PHY 18L Lab 1: Photoelectric Effect and Plank's Constant

S. Yu

Professor: Huang Huan

TA: Changling Zhao

(Dated: January 7, 2018)

Abstract. In our experiment, we investigate the relationship between frequency of light and the stopping potential of photoelectrons in photoelectric effect, using light spectra from two separate sources: sodium and mercury vapors. This relationshipallows us to experimentally measure fundamental constant h/e, where h is the plank constant and e the electron charge. Our precision measurement using mercury vapor light yields $h/e = (3.9574 \pm 0.1203) \times 10^{-15}$V/s, as well as the work function of metal to be $W_o/e = -1.3432 \pm 0.0811$V. Comparing to the NIST value [3], our measurement has a 4.3% error.

I. INTRODUCTION AND EXPERIMENTAL OBJECTIVES

The photoelectric effect proposed by Albert Einstein 1905 and later experimentally confirmed by Robert Millikan in 1924 contributed directly and significantly to the development of the quantization of light, and, eventually, quantum mechanics. In the photoelectric effect, photons incident on metals and, if the photon's energy is high enough to overcome the potential (known as the work function) that binds the electrons to the metal's atoms, the electrons will be liberated. The photoelectric experiment brings out the then contrasting physical interpretations of the energy of EM radiation. In a classical view, the instantaneous energy flux of EM radiation given by the Poynting vector is continuous, whereas the quantizaiton of light tells us that in the quantum regime, there must be a threshold frequency (or energy) f_s for photoelectrons emission. Therefore, once a single photon with frequency $f > f_s$ hits the metal, a photoelectron will come off without delay. The kinetic energy of this photoelectron is the photon energy hf (where h is the Plank's constant) minus the work function W_o:

$$\frac{1}{2}m_e v^2 = hf - W_o \tag{1}$$

One can see that the higher intensity of incident photons hitting the metal, the more emitted photoelectrons, and thus producing a higher photoelectric current. However, the intensity of incident light does not affeact the kinetic energy of the emitted photoelectrons, as their energy is only dependent on the energy (or frequency) spectrum of the light as shown in Eqn 1.

The produced photoelectric current I is extremely small to be measured precisely in typical laboratory setup, so we will measure instead, the stopping potential V_s using an op-amp. The stopping voltage is produced by sending the photoelectrons through a photodiode, where by applying a reverse voltage to the anode, one can stop the photoelectrons from reaching the anode, and the minimum voltage needed to reverse the photoelectrons is our "stopping voltage" V_s. This stopping voltage is roughly proportional (see Sect. II and Eqn 5) of photoelectrons.

The stopping potential of one electron, hence, is eV_s. Equating this with the kinetic energy of photoelectrons in Eqn (1):

$$eV_s = \frac{1}{2}m_e v^2 = hf - W_o \tag{2}$$

$$\Rightarrow V_s = \frac{h}{e}f - \frac{W_o}{e} \tag{3}$$

Specifically, in this lab, we investigate the relationship between frequency of light and the stopping potential as expressed in Eqn (3) by using the light spectra from mercury and sodium vapors to vary the frequency of incident light (f) and measure the corresponding stopping potential (V_s). According the Eqn (3), we expect the plot of frequencies vs. measured stopping voltages will yield linear relation with slope = $\frac{h}{e}$.

II. EXPERIMENTAL SETUP AND PROCEDURE

A. The light sources and diffraction grating

In the experiment, we use two sources of monochromatic light with each glass tube containing sodium or mercury vapor sources. In order to obtain different frequencies of light, we use a blazed diffraction grating with ruling spacing of $d \approx 1.67 \times 10^{-6}$m to resolve the spectral lines, as different frequencies of light will be diffracted at different angles according to Eqn 4:

$$d\sin(\theta) = m\lambda \tag{4}$$

where d is the slit spacing, θ is the diffraction angle, m is the order number of the spectrum, and λ the wavelength of light.

The two wavelengths in sodium's spectrum from diffraction grating are 588.995nm and 589.294nm. The wavelengths in mercury's resolved spectrum used in our experiment for stopping voltage measurements are shown in Table 1.

