatives

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Process Dynamics and Control

Laplace transform of derivatives

Ex. Evaluate $\mathcal{L}[f'(t)]$

Ex. Evaluate $\mathcal{L}[f''(t)]$

Ex. Evaluate by induction $\mathcal{L}[f'''(t)]$

Laplace transform of a derivative

$$\mathcal{L}[f^{(n)}(t)] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - s f^{(n-2)}(0) - f^{(n-1)}(0)$$

Final value theorem

Ex. Proof the final value theorem.

$$\lim_{t \rightarrow \infty} y(t) = \lim_{s \rightarrow 0} [sY(s)]$$

Initial value theorem

Ex. Proof the initial value theorem.

$$\lim_{t \rightarrow 0} y(t) = \lim_{s \rightarrow \infty} [sY(s)]$$