前言

TOEFL、GRE 是当代大学生耳熟能详的字眼。在前几年，好的 TOEFL、GRE 成绩可以使申请者较轻松地获得美国、加拿大大学的全额奖学金。但随着中国学生 G、T 分数的不断提高，美国大学已经很难完全凭此来判断申请者的潜质。于是，更多的大学将目光投向了申请者在校的平时成绩（GPA）。但由于各校的教学水平不同，评分的松紧程度不同，录取人很难对不同学校毕业的申请者进行比较。专项 GRE（SUB）考试帮助美、加大学的录取人很好地解决了这个问题。SUB 的考题基本上覆盖了美、加大学要求本科生掌握的所有专业内容。因此，在 SUB 考试中取得好的成绩，可以证明你已经掌握了该学科的基本内容，并具备进一步在该学科进行深造的潜力和能力。根据笔者的粗略统计，美国前 20 名的大学，基本上或是要求申请者必须有 SUB 成绩，或是强烈推荐申请者参加 SUB 考试，并指出好的 SUB 成绩将使申请者处于更为有利的位置。从近几年留学美、加的情况来看，好的 SUB 成绩将大大提高申请者的录取机会。那些并不要求申请者参加 SUB 考试的排名相对靠后的美、加大学对提供了 SUB 成绩的申请者也是另眼相待。此外，对于大学期间成绩（GPA）并不理想的申请者来说，成绩已无法更改了，但如果通过自身的努力而在 SUB 考试中取得优异的成绩的话，无疑是对自身学术能力的一种有力的证明，更是对平庸的 GPA 的一种弥补。毫不夸张地讲，SUB 成绩往往可以决定一个申请者的命运。

- 本丛书的创作动机

作为北京大学的学生，我们基本上都参加了 SUB 考试，并获得了满分的成绩。在准备这项考试的过程中，我们深感到还没有一本好的参考书能帮助考生避免走弯路，事半功倍地去面对 SUB。我们在准备考试的过程中，通过在美国的朋友以及其它一些途径，得到了一些宝贵的学习资料。这些资料帮助我们很轻松的抓住了 SUB 考试的重点，理清了 SUB 考试的出题策略，也使得我们轻松地拿到了满分。现在，我们身边的许多考生面对 SUB 考试无从下手，甚至连 SUB 考试的考试范围、考试重点都不了解，我们觉得有责任去帮助身边的人去实现他们的梦想。毕竟，我们手中有大量的资料，有大家在准备考试以及实战过程中总结出的考试重点，更有大家共同研究出的应试策略。我们决定将这一切都奉献给所有的将来的考生。新东方学校的俞敏洪校长对我们的想法给予了充分的肯定，并提出了很多好的建议与意见，为我们创作此丛书奠定了坚实的基础。

- 本书的特点

本书由三部分组成。

第一部分，我们对物理 SUB 进行了详细的介绍。其中包括：如何报名参加物理 SUB 考试，考试的命题范围，考试的重点，常用的公式等等。读过这一部分后，读者将对物理 SUB 有一个较为完整的认识。

第二部分，我们将所考的内容按具体的分支进行了分类，并对每个分支的考题出现的频率进行了统计，以帮助读者有针对性地进行准备。在这一部分中，我们对每一道题都作了详尽的解释，并结合具体题目给出了一些应试的技巧。对于某些类问题，我们或是给出了经典的快速、准确的算法，或是给出了多种解法，使得读者可以根据自己的知识面来准备这项考试。相信读者在对本部分的题目进行了分析并结合我们给出的方法进行了体会之后，定会在 SUB 考试中做到游刃有余。

第三部分，我们提供了两套完整的题目来帮助读者检验自身的水平和学习的效果，并在后面给出了详细的解答。

● 致谢

我们身边的很多同学都参与到了这本书的创作中来。感谢华明同学帮助录入了一套模拟题。感谢谢平和冉鹰同学帮助解决一些疑难问题。感谢钟宇同学提供硬盘作数据备份。感谢高颖佳同学自愿放弃自己的机时。我们的很多其他同学在答案的准确性、方法的合理性、技巧的总结等很多方面都给予了我们很大的帮助，恕不一一列出，在此一并表示感谢。

由于作者水平有限，再加上物理 SUB 考题的覆盖面较广，本书难免有不足和疏漏之处，恳请读者指教。

物理 SUB 创作组

二〇〇〇年七月二十五日

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关于 SUB

什么是 GRE：

美国（及一些其他国家，如加拿大）绝大多数研究生院及专业学校都要求申请入学者参加 GRE 考试（Graduate Record Examination）。该考试由美国教育考试服务中心 ETS（Educational Testing Service）举行，命题有关原则由 GRE 理事会拟定。该理事会与研究生院联合会，研究生院审议会相协商。

GRE 考试是为向有关学校提供申请者教育背景及学术能力的有关信息，以使学校更好地衡量申请者的学术履历和资格，便于奖学金发放单位选择合适的资助对象。

GRE 考试共分为两种。普通能力测试（general test）和专项考试（subject test）。

什么是 SUB：

SUB 即 GRE 专项考试（Subject Tests）的简称，目前有专项考试的专业有：
生物 计算机科学 化学 经济学 教育学 工程学 法语 地理 地质学 德语 历史 英语文学 数学 音乐 哲学 体育 物理学 政治 心理学 社会学 西班牙语 （按英文字母顺序排列）

专项考试长度为 170min，考试目的为测试你对要申请的专业或领域内最基础知识的理解程度。同时也测试你对该领域内最重要的概念、方法的掌握情况。

专项考试涵盖专业内的许多事实和原理，其题目的解答即需要对这些事实、原理的正确把握及运用。每个学生并不一定对考试的所有内容都非常熟悉，因为命题原则是尽可能就某一领域广泛地搜取资料进行考查。

考生在每次考试当天只能参加一种专项考试，额外的考试可以在另外的日期参加，允许重考。

考试的所有题目及相关指示语均为英文（除法语，德语及西班牙语专项考试）。

对每一次专项考试，考生都会得到一个总分（某些考试中还会得到不同题目的分数），由于考试是标准化的，成绩单上给出了考生分数在一定人群中的比例，因此不同学历背景学生的分数可以进行比较。成绩可以在考试一周后花十美元电话查询。

GRE 理事会提倡将专项考试成绩与申请者其他信息联合起来对申请者的综合科研素质进行评价。这些信息包括本科专业课程及成绩（GPA），推荐信，GRE 普通考试成绩等等。

为什么要考物理专项：

对于以物理专业申请出国的本科生，专项成绩是必不可少的。因为现在全美绝大多数的研究生院要求物理专项成绩，（且数目仍在扩大）那些不要求专项成绩的学校对专项成绩也十分欢迎，而且往往会对有专项成绩的申请者更加青睐。北京大学物理系在过去三年里申请出国者基本上都参加了专项考试。而且在所有专业中物理专业的专项考试是相对比较容易通过准备而取得高分的，所以最好要考。

物理专项考试命题：

物理专项考试试题是由美国不同地区学校的负责教学、研究的教授共同拟定的。GRE 命题组是在美国物理会的推荐基础上选择这些教授的。

考试的内容和范围要由命题者（包括一些专业的教研工作者）定期监督、讨论。所有问题都由理事会审定并予以适当修改，组卷工作也由理事会进行，其间要确保内容的覆盖面，但不偏重某一分支，此后，考卷还要由理事会审定并通过。

美国教育考试服务中心有关人员协助理事会进一步协调考试工作，并对题目中语言、符号及有关态度甚至歧视性等问题进行调整，试题也要适合不同职业的不同考生。

由于物理教育范围的差异，一次考试并不能涵盖考生所有学过的知识，因此命题者主要选最基础及最重要的内容进行命题、题目也要不断更换，以确保内容随教学改革的稳定变化。所有新的考题均要与旧题进行难度对照评分，以使不同试题的分数可以平行对照，完全评价考生的知识水平与技能。每一套新题考后，考生的答案将被用来检验试题的合理性。这种检验可能暴露出某些题目是模棱两可的或是超出考试范围的，或对某一考生群是不适合的。这样的题目将不计分。

怎样参加物理专项考试：

GRE 专项考试在中国大陆地区每年举行一次，一般在十一月上旬，要求参加考试者事先交纳报名费（150$）报名参加。考试有关事务由国家教育部考试中心办理（可以电话咨询，号码是：010-62510951），一般报名程序为领表→填表→交报名费→交表。交完表即可认为报了名。按指示到指定考点领取准考证并按时到指定考点参加考试即可。

常见问题：

Q：领表一般在什么时间开始？

A：每年八月到九月之间，可以拨打电话查询。

Q：考试大约准备多久？

A：因人而宜，对一般物理专业的本科生，大四上学期开始准备一个月左右足够。

Q：在此期间怎样准备？

A：使用本书，并查阅相关内容的教材即可（后已列出参考书籍）。不必将所学专业内容全部复习一遍。

Q：考试内容、范围？

A：详见“物理专项考试内容”。
Q：考试题型？
A：全部为单项选择题，五选一。
Q：需要哪些参考书？
A：见后面参考书目。
Q：成绩反馈？
A：两个月后可以收到自己的成绩通知，如果你愿意还可将成绩免费送至三所学校研究生院。

近年成绩及趋势：

前几年绝大多数中国考生成绩均在 90%以上（意味着 90%以上的人成绩在你之下），但近几年获此成绩难度加大，同时也就更能检验考生的水平。因此参加考试并能获好成绩就更有利于申请。

考试重点

物理 SUB 考试由 100 道五选一单选题组成，其中一些以体组的形式出现。每道题中含有图、表、实验数据和对具体物理内容的描述。

考试目的是检验学生对基本原理的理解程度和应用这些基本原理的能力。绝大多数的考题只用到本科头三年的物理知识。

方面	占总分的百分比
1. 经典力学 （包括动力学、Newton 定律、功和能、振动、绕固定轴的旋转、质点组动力学、中心力场、天体力学、三维质点动力学、Lagrange 和 Hamilton 形式、非惯性系、流体力学初步）	20
2. 电磁学 （包括静电学、电流和直流电路、自由空间中的磁场、Lorentz 力、电磁感应、Maxwell 方程及其应用、电磁波、交流电路、物质中的电磁场）	18
3. 光学和波动 （包括波的基本性质、波的叠加、波的干涉衍射、几何光学、偏振、Doppler 效应）	9
4. 热力学和统计力学 （包括热力学定律、热力学过程、物态方程、理想气体、分子运动论、系综、统计概念、热力学量的计算、热膨胀、热传导）	10
5. 量子力学 （包括基本概念、Schrodinger 方程的解（包括方势阱、谐振子、氢原子）、自旋、角动量、波函数的对称性、微扰论初步）	12
6. 原子物理 （包括电子的性质、Bohr 模型、能量量子化、原子结构、原子光谱、选择定则、黑体辐射、X-射线、电场和磁场中的原子）	10
7. 狭义相对论 （包括基本概念、钟慢尺缩、同时性、能量和动量、四维矢量、Lorentz 变换、速度叠加）	6
8. 实验方法 （包括数据误差分析、电子线路、辐射检测、计数统计、带电粒子与物质相互作用、激光、光学干涉仪、数量级估计、概率统计的基本应用）	6
9. 特殊专题：核物理和粒子物理（包括核的性质、辐射衰变、裂变和聚变、反应、基本粒子的主要性质）、凝聚态（包括晶体结构、X-射线衍射、热性质、金属电子论、半导体、超导）、杂项（包括宇宙学、数学方法、计算机应用）	9

考生应能熟练掌握一定的数学知识，并能将其应用于具体的物理问题。这些数学知识包括：一元和多元微积分、坐标系统（直角、柱、球），矢量代数和矢量微分算子、Fourier 级数、偏微分方程、边值问题、矩阵和行列式、复变函数。

每次考试都从上述范围内出题，但一套题并不能涵盖所有内容。由于不同学校对本科物理教学的内容有差异，很少有考生能对所有内容了如指掌。考生并不一定要能回答所有问题。事实上，对 80%左右的问题即可得到相当高的标准分数。

考前必读

每一种考试的应试技巧总是由这种考试自身的特点决定的。物理 SUB 不同于一般的物理考试：它全部由选择题组成，并且它的题量比一般的物理考试要大得多。选择题的好处是用不着把每道题都按部就班地做到底，必要的时候可以用排除法。SUB 题量大，要在 170 分钟内做完 100 道题，就是说平均每 170 / 100 = 1.7 分钟就要做完一道题，时间非常紧迫。但是从另外一方面看，这恰恰说明 SUB 不可能有特别繁难的题目。以往的全部所有试题都证明：**SUB 的任一道题目都应该而且可以在 5 分钟内完成。**换句话说，**如果你认为有一道题目自己不可能在 5 分钟内做出，那一定是你的方法有问题**，这时你应该想想有没有更简单的方法，或是先把这题跳过往下做。

物理 SUB 的几分规则是做对得 1 分，不做不得分，做错倒扣 0.25 分。这就涉及到是否有必要猜题。假如五个选项你一个也排除不了，这时猜题的平均得分按概率计算为 $0.2 \times 1 - 0.8 \times 0.25 = 0$，就是说猜和不猜的效果是一样的。假如五个选项你至少能排除其中的一个，那么猜题的平均得分就会大于零。因此我们说**，假如五个选项你至少能排除一个，一定要猜题**。这样做的后果，虽然就每道题来说有可能对有可能错，但是总的来说只赚不赔。

物理 SUB 考试不让用计算器，因此大家在做练习题的时候也不要用计算器。一定要学会估算的方法，事实上这往往比摁计算器节约时间。

我们将在后面的练习题和模拟题中针对具体的问题给出一些常用的实战技巧。

常数表

下面是考卷上会给的一些常数，应尽可能多地记住以便考试时节约时间。

Rest mass of the electron	$m_e = 9.11 \times 10^{-31}$ kilogram $= 9.11 \times 10^{-28}$ gram
Magnitude of the electron charge	$e = 1.60 \times 10^{-19}$ coulomb $= 4.80 \times 10^{-10}$ statcoulomb (esu)
Avogadro's number	$N_0 = 6.02 \times 10^{23}$ /mole
Universal gas constant	$R = 8.31$ joules/(mole·K)
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ joule/K $= 1.38 \times 10^{-16}$ erg/K
Speed of light	$c = 3.00 \times 10^{8}$ m/s $= 3.00 \times 10^{10}$ cm/s
Plank's constant	$h = 6.63 \times 10^{-34}$ joule·second $= 4.14 \times 10^{-15}$ eV·second
	$\hbar = h / 2\pi$
Vacuum permittivity	$\varepsilon_0 = 8.85 \times 10^{-12}$ coulomb^2/(newton·meter^2)
Vacuum permeability	$\mu_0 = 4\pi \times 10^{-7}$ weber/(ampere·meter)
Universal gravitational constant	$G = 6.67 \times 10^{-11}$ meter^3/(kilogram·second^2)
Acceleration due to gravity	$g = 9.80 \times$ m/s^2 $= 980$ cm/s^2
1 atmosphere pressure	1 atm $= 1.0 \times 10^{5}$ newton/meter^2 $= 1.0 \times 10^{5}$ pascals(Pa)
1 angstrom	1 A $= 1 \times 10^{-10}$ meter
	1 weber/m^2 = 1 tesla = 10^4 gauss

Moments of inertia about center of mass

Rod	$\frac{1}{12}Ml^2$
Disc	$\frac{1}{2}MR^2$
Sphere	$\frac{2}{5}MR^2$

推荐参考书

《新概念物理教程 力学》	赵凯华、罗蔚茵	高等教育出版社
《新概念物理教程 热学》	赵凯华、罗蔚茵	高等教育出版社
《电磁学》（第二版）	赵凯华、陈熙谋	高等教育出版社
《光学》（第二版）	赵凯华、钟锡华	北京大学出版社
《新概念物理教程 量子物理》	赵凯华、罗蔚茵	高等教育出版社
《理论力学基础教程》	林纯镇、胡慧玲、吴惟敏	高等教育出版社
《热力学·统计物理》（第二版）	汪志诚	高等教育出版社
《电动力学简明教程》	俞允强	北京大学出版社
《量子力学导论》（第二版）	曾谨言	北京大学出版社
《固体物理讲义》	高政祥	北京大学内部资料
《近代物理实验》	吴思诚、王祖铨	北京大学出版社

第二部分 分类练习题

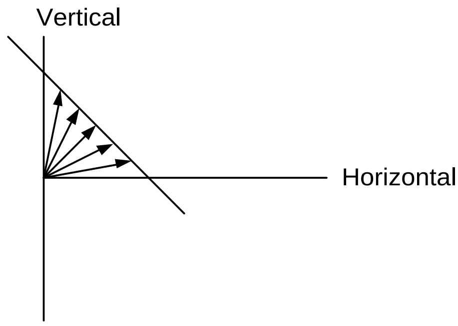
第一节 经典力学

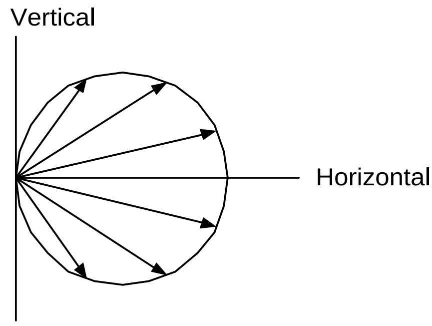
1. A satellite orbits the Earth in a circular orbit. An astronaut on board perturbs the orbit slightly by briefly firing a control jet aimed toward the Earth's center. Afterward, which of the following is true of the satellite's path?

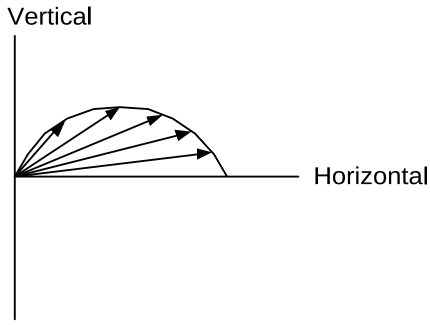
(A) It is a ellipse.
(B) It is a hyperbola.
(C) It is a circle with larger radius.
(D) It is a spiral with increasing radius.
(E) It exhibits many radial oscillations per revolution.

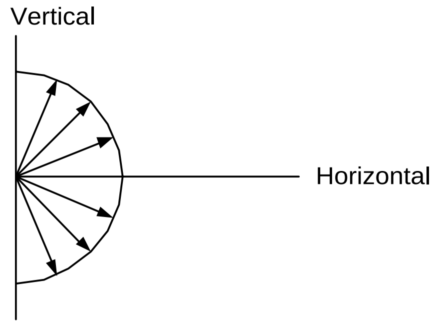
解：圆轨道受到小扰动后仍是闭合轨道，因此是椭圆。选（A）。

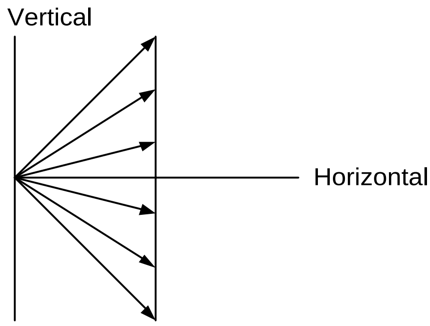
2. A person standing on the surface of the Earth throws a ball. The ball leaves the thrower's hand with initial velocity v_i and has final velocity v_f just before it is caught. If air resistance is negligible, which of the following diagrams correctly represents a possible sequence of velocity vectors for the ball?

(A)


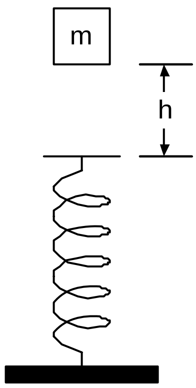
(B)


(C)


(D)


(E)


解：因为球在水平方向不受力，所以其水平动量不变，即速度的水平分量不变，只有（E）符合。选(E)。



3. As shown above, a block of mass m is released from rest at a distance h above a vertical massless spring with spring constant k, what is the maximum kinetic energy of the block?

(A) mgh

(B) $mgh+\frac{1}{4}\frac{m^2g^2}{k}$

(C) $mgh+\frac{1}{2}\frac{m^2g^2}{k}$

(D) $mgh+\frac{m^2g^2}{k}$

(E) $2mgh$

解：物体速度最大时 $\frac{dv}{dt}=0$，即加速度为零，此时物体受力平衡。设速度最大时弹簧压缩量为 x，则

$$mg = kx\text{，}$$

$$x=\frac{mg}{k}\text{。}$$

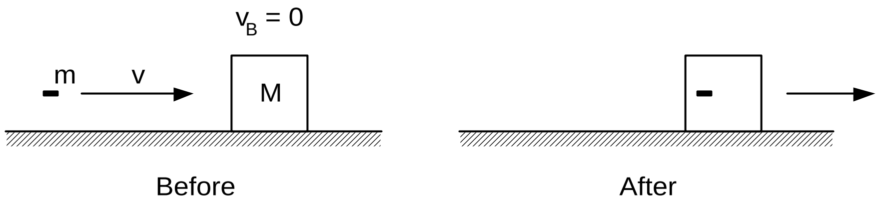
由机械能守恒

$$mg(h+x)=\frac{1}{2}mv^2+\frac{1}{2}kx^2\text{。}$$

将 x 的表达式代入，

$$E_k=\frac{1}{2}mv^2=mg(h+x)-\frac{1}{2}kx^2=mgh+\frac{m^2g^2}{2k}\text{。}$$

选（C）。



4. A bullet of mass m traveling at speed v strikes a block of mass M, initially at rest, and is embedded in it as shown above. How far will the block with the bullet embedded in it slide on a rough horizontal surface of coefficient of kinetic friction μ_k before it comes to rest?

(A) $(\frac{m+M}{m})(\frac{v^2}{2\mu_k g})$

(B) $(\frac{m+M}{M})(\frac{v^2}{2\mu_k g})$

(C) $(\frac{m+M}{M})^2(\frac{v^2}{2\mu_k g})$

(D) $(\frac{m}{m+M})(\frac{v^2}{2\mu_k g})$

(E) $(\frac{m}{m+M})^2(\frac{v^2}{2\mu_k g})$

解：子弹射入过程很短，时间及此过程中所走过距离可忽略不计。由动量守恒，

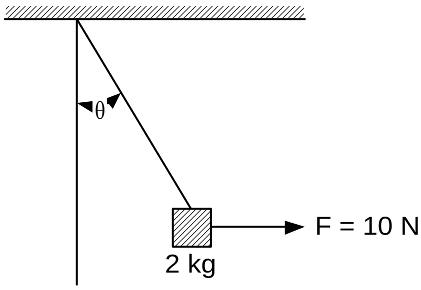
$$mv = (m+M)V \text{。}$$

之后由于摩擦做匀减速运动，机械能全部转换为摩擦热，则

$$\frac{1}{2}(m+M)V^2 = \mu_k (m+M)gs \text{，}$$

$$s = \frac{V^2}{2\mu_k g} = (\frac{v^2}{2\mu_k g})(\frac{m}{m+M})^2 \text{。}$$

选（E）。



5. A 2-kilogrom box hangs by a massless rope from a ceiling. a force slowly pulls the box horizontally to the side until the horizontal force is 10 newtons. The box is then equilibrium as shown above. The angle that the rope makes with the vertical is closest to

(A) arctan 0.5
(B) arcsin 0.5
(C) arctan 2.0
(D) arcsin 2.0
(E) 45°

解：物体重 20N，角度

$$\theta = \arctan\frac{10}{20} = \arctan 0.5 \text{。}$$

选（A）。

6. A 5-kilogram stone is dropped on a nail and drives the nail 0.025 meter into a piece of wood. If the stone is moving at 10 meters per second when it hits the nail, the average force exerted into the wood is most nearly

(A) 10 N
(B) 100 N
(C) 1000 N
(D) 10,000 N
(E) 100,000 N

解：力对石头做功等于其动能减少，

$$F=\frac{1}{2}mv^2/h=\frac{1}{2}\times5\times10^2/0.025=10000\text{N}$$ 。

选（D）。

7. A machine gun fires bullets of mass 20 grams each at a rate of 1200 bullets per minute. The bullets hit a thick wooden target at a speed of 600 meters per second and are stopped in the target. The average force exerted on the target by the bullets striking it is

(A) 144N

(B) 240N

(C) 14.4×10^3 N

(D) 24.0×10^3 N

(E) 14.4×10^3 N

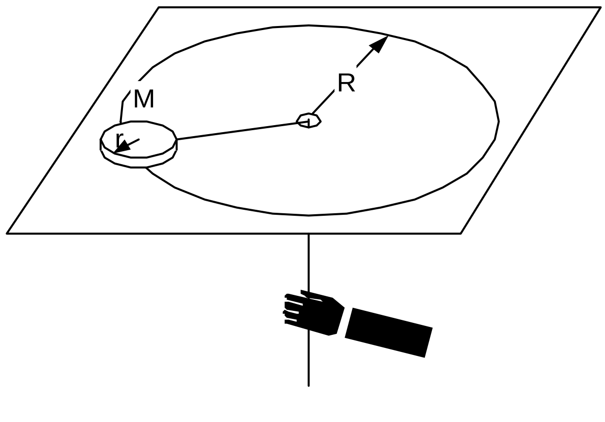
解：由冲量定理

$$\int Fdt=\Delta P,$$

$$F=\frac{\Delta mV}{\Delta t}=\frac{20\times10^{-3}\times1200\times600}{60}=240\text{ N}$$ 。

选(B)。

<u>Questions 8-9</u>



A uniform cylindrical puck of radius r and mass M is attached to the end of a cord that passes through a hole in a fixed horizontal frictionless table as shown above. The center of the puck moves in a circle of radius R with angular speed ω

8. The magnitude of the angular momentum of the puck about the hole is

(A) $\frac{3}{2}M\omega R^2$

(B) $M(R^2+r^2)\omega$

(C) $M(R^2+r^2)^{\frac{1}{2}}\omega^2$

(D) $M(R^2+\frac{1}{2}r^2)\omega$

(E) $M(R^2+\frac{1}{2}r^2)^{\frac{1}{2}}\omega^2$

解：首先，圆盘绕自身圆心的转动惯量为

$$I=\frac{1}{2}Mr^2,$$

由平行轴定理，圆盘绕桌子中心的转动惯量为

$$I=I_0+MR^2=\frac{1}{2}Mr^2+MR^2,$$

所以它相对于洞的角动量为

$$L=I\omega=M(R^2+\frac{1}{2}r^2)^{\frac{1}{2}}\omega^2。$$

选（E）。

9. If the string is pulled slowly downward so that the center of the puck moves in a circle of smaller radius, quantities that are conserved include which of the following?

I. Angular momentum
II. Linear momentum
III. Kinetic energy

(A) I only
(B) III only
(C) I and II only
(D) II and III only
(E) I, II, and III

解：由于是有心力，所以冰球相对洞的角动量守恒，I 正确。在圆运动中动量本身就不是一个守恒量，变化中冰球又显然受力，所以动量守恒无从谈起，II 不对。变化中冰球受到了指向洞的力，沿此方向又有位移，所以绳子对其做功，动能不守恒。或者由动能为 $\frac{1}{2}I\omega^2$，而角动量守恒，$I\omega=\text{constant}$，动能显然不能守恒，III 不对。选（A）。

10. An expression for the potential energy of two ions is

$$P.E.=\frac{-kq^2}{r}+\frac{b}{r^9}.$$

What is the constant b as a function of the equilibium spacing r_0?

(A) $\frac{kq^2}{r_0^{10}}$

(B) $\frac{kq^2}{r_0}$

(C) $\frac{8kq^2}{9r_0}$

(D) $\dfrac{kq^2r_0}{10}$

(E) $\dfrac{kq^2r_0^8}{9}$

解：因为 r_0 处势能最小，由

$$\frac{dU_{P.E.}}{dr}=\frac{kq^2}{r^2}-\frac{9b}{r^{10}}=0$$

解出

$$r_0^{\ 8}=\frac{9b}{kq^2},$$

$$b=\frac{kq^2r_0^{\ 8}}{9}\text{。}$$

选（E）。

11. A rigid cylinder rolls at constant speed without slipping on top of a horizontal plane surface. The acceleration of a point on the circumference of the cylinder at moment when the point touches the plane is

(A) directed forward
(B) directed backward
(C) directed up
(D) directed down
(E) zero

解：因为触地点相对于圆心做匀速圆周运动，所以相对于圆心的加速度为竖直向上方向，而圆心做匀速直线运动，加速度为零，所以触地点相对于圆心的加速度就是其对地的加速度。选(C)。

<u>Questions 12-13</u>

A cylinder with moment of inertia 4 kg·m^2 about a fixed axis initially rotates at 80 radians per second about this axis. A constant torque as applied to slow it down to 40 radius per second.

12. The kinetic energy lost by the cylinder is

(A) 80 J
(B) 800 J
(C) 4000 J
(D) 9600 J
(E) 19,200 J

解：刚体定轴转动的动能为

$$T=\frac{1}{2}I_{\omega}\omega^2,$$

动能变化为

$$\Delta T=\frac{1}{2}I\left(\omega_0{}^2-\omega'^2\right)=\frac{1}{2}\times4\times\left(80^2-40^2\right)=9600\text{J}\ 。$$

选(D)。

13. If the cylinder takes 10 seconds to reach 40 radius per second, the magnitude of the applied torque is
(A) 80 N·m
(B) 40 N·m
(C) 32 N·m
(D) 16 N·m
(E) 8 N·m

解：由刚体定轴转动的动力学方程

$$M=I\beta\ ,$$

其中 M 为外力矩，β为刚体的角加速度。对于本题

$$M=I\beta=I\frac{\omega_0-\omega'}{t}=4\times\frac{80-40}{10}=16\text{N}\cdot\text{m}\ 。$$

选（D）。

<u>Questions 14-15</u>

A nonrelativistic particle of mass m moves in a plane. Its position is described by the polar coordinates r and θ, with time derivatives $\dot{r}$ and $\dot{\theta}$. There exists a potential energy U = kr^2, where k is a constant.