To investigate how the intensity of the incident photons affects the measured stopping voltage (or the kinetic

	Wavelength (nm)
Ultraviolet (UV)	365.0158
Violet	404.6565
Blue	433.922
Green	546.0700
Yellow	579.067

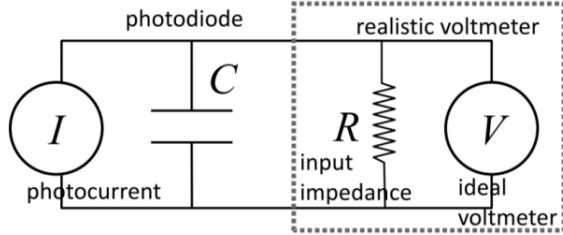TABLE 1.   Wavelengths of the brightest spectral lines of mercury vapor used in the experiment.

FIG. 1.   Measuring rise time of the RC circuit

FIG. 2. The full scheme of setup used in the experiment, with $\frac{h}{e}$ photodiode apparatus and light source box.

energy of the emitted photoelectrons), we use a neutral density filter that allows us to vary the relative transmission percentage of incident light by 20%, 40%, 60%, 80%.

B.   The $\frac{h}{e}$ photodiode Apparatus

The cathode of photodiode has a clean metal surface on which the incoming light hits to liberate photoelectrons and produce a photoelectric current. The internal circuit of the photodiode apparatus is shown in Figure 1, and taking into account the small capacitance C associated with the photodiode and input impedance R of the op-amp, the photoelectric current I is related to the stopping voltage V_s by:

$$V(t) = \frac{IR}{1 + IR/Vs}(1 - e^{\frac{-t}{RC}(1+IR/Vs)}) \qquad (5)$$

We see that their relationship is not directly proportional, and so we expect to see some non-linear behaviors in our measurements.

C.   Experimental Procedure and Method

To direct the light beam into the photodiode, we mount a lens in front of the light source and move its position until a sharpest image is formed on the photodiode detector, as illustrated in Figure 2. A slit aperture is mounted over an orifice in the front cover of the photodiode apparatus box, and we align the box such that only the desired spectral line passes through and hits the photodiode detector. Misalignment of the light with the photodiode
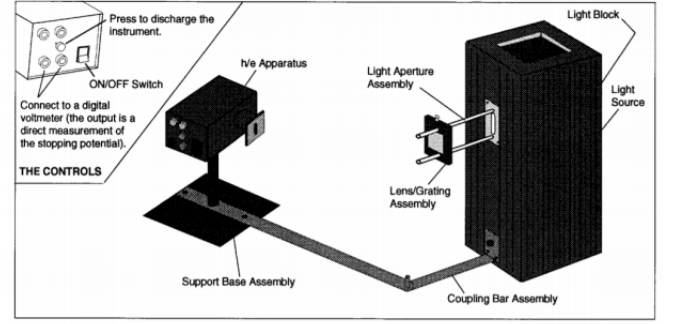
will result in shifted and unstable stopping voltages as we later observed in our first lab session. Color filters are also used to further minimize unwanted lines scattering into photodiode.

With the light sources and photodiode assembled and properly aligned, we check the battery of the $\frac{h}{e}$ photodiode apparatus box to be $\pm 6V$.

1.   Measuring Na vapor light's stopping voltages

The sodium vapor light gives off a visible yellow light, and we use the diffraction grating to resolve the sodium light's spectrum. We measure the stopping voltage of the first two diffraction orders light by recording the stable voltage value given by the $\frac{h}{e}$ apparatus, and we repeat the measurement after adding a yellow filter. Note that after each measurement, we discharge the built-up charges on the cathode's metal plate by using the "Push to Zero" feature on the photodiode apparatus. We first measure the stopping voltage of the first and second order of the main sodium lines on both sides of the diffraction grating, with and without a yellow filter. The eight measurements are shown in Table 2. Next, we vary the transmission percentage of the incident sodium vapor light by using a neutral density filter, and measure the time needed to reach 90% of the corresponding stopping voltages of the first two orders on both sides of grating. The stopping voltages are then plotted against the charging times in Figure 3.

2.   Measuring Hg vapor light's stopping voltages

We measure the stopping voltages for the five first-order lines (see Table 1) on the dominate side of the diffraction grating. The results are shown in Table 5. Plotting the stopping voltages against frequency of incident photons, the slope obtained from linear fit corresponds to h/e and the y-intercept corresponds to W_o/e,

according to Eqn (4).

