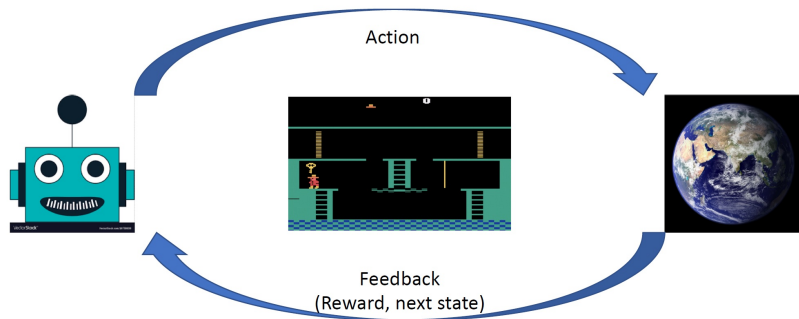


Provably Robust Offline Reinforcement Learning via Smoothed Policy Iteration

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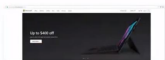
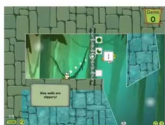
Reinforcement Learning



Learning to Make Good Decisions under Uncertainty

Motivations

- ▶ RL has been used in many *safety-critical* applications (e.g., **medical treatment design**, **autonomous driving**).



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Motivations

- ▶ RL has been used in many *safety-critical* applications (e.g., medical treatment design, autonomous driving).
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Goal: Robust RL against (adversarial) perturbations on state observations.

Problem Setup

- ▶ Typical RL: Observe state s , take action $a \sim \pi(\cdot|s)$, receive reward r and next state s' .
- ▶ This paper: Observe **perturbed** state $s + \mathbf{v}(s)$, take action $a \sim \pi(\cdot|s + \mathbf{v}(s))$, receive reward r and next **perturbed** state $s' + \mathbf{v}(s')$.

($\mathbf{v}(s)$ — arbitrary L^2 bounded perturbation)

Challenge

For any given π , what we actually executed is $\pi \circ \nu$.

Because the observation can be always perturbed like this —



Proposed Method

Policy Iteration + Lipschitz

Why policy iteration (PI)?

PI is the prototype of all policy-based methods, e.g., actor-critic, TRPO, PPO...

Basic framework of PI: At every iteration t , repeat

- (1) Estimate Q^{π_t} .
- (2) Set the greedy policy of Q^{π_t} as π_{t+1} .

Proposed Method

Policy Iteration + Lipschitz

Why Lipschitz?

Lipschitz continuity measures the robustness for supervised learning [Salman et al., 2019; Bubeck et al., 2020].

Open Question:

Can this principle (Lipschitz \Rightarrow robustness) apply to RL? How?

Our Algorithm

Given perturbed batch data $\mathcal{D} = \{s_i + \mathbf{v}(s_i), a_i, s'_i + \mathbf{v}(s'_i), r_i\}_{i=1}^n$, where $r_i \sim \mathcal{R}(s, a)$ and $s' \sim \mathcal{P}(\cdot|s, a)$.

1. Initialize policy as π_1 .
2. For $t = 1, 2, \dots, T$, do
 - 2.1 Estimate Q^{π_t} as f_t using \mathcal{D} , where f_t satisfies i) $\|f_t - Q^{\pi_t \circ \mathbf{v}}\|_\infty = \varepsilon_1$ and, ii) f_t is L -Lipschitz.
 - 2.2 $\pi_{t+1} \leftarrow$ greedy policy of f_t .
3. Output π_{T+1}

Robust Policy Improvement

Theorem (Robust Policy Improvement). Assume (Q, π) pair satisfies:

(i) $\|Q - Q^\pi\|_\infty = \varepsilon_1$.

(ii) Q is L -Lipschitz.

Let $\pi_Q \circ \nu_{\varepsilon_2}$ be the perturbed greedy policy w.r.t. Q and an ε_2 -bounded perturbation ν_{ε_2} , then for any $s \in S$,

$$V^{\pi_Q \circ \nu_{\varepsilon_2}}(s) \geq V^\pi(s) - \frac{2\varepsilon_1 + 2L\varepsilon_2}{1 - \gamma},$$

where $\pi_Q \circ \nu_{\varepsilon_2}(s) = \operatorname{argmax}_{a \in \mathcal{A}} Q(\nu_{\varepsilon_2}(s), a)$.

Performance Bound

Theorem (Performance Bound). Let π_{T+1} be the output of our algorithm. Then,

$$\begin{aligned} & J(\pi^*) - J(\pi_{T+1}) \\ & \leq \gamma^T V_{\max} + \frac{1 - \gamma^T}{1 - \gamma} \left(2L\varepsilon_2 + 2\varepsilon_1 + \frac{\gamma(2\varepsilon_1 + 2L\varepsilon_2)}{1 - \gamma} \right), \end{aligned}$$

where ε_1 is the estimation error in the policy evaluation step ($\varepsilon_1 = \varepsilon_Q + O(n^{-1/2})$ or $\varepsilon_1 = O(n^{-1/d})$ depends on the evaluation algorithm), and ε_2 is the perturbation scale.

