Provably Robust Offline Reinforcement Learning via Smoothed Policy Iteration

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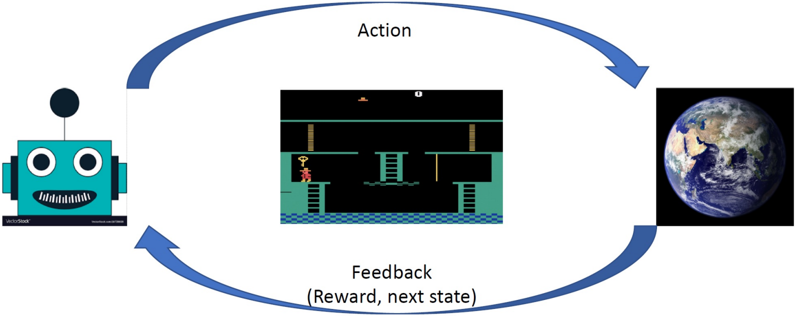
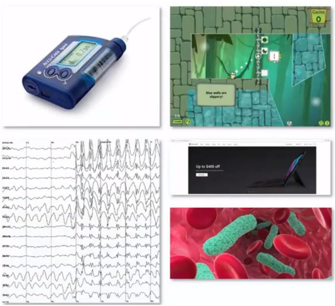
Reinforcement Learning



Sequential Decision-Making under Uncertainty

Motivations

- RL has been used in many *safety-critical* applications (e.g., medical treatment, autonomous driving).



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- RL has been used in many *safety-critical* applications (e.g., medical treatment design, autonomous driving).
- Observation from real-world applications contains unavoidable perturbation
  (e.g., sensor errors or adversarial attack).

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45 MPH Sign

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- RL has been used in many *safety-critical* applications (e.g., medical treatment design, autonomous driving).
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  (e.g., sensor errors or adversarial attack).

Goal: Robust RL against (adversarial) perturbations on state observations.

Problem Setup

- Typical RL: Observe state s, take action $a \sim \pi(\cdot|s)$, receive reward r and next state s'.
- This paper: Observe perturbed state $s+\nu(s)$, take action $a \sim \pi(\cdot|s+\nu(s))$, receive reward r and next perturbed state $s'+\nu(s')$.

($\nu(s)$ — arbitrary L^2 bounded perturbation)

Challenge

For any given π, what we actually execute is $\pi \circ \nu$.

Because the observation can be always perturbed like this —

Proposed Method

Policy Iteration + Lipschitz

Why policy iteration (PI)?

PI is the foundation of all policy-based methods, e.g., actor-critic, TRPO, PPO...

Basic framework of PI: At every iteration t, repeat

(1) Estimate Q^{π_t}.

(2) Set the greedy policy of Q^{π_t} as π_{t+1}.

Proposed Method

Policy Iteration + Lipschitz

Why Lipschitz?

Lipschitz continuity measures the robustness for supervised learning [Salman et al., 2019; Bubeck et al., 2020].

Open Question:

Can the same principle (*Lipschitz* $\Rightarrow$ *robustness*) apply to RL? If so, how?

Our Algorithm

Given perturbed batch data $\mathcal{D} = \{s_i + \nu(s_i), a_i, s_i' + \nu(s_i'), r_i\}_{i=1}^n$, where $r_i \sim \mathcal{R}(s,a)$ and $s' \sim \mathcal{P}(\cdot|s,a)$.

1. Initialize policy as π_1.
2. For $t = 1, 2, \ldots, T$, do
   - 2.1 (*Policy Evaluation*) Estimate Q^{π_t} as f_t using $\mathcal{D}$, where f_t satisfies:
     i) $\|f_t - Q^{\pi_t \circ \nu}\|_\infty = \varepsilon_1$,
     ii) f_t is L-Lipschitz.
   - 2.2 (*Policy Improvement*) $\pi_{t+1} \leftarrow$ greedy policy of f_t.
3. Output π_{T+1}.

Robust Policy Improvement

Theorem (Robust Policy Improvement). Assume (Q, π) pair satisfies:

(i) $\|Q - Q^{\pi}\|_{\infty} = \varepsilon_1$.

(ii) Q is L-Lipschitz.

Let $\pi_Q \circ \nu_{\varepsilon_2}$ be the perturbed greedy policy w.r.t. Q and an ε_2-bounded perturbation ν_{ε_2}, then for any $s \in \mathcal{S}$,

$$V^{\pi_Q \circ \nu_{\varepsilon_2}}(s) \geq V^{\pi}(s) - \frac{2\varepsilon_1 + 2L\varepsilon_2}{1-\gamma},$$

where $\pi_Q \circ \nu_{\varepsilon_2}(s) = \operatorname{argmax}_{a \in \mathcal{A}} Q(\nu_{\varepsilon_2}(s), a)$.

