Calculations with Discount Factors

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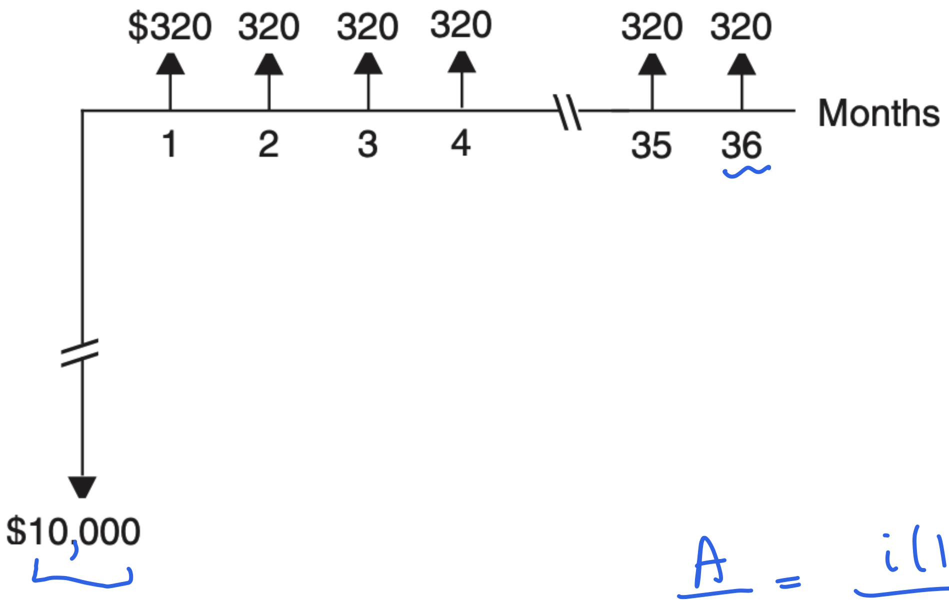
Process Design

Calculating interest rates using discount factors

Turton Ex. 9.15 Given discrete cash flow diagram from the bank's point of view, what is the interest rate the bank is charging for this loan?



compounded monthly

$A = \$320/\text{month}$, $n = 36$ month } annuity

$P = \$10{,}000$

$i = ?$

$$\frac{A}{P} = \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$\frac{320}{10{,}000} = \frac{i(1+i)^{36}}{(1+i)^{36} - 1}$$ ← month

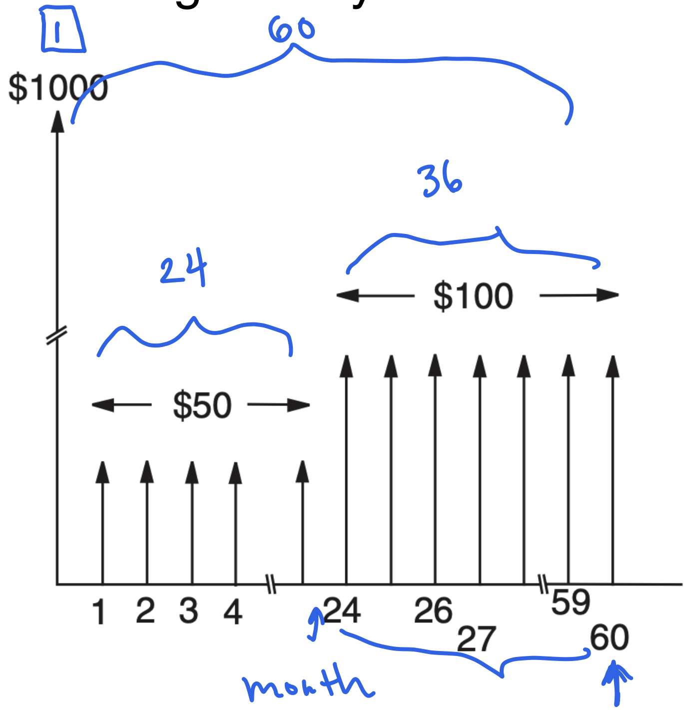
↑ monthly interest rate

$\Rightarrow i = 0.00786 = 0.786\%$ ↗

yearly : $12i = 12(0.786\%) = 9.432\% \Rightarrow \boxed{9.4\%}$

Calculating future values using discount factors

Turton Ex. 9.16 Money is invested in a savings account that pays a nominal interest rate of 6% p.a. compounded monthly. The account is opened with a deposit of $1000, and then deposits of $50 at the end of each month are made for a period of two years, followed by a monthly deposit of $100 for the following three years. What will the value of the savings account be at the end of the five-year period?