3. Precision measurement of $\frac{h}{e}$

Finally, we perform a more precise measurement of h/e using the dependence of stopping potential on frequency of incident light. For each spectral line, we use the appropriate color filter to block out unwanted wavelengths, and measure the voltages for as many visible lines.

III. MEASUREMENT DATA AND ANALYSIS

The measured ambient noise on the stopping potential values is 0.05 ± 0.01mV. In addition, during data taking, we observed that the desk lamp radiating yellow light from the neighboring stations would make our voltage reading noticeably less stable.

A. Sodium vapor light's stopping voltages

The data of the final stopping voltage using first two orders of sodium lines on both sides of grating, $m = \pm 1, \pm 2$, with and without using a yellow filter is shown in Table 2. The standard deviation of the measurements is 0.0548, which is in agreement with the measured ambient light level and also indicates the presence of systematic errors such as finite impedance of amplifier.

Taking the two $m = -1, -2$ measurements with yellow filter as our best data for sodium, we plot the stopping voltage against frequency of lines in Figure 4. The slope and y-intercept obtained from linear regression are 4.3918×10^{-13} and -223.0932, respectively. Since it is a two-point fit, there is not error on the slope.

	m=1 (V)	m=-1 (V)	m=2 (V)	m=-2 (V)
No filter	0.6490 ± 0.0020	0.6200 ± 0.0030	0.5775 ± 0.0020	0.5140 ± 0.0030
With filter	0.6446 ± 0.0020	0.6020 ± 0.0002	0.5635 ± 0.0020	0.4885 ± 0.0030

TABLE 2. The eight measurements of stopping voltages of the first two orders of $m = \pm 1, \pm 2$ sodium light with and without yellow filter. The standard deviation is 0.0548.

We then measure for m= -1 and -2 the charging times of h/e apparatus to 90% of final stopping voltage at each transmission intensity level. The results are shown in Table 3 and Table 4 for m=-1 and -2, respectively. During data taking, for certain transmission levels, the measured stopping potential never reach 90% of the full potential, so in such cases we record, instead, the time for the voltage reading to become stable which is longer than the charging time. The 90% stopping voltages as a function of charging times are plotted in Figure 3, which shows a
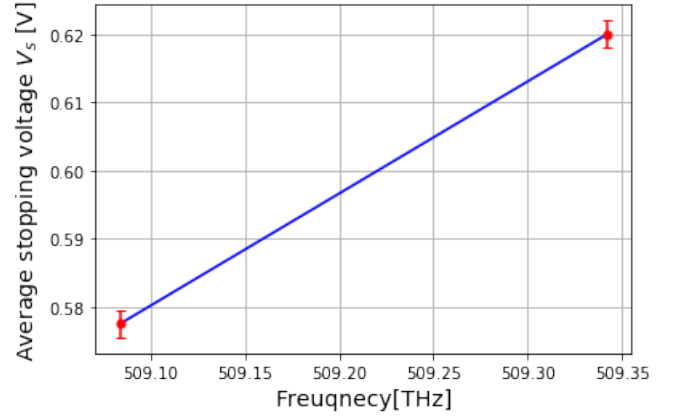

FIG. 3. Dependence of stopping voltage on frequency of incident sodium light. The slope is 4.3918×10^{-13} V/s and y-intercept is -223.0932 V.

non-linear trend with a saturation voltage of 0.6V. One possible explanation is the fact that the capacitance is connected to ground and there is leakage of charge to ground. Another possible cause is that if we observe Eqn (5), we see that the voltage reading V is not exactly linearly proportional to the stopping voltage.

Intensity	Full Stopping Voltage (V)	90% Time (sec)
20	0.5420 ± 0.015	10.21
40	0.5850 ± 0.015	7.05
60	0.6000 ± 0.010	6.30
80	0.5970 ± 0.010	5.80
100	0.6070 ± 0.010	2.41

TABLE 3. The eight measurements of stopping voltages of $m = -1$ order of sodium light with and without yellow filter.

