PHYSICS 18L Lab 2: The Charge-to-Mass Ratio ($\frac{e}{m}$) of Electrons

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Abstract. In this experiment, we experimentally determine the electron's charge to mass $\frac{e}{m}$ ratio by sending a beam of electron to a uniform magnetic field produced by a pair of Helmholtz coils, and measuring the radius electron beam's circular orbit. Our measured value of $\frac{e}{m}$ yields $(1.758 \pm 0.06) \times 10^{11}$ C/kg, which corresponds to a 0.05% percent error compared to the expected value of 1.75888×10^{11}C/kg given by CODATA.

I. INTRODUCTION AND OBJECTIVES OF EXPERIMENT

The ratio of two fundamental constants $\frac{e}{m}$, where e is the electron's charge and m is its mass, can be measured with high precision in a typical laboratory setup by accelerating a beam of electrons with a potential into an uniform external magnetic field and looking at their motion. The force acting on an electron moving in a homogeneous magnetic field is given by the Lorentz force (in absence of the electric field) $\bar{F}$:

$$\bar{F} = -e(\bar{v} \times \bar{B}) \tag{1}$$

where $\bar{v}$ is the velocity of the electron and $\bar{B}$ is the magnetic field. We see from the cross prodcut that the trajectory of electron motion is a helix. A special case is when the electron's orbit is perpendicular to the direction of magnetic field, the trajectory becomes circular, and the Lorenz force acts as the centripetal force in this motion:

$$|\bar{F}| = F_c \tag{2}$$

$$\Rightarrow evB = \frac{mv^2}{R} \tag{3}$$

$$\Rightarrow \frac{e}{m} = \frac{mv^2}{BR} \tag{4}$$

where R is the radius of the circular orbit and m is the electron's mass.
In this lab, a beam of electrons is accelerated with a potential V, and thus we know the kinetic energy of an electron is:

$$eV = \frac{1}{2}mv^2 \Rightarrow v^2 = \frac{2eV}{m} \tag{5}$$

Using the Eqn (6) expression of electron velocity v in Eqn (4), we have the following relationship:

$$\frac{e}{m} = \frac{2V}{R^2B^2} \tag{6}$$

In this lab we will measure the applied voltage$V$, the magnetic field B, and radius of electron's orbit R in the field in order to determine the charge-mass ratio of electron. Using Eqn (6) and error propagation, the uncertainty of e/m is:

$$\delta(e/m) = \sqrt{(\frac{\delta V}{V})^2 + 4(\frac{\delta B}{B})^2 + 4(\frac{\delta R}{R})^2} \tag{7}$$

II. EXPERIMENTAL SET-UP AND PROCEDURE

The experiment is performed with a Kent $\frac{e}{m}$ apparatus (Model Tg-13), and its set up is illustrated in Figure 1. In our set up, the homogeneous magnetic field is produced by placing a pair of identical Helmholtz coils with N turns and radius R at a distance $l = R$, and connecting them in series. This configuration produces an uniform magnetic field in the space between the coils. The Kent $\frac{e}{m}$ apparatus contains an electron-beam bulb (EB-bulb) in which an electron beam is produced, and travels in circular orbit in the plane of coils. An LED-lit ruler is held behind the bulb "for measuring the radius of orbit. An electron beam is produced when the heated cathode in the EB-bulb gives off electrons, which are then attracted to the anode and accelerated by the electric potential between the cathode and anode.

To properly assemble and supply power to the $\frac{e}{m}$ apparatus, we use a DC supply (5-15V and 1-2 A) as input power for the Helmholtz coils, a DC/AC high-voltage power supply to provide 150V-300V for the EB-bulb to create the electric potential between cathode and anode, and an AC power supply (6.3V) for the electron gun cathode heater. A multimeter is connected in series to the apparatus to read out the input current of the Helmholtz coils, and another multimeter is connected in parallel to measure the applied accelerating voltage, and a Gaussmeter is used to measure the Earth's magnetic field and other stray fields in vicinity of the apparatus.

