Towards Optimal Off-Policy Evaluation for Reinforcement Learning with Marginalized Importance Sampling

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Challenges

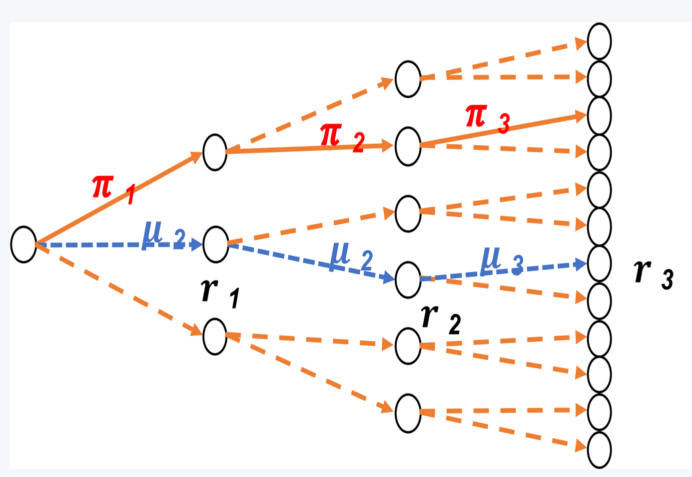
Off-policy evaluation: Evaluating the performance of *target policy* using data sampled by a *behavior policy*.

Importance: Crucial for using reinforcement learning (RL) algorithms responsibly in many real-world applications, e.g., medical treatment and digital marketing.

Challenge: The variance of importance sampling (IS)-based approaches tends to be too high to be useful for long-horizon problems because the variance of the *cumulative product of importance weights* is exploding exponentially.

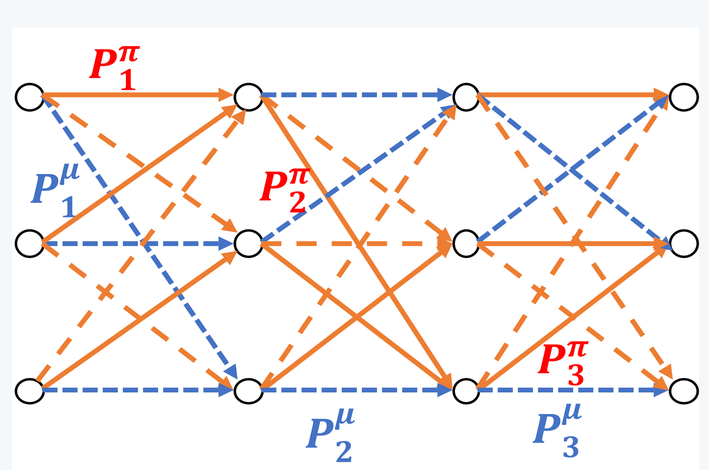
Our Solution

- Cumulative product of importance weights is only necessary without state observability.
- Reduce variance by marginalizing the actions to get the state distribution at every step.



(a) Vanilla IS

$$\hat{v}_{IS}^{\pi} = \frac{1}{n}\sum_{i=1}^{n}\sum_{h=1}^{H}\left[\prod_{t=1}^{h}\frac{\pi(a_t^{(i)}|s_t^{(i)})}{\mu(a_t^{(i)}|s_t^{(i)})}\right] r_h^{(i)}$$



(b) Our MIS

$$\hat{v}_{MIS}^{\pi} = \frac{1}{n}\sum_{i=1}^{n}\sum_{t=1}^{H}\frac{\hat{d}_t^{\pi}(s_t^{(i)})}{\hat{d}_t^{\mu}(s_t^{(i)})}\hat{r}_t^{\pi}(s_t^{(i)})$$

Figure: Vanilla IS versus our MIS. MIS changes mean-of-products to product-of-means.

The marginalized estimators work in the ***space of possible states***, instead of the ***space of trajectories***, resulting in a significant potential for variance reduction.

link: `https://arxiv.org/abs/1906.03393`

Scan QR code to view the paper

Marginalized Importance Sampling (MIS)

Notations: behavior and target policy $\mu_t(a_t|s_t)$ and $\pi_t(a_t|s_t)$, resp.; transition function $T(s_{t+1}|s_t,a_t)$; state distribution $d_t^{\mu}(s_t)$ and $d_t^{\pi}(s_t)$.

