Laplace Transforms of Integrals

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Laplace transforms of integrals

Ex. Proof that the Laplace transform of an integral is

$$\frac{d}{dt}\int_a^x f(x^*)\,dx^* = f(x)$$

$$\mathcal{L}\left[\int_0^t f(t^*)dt^*\right] = \frac{1}{s}F(s)$$

$$\mathcal{L}\left[\int_0^t f(t^*)\,dt^*\right]$$

$$u = \int_0^t f(t^*)dt^* \qquad dv = e^{-st}$$

$$du = f(t)\,dt \qquad v = -\frac{1}{s}e^{-st}$$

$$= \int_0^\infty e^{-st}\left[\int_0^t f(t^*)\,dt^*\right]dt$$

$$= \left[\int_0^t f(t^*)dt^* \cdot \left(-\frac{1}{s}e^{-st}\right)\right]_0^\infty - \int_0^\infty -\frac{1}{s}e^{-st} f(t)\,dt$$

$$= \left[0 - 0\right] + \frac{1}{s}\underbrace{\int_0^\infty e^{-st} f(t)\,dt}_{\mathcal{L}[f(t)] = F(s)}$$

$$= \frac{1}{s}F(s)$$

Laplace transforms of derivatives and integrals

initial cond.

Derivative: $$\mathcal{L}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0)$$

Integral: $$\mathcal{L}\left[\int_0^t f(t^*)dt^*\right] = \frac{1}{s}F(s)$$