Optical Resonantors and Cavity

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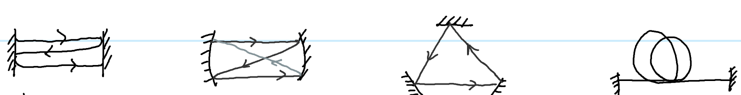
1. Introduction

Optical counterpart of electronic resonator circuit = optical resonator that confines & stores light at certain resonance frequencies.

Laser is also just an optical resonator that contains a medium that amplifies light. The resonator determines the frequency & spatial distribution of output beam.

- Different approaches to describe optical resonator:
  - → Ray optics : trace optical rays as they get reflected. Use geometry to determine <u>confinement</u> condition.
  - → Wave optics : Determine <u>Modes</u> of the resonator i.e. which wavefunctions & frequencies are supported by the cavity self-consistently. Longitudinal
  - → Beam optics: used to look at <u>beams/modes</u> inside spherical-mirror resonator. Transverse
    ↑ Gaussian or Hermite-Gaussian
  - → Fourier optics: used for understanding effect of finite size of resonator mirrors on its loss on spatial distribution of modes

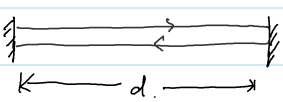
- Types of resonators:



planar mirror | Spherical Mirror (Bowtie) | Ring Resonator | Optical Fiber Resonator

2. <u>planar-mirror Resonator</u>

2 parallel, highly-reflective, flat mirrors. Separated by distance d.
" Febry-perot etalon



- what are the resonator modes?

Incident light : monochromatic at ν in $\hat{z}$, polarized in $\hat{x}$.

Forward: $\vec{E}(z,t) = \hat{x} \cdot E_+ \, e^{i(-kz+\omega t)}$

Reverse: $\vec{E}_{0-}(z,t) = \hat{x} E_- e^{i(kz+\omega t)}$

Forward

$$\vec{E}_{For}(z,t) = \hat{x}\cdot E_+ \, e^{i(-kz+\omega t)}$$

$$\vec{E}_{for}(z,t) = \hat{y}\,(E_+/\eta)\, e^{i(+kz+\omega t)}$$

Reverse

$$\vec{E}_{Re}(z,t) = \hat{x}\, E_- \, e^{i(kz+\omega t)}$$

$$\vec{E}_{Re}(z,t) = -\hat{y}\,(E_-/\eta)\, e^{i(kz+\omega t)}$$

Apply appropriate boundary condition:

$$\begin{cases} z=0: & E_+e^{-ikz} + E_-e^{ikz} = 0 \\ z=d: & E_+e^{-ikz} + E_-e^{ikz} = 0 \end{cases}$$

zero fields

$\overset{z=0}{\Rightarrow} \quad E_+ = E_-$

$\overset{z=d}{\Rightarrow} \quad \cancel{E_+}e^{-ikd} + \cancel{E_-}e^{ikd} = 0$

$\Rightarrow \frac{1}{2}(e^{ikd} + e^{-ikd}) = 0$

$\Rightarrow \sin(kd) = 0$

$\Rightarrow kd = n\cdot\pi \quad (n=1,2,3\ldots)$

$k = 2\pi/\lambda$

$\therefore \boxed{K_n = \frac{n\pi}{d}}$ or $\boxed{d = \frac{n\pi}{2}}$

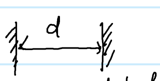
Now, frequency $f = c/\lambda \Rightarrow f_n = \frac{nc}{2d} \Rightarrow \boxed{\Delta f = \frac{c}{2d}}$

$\frac{kc}{2\pi}$

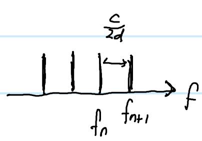
$k = \frac{n\pi}{d}$

Free spectral Range.

(freq spacing of 2 adjacent modes)



Distance of half of round trip.



(ex) 15cm Long $\Rightarrow$ $\underline{\Delta F = 1\text{ GHz}}$

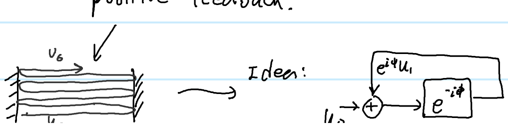
- Another simpler way to look at resonator modes:

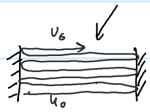
Resonator modes are travelling waves (transverse). A <u>mode</u> is a self-reproducing wave (= wave that reproduces itself after a single round trip)

Each mirror reflection causes phase shift π $\Rightarrow$ Total phase shift in a round trip imparted by 2 mirrors $= \underline{2\pi}$

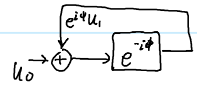
so $\phi = k\cdot(2d) \overset{\text{must}}{=} n\cdot(2\pi)$, $n = 1, 2, \ldots$ # of round trips.

$\Rightarrow kd = n\pi$. Same result. This can be considered as the condition for positive feedback.





Idea:



Optical feedback system.