14. Which of the following is the Lagrangian of the particle?
(A) $L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2\right)-kr^2$
(B) $L=\frac{1}{2}m\left(\dot{r}^2+\theta^2\right)+kr^2$
(C) $L=\frac{1}{2}m\left(\theta^2\dot{r}^2+r^2\dot{\theta}^2\right)-kr^2$
(D) $L=\frac{1}{2}m\left(\dot{r}^2+r\dot{r}\dot{\theta}+r^2\dot{\theta}^2\right)-kr^2$
(E) $L=\frac{1}{2}m\left(\dot{r}^2+2r\dot{r}\dot{\theta}+r^2\dot{\theta}^2\right)-kr^2$

解：对于有势力系，Lagrange 函数定义为

$$L=T-V\ 。$$

本题中，由于极坐标系下的速度表达式为

$$\mathbf{v}=\dot{r}\mathbf{e}_r+r\dot{\theta}\,\mathbf{e}_\theta\ 。$$

而两分量之间垂直，所以

$$L=T-V=\frac{1}{2}m\left(v^2{}_r+v^2{}_\theta\right)-V=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2\right)-kr^2\ 。$$

选(A)。

15. Which of the following quantities remains constant?

(A) $m\left(\dot{r}^2 + r^2\dot{\theta}^2\right)$

(B) $mr^2\dot{\theta}^2$

(C) kr^2

(D) $mr\dot{\theta}$

(E) $mr^2\dot{\theta}$

解：$\mathbf{F} = \frac{dU}{dr} = 2kr\hat{\mathbf{r}}$，或者由于$U = kr^2$与θ无关，为有心力场，角动量守恒。利用上题中速度的表达式，径向速度为$r\dot{\theta}$，所以角动量表达式为$mr^2\dot{\theta}$。答案选(E)。

16. A ball dropped from a height h. As it bounces off the floor, its speed is 80 percent of what it was just before it hit the floor. The ball will then rise to a height of most nearly

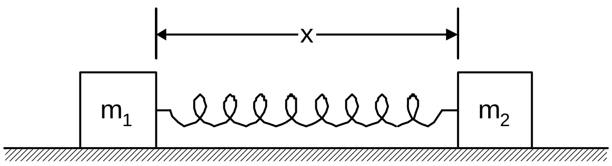
(A) 0.94 h
(B) 0.80 h
(C) 0.75 h
(D) 0.64 h
(E) 0.50 h

解：由机械能守恒

$$\frac{1}{2}mv^2 = mgh,$$

$$\frac{h'}{h} = \left(\frac{v'}{v}\right)^2 = 0.8^2 = 0.64。$$

选(D)。



17. Two masses, m_1 and m_2 , are joined by a massless spring of force constant k and placed on a horizontal frictionless surface as shown above. The system is released from rest when the separation between the masses is x. If the unstretched length of the spring is x_0, the speed of mass m_1 when the two masses are a distance x_0 apart is

(A) $\sqrt{\frac{k}{m_1}(x-x_0)^2}$

(B) $\sqrt{\frac{k}{m_2}(x-x_0)^2}$

(C) $\sqrt{\frac{k}{m_1+m_2}(x-x_0)^2}$

(D) $\sqrt{\frac{k}{m_1}\cdot\frac{m_2}{m_1+m_2}(x-x_0)^2}$

(E) $\sqrt{\frac{k}{m_2}\cdot\frac{m_1}{m_1+m_2}(x-x_0)^2}$

解：设恢复原长时两物体速度为 v_1、v_2，由动量守恒

$$m_1v_1=m_2v_2\text{。}$$

由机械能守恒

$$\frac{1}{2}k\left(x-x_0\right)^2=\frac{1}{2}m_1{v_1}^2+\frac{1}{2}m_2{v_2}^2\text{。}$$

联立以上二式，解得

$$v_1=\sqrt{\frac{k}{m_1}\cdot\frac{m_2}{m_1+m_2}(x-x_0)^2}\text{。}$$

选(D)。

17. What is the number of degrees of freedom for 6 particles moving freely in one plane?

(A) 6

(B) 8

(C) 10

(D) 12

(E) 18

解：二维运动，每个粒子的自由度为 2，而且相互之间无关联，没有限制条件，所以总数为 2×6=12。答案选（D）。

18. A mass m_1 at the end of a spring executes simple harmonic motion with a period T_1. The period of oscillation of a different mass m_2 on the same spring is

(A) $T_1\sqrt{\frac{m_2}{m_1}}$

(B) $T_1\sqrt{\frac{m_1}{m_2}}$

(C) T_1

(D) $2\pi\sqrt{\frac{m_1}{m_2}}$

(E) $2\pi\sqrt{\dfrac{m_2}{m_1}}$

解：一维谐振子公式，$T = 2\pi\sqrt{\dfrac{m}{k}}$ 。答案选（A）。

19. Under the influence of a mutual interaction, an object orbits another object that is fixed. The orbit lies in a plane and the areas swept out by the radius vector in equal times are equal. What can be correctly concluded about the force between the objects?

(A) It is central.

(B) It is inverse-square.

(C) It is conservative.

(D) It is gravitational.

(E) None of these conclusions is justified.

解：角动量

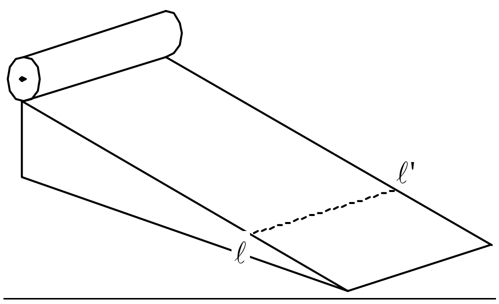
$$|\mathbf{L}| = |\mathbf{r}\times m\mathbf{v}| = m\left|\mathbf{r}\times\frac{d\mathbf{r}}{dt}\right| = 2m\frac{d\sigma}{dt}\ ,$$

其中

$$d\sigma = \frac{1}{2}|\mathbf{r}\times d\mathbf{r}|$$

为矢径 **r** 在 dt 时间内扫过的面积。由本题所给条件，$\dfrac{d\sigma}{dt}$ 为常量，故角动量守恒。有心力可保证角动量守恒。选（A）。

Questions 20-21



In two experiments, two cylinders, X and Y, are released from rest at the top of the same inclined ramp and roll down without slipping. Let t_X and t_Y be the respective times taken for the cylinders to reach a particular line ll’ on the ramp shown above.

20. In the first experiment, both cylinders are solid, uniform, and of identical dimensions; but they are made of different materials so that the mass of X is twice that of Y, $M_X = M_Y$. Which of the following relationships between t_X and t_Y is correct?

(A) $t_X \geq 2t_Y$

(B) $2t_Y > t_X > t_Y$

(C) $t_X = t_Y$

(D) $2t_X > t_Y > t_X$

(E) $t_Y \geq 2t_X$

解：设斜面倾角为θ，圆柱体受摩擦力为 f，则

$$mg\sin\theta - f = ma\text{。}$$

因为摩擦力 f 提供了力矩，所以

$$fr = I\beta\text{。}$$

无滑滚动条件

$$\beta r = a\text{。}$$

由以上三式解得

$$a = \frac{mg\sin\theta}{m + \frac{I}{r^2}}\text{。}$$

圆柱体转动惯量 $I = \frac{1}{2}mr^2$，代入上式得

$$a = \frac{2}{3}g\sin\theta\text{。}$$

与 m、r 无关。选（C）。

21. In the second experiment, both cylinders are solid, uniform, of identical length, and of the same density; but their radii are different so that the mass of X is twice that of Y, $M_X = 2M_Y$. Which of the following relationships between t_X and t_Y is correct?

(A) $t_X \geq 2t_Y$

(B) $2t_Y > t_X > t_Y$

(C) $t_X = t_Y$

(D) $2t_X > t_Y > t_X$

(E) $t_Y \geq 2t_X$

解：利用上一问的结论。选(C)。

22. A golf ball is hit from ground level with an initial velocity v_0 at an angle θ with respect to the ground. If air resistance is negligible and the magnitude of the gravitational acceleration is g, the ball hits the ground at what distance from the point at which it was hit?

(A) $\frac{v_0^2}{g}$

(B) $\frac{v_0^2\sin\theta}{g}$

(C) $\frac{v_0^2\cos\theta}{g}$

(D) $\dfrac{2{v_0}^2\tan\theta}{g}$

(E) $\dfrac{2{v_0}^2\sin\theta\cos\theta}{g}$

解：飞行时间为

$$t=\frac{2v_0\sin\theta}{g},$$

水平位移为

$$S=v_0\cos\theta\, t=\frac{2{v_0}^2\sin\theta\cos\theta}{g}。$$

选（E）。

23. A spherical neutron star has a uniform mass density ρ. What is the period of rotation below which material will fly off the equator? (Use nonrelativistic mechanics and let G be the universal gravitational constant.)

(A) $\dfrac{3}{4\pi G}$

(B) $\dfrac{4}{3\pi G}$

(C) $\left(\dfrac{3}{8\pi\rho G}\right)^{\frac{1}{2}}$

(D) $\left(\dfrac{\pi}{\rho G}\right)^{\frac{1}{2}}$

(E) $\left(\dfrac{3\pi}{\rho G}\right)^{\frac{1}{2}}$

解：临界状态质点所受万有引力全部用来提供所需向心力：

$$\frac{GMm}{r^2}=m\omega^2 r,$$

$$\omega=\sqrt{\frac{GM}{r^3}}=\sqrt{\frac{4\pi G\rho}{3}},$$

所以

$$T=\frac{2\pi}{\omega}=\left(\frac{3\pi}{\rho G}\right)^{\frac{1}{2}}。$$

选（E）。

24. A particle of mass M moves along the x-axis under the influence of a conservative field with the potential energy $V = \frac{b}{2}x^2$. If the particle starts from rest at x = 1, its maximum velocity is

(A) $\sqrt{\frac{Mb}{2}}$

(B) $\sqrt{Mb}$

(C) $\sqrt{2Mb}$

(D) $\sqrt{\frac{b}{2M}}$

(E) $\sqrt{\frac{b}{M}}$

解：因为所受力为保守力，机械能守恒。势能最小时速度最大：

$$\frac{b^2}{2}x_{\max} = \frac{1}{2}mv_{\max}{}^2,$$

解得

$$v_{\max} = \sqrt{\frac{b}{M}}。$$

选(E)。

25. A particle moves in a force field given by **F** = r^2 **r**, where **r** is the position vector. If there are no other forces, quantities that remain constant include which of the following?

I. Total energy

II. Torque about the origin

III. Angular momentum about the origin

(A) I only

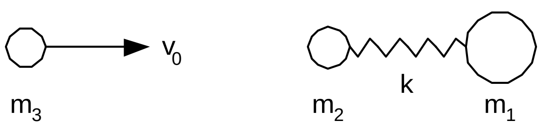
(B) III only

(C) I and II only

(D) II and III only

(E) I, II, and III

解：力场为保守场，能量守恒，I 正确。又为有心力场，且 **F** 指向原点，相对于原点的力矩为零，II 正确。显然相对于原点的角动量守恒，III 正确。选（E）。



26. Particle 1 of mass m_1 and particle 2 of mass $m_2 = \frac{1}{2}m_1$ are coupled by a massless spring of

force constant k and lie at rest on a horizontal frictionless surface as shown above. A third particle of mass $m_3 = m_2 = \frac{1}{2}m_1$ and speed v_0 strikes particle 2 along the axis of the spring and sticks to particle 2. The speed of the center of mass of the system after the collision is

(A) $\frac{v_0}{4}$

(B) $\frac{v_0}{3}$

(C) $\frac{v_0}{2}$

(D) v_0

(E) $v_0 + \sqrt{\frac{8k}{3m_1}}$

解：质心动量等于各部分动量之和。碰撞前后动量守恒，

$$m_3 v_0 = (m_1 + m_2 + m_3)V_{MC}，$$

$$V_{MC} = \frac{m_3}{m_1 + m_2 + m_3}v_0 = \frac{v_0}{4}\ 。$$

选(A)。

27. A planet of mass m moves about the Sun of mass M. G is Newton's constant, r is the planet's distance from the Sun, and v is the planet's speed. Except for an additive constant the planet's potential energy is

(A) $\frac{1}{2}mv^2 + \frac{GMm}{r}$

(B) $\frac{1}{2}mv^2 - \frac{GMm}{r}$

(C) $-\frac{GMm}{r}$

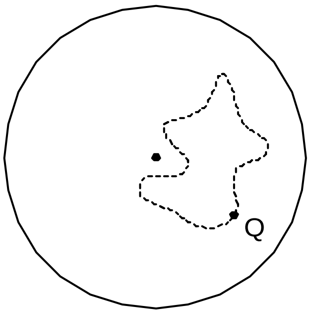
(D) $-\frac{GMm}{r^2}$

(E) mgr

解：取无穷远为势能零点，则万有引力势为

$$U = \int_r^{\infty} -\frac{GM}{r^2}dr = -\frac{GM}{r}\ 。$$

选（C）。



28. The diagram above show a top view of a phonograph turntable mounted on a frictionless bearing. Initially, the turntable is at rest and a massive bug is asleep at point Q. The bug wakes up, takes a walk such as the one indicated by the dotted line, returns to Q, and goes back to sleep. Afterward, the turntable is

(A) again at rest in the same position

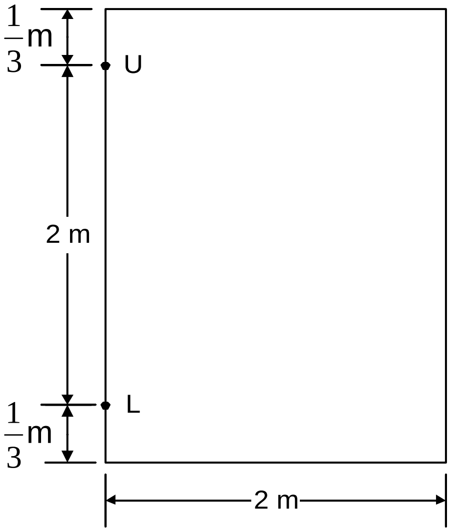
(B) at rest, but in the same position only if the walk did not encircle the pivot point

(C) at rest, but not necessarily in the same position whether or not the walk encircled the pivot point

(D) at rest only if the walk did not encircle the pivot point

(E) not necessarily at rest, whether or not the pivot point was encircled

解：因为系统没受到相对圆心的外力矩，所以相对于圆心角动量守恒，最后转桌必处于静止状态。小虫和转桌间有相对运动，转桌可有任意的角位移。答案选（C）。



29. The weight of a door is entirely supported by the upper hinge U which is 2 meters from the lower hinge L as shown above. Assume the mass per unit area of the door is constant and the hinges have negligible size. If the door weighs 200 newtons and is 2 meters wide, of what magnitude is the horizontal force exerted by the lower hinge L?

(A) 80 N

(B) 100 N

(C) 120 N

(D) 140 N

(E) 200 N

解：因为门的重量完全由 U 点支撑，所以 L 点无竖直方向的力，L 处受力沿水平方向。系

统对 U 点力矩平衡，

$$G \cdot 1\text{m} = F_L \cdot 2\text{m}\ ,$$

$$F_L = 100\text{N}\ 。$$

选（B）。

30. A thin uniform rod of mass m and length l is hinged at one end to a level floor and stands vertically. If allowed to fall, the rod will strike the floor with an angular speed ω. If the same rod were cut in half to length l/2 and the initial conditions remained unchanged, it would strike the floor with an angular speed most nearly equal to

(A) 2ω

(B) $\omega\sqrt{2}$

(C) ω

(D) $\omega/\sqrt{2}$

(E) ω/2

解：由能量守恒，势能转换为转动能量

$$mg\frac{l}{2} = \frac{1}{2}I\omega^2\ 。$$

对于质量为 m 长度为 l 的均匀杆，相对一端的转动惯量为

$$I = \int_0^l x^2\frac{m}{l}dx = \frac{1}{3}ml^2\ 。$$

解得

$$\omega = \sqrt{\frac{3g}{l}}\ ,$$

可见只与杆长有关。第二次杆长变为一半，$\omega' = \sqrt{2}\omega$ 。选（B）。

31. An asteroid has a radius of 5×10^5 meters and an acceleration due to gravity of $\frac{1}{4}$ that on Earth. The velocity of escape for an object starting on the surface of the asteroid is most nearly

(A) 150 m/s

(B) 720 m/s

(C) 1560 m/s

(D) 5550 m/s

(E) 11,200 m/s

解：所谓逃逸速度，是恰好使物体的机械能为 0。所以

$$-\frac{GM_a m}{r} + \frac{1}{2}mv_e^{\ 2} = 0\ ,$$

$$v_e = \sqrt{\frac{2GM_a}{r}}\ 。$$

而由重力加速度

$$\frac{GM_a}{r^2}=\frac{1}{4}g\text{，}$$

$$\mathrm{GM_a}=\frac{1}{4}gr^2\text{，}$$

所以

$$v_e=\sqrt{\frac{gr}{2}}=\sqrt{\frac{9.8\times5\times10^5}{2}}=1565\text{。}$$

选(C)。

32. A solid ball weighs 5.0 newtons in air and 3.0 newtons submerged in water. If the ball weighs 2.0 newtons submerged in an unknown liquid, the specific gravity of the unknown liquid is most nearly

(A) 0.66
(B) 1.00
(C) 1.25
(D) 1.50
(E) 1.75

解：由重力与浮力和拉力平衡

$$G-F_{水}=3.0\mathrm{N}\text{，}$$

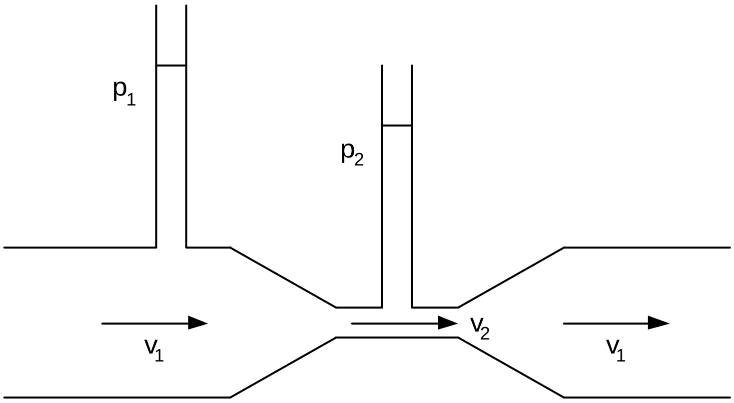
$$G-F_{液}=2.0\mathrm{N}\text{。}$$

由阿基米德定律 $F_{浮}=\rho gV$，得

$$\frac{F_{液}}{F_{水}}=\frac{\rho_{液}}{\rho_{水}}\text{，}$$

$$\rho_{液}=\frac{F_{液}}{F_{水}}\rho_{水}=\frac{G-2.0}{G-3.0}\rho_{水}=1.5\text{。}$$

选（D）。



33. Water flows through a Venturi tube as shown in the diagram above. The radius of the large cross section of the pipe is 2 centimeters and the raduis of the constricted portion of the pipe is 1 centimeter. If the speed of the water in the large cross section is 1 meter per second, the pressure difference ($p_1 - p_2$) is most nearly

(A) 0.6×10^2 N/m

(B) 3×10^2 N/m

(C) 1.5×10^3 N/m

(D) 7.5×10^3 N/m

(E) 37.5×10^3 N/m

解：由流量守恒

$$v_1 S_1 = v_2 S_2 \text{，}$$

$$v_2 = 4v_1 \text{。}$$

由 Bernoulli 方程，

$$\frac{1}{2}\rho {v_1}^2 + P_1 + \rho g h_1 = \frac{1}{2}\rho {v_2}^2 + P_2 + \rho g h_2 \text{，}$$

$$P_1 - P_2 = \frac{1}{2}\rho({v_2}^2 - {v_1}^2) + \rho g(h_2 - h_1) \text{。}$$

代入数字计算，选（B）。考试现场有可能不记得 Bernoulli 方程，其实它是机械能守恒在流体力学中的一种体现。再结合量纲分析，很容易推出这个形式。

34. A rock is thrown vertically upward with initial speed v_0. Assume a friction force proportional to –**v**, where **v** is the velocity of the rock, and neglect the buoyant force exerted by air. Which of the following is correct?

(A) The acceleration of the rock is always equal to **g**.

(B) The acceleration of the rock is equal to **g** only at the top of the flight.

(C) The acceleration of the rock is always less than **g**.

(D) The speed of the rock upon return to its starting point is v_0.

(E) The rock can attain a terminal speed greater than v_0 before it returns to its starting point.

解：在顶点，石块的速度为零，此时空气摩擦力为零，故加速度为 **g**，选（B）。对于选项（E），石块有可能获得收尾速度，但由于有阻力，必小于 v_0。

<u>Question 35-37</u>

The potential energy of a body constrained to move on a straight line is kx^4, where k is a constant. The position of the body is x, its speed v, its linear momentum p, and its mass m.

35. The force on the body is

(A) $\frac{1}{2}mv^2$

(B) $-4kx^3$

(C) kx^4

(D) $-\frac{kx^5}{5}$

(E) mg

解：直接对势能求导得

$$F = -\frac{dU}{dx} = -4kx^3\text{。}$$

选(B)。从量纲上看，（A）、（C）为能量，（D）为能量乘以距离，均不是力的单位。

36. The Hamiltonian function for this system is

(A) $\frac{p^2}{2m} + kx^4$

(B) $\frac{p^2}{2m} - kx^4$

(C) kx^4

(D) $\frac{1}{2}mv^2 - kx^4$

(E) $\frac{1}{2}mv^2$

解：对于变换方程不显含 t 的有势力系（这一条件在 SUB 考试中几乎必然满足），H=T+V。选（A）。

37. The body moves from x_1 at time t_1 to x_2 at time t_2. Which of the following quantities is an extremum for the x–t curve corresponding to this motion, if end points are fixed?

(A) $\int_{t_1}^{t_2}\left(\frac{1}{2}mv^2 - kx^4\right)dt$

(B) $\int_{t_1}^{t_2}\left(\frac{1}{2}mv^2\right)dt$

(C) $\int_{t_1}^{t_2}(mxv)dt$

(D) $\int_{x_1}^{x_2}\left(\frac{1}{2}mv^2 + kx^4\right)dx$

(E) $\int_{x_1}^{x_2}(mv)dx$

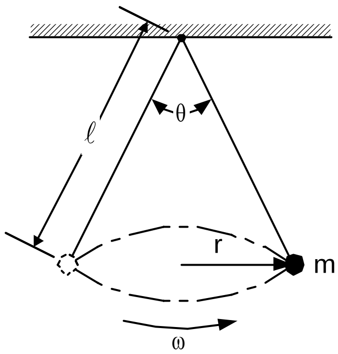
解：本题涉及分析力学的变分原理部分。由 Hamilton 原理，定义 Hamilton 作用量

$$S=\int_{t_1}^{t_2}Ldt\ ,$$

则真实运动使 S 取极值，其中 L 为拉格朗日量，

$$L=T-V\ 。$$

选（A）。



38. The figure above represents a point mass m attached to the ceiling b a cord of fixed length l. If the point mass moves in a horizontal circle of radius r with uniform angular velocity ω, the tension in the cord is

(A) $mg\left(\frac{r}{l}\right)$

(B) $mg\cos\left(\frac{\theta}{2}\right)$

(C) $\frac{m\omega r}{\sin\left(\frac{\theta}{2}\right)}$

(D) $m\left(\omega^2r^2+g^2\right)^{\frac{1}{2}}$

(E) $m\left(\omega^4r^2+g^2\right)^{\frac{1}{2}}$

解：竖直方向受力平衡

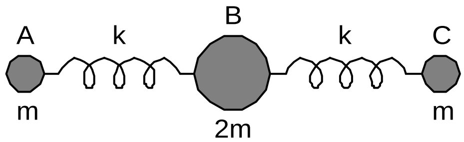
$$F\cos\theta/2=mg\ ,$$

$$F=\frac{mg}{\cos\theta/2}=\frac{mgl}{\sqrt{l^2-r^2}}\ 。$$

从另一个角度看 F 等于重力和离心力的合力

$$F = m\left(\omega^4 r^2 + g^2\right)^{\frac{1}{2}}。$$

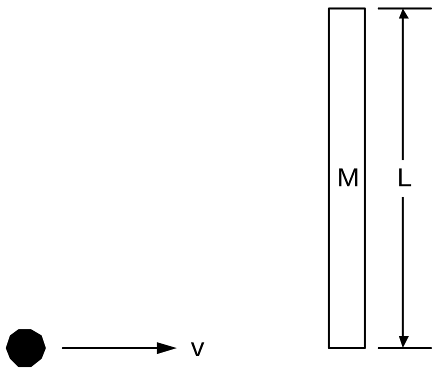
选（E）。



39. Three masses are connected by two springs as shown above. A longitudinal normal mode with frequency $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ is exhibited by

(A) A, B, C all moving in the same direction with equal amplitude
(B) A and C moving in opposite directions with equal amplitude, and B at rest
(C) A and C moving in the same direction with equal amplitude, and B moving in the opposite direction with the same amplitude
(D) A and C moving in the same direction with equal amplitude, and B moving in the opposite direction with twice the amplitude
(E) none of the above

解：（B）所属情况是可实现的，A、C 对 B 的作用力始终大小相等方向相反，B 保持静止，A、C 与普通弹簧振子完全相等。选（B）。



View from above

40. A uniform stick of length L and mass M lies on a frictionless horizontal surface. A point particle of mass m approaches the stick with speed v an a straight line perpendicular to the stick that intersects the stick at one end, as shown above. After the collision, which is elastic, the particle is at rest. The speed V of the center of mass of the stick after the collision is

(A) $\frac{m}{M}v$

(B) $\frac{m}{M+m}v$

(C) $\sqrt{\frac{m}{M}}v$

(D) $\sqrt{\dfrac{m}{M+m}}v$

(E) $\dfrac{3m}{M}v$

解：由动量守恒，碰撞后杆的动量为 mv。由于质点组相对质心动量为零，所以质点组的动量等于质心速度乘以总质量：

$$mv = (M+m)V_c\text{，}$$

$$V_c = \frac{m}{M+m}v\text{。}$$

选（B）。

41. A particle of mass m that move s along the x-axis has potential energy $V(x)=a+bx^2$，where a and b are positive constants. Its initial velocity is v_0 at x=0. It will execute simple harmonic motion with a frequency determined by the value of

(A) b alone

(B) b and a alone

(C) b and m alone

(D) b, a, and m alone

(E) b, a, m, and v_0

解：由一维谐振子公式，$\omega = \sqrt{\dfrac{2b}{m}}$。选（C）。ω显然与 a 无关，因为势能的零点选取有任意性，或者说质点受力 $F = -\dfrac{dV}{dx} = -2bx$ 与 a 无关。

<u>Questions 42-43</u>

The equation of motion of a rocket in free space can be written

$$m\frac{dv}{dt} + u\frac{dm}{dt} = 0$$

where m is the rocket's mass, v is its velocity, t is time, and u is a constant.

42. The constant u represents the speed of the

(A) rocket at t=0

(B) rocket after its fuel spent

(C) rocket in its instantaneous rest frame

(D) rocket's exhaust in a stationary frame

(E) rocket's exhaust relative to the rocket

解：由动量守恒

$$(m+dm)v = m(v+dv) + Vdm\text{，}$$

其中 V 为燃料被抛出后的速度。

$$mdv + (V-v)dm = 0\text{，}$$

令$u = V - v$为燃料的相对抛出速度，则

$$m\frac{dv}{dt} + u\frac{dm}{dt} = 0 \text{。}$$

选(E)。

43. The equation can be solved to give b as a function of m. If the rocket has m=m_0 and v=0 when it starts, what is the solution?

(A) um_0 / m

(B) $u\exp(m_0 / m)$

(C) $u\sin(m_0 / m)$

(D) $u\tan(m_0 / m)$

(E) None of the above.

解：由前题结论

$$m\frac{dv}{dt} + u\frac{dm}{dt} = 0$$

得

$$\frac{dm}{m} = -\frac{dv}{u} \text{。}$$

两边积分得

$$m(t) = m_0 e^{-\frac{v(t)-v_0}{u}},$$

因此

$$v(t) = v_0 + u\ln\frac{m_0}{m(t)} = u\ln\frac{m_0}{m(t)} \text{。}$$

选（E）。

44. The period of a hypothetical Earth satellite orbiting at sea level would be 80 minutes. In terms of the Earth's radius R_e, the radius of a synchronous satellite orbit (period 27 hours) is most nearly

(A) 3 R_e

(B) 7 R_e

(C) 18 R_e

(D) 320 R_e

(E) 5800 R_e

解：由向心力公式

$$m\omega^2 r = \frac{GMm}{r^2},$$

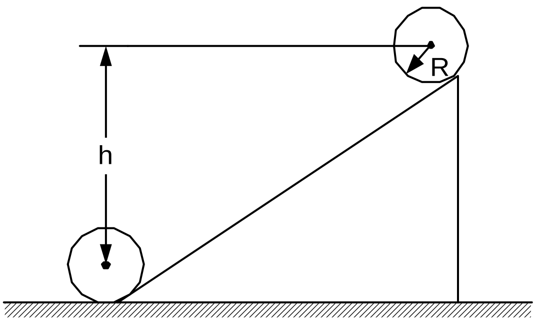
其中$\omega = \frac{2\pi}{T}$，得

$$T^2 = \frac{4\pi^2}{GM} r^3,$$

周期比为

$$\frac{27 \times 60}{80} = \frac{81}{4},$$

由此得半径之比约为 7.4。选(B)。或者直接用 Kepler 第三定律，周期的平方与半径的立方之比为常数。但是要切忌，这只是对绕同一天体运行的星体适用。



45. A hoop of mass M and radius R is at rest at the top of an inclined plane as shown above. The hoop rolls down the plane without slipping. When the hoop reaches the bottom, its angular momentum around its center of mass is

(A) $MR\sqrt{gh}$

(B) $\frac{1}{2}MR\sqrt{gh}$

(C) $M\sqrt{2gh}$

(D) Mgh

(E) $\frac{1}{2}Mgh$

解：圆环的转动惯量为 $I = MR^2$ 。由能量守恒，

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = Mgh,$$

由无滑滚动，

$$\omega R = v,$$

联立上面两式，解得

$$\omega = \frac{\sqrt{gh}}{R}。$$

所以角动量

$$L = I\omega = MR^2 \cdot \frac{\sqrt{gh}}{R} = MR\sqrt{gh}。$$

选（A）。

46. A particle is constrained to move along the x-axis under the influence of the net force **F**=–k**x** with amplitude A and frequency f, where k is a positive constant. When x=A/2, the particle's speed is

(A) $2\pi fA$

(B) $\sqrt{3}\pi fA$

(C) $\sqrt{2}\pi fA$

(D) πfA

(E) $\frac{1}{3}\pi fA$

解：质点显然做简谐振动，势能为 $\frac{1}{2}kx^2$。由能量守恒，

$$\frac{1}{2}kA^2 = \frac{1}{2}k(\frac{A}{2})^2 + \frac{1}{2}mv^2,$$

解得

$$v = \sqrt{\frac{3k}{4m}}A = \frac{\sqrt{3}}{2}\omega A。$$

因为

$$f = \frac{1}{T} = \frac{\omega}{2\pi},$$

所以

$$v = \sqrt{3}\pi fA。$$

选(B)。

47. A system consists of two charged particles of equal mass. Initially the particles are far apart, have zero potential energy, and one particle has nonzero speed. If radiation is neglected, which of the following is true of the total energy of the system?

(A) It is zero and remains zero.