$i_y = 6\% = 0.06$

$F = ?$

$i_m = \frac{i_y}{12} = \frac{0.06}{12} = 0.005$

$\boxed{1}$ $P = \$1000$, $n = 60$

$F = P\left(\frac{F}{P}\right) = P(1+i)^n = (1000)(1+0.005)^{60} =$

$\boxed{2}$ $A = 50$, $n_1 = 24$, $n_2 = 36$

(a) calculate $F(24\ \text{month}) = A\left(\frac{F}{A}\right) = A\frac{(1+i)^{n_1}-1}{i}$ ← Annuity

$F(60\text{month}) = F(24\text{moth})\left(\frac{F}{P}\right) = F(24\text{month})(1+i)^{n_2}$

(present) compound interest

$= A\frac{(1+i)^{n_1}-1}{i}(1+i)^{n_2} =$

$\boxed{3}$ $A = 100$, $n = 36$

$F = A\left(\frac{F}{A}\right) = A\frac{(1+i)^n-1}{i}(1+i)^n$

$\boxed{4}$ $F = F_1 + F_2 + F_3 = \$6804$

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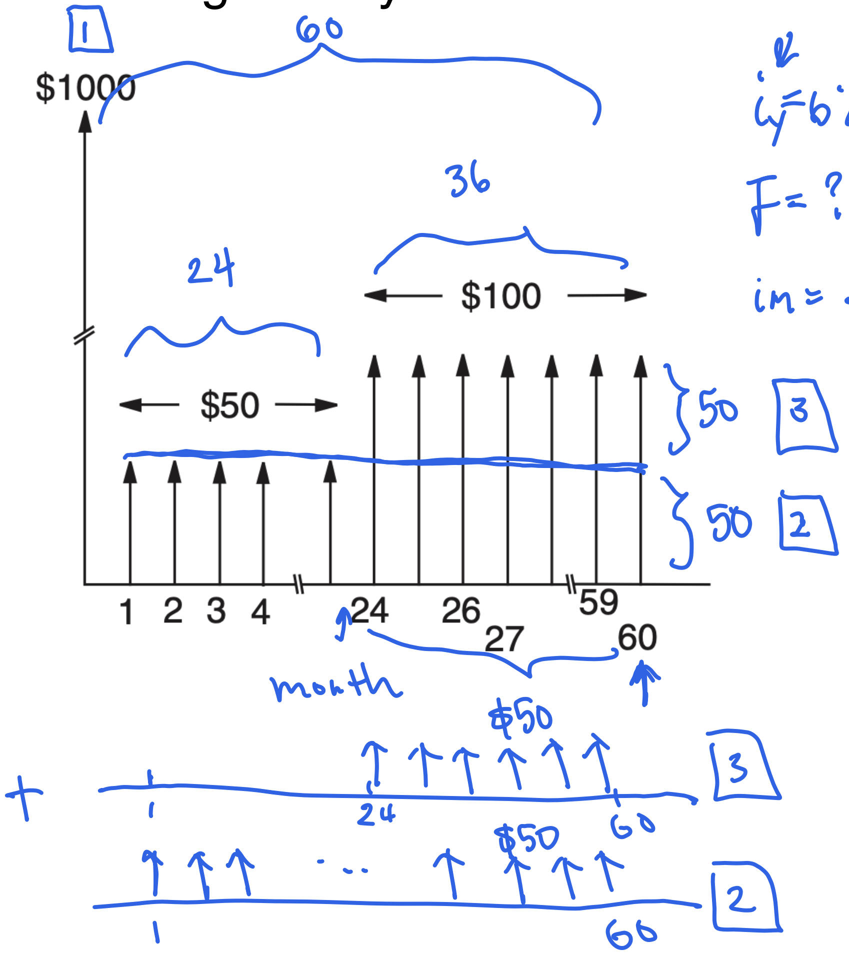

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Calculating interest rates and future values using discount factors

Turton Ex. 9.17 An investment plan involved investing \$8000/year for 40 years leading to retirement. The plan then provided \$106,667/year for 20 years of retirement income.

A= annuity

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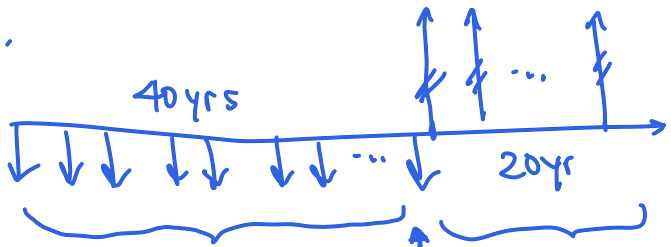
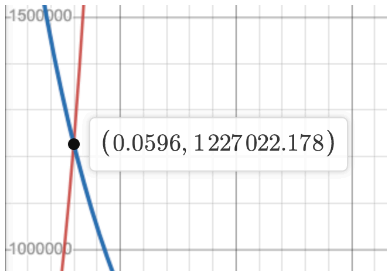
(**a**) What yearly interest rate was used in this evaluation?
(**b**) How much money was invested in the retirement plan before withdrawals began?

(a)



value at retirement value



$n_1 = 40$
$A_1 = \$8000$
$i = ?$

$A_2 = \$106{,}667$
$i = ?$
$n_2 = 20$

$$F_{40} = P$$

$$A_1\left(\frac{F}{A_1}\right) = A_2\left(\frac{P}{A_2}\right)$$

$$A_1 \frac{(1+i)^{n_1}-1}{i} = A_2 \frac{(1+i)^{n_2}-1}{i(1+i)^{n_2}}$$

$i = 0.0596 = 5.96\%$

(b) $F_{40} = \$1.23 \times 10^6$

$$F_{40} = A_1 \frac{(1+i)^{n_1}-1}{i}$$

$A_1 = 8000$
$i = 0.0596$
$n_1 = 40$

$= \$1.23 \times 10^6$