∴ 2 peспective of mode
- Standing wave: write down steady state E-fields in cavity, apply B.C. ⇒ get $kd = n\pi$. ✓
- Traveling wave: write down total phase shift experienced from n round-trips ⇒ get $kd = n\pi$. ✓

• Mode density: # of modes per frequency: $\frac{1}{\Delta f} = \frac{2d}{c}$ in each of 2 polarizations ⇒ $2 \times \frac{2d}{c} = \underline{\frac{4d}{c}}$ total # of modes

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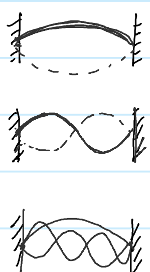
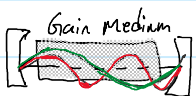
Frequency | Spatial

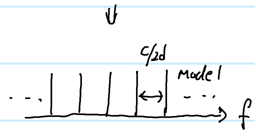
Longitudinal Mode ⟨ standing / traveling



↑ Different Longitudinal modes (different supported wavelength / freq)

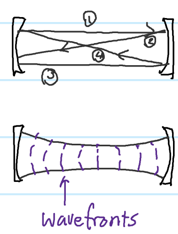


↑ Two distinct longitudinal modes
‖
2 supported frequencies.
⇓



Longitudinal modes → relate to different resonances along ↔ optical axis

Transverse mode (TEM)

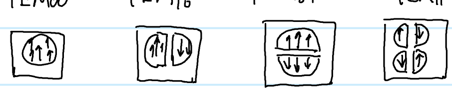


Off-axis transverse mode supported by cavity & self-replicate

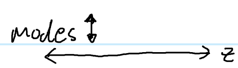
Transverse mode → relates to cross-sectional profile / waveform of input beam.

Eg. Transverse cavity modes & their fields

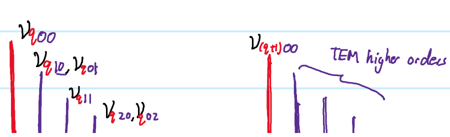
TEM$_{00}$ TEM$_{10}$ TEM$_{01}$ TEM$_{11}$

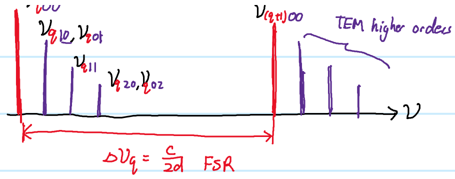


Directions are all transverse ⊥ to propagation!

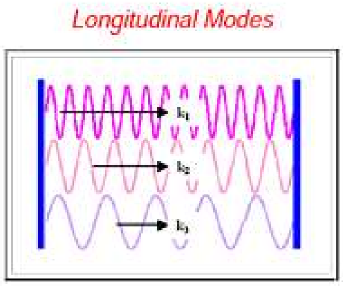


One "mode" = 3 parameters ν_{qmn} $\begin{cases} m,n = \text{Transverse} \\ q = \text{Longitudinal} \end{cases}$



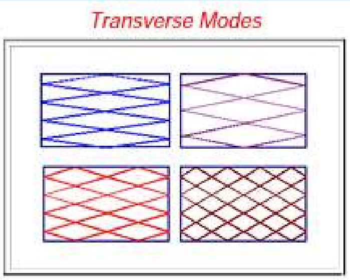


Intensity distribution on along propagation axis (longitudinal) And in plane ⊥ to the propagation axis (Transverse)



Many standing waves – with different wavelengths and k-vector directions satisfy the resonance condition. (i.e. fit inside, given BC.): $\frac{m\lambda}{2} = d$.
ie. common lingo: the laser (cavity) contains/supports many modes and thus does not automatically give monochromatic light in single direction.
↳ laser can $\boxed{\text{mode hop}}$ to different longitudinal modes

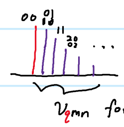
making laser single-mode (monochromatic) is all where all the technique lies.



Many transvese modes. → most are undesired and to be avoided.

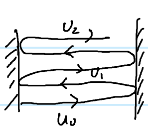
→ Laser can $\boxed{\text{multi-modes}}$ = multiple spatial modes.

At a single frequency (ie. monochromatic, single longitudinal mode) → there can be multimode.



3. Losses of cavity ⇒ spectral width in longitudinal modes.

In presence of loss, mode condition inside a resonator is relaxed



$\varphi = n \cdot 2\pi$. $\quad \vec{U_0}\,\vec{U_1}\,\vec{U_2}$ phasor diagram

$\varphi \neq n \cdot 2\pi$ $\quad U_0, U_1, U_2$

With loss: phasors don't have exactly same magnitude. ⇒ So we use an attenuation factor r

E-fields:

$U_1 = re^{-i\varphi} U_0$

$U_2 = re^{-i\varphi} U_1$

$$U_1 = re^{-i\varphi}U_0$$
$$U_2 = re^{-i\varphi}U_1$$
$$\vdots$$
$$U = U_0 + U_1 + U_2 \quad \text{Sum of E-fields at each leg.}$$

Intensity:

$$I = |U|^2 = \frac{|U_0|^2}{|1-re^{-i\varphi}|^2} = \frac{I_0}{(1-r\cos\varphi)^2 + (r\sin\varphi)^2}$$

...

$$\Rightarrow I = \frac{I_{max}}{1+\left(\frac{2\mathcal{F}}{\pi}\right)^2 \sin^2\left(\frac{\varphi}{2}\right)}$$

↳ periodic with φ.