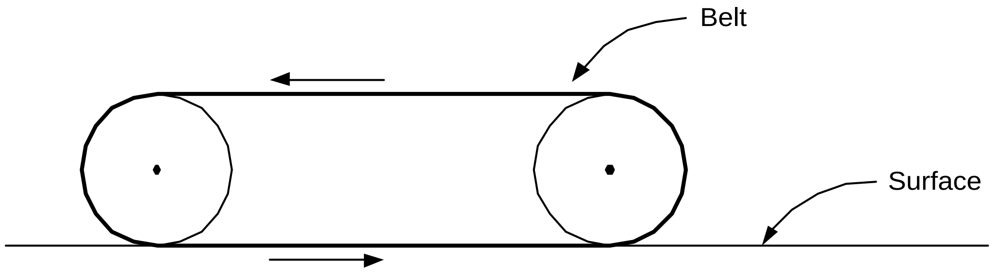
(B) It is negative and constant.

(C) It is positive and constant.

(D) It is constant, but the sign cannot be determined unless the initial velocities of both particles are known.

(E) It cannot be a constant of the motion because the particles exert force on each other.

解：开始时系统势能为 0，动能不为 0，从而动能为正值，总能量为正。不考虑辐射，则系统与外界无相互作用，能量守恒。选(C)。



48. An electric sander has a continuous belt that rubs against a wood surface as shown schematically above. The sander is 100 percent efficient and draws a current of 9 amperes from a 120-volt line. The belt speed is 10 meter per second. If the sander is pushing against the wood with a normal force of 100 newtons, the coefficient of friction is most nearly

(A) 0.02

(B) 0.2

(C) 0.4

(D) 1.1

(E) 10

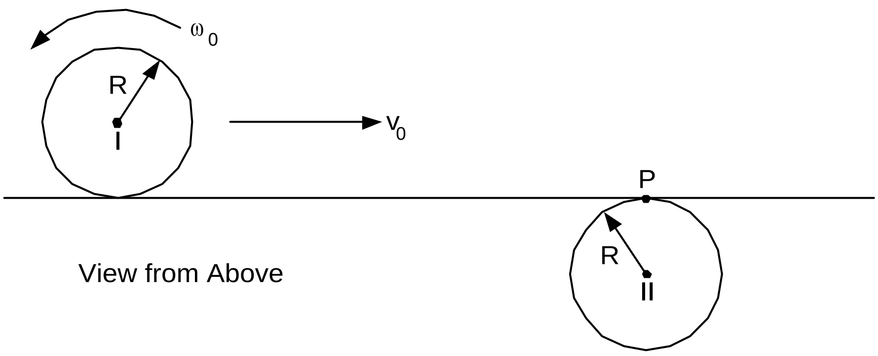
解：功率全消耗在克服摩擦力上，所以

$$f=\frac{P}{v}=\frac{UI}{v}=\frac{120\times 9}{10}=108\ \mathrm{N},$$

摩擦系数为

$$\mu=\frac{f}{N}=\frac{108}{100}=1.08\approx 1.1。$$

选(D)。



49. Two uniform cylindrical disks of identical mass M, radius R, and moment of inertia $\frac{1}{2}MR^2$, as shown above, collide on a frictionless, horizontal surface. Disk I, having an initial counterclockwise angular velocity ω_0 and a center-of-mass velocity $v_0=\frac{1}{2}\omega_0 R$ to the right, makes a grazing collision the two disks stick together, the magnitude of the total angular momentum about the point P is

(A) zero

(B) $\frac{1}{2}MR^2\omega_0$

(C) $\frac{1}{2}MRv_0$

(D) MRv_0

(E) dependent on the time of the collision

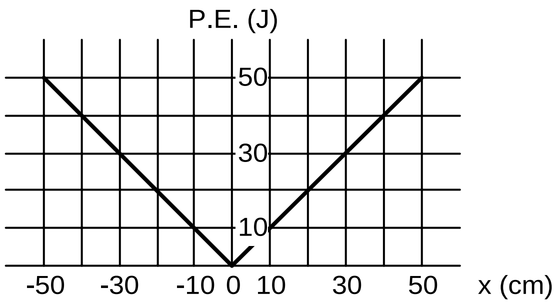
解：碰撞前对于盘 I，质心角动量为

$$MV_0R = \frac{1}{2}M\omega_0R^2，$$

方向垂直纸面向里；而转动部分角动量为

$$I\omega_0 = \frac{1}{2}M\omega_0R^2。$$

方向垂直纸面向外。两部分角动量大小相等方向相反，故总角动量为 0。因为碰撞前后相对 P 点的角动量守恒，所以碰撞后仍为 0。选（A）。注意仔细看图，盘 I 不是做无滑滚动。



50. The graph above shows the potential energy (P.E.) in joules of an object of mass m moving horizontally in a conservative one-dimensional force field. If the initial total energy of the object is 30 joules, which of the following statements is NOT correct?

(A) The magnitude of the force acting on the object when it is at x = −15 cm is 100 N

(B) The kinetic energy of the object when it is at x = 5 cm is 25 J.

(C) The speed of the object is a maximum at x = 0.

(D) The motion of the object is periodic in time and has an amplitude of 30 cm.

(E) The period of the motion is independent of the mass of the object.

解：$F = \frac{dU}{dx} = 100\ \mathrm{N}$，为恒力，且为恢复力，方向与位移相反，故（A）正确。（B）、（C）、（D）显然正确。因为质点受力大小保持不变，质量不同时，加速度不同，而振幅 A 相同，均为 30 cm，所以周期将不同。选（E）。

51. A thin uniform steel chain is 10 meters long with a mass density of 2 kilograms per meter. One end of the chain is atteched to a horizontal axle having a radius that is small compared to the length of the chain. If the chain initially hangs vertically, the work required to slowly wind it up on to the axle is closest to

(A) 100 J

(B) 200 J

(C) 1,000 J

(D) 2,000 J

(E) 10,000 J

解：铁链从垂吊状态变到缠卷状态，重力势能改变为

$$\Delta E_P = mg\Delta h = l\rho gl/2 = 2\times10\times10^2/2 = 1000\ \text{J}\ 。$$

外界对铁链做功 W ≥ E_P。选（C）。

52. Two circular hoops, X and Y, are hanging on nails in a wall. The mass of X is four-times shat of Y, and thr diameter of X is also four times that of Y. If the period of small oscilations of X is T, the period of small oscillations of Y is

(A) T
(B) T/2
(C) T/4
(D) T/8
(E) T/16

解：圆环相对于其圆心的转动惯量为 $I_0 = MR^2$ 。由平行轴定理，相对环上一点的转动惯量为

$$I = I_0 + MR^2 = 2MR^2\ 。$$

所以 X 、Y 相对于悬挂点的转动惯量之比为

$$\frac{I_X}{I_Y} = \frac{2M_X {R_X}^2}{2M_Y {R_Y}^2} = 64\ 。$$

由复摆周期公式

$$T = 2\pi\sqrt{\frac{I}{Gl}}\ ,$$

其中 l 为质心到转动轴的距离，X、Y 周期之比为

$$\frac{T_X}{T_Y} = \sqrt{\frac{I_X}{I_Y}\frac{G_Y l_Y}{G_X l_X}} = \sqrt{\frac{64}{1}\frac{1}{4}\frac{1}{4}} = 2\ 。$$

所以

$$T_Y = \frac{T_X}{2}\ 。$$

选（B）。

53. A certain 100-meter length of piano wire has a mass of 1 kilogram. If transverse waves move on the wire with a speed of 300 meters per second, the tension in the wire is closest to

(A) 10^1 N
(B) 10^2 N
(C) 10^3 N
(D) 10^4 N
(E) 10^5 N

解：由连续介质中的波速公式

$$v = \sqrt{\frac{T}{\rho}},$$

$$T = \rho v^2 = \frac{1}{100} \times 300^2 = 900\text{N},$$

选（C）。这个公式即使考试时记不清，也很容易由量纲分析得到。速度的量纲为米·秒$^{-1}$，线密度的量纲为千克·米$^{-1}$，张力量纲为千克·米·秒$^{-2}$，张力和线密度之比的量纲为米^2·秒$^{-2}$，为速度量纲的平方，于是可得公式的形式。

54. Which of the following is true of an automobile that is accelerated from rest along a straight level road by an engine that supplies constant power?

(A) Its acceleration is constant.

(B) Its acceleration decreases with time.

(C) Its acceleration is directly proportional to its velocity.

(D) Its velocity is proportional to the distance travelled.

(E) Its kinetic energy is proportional to the distance travelled.

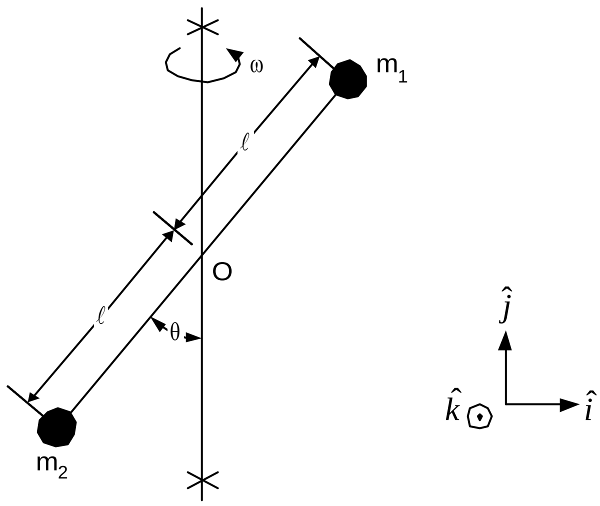
解：由功率的表达式

$$P = F \cdot v 。$$

功率恒定，则速度从 0 开始变大时，牵引力将减小。而摩擦力 f 不变，所以加速度

$$a = \frac{F - f}{m}$$

随速度的增加变小，直到减小为零。此后汽车作匀速直线运动。选（B）。



55. A rigid dumbbell consists of point masses m_1 and m_2 joined together by a massless rod of length 2l.The dumbbell is fixed at an angle to a shaft about which it rotates with angular velocity ω $\hat{j}$ in the coordinate system shown above. At the instant shown, the velocity of m_1 is directed into the paper, and the velocity of m_2 is directed out of the paper. The vector angular momentum L of the system about the point O is

(A) $(m_1 + m_2) l^2 \omega \hat{j}$

(B) $(m_1 + m_2)(l\cos\theta)^2 \omega\hat{j}$

(C) $(m_1 + m_2)(l\sin\theta)^2 \omega\hat{j}$

(D) $(m_1 + m_2)l^2\omega\sin\theta(\sin\theta\ \hat{j} - \cos\theta\ \hat{i})$

(E) $(m_1 + m_2)l^2\omega\sin\theta(\cos\theta\ \hat{j} + \sin\theta\ \hat{i})$

解：m_1 相对于 O 点的角动量为

$$\mathrm{L}_1 = \mathrm{r} \times m_1\mathrm{v} = lm_1v_1(-\cos\theta\ \mathrm{i} + \sin\theta\ \mathrm{j}) = m_1 l^2\omega\sin\theta(-\cos\theta\ \mathrm{i} + \sin\theta\ \mathrm{j})。$$

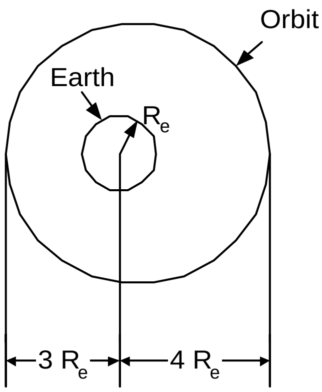
同理 m_2 相对于 O 点的角动量为

$$\mathrm{L}_1 = \mathrm{r} \times m_2\mathrm{v} = m_2 l^2\omega\sin\theta(-\cos\theta\ \mathrm{i} + \sin\theta\ \mathrm{j})$$

刚体角动量是各质点角动量之和，

$$\mathrm{L} = \mathrm{L}_1 + \mathrm{L}_2 = (m_1 + m_2)l^2\omega\sin\theta(-\cos\theta\ \mathrm{i} + \sin\theta\ \mathrm{j})。$$

选（D）。



56. A satellite is in elliptical orbit about the Earth as shown above with maximum distance from the center of the Earth of $4R_e$ and minimum distance of $3R_e$, where R_e is the radius of the Earth. If g is the acceleration due to gravity at the Earth's surface, the maximum speed of the satellite is

(A) $\sqrt{\frac{8}{21}gR_e}$

(B) $\sqrt{\frac{4}{7}gR_e}$

(C) $\sqrt{\frac{3}{4}gR_e}$

(D) $\sqrt{gR_e}$

(E) $\sqrt{\frac{4}{3}gR_e}$

解：在近地点和远地点，径向速度为零，所以速度 V_a、V_b 沿角向。由角动量守恒

$$V_a \cdot 3R_e = V_b \cdot 4R_e\text{；}$$

由机械能守恒

$$\frac{1}{2}m{v_a}^2 - \frac{GMm}{3R_e} = \frac{1}{2}m{v_b}^2 - \frac{GMm}{4R_e}\text{。}$$

联立以上两式，解得

$$v_a = \sqrt{\frac{8}{21}gR_e}\text{ 。}$$

因为在近地点时引力势能最小（绝对值最大），所以动能最大。所以近地点速度就是速度的最大值。选（A）。

57. A metal sphere of mass m falls from rest in a tall vertical tube containing a viscous fluid. The magnitude of the retarding force on the sphere is given by βv where v is the speed of the sphere and β is a constant. If the buoyant force on the sphere is negligible, the maximum (terminal) speed of the sphere as it falls through the fluid is most nearly

(A) $\frac{1}{2}mg\beta$

(B) mg β

(C) $\frac{1}{2}\frac{mg}{\beta}$

(D) $\frac{mg}{\beta}$

(E) $2\frac{mg}{\beta}$

解：达到最大（收尾）速度时，小球所受的合力为零，做匀速直线下滑运动。根据题意，忽略浮力有

$$mg = \beta v\text{ ,}$$

由此解得收尾速度

$$v = \frac{mg}{\beta}\text{。}$$

选（D）。

58. Under the influence of a mutual interaction, an object orbits another object that is fixed. The

orbits lies in a plane and the areas swept out by the radius vector in equal times are equal. What can be correctly concluded about the force between the objects?

(A) It is central.

(B) It is inverse-square.

(C) It is conservative.

(D) It is gravitational.

(E) None of these conclusions is justified.

解：面积速度

$$\frac{ds}{dt}=\left|\frac{1}{2}\mathbf{v}\times\mathbf{r}\right|=\frac{J}{2m}=const.,$$

又由于题中指明是平面运动，因此角动量 **J** 的方向也固定，所以角动量 **J** 守恒。是中心力场。选（A）。

59. A machine gun fires bullets of mass 20 grams each at rate of 1200 bullets per minute. The bullets hit thick wooden target at a speed of 600 meters per second and are stopped in the target. The average force exerted on the target by the bullets striking it is

(A) 144N

(B) 240N

(C) 14.4×10^{3} N

(D) 24.0×10^{4} N

(E) 14.4×10^{6} N

解：直接利用冲量原理即可写出靶子的受力

$$F=\frac{dp}{dt}=\frac{1200\times600\times0.020}{60}=240\text{N}。$$

选（B）。

<u>Questions 60-61</u> relate to a particle of mass M that is moving in an attractive central force field. The potential energy function representing the attractive central force field can be written as V(r) = −k/r. At a certain time the particle has angular momentum **L** and total energy E.

60. At some later time, which of the following statements will be true of the angular momentum **L** and total energy E of the particle?

(A) **L** will have changed, but E will not.

(B) E will have changed, but **L** will not.

(C) Neither **L** nor E will have changed.

(D) Both **L** and E will have changed.

(E) It is not possible to say what will happen to **L** and E.

解：由于为有心力，所以角动量 **L** 守恒；由于有势函数存在，说明其为保守力场，能量 E 守恒。选（C）。

61. For a given nonzero angular momentum, there is a minimum energy for which it is possible to find a solution to the equations of motion. At this minimum energy, the particle is moving in a

(A) circular orbit

(B) noncircular elliptical orbit

(C) parabolic orbit
(D) hyperbolic orbit
(E) straight line
解：题目中所给势场与万有引力相同，很多结论可以借用。由于物体运动轨道为抛物线时，机械能为 0；轨道为双曲线时，机械能为正。均比椭圆轨道能量高。（C）、（D）不对。当物体做圆运动时，在不改变其角动量情况下，给它一个径向速度，增加它的机械能，物体将变为做椭圆运动，而角动量并没有改变。同理通过减少径向速度为零，不改变角动量，也可将一物体从椭圆轨道变为圆轨道，机械能降低。或者说对任一椭圆轨道，总有一圆轨道，二者角动量相同，单元轨道能量低。选（A）。

62. Two particles, each with mass m, are attached to each other by a spring having a spring constant k. The composite molecule like object is traveling freely in space and, at the same time, it is vibrating. The frequency of vibration is

(A) $\frac{1}{4\pi}\sqrt{\frac{k}{m}}$

(B) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(C) $\frac{1}{\pi}\sqrt{\frac{k}{m}}$

(D) $\frac{1}{2\pi}\sqrt{\frac{k}{2m}}$ t

(E) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

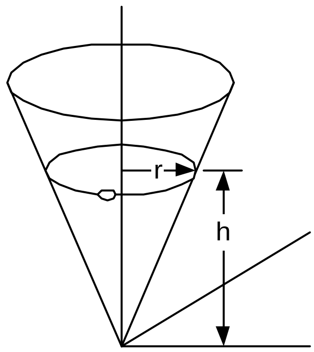
解：两个质点的质心不受外力，因此质心系是惯性参考系，在质心系中

$$\begin{aligned} m\dot{x}_1 + m\dot{x}_2 &= 0 \\ m\ddot{x}_1 &= -k(x_1 + x_2 - x_0) \end{aligned},$$

消元得标准波动方程形式

$$m\ddot{x}_1^{'} + 2kx_1^{'} = 0,$$

x_1’与 x_1 仅相差某个平移的常数。选（E）。
另一种看法：质心固定的话，每个质点相当于在半根弹簧的作用下独立振动，两根串联弹簧的等效弹性系数均为 2k。



63. A particle of mass m moves in a horizontal circle of radius r on the inside of a frictionless cone with a vertical axis as shown above. If the plane of the particle is a distance h above the apex of the cone, what is the speed of the particle?

(A) $\sqrt{gr}$

(B) $\frac{1}{2}\sqrt{gr}$

(C) $\sqrt{gh}$

(D) $\frac{1}{2}\sqrt{gh}$

(E) $r\sqrt{\frac{g}{h}}$

解：由于内壁光滑，因此根据受力分析易得重力与法向支持力的合力提供向心力。

$$mgctg\theta = \frac{mv^2}{r},$$

$$ctg\theta = \frac{h}{r}。$$

解以上方程，选（C）。

64. A ball rises after one bounce to 80 percent of the height from which it is released from rest. If air resistance is negligible and the ball is released from rest from a height H_0, what is its speed just before it hits the floor for its Nth bounce?

(A) $(0.8)^{2N}\sqrt{2gH_0}$

(B) $(0.8)^{N}\sqrt{2gH_0}$

(C) $(0.8)^{N-\frac{1}{2}}\sqrt{2gH_0}$

(D) $(0.8)^{\left(\frac{N-1}{2}\right)}\sqrt{2gH_0}$

(E) $(0.8)^{N-1}\sqrt{2gH_0}$

解：每次碰撞前后小球损失的能量为 80%，因此每次碰后返回的速度值是碰前的 $0.8^{1/2}$。第一次着地前小球的速度大小又机械能守恒不难算出

$$v_0 = \sqrt{2gH_0}\ 。$$

由等比级数的通项公式可以写出第 N 次着地前小球的速度

$$v_N = (0.8)^{\left(\frac{N-1}{2}\right)}\sqrt{2gH_0}\ 。$$

选（D）。

65. A particle of mass m is located at $x_0 = \pi/a$, a minimum of the periodic potential $U(x) = U_0\cos\alpha x$. If the particle is displaced slightly from x_0 and released, what is the approximate angular frequency of its oscillations?

(A) $\alpha\sqrt{\dfrac{U_0}{m}}$

(B) $\alpha\sqrt{\dfrac{U_0}{2m}}$

(C) $\dfrac{\alpha}{2}\sqrt{\dfrac{U_0}{m}}$

(D) $\sqrt{\dfrac{\alpha U_0}{m}}$

(E) $\sqrt{\dfrac{\alpha U_0}{2m}}$

解：将势能在 x_0 附近作 Taylor 展开

$$U(x_0 + \Delta x) = U_0\left(1 - \frac{1}{2}(\alpha\Delta x)^2\right) + o(\Delta x^2),$$

由其中二次项的系数求得等效弹性系数为

$$k = U_0\alpha\ ,$$

因此振动圆频率

$$\omega = \sqrt{\frac{k}{m}}\ 。$$

选（A）。

66. An alpha particle with nonrelativistic kinetic energy K collides head-on with a massive gold nucleus with charge Ze. The distance of slowest approach is

(A) $\dfrac{2Ze^2}{K}$

(B) $\dfrac{Z^2e^4}{K^2}$

(C) $\dfrac{2K}{Ze^2}$

(D) $\dfrac{Ze^2}{137K}$

(E) $2Ze^2K$

解：直接利用经典力学中能量守恒的观点求解，由题意，可忽略 Au 原子的动能，当初始时 α 粒子的动能完全转换为两者之间的电势能时，两者最接近，且接近速度减小并反向。

$$K = \frac{2Ze^2}{R_{\min}}\text{。}$$

选（A）。

Question 67-68

A nonrelativistic particle of mass m moves in a plane. Its position is described by the polar coordinates r and θ, with time derivatives $\dot{r}$ and $\dot{\theta}$. There exists a potential energy U = kr^2, where k is a constant.

67. Which of the following is the Lagrangian L of the particle?

(A) $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - kr^2$

(B) $L = \frac{1}{2}m(\dot{r}^2 + \theta^2) + kr^2$

(C) $L = \frac{1}{2}m(\theta^2\dot{r}^2 + r^2\dot{\theta}^2) - kr^2$

(D) $L = \frac{1}{2}m(\dot{r}^2 + r\dot{r}\dot{\theta} + r^2\dot{\theta}^2) - kr^2$

(E) $L = \frac{1}{2}m(\dot{r}^2 + 2r\dot{r}\dot{\theta} - r^2\dot{\theta}^2) - kr^2$

解：粒子的 Lagrange 函数

$$L = T - V = T_n + T_\tau - V = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - kr^2,$$

选（A）。

68. Which of the following quantities remain constants?

(A) $m(\dot{r}^2 + r^2\dot{\theta}^2)$

(B) $mr^2\dot{\theta}^2$

(C) kr^2

(D) $mr\dot{\theta}$

(E) $mr^2\dot{\theta}$

解：接上题，Lagrange 函数 L 中不显含广义坐标θ，因此根据 Lagrange 方程

$$\frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta} = p_\theta = const\text{。}$$

选（E）。

69. A particle with mass 10 kg is moving in a straight line under the influence of a force applied opposite to the motion along the same straight line. The equation F = $2v^2$ represents the magnitude of the force F in newtons as a function of the instantaneous speed of the object at time t = 0 is 10 meters per second. At time t = –2 seconds, the speed of the object is most nearly

(A) 2.0 m/s

(B) 2.5 m/s

(C) 3.3 m/s

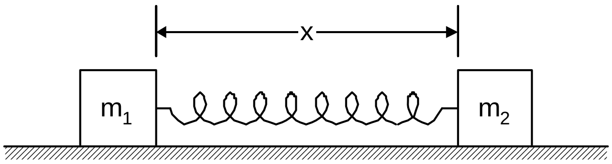
(D) 4.0 m/s

(E) 5.0 m/s

解：根据 Newton 运动定律 **F** = m**a**，直接可对时间反演积分求解初速度

$$\frac{dv}{dt} = \frac{F}{m} = \frac{2v^2}{10} \Rightarrow \int_{10}^{v_{-2}} \frac{dv}{v^2} = \int_0^{-2} \frac{1}{5}dt\text{，}$$

积分可求得结果。选（A）。



70. Two masses, m_1 and m_2, are joined by a massless spring of force constant k and placed on a horizontal frictionless surface as shown above. The system is released from rest when the separation between the masses is x. If the unstretched length of the spring is x_0, the speed of mass m_1 when the two masses are at the distance x_0 apart is

(A) $\sqrt{\frac{k}{m_1}(x-x_0)^2}$

(B) $\sqrt{\frac{k}{m_2}(x-x_0)^2}$

(C) $\sqrt{\frac{k}{m_1-m_2}(x-x_0)^2}$

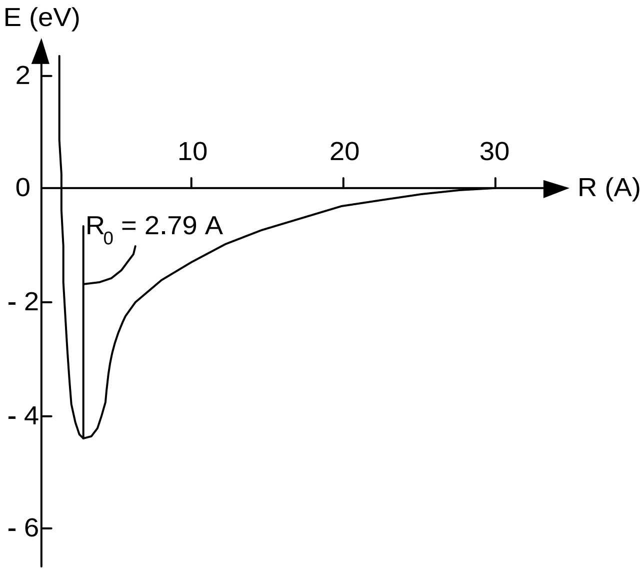
(D) $\sqrt{\frac{k}{m_1}\cdot\frac{m_2}{m_1+m_2}(x-x_0)^2}$

(E) $\sqrt{\frac{k}{m_2}\cdot\frac{m_1}{m_1+m_2}(x-x_0)^2}$

解：直接利用动量守恒定律和机械能守恒定律可以联合求解

$$m_1v_1 + m_2v_2 = 0$$
$$\frac{1}{2}k(x-x_0)^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2,$$

可求得 v_1。选（D）。



71. The diagram above shows the energy E of a KCl molecule as a function of the internuclear distance R. True statements include which of the following?

I. The energy required to dissociate a KCl molecule is about 4.4 eV.

II. R_0 refers to the equilibrium separation of the atoms in the molecule when it is in its ground state.

III. A KCl molecule in all of its bound electronic states would have the same E vs. R curve.

(A) I only

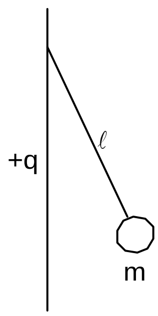
(B) III only

(C) I and II only

(D) II and III only

(E) I, II, and III

解：在势能曲线的最低点，对应的能量负值即将 KCl 原子分离所需要的最小能量，相应的距离是振动的平衡位置，即处于基态的平衡距离。I、II 都对，III 不对。选（C）。



72. A small sphere of mass m and charge q (cgs units) is attached by a string of length l to an

infinite vertical plane that has a surface charge density σ. If q > 0 and σ > 0, what is the tension in the string?

(A) mg

(B) $2\pi\sigma q$

(C) $(mg)^2/2\pi\sigma q$

(D) $(2\pi\sigma q)^2/mg$

(E) $\sqrt{(mg)^2+(2\pi\sigma q)^2}$

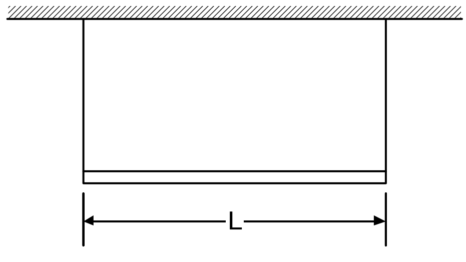
解：拉力与电场斥力和重力的合力平衡，无穷大带电平面附近的电场强度公式应掌握。

$$T^2=(mg)^2+(Eq)^2,$$

$$E=2\pi\sigma\text{。}$$

选（E）。

Questions 73-74



A thin uniform stick of length L and mass M is supported in a horizontal position by vertical strings on its ends, as shown above. One of the strings is cut at time t. The following questions pertain to the time (t + δ) where $\delta \to 0$.

73. The angular acceleration of the stick is

(A) 0

(B) $\dfrac{g}{2L}$

(C) $\dfrac{g}{L}$

(D) $\dfrac{3g}{2L}$

(E) $\dfrac{2g}{L}$

解： 在(t + δ) 时刻杆刚开始运动，相对于悬点的转动仅受重力矩的影响。

$$Mg\frac{L}{2}=I\beta=\frac{1}{3}ML^2\beta\Rightarrow\beta=\frac{3g}{2L}\text{。}$$

选（D）。

74. The tension in the remaining string is

(A) 0

(B) $\dfrac{1}{4}Mg$

(C) $\frac{1}{2}Mg$

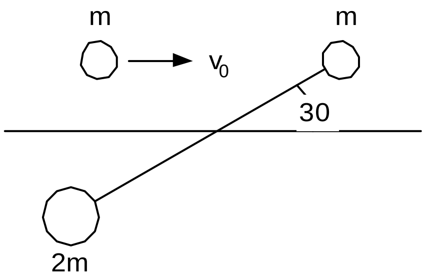
(D) $\frac{3}{4}Mg$

(E) Mg

解：考察质心的运动，根据 Newton 方程可得

$$Ma_c = M\beta\frac{L}{2} = Mg - T \text{，}$$

解得 T = Mg / 4。选（B）。



75. A puck of mass m slides with velocity v_0 on a horizontal frictionless surface as shown in the top view above. Lying on the surface at rest is a dumbbell formed by two other pucks, one of mass m and one of mass 2m, joined by a massless rod of length l. The axis of the dumbbell is inclined at a angle of 30° with respect to v_0 as shown. The moving puck collides and sticks to the lighter end of the dumbbell. The kinetic energy (translation and rotation) of the system after the collision is

(A) $\frac{5}{8}mv_0{}^2$

(B) $\frac{1}{2}mv_0{}^2$

(C) $\frac{5}{32}mv_0{}^2$

(D) $\frac{1}{8}mv_0{}^2$

(E) $\frac{1}{64}mv_0{}^2$

解：此题计算稍繁，比较简单一点是选择在质心系里计算。

$$v_c = \frac{v_0}{4} \text{，}$$

角动量守恒，

$$\left(m\frac{3}{4}v_0 - m\frac{1}{4}v_0 + 2m\frac{1}{4}v_0\right)\frac{L}{2}\cdot\sin 30 = 2\left(2m\left(\frac{L}{2}\right)^2\right)\omega \text{，}$$

从而解得

$$\omega = \frac{v_0}{4l} \text{。}$$

碰后系统的总能量

$$E = \frac{1}{2}4mv_c^2 + \frac{1}{2}I\omega^2 = \frac{5}{32}mv_0^2 。$$

选（C）。

76. An asteroid has a radius of 5×10^5 meters and an acceleration due to gravity of 1/4 that on Earth. The velocity of escape for an object starting on the surface of the asteroid is most nearly

(A) 150 m/s

(B) 720 m/s

(C) 1560 m/s

(D) 5550 m/s

(E) 11,200 m/s

解：星球的逃逸速度（第一宇宙速度）为

$$v_1 = \sqrt{Rg'} = \sqrt{\frac{1}{4}Rg'} \approx 5550\text{m/s} 。$$

选（D）。

77. An object of mass m is released from rest far above the Earth, at a distance r from the center of the Earth. The Earth’s radius is R and the acceleration due to gravity at the Earth’s surface is g. If air resistance is neglected and nonrelativistic motion is assumed, what is the speed of the object when it strikes the Earth’s surface?

