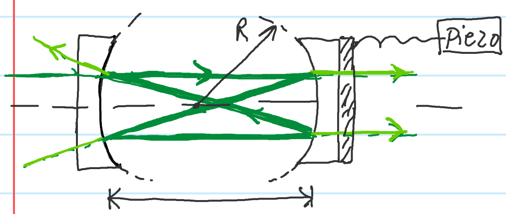
Fabry-Perot Cavity

Monday, February 21, 2022 3:07 PM

Scanning FP Confocal Cavity



$L = R_1 = R_2$

Distance of cavity = Radius of curvature of Mirror.

Components:
- Invar/Al tube for housing + orings
- Confocal mirrors (high quality dielectric with reflection ~ 99.5%)
- PZ with center hole

1. $FSR = \frac{c}{2d} = \frac{c}{2R}$.

2. Finesse, mode width

The mirror reflectivity determines the cavity loss ⇒ longitudinal mode width.
The higher quality the mirrors → narrower transmission peaks → your FP cavity can better distinguish features of transmission spectrum.

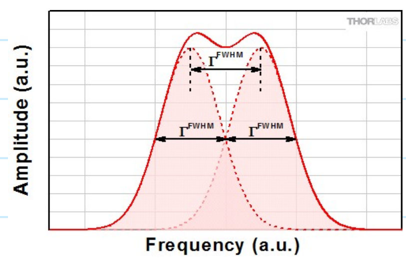
As from previous section, Finesse for mirrors with reflectivity coef r:

$$\boxed{\mathcal{F} = \frac{\pi\sqrt{r}}{(1-r)}}$$

For confocal cavity: $\boxed{\mathcal{F}_{conf} = \frac{\mathcal{F}}{2} = \frac{\pi\sqrt{r}}{(1-r)}}$.

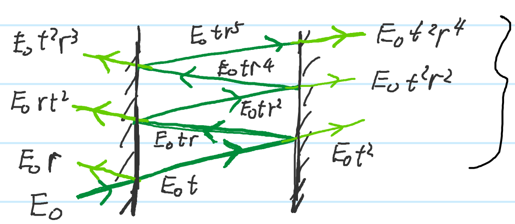
⇒ FWHM mode width of the Lorenzian peaks:

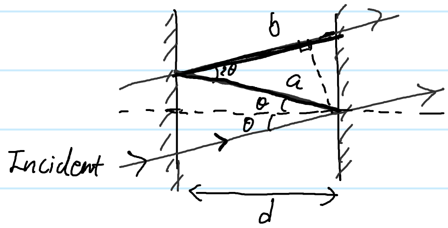
$$\boxed{\Gamma_{FWHM} = \frac{\Delta f_{FSR}}{\mathcal{F}}}$$



- More details:

Simplified geometry





Consider light incident on mirror at angle θ.
The reflecting light back and forth in the cavity are at 2θ.
The reflected light follows additional path length of $a+b$.

Additional length $= a+b$.

$$a = \frac{d}{\cos\theta}, \quad b = a\cdot\cos(2\theta)$$

$$\therefore a+b = \frac{d}{\cos\theta}(1+\cos 2\theta) = \frac{d}{\cos\theta}(1+2\cos^2\theta - 1) = \underline{2d\cos\theta}.$$

This additional path inputs phase delay on the incident plane wave e^{ikz}

$$\text{So } \phi = kz = k(a+b)$$
$$= \underline{2kd\cos\theta}$$

The overall transmitted E-field is the sum of each leg:

$$E_t = E_0 t^2 + (E_0 t^2 r^2)\cdot e^{i\phi} + (E_0 t^2 r^4)\cdot e^{i2\phi} + \cdots$$
$$= E_0 t^2 (1 + r^2 e^{i\phi} + r^4 e^{2i\phi} + \cdots)$$
$$= E_0 t^2 \underbrace{\sum_{n=0}^{\infty} r^{2n} e^{in\phi}}_{\frac{1}{1-r^2 e^{i\phi}}}$$

$$\Rightarrow E_t = \frac{E_0 t^2}{1 - r^2 e^{i\phi}}$$

Transmitted Intensity:

$$I = |E_t|^2 = E_0^2 \frac{|t|^4}{|1-r^2 e^{i\phi}|^2} = \frac{I_0 |t|^4}{|1-r^2 e^{i\phi}|^2}$$