Observation: Policy-induced state transitions are temporally independent

$$d_t^{\pi}(s_t) = \sum_{s_{t-1}} P_t^{\pi}(s_t|s_{t-1}) d_{t-1}^{\pi}(s_{t-1}),$$

$$\text{where } P_t^{\pi}(s_t|s_{t-1}) = \sum_{a_{t-1}} T_t(s_t|s_{t-1},a_{t-1})\pi(a_{t-1}|s_{t-1}).$$

Off-policy evaluation with MIS

$$\hat{v}_{\text{MIS}}^{\pi} = \frac{1}{n}\sum_{i=1}^{n}\sum_{t=1}^{H}\frac{\hat{d}_t^{\pi}(s_t^{(i)})}{\hat{d}_t^{\mu}(s_t^{(i)})}\hat{r}_t^{\pi}(s_t^{(i)}),$$

where the functions of the states are estimated as

$$\hat{d}_t^{\mu}(s_t) = \frac{1}{n}\sum_i \mathbf{1}(s_t^{(i)} = s_t); \quad \hat{d}_t^{\pi}(s_t) = \sum_{s_{t-1}} \hat{P}_t^{\pi}(s_t|s_{t-1})\hat{d}_{t-1}^{\pi}(s_{t-1}), \text{ where}$$

$$\hat{P}_t^{\pi}(s_t|s_{t-1}) = \frac{1}{n_{s_{t-1}}}\sum_{i=1}^{n}\frac{\pi(a_{t-1}^{(i)}|s_{t-1})}{\mu(a_{t-1}^{(i)}|s_{t-1})}\mathbf{1}(s_{t-1}^{(i)} = s_{t-1}, s_t = s_t^{(i)});$$

$$\text{and } \hat{r}_t^{\pi}(s_t) = \frac{1}{n_{s_t}}\sum_{i=1}^{n}\frac{\pi(a_t^{(i)}|s_t)}{\mu(a_t^{(i)}|s_t)} r_t^{(i)}\mathbf{1}(s_t^{(i)} = s_t).$$

Theoretical Analysis -- Methodology

Why MIS breaks the exponential dependency on horizon:

- Ergodicity - all states are visited with probability at least $d_t^{\mu} > d_m > 0$.
- Sufficient data $n > O(d_m^{-1})$, so every state is empirically visited with high probability.
- Let $\tau_a\tau_s$ be the max importance weight; $n > O(\tau_a\tau_s)$ controls the variance.

Bellman equation for variance decomposition (Lemma B.3)

- Define $\tilde{d}(s)$, $\tilde{V}(s)$, and $\tilde{r}(s)$ to be "fictitious" tail-clipped estimators;
- Their vector forms include their values on all the states.

$$\text{Bellman equation} \quad V_t^{\pi}(s_t) = r_t^{\pi}(s_t) + \sum_{s_{t+1}} P_t^{\pi}(s_{t+1}|s_t)V_{t+1}^{\pi}(s_{t+1})$$

$$\Rightarrow \quad \mathrm{Var}[\tilde{v}^{\pi}] = \frac{\mathrm{Var}[V_1^{\pi}(s_1^{(1)})]}{n}$$

$$+ \sum_{h=1}^{H}\sum_{s_h}\mathbb{E}\left[\frac{\tilde{d}_h^{\pi}(s_h)^2}{n_{s_h}}\mathbf{1}(E_h)\right]\mathrm{Var}_{\mu}\left[\frac{\pi(a_h^{(1)}|s_h)}{\mu(a_h^{(1)}|s_h)}(V_{h+1}^{\pi}(s_{h+1}^{(1)}) + r_h^{(1)})\middle| s_h^{(1)} = s_h\right]$$

Theoretical Analysis -- Optimality

MIS has optimal sample complexity upto a factor of H (Theorem 4.1)

- Define $\mathcal{P}$ be the projection to feasible policy values.
- Let $\tau_a := \max_{t,s_t,a_t}\frac{\pi(a_t|s_t)}{\mu(a_t|s_t)}$ and $\tau_s := \max_{t,s_t}\frac{d_t^{\pi}(s_t)}{d_t^{\mu}(s_t)}$.
- H is total horizon, σ is observation noise, and $R_{\max}$ is maximal immediate reward.

$$\text{then} \quad \mathbb{E}[(\mathcal{P}\hat{v}_{\text{MIS}}^{\pi} - v^{\pi})^2]$$

$$\leq \frac{1}{n}\sum_{t=1}^{H}\mathbb{E}_{\mu}\left[\frac{d_t^{\pi}(s_t)^2}{d_t^{\mu}(s_t)^2}\mathrm{Var}_{\mu}\left[\frac{\pi_t(a_t|s_t)}{\mu_t(a_t|s_t)}(V_{t+1}^{\pi}(s_{t+1}) + r_t)\middle| s_t\right]\right] + \tilde{O}(n^{-1.5}).$$

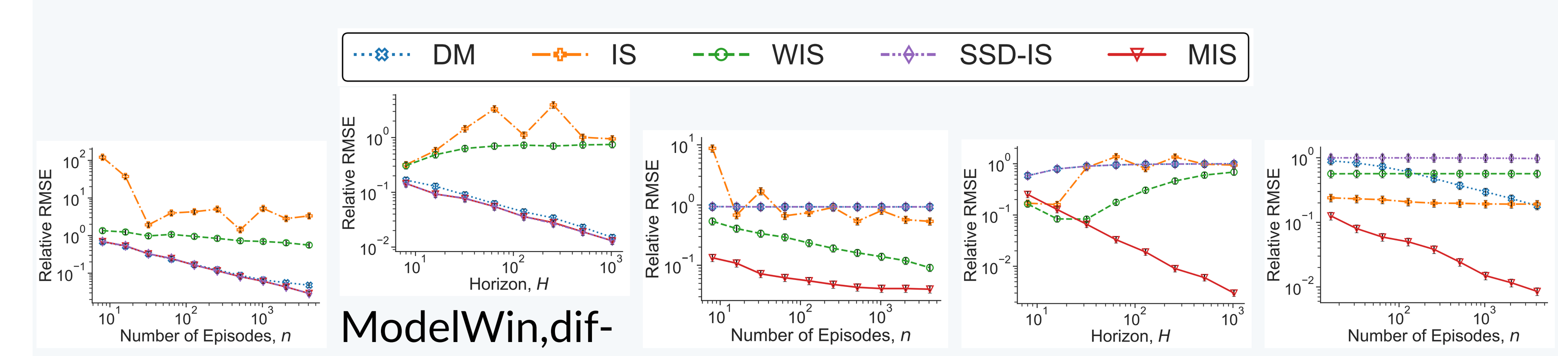
Our paper shows the worst-case bound of our estimator is $\mathbb{E}[(\mathcal{P}\hat{v}_{\text{MIS}}^{\pi} - v^{\pi})^2] \leq \frac{4}{n}\tau_a\tau_s(H\sigma^2 + H^3R_{\max}^2)$, which is optimal upto a factor of H compared with the CR lower bound [Jiang and Li, 2016].

Experimental Study

Tabular MDPs: (common) start with State s_1, choose between Action a_1/a_2, where $\mu(a_1) = 0.5; \pi(a_1) = 0.2$. Random transition to state s_2/s_3. ModelWin and ModelFail MDPs are described as follows:

- ModelWin: rewards decided by the action on the observable state s_1. Set $p = 0.4$.
- ModelFail: actions lead to unobservable states, where rewards are decided; set $p = 1$.

MountainCar: drive back and forth until at top of the hill. State = (position, speed); action=acceleration. Evaluate Q-learning policy π from soft-Q-policy for μ.



ModelWin, different n. ModelWin, different H. ModelFail, different n. ModelFail, different H. MountainCar, different n.

Figure: Relative RMSE error for policy evaluation. MIS matches DM on ModelWin and outperforms IS/WIS on ModelFail and MountainCar, both of which are the best existing methods on their respective domains.

References

Nan Jiang and Lihong Li. Doubly robust off-policy value evaluation for reinforcement learning. In *International Conference on Machine Learning*, pages 652--661, 2016.