(A) $\sqrt{2gr^2/R}$

(B) $\sqrt{2g(r^2-R^2)/r}$

(C) $\sqrt{g(r-R)}$

(D) $\sqrt{2g(r-R)}$

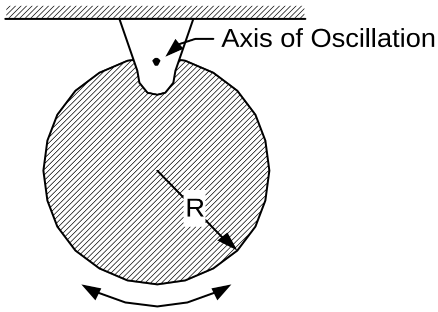
(E) $\sqrt{2gR(r-R)/r}$

解：物体的机械能守恒

$$-\frac{GMm}{r} = -\frac{GMm}{R} + \frac{1}{2}mv^2 ,$$

$$-g\frac{R}{r} = -g + \frac{1}{2}v^2 。$$

由此可解得速度 v。选（E）。



78. A uniform disk of radius R is suspended at its edge as shown above. It is free to swing back and forth in the plane of the disk. What is the frequency of small oscillations of this disk?

(A) $\frac{1}{2\pi}\sqrt{\frac{2g}{R}}$

(B) $\frac{1}{2\pi}\sqrt{\frac{2g}{3R}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{3g}{4R}}$

(D) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$

(E) $\frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$

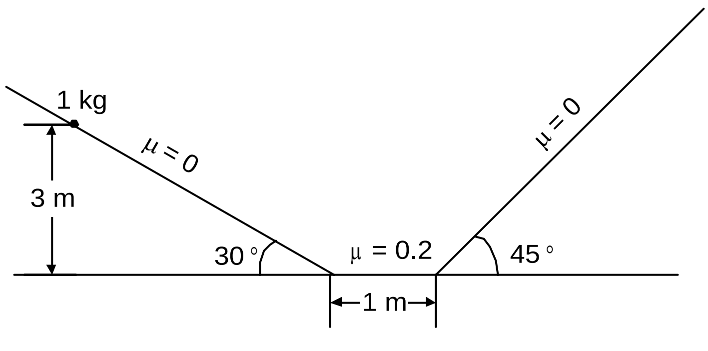
解：相当于一个复摆。圆盘相对于悬点的转动惯量

$$I = mR^2 + \frac{1}{2}mR^2 = \frac{3}{2}mR^2 \text{。}$$

小振动的频率为

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{mgR}{I}} = \frac{1}{2\pi}\sqrt{\frac{2g}{3R}} \text{。}$$

选（B）。



79. A 1-kilogram mass is initially held at rest at a height of 3 meters from the bottom of a 30

frictionless inclined plane, as shown above. The mass is released, slides down the plane, and comes to a 1-meter horizontal table with coefficient of friction μ = 0.2. After crossing this rough table, the mass slides up a 45° frictionless inclined plane. How many times does the mass cross the table before stopping?

(A) 1
(B) 5
(C) 15
(D) 10
(E) 100

解：最初的机械能完全被摩擦所损耗，每滑过一次水平的桌面，克服摩擦所损耗的能量

$$\Delta E = \mu mgs = 0.2g\text{，}$$

故滑块静止前滑过桌面的次数

$$n = \frac{mgH}{\Delta E} = \frac{3g}{0.2g} = 15\text{。}$$

选（C）。

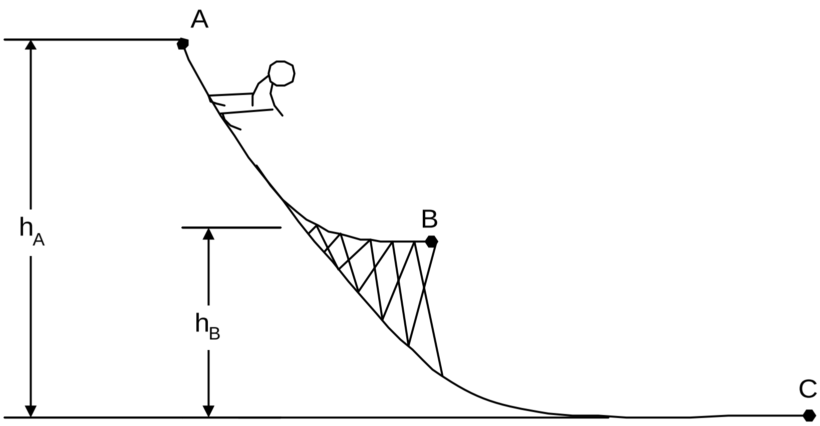
80. The speed v of a particle moving in a straight line is given by $v = \dfrac{a}{bt + c}$, where t is time and a, b, c are constants. The magnitude of the resultant force on the particle is directly proportional to

(A) b^{-2}
(B) b^{-1}
(C) b^{0}
(D) b^{1}
(E) b^{2}

解：直接根据 Newton 第二定律可以求出质点所受的阻力

$$f = m\frac{dv}{dt} = -\frac{mab}{(bt + c)^2}\text{。}$$

选（D）。



81. The skier shown above leaves point A from rest, skies down a frictionless ski jump, leaves the

jump at point B, and hits the ground at point C. The height at A and B are h_A and h_B. What is the skier's speed just before hitting the ground?

(A) $\sqrt{2gh_B}$

(B) $\sqrt{2gh_A}$

(C) $\sqrt{2g(h_A-h_B)}$

(D) $\sqrt{2gh_A h_B/(h_A+h_B)}$

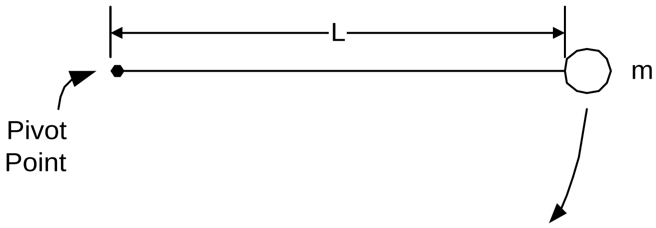
(E) $\sqrt{2g(h_A+h_B)}$

解：整个下滑、跳跃过程是满足机械能守恒的

$$mgh_A = \frac{1}{2}mv^2 \text{，}$$

$$v = \sqrt{2gh_A} \text{。}$$

选（B）。



82. A massless rod of length L is pivoted about a horizontal axis through one end. A small object of mass is attached to the other end. The rod is released from rest in a horizontal position, as shown above, and swings as a pendulum. Assume there is no friction and no air resistance. Which of the following quantities remains constant throughout the motion of the pendulum?

(A) Linear momentum of m

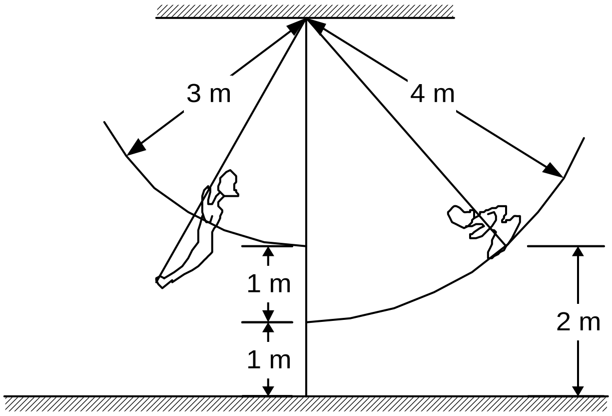
(B) Angular momentum of m relative to the pivot point

(C) Gravitational potential energy

(D) Kinetic energy

(E) None of the above

解：小球的机械能守恒。而下落过程中小球所受的合力、合力矩都不为零，动量和角动量都不守恒。选（E）。



83. A girl is initially crouching on a massless swing that is held at rest so that her center of mass is 2 meters above the ground. Assuming that the girl weighs 500 newtons, her center of mass is 4 meters from the pivot of the swing. The swing is released from rest. At the bottom of the arc the girl stands up, instantaneously raising her cernter of mass by 1 meter. The dotted line in the figure above represents the path of her center of mass. What is the ratio of her velocity just after she stands up to her velocity just before she stands up?

(A) 6/5

(B) 5/4

(C) 4/3

(D) 3/2

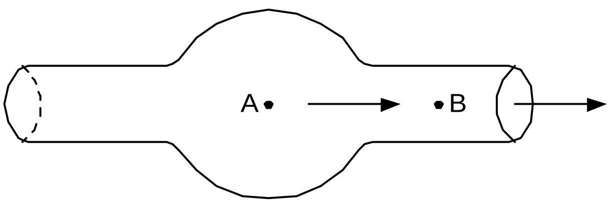
(E) 2

解：在女孩的下蹲过程中，由于重力沿竖直方向，整个系统没有受到外力矩，相对悬点的角动量守恒。

$$ml_0v_0 = mlv\ ,$$

$$\frac{v}{v_0} = \frac{l_0}{l} = \frac{4}{3}\ 。$$

选（C）。



84. Water flows through a horizontal pipe of varying cross section, as shown in the figure above. The pressure is p_A at A and p_B at B. Which of the following statements is true?

(A) p_A is greater than p_B because the velocity is greater at B.

(B) p_A is greater than p_B because A is upstream from B.

(C) p_A is less than p_B because A is upstream from B.

(D) p_A equals p_B according to Pascal’s theorem.

(E) p_A equals p_B according to Bernoulli’s theorem.

解：首先根据流体的连续性原理

$$v_A s_A = v_B s_B,$$

故 $v_A < v_B$。然后由 Bernoulli 方程

$$p_A + \frac{1}{2}\rho v_A^2 + \rho gh = p_B + \frac{1}{2}\rho v_B^2 + \rho gh,$$

得 $p_A > p_B$。选（A）。

85. A satellite of mass m circles the Earth at rasius r. Its period T is proportional to

(A) r^3 / m

(B) $(r^3 / m)^{1/2}$

(C) $r^{-3/2} m^0$

(D) $r^{3/2} m^0$

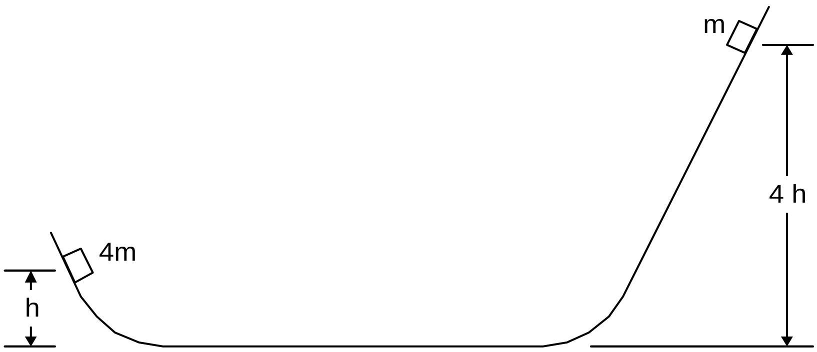
(E) $(m / r^3)^{1/2}$

解：地球与卫星间的万有引力提供卫星的向心加速度，

$$\frac{GMm}{r^2} = m\omega^2 r,$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r^3}{GM}} \propto r^{3/2}\text{。}$$

选（D）。



86. The figure above shows two objects of mass m and 4m placed on a frictionless track at heights of 4h and h, respectively. They start from rest, slide along the track, collide at the bottom, and stick together. The speed of the objects just after the collision is

(A) 0

(B) $\frac{2}{5}\sqrt{2gh}$

(C) $\sqrt{gh}$

(D) $\sqrt{2gh}$

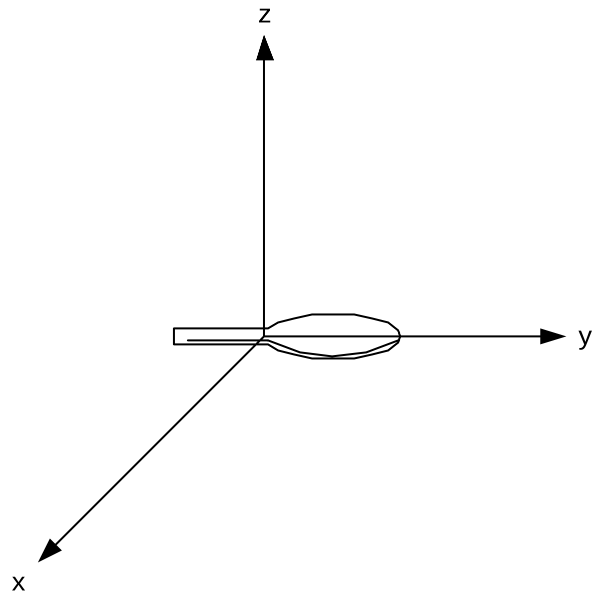
(E) $2\sqrt{2gh}$

解：是二体的完全非弹性碰撞，利用动量守恒，

$$4m\sqrt{2gh} - m\sqrt{2g \cdot 4h} = 5mv,$$

$$v = \frac{2}{5}\sqrt{2gh}。$$

选（B）。



87. A table tennis paddle lies in the x-y plane, as shown above. Its center of mass is at the origin of the coordinate system. If I_x, I_y,and I_z are the moments of inertia about the tthree axes, which of the following is true?

(A) $I_x = I_y = I_z$

(B) $I_x < I_y < I_z$

(C) $I_x < I_z < I_y$

(D) $I_y < I_x < I_z$

(E) $I_z < I_y < I_x$

解：根据转动惯量的正交轴定理，$I_z = I_x + I_Y$ 最大。而圆盘和把两部分的质心都在 Y 轴上，而不在 X 轴上，说明质量的分布相对 X 轴更分散，因此 $I_X > I_Y$。选（D）。

88. A spaceship of mass m is initially in a circular orbit around an isolated star of mass M. The radius of the orbit is R. If the kinetic energy of the spaceship is doubled by a short burst of its engine, what is the final total energy of the spacship? (The potential energu is taken to be zero when the spaceship is at infinite distance from the star. G is the gravitational constant.)

(A) $-\frac{1}{2}\frac{GMm}{R}$

(B) $-\frac{1}{4}\frac{GMm}{R}$

(C) 0

(D) $\frac{1}{4}\frac{GMm}{R}$

(E) $\frac{1}{2}\frac{GMm}{R}$

解：初始时，宇宙飞船的动能

$$T_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}\frac{GMm}{R} = -\frac{1}{2}V(R),$$

因此，加速后的总能量

$$E = 2T_0 + V(R) = 0\text{。}$$

选（C）。

89. The mass of the Moon is 1.2 percent of the mass of the Earth. The radius of the Moon is 27 percent of the radius of the Earth. Approximately how long does it take for a rock released from rest from a height of 0.5 meter above the Moon's surface to strike the surface of the Moon?

(A) 0.25 s

(B) 0.8 s

(C) 2.5 s

(D) 8 s

(E) 25 s

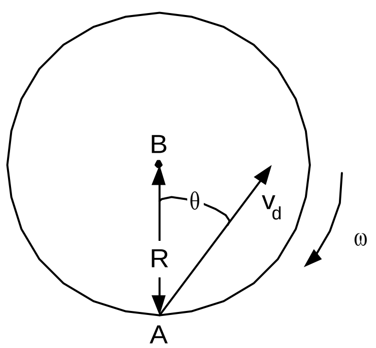
解：月球上的重力加速度

$$g' = \frac{GM'}{R'^2} = \frac{G\cdot 0.012M}{(0.27R)^2} \approx \frac{1}{6}g\text{，}$$

故自由落体的下落时间

$$t = \sqrt{\frac{2h}{g'}} = \sqrt{\frac{12\times 0.5}{10}} \approx 0.8\text{s}\text{。}$$

选（B）。



90. A person at A is rotating at a radius R and angular speed ω about a vertical axis B, as shown above. The person throws a ball with horizontal speed v_0 with respect to the person. If the ball is to hit B, at what angle θ must the person aim?

(A) 0

(B) $\omega R / v_0$

(C) $\tan^{-1}(\omega R / v_0)$

(D) $\tan^{-1}(v_0 / \omega R)$

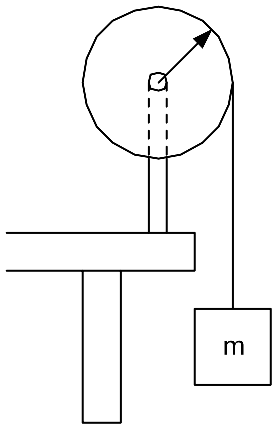
(E) $\sin^{-1}(\omega R / v_0)$

解：小球抛出的速度在静止系中看来应该是沿径向的，根据相对速度合成的原理

$$\sin\theta = \frac{\omega R}{v_0},$$

$$\theta = \sin^{-1}(\omega R / v_0)\text{。}$$

选（E）。



91. A mass m is tied to a light string wound around a pulley of moment of inertia I and radius R, as shown above. The pulley bearing is frictionless. The tension in the string as the mass falls is

(A) mg

(B) $(I / R^2)g$

(C) $(I / R^2 + m)g$

(D) $(m - I / R^2)g$

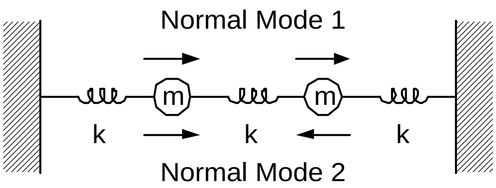
(E) $\dfrac{I / R^2}{m + I / R^2} mg$

解：假设绳中张力为 T，对滑轮和滑块的运动写出受力方程为

$$TR = I\beta = I\frac{a}{R},$$

$$mg - T = ma$$

以上两式联立消去 a 可解得张力 T。选（E）。



92. Two objects of mass m are connected by three identical springs of negligible mass, as shown above. Two normal modes of oscillation are shown by the arrows. The ratio ω_1/ω_2 of the frequencies of the two normal modes is

(A) $\sqrt{1/3}$

(B) $\sqrt{2/3}$

(C) 1

(D) $\sqrt{3/2}$

(E) $\sqrt{3}$

解：振动模式一中两个物体同相振动，中间的那根弹簧没有作用。振动模式二中两个物体反向振动，它们的质心静止，等效的弹性系数

$$k' = k + 2k = 3k 。$$

$$\omega_1 = \sqrt{\frac{k}{m}} ,$$

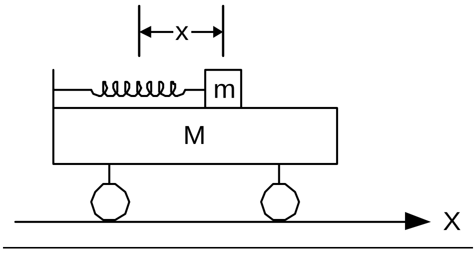
$$\omega_2 = \sqrt{\frac{k'}{m}} = \sqrt{\frac{3k}{m}} ,$$

相比得

$$\frac{\omega_1}{\omega_2} = \frac{1}{\sqrt{3}} 。$$

选（A）。

Questions 93-94



A car of mass M moves horizontally along the X-axis, as shown above. A mass m is attached to the car by a spring having spring constant k. The x-coordinate measures of the position of m; x = 0 corresponds to the position of m in which the spring in neither stretched nor compressed. The axes x and X are parallel.

93. What is the Lagrangian of this system?

(A) $\frac{1}{2}M\dot{X}^2+\frac{1}{2}m\dot{x}^2+\frac{1}{2}kx^2$

(B) $\frac{1}{2}M\dot{X}^2+\frac{1}{2}m\dot{x}^2-\frac{1}{2}kx^2$

(C) $\frac{1}{2}M\dot{X}^2+\frac{1}{2}m(\dot{X}^2+\dot{x}^2)-\frac{1}{2}kx^2$

(D) $\frac{1}{2}M\dot{X}^2+\frac{1}{2}m(\dot{X}^2+2\dot{x}\dot{X}+\dot{x}^2)-\frac{1}{2}kx^2$

(E) $\frac{1}{2}(M+m)(\dot{X}^2+\dot{x}^2)-\frac{1}{2}kx^2$

解：根据整个体系的动量守恒，

$$MX+m\left(X+x\right)=\text{const.},$$

$$v_m=-\frac{M\dot{X}}{m}=-\left(\dot{X}+\dot{x}\right)。$$

系统的 Lagrange 量

$$L=\frac{1}{2}Mv_M^2+\frac{1}{2}mv_m^2-\frac{1}{2}kx^2。$$

选（D）。

94. What is ω, the angular frequency of oscillations of m, as determined by the differential equation $\ddot{x}+\omega^2x=0$

(A) $\sqrt{\frac{k}{m}}$

(B) $\sqrt{\frac{k}{M}}$

(C) $\sqrt{\frac{k}{M+m}}$

(D) $\sqrt{\dfrac{k}{M-m}}$

(E) $\sqrt{\dfrac{k\,(M\;+\;m\,)}{Mm}}$

解：由上问

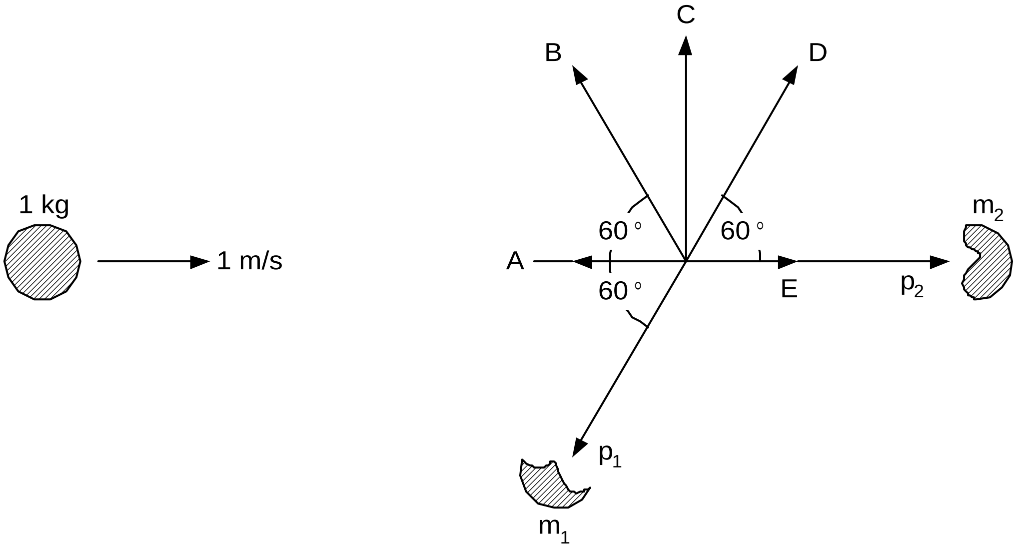
$$v_m = -\left(\dot{X} + \dot{x}\right),$$

列出 Newton 力学方程为

$$kx + m\dot{v}_m = kx - m\left(\ddot{X} + \ddot{x}\right) = kx - m\left(\ddot{x} - \frac{m}{M+m}\ddot{x}\right) = kx - \frac{mM}{M+m}\ddot{x} = 0\,,$$

$$\omega = \sqrt{\frac{k(M+m)}{Mm}}\ 。$$

选（E）。



95. A body of mass 1 kilogram moves to the right with velocity 1 meter per second, as shown above on the left. The body explodes into three parts: m_1, m_2, and m_3. After the explosion, m_1 and m_2 have momenta p_1 and p_2, both of magnitude 2 kilogram·meter per second, oriented as shown above on the right. If all vectors shown start at the origin and are in the plane of the paper, the momentum of m_3 is represented by the vector

(A) A

(B) B

(C) C

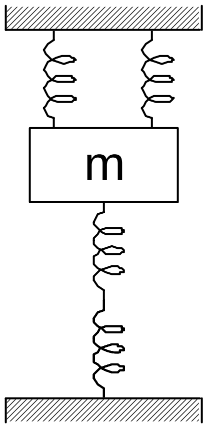
(D) D

(E) E

解：整个爆炸过程前后的动量守恒，写成水平和垂直的分量形式有

$$\begin{aligned}&p_2 - p_1 \cos 60° + p_{3x} = p_0 \Rightarrow p_{3x} = 0 \\ &p_{3y} = p_2 \cos 60° > 0\end{aligned}\text{。}$$

选（C）。



96. An object of mass m is connected to two fixed surfaces by four identical springs, as shown above. The springs are of negligible mass and have spring constant k. The period of vertical oscillation is

(A) $2\pi\sqrt{2m/k}$

(B) $2\pi\sqrt{m/k}$

(C) $2\pi\sqrt{2m/5k}$

(D) $\pi\sqrt{m/k}$

(E) $\pi\sqrt{m/2k}$

解：整个体系的等效弹性系数

$$k' = 2k + \frac{k}{2} = \frac{5}{2}k\text{，}$$

周期

$$T = 2\pi\sqrt{\frac{m}{k'}} = 2\pi\sqrt{\frac{2m}{5k}}\text{。}$$

选（C）。

<u>Questions 96-97</u>

The Lagrangian for a system with generalized coordinate q is

$$L = m\dot{q}^4 - g(q)$$

where $\dot{q} = dq/dt$ (t is the time) and g(q) is an arbitrary function of the coordinate.

96. The canonical momentum conjugate to q is

(A) $m\dot{q}$

(B) $m\dot{q}^3$

(C) $g(q)\dot{q}$

(D) $m\dot{q}^2/3$

(E) $4m\dot{q}^3$

解：广义动量

$$p_q = \frac{\partial L}{\partial \dot{q}} = 4m\dot{q}^3 \text{。}$$

选（E）。

97. Which of the following is a constant of the movement for this system?

(A) $\frac{1}{2}m\dot{q}^2 + g(q)$

(B) $m\dot{q}^2 + g(q)$

(C) $m\dot{q}^4 + g(q)$

(D) $3\,m\dot{q}^4 + g(q)$

(E) $8\,m^2\dot{q}^6 + g(q)$

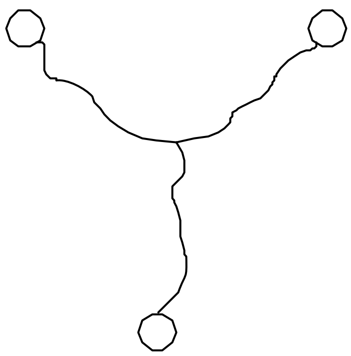
解：因为 L 不显含 t，故能量初积分

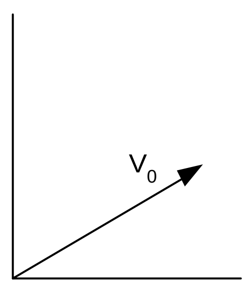
$$T + V = 2T - L$$

为常量。

$$\frac{dL}{d\dot{q}}\dot{q} - L = 3m\dot{q}^4 + g(q) = \text{const.} \text{。}$$

选（D）。





98. Three balls, each of mass m, are attached to massless rubber cords knotted together, as shown above left. The assembly is then thrown upward so that the initial velocity of the center-of-mass (CM) is as shown above right. The CM speed V has an initial value V_0. Which of the following is NOT true? (KE = kinetic energy)

(A) The CM moves like a projectile of mass 3m under the influence of gravity.

(B) $\text{Total KE} = \frac{3}{2}mV^2 + (\text{KE relative to the CM})$.

(C) The angular momentum of the system about its CM is constant.

(D) KE relative to the CM may change with time.

(E) Potential energy can be stored in the stretched rubber cords.

解：（A）是质心运动定理的结果。总动能等于质心动能加上系统相对于质心的动能，（B）正确。系统受到三个小球的重力的作用，相对于质心的合力矩为零，因此相对于质心的角动量守恒，（C）正确。系统总动能改变量等于重力做功，而重力做功等于质心动能改变量，因此系统相对于质心的动能不变，（D）错。选（D）。

99. The weight of an object on the Moon is 1/6 of its weight on the Earth. A pendulum clock that ticks once per second on the Earth is taken to the Moon. On the Moon the clock would tick once every

(A) 1/6 s

(B) $1/\sqrt{6}$ s

(C) 1 s

(D) $\sqrt{6}$ s

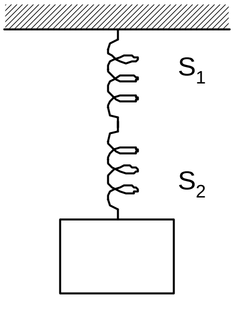
(E) 6 s

解：由单摆周期公式

$$T = 2\pi\sqrt{\frac{l}{g}},$$

$$T_M = \sqrt{\frac{g_E}{g_E}}T_E = \sqrt{6}T_E = \sqrt{6}\text{s}。$$

选（D）。



100. Two springs, S_1 and S_2, have negligible masses and the spring constant of S_1 is 1/3 that of S_2. When a block is hung from the springs as shown above and the springs come to equilibrium again,

the ratio of the work done in stretching S_1 to the work done in stretching S_2 is

(A) 1/9

(B) 1/3

(C) 1

(D) 3

(E) 9

解：平衡时两弹簧上拉力均等于重物的重力，即

$$k_1\Delta x_1 = k_2\Delta x_2 = G，$$

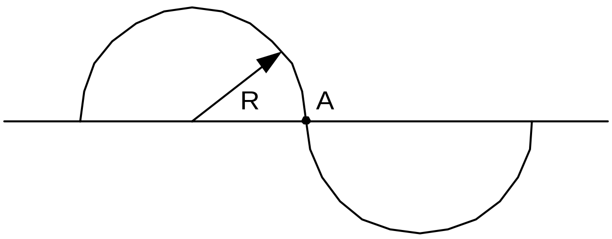
则伸长量之比为

$$\frac{\Delta x_1}{\Delta x_1} = \frac{k_2}{k_1}。$$

所以势能之比为

$$\frac{E_{1p}}{E_{2p}} = \frac{\frac{1}{2}k_1\Delta {x_1}^2}{\frac{1}{2}k_2\Delta {x_2}^2} = \frac{k_1}{k_2}\left(\frac{k_2}{k_1}\right)^2 = \frac{k_2}{k_1} = 3。$$

选（D）。



101. The S-shaped wire shown above has a mass M, and the radius of curvature of each half is R. The moment of inertia about an axis through A and perpendicular to the plane of the paper is

(A) $\frac{1}{2}MR^2$

(B) $\frac{3}{4}MR^2$

(C) MR^2

(D) $\frac{3}{2}MR^2$

(E) $2\ MR^2$

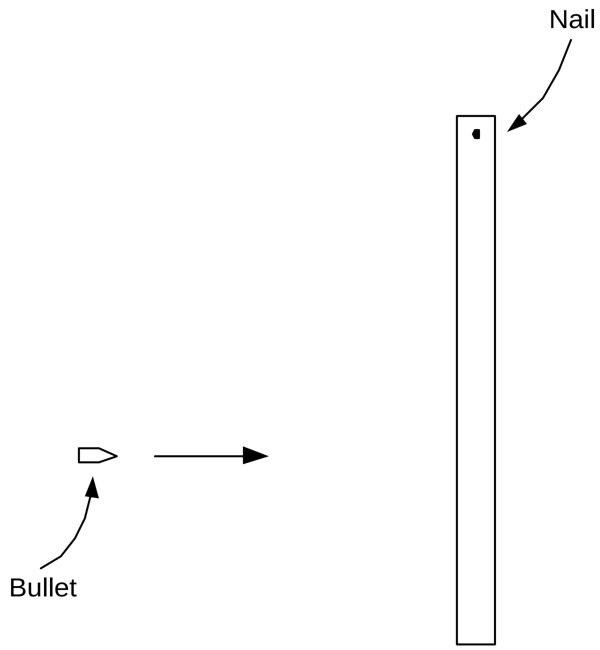
解：圆环相对其中心的转动惯量为 MR^2，由对称性每个半圆环相对其中心的转动惯量为

$$I_0 = \frac{1}{2}MR^2。$$

由平行轴定理，每个半圆环相对于 A 点的转动惯量为

$$I = I_0 + \frac{1}{2}MR^2 = MR^2。$$

所以整体相对于 A 点的转动惯量等于两个半圆环之和，$2MR^2$。选（E）。



102. The speed of rifle bullets is to be determined by using a ballistic pendulum consisting of a plank suspended from a nail, as shown above. If the friction is negligible, for any point of impact, quantities conserved during the collision for the system of the bullet and the plank include which of the following?