With all input and readout connections in place, we rotate the EB-bulb to ensure the electron beam is in the plane of the Helmholtz coils and the beam's orbit forms a circle. To perform precision measurement of $\frac{e}{m}$, we vary the applied accelerating voltages, and measure the corresponding orbit radius at different voltages. This allows us to plot voltage V against the orbit radius R and obtain the $\frac{e}{m}$ value from the slope of the plot. We also vary the input current of coils (hence, varying the magnetic field B) and measure the corresponding orbit radius, allowing us to plot B^2 against $\frac{1}{R^2}$, and obtaining $\frac{e}{m}$ value from the slope of plot. The measurements and plots are discussed in Section 3.

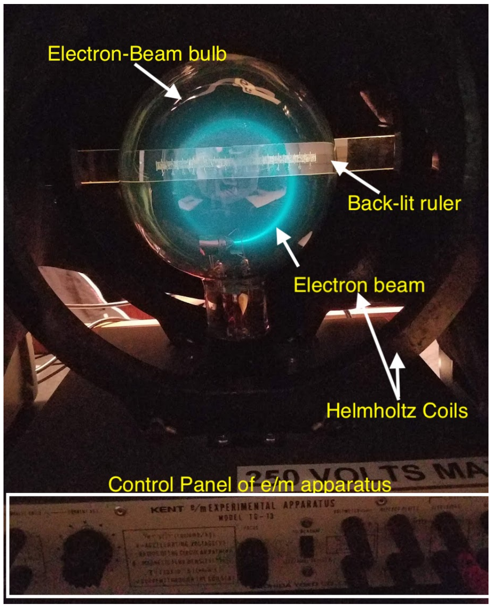

FIG. 1. The main experimental set up consists of an Electron-Beam (EB) bulb that heats off and creates an electron beam, a pair of Helmholtz coils for generating a uniform magnetic field in which the electron beam forms a circular orbit, a lit ruler held behind the bulb for measuring the radius of orbit, and an $\frac{e}{m}$ apparatus providing user controls for the input voltages and currents to the coils and EB-bulb.

Lastly, to investigate the presence of other magnetic fields around our set up, we measure the Earth's magnetic field using a Gaussmeter, as well as the fields at different locations around the set up.

III. MEASUREMENT DATA AND ANALYSIS

A preliminary measurement of e/m is first made by setting the coil current to $I = (1.50 \pm 0.05)A$, accelerating voltage to $V = (143.0 \pm 0.5)V$. The magnetic field is then estimated to be: $B = 7.8 \times 10^{-4}T \times I = (1.17 \pm 0.004) \times 10^{-3}T$, where the uncertainty of B is derived from δI using error propagation . We use a ruler to measure the radius of electron beam's orbit to be $R = 3.9 \pm 0.6$ cm. Using Eqn (6) and Eqn (7), this preliminary measurement yields $e/m = (1.37 \times 10^{11} \pm 0.31)$ C/kg. Taking the literature value of e/m from CODATA to be 1.76×10^{11} C/kg, our measurement has a percent error of 19%.

To perform precision measurement of e/m, we take five measurements by varying the accelerating voltages and measuring the electron beam's radius. The measurements and their corresponding calculated e/m values are shown in Table 1. By plotting applied voltage V against square of orbit's radius R^2, we obtain a more precise value of e/m using the slope from linear regression analysis (Figure 2). Taking the slope of linear fit and dividing it by $\frac{B^2}{2}$, the resulting e/m value is $(1.72 \pm 0.06) \times 10^{11}$ C/kg.

V [V]	R[cm]	δR [cm]	Calculated $\frac{e}{m}$ [C/kg]	$\delta\frac{e}{m}$ [C/kg]	Percent error
100	2.65	0.60	2.08E+11	0.46	22%
120	3.25	0.60	1.66E+11	0.38	2%
132	3.50	0.60	1.57E+11	0.35	7%
140	3.90	0.60	1.34E+11	0.32	21%
165	4.05	0.60	1.47E+11	0.30	14%

TABLE I. Five measurements of e/m corresponding to different applied accelerating voltages.