The exponential term represents phase change upon 2 reflections

$$r = |r| e^{\frac{i\phi_r}{2}}$$

Let Mirror Reflectivity be R, Transmission be T, then

$R = |r|^2$, $T = |t|^2$, $R+T = 1$

$$\Rightarrow T_\perp = \frac{I_0 \cdot T}{}$$

$R = |r|^2, \quad T = |t|^2 \quad , R + T = 1$

$$\Rightarrow I_t = \frac{I_0 \cdot T}{\underbrace{|1 - Re^{i\phi}|^2}}$$

$$|1 - Re^{i\phi}|^2 = (1 - Re^{i\phi})(1 - Re^{-i\phi})$$

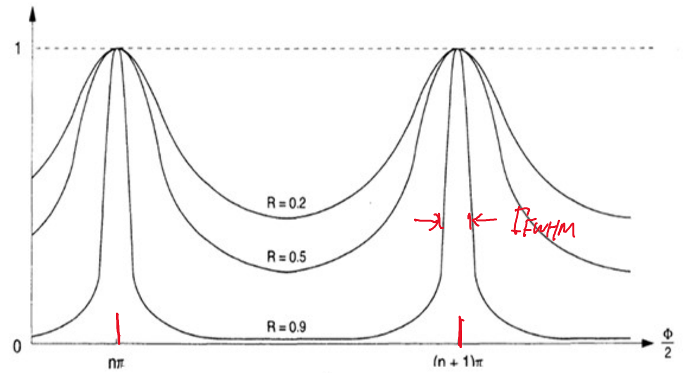
$$= 1 - 2R\cos\phi + R^2$$

$$= (1-R)^2 \left[1 + \underbrace{\frac{4R}{(1-R)^2}}_{\text{let it be } \mathcal{F}'} \sin^2\left(\frac{\phi}{2}\right)\right]$$

$$\therefore \boxed{\frac{I_t}{I_0} = \frac{1}{1 + \mathcal{F}' \sin^2(\phi/2)}} \qquad \text{Since } \varphi = 2kd \quad = \frac{4\pi}{\lambda} d\cos\theta$$

↖ Airy function. (periodic)

plot



- Each of the airy function peak is identical in shape (Lorenzian). So to get FWHM we only need to solve for $n=0$ simpliest case:

  For $R > 0.6 \Rightarrow \sin\left(\frac{\phi}{2}\right) \sim \frac{\phi}{2}$. small incident angle

$$\Rightarrow \text{At Half Max}: \quad \frac{1}{1 + \mathcal{F}'\left(\frac{\phi_{Half}}{2}\right)} = \frac{1}{2}$$

$$\Rightarrow \mathcal{F}'\left(\frac{\phi_{Half}}{2}\right) = 1$$

$$\Rightarrow \phi_{\substack{Half\\max}} = \frac{2}{\sqrt{\mathcal{F}'}}$$

$$\Rightarrow \text{FWHM} = 2\phi_{\substack{Half\\max}} = \frac{4}{\sqrt{\mathcal{F}'}}$$

Define finesse as:

$$\boxed{\mathcal{F} = \frac{\Delta\phi}{\text{FWHM}}} = \frac{2\pi}{4/\sqrt{\mathcal{F}'}} = \boxed{\frac{\pi\sqrt{R}}{1-R}}.$$

3. Transmission Spectrum. of FP cavity

- First, let's see the transverse spatial modes in the resonator:

It can be shown that only the following frequencies ν_{qmn} can exist in the resonator of given config (Length L, 2 confocal mirors with R_1, R_2 reflectivity)

$$\boxed{\nu_{qmn} = \frac{c}{2L}\left[q + \frac{1}{\pi}(m+n+1)\cos^{-1}\sqrt{g_1 g_2}\right]}$$ where $q, m, n = 0, 1, 2 \ldots$ mode numbers.