I. Linear momentum

II. Angular momentum about the nail

III. Angular momentum about the center of mass

(A) I only

(B) II only

(C) III only

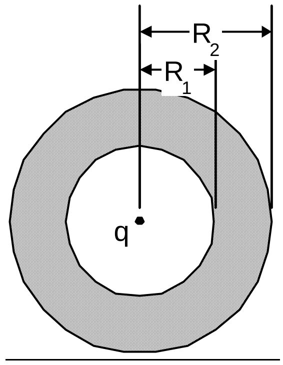
(D) I and II

(E) I and III

解：由于钉子的固定作用，碰撞过程的动量不守恒。相对质心的合外力矩不为零，角动量也不守恒。只有相对钉子的角动量守恒。选（B）。

第二节 电磁学

Questions 1-2



A conducting sphere of radius R_2 has a concentric spherical cavity of radius R_1, as shown above. The sphere has no charge. However, there is a charge q in the center of the cavity.

1. What is the surface charge density on the inner surface of the sphere?

(A) $-\dfrac{q}{4\pi R_1^2}$

(B) $-\dfrac{q}{8\pi R_1^2}$

(C) $-\dfrac{q}{4\pi R_2^2}$

(D) $-\dfrac{q}{8\pi R_2}$

(E) $\dfrac{R_2^2 q}{4\pi R_1^3}$

解：电荷 q 发出的电力线均终止在球壳内表面上（或终止于 q 的电力线全来自于球壳内表面），所以球壳内表面电量为–q。由于球对称性，电量均匀分布，面电荷密度

$$\sigma = -\frac{q}{4\pi R_1{}^2}\text{。}$$

选(A)。

2. What is the potential of the sphere? (Assume that the potential is zero at an infinite distance away.)

(A) $\dfrac{q}{R_1}$

(B) $\frac{q}{R_2}$

(C) $\frac{qR_1}{R_2}$

(D) $\frac{qR_2}{R_1}$

(E) $q(\frac{1}{R_1}+\frac{1}{R_2})$

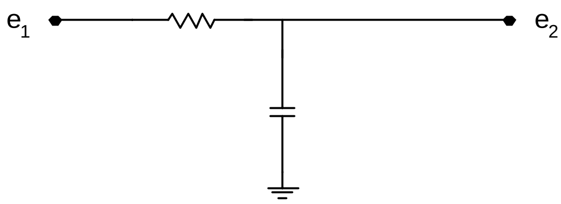
解：由导体球壳电量守恒，外表面电量为 q。所以球壳外电场强度为

$$E=\frac{q}{4\pi\varepsilon_0 r^2},$$

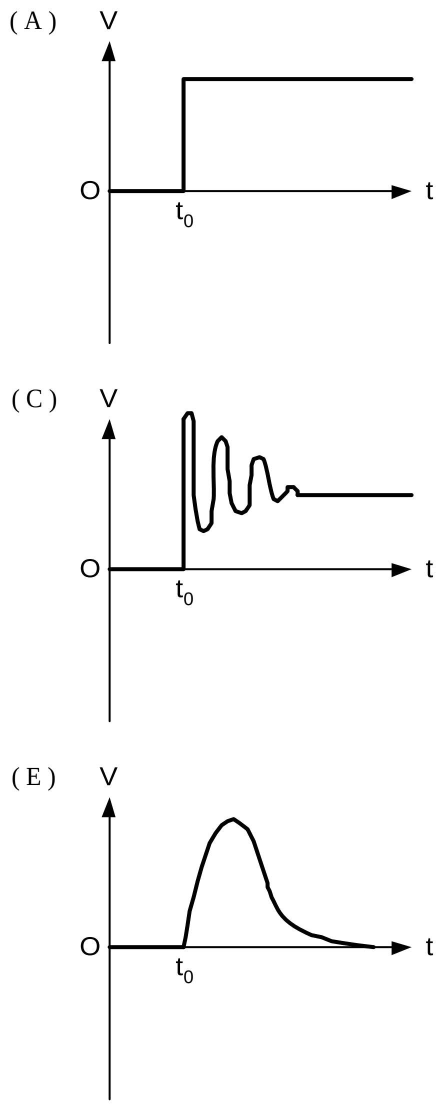
电势为

$$U=\int_{R_2}^{\infty}\frac{q}{4\pi\varepsilon_0 r^2}dr=\frac{q}{R_2},$$

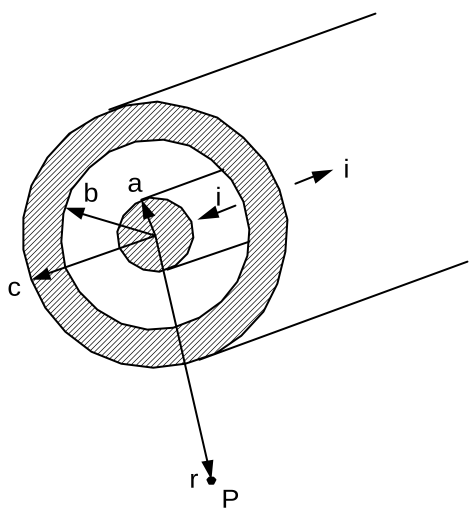
注意答案为 Gauss 单位制。选（B）。



3. In the circuit illustrated above, the point marked e_1 is raised instaneously at time t_0 from zero to a fixed positive potential. If e_2 is connected to a high-impedance D.C. oscilloscope input, the gragh of its voltage V as a function of time t will most resemble which of the following?



解：t_0瞬间电容上电量无法突变，所以示波器信号电压为 0，可排除(A)、(B)、(C)三种情况；当 $t \to \infty$ 时，电路处于稳定状态，电容相当于短路，电阻上无压降，示波器信号电压为 V_0。选（E）。



4. a coaxial cable having radii a, b, andc carries equal and opposite currents of magnitude i on the inner and outer conductors.What is the magnitude of the magnetic induction at point P outside of the cable at a distance r from the axis?

(A) Zero

(B) $\dfrac{\mu_0 ir}{2\pi a^2}$

(C) $\dfrac{\mu_0 i}{2\pi r}$

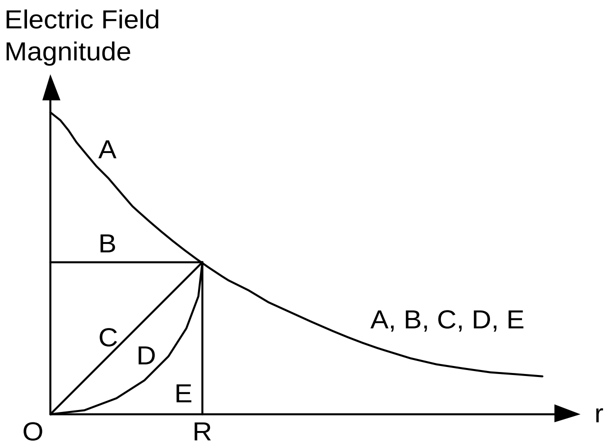
(D) $\dfrac{\mu_0 i}{2\pi r}\dfrac{c^2-r^2}{c^2-b^2}$

(E) $\dfrac{\mu_0 i}{2\pi r}\dfrac{r^2-b^2}{c^2-b^2}$

解：系统具有轴对称性，r 相同的地方磁感应强度相等。由 Ampere 环路定理，

$$B=\frac{1}{2\pi r}\oint \mathbf{B}\cdot\mathbf{dr}=\frac{\mu_0}{2\pi r}\iint \mathbf{j}\cdot\mathbf{ds}=\frac{\mu_0}{2\pi r}I\text{ 。}$$

因为电流总量为 0，所以磁感应强度为 0。选(A)。



5. A isolated sphere of radius R contains a uniform volume distribution of positive charge. Which of the curves on the graph above correctly illustrates the dependence of the magnitude of the electric field of the sphere as function of the distance r from its center?

(A) A

(B) B

(C) C

(D) D

(E) E

解：由 Gauss 定理，在球内部

$$E4\pi r^2=\frac{1}{\varepsilon_0}\rho\frac{4}{3}\pi r^3\text{，}$$

故 E ∝ r。选（C）。

6. Which of the following equations is a consequence of the equation $\nabla\times\mathbf{H}=\dot{\mathbf{D}}+\mathbf{J}$?

(A) $\nabla\cdot(\dot{\mathbf{D}}+\mathbf{J})=0$

(B) $\nabla\times(\dot{\mathbf{D}}+\mathbf{J})=0$

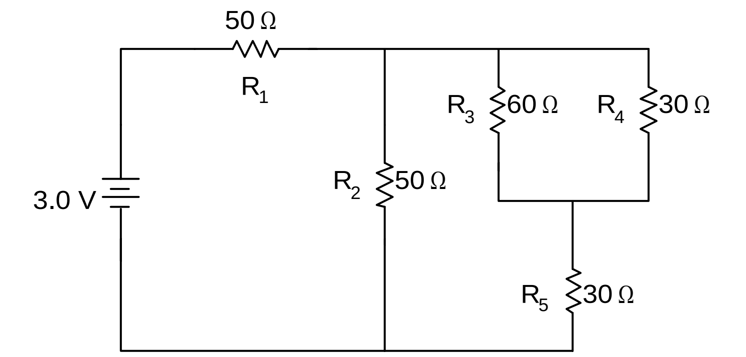
(C) $\nabla(\dot{\mathbf{D}}\cdot\mathbf{J})=0$

(D) $\dot{\mathbf{D}}+\mathbf{J}=0$

(E) $\dot{\mathbf{D}}\cdot\mathbf{J}=0$

解：旋度场无散，选（A）。

Questions 7-8



In the circuit shown above, the resistances are given in ohms and the battery is assumed ideal with emf equal to 3.0 volts.

7. The resistor that dissipates the most power is

(A) R_1
(B) R_2
(C) R_3
(D) R_4
(E) R_5

解：比较简单的方法是设 R_3 上电流为 I_0，利用串并联的关系易得：（R_4，$2I_0$）；（R_5，$3I_0$）；（R_2，$3I_0$）；（R_1，$6I_0$）。显然 R_1 上功率 $P=I^2R$ 最大，选(A)。

8. The voltage across resistor R_4 is

(A) 0.4 V
(B) 0.6 V
(C) 1.2 V
(D) 1.5 V
(E) 3.0 V

解：由上一问的分析，且总电阻为 75 Ω ，则

$$I_0 = \frac{I}{6} = \frac{1}{6}\frac{U}{R} = \frac{1}{6}\frac{3.0}{75} = \frac{1}{150}\text{ A},$$

R_4上的电压为

$$2I_0R_4 = 2\times\frac{1}{150}\times 30 = 0.4\text{ V}。$$

选（A）。

9. A conducting cavity is driven as an electromagnetic resonator. If perfect conductivity is assumed, the transverse and normal field components must obey which of the following conditions at the inner cavity walls?

(A) $E_n = 0, B_n = 0$

(B) $E_n = 0, B_t = 0$

(C) $E_t = 0, B_t = 0$

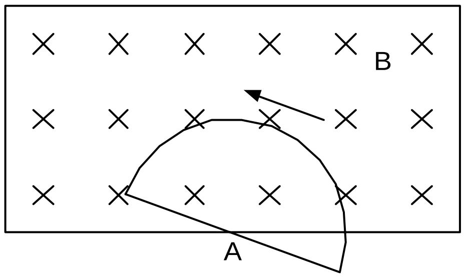
(D) $E_t = 0, B_n = 0$

(E) None of the above

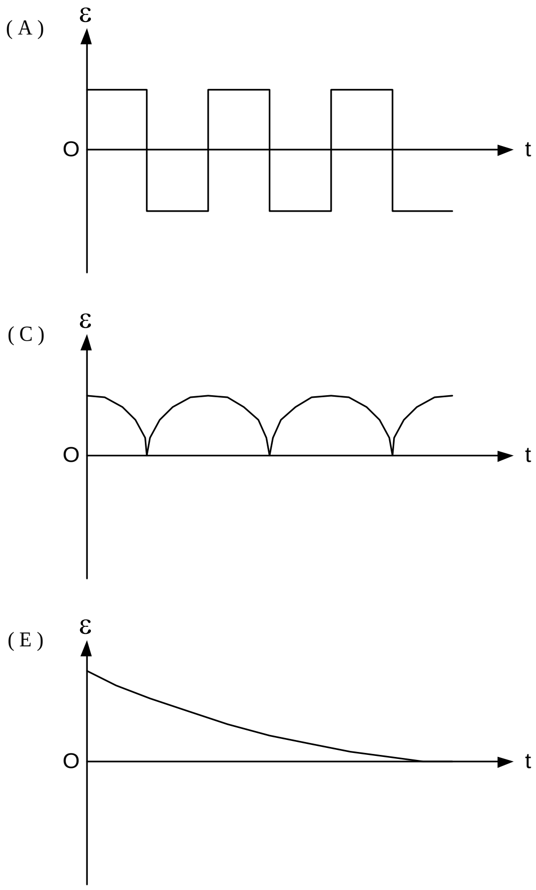
解：由电磁场的边界条件，

$$E_{1t} = E_{2t}, \qquad B_{1n} = B_{2n} 。$$

因为理想导体内部电磁场为 0，$E_{2t} = 0$，$B_{2n} = 0$，所以 $E_{1t} = 0$，$B_{1n} = 0$。选(D)。



10. A uniform and constant magnetic field B is directed perpendicularly into the plane of the page everywhere within a rectangular region as shown above. A wire circuit in the shape of a semicircle is rotated counterclockwise in the plane of the page about an axis A. The axis A is perpendicular to the page at the edge of the field and directed through the center of the straight-line portion of the circuit. Which of the following graphs best approximates the emf ε induced in the circuit as a function of time t?



解：由 Faraday 电磁感应定律

$$\mathrm{E}=-\frac{d\Phi}{dt}\text{，}$$

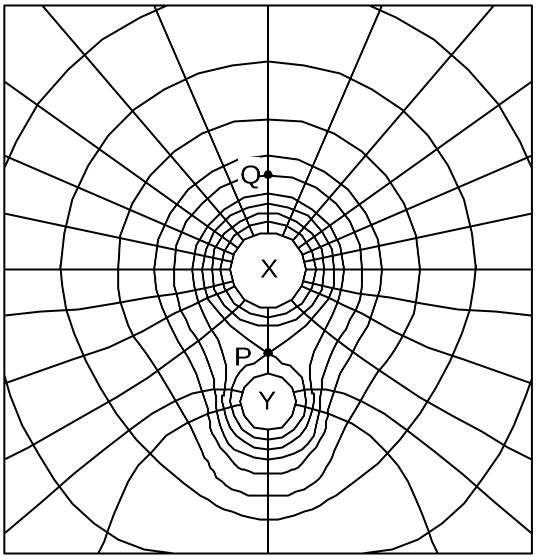
其中Φ为磁通量，对于本题中的均匀磁场，

$$\Phi=BS\text{，}$$

所以感生电势为

$$\mathrm{E}=-\frac{d\Phi}{dt}=-\frac{BdS}{st}=\pm\frac{1}{2}BR^{2}\omega\text{。}$$

感生电势的绝对值与时间无关，感生电势为交替的两个互为相反数的值，图形应为方波。选（A）。



11. The diagram above represents the electric field and the equipotential line for two charged conductors, X and Y. The charge on X is positive. Which of the following statements is NOT correct?

(A) The charges on X and Y are of the same sign.

(B) The charge on X is greater than that on Y.

(C) The electric field lines and the equipotential lines always intersect each other at an angle of 90 degrees.

(D) A test charge placed at point P would be in equilibrium under the action of the electrical forces.

(E) The electrical potential at point P is less than it is at point Q, which is symmetrically located on the other side of X.

解：X、Y 之间没有被电力线连接，说明二者电荷同号，（A）正确。由于 Y 周围的等势线比较密集，或者说相同的等势线距离 X 的平均距离比 Y 长，由电势的公式，

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q}{r},$$

X 电量大，（B）正确。（C）显然正确。P 是两组等势面交汇的地方，该点电场强度必须和两组等势面都垂直，因此只能是零，（D）正确。对于选项（E），Q 处在比 P 远离电荷的等势线上，但由于不知道电荷的正负，无法判断接近电荷的地方电势高还是低，所以（E）不正确。选（E）。

12. The potential difference across the terminals of a battery is 10 volts if no current is drawn from it. When a 4-ohm resistor is connected across the terminals, a current of 2 amperes is drawn from the battery. This resistor is then removed and replaced with a variable resistor that can have any value of resistance from 0 to ∞. If the internal resistance of the battery is constant, the maximum power P the battery can deliver to the variable resistor is

(A) 16 Watts

(B) 25 Watts

(C) 50 Watts

(D) 100 Watts

(E) arbitrarily large as the resistance goes to zero

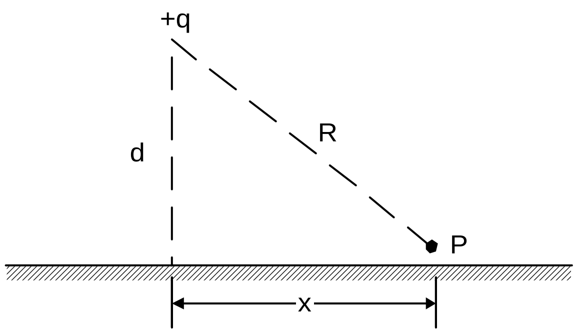
解：电池的内阻为

$$r=\frac{U}{I}-R_{外}=\frac{10}{2}-4=1\ \Omega。$$

当外接电阻与电池内阻相同即 R = r 时，电池输出功率最大，为

$$W=I^2R_{外}=\left(\frac{U}{r+r}\right)^2 r=25\ \text{watts}。$$

选（B）。



13. A point charge +q is located a distance d above a grounded conducting plane. The magnitude, in MKS units, of the electric field at point P just above the surface is

(A) $\frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

(B) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{R^2}$

(C) $\frac{1}{4\pi\varepsilon_0}\frac{2qd}{R^3}$

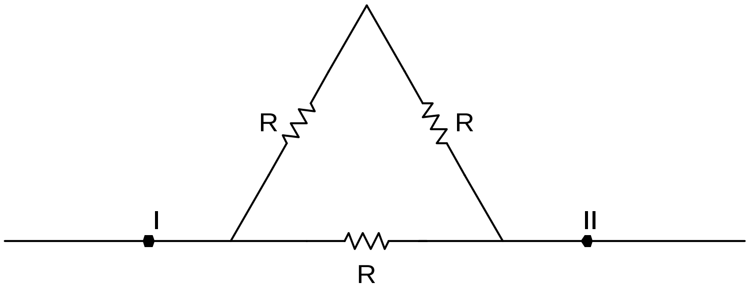
(D) $\frac{1}{4\pi\varepsilon_0}\frac{2qx}{R^3}$

(E) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{xd^2}$

解：导电平面接地，电势为零。用镜像法，在−d 处放置一像电荷−q。则

$$E_P=2\times\frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\frac{d}{R}=\frac{1}{4\pi\varepsilon_0}\frac{2qd}{R^3}。$$

答案选（C）。



14. If V is the potential difference between points I and II in the diagram above and all three resistors have the same resistance R, what is the total current between I and II.

(A) $\frac{V}{3R}$

(B) 3VR

(C) $\frac{2V}{3R}$

(D) $\frac{3VR}{2}$

(E) $\frac{3V}{2R}$

解：混联电路，等效电阻

$$R_E = \frac{2R \cdot R}{2R + R} = \frac{2}{3} R \text{，}$$

电流为

$$I = \frac{U}{R_E} = \frac{3V}{2R} \text{。}$$

选（E）。

<u>Questions 15-16</u>

An ion with charge +q travels in the +x direction relative to a right-handed coordinate system. A magnetic field of intensity B webers per square meter is applied in the +y direction.

15. For the ion to be undeflected if its velocity is v = 0.01c, there must also be an electric field E of magnitude in volts per meter and direction equal to

(A) B, in the –y direction

(B) vB, in the –y direction

(C) B, in the +z direction

(D) vB, in the +z direction

(E) vB, in the –z direction

解：由 Lorentz 力公式

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \text{，}$$

而

$$\mathbf{v} \times \mathbf{B} = vB\hat{\mathbf{z}} \text{。}$$

所以为使 $\mathbf{F} = 0$，电场强度 $\mathbf{E} = -vB\,\hat{\mathbf{z}}$。答案选（E）。

16. For the ion to be undeflected if its velocity is v = 0.99c, the magnitude in volts per meter of the required electric field E is equal to

(A) B

(B) vB

(C) $\dfrac{B}{\sqrt{1-\dfrac{v^2}{c^2}}}$

(D) $\dfrac{vB}{\sqrt{1-\dfrac{v^2}{c^2}}}$

(E) $\dfrac{vB}{\left(1-\dfrac{v^2}{c^2}\right)}$

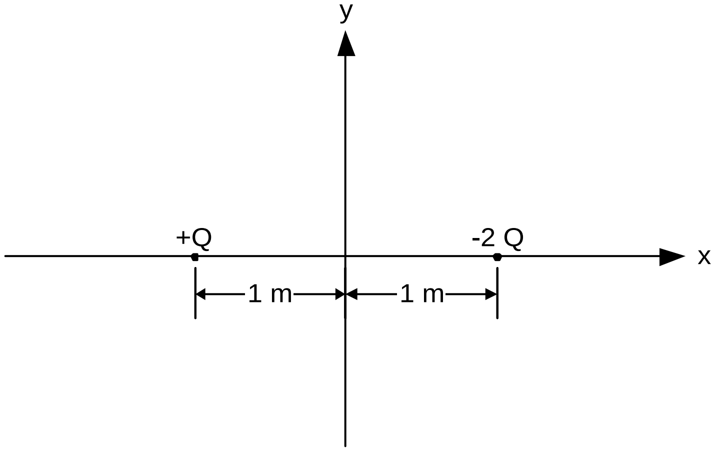
解：其实答案与上一问完全相同，因为 Lorentz 力公式总是适用的。许多人由于对相对论的惧怕而不敢选。我们换一个参照系看问题。选带电粒子的静止参考系。此时带电粒子只受电场力。设两参考系之间相对速度为 **v**，沿 x 轴正方向，由变换公式（只写出了电场的变换）

$$\bar{E}_x = E_x$$
$$\bar{E}_y = \gamma(E_y - vB_z),$$
$$\bar{E}_z = \gamma(E_z + vB_y)$$

未加电场前，**B** =（0, B, 0）。显然如果电场为 **E** =（0, 0, –vB），则

$$\bar{E}_x = \bar{E}_y = \bar{E}_z = 0,$$

带电粒子不受力。选（B）。



17. A charge of +Q coulombs is placed on the x-axis at x = –1 meter and a charge of –2Q coulombs is placed at x = +1 meter as shown above. A test charge of +q coulombs will experience zero net force if placed on the x-axis at x equals

(A) $-\left(3+\sqrt{8}\right)m$

(B) $-\dfrac{1}{3}m$

(C) 0 m

(D) $\dfrac{1}{3}m$

(E) $\left(3+\sqrt{8}\right)m$

解：首先推断一下 x 的大概范围。在两电荷之间二者电场方向相同，不可能有零点。在二者之外电场相反，由于左边电荷电量小，试验电荷应放在+Q 的左侧，x < –1。由受力平衡

$$\frac{Q}{(x+1)^2}=\frac{2Q}{(x-1)^2}\text{，}$$

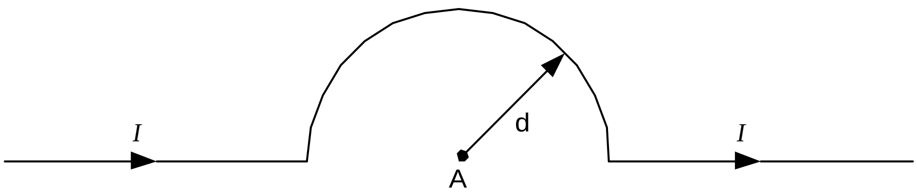
$$\left|\frac{x-1}{x+1}\right|=\sqrt{2}\text{。}$$

因为 x < –1，所以 $\dfrac{x-1}{x+1}>0$，进而有

$$\frac{x-1}{x+1}=\sqrt{2}\text{，}$$

$$x=-\left(3+\sqrt{8}\right)\text{。}$$

选（A）。



18. A circular wire loop having radius d and carrying current I has at its center a magnetic field of magnitude B_0. If another wire carrying current I is curved to form a semicircle of radius d as shown above, the magnitude of the magnetic field at point A is

(A) 4 B_0

(B) 2 B_0

(C) B_0

(D) $\dfrac{B_0}{2}$

(E) $\dfrac{B_0}{4}$

解：由 Biot-Savart 定律

$$\mathbf{B}=\int\frac{Id\mathbf{l}\times\hat{\mathbf{r}}}{r^2}$$

知，直线部分 I 与 **r** 同向，对原点处的磁场贡献为零。圆环上各点贡献均等。因此后来的磁

感应强度为整个圆环产生的一半。选（D）。

19. If the currents in two parallel wires are in the same direction, the force between them

(A) is zero

(B) is repulsive and in the plane containing the wires

(C) is attractive and in the plane containing the wires

(D) is perpendicular to the plane containing the wires

(E) depends on the sign of the current-carrying charges

解：由右手定则可判断一根导线在另一根导线处产生的磁感应强度的方向。由 Ampere 力公式

$$d\vec{F} = Id\vec{l} \times \vec{B}$$

知二者之间为吸引力，方向在纸面内。这一结论与导线中电流方向无关。选（C）。

20. The most general motion of a charged particle in a uniform magnetic field is

(A) a parabola

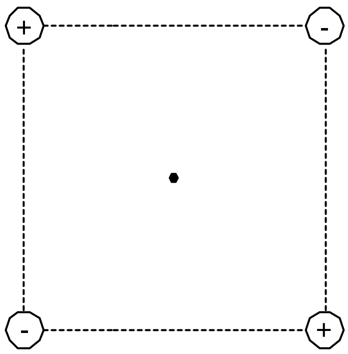
(B) a hyperbola

(C) a helix

(D) an ellipse

(E) a cycloid

解：垂直磁场方向做匀速圆周运动，沿磁场方向做匀速直线运动。整体为螺旋线型运动。答案选(C)。认识一下选项中的单词，(A)为抛物线，（B）为双曲线，(D)为椭圆，（E）为悬轮线。



21. Four equal-magnitude point charges, two positive and two negative, are arranged at the corners of a square as shown above. Let V and E be the potential and the electric field magnitude, respectively, at the center of the square, and let V_0 and E_0 be the same quantities at the same point if the upper left positive charge only were present. Which of the following expresses the values of V and E?

(A) $V = V_0$, $E = E_0$

(B) $V = 4\ V_0$, $E = 0$

(C) $V = 0$, $E = 4\ E_0$

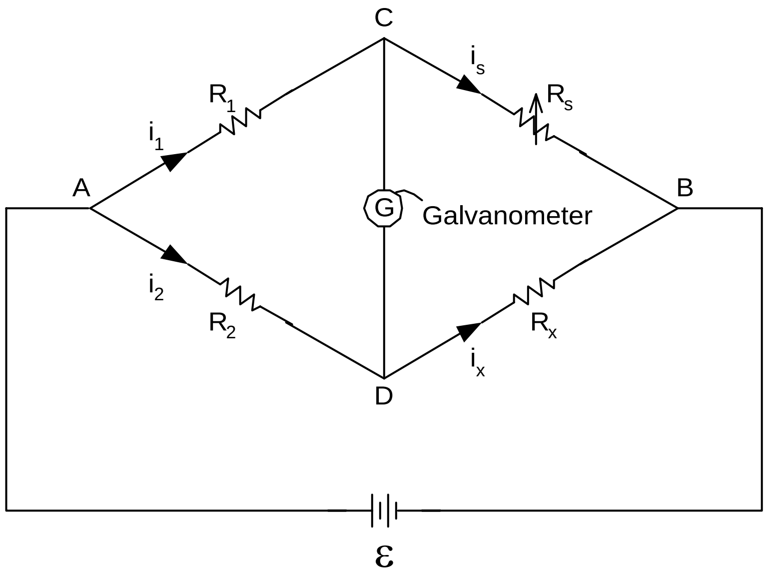
(D) $V = 4\ V_0$, $E = 4\ E_0$

(E) $V = 0$, $E = 0$

解：四个电荷在中心处产生的电势大小相等，两正两负，总量为 0。对角线上两同号电荷在

中心处产生的电场强度大小相等方向相反，合矢量为 0。所以中心处场强为 0。选（E）。

<u>Questions 22-23</u> relate to the following Wheatstone bridge circuit used for determining the unknown resistance R_x.