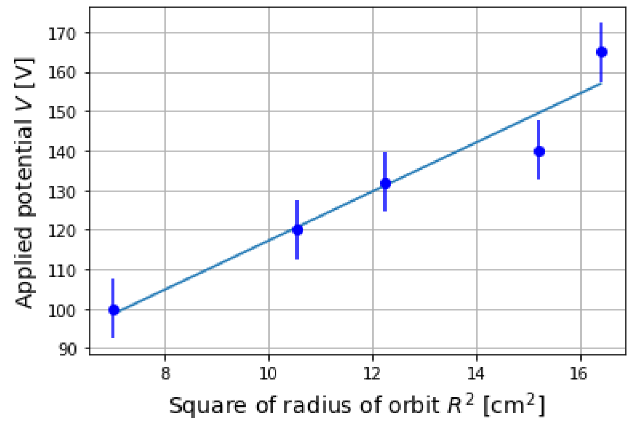

FIG. 2. Plotting V against R^2 using data from in Table 1. A linear fit yields a slope of 62017, and dividing this slope by $B^2/2$, where B is calculated using $I = (1.05 \pm 0.02)$ A, gives an e/mvalue of $(1.72 \pm 0.06) \times 10^{11}$ C/kg. Note that the error bars have been exaggerated by factor of 25 for visibility on the plot.

we take another five measurements by varying the current of the coils (and hence B-field) and measuring the electron beam's radius. The measurements and their corresponding calculated e/m values are shown in Table 2. By plotting the square of B-field B^2 against $1/R^2$, we obtain a value of $e/m = (1.45 \pm 0.25) \times 10^{11}$ C/kg using the slope from linear regression analysis (Figure 3).

I [A]	R[cm]	δR [cm]	Calculated $\frac{e}{m}$ [C/kg]	$\delta\frac{e}{m}$ [C/kg]	Percent error
1	5.60	0.60	1.57E+11	0.12	8%
1.2	5.00	0.60	1.37E+11	0.13	19%
1.4	4.00	0.60	1.57E+11	0.15	8%
1.6	3.75	0.60	1.37E+11	0.16	19%
2	3.00	0.60	1.37E+11	0.20	19%

TABLE II. Five measurements of e/m corresponding to different applied current to the coils.

The Helmholtz coils are measured to have a separation of 152 ± 2mm, and a mean radius of 150 ± 2mm. Since each individual wire has a finite cross section, so

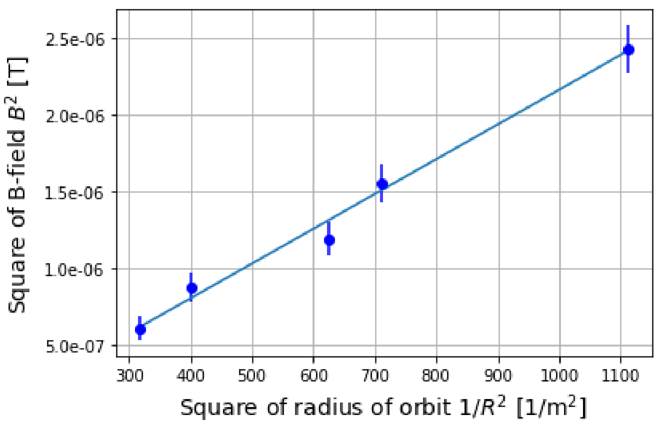

FIG. 3. Plotting B^2 against $1/R^2$ using data from in Table 2. Calculating $\frac{2V}{k}$, where k is the slope obtained by linear fit, yields an e/mvalue of $(1.45 \pm 0.25) \times 10^{11}$ C/kg. Note that the error bars have been exaggerated by factor of 25 for visibility on the plot.

the coil itself is actually hallow cylinder. The field produced by such pair of non-ideal H-coils at a given point in space can be approximated by using superposition principal and calculating the magnetic field of two cylinders with hole.