Performance Bound

Theorem (Performance Bound). Let π_{T+1} be the out put of our algorithm. Then,

$$J(\pi^\star) - J(\pi_{T+1}) \\ \leq \gamma^T V_{\max} + \frac{1-\gamma^T}{1-\gamma}\left(2L\varepsilon_2 + 2\varepsilon_1 + \frac{\gamma(2\varepsilon_1 + 2L\varepsilon_2)}{1-\gamma}\right),$$

where ε_1 is the estimation error in the policy evaluation step ($\varepsilon_1 = \varepsilon_Q + O(n^{-1/2})$ or $\varepsilon_1 = O(n^{-1/d})$ depends on the evaluation algorithm), and ε_2 is the perturbation scale.

Policy Evaluation Subroutine

Why is Lipschitz policy evaluation possible?

Lipschitz MDP $\Longrightarrow$ Lipschitz Q-Function [Asadi et al., 2018]

Options for us:

(i) Non-parametric policy evaluation over a Lipschitz function class.

(ii) General function class + randomized smoothing.

(i) Non-Parametric Policy Evaluation over a Lipschitz Function Class

Inspired by [Tang et al., 2020].

Algorithm:

Given batch data $\mathcal{D} = \{(s_i, a_i, r_i, s_i')\}_{i=1}^n$ (all the states are perturbed by ν_{ε_2}) and target policy π.

1. Set $q_{0,i} = r_i - \gamma L d(s_i, s_i'), \forall i \in [i]$
2. For $t = 1, 2, \ldots, T$
   - 2.1 $Q_t(s, a) = \max_{i \in [n]: a_i = a} (q_{t-1,i} - Ld(s, s_i) - L\varepsilon_2), \ \forall (s, a) \in \mathcal{S} \times \mathcal{A}$
   - 2.2 $q_{t,i} = r_i + \gamma [\widehat{T}^\pi Q_t](s_i, a_i), \forall i \in [i]$

Issue 1: Need to know/estimate Lipschitz constant.

(i) Non-Parametric Policy Evaluation over a Lipschitz Function Class

Theorem

Let $Q = Q_T$, $T \to \infty$, *we have*

$$\|Q_T - Q^\pi\|_\infty \leq \frac{2L(\varepsilon_{\mathcal{S}} + \varepsilon_2)}{1-\gamma},$$

where $\varepsilon_{\mathcal{S}} \coloneqq \sup_{(s,a)\in\mathcal{S}\times\mathcal{A}} \min_{i\in[n]:a_i=a} d(s, s_i)$, *and* ε_2 *is the perturbation bound.*

Issue 2: $\varepsilon_{\mathcal{S}} = O(n^{-1/d})$ — curse of dimensionality, inevitable for non-parametric approaches.

(ii) General Function Class + Randomized Smoothing

For any Q-function class $\mathcal{F}$, construct $\widetilde{\mathcal{F}}$ as

$$\widetilde{\mathcal{F}} = \{f \circ \mathcal{N}(0, \sigma I); f \in \mathcal{F}\}, \text{ where}$$
$$(f \circ \mathcal{N}(0, \sigma I))(s, a) := \mathop{\mathbb{E}}_{\epsilon \sim \mathcal{N}(0, \sigma I)} [f(s + \epsilon, a)] .$$

Lipschitz constant is controlled by $O(V_{\max}/\sigma)$.

Theorem. For any function $\mathcal{F}$, if $f(s, a) \in [0, V_{\max}]$ for any $f \in \mathcal{F}$ and any $(s, a) \in \mathcal{S} \times \mathcal{A}$, then $f \circ \mathcal{N}(0, \sigma I)$ is $\frac{V_{\max}}{\sigma}\sqrt{2/\pi}$-Lipschitz.

Approximation error scales with $O(\sigma)$.

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Theorem. If $Q^\pi \in \mathcal{F}$, then

$$\min_{\widetilde{f} \in \widetilde{\mathcal{F}}} \left\| \widetilde{f} - \mathcal{T}^\pi \widetilde{f} \right\|_\infty \leq \sigma L_R \sqrt{\frac{2}{\pi}} + \sigma (L_{\mathcal{P}} + 1) L_Q \sqrt{\frac{2}{\pi}}.$$

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Theorem[a] (informal). Let π_{T+1} be the output of the algorithm and the policy evaluation subroutine uses the randomized smoothed function class.
If $\sigma = \Theta(\sqrt{V_{\max}\varepsilon_2})$ (ε_2 is the perturbation bound), then,

$$J(\pi^\star) - J(\pi_{T+1}) \le O\left(n^{-1/2} + \sqrt{V_{\max}\varepsilon_2}\right).$$

[a] proof uncompleted due to limited time

Conclusion

We provide a novel RL algorithm that:

- Provably robust against perturbations on state observations.
- The performance is guaranteed with a finite-sample error bounds.

Messages from our results:

- The principle of *Lipschitz* $\Rightarrow$ *robustness* also applies to RL indeed.
- Randomized smoothing still works well in RL against perturbations on state observations.

Future Directions

Next plan for this project:

- Complete the proof of current randomized smoothing part.
- The theoretical analysis can be further improved by $\|\cdot\|_\infty \to C\|\cdot\|_{2,\mu}$[1], which is more suitable for capturing rich observation with function approximation.
- Experiments.

Future work:

- Policy-gradient based approaches.
- Online setting with exploration.

[1] C is the concentrability coefficient for capturing the distribution shift.

Reference I

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