Policy Evaluation Subroutine

Why Lipschitz policy evaluation (PE) is doable?

Lipschitz MDP \implies Lipschitz Q-Function [[Asadi et al., 2018](#)]

Options for us:

- (i) Non-parametric policy evaluation over a Lipschitz function class.
- (ii) General function class + randomized smoothing.

(i) Non-Parametric Policy Evaluation over a Lipschitz Function Class

Inspired by [Tang et al., 2020].

Algorithm:

Given batch data $\mathcal{D} = \{(s_i, a_i, r_i, s'_i)\}_{i=1}^n$ (all the states are perturbed by v_{ε_2}) and target policy π .

1. Set $q_{0,i} = r_i - \gamma Ld(s_i, s'_i), \forall i \in [i]$

2. For $t = 1, 2, \dots, T$

$$2.1 \quad Q_t(s, a) = \max_{i \in [n]: a_i = a} (q_{t-1,i} - Ld(s, s_i) - L\varepsilon_2), \quad \forall (s, a) \in \mathcal{S} \times \mathcal{A}$$

$$2.2 \quad q_{t,i} = r_i + \gamma[\widehat{T}^\pi Q_t](s_i, a_i), \quad \forall i \in [i]$$

(i) Non-Parametric Policy Evaluation over a Lipschitz Function Class

Theorem

Let $Q = Q_T$, $T \rightarrow \infty$, we have

$$\|Q_T - Q^\pi\|_\infty \leq \frac{2L(\varepsilon_S + \varepsilon_2)}{1 - \gamma},$$

where $\varepsilon_S := \sup_{(s,a) \in S \times \mathcal{A}} \min_{i \in [n]: a_i = a} d(s, s_i)$, and ε_2 is the perturbation bound.

Open Issue: $\varepsilon_S = O(1/n^{1/d})$ — inevitable **curse of dimensionality** for non-parametric approaches

(ii) General Function Class + Randomized Smoothing

Lipschitz constant is controlled by $O(V_{\max}/\sigma)$.

Theorem. For any function \mathcal{F} , if $f(s, a) \in [0, V_{\max}]$ for any $f \in \mathcal{F}$ and any $(s, a) \in \mathcal{S} \times \mathcal{A}$, then $f \circ \mathcal{N}(0, \sigma I)$ is $\frac{V_{\max}}{\sigma} \sqrt{2/\pi}$ -Lipschitz.

Approximation error scales with $O(\sigma)$.

(ii) General Function Class + Randomized Smoothing

Lipschitz constant is controlled by $O(V_{\max}/\sigma)$.

Approximation error scales with $O(\sigma)$.

Theorem. Let the smoothed function class $\tilde{\mathcal{F}}$ be defined by

$$\tilde{\mathcal{F}} = \{f \circ \mathcal{N}(0, \sigma I); f \in \mathcal{F}\},$$

If $Q^\pi \in \mathcal{F}$, then

$$\min_{\tilde{f} \in \tilde{\mathcal{F}}} \left\| \tilde{f} - \mathcal{T}^\pi \tilde{f} \right\|_\infty \leq L_R \sigma \sqrt{\frac{2}{\pi}} + (L_{\mathcal{P}} + 1) L_Q \sigma \sqrt{\frac{2}{\pi}}.$$

(ii) General Function Class + Randomized Smoothing

Lipschitz constant is controlled by $O(V_{\max}/\sigma)$.

Approximation error scales with $O(\sigma)$.

Theorem^a (informal). Let π_{T+1} be the output of the algorithm and the policy evaluation subroutine uses the randomized smoothed function class.

If $\sigma = \Theta(\sqrt{V_{\max}\varepsilon_2})$ (ε_2 is the perturbation bound), then,

$$J(\pi^*) - J(\pi_{T+1}) \leq O\left(n^{-1/2} + \sqrt{V_{\max}\varepsilon_2}\right).$$

^aproof uncompleted due to limited time

Conclusion

We provides a novel RL algorithm that:

- ▶ Provably robust against against perturbations on state observations.
- ▶ The performance is guaranteed with a finite-sample error bounds.

Messages from our results:

- ▶ The principle of Lipschitz \Rightarrow robustness also applies to RL indeed.
- ▶ Randomized smoothing still works well in RL against perturbations on state observations.

Next Plan

- ▶ Complete the current randomized smoothing part.
- ▶ The theoretical analysis can be improved by $\|\cdot\|_\infty \rightarrow C\|\cdot\|_\mu$ ¹, which is more practical for capturing rich observation with function approximation.
- ▶ Experiments.
- ▶ Extend to policy-gradient based approaches. (future work)

¹ C is the concentrability coefficient for capturing the distribution shift.

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