We measure and calibrate the B-field at the center of H-coils with five different current values. The measurements are shown in Table III, and the current vs. B-field is plotted in Figure 3, which shows a clear linear relationship between the current and produced B-field with a slope of 7.26. We also take off the EB-

I [A]	B [G] (Scale = 30)	B [G] (Scale = 100)
1	0.80 ± 0.20	0.08 ± 0.050
1.2	0.94 ± 0.20	0.096 ± 0.050
1.4	1.10 ± 0.20	0.110 ± 0.050
1.6	1.25 ± 0.20	0.120 ± 0.050
2	1.52 ± 0.20	0.160 ± 0.050

TABLE III. Five measurements of magnetic fields corresponding to different coil current values. The measurements are repeated using two reading scales of the Gaussmeter: 30 and 100.

bulb and measure the B-fields around the coils in order to account for Earth's magnetic field and other stray fields. The measured fields are: $B_z = (0.5 \pm 0.05)G$, $B_x = (0.025 \pm 0.05)G$, and $B_y = (0.4 \pm 0.05)G$, where $\hat{z}$ is pointing North of the apparatus $\hat{x}$ is in horizontal direction and in plane of H-coil, and $\hat{y}$ is in perpendicular direction to the H-coil. These fields allow us to account for a background field, and since B_z is parallel to the electron's orbit, the field's effect can be neglected. We also orient the Gaussmeter's probe in other directions to test the profile of magnetic field along the central axis of the H-coils by placing the probes at different dis-

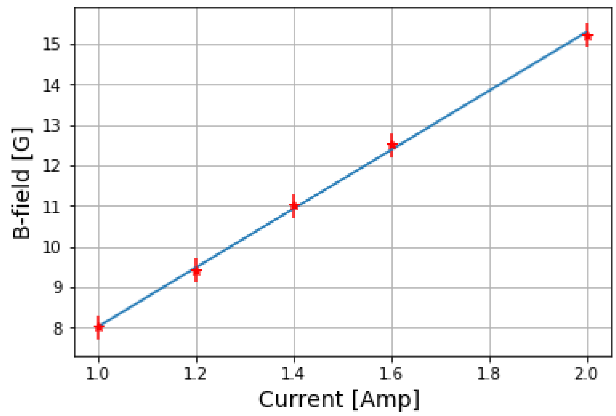

FIG. 4. Calibration plot of magnetic field at the center of H-coils as a function of applied current, using measurements from Table III. The linear regression yield a slope of 7.26.

tances away from the center of coils, the results are shown in Table VI. Finally, taking our most precise measured value of e/m obtained from linear regression in Figure 2 and correcting it with the background field in B_y, the $(1.758 \pm 0.06) \times 10^{11}$ C/kg.

Probe Position	B [G] (Scale = 10)
$x = 2l$	0.12 ± 0.05
$x = l$	0.35 ± 0.05
$x = \frac{l}{2}$	0.28 ± 0.05
$x = 0$	0.42 ± 0.05
$x = -\frac{l}{2}$	0.25 ± 0.05
$x = -l$	0.30 ± 0.05
$x = -2l$	0.16 ± 0.05

TABLE IV. Measured magnetic field profile along the central axis of the H-coils.

IV. CONCLUSION AND OUTLOOK

We have experimentally determined the electron's charge to mass ratio $\frac{e}{m}$, with our most accurate measurement yielding $(1.758 \pm 0.06) \times 10^{11}$ C/kg. This measured value is in extremely well agreement with the expected value of 1.75888×10^{11}C/kg given by CODATA, with only 0.05% percent error. We also remark that a significant part of the error might come from the reading of the orbit's radius on a ruler, as the center of the orbit was not referenced in our experimental set up so we used our eyes to gauge the center.