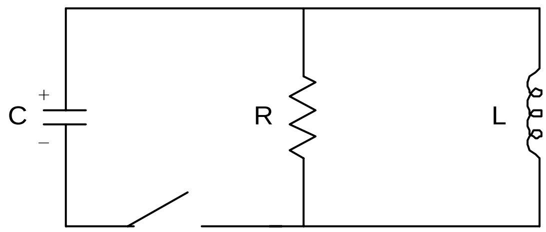
22. When the bridge is balanced by adjusting R_s, which of the following statements is FALSE?
(A) The current through the galvanometer is zero.
(B) $i_2 = i_x$.
(C) $i_sR_s = i_xR_x$.
(D) The potential at A is the same as that at B.
(E) The potential at C is the same as that at D.
解：Wheatstone 电桥是我国高中物理课程的知识。其关键点是电桥平衡时 C、D 两点电势相等，检流计示数为 0。从而 $i_1 = i_s$ ，$i_2 = i_x$ 。选（B)。

23. If $R_1 = 10\ \Omega$, $R_2 = 2000\ \Omega$, and $R_s = 50\ \Omega$, what is R_x if the bridge is balance?
(A) 0.025 Ω
(B) 0.25 Ω
(C) 4 Ω
(D) 400 Ω
(E) 10,000 Ω
解：如图所示的 Wheatstone 电桥的平衡条件是

$$R_1 \cdot R_x = R_2 \cdot R_s，$$

$$R_x = \frac{R_2}{R_1} R_s = \frac{2000}{10} 50 = 10000。$$

选(E)。



24. The capacitor C in the circuit shown above is initially charged. Neglect the resistance, capacitance, and inductance of the connecting wires. Immediately after the switch S is closed, which of the following is correct?

(A) The voltage across the resistor is zero.

(B) The voltage across the inductor is zero.

(C) The current through the inductor is zero.

(D) The current through the switch is zero.

(E) The current through the inductor is a maximum.

解：电感 L 上感生电压为

$$\mathrm{E} = L\frac{dI}{dt},$$

阻碍电流突变。所以 S 闭合瞬间通过电感的电流为 0。由于电感 L 两端有电压，而电阻 R 与电感电压相同，所以电阻有电流通过，且为

$$I = \frac{U}{R} = \frac{L\frac{dI}{dt}}{R} = \frac{\frac{Q}{C}}{R} = \frac{QC}{R},$$

其中由于电容上电荷无法突变，保持为 Q，所以电压为 Q/C。选(C)。

25. A metallized balloon of radius R carries a constant charge Q. The total energy in the electrostatic field is proportional to

(A) R^2

(B) R

(C) R^0 (i.e., constant)

(D) 1/R

(E) $1/R^2$

解：电场的能量密度为 $\frac{\varepsilon_0}{2}E^2$，球壳产生的电场为

$$E = \begin{cases} 0 & \mathrm{r} < \mathrm{R} \\ \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} & \mathrm{r} > \mathrm{R} \end{cases}$$

所以电场的总能量为

$$U = \frac{\varepsilon_0}{2}\iiint E^2 d\tau = \frac{\varepsilon_0}{2}\int_R^\infty \left(\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\right) 4\pi r^2 dr = \frac{Q^2}{8\pi\varepsilon_0 R}。$$

选（D）。考试现场如果不会，可以想一下极限情况。如果 R→∞ ，因为球壳内部电场为 0，则此时没有电场能，答案只能选（D）或（E），这样去猜答案比较容易猜对。

26. A region of space contains a uniform electric field in the +z-direction and a uniform magnetic field in the –z-direction. Which of the following statements is true?

(A) A particle could have a positive charge and initial velocity such that it would pass unaccelerated through this region.
(B) The Poynting vector of this distribution of fields is zero.
(C) This arrangement of fields can produce the Hall effect.
(D) The vector potential of the magnetic field is also parallel to the x-axis.
(E) Since the divergence and curl of both fields are zero, the fields must be antiparallel everywhere in space.

解：（A）、（C）不对，均要求电场和磁场相互垂直。答案（D）也不对，因为矢势有多种取法，例如本题情况可取（yB, 0, 0）、（ 0, –xB, 0）等等。由 Poynting 矢量的定义式 $\mathbf{S}=\mathbf{E}\times\mathbf{H}$，而本题中 **E** 与 **H** 反平行，叉乘为 0，（B）正确。（E）不对，电场可以在一个局部区域内均匀，但在一个更大的区域内不均匀，比如一个无穷大均匀带电平面两侧的电场，方向就是相反的。

27. A hollow metal sphere X of radius 10 centimeters is supported on an insulating stand A second hollow metal sphere Y of radius 2 centimeters is supported on an insulating rod. A hole is made in sphere X just large enough so that sphere Y can be introduced into the interior of X without toughing it. With the two spheres separated, X is given a positive charge of 2000 picocoulombs and Y is given a negative charge of 500 picocoulombs. Y is now introduced into the interior of X so that the two spheres do not touch. After Y is in the interior of X the two spheres are brought into contact and held there. Which of the following statements is true?

(A) There is an electric field inside sphere Y.
(B) The potential of sphere X is equal to that of sphere Y.
(C) The charge redistributes itself so that the charge on sphere X is 5 times the charge on sphere Y.
(D) All of the charge is transferred from sphere X to sphere Y.
(E) No charge is transferred between the two spheres.

解：导体 X、Y 相接触，电势相等。电荷只处在 X 壳上，而壳内无电场。所以答案选（B）。

28. Which of the following pairs of macroscopic electromagnetic field components is continuous across a boundary between two media of different dielectric constants and different magnetic permeablities?

(A) $B_\perp$ and $B_{//}$
(B) $B_\perp$ and $E_{//}$
(C) $D_\perp$ and $B_{//}$
(D) $E_\perp$ and $H_{//}$
(E) $H_\perp$ and $D_{//}$

解：再重复一遍电磁场的边界连接方程

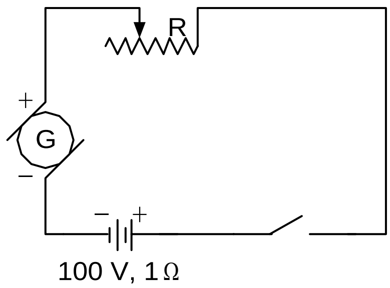
$$\mathbf{n}\cdot\left(\mathbf{D}_2-\mathbf{D}_1\right)=\Sigma$$

$$\mathbf{n}\times\left(\mathbf{E}_2-\mathbf{E}_1\right)=0$$

$$\mathbf{n}\cdot\left(\mathbf{B}_2-\mathbf{B}_1\right)=0$$

$$\mathbf{n}\times\left(\mathbf{H}_2-\mathbf{H}_1\right)=\mathbf{K}$$

其中Σ为自由面电荷密度，K为传导电流的面密度。由第二、三式，$B_\perp$和 $E_{//}$保持不变。选（B）。



29. The battery in the diagram above is to be charged by the generator G. The generator has a terminal voltage of 120 volts and when the charging current is 10 amperes. The battery has a emf of 100 volts and an internal resistance of 1 ohm. In order to charge the battery at 10 amperes charging current, the resistance R should be set at

(A) 0.1 Ω

(B) 0.5 Ω

(C) 1.0 Ω

(D) 5.0 Ω

(E) 10.0 Ω

解：注意电池与发电机 G 反接，所以

$$R=\frac{120-100}{10}-1=1.0(\Omega)\text{。}$$

选（C）。

30. A charged particle is released from rest in a region where there is a constant electric field and a constant magnetic field. If the two fields are parallel to each other, the path of the particle is a

(A) circle

(B) parabola

(C) helix

(D) cycloid

(E) straight line

解：带电体由静止释放，初始受力沿电场方向，使得 $\mathbf{v}//\mathbf{E}$，从而也平行于磁场 **B**。由 Lorentz 力公式，$\mathbf{F}=q(\mathbf{v}\times\mathbf{B})$，粒子不受磁场力作用，只受电场作用，直线运动。选（E）。

31. A negative test charge is moving near a long straight wire in which there is a current. A force will act on the test charge in a direction parallel to the direction of the current if the motion of the charge is in a direction

(A) toward the wire

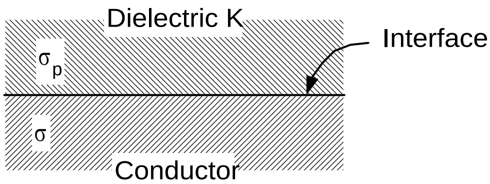
(B) away from the wire

(C) the same as that of the current
(D) opposite to that of the current
(E) perpendicular to both the direction of the current and the direction toward the wire

解：磁场方向环绕电流，服从右手定则。由 Lorentz 公式，$\mathbf{F}=q(\mathbf{v}\times\mathbf{B})$。本题关键是注意到电量为负，$\mathbf{F}=-|q|(\mathbf{v}\times\mathbf{B})$。选（A）。

32. A cube has a constant electric potential V on its surface. If there are no charges inside the cube, the potential at the center of the cube is
(A) zero
(B) V/8
(C) V/6
(D) V/2
(E) V

解：立方体内无电荷，没有电力线，电势相等。选(E)。



33. A dielectric of dielectric constant K is placed in contact with a conductor having surface charge density σ, as shown above. What is the polarization (bound) charge density σ_p on the surface of the dielectric at the interface between the two materials?

(A) $\sigma\dfrac{K}{1-K}$
(B) $\sigma\dfrac{K}{1+K}$
(C) σK
(D) $\sigma\dfrac{1+K}{K}$
(E) $\sigma\dfrac{1-K}{K}$

解：由边界条件

$$\mathbf{n}\cdot(\mathbf{D}_2-\mathbf{D}_1)=\sigma,$$

由于导体内部电场为零，

$$\mathbf{D}_1=0,$$

所以介质内部

$$D_{2n}=\sigma。$$

而

$$\mathbf{D}_2 = \varepsilon_0 \mathbf{E}_2 + \mathbf{P} = K\varepsilon_0 \mathbf{E}_2$$

$$P_{2n} = \mathbf{n} \cdot \mathbf{P}_{2n} = \mathbf{n} \cdot (\mathbf{D}_2 - \varepsilon_0 \mathbf{E}_2) = \sigma - \frac{\sigma}{K} = \frac{K-1}{K}\sigma ,$$

所以束缚电荷密度

$$\sigma_P = -P_{2n} = \frac{1-K}{K}\sigma 。$$

选（E）。

34. A point charge –q coulombs is placed at a distance d from a large grounded conducting plane. The surface charge density on the plane a distance D from the point charge is

(A) $\frac{q}{4\pi D}$

(B) $\frac{qD^2}{2\pi}$

(C) $\frac{qd}{2\pi D^3}$

(D) $\frac{qd}{2\pi D^4}$

(E) $\frac{qd}{4\pi\varepsilon_0 D^2}$

解：因为导体板电势为 0，用镜像法。在–d 放置一镜电荷+q，易得在 D 处

$$E = 2 \times \frac{q}{4\pi\varepsilon_0 D^2} \times \frac{d}{D} = \frac{qd}{2\pi\varepsilon_0 D^3} 。$$

由边界条件

$$\vec{n} \cdot (E_2 - E_1) = \frac{\sigma}{\varepsilon_0} ,$$

并考虑到导体内部电场为 0，有

$$\sigma = \frac{qd}{2\pi D^3} 。$$

选（C）。

35. A current i in a circular loop of radius b produces a magnetic field. At a fixed point far from the loop, the strength of the magnetic field is proportional to which of the following combinations of i and b?

(A) ib

(B) ib^2

(C) i^2b

(D) $\frac{i}{b}$

(E) $\dfrac{i}{b^2}$

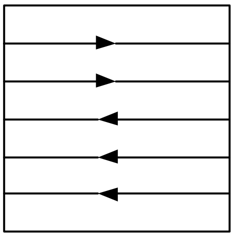
解：取一特殊点，计算环轴线上距中心距离为 L 处的磁感应强度 **B**：

$$\mathbf{B}=\frac{\mu_0}{4\pi}\oint\frac{Id\mathbf{l}\times\hat{\mathbf{r}}}{r^2}=\frac{\mu_0}{4\pi}\frac{i2\pi b}{L^2+b^2}\frac{b}{\sqrt{L^2+b^2}}=\frac{\mu_0 i}{2}\frac{b^2}{\left(L^2+b^2\right)^{\frac{3}{2}}},$$

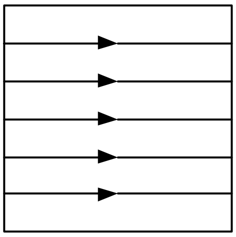
当 L>>b 时，$B\propto ib^2$。选（B）。

36. One of Maxwell's equations is $\nabla\cdot\mathbf{B}=0$. Which of the following sketches shows magnetic field lines that clearly violate this equation within the region bounded by the dashed lines?

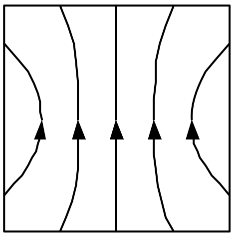
(A)



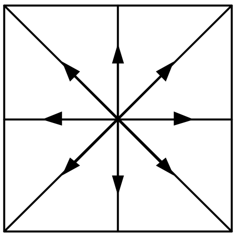
(B)



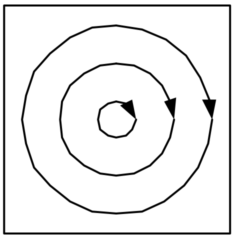
(C)



(D)



(E)



解：（A）、（B）、（C）、（E）为常见的磁力线。（D）给人的第一印象是点电荷的电力线，由

$$\nabla\cdot\mathbf{E}=\frac{\rho}{\varepsilon_0},$$

散度不为零，所以它代表磁力线时将导致$\nabla\cdot\mathbf{B}\neq 0$。选（D）。

37. Which of the following electric fields could exist in a finite region of space that contains no charges? (In these expressions, A is a constant, and **i**, **j**, and **k** are unit vectors pointing in the x, y, and z directions, respectively.)

(A) A(2xy**i**–xz**k**)

(B) A(–xy**j**+xz**k**)

(C) A(xz**i**+xz**j**)

(D) Axyz(**i**+**j**)

(E) Axyz**i**

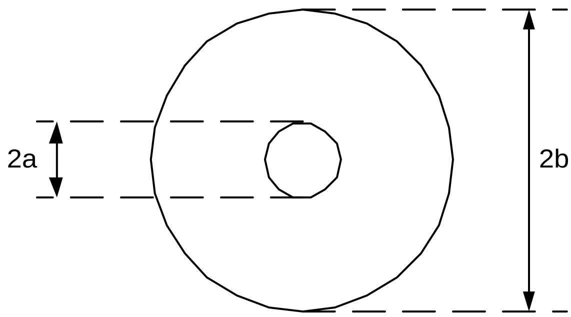
解：由 Maxwell 方程组，

$$\nabla \cdot \vec{E} = \rho / \varepsilon_0 ,$$

本题中只有选项（B）

$$\nabla \cdot \left(-xy\vec{j} + xz\vec{k} \right) = -x + x = 0 ,$$

空间处处无电荷。选（B）。



38. A small circular wire loop of radius a is located at the center of a much larger circular wire loop of radius b sa shown above. The larger loop carries an alternating current $I=I_0\cos\omega t$, where I_0 and ω are constants. The magnetic field degenerated by the current in the large loop induces in the small loop an emf that is approximately equal to which of the following? (Either use mks units and let μ_0 be the permeability of free space, or use Gaussian units and let μ_0 be $4\pi/c^2$.)

(A) $\left(\dfrac{\pi\mu_0 I_0}{2}\right)\dfrac{a^2}{b}\omega\cos\omega t$

(B) $\left(\dfrac{\pi\mu_0 I_0}{2}\right)\dfrac{a^2}{b}\omega\sin\omega t$

(C) $\left(\dfrac{\pi\mu_0 I_0}{2}\right)\dfrac{a}{b^2}\omega\sin\omega t$

(D) $\left(\dfrac{\pi\mu_0 I_0}{2}\right)\dfrac{a}{b^2}\omega\cos\omega t$

(E) $\left(\dfrac{\pi\mu_0 I_0}{2}\right)\dfrac{a}{b}\omega\sin\omega t$

解：由 Biot-Savart 定律

$$\vec{B} = \frac{\mu_0}{4\pi}\oint\frac{Id\bar{l} \times \hat{\vec{r}}}{r^2} ,$$

外环上各点在中心产生的磁感应方向相同，均垂直于圆环平面，易得

$$B=\frac{\mu_0}{4\pi}\frac{2\pi b}{b^2}I=\frac{\mu_0}{2}\frac{I}{b}\text{。}$$

因为 a << b，故内环磁感应强度近似为 B，磁通量为

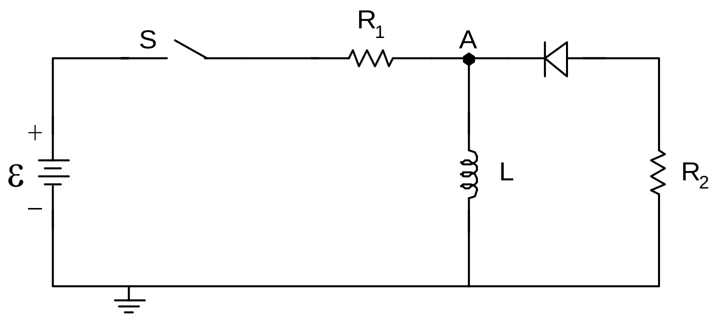
$$\Phi=\pi a^2 B=\frac{\pi\mu_0}{2}\frac{a^2}{b}I=\frac{\pi\mu_0}{2}\frac{a^2}{b}I_0\cos\omega t\text{，}$$

所以感生电场

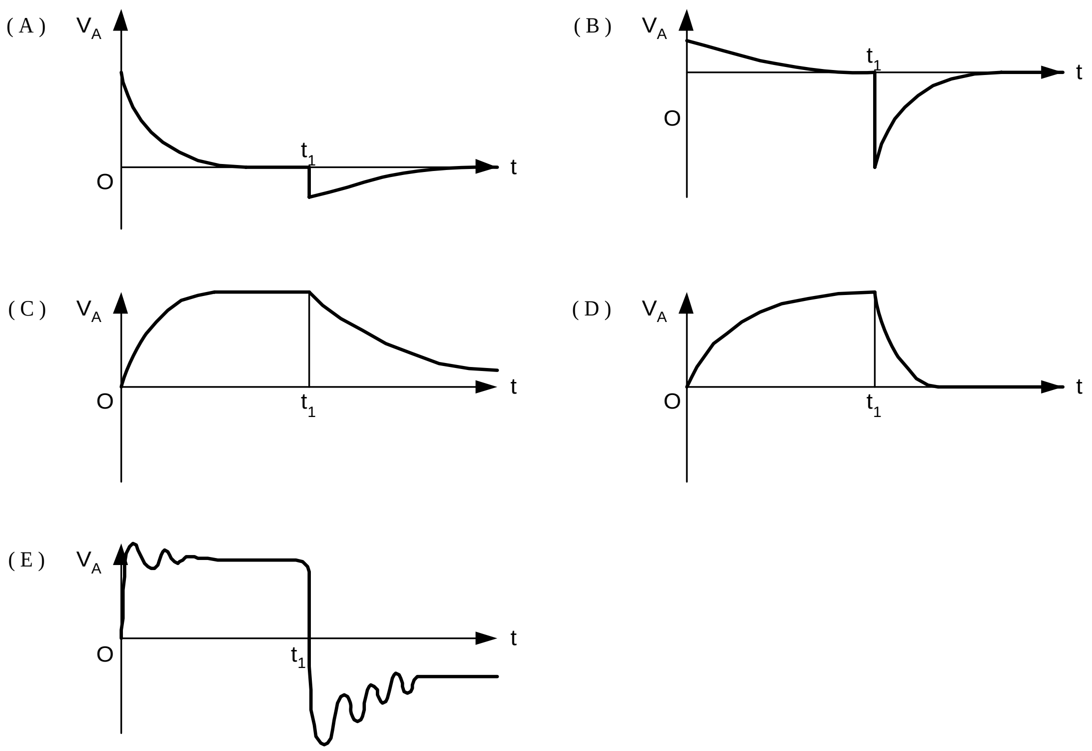
$$\mathrm{E}=\left|\frac{d\Phi}{dt}\right|=\left(\frac{\pi\mu_0 I_0}{2}\right)\frac{a^2}{b}\omega\sin\omega t\text{。}$$

选（B）。

39. The exponent in Coulomb’s inverse square law has been found to differ from two by less than one part in a billion by measuring which of the following?
(A) The charge on an oil drop in the Millikan experiment
(B) the deflection of an electron beam in an electric field
(C) The neutrality of charge of an atom
(D) The electric force between two charged objects
(E) The electric field inside a charged conducting shell
解：如果 Coulomb 平方反比定律严格成立，则可以用 Gauss 定理证明在均匀带电球壳内部电场处处为零。选（E）。



40. In the circuit shown above, $R_2=3R_1$ and the battery of emf E has negligible internal resistance. The resistance of the diode when it allows current to pass through it is also negligible internal resistance. At time t=0, the switch S is closed and the currents and voltages are allowed to reach their asymptotic values. Then at time t_1, the switch is opened. Which of the following curves most nearly represents the potential at point A as a function of time t?



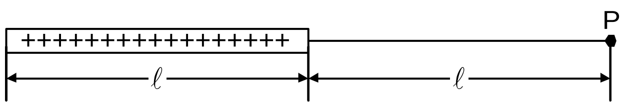
解：技巧如下：在 t_1 前瞬间，系统处于稳定状态，电感不起作用（$\frac{dI}{dt}=0$），所以 A 点电压为 0，答案只能为（A）或（B）；断开 S 前瞬间，

$$I=\frac{\varepsilon}{R_1}。$$

故断开 S 后瞬间 A 点电压为

$$U_A=-IR_2=-3\varepsilon。$$

选（B）。



41. The long thin cylindrical glass rod shown above has length l and is insulated from its surroundings. The rod has an excess charge Q uniformly distributed along its length. Assume the electric potential to be zero at infinite distances from the electric potential at a point P along the axis of the rod and a distance l from one end is $\frac{kQ}{l}$ multiplied by

(A) $\frac{4}{9}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

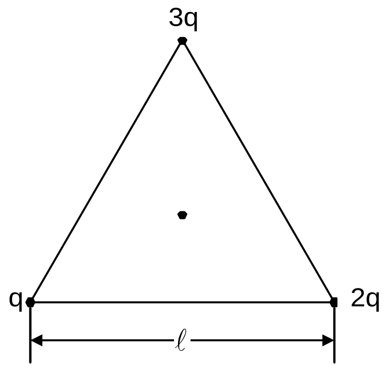
(D) $\ln 2$

(E) 1

解：电势沿杆积分得

$$U=\int_0^l K\frac{\frac{Q}{l}dx}{l+x}=K\frac{Q}{l}\ln 2\text{。}$$

选（D）。



42. Positive charge of magnitude q, 2q, and 3p are located at the corners of an equilateral triangle as shown above. The sides of the triangle have a length l. What is the minimum amount of work requires to bring a positive charge Q from infinity to the center of the triangle?

(A) $\dfrac{2\sqrt{3}qQ}{l}$

(B) $\dfrac{4\sqrt{3}qQ}{l}$

(C) $\dfrac{6\sqrt{3}qQ}{l}$

(D) $\dfrac{8\sqrt{3}qQ}{l}$

(E) $\dfrac{12\sqrt{3}qQ}{l}$

解：无穷远处电势为零，则

$$U=\frac{q+2q+3q}{r}=\frac{6q}{\sqrt{3}l/3}=\frac{6\sqrt{3}q}{l},$$

所以需要

$$W=QU=\frac{6\sqrt{3}Qq}{l}\text{。}$$

选（C）。

43. If an electric field is given in a certain region by $E_x = 0$, $E_y = 0$, $E_z = kz$, where k is a nonzero constant, which of the following is true?
(A) There is a time-varying magnetic field
(B) There is charge density in the region
(C) The electric field cannot be constant in time
(D) The electric field is impossible under any circumstances
(E) None of the above
解：由 Maxwell 方程组

$$\nabla \cdot \mathrm{E} = \rho / \varepsilon_0$$

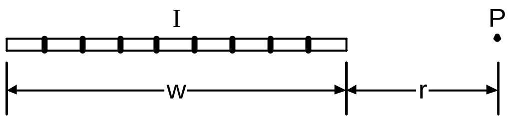
所以电荷密度为

$$\rho = \varepsilon_0 \nabla \cdot \mathrm{E} = \varepsilon_0 \left(\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\right) = \varepsilon_0 k \text{ ,}$$

选（B）。由于

$$\nabla \times \mathrm{E} = -\frac{\partial \mathrm{B}}{\partial t} ,$$

而本题中$\nabla \times \mathrm{E} = 0$，所以并不一定存在变化的磁场，（A）不对。



44. The diagram above represent the cross section, a very long thin strip of conductor of width.The strip carries a total current I uniformly distributed and directed into the plane of the page. What is the magnitude of the magnetic field due to I at point P in the plane of the strip a distance r from the closer edge?

(A) $\dfrac{\mu_0 I}{2\pi r}$

(B) $\dfrac{\mu_0 I}{2\pi (r + w)}$

(C) $\dfrac{\mu_0 I}{2\pi (r + w/2)}$

(D) $\dfrac{\mu_0 I}{2\pi w} \ln(\dfrac{r + w}{r})$

(E) $\dfrac{\mu_0 I}{2\pi r} \ln(\dfrac{r - w}{r})$

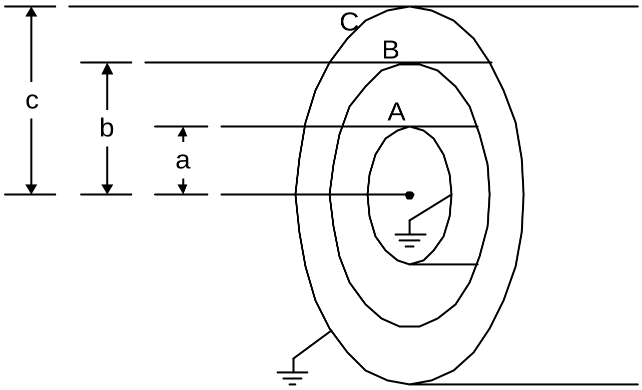
解：由 Ampere 环路定理易得，载流为 I 的无穷长导线在距离 r 处产生的磁感应强度为

$$B = \frac{\mu_0 I}{2\pi r},$$

所以对于本题

$$B = \int_0^w \frac{\mu_0}{2\pi(r+x)} \frac{I}{w} dx = \frac{\mu_0 I}{2\pi w} \ln(r+x)\Big|_0^w = \frac{\mu_0 I}{2\pi w} \ln\left(\frac{r+w}{r}\right)。$$

选（E）。



45. Three long concentric conducting cylinders A, B, and C have radii a, b, and c, respectively, as shown above. The thickness of each cylinder is much smaller than a. The innermost and ountermost cylinders are grounded. A charge Q is placed on cylinder B. If end effects are negligible, the ratio of the charge on the inner surface of B to the charge on the outer surface of B, Q_{inner}/Q_{outer}, is

(A) 0

(B) 1

(C) $\dfrac{\ln(b/a)}{\ln(c/b)}$

(D) $\dfrac{\ln(c/b)}{\ln(b/a)}$

(E) $\dfrac{\ln(c/a)}{\ln(b/c)}$

解：圆柱体 A 和 B、C 和 B 分别组成同轴柱形电容。因为 A、C 电势相等同为 0，所以系统可看为这两个电容的并联。由电容并联的性质，

$$\frac{Q_A}{Q_C} = \frac{C_{AB}}{C_{CB}}。$$

为计算同轴柱形电容的值，设长度为 L，每个电极电荷的线密度的绝对值为 λ，则由 Gauss 定理，电极间的电场强度为

$$E = \frac{\lambda}{2\pi\varepsilon_0 r},$$

所以电势差为

$$U_{AB}=\int_A^B \mathrm{E}\cdot \mathrm{d}\mathrm{l}=\int_{R_A}^{R_B}\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}dr=\frac{\lambda}{2\pi\varepsilon_0}\ln\frac{R_B}{R_A}\text{。}$$

而电极上总电量为 q = λ L，所以

$$C=\frac{q}{U_{AB}}=\frac{2\pi\varepsilon_0 L}{\ln\frac{R_B}{R_A}}\text{。}$$

$$\frac{Q_A}{Q_C}=\frac{C_{AB}}{C_{CB}}=\frac{\ln\frac{R_C}{R_B}}{\ln\frac{R_B}{R_A}}=\frac{\ln(c/b)}{\ln(b/a)}\text{。}$$

选（D）。

46. Two infinite conducting planes are located at x = 0 and y = 0, respectively. These planes intersect each other at right angles along the z-axis. The planes are grounded. A point charge q is located at x = c, y = c. What is the magnitude of the force on the charge?

(A) $\frac{3}{8}\frac{q^2}{a^2}$

(B) $(\frac{1}{4}+\frac{\sqrt{2}}{8})\frac{q^2}{a^2}$

(C) $(\frac{1}{4}-\frac{\sqrt{2}}{8})\frac{q^2}{a^2}$

(D) $(\frac{\sqrt{2}}{4}+\frac{1}{8})\frac{q^2}{a^2}$

(E) $(\frac{\sqrt{2}}{4}-\frac{1}{8})\frac{q^2}{a^2}$

解：用镜像法。因为两导体板接地电势为 0，所以要放置三个像电荷，分别是在（–a，a）、（a，–a）放置 电量为–q 的 电荷，在（–a，–a）放置电量为 q 的电荷。则电荷受到导体板的作用力可等效为这三个像电荷对它的作用力，

$$\mathrm{F}=\mathrm{F}_1+\mathrm{F}_2+\mathrm{F}_3=\frac{q^2}{a^2}\left(-\frac{1}{4}\mathrm{i}-\frac{1}{4}\mathrm{j}+\frac{\sqrt{2}}{16}\mathrm{i}+\frac{\sqrt{2}}{16}\mathrm{j}\right)=-\frac{q^2}{4a^2}\left(1-\frac{\sqrt{2}}{4}\right)(\mathrm{i}+\mathrm{j})\text{。}$$

力的大小为

$$F=\frac{q^2}{4a^2}\left(1-\frac{\sqrt{2}}{4}\right)\sqrt{2}=\left(\frac{\sqrt{2}}{4}-\frac{1}{8}\right)\frac{q^2}{a^2}\text{。}$$

选（E）。

47. An ideal transformer has a primary coil of N_p turns and a secondary coil of N_s turns. An altering voltage V_p is applied to the primary coil of the transformer. Which of the following statements is NOT correct?

(A) In the primary coil of the transformer, the voltage lags the current.

(B) The coefficient of mutual inductance between the primary and secondary coils is proportional to the product N_pN_s .

(C) When the secondary coil is open, the power factor of the transformer is zero.

(D) When the secondary coil is open, the secondary voltage is $V_s = V_p(\frac{N_p}{N_s})$.

(E) If a resistance R is placed across the secondary coil, the reflected impedance at the terminals of the primary coil will be $Z_p = R(\frac{N_p}{N_s})^2$.