Intensity	Full Stopping Voltage (V)	90% Time (sec)
20	0.38100 ± 0.013	14.50
40	0.43400 ± 0.015	17.01
60	0.46037 ± 0.015	10.29
80	0.48000 ± 0.013	9.64
100	0.51400 ± 0.015	5.04

TABLE 4. The eight measurements of stopping voltages of $m = -2$ order of sodium light with and without yellow filter

B. Mercury vapor light's stopping voltages

The stopping voltages for the five first-order lines $m = 1$ of mercury lamp are shown in Table 5, and each measurement is repeated three times. We take the average of the three voltage measurements for each frequency and plot the stopping voltages against frequency in Figure 5. The linear regression analysis yields the slope of
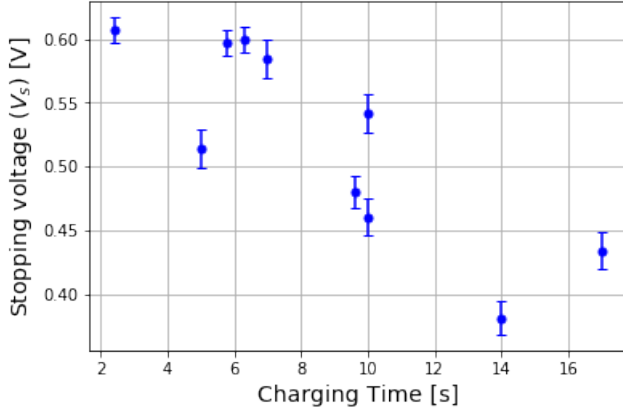
3

FIG. 4. Dependence of charging time (to 90% final stopping voltage) on relative intensity of incident light.
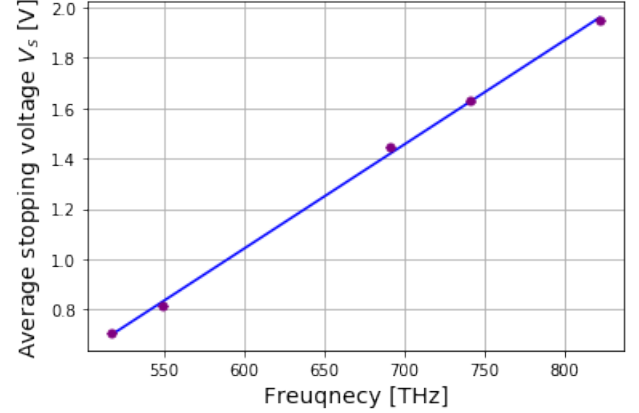

FIG. 5. Dependence of stopping voltage on frequency of incident Mercury light. The slope is $(4.1361 \pm 0.1243) \times 10^{-13}$ V/s and y-intercept is -1.4390 ± 0.0837V.

the plot to be $(4.1361 \pm 0.1243 \times 10^{-15}$ and the y-intercept -1.4390 ± 0.0838, where the covariance matrix is used to calculate the respective errors.

Comparing to the literature value $h/e = 4.1357 \times 10^{-15}$ V/s given by the National Institute of Standards and Technology [2], our derived h/e has $< 0.01\%$ percent error, which is significantly small. The sources of error might both systematic and random, including voltmeter calibration and offset, and background light scattering.

The work function W_o per eV derived from Fig. 5 is -1.439 ± 0.0838. Looking up the work function table of metals in literature [3], our value is closest the work function of pure cesium metal $W_{Cs} = 1.95$. Therefore, our measurements suggest the metal used in the h/e photodiode is of cesium alloy, which is in agreement with the PASCO h/e apparatus' document that the metal used is C_{S_3}Sb [1].

Repeat	Yellow	Green	Blue	Violet	UV
#1	0.7080 ± 0.0005	0.8150 ± 0.0003	1.4428 ± 0.0003	1.6280 ± 0.0002	1.9500 ± 0.0003
#2	0.7078 ± 0.0003	0.8142 ± 0.0002	1.4427 ± 0.0003	1.6280 ± 0.0002	1.9500 ± 0.0005
#3	0.7078 ± 0.0005	0.8138 ± 0.0003	1.4428 ± 0.0003	1.6280 ± 0.0002	1.9495 ± 0.0005

TABLE 5. Stopping voltages for the five dominant 1st-order lines $m = 1, 2, 3, 4, 5$ of mercury vapor light, with each voltage measurement repeated three times.

C. Precision measurement of h/e

According to Eqn (4), the 3rd order diffraction angle of UV light (365nm) is $\theta = \sin^{-1}(\frac{3 \times 365^1 0^{-9}}{1.67 \times 10^{-6}}) = 40.97$ degrees. The 2nd order diffraction angle of yellow light (578nm) is $\theta = \sin^{-1}(\frac{2 \times 578^1 0^{-9}}{1.67 \times 10^{-6}}) = 43.81$ degrees. The

calculated results are in good agreement with our observations in the experiment. Proceeding to the precision measurement of h/e, we take 4 measurements (corresponding to $m = \pm 1, \pm 2$) of stopping voltages for each lines in the spectrum of mercury vapor light (see Table 1), amounting to 20 measurements in total. The stopping voltages are plotting against the frequencies in Figure 6. The slope obtained from linear regression is $h/e = (3.9272 \pm 0.2310) \times 10^{-15}$V/s, and the y-intercept $W_o/e = -1.3454 \pm 0.155$V, with the errors derived from the covariance matrix. Looking at
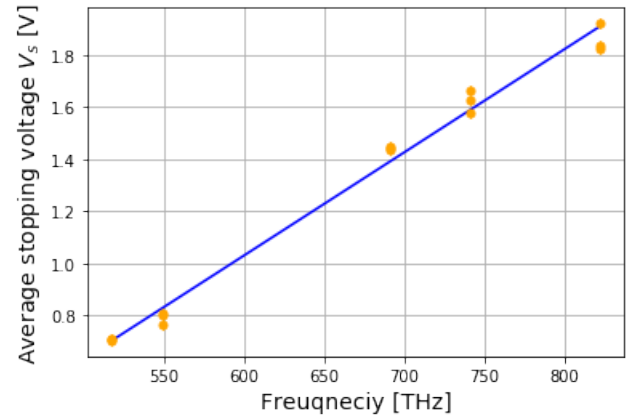

FIG. 6. Precision measurement of h/e using the dependence of stopping voltage on frequency of incident mercury light. The slope is $(3.9574 \pm 0.1203) \times 10^{-15}$ V/s and y-intercept is -1.3432 ± 0.0811V.

the measurements and our log book, we post-select and throw out some data points that we noted to be highly fluctuated by the background during data-taking and those are were taken with incomplete discharge of the h/e apparatus (random errors). This reduces our sample size from the original 20 measurements in Figure

6 to 16 total measurements in Figure 7, with the final $h/e = (3.9574 \pm 0.1203) \times 10^{-15}$V/s, and the y-intercept $W_o/e = -1.3432 \pm 0.0811$, yielding $\delta\frac{h}{e} = 4.3\%$.
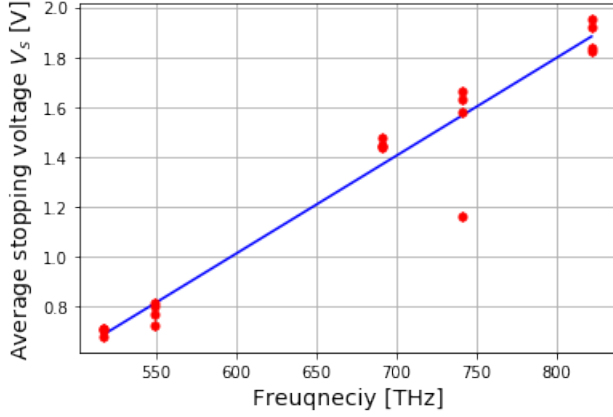

FIG. 7. Precision measurement of h/e using the dependence of stopping voltage on frequency of incident mercury light. The slope is $(3.9272 \pm 0.2310) \times 10^{-15}$ V/s and y-intercept is -1.3454 ± 0.1557V.

IV. CONCLUSION AND OUTLOOK

We experimentally determined the fundamental constant h/e, where h is the plank constant and e the electron charge, by measuring stopping potentials of electron in C_{S_3}Sb metal using the resolved spectrum of mercury vapor lamp. Our precision measurement yields $h/e = (3.9574 \pm 0.1203) \times 10^{-15}$V/s, as well as the work function of $W_o/e = -1.3432 \pm 0.0811$V. Comparing to the NIST value [3], our measurement has a 4.3% error. The uncertainties in measurements result from both random errors of background scattering, and systematic errors of incomplete filter of wavelengths, finite impedance of amplifier and leakage of charge in the capacitance in the photodiode. Therefore, better filtering and a more stable environment can be implemented to improve our precision.

References
[1] UCLA Physics 18L Manual.
[2] National Institute of Standards and Technology (NIST), *Fundamental Physical Constants–Extensive Listing.*
[3]National Physics Laboratory, *Kaye and Laby Online Library.*