$g_{1,2} \equiv 1 - \frac{L}{R_{1,2}}$. g-parameter of resonator.

These frequencies are referred to Gaussian TEM modes of order (m,n).
OR. "Hermite – Gaussian modes".

q labels the longitudinal mode
m, n label the # of intensity distributions in y; x-axis.

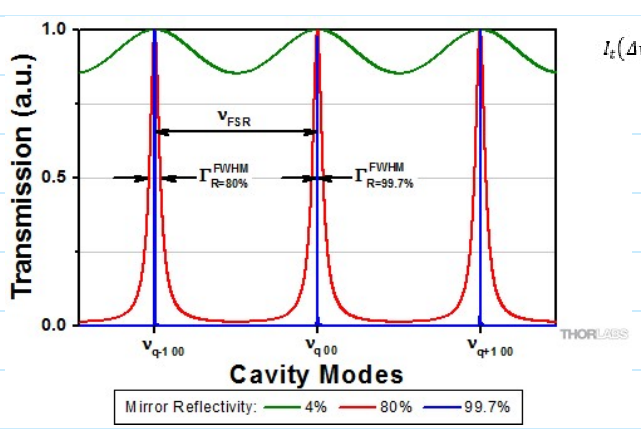
- Now, We just assume the alignment is such that the light is spatially mode-matched to $\underline{TEM_{00}}$ (i.e. wavefronts of Gaussian beam Match with Mirror Surfaces & incident beam is aligned to optical axis). $\Rightarrow$ So all higher modes $(m, n > 0)$ are not involved.

$\Downarrow$

Adjacent modes are: ν_{q00}, $\nu_{(q\pm1)00}$.

$$\Delta\nu_{FSR} = \nu_{q00} - \nu_{(q\pm1)00} = \frac{c}{2L}.$$

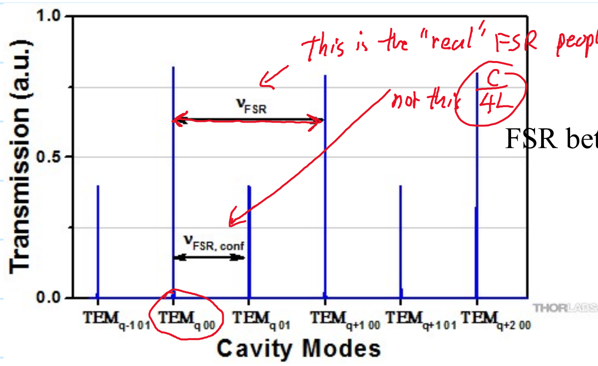
- With all that, we get the transmission mode spectrum



$$I_t(\Delta\nu_q) = \frac{t_1 t_2}{(1-\sqrt{r_1 r_2})^2} \frac{I_0}{1 + \frac{4\sqrt{r_1 r_2}}{(1-\sqrt{r_1 r_2})^2} sin^2\left(\frac{\pi \Delta\nu_q}{\nu_{FSR}}\right)}, \qquad (4)$$

- If our alignment is not perfect and we have a bit of higher modes

• If our alignment is not perfect and we have a bit of higher modes other than the fundamental TEM_{00} mode, then we will see more peaks:



This is the "real" FSR people usually refer to, $\frac{c}{2L}$

FSR btw adjacent Longitudinal modes

not this $\frac{c}{4L}$

FSR between adjacent Transverse modes

For confocal cavity: $g_1 g_2 = (1-\frac{L}{R})(1-\frac{L}{R})$, $R=L$. config.

$$\Rightarrow \boxed{\nu_{qmn} = \frac{c}{2L}\left[q+\frac{1}{2}(m+n+1)\right]}$$ simplifies to this..

$$\Rightarrow \nu_{FSR,conf} = |\nu_{q00} - \nu_{q01}| = \frac{c}{2L}\left[(q+\tfrac{1}{2})-(q+1)\right] = \frac{c}{4L}$$

Real $\nu_{FSR} = \frac{c}{2L}$.