↳ Higher $\mathcal{F} \Rightarrow I$ has sharper peak.

corresponding to $\varphi = n \cdot 2\pi$.

where $I_{max} \equiv \frac{I_0}{(1-r)^2}$

$$\boxed{\mathcal{F} \equiv \frac{\pi\sqrt{r}}{1-r}}$$ "Finesse" of the resonator

$\mathcal{F} \to \infty$ when $r \ll 1$

$\mathcal{F} = 0$ when $r = 0$

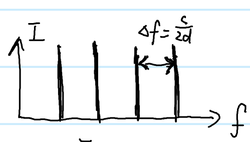
$\mathcal{F} \gg 1$ when r is large.

Since $\varphi = 2kd = \frac{2\cdot 2\pi}{\lambda} d = \frac{4\pi}{c/f} d = \frac{4\pi f d}{c}$; $\Delta f = \frac{c}{4d}$

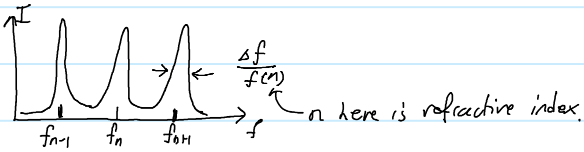
$$\Rightarrow \varphi = \frac{4\pi \cdot f}{4\Delta f} = \frac{\pi f}{\Delta f}$$

then $$\boxed{I = \frac{I_{max}}{1+\left(\frac{2\mathcal{F}}{\pi}\right)^2 \sin^2\left(\frac{\pi f}{\Delta f}\right)}}$$ Spectral resonances (periodic).

width $\frac{\Delta f}{\mathcal{F}}$ $\frac{FSR}{freq}$



$\mathcal{F} \to \infty$

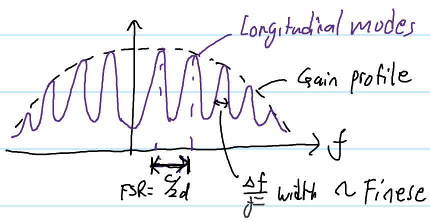


$\mathcal{F} \gg 1$

- So, realistically, laser cavity has ① a specific length $\Rightarrow$ FSR

  FSR (= freq separation btw 2 adjacent modes): $\Delta f = \frac{c}{2d}$.

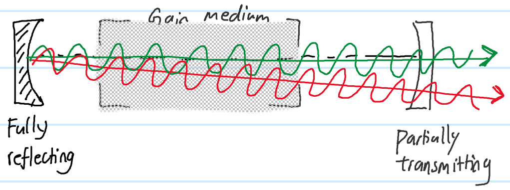
  ② Bandwidth of net gain $\frac{\Delta f}{\mathcal{F}}$ $\frac{FSR}{Finesse}$



4 More on Transverse Mode

4. More on Transverse Mode.

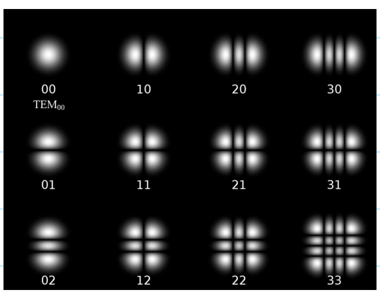
The transverse intensity distribution depends on the specific resonator configuration, as the beam size $W(z)$ depends on mirror curvature R and resonator length d. $\Rightarrow$ shape of the mode changes as beam propagates along the resonator axis.



2 distinct transverse modes emerging in different directions

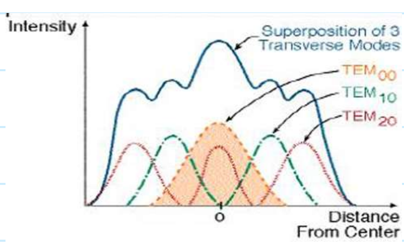
Different transverse modes can occur simultaneously within a laser cavity.
These different transverse modes have slightly different optical paths inside
$\Rightarrow$ thus different frequencies (as d cavity length is different for them)
ofc, we can see that each transverse mode with unique optical path can have several longitudinal modes, separated by FSR in frequency.

- The intensity distribution is characterized by $m, n \to TEM_{nm}$.
  m, n refer to the # of intensity modes in along y and x-axis.



Transverse Gaussian Modes

- when there are several transverse modes (laser multimoding), the total intensity profile is the superposition of all existing transverse modes:



$\leftarrow$ lower transverse mode has smaller diameter

To make laser operate in a single transverse mode : Choose a pinhole diameter inside the laser cavity = diameter of TEM_{00} mode. Thus only TEM_{00} mode can pass through and be amplified, all other modes attenuated.