解：由于原线圈具有自感系数，而没有电容，所以总阻抗是电感性的。所以电压的相位超前于电流。选项（A）不对。（B）、（C）、（D）为明显的结论。对于（E），因为

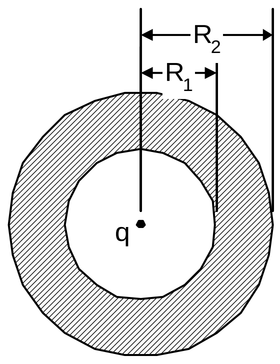
$$\frac{U_P}{U_S} = \frac{N_P}{N_S}, \quad \frac{I_P}{I_S} = \frac{N_S}{N_P},$$

所以

$$Z_p = \frac{U_P}{I_P} = \left(\frac{N_P}{N_s}\right)^2 \frac{U_S}{I_S} = R\left(\frac{N_P}{N_s}\right)^2 。$$

选（A）。

<u>Questions 48-49</u>



A conducting sphere of radius R_2 has a concentric spherical cavity of radius R_1, as shown above. The sphere has no net charge. However, there is a charge q in the center of the cavity.

48. What is the surface charge density on the inner surface of the sphere?

(A) $-\frac{q}{4\pi R_1^2}$

(B) $-\dfrac{q}{8\pi R_1^{\ 2}}$

(C) $-\dfrac{q}{4\pi R_2^{\ 2}}$

(D) $-\dfrac{q}{8\pi R_2^{\ 2}}$

(E) $-\dfrac{R_2^{\ 2}q}{4\pi R_1^{\ 2}}$

解：导体球内壁上总的感应电荷为–q，由于导体球净电荷为零，相应的球外壳上分布又总的电荷为+q。由球对称性的分析可以得知，球壁上的感应电荷都是均匀分布的。因此球内的面电荷密度为

$$\sigma = \frac{1}{4\pi R_1^{\ 2}}\text{。}$$

选（A）。

49. What is the potential of the sphere? (Assume that the potential is zero at an infinite distance away)

(A) $\dfrac{q}{R_1}$

(B) $\dfrac{q}{R_2}$

(C) $\dfrac{qR_2}{R_1^2}$

(D) $\dfrac{qR_1}{R_2^2}$

(E) $\dfrac{qR_2}{R_1}$

解：球外电场分布等价于球心处点电荷产生的电场分布，导体球的电势等于点电荷在距自身 R_2 远处产生的电势。选（B）。
注意此题由答案的四个选项来看，显然命题者采用的是 Gauss 单位制，不必细究。

50. In which of the following circumstances must the displacement current be zero?
(A) If the magnetic field is zero
(B) If the electric field is constant in time

(C) When there is an open circuit

(D) In a metal

(E) In an insulator

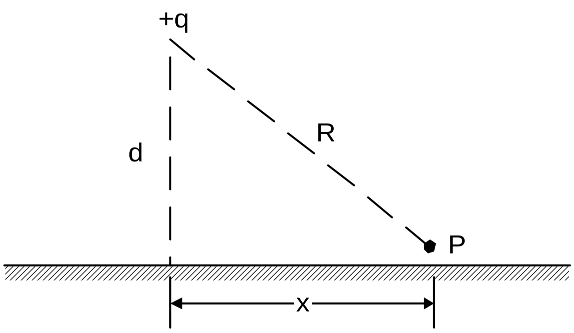
解：根据位移电流的定义式

$$J_D = \frac{\partial D}{\partial t},$$

显然当静场时

$$\frac{\partial D}{\partial t} = 0 \text{ 或 } \frac{\partial E}{\partial t} = 0,$$

位移电流为零。选（B）。



51. A point charge +q is located a distance d above a grounded conducting plane. The magnitude , in MKS units, of the electric field at point P just above the surface is

(A) $\frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

(B) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{R^2}$

(C) $\frac{1}{4\pi\varepsilon_0}\frac{2qd}{R^3}$

(D) $\frac{1}{4\pi\varepsilon_0}\frac{2qx}{R^3}$

(E) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{xd^2}$

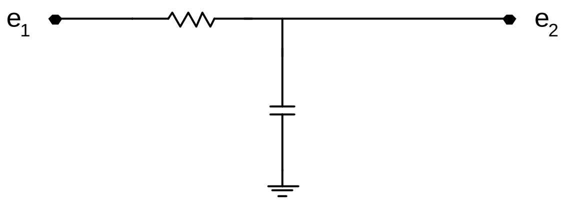
解：显然是用电像法。q 的感应电荷的电像带电荷–q，位置与 q 相对于接地平面成镜像对称。再根据对称性分析，P 点的电场强度水平分量为零，只有垂直向下的分量。

$$E_P = 2\cdot\frac{q}{4\pi\varepsilon_0 R^2}\cdot\frac{d}{R},$$

选（C）。

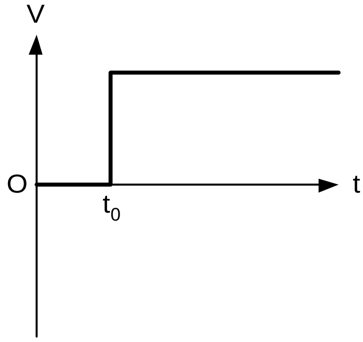
关于电像法：无穷大平面附近点电荷的电像法一定要熟练掌握，带点球壳的共轭电像法只需

了解。

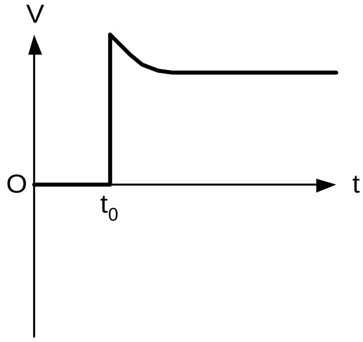


52. In the circuit illustrated above, the point marked c_1 is raised instantaneously at time t_0 from zero to a fixed positive potential. If c_2 is connected to a high-impedance D.C. oscilloscope input, the graph of its voltage V as a function of time t will most resemble which of the following?

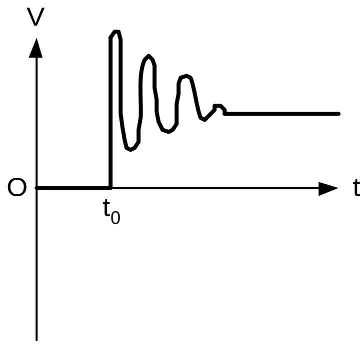
(A)



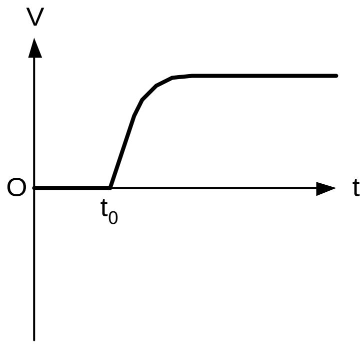
(B)



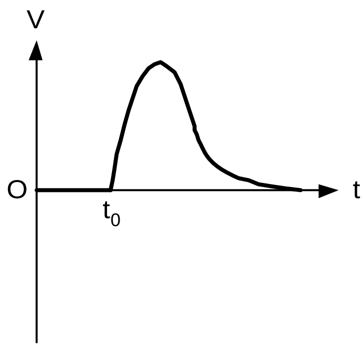
(C)



(D)



(E)



解：这是电容的直流放电暂态线路，电路的时间常数为 RC，电容两端的电压随时间指数增长，定性就可判断应选（D）。

严格求解电路微分方程的结果是

$$V = V_0\left(1 - e^{-\frac{t-t_0}{RC}}\right),$$

其中 $t > t_0$。

53. A parallel-plate capacitor has a charge of 10^{-5} coulomb. The potential difference across the capacitor is 10^4 volts when the plates have a vacuum between them. If a second capacitor has the identical dimensions and the same charge as the first one but has a dielectric of K = 5 between the plates, the potential difference across this second capacitor would be most nearly

(A) 0V

(B) 2000V

(C) 10,000V

(D) 20,000V

(E) 50,000V

解：平行板电容器的电容

$$C=\frac{\varepsilon_r\varepsilon_0 S}{d},$$

而$U=\dfrac{Q}{C}$。对电介质 K = 5 的第二个电容器，其电容为第一个的 5 倍。带同样多电荷的情况下，相应两极板间的电势差是第一个电容器的 1/5。选（B）。

54. A spherical cloud of charge of radius R has a uniform charge density ρ. At a distance r from the center of the charge distribution, with r < R, the magnitude of the electric field is

(A) $\dfrac{\rho r^3}{3\varepsilon_0 R^2}$

(B) $\dfrac{\rho r^2}{3\varepsilon_0 R}$

(C) $\dfrac{\rho r}{3\varepsilon_0}$

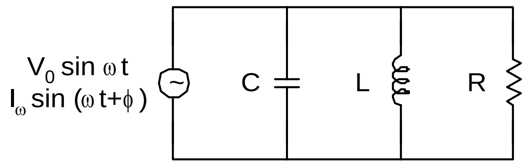
(D) $\dfrac{\rho R^2}{3\varepsilon_0 r}$

(E) $\dfrac{\rho R^3}{3\varepsilon_0 r^2}$

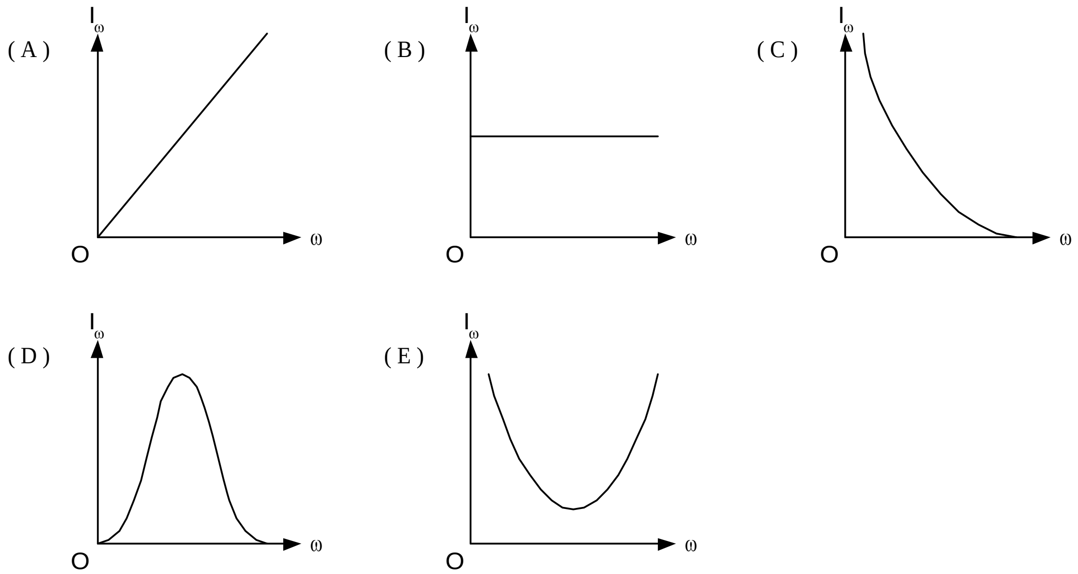
解：在球的内部，只有半径 r 以内的电荷对 r 远处的电场有贡献，其余均匀电荷的分布总贡献为零。因此

$$E=\frac{\rho\frac{4}{3}\pi r^3}{4\pi\varepsilon_0 R^2}=\frac{\rho r^3}{3\varepsilon_0 R^2}。$$

选（A）。



55. In the circuit shown above, the maximum generator voltage V_0 is fixed. Which of the following graphs best represents the amplitude I of the current in the generator as a function of the frequency ω?



解：RCL 并联谐振电路。简单定性分析一下很快就能得到答案：ω→0 时，电感相当于短路，因此 I→∞；类似当 ω→∞时，电容相当于短路，因此 I→∞；当中某个频率 ω 的时候曲线必然存在拐点。因此选（E）。

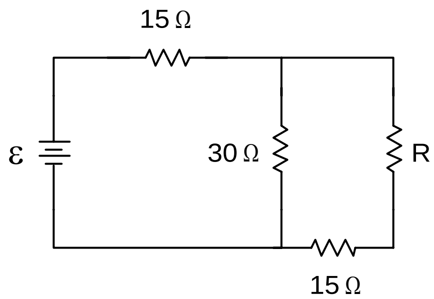
定量的计算结果，等效阻抗

$$\frac{1}{z}=\frac{1}{R}+\frac{1}{j\omega L}+j\omega C,$$

电流

$$I=\frac{V}{Z}=\frac{Z}{R}+iZ\left(\omega C-\frac{1}{\omega L}\right),$$

当 $\omega=\dfrac{1}{\sqrt{LC}}$ 时回路中的总电流最小。



56. A battery with emf ε and negligible internal resistance is connected in the circuit shown above. The value of the resistance R required to maximize the power dissipated in R is

(A) 10 Ω
(B) 15 Ω
(C) 25 Ω
(D) 30 Ω
(E) 45 Ω

解：此题的最佳解法应当是利用等效电压源定理，将电路中除 R 以外的部分看成一个等效电压源，其内阻为

$$15+15//30=25\Omega。$$

然后根据外接电阻与电源内阻相等时，输出功率最大，选（C）。

一般的解法，是求函数

$$P_R=\left(\varepsilon\frac{30//(15+R)}{15+30//(15+R)}\frac{R}{15+R}\right)^2/R$$

的极大值，较繁。

57. A certain electric field is given in spherical coordinate by

$$\vec{E}(t)=Ar^{-\frac{3}{2}}\vec{r}$$

where A is a constant and $\vec{r}$ is a unit vector radically outward. The charge density ρ(r) is proportional to

(A) $r^{\frac{1}{2}}$

(B) $r^{-\frac{1}{2}}$

(C) $r^{-\frac{3}{2}}$

(D) $r^{-\frac{5}{2}}$

(E) $r^{-\frac{7}{2}}$

解：直接由 Maxwell 方程的微分形式求解电荷密度

$$\rho=\nabla\cdot\vec{D}=\varepsilon\nabla\cdot\vec{E}\propto\nabla\cdot\frac{\vec{r}}{r^{3/2}}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\cdot r^{-3/2}\right)\propto r^{-5/2}。$$

选（C）。

58. A circular ring of radius 10 centimeters is made out of a wire of radius 1 millimeter. The ring carries a steady current. The magnetic field due to that current is measured at the center of the ring, where its magnitude is found to B_c, and at the surface of the wore, where its magnitude is found to be B_s. The ratio B_s/B_c is most nearly

(A) 300
(B) 30
(C) 3

(D) 1/3

(E) 1/30

解：电流线圈中心的磁场方向垂直于线圈平面，

$$B_c = \frac{\mu_0 I}{2R}\text{。}$$

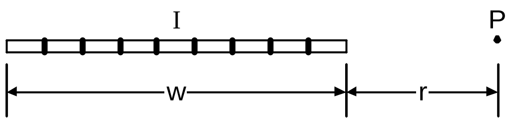
电缆线圈表面的磁场沿切向，可由 Ampere 环路定理求出

$$B_S = \frac{\mu_0 I}{2\pi r}\text{。}$$

所以

$$\frac{B_S}{B_C} = \frac{2R}{2\pi r} = \frac{100}{\pi} \approx 30\text{。}$$

选（B）。



59. The diagram above represents the cross section of a very long, thin strip of conductor of width w. The strip carries a total current I uniformly distributed and directed into the plane of the page. What is the magnitude of magnetic field due to I at point P in the plane of the strip a distance r from the closer edge?

(A) $\dfrac{\mu_0 I}{2\pi r}$

(B) $\dfrac{\mu_0 I}{2\pi(r-w)}$

(C) $\dfrac{\mu_0 I}{2\pi(r+w/2)}$

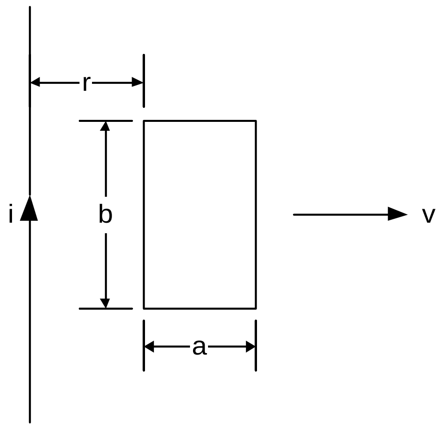
(D) $\dfrac{\mu_0 I}{2\pi w}\ln\left(\dfrac{r+w}{r}\right)$

(E) $\dfrac{\mu_0 I}{2\pi r}\ln\left(\dfrac{r-w}{r}\right)$

解：只能利用微元法积分求解。以到替代的最左端为原点建立向右的 x 轴。

$$B = \frac{\mu_0}{2\pi}\int_0^w \frac{I}{w(w+r-x)}dx = -\frac{\mu_0 I}{2\pi w}\ln\left(w+r-x\right)\Big|_0^w\text{。}$$

实际计算中只需判断是 r 的指数关系即可，（E）为负值显然不对。选（D）。



60. A rectangular loop of wire resistance R has the dimensions shown above. A long wire with current i is located in the plane of the loop a distance r from one side of the loop as shown. If the loop is pulled radically away from the wire at constant speed v , the current in the loop is

(A) $\dfrac{\mu_0 ivab}{2\pi R}\cdot\dfrac{1}{r}$

(B) $\dfrac{\mu_0 ivab}{2\pi R}\cdot\dfrac{1}{r^2}$

(C) $\dfrac{\mu_0 ivab}{2\pi R}\cdot\dfrac{1}{(r+a)}$

(D) $\dfrac{\mu_0 ivab}{2\pi R}\cdot\dfrac{1}{r(r+a)}$

(E) $\dfrac{\mu_0 ivab}{2\pi R}\cdot\dfrac{1}{r^2(r+a)}$

解：矩形线圈的两条短边运动没有切割磁力线，不产生感生电动势。两条长边产生的感生电动势方向相反，总和为

$$\varepsilon = vb(B_1 - B_2) = \frac{\mu_0 i}{2\pi} vb\left(\frac{1}{r} - \frac{1}{r+a}\right)。$$

选（D）。

61. The plane of a square wire loop 0.2 meter on a side is perpendicular to a 0.008 tesla magnetic field. If the magnetic field reduced to zero in 0.04 second, the average voltage induced in the loop during the time interval is

(A) 0.008 V

(B) 0.04 V

(C) 0.2 V

(D) 3.2 V

(E) 32 V

解：根据 Faraday 的电磁感应定律，感生电动势的大小

$$\bar{\varepsilon}=\frac{\Delta\Phi}{\Delta t}=\frac{S\Delta B}{\Delta t}=\frac{0.2^2\times 0.008}{0.04}\approx 0.008\text{V}\ 。$$

选（A）。

62. A parallel-plate capacitor containing a removable dielectric is fully charged . If the dielectric is removed with the charging battery disconnected, how do the total charge and energy stored by the capacitor change?

	Charge	Energy
(A)	Decreases	Decreases
(B)	Decreases	Increases
(C)	Stays the same	Decreases
(D)	Stays the same	Increases
(E)	Increases	Decreases

解：由于电路是断开的，因此电容器上储存的电量保持不变。除去极板间电介质后，电容器的电容变为原来的 1/ ε_r，而总能量

$$E=\frac{Q^2}{2C}$$

变大。选（D）。

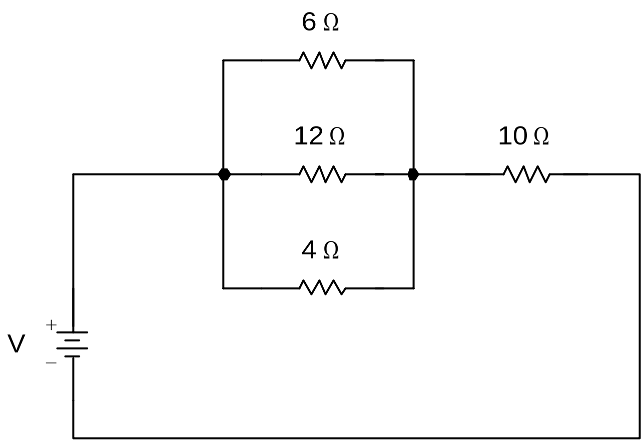
63. An electron moves at constant speed in auniform magnetic field. If the initial velocity of the electron is parallel to the magnetic field, the electron describes a path that is a

(A) straight line
(B) helix
(C) circle
(D) parabola
(E) hyperbola

解：电子的初速度平行于磁场，因此所受的 Lorentz 力

$$\mathbf{F}=-e\mathbf{v}\times\mathbf{B}=0\ 。$$

因此电子不受力，做匀速直线运动。选（A）。



64. The current in the 6-ohm resistpr in the circuit shown above is 0.5 ampere. The voltage V applied to the circuit is most nearly

(A) 6 V

(B) 8 V

(C) 16 V

(D) 18V

(E) 23 V

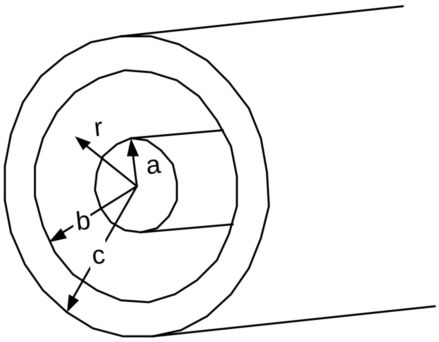
解：6Ω电阻上的电压降为 3V，因此，回路中的总电流

$$I = 0.5 + \frac{3}{12} + \frac{3}{4} = 1.5\text{A}\ ,$$

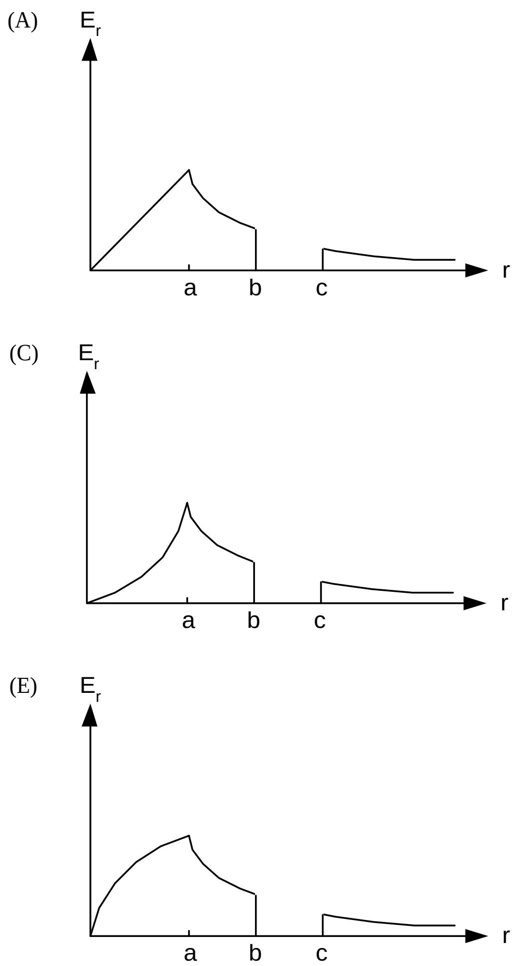
电源总电压

$$V = 3 + 1.5 \times 10 = 18\text{V}\ 。$$

选（D）。



65. The cylinder shown above has radius a and uniform volume density ρ. It is surrounded by a concentric cylinderical shell, which is a conductor with inner and outer radii b and c, respectively. Which of the following graphs displays the radial electric field E_r as a function of r, the distance from the axis?

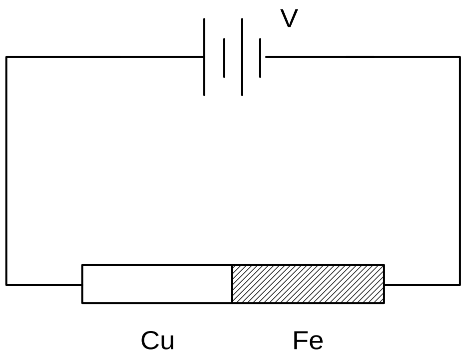


解：b < r < c 是在导体内部，因此 $E_r = 0$。在 r < a 的区域内，由对称性电场强度沿径向，其大小可由 Gauss 定理求出

$$E = \frac{\rho r}{2\varepsilon_0} \propto r \text{ 。}$$

选（A）。

<u>Questions 66-67</u>



Two rods are joined end-to-end, as shown above. Both have a cross-sectional area of 0.01 cm^2. Each is I meter long. One rod is copper with a resistivity of 1.7×10^{-6} ohm·centimeter; the other is iron with a resistivity of 10^{-5} ohm·centimeter.

66. What voltage is required to produce a current of 1 ampere in the rods?

(A) 0.117 V

(B) 0.0145 V

(C) 0.0117 V

(D) 0.00145 V

(E) 1.7×10^{-6} V

解：两段金属棒串联的总电阻

$$R = \rho_{Cu}\frac{l}{s} + \rho_{Fe}\frac{l}{s} = \left(1.7\times10^{-6} + 10^{-5}\right)\times\frac{100}{0.01} = 0.117\Omega,$$

由 Ohm 定理可求出电流 I = U/R。选（A）。

67. What is the surface charge density that builds up at the copper-iron interface when there is a current of 1 ampere in the rods?

(A) Zero

(B) 1.7×10^{-8} coulomo/m^2

(C) 10^{-10} coulomo/m^2

(D) 7.0×10^{-13} coulomo/m^2

(E) 2.7×10^{-13} coulomo/m^2

解：在界面两端分别应用 Ohm 定理的微分形式有

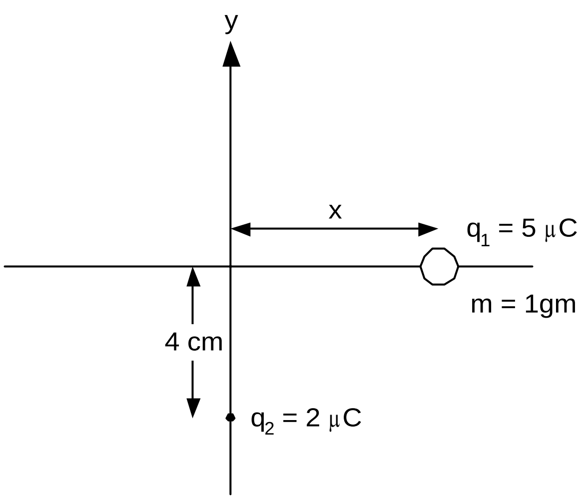
$$j = \frac{I}{s} = \sigma E \Rightarrow E = \rho\frac{I}{s},$$

根据边界关系有

$$\sigma_s = D_{Fe} - D_{Cu} = \varepsilon_0\left(E_{Fe} - E_{Cu}\right) = \varepsilon_0\left(\rho_{Fe} - \rho_{Cu}\right)\frac{I}{s}。$$

$$\sim 10^{-11}\times10^{-7}/10^{-6} = 10^{-12}$$

选（D）。

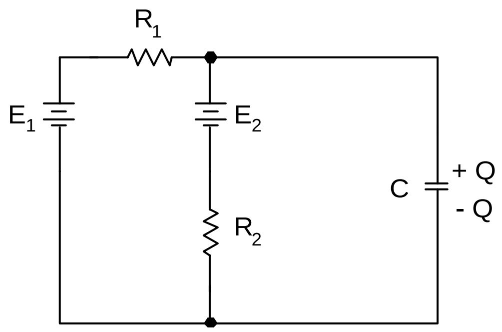


68. A bead of charge q_1 = 5 microcoulombs and mass 1 gram slides without friction on a wire along the x-axis, as shown above. Another charge q_2 = 2 microcoulombs is fixed at x = 0, y = –4 centimeters. If q_1 is released from rest at x = 3 centimeters, its speed at x = ∞ is most nearly

(A) 6 m/s
(B) 60 m/s
(C) 190 m/s
(D) 1900 m/s
(E) 6000 m/s
解：根据能量守恒，当 q_1 运动到无穷远处时初始的电场势能完全转化为动能。

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\sqrt{x^2+y^2}}=\frac{1}{2}mv^2,$$

由此可解得 v。选（C）。



69. A capacitor is charged with two batteries, as shown above. If initially there is no charge on C, which of the following expressions gives the time dependence of Q?

(A) $Q_0[1-e^{-t(R_1+R_2)C}]$

(B) $Q_0[1-e^{-t(R_1+R_2)/R_1R_2C}]$

(C) $Q_0e^{-t(R_1+R_2)/R_1R_2C}$

(D) $Q_0e^{-t(R_1+R_2)C}$

(E) $Q_0e^{-t/\sqrt{R_1R_2}C}$

解：利用等效电压源定理简化电路，等效的电源内阻

$$R=R_1 // R_2=\frac{R_1R_2}{R_1+R_2}\text{。}$$

电容充电的时间常数

$$\tau=RC=\frac{R_1R_2C}{R_1+R_2}\text{。}$$

选（B）。

70. Positive charge is brought from far away and gradually assembled on the surface of an initially uncharged sphere of radius R. The work required to place total charge Q on a sphere in this

manner is

(A) $\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{R^2}$

(B) $\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{R^2}$

(C) $\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{R}$

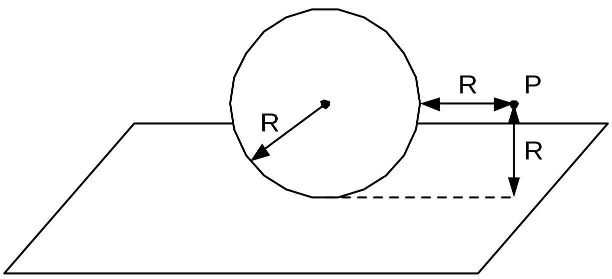
(D) $\frac{3}{5}\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{R}$

(E) $\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{R}$

解：设想每次移动很小的一份电荷 dq 到球壳上，积分可得总共做的功

$$W=\int_0^Q\frac{1}{4\pi\varepsilon_0}\frac{qdq}{R}=\frac{1}{8\pi\varepsilon_0}\frac{q^2}{R}\Big|_0^Q=\frac{1}{8\pi\varepsilon_0}\frac{Q^2}{R}\text{。}$$

选（A）。



71. A thin insulating spherical shell of radius R just touches a thin infinite insulating plane, as shown above. Both the shell and the plane have a uniform charge density of σ coulombs per square meter. What is the magnitude of the electric field at point P, which is a distance R above the plane and a distance R from the outside of the sphere?

