

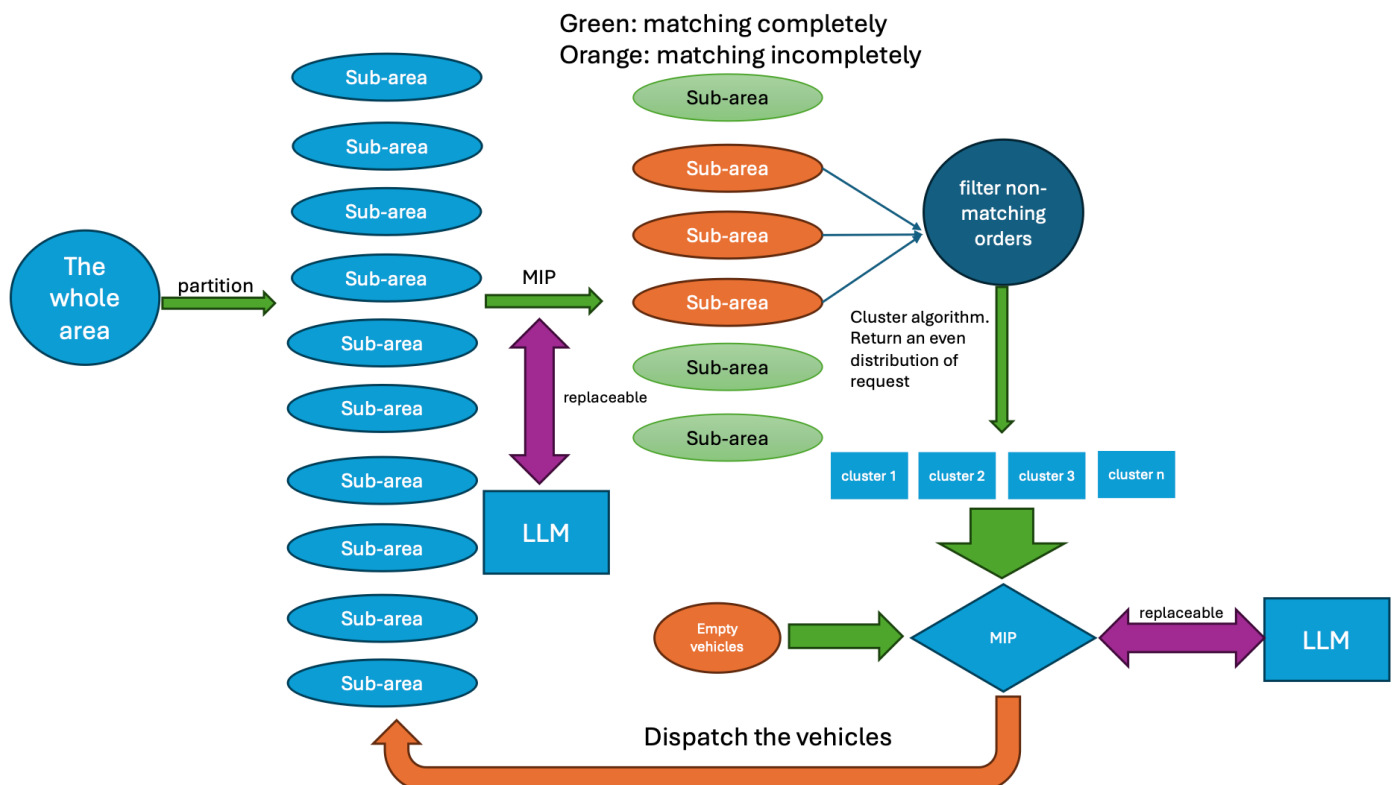
## The simulation process is as follows:

1. initialize all of vehicles in the city, generate the user requests based on the record from 滴滴 / NYC taxi, and init the map of Chengdu/ New York City.
2. in every time range (like 2 seconds), update vehicle's request and current situation (like, current position, if accept request, if accept car sharing, etc.).
3. Calculate the waiting time to pickup for every user request and the money they spend on the trip.

## The rule for vehicle-order:

1. One order is only for one user.
2. One car can accept up to two orders.
3. Three behaviors:
  1. Two people simultaneously hail a ride within a certain time range and decide to share the ride. The car will pickup Person A first, then Person B, and drop off Person A first, followed by Person B.
  2. A person willing to share a ride, Person A, is already in the car and the car has started the trip. Suddenly, a new carpool request from Person B is received. The car will then go from its current location to pickup Person B, drop off Person A, and finally drop off Person B.
  3. A person takes a ride from start to finish without any carpooling.

## The workflow:



## Mathematical Formulation for Dispatch

Since the cost of calculating the distance using dijkstra algorithm is very expensive and unrealistic, we only calculate the manhattan distance when we build the MIP model. In the simulation process, using dijkstra algorithm to simulate car move.

### Objective Function

The objective is to minimize the total Manhattan distance between vehicles and users, considering all segments of the vehicle paths:

$$\min \sum_{i=1}^{n_1} \sum_{j=1}^m x_{ij} \cdot d_{ij} + \sum_{i=1}^{n_1} \sum_{j=1}^m \sum_{k=1, k \neq j}^m y_{ijk} \cdot (d_{ij} + d_{jk}) + \sum_{i=1}^{n_2} \sum_{j=1}^m z_{ij} \cdot d_{ij} \quad (1)$$

where:

- $x_{ij}$  is a decision variable that indicates whether empty car  $i$  is assigned to user  $j$  (1 if yes, 0 if no).
- $y_{ijk}$  is a decision variable that indicates whether empty car  $i$  is assigned to pick up user  $j$  and then user  $k$  (1 if yes, 0 if no).
- $z_{ij}$  is a decision variable that indicates whether car  $i$  with one passenger willing to share is assigned to user  $j$  (1 if yes, 0 if no).
- $d_{ij}$  is the Manhattan distance between vehicle  $i$  and user  $j$ .
- $d_{jk}$  is the Manhattan distance between user  $j$  and user  $k$ .

### Constraints

#### Situation 1

the number of empty vehicles \* 2 + the number of vehicles with one sharing order <= the number of coming order

1. **Each empty car either not assigned to any user or assigned to exactly two users:**

$$\sum_{j=1}^m \sum_{k=1, k \neq j}^m y_{ijk} = 2b_i \quad \forall i = 1, \dots, n_1 \quad (2)$$

2. **Each empty car can be assigned to at most one user directly:**

$$\sum_{j=1}^m x_{ij} \leq 1 \quad \forall i = 1, \dots, n_1 \quad (3)$$

3. **Each car with one passenger willing to share can be assigned to at most one user:**

$$\sum_{j=1}^m z_{ij} \leq 1 \quad \forall i = 1, \dots, n_2 \quad (4)$$

4. Each user can be assigned to at most one vehicle:

$$\sum_{i=1}^{n_1} x_{ij} + \sum_{i=1}^{n_1} \sum_{k=1, k \neq j}^m y_{ijk} + \sum_{i=1}^{n_2} z_{ij} \leq 1 \quad \forall j = 1, \dots, m \quad (5)$$

5. Each empty car must be assigned to at least one user if it is used:

$$\sum_{j=1}^m x_{ij} + \sum_{j=1}^m \sum_{k=1, k \neq j}^m y_{ijk} \geq b_i \quad \forall i = 1, \dots, n_1 \quad (6)$$

6. Linearization Constraints for  $y_{ijk}$ :

$$y_{ijk} \leq x_{ij} \quad \forall i = 1, \dots, n_1, \forall j, k = 1, \dots, m, j \neq k \quad (7)$$

$$y_{ijk} \leq x_{ik} \quad \forall i = 1, \dots, n_1, \forall j, k = 1, \dots, m, j \neq k \quad (8)$$

$$y_{ijk} \geq x_{ij} + x_{ik} - 1 \quad \forall i = 1, \dots, n_1, \forall j, k = 1, \dots, m, j \neq k \quad (9)$$

## situation 2

the number of empty vehicles \* 2 + the number of vehicles with one sharing order > the number of coming order

1. Each empty car either not assigned to any user or assigned to exactly two users:

$$\sum_{j=1}^m \sum_{k=1, k \neq j}^m y_{ijk} = 2b_i \quad \forall i = 1, \dots, n_1 \quad (10)$$

2. Each empty car can be assigned to at most one user directly:

$$\sum_{j=1}^m x_{ij} \leq 1 \quad \forall i = 1, \dots, n_1 \quad (11)$$

3. Each car with one passenger willing to share can be assigned to at most one user:

$$\sum_{j=1}^m z_{ij} \leq 1 \quad \forall i = 1, \dots, n_2 \quad (12)$$

4. Each user must be assigned to exactly one vehicle:

$$\sum_{i=1}^{n_1} x_{ij} + \sum_{i=1}^{n_1} \sum_{k=1, k \neq j}^m y_{ijk} + \sum_{i=1}^{n_2} z_{ij} = 1 \quad \forall j = 1, \dots, m \quad (13)$$

5. Linearization Constraints for  $y_{ijk}$ :

$$y_{ijk} \leq x_{ij} \quad \forall i = 1, \dots, n_1, \forall j, k = 1, \dots, m, j \neq k \quad (14)$$

$$y_{ijk} \leq x_{ik} \quad \forall i = 1, \dots, n_1, \forall j, k = 1, \dots, m, j \neq k \quad (15)$$

$$y_{ijk} \geq x_{ij} + x_{ik} - 1 \quad \forall i = 1, \dots, n_1, \forall j, k = 1, \dots, m, j \neq k \quad (16)$$

## Mathematical Formulation for Unmatching Vehicle-Order.

### Objective Function

The objective is to minimize the total distance from vehicles to high-demand areas:

$$\min \sum_{i=1}^m \sum_{j=1}^n x_{ij} \cdot d_{ij} \quad (17)$$

where:

- $x_{ij}$  is a binary variable indicating whether vehicle  $i$  is assigned to area  $j$  (1 if yes, 0 if no).
- $d_{ij}$  is the Manhattan distance between vehicle  $i$  and area  $j$ .

### Constraints

1. Each car must be assigned to one area:

$$\sum_{j=1}^n x_{ij} = 1 \quad \forall i = 1, \dots, m \quad (18)$$

2. Each area can receive multiple cars, up to its demand:

$$\sum_{i=1}^m x_{ij} \leq q_j \quad \forall j = 1, \dots, n \quad (19)$$

where  $q_j$  is the demand (number of orders) for area  $j$ .