Output Response from Transfer Functions

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Process Dynamics and Control

Evaluating nominal steady-state condition

Seborg Ex. 4.5a Given constant liquid density ρ, volume V, mass flow rates w_1, w_2, and w, the governing equation for a continuous blending process is

$$\cancel{\rho V \frac{dx}{dt}}^{\,0} = w_1 x_1 + w_2 x_2 - wx$$

output

input

const

where x are compositions. When output x varies upon change in input x_1 while x_2 is held constant, the transfer function is

$$G(s) = \frac{K_1}{\tau s + 1}, \quad K_1 \equiv \frac{w_1}{w}, \quad \tau \equiv \frac{\rho V}{w}$$

Determine the nominal exit concentration $\overline{x}$, given $w_1 = 600$ kg/min, $w_2 = 2$ kg/min, $x_1 = 0.05$, $x_2 = 1$.

steady-state

$w = w_1 + w_2$

$$0 = w_1 \overline{x}_1 + w_2 x_2 - w\overline{x}$$

$$\overline{x} = \frac{w_1 \overline{x}_1 + w_2 x_2}{w} = \frac{(600)(0.05) + (2)(1)}{600 + 2} = \boxed{0.053}$$

Output response upon step input change can be determined from transfer functions

Seborg Ex. 4.5b Derive an expression of the output response $x(t)$ given the transfer function

$$\frac{X'(s)}{X_1'(s)} = G(s) = \frac{K_1}{\tau s + 1}, \quad K_1 \equiv \frac{w_1}{w}, \quad \frac{600}{602} = 0.997, \quad \tau \equiv \frac{\rho V}{w} = \frac{(900)(2)}{600+2} = 2.99 \text{ min}$$

and sudden input change in x_1 from 0.050 to 0.075 at $t = 0$. Assume the process is initially at steady-state. Given $w_1 = 600$ kg/min, $w_2 = 2$ kg/min, $x_1 = 0.05$, $x_2 = 1$, $V = 2\ \mathrm{m}^3$, $\rho = 900\ \mathrm{kg/m^3}$.

step

$$X'(s) = G(s) X_1'(s) = G(s) \mathcal{L}[x_1'(t)] = G(s) \mathcal{L}[0.075 - 0.050] = G(s) \frac{0.25}{s}$$

$$= \frac{K_1}{\tau s + 1} \cdot \frac{0.25}{s}$$

$$\boxed{\mathcal{L}^{-1}\left[\frac{1}{(s+b_1)(s+b_2)}\right] = \frac{1}{b_1 - b_2}\left(e^{-b_2 t} - e^{-b_1 t}\right)}$$

$$b_1 = \frac{1}{\tau}, \quad b_2 = 0$$

$$x'(t) = \mathcal{L}^{-1}[X'(s)] = \mathcal{L}^{-1}\left[\frac{0.25 K_1}{(\tau s+1) s}\right] = 0.25 K_1 \mathcal{L}^{-1}\left[\frac{1/\tau}{(s + 1/\tau) s}\right]$$

analytic

$$x(t)$$

$$= \frac{0.25 K_1}{\tau} \frac{1}{\tau} \left(e^0 - e^{-t/\tau}\right)$$

$$= \frac{0.25 K}{\tau^2} \left(1 - e^{-t/\tau}\right)$$

substitute

$$x(t) = x'(t) + \bar{x} = \frac{0.25 K}{\tau^2}\left(1 - e^{-t/\tau}\right) + \bar{x} = \frac{(0.25)(0.997)}{(2.99)^2}\left(1 - e^{-t/2.99}\right) + 0.053$$

$$= \boxed{0.0249\left(1 - e^{t/2.99}\right) + 0.053}$$

Output response upon step input change can be determined from transfer functions

Seborg Ex. 4.5b Derive an expression of the output response $x(t)$ given the transfer function

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