(A) $\frac{\sigma}{4\varepsilon_0}$

(B) $\sqrt{\frac{2\sigma}{4\varepsilon_0}}$

(C) $\frac{2\sigma}{4\varepsilon_0}$

(D) $\frac{\sigma}{4\varepsilon_0}$

(E) $\dfrac{\sqrt{5}\sigma}{4\varepsilon_0}$

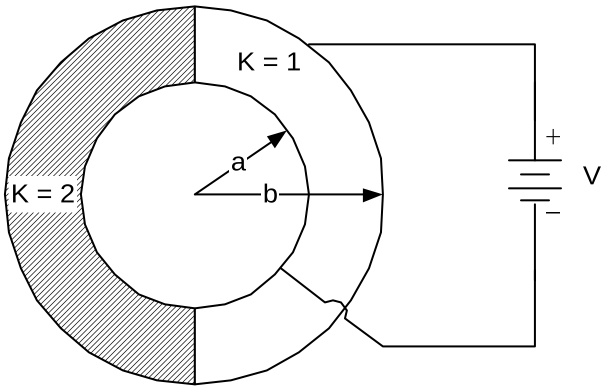
解：P 点的电场强度由两个部分组成：球壳的贡献沿水平方向向右，

$$E_{//}=\frac{1}{4\pi\varepsilon_0}\frac{\sigma 4\pi R^2}{(2R)^2}=\frac{\sigma}{4\varepsilon_0};$$

无穷大带电平面的贡献垂直向上，

$$E_{\perp}=\frac{\sigma}{2\varepsilon_0}。$$

合场强的大小由二者的合成计算。选（E）。



72. A capacitor is formed of two concentric conducting spherical shells. Half of the space between the shells is filled with air, and the other half is filled with a dielectric of dielectric constant K = 2. The capacitor is charged with a battery of emf V connected as shown above. The total charge stored on the inner sphere is

(A) $-V6\pi\varepsilon_0 ab/(b-a)$

(B) $+V6\pi\varepsilon_0 ab/(b-a)$

(C) $-V6\pi\varepsilon_0 ab/(b+a)$

(D) $-V6\pi\varepsilon_0/\ln(b/a)$

(E) $+V6\pi\varepsilon_0/\ln(b/a)$

解：由对称性可以判断，极板间的电场是沿径向的，且在两种介质中的分布是一致的：

$$E(r)=\frac{a\sigma_1}{K_1\varepsilon_0 r}=\frac{a\sigma_2}{K_2\varepsilon_0 r},$$

由

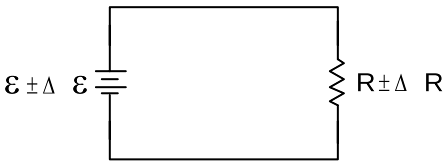
$$\int_a^b E dr=V,$$

$$\frac{a\sigma_1}{K_1\varepsilon_0}\ln\frac{b}{a}=\frac{a\sigma_2}{K_2\varepsilon_0}\ln\frac{b}{a}=V\text{，}$$

解得

$$Q=-2\pi R^2(\sigma_1+\sigma_2)=-\frac{V6\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right)}\text{。}$$

选（D）。



73. The circuit shown above contains a resistor whose resistance is R with an uncertainty ΔR and a battery of emf ε with an uncertainty Δε. The uncertainties are uncorrelated. What is the uncertainty in the power dissipated in the resistor?

(A) $(\frac{\Delta\varepsilon}{\varepsilon}+\frac{\Delta R}{R})\frac{\varepsilon^2}{R}$

(B) $\sqrt{(\frac{\Delta\varepsilon}{\varepsilon})^2+(\frac{\Delta R}{R})^2}\,\frac{\varepsilon^2}{R}$

(C) $(\frac{\Delta\varepsilon}{\varepsilon}-\frac{\Delta R}{R})\frac{\varepsilon^2}{R}$

(D) $[2(\frac{\Delta\varepsilon}{\varepsilon})+\frac{\Delta R}{R}]\frac{\varepsilon^2}{R}$

(E) $\sqrt{4(\frac{\Delta\varepsilon}{\varepsilon})^2+(\frac{\Delta R}{R})^2}\,\frac{\varepsilon^2}{R}$

解：功率

$$P=\frac{\varepsilon^2}{R}\text{，}$$

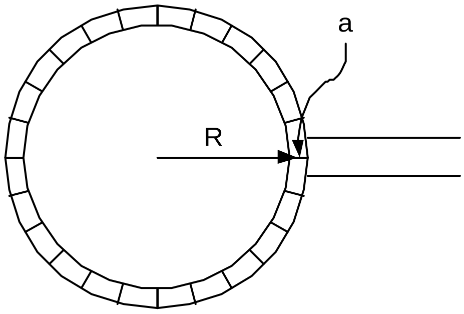
写成对数形式后求导得

$$\frac{dP}{P}=2\frac{d\varepsilon}{\varepsilon}+\frac{dR}{R}\text{。}$$

利用相对误差的合成公式有

$$\Delta P=P\sqrt{\left(2\frac{d\varepsilon}{\varepsilon}\right)^2+\left(\frac{dR}{R}\right)^2}\text{。}$$

选（E）。



74. The inductor shown above is formed by winding wire N times around a toroidal piece of iron of permeability μ. The major radius of the toroid is R and the minor radius is a, with a << R. What is the inductance if this inductor?

(A) $\dfrac{\mu_0 N a^2}{2R}$

(B) $\dfrac{\mu N a^2}{2R}$

(C) $\dfrac{\mu N^2 a^2}{2R}$

(D) $\dfrac{\mu N^2 R}{2}$

(E) $\dfrac{\mu_0 N^2 a^2}{2R}$

解：环内的磁场沿切向，其大小可由 Ampere 环路定理求出，

$$B = \frac{\mu N I}{2\pi R}。$$

利用电感的定义式之一

$$\Phi = LI$$

可得

$$L = \frac{\Phi}{I} = \frac{NBs}{I} = \frac{\mu N^2 a^2}{2R}。$$

选（C）。

75. An alpha particle and a proton follow the same circular path in a uniform magnetic field. What is the ratio, v_α/v_p , of their nonrelativistic velocities?

(A) 1/4

(B) 1/2

(C) 1

(D) 2

(E) 4

解：在非相对论情形下，Lorentz 力提供粒子作圆周运动所需的向心力

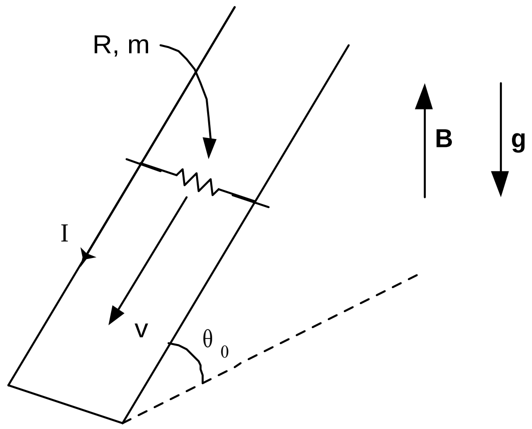
$$m\frac{v^2}{R}=qvB\,,$$

$$v=\frac{qBR}{m}\,,$$

因此两种粒子的回旋速度比

$$\frac{v_\alpha}{v_p}=\frac{q_\alpha m_p}{q_p m_\alpha}=\frac{1}{2}\,。$$

选（B）。



76. A resistor R of mass m is released from rest and slides without friction on conducting rails connected to form a complete circuit, as shown above. Initially, the current I is zero. The rails are inclined at an angle θ_0 to the horizontal. A vertical magnetic field B is present, as well as gravity. Consider the time dependences

(1) $\alpha(1-e^{-\beta t})$, (2) $\alpha t-\beta$, (3) $\alpha(1+e^{-\beta t})$

where α and β are positive and time-independence. What are the time dependence of the current I and the speed v of the resistor?

(A) v and I both have time dependence (1)

(B) v and I both have time dependence (2)

(C) v and I both have time dependence (3)

(D) v has time dependence (1), I has time dependence (3)

(E) v has time dependence (2), I has time dependence (3)

解：由于杆下滑产生的感生电动势为

$$E=Blv\,,$$

感生电流

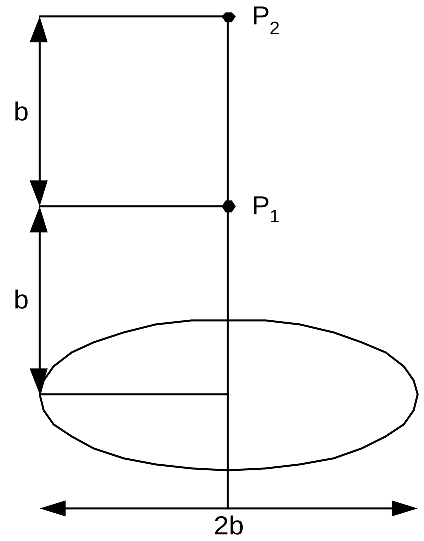
$$I=\frac{Blv}{R}\propto v\,。$$

杆所受的 Ampere 力沿水平方向，其分量形成阻止杆加速下滑的阻力。最终杆会达到一个收尾速度匀速下滑。定性的分析即可得结果。一般的运动方程形式如下

$$m\frac{dv}{dt}=mg\sin\theta_0-\frac{B^2l^2v}{R}\cos\theta_0\ ,$$

解为指数形式。选（A）。

Questions 77-78 concern a uniformly charged wire that has the form of a circular loop with radius b. Consider two points on the axis of the loop. P_1 is at a distance b from the loop's center, and P_2 is at a distance 2b from the loop's center. The potential V is zero, very far from the loop. At P_1 and P_2 the potentials are V_1 and V_2, respectively.



77. What is V_2 in terms of V_1?

(A) $\frac{V_1}{3}$

(B) $\frac{2V_1}{5}$

(C) $\frac{V_1}{2}$

(D) $\sqrt{\frac{2}{5}}V_1$

(E) $4\pi V_1$

解：显然环上每一点对轴线上电势的贡献相同。所以轴线上距离环心为 L 的点的电势为

$$V=\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{L^2+b^2}}\ 。$$

所以 V_2 和 V_1 的关系为

$$\frac{V_2}{V_1}=\frac{\sqrt{{L_{p1}}^2+b^2}}{\sqrt{{L_{p2}}^2+b^2}}=\frac{\sqrt{2}}{\sqrt{5}}=\sqrt{\frac{2}{5}}\text{。}$$

选（D）。

78. How much work would be required to move a charge q from P_1 to P_2?

(A) $\frac{qV_2}{V_1}$

(B) qV_2

(C) $q\log_e\left(\frac{V_2}{V_1}\right)$

(D) qV_1V_2

(E) $q(V_2-V_1)$

解：从 P_2 点到 P_1 点，电势能改变为 $q(V_2-V_1)$。而机械能没有发生变化，所以外界做功应等于电势能的变化。选（E）。

79. In electrostatic problems, the electric field always satisfies the equation

(A) $\nabla\cdot\mathbf{E}=\nabla\times\mathbf{E}$

(B) $\nabla\cdot\mathbf{E}=0$

(C) $\nabla\times\mathbf{E}=0$

(D) $\nabla(E^2)=0$

(E) $\nabla(\nabla\cdot\mathbf{E})=\nabla\times\mathbf{E}$

解：由 Maxwell 方程组

$$\nabla\times\mathrm{E}=-\frac{\partial\mathrm{B}}{\partial t}\text{。}$$

而对于静电场，

$$\frac{\partial\mathrm{B}}{\partial t}=0\text{，}$$

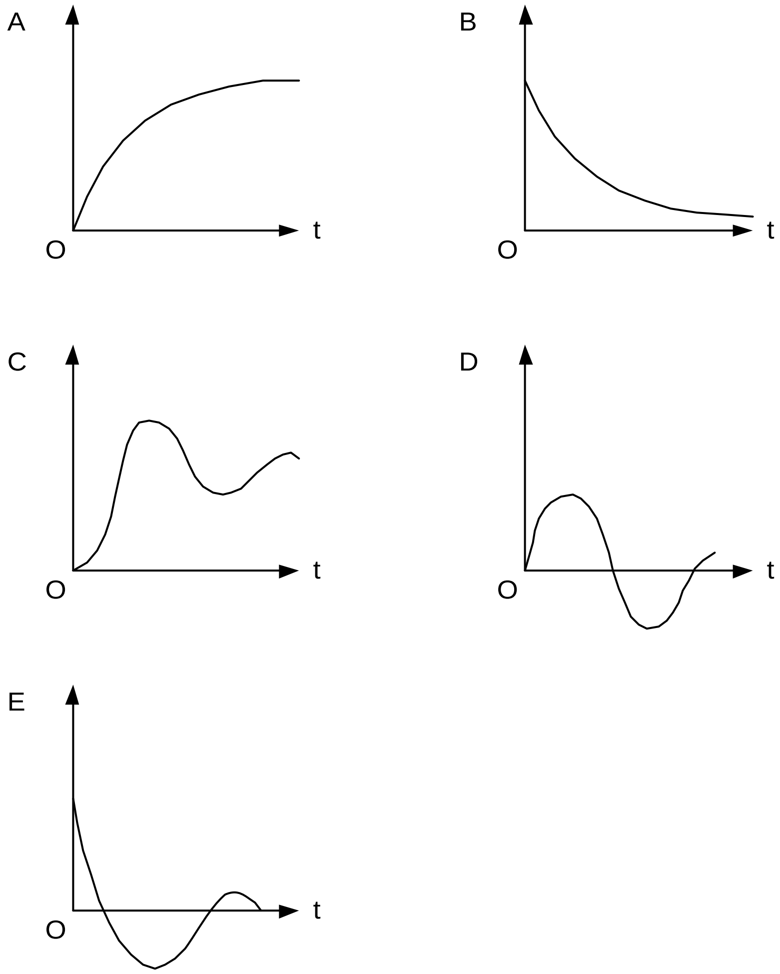
所以

$$\nabla\times\mathrm{E}=0\text{。}$$

选（C）。

Questions 80-81

The graphs below represent variables of an electrical circuit as functions of time t after the circuit switch is closed. In each case the circuit specified contains circuit elements connected in series with each other and with a battery. Any capacitor is uncharged at the beginning. Select the graph that most nearly shows the nature of the time dependence of the indicated variable.



80. Which graph represents the potential drop across the resistor as a function of time in an inductance-resistance circuit?

(A) A

(B) B

(C) C

(D) D

(E) E

解：由于电感在电流变化时会产生感应电压，阻碍电流改变。所以当开关合上瞬间，电流无法突变，保持为 0。电阻上无电流通过，电压为零。之后电流逐渐增大。当开关合上很长时间以后，电路趋于稳定。电感上不再有压降，电阻上电压趋于电源电压。选（A）。

或者进行计算，

$$L\frac{dI}{dt}+RI=\mathrm{E}\text{ 。}$$

解此微分方程，并考虑初条件 I（0）=0，得

$$I=\frac{\mathrm{E}}{R}(1-e^{-\frac{R}{L}t})\text{ 。}$$

正是图线（A）所代表的函数关系。

81. Which graph represents the charge on the capacitor as a function of time in an underdamped inductance-resistance-capacitance circuit?

(A) A

(B) B

(C) C

(D) D

(E) E

解：由于电容的初始电荷为 0，排除（B）、（C）。当电路趋于稳定时，电池的全部点压降在电容上，有一个稳定的电荷分布，排除（D）。由于欠阻尼，电容上的电荷有振荡，排除（A）。选（C）。

Questions 82-83 are based on the following information.

A long, thin, vertical wire has a net positive charge λ per unit length. In addition, there is a current I in the wire. A charged particle moves with speed u in a straight-line trajectory, parallel to the wire and at a distance r from the wire. Assume that the only forces on the particle are those that result from the charge on and the current in the wire and that u is much less than c, the speed of light.

82. Suppose that the current in the wire is reduced to 1/2. Which of the following changes, made simultaneously with the change in the current, is necessary if the same particle is to remain in the same trajectory at the same distance r from the wire?

(A) Doubling the charge per unit length on the wire only

(B) Doubling the charge on the particle only

(C) Doubling both the charge per unit length on the wire and the charge on the particle

(D) Doubling the speed on the particle

(E) Introducing an additional magnetic field parallel to the wire

解：先计算一下导线所产生的电磁场。对于 r 处的电场强度，由高斯定理，作一包围导线的圆柱体面，高度为 h，则

$$2\pi rh \cdot E = \lambda h / \varepsilon_0 ,$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} 。$$

方向垂直导线朝外。对于磁场，由安培环路定理

$$2\pi rB = \mu_0 I \ ,$$

$$B = \frac{\mu_0 I}{2\pi r} 。$$

方向为在垂直于导线的平面内，符合右手定则。带电粒子做直线运动，受力平衡，

$$\mathrm{E}q = q\mathrm{v} \times \mathrm{B} 。$$

将 E 和 B 的表达式带入，得

$$\frac{\lambda}{\varepsilon_0} = \mu_0 I v ,$$

这就是带电粒子平衡的条件。

对于本小题，电流减半。为继续维持平衡，可以把粒子的速度加倍，或把电荷密度减半。选（D）。

83. The particle is later observed to move in a straight-line trajectory, parallel to the wire but at a distance 2r from the wire. If the wire carries a current I and the charge per unit length is still λ, the speed of the particle is

(A) 4u

(B) 2u

(C) u

(D) u/2

(E) u/4

解：在前面的推导中看到，带电粒子平衡条件并不涉及它所处的位置 r。原因是电场和磁场随距离衰减的函数关系是相同的。所以速度应维持不变。选（C）。

84. If the outward electric flux through the surface of a cube is negative, then one can be certain that the cube contains

(A) a net negative charge

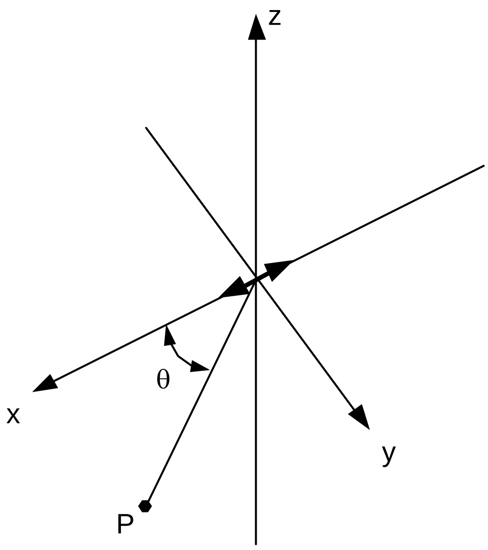
(B) a net positive charge

(C) only negative charges

(D) only positive charges

(E) both positive and negative charges

解：选（A）。Gauss 定理的必然结果。



85. A charged particle oscillates harmonically along the x-axis as shown above. The radiation from the particle is detected at a distant point P, which lies in the xy-plane. The electric field at P is in the

(A) ±z direction and has a maximum amplitude at θ=90°

(B) ±z direction and has a minimum amplitude at θ=90°

(C) xy-plane and has a maximum amplitude at θ=90°

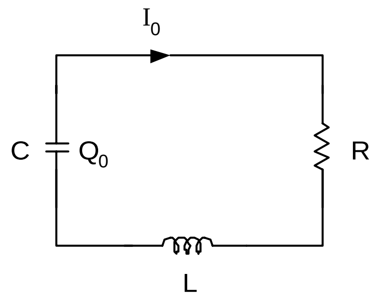
(D) xy-plane and has a minimum amplitude at θ=90°

(E) xy-plane and has a maximum amplitude at θ=45°

解：取 x 轴为极轴（$\mathbf{e}_r$ 的方向），则空间中某点的电场强度可表为

$$\mathbf{E}(\mathbf{r},t) = -\frac{\mu_0 q\omega^2 a}{4\pi}\frac{\cos(kr-\omega t)}{r}\sin\theta\mathbf{e}_\theta,$$

可见 **E** 的方向（平行于 $\mathbf{e}_\theta$）在 xy 平面内，并且θ=90°有最大值。选（C）。



86. The circuit above is shown at time t = 0, when a charge Q_0 is stored on the capacitpr C and a current I_0 is in the inductor L. Which of the following statements is NOT true?

(A) If $\frac{R}{2L} < \frac{1}{\sqrt{LC}}$, the charge on the capacitor will undergo a damped oscillation in time.

(B) If $\frac{R}{2L} >> \frac{1}{\sqrt{LC}}$, the current in the inductor will decay to zero with time as a simple exponential $e^{-(R/2L)t}$.

(C) If $\frac{R}{2L} << \frac{1}{\sqrt{LC}}$, and effects of order $\frac{R}{2L\sqrt{LC}}$ are negligible, the total energy stored in the circuit will decay with time as $e^{-(R/L)t}$

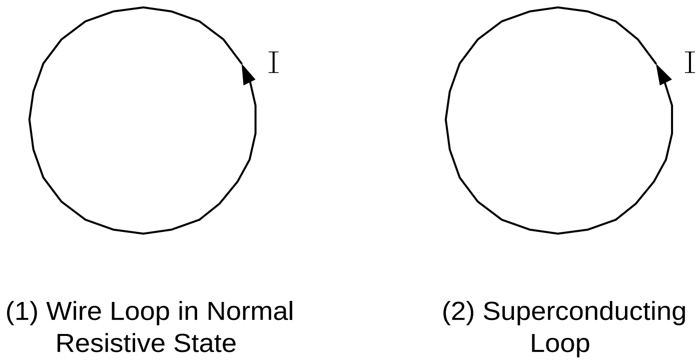
(D) If $\frac{R}{2L} > \frac{1}{\sqrt{LC}}$, the current in the resistor can change sign once as it decays, or can decay to zero without changing sign, depending on the values of Q_0 and I_0.

(E) If $I_0 = 0$, then for fixed values of L and C, the charge will decay to zero as quickly as possible without oscillation when $R^2 = \frac{4L}{C}$.

解：RLC 电路的阻尼度定义为

$$\lambda = \frac{R}{2}\sqrt{\frac{C}{L}}。$$

$\lambda > 1$、$\lambda = 1$、$\lambda < 1$ 分别对应过阻尼、临界阻尼和阻尼振荡。（D）选项条件保证$\lambda > 1$，是过阻尼情形，电流不可能改变符号。选（D）。



87. The diagrams above show two noninteracting wire loops of identical size each carrying a current I in the counterclockwise direction as indicated. The only difference between (1) and (2) is that loop (2) is superconducting. Which of the following is a correct statement about the magnetic induction vectors B_1 and B_2 at the respective centers of loop (1) and loop (2)?

(A) B_1 is identical to B_2 and both are directed out of the page.

(B) B_1 is identical to B_2 and both are directed into the page.

(C) B_2 is zero because the magnetic field cannot penetrate a superconductor.

(D) B_1 is directed into the page but B_2 is directed out of the page to satisfy boundary conditions on the surface of the superconductor.

(E) B_1 is directed out of the page but B_2 is directed into the page to satisfy boundary conditions on the surface of the superconductor.

解：磁场分布只由电流分布决定，与线圈超导不超导没有关系，当然如果是在线圈内部就另当别论。选（A）。

88. A monochromatic plane wave of visible light is normally incident from air on a thick highly polished metal sheet. Which of the following statements is NOT correct?

(A) The 1/e penetration depth of radiation in the metal is less than the wavelength of visible light.

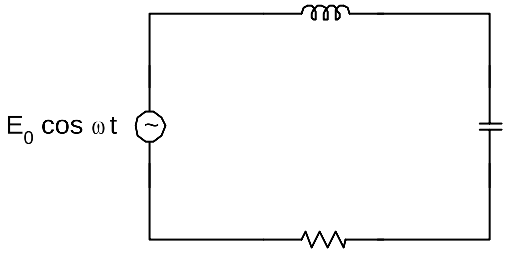
(B) The reflected wave is 180° out of phase with the incident wave.

(C) More than half the energy of the wave in the metal is stored in the magnetic field.

(D) The pressure of the wave on a copper sheet is decreased if the sheet is blackened.

(E) The fraction of energy absorbed in a silver sheet is larger than that absorbed in a zinc sheet of the same dimensions.

解：导体越接近于理想情况，入射波被吸收部分所占的比例越小，从而吸收的能量少。由于银的导电能力强于锌，更接近于理想导体，所以应吸收较少的能量。选（E）。

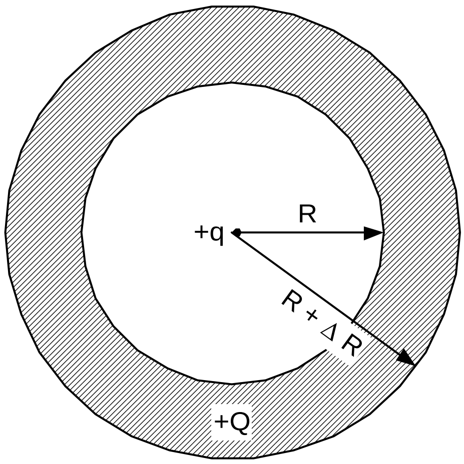


89. An AC source is connected to an LRC circuit, as shown above. Energy is stored in the

capacitor and inductor, and is dissipated in the resistor. At what times is the energy dissipated at the maximum rate?

(A) When the stored energy is all in electric fields.
(B) When the stored energy is all in magnetic fields.
(C) When the stored energy is equally divided between electric fields and magnetic fields.
(D) When no energy is stored.
(E) Under none of the conditions above.

解：能量全部耗散在电阻上，电路中电流最大时能量耗散率最大，电流最大的时候磁场能量最大。选（B）。



90. A particle with positive charge q is at the center of a spherical shell of inner radius R and outer radius R+ΔR, as shown above. The shell carries a positive charge Q that is uniformly distributed throughout the shell. What is the work done on the particle in bringing it from r = 0 to r = R?

(A) 0

(B) $\frac{1}{4\pi\varepsilon_0}\frac{qQ}{R}$

(C) $-\frac{1}{4\pi\varepsilon_0}\frac{qQ}{R}$

(D) $\frac{1}{8\pi\varepsilon_0}\frac{qQR^2}{(R+\Delta R)^3-R^3}$

(E) $-\frac{1}{8\pi\varepsilon_0}\frac{qQR^2}{(R+\Delta R)^3-R^3}$

解：壳上电荷+Q 对内部电场无贡献，q 不受力，做功为零。选（A）。

第三节 光学和波动

1. In a hologram a photographic plate contains a wave pattern that produces a three-dimensional picture when illuminated with monochromatic coherent light from a laser. When only half of the

photographic plate is illuminated, which of the following is true of the resulting picture?
(A) Only half of the picture is seen.
(B) The picture is still seen, but is less distinct than before.
(C) The picture is still seen, but is smaller than before.
(D) The color of the picture is changed.
(E) The picture is inverted.
解：全息成像的一大特点是由全息片的任一部分均可成像，因为全息片上纪录的内容为干涉条纹，而条纹以所载信息的频率在全息片上重复排布。但因为用全息片的局部成像会丢失高频信息，所以图像将会变模糊。选（B）。

2. Waves on a string propagate with a speed v and are represented by giving the displacement y as a function of x and t. Which of the following is NOT a possible wave or pulse? (c, b, and k are constants.)

(A) $y = ac^{-b(x-vt)^2}$

(B) $y = ac^{-b(x-vt)^2}\sin k(x+vt)$

(C) $y = a\cos k(x-vt)$

(D) $y = ae^{-bx^2}e^{-b(vt)^2}$

(E) $y = a/\cosh b(x-vt)$

解：由偏微分方程理论，函数 f (x ± vt)满足波动方程

$$\frac{\partial f^2}{\partial t^2} = v^2\frac{\partial f^2}{\partial x^2}。$$

所以答案选（D）。

3. A converging lens of focal length 4 centimeters is used as a magnifier. If an object is place 3 centimeters from the lens,what is the magnification?
(A) 3
(B) 4
(C) 6
(D) 12
(E) 24
解：由凸透镜成像公式

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f},$$

得像距为

$$v=\frac{uf}{u-f}=\frac{3\times 4}{3-4}=-12\,\mathrm{cm}。$$

放大倍数为

$$\left|\frac{v}{u}\right| = 4 \text{。}$$

选（B）。

4. A source of 1-kilohertz sound is moving straight toward you at a speed 0.9 times the speed on sound. The frequency you receive is

(A) 0.1 kHz

(B) 0.5 kHz

(C) 1.1 kHz

(D) 1.9 kHz

(E) 10 kHz

解：Doppler 效应

$$\nu' = \frac{c}{c-v}\nu = \frac{1}{1-0.9}\times 1 = 10\text{kHz} \text{。}$$

选（E）。

5. Two coherent sources of visible monochromatic light form an interference pattern on a screen. If the relative phase of the sources is varied from 0 to 2π at a frequency of 500 hertz, which of the following best describes the effect, if any, on the interference pattern?

(A) It is unaffected because the frequency of the phase change is very small compared to the frequency of visible light.

(B) It is unaffected because the frequency of the phase change is an integral multiple of π.

(C) It is destroyed except when the phase difference is 0 or π.

(D) It is destroyed for all phase differences because the monochromaticity of the sources is destroyed.

(E) It is not destroyed but simply shifts positions at a rate too rapid to be detected by the eye.

解：干涉光源的相对相位改变会使干涉条纹移动，只要想一下零级条纹就知道了。500Hz 的改变频率人眼无法分辨，因为人眼的视觉暂留时间约为 25s。选（E）。

Questions 6-7

Light of wavelength 5200 Angstroms is incident normally on a transmission diffraction grating with 2000 lines per centimeter.

6. The first-order diffraction maximum is at an angle, with respect to the incident beam, that is most nearly

(A) 3°

(B) 6°

(C) 9°

(D) 12°

(E) 15°

解：由光栅公式

$$\sin\theta = k\frac{\lambda}{d}$$

对于一级衍射，k = 1，所以

$$\sin\theta = \frac{\lambda}{d} = \frac{5200\times 10^{-10}}{0.01/2000} = 0.104\text{。}$$

角度很小，近似

$$\theta \approx \sin\theta = 0.104 = 5.96^0\text{。}$$

选（B）。

transmission diffraction grating：透射光栅。

7. If the width of one slit is reduced to $\frac{w}{2}$, what happens to the interference pattern of the light from the two slits?

(A) It remains the same except that it has lower intensity.

(B) It remains the same except that it is replaced.

(C) It still has intensity I_0 at θ = 0.

(D) It no longer has minima with zero intensity.

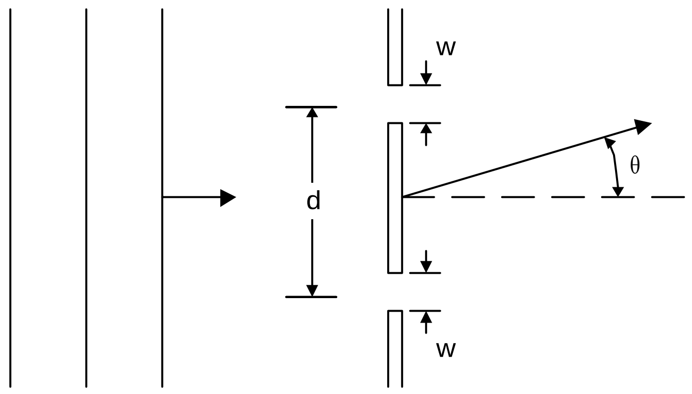
(E) It has zero intensