

PHYSICS 18L Lab 4: Spectroscopy and Rydberg Constant using Atomic Gaslight

S. Yu

Lab Partner: Mihai Bibireata

Professor: H. Huang

TA: P. Chin (Section B)

University of California, Los Angeles

(Dated: January 13, 2018)

Abstract. In this lab, we experimentally measure the diffraction angles of the 0th, 1st and 2nd orders of sodium vapor light, and using these measurements to determine the slit width used in this lab to be $d = (1.694 \pm 0.426) \times 10^{-6} \text{m}$. We also experimentally obtain the Rydberg constant from diffraction angle measurements on Hydrogen vapor light to be $R_H = (10.715 \pm 0.059) \mu\text{m}^{-1}$.

I. INTRODUCTION AND OBJECTIVES OF EXPERIMENT

One of the fundamental postulates of quantum mechanics is the quantization of energy as expressed in the following relationship:

$$E = hf \quad (1)$$

The quantized atomic transitions along with the produced emission/absorption spectrum provide an important experimental confirmation for this postulate. In 1884, Balmer imperially derived a formula to predict four visible spectrum lines of hydrogen atom, and now this is known as the Balmer series:

$$\lambda_n = 364.6 \frac{n^2}{n^2 - 2^2} \times 10^{-9} \text{ [meters]} \quad (2)$$

where n is any positive integer greater than 3, and $\lambda = c/f$ is the wavelength of photons. In 1913 Niels Bohr further applied quantization of energy model to the electron orbitals in hydrogen, generalizing Eq(2) to :

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_2^2} - \frac{1}{n_i^2} \right) \quad (3)$$

where n_i is any positive integer greater than 3, R is a proportionality constant called the “Rydberg” constant, and Z is the atomic number of the atom.

The objective of this lab is to experimentally determine the Rydberg constant R_H of hydrogen by measuring the wavelength λ of each Balmer lines seen in hydrogen vapor light spectrum, and comparing the measured R_H to the theoretically calculated value.

To resolve the spectrum of hydrogen, we will use a narrow slit and a diffraction grating. We will first determine the slit width d by measuring the diffraction angles of the first and second orders of sodium light with a spectroscope, and using the single-slit relation:

$$d \sin \theta = m\lambda \quad (4)$$

Given the slit width d and diffraction angle measurements of hydrogen lines, we can calculate the wavelength corresponding to each line, which allows us to find R_H using Eq (3).

II. EXPERIMENTAL SET-UP AND PROCEDURE

A. Spectrum of sodium

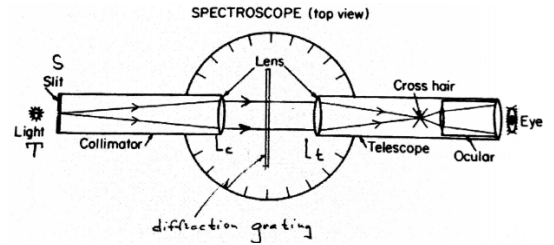


FIG. 1. Sketch diagram of the spectroscope used to measure the diffraction angle of a light source through a diffraction grating.

We tune and align the collimator and telescope on the spectrometer to be collinear, see Figure 1. The slit width is initially set to $\frac{1}{2} \text{mm}$, and the eyepiece focus is adjusted until a sharp image of the slit is formed on the cross hair of the eyepiece. We then zero the angle reading on the spectrometer’s scale. A diffraction grating is installed such that it is perpendicular to the collimator-telescope axis, and a sodium gas tube is put in front of the collimator such that the light source, the slit, the telescope are collinear.

We measure the angle of $m = 1$ straight-through spectrum and $m = \pm 1$ spectra on both sides, respectively. We fine-tune to narrow the slit width until we have a better resolved spectrum and observe a clear doublet in the diffracted light, which correspond to $\lambda_1 = 588.995 \text{nm}$ and $\lambda_2 = 589.592 \text{nm}$, respectively. We measure the $m = \pm 2$ doublet spectra on both sides. We take the average of the two 2nd-order angles measured for λ_2 , and use Eqn (4) to calculate the slit width d .

B. Spectra of mystery gases

Replacing the sodium tube with an “unknown” gas tube, we first observe the spectrum features of the gas

through a hand-held spectrometer, and then use the spectroscope to measure the angles of brightest lines. We repeat this for two “unknown” tubes in total and use the Eq (4) to calculate the wavelengths of the lines, and then compare our results with the wall charts in lab to identify the gases.

C. Spectrum of hydrogen

Switching to hydrogen gas tube, we observe and measure the first and second order spectral lines of the hydrogen vapor on both sides. The hydrogen diffracted light is known to have three bright lines, known as the Balmer series: $H_\alpha(n_3 \rightarrow n_2)$, $H_\beta(n_4 \rightarrow n_2)$, $H_\gamma(n_5 \rightarrow n_2)$, and a weak fourth line in blue region $H_\delta(n_6 \rightarrow n_2)$. By measuring the angles and calculating the corresponding wavelength using Eq (4), we can obtain an experimentally-derived Rydberg constant according to Eq (3) by plotting $\frac{1}{\lambda}$ vs. $(\frac{1}{n^2} - \frac{1}{n_i^2})$ for $i = 3, 4, 5, 6$, and obtaining the slope.

III. MEASUREMENT DATA AND ANALYSIS

A. Spectrum of sodium

Table I shows our measurements of the diffracted angle for $m = 0, \pm 1, \pm 2$ sodium spectra. The angles for $m = -2$ are subtracted from 360° . Taking the average of the two $m = \pm 2$ angles measured for $\lambda_2 = 589.592\text{nm}$ doublet, we calculate the slit width to be

$$d = \frac{m\lambda}{\sin(\theta)} = \frac{2 \times 589.592\text{nm}}{\sin(\frac{43.583+43.683}{2})} = 1.696 \times 10^{-6} \text{ m} \quad (5)$$

The error on the slit width derived from uncertainty propagation is $\delta d = \pm 0.424 \times 10^{-6}\text{m}$. Similar calculation using the measured $\lambda_1 = 588.995\text{nm}$ doublet yields slit width to be $d = (1.694 \pm 0.426) \times 10^{-6}\text{m}$. Our results for d is very close to the manufacturer’s specification of $590 \text{ lines/mm} = 1.695 \times 10^{-6}\text{m}$, so using either values would not be not significantly different. The systematic errors involved in this measurement may be that the vernier scale on the spectrometer is slightly off, or the zero of the vernier scale is not centered at the zero-th order.

B. Spectra of mystery gases

We replace the sodium gas tube with the first “unknown” tube and use the hand-held spectrometer to measure the wavelength of the brightest visible lines. The spectroscope is then used to measure the angles of the lines more precisely. Table II shows the observed wavelengths as read off from the spectrometer and the angle measurements of the first tube.

Order m	Angle θ (deg)
0	20.500 ± 0.025
1	33.250 ± 0.025
-1	44.517 ± 0.025
2 (λ_1)	44.483 ± 0.025
2 (λ_2)	44.517 ± 0.025
-2 (λ_1)	43.583 ± 0.025
-2 (λ_2)	43.683 ± 0.025

TABLE I. Measured angles of the $m = 0, \pm 1, \pm 2$ spectra of diffracted sodium light.

Referencing the wall charts in lab and literature [2], we find that our wavelength measurements of the first tube agree well with the Hydrogen spectral lines: $H_\alpha = 656.3\text{nm}$, $H_\beta = 486.1\text{nm}$, $H_\gamma = 434.1\text{nm}$, thus we identify our first “unknown” gas to be hydrogen.

Spectrometer λ (nm)	Angle θ (deg)	Calculated λ (nm)
635 ± 5	22.917 ± 0.025	660.374 ± 0.711
485 ± 5	16.717 ± 0.025	487.810 ± 0.729
435 ± 5	14.950 ± 0.025	437.504 ± 0.733

TABLE II. Measured angles of the brightest first-order spectral lines seen in diffracted light of the first “unknown” tube.

We observe many closely-spaced lines in the spectrum of the second “unknown tube”, which almost looks continuum, so we pick the brightest five lines and measure the angles of each of them. Table III shows the angle measurements for spectral lines of the second tube. Comparing our measured spectrum and wavelengths with different gas spectra, we find that our measurements of the second tube agree well with the absorption/emission spectrum of the carbon dioxide gas [3].

Color	Angle θ (deg)	Calculated λ (nm)
Purple	15.483 ± 0.025	452.481 ± 0.732
Cyan	16.683 ± 0.025	486.590 ± 0.729
Green	18.000 ± 0.025	523.769 ± 0.725
Yellow	19.483 ± 0.025	565.322 ± 0.721
Red	21.183 ± 0.025	612.477 ± 0.716

TABLE III. Measured angles of the brightest first-order spectral lines seen in diffracted light of the second “unknown” tube.

C. Spectrum of hydrogen

We measure the diffraction angles of brightest 3 lines for $m = \pm 1, \pm 2$ spectra in hydrogen vapor light on both sides. The fourth line H_δ is too weak that we only observe it in the $m = 1$ diffraction. Note that narrower slit width would make the lines dimmer and finer but resolve the lines better, thus providing higher resolution and accuracy of the angle measurement. The angle measurements for all the Balmer series line we observe are shown in Ta-

ble IV, along with the wavelengths corresponding to the angle calculated using Eq (4).

Order m	Line	Angle θ (deg)	Experimental λ (nm)
1	α	23.000 ± 0.025	621.202 ± 0.710
	β	16.717 ± 0.025	487.534 ± 0.729
	γ	14.850 ± 0.025	434.875 ± 0.733
	δ	14.033 ± 0.025	411.002 ± 0.735
-1	α	22.683 ± 0.025	653.927 ± 0.711
	β	17.567 ± 0.025	483.584 ± 0.729
	γ	14.783 ± 0.025	432.795 ± 0.733
	δ	Not Observed	/
2	α	51.667 ± 0.025	661.709 ± 0.562
	β	35.133 ± 0.025	486.901 ± 0.659
	γ	Not Observed	/
	δ	Not Observed	/
-2	α	50.750 ± 0.025	656.383 ± 0.566
	β	34.833 ± 0.025	484.201 ± 0.660
	γ	30.700 ± 0.025	432.809 ± 0.680
	δ	Not Observed	/

TABLE IV. Measured angles of the $m = \pm 1, \pm 2$ spectra of hydrogen gas, displayed with the wavelength of each line calculated from the angles.

Table V shows the theoretically calculated wavelength for each of the Balmer series using $R_\infty = 10.974 \mu\text{m}^{-1}$ as given by the lab manual [1], which are also compared against the experimentally obtained wavelengths in Table IV to obtain the percent errors. The theoretical calculations of λ are all within the uncertainty of the experimental values, and the percent errors are all under 1%, allowing us to unambiguously identify each of the observed lines corresponds to a Balmer series line

According to Eqn (3), the Rydberg constant can be derived from the slope of $\frac{1}{\lambda}$ plotted against $(\frac{1}{n^2} - \frac{1}{n_i^2})$ for $i = 3, 4, 5, 6$. In Figure 2 we plot each of the experimental λ in Table IV, average the wavelength measurements for each of the Balmer line: $H_\alpha, H_\beta, H_\gamma$ and H_δ , and then perform a linear fit to the averaged wavelengths. The slope obtained in Figure 2 is $R_H = (10.715 \pm 0.059) \mu\text{m}^{-1}$. Comparing to $R_\infty = 10.974 \mu\text{m}^{-1}$ in the lab manual [1], the upperbound of our experimental value of R_H has a percent error of 0.0018%. This is very small and further justified by the fact that the R_∞ value assumes infinite nucleus mass, which is not physically true, and thus must include a correction term.

IV. CONCLUSION AND OUTLOOK

We experimentally measure the diffraction angles of the 0th, 1st and 2nd orders of sodium vapor light, and using these measurements to determine the slit width used in this lab to be $d = (1.694 \pm 0.426) \times 10^{-6}\text{m}$. We further identify two “unknown” gases by measuring

the wavelengths of their spectra with a hand-held spectrometer, as well as the diffraction angles of the brightest observable lines using a spectroscope. Finally we experimentally obtain the Rydberg constant from

Order m	Line	Theoretical λ (nm)	% err of λ
1	α ($n_{3 \rightarrow 2}$)	656.112	0.939%
	β ($n_{4 \rightarrow 2}$)	486.009	0.314%
	γ ($n_{5 \rightarrow 2}$)	433.937	0.106%
	δ ($n_{6 \rightarrow 2}$)	410.070	0.227%
-1	α ($n_{3 \rightarrow 2}$)	656.112	0.333%
	β ($n_{4 \rightarrow 2}$)	486.009	0.499%
	γ ($n_{5 \rightarrow 2}$)	433.937	0.263%
2	α ($n_{3 \rightarrow 2}$)	656.112	0.853%
	β ($n_{4 \rightarrow 2}$)	486.009	0.183%
-2	α ($n_{3 \rightarrow 2}$)	656.112	0.041%
	β ($n_{4 \rightarrow 2}$)	486.009	0.372%
	γ ($n_{5 \rightarrow 2}$)	433.937	0.260%

TABLE V. Theoretical wavelength of each of the four Balmer series lines calculated using $R_\infty = 10.974 \mu\text{m}^{-1}$ [1]. The percent error is calculated using experimental values of λ from Table IV and the corresponding theoretical values in Table V.

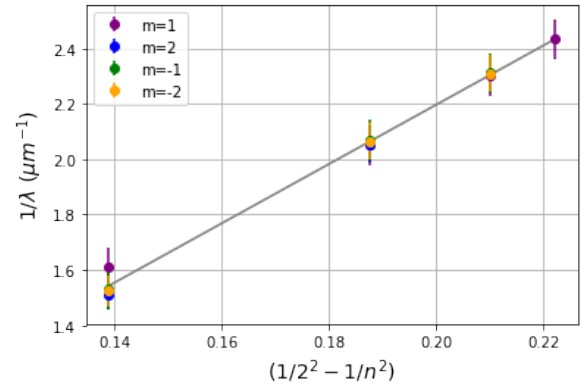


FIG. 2. The inverse of wavelength values in Table IV are plotted as a function of $(\frac{1}{n^2} - \frac{1}{n_i^2})$ for $i = 3, 4, 5, 6$. The average values of the Balmer lines are used to obtain a linear regression, which yields a slope of $R_H = (10.715 \pm 0.059) \mu\text{m}^{-1}$.

diffraction angle measurements on Hydrogen vapor light to be $R_H = (10.715 \pm 0.059) \mu\text{m}^{-1}$.

References

- [1] Huang, H. *Modern Physics Lab Manual*. January, 2019.
- [2] National Institute of Standards and Technology (NIST), *Basic Atomic Spectroscopic Data*. Accessed February, 2019.
- [3] W.S. Benedict and E. K. Plyler, *Journal of Research of the National Bureau of Standards*, Volume **46**, 1950.