Wicking Flow in Porous Media

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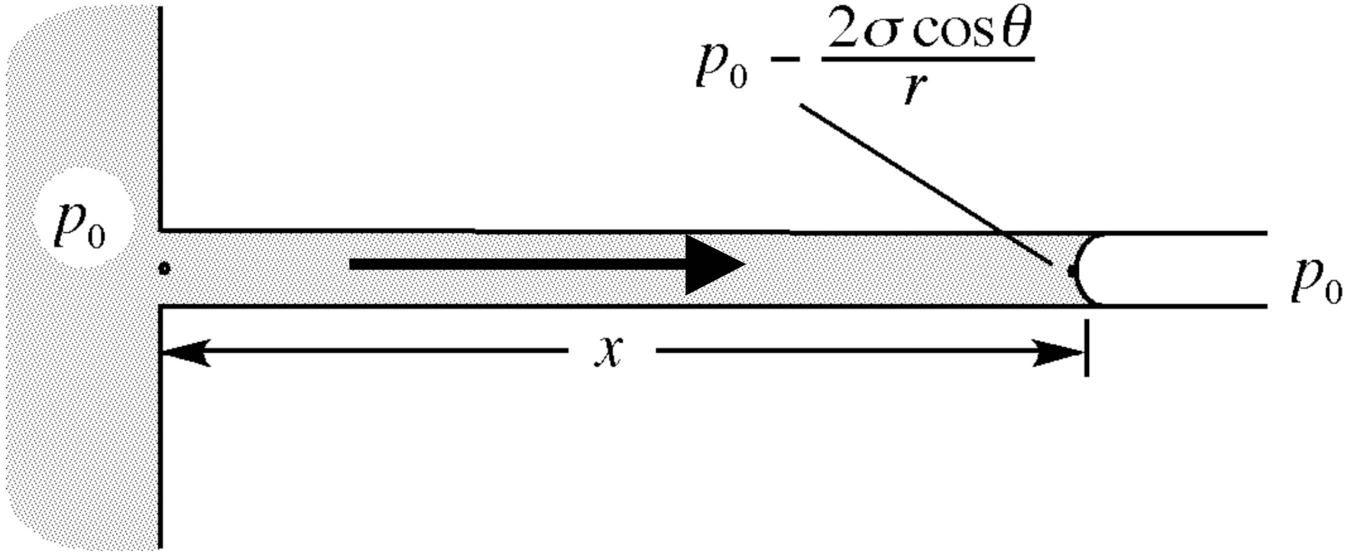
Wicking flow in a horizontal tube is driven by Young-Laplace pressure gradient



- Young-Laplace equation

$$\boxed{\Delta p_c = \frac{2\sigma}{R_m}}$$

  - Part-of-sphere approx

$$R_m = \frac{r}{\cos\theta}$$

Washburn equation is derived from Hagen-Poiseuille equation

- Hagen-Poiseuille equation

$$\boxed{v = \frac{r^2}{8\mu}\frac{dp}{dx}}$$

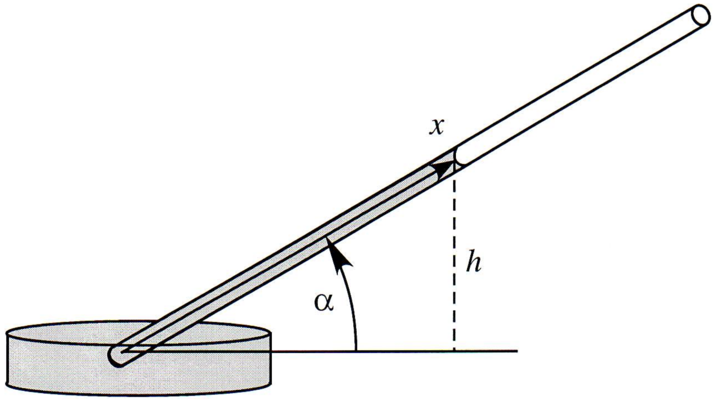
- Washburn equation

$$\boxed{x = k_W\sqrt{t}}$$

  - Washburn constant

$$k_W = \sqrt{\frac{r\sigma\cos\theta}{2\mu}}$$

Wicking flow in an inclined tube is affected by gravity



- Pressure drop

$$\Delta p = \Delta p_c - \rho g h$$

- Rise height

$$\boxed{H = \frac{2\sigma\cos\theta}{\rho g r}}$$

- Wicking distance

$$X = \frac{H}{\sin\alpha} = \frac{2\sigma\cos\theta}{\rho g r \sin\alpha}$$

Wicking distance with respect to time in an inclined tube

- Hagen-Poiseuille equation

$$\boxed{v = \frac{r^2}{8\mu}\frac{dp}{dx}}$$

- Integrate

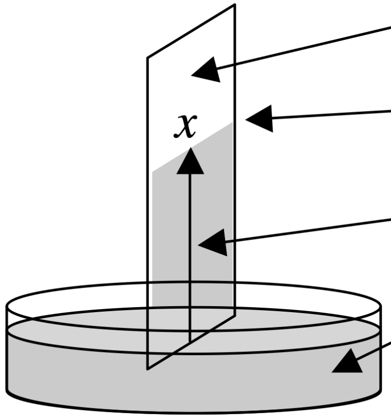
$$t = \frac{8\mu X}{\rho g r^2}\frac{1}{\sin\alpha}\left[-\ln\left(1-\frac{x}{X}\right)-\frac{x}{X}\right]$$

- Taylor series approximation

$$t \approx \frac{8\mu X}{\rho g r^2}\frac{1}{\sin\alpha}\left[\frac{1}{2}\left(\frac{x}{X}\right)^2+\mathcal{O}\left(\frac{x}{X}\right)^3\right]$$

Reduces to Washburn equation when x/X is small

Wicking flow in porous media can be approximated by Washburn analysis



- Δp_c varies point to point, but Washburn analysis is good approx

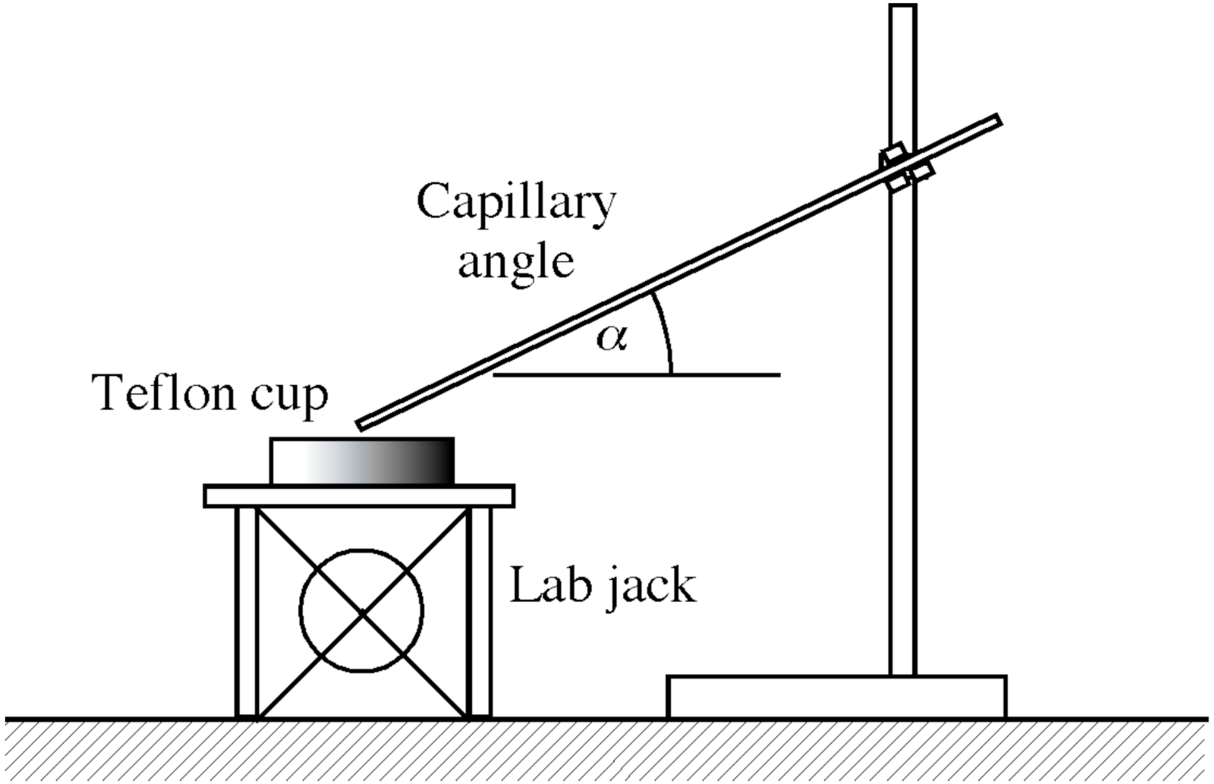
$$k_W = \sqrt{\frac{r_W \sigma \cos\theta}{2\mu}}$$

  - r_W - Wicking equivalent radius, effective cylindrical pore radius for Washburn analysis
    - One order of magnitude smaller than actual pore radius
    - Tortuosity correction - replace x as τx

$$x = \sqrt{\frac{r\sigma\cos\theta}{2\tau\mu}t} = \sqrt{\frac{r_W\sigma\cos\theta}{2\mu}t} \implies r_W = \frac{r}{\tau}$$

- Gravity effect negligible due to small pore radius

Experimental setup



- **Wear safety goggles at all times!**
- Variables: liquid, capillary radius, tilt angle