Derivation of the Wicking Equation for Inclined Capillary

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Derivation of wicking equation for inclined capillary

- Given the wicking distance

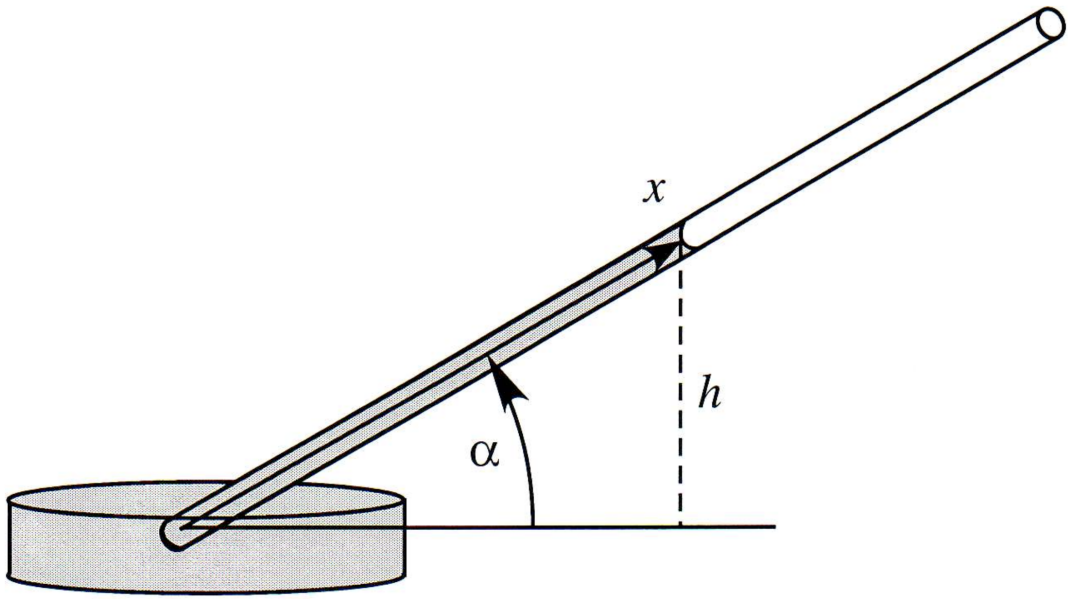
$$X = \frac{H}{\sin\alpha} = \frac{2\sigma\cos\theta}{\rho g r \sin\alpha}$$

- Use the Hagen-Poiseuille equation for inclined capillary

$$\frac{dx}{dt} = \frac{r^2}{8\mu}\left[\frac{2\sigma\cos\theta}{rx} - \rho g\sin\alpha\right]$$

- Verify that the wicking equation for inclined capillary is

$$\boxed{t = \frac{8\mu X}{\rho g r^2}\frac{1}{\sin\alpha}\left[-\ln\left(1-\frac{x}{X}\right) - \frac{x}{X}\right]}$$



Derivation of wicking equation for inclined capillary

Washburn equation is recovered at small x/X (far from equilibrium)

- Verify that at small $x/X < 0.3$, the wicking equation reduces to the Washburn equation

$$x = \sqrt{\frac{r\sigma\cos\theta}{2\mu}t}$$

- with Taylor series approximation

$$\ln(1-x) = \sum(-1)^n \frac{x^n}{n}$$

Washburn equation is recovered at small x/X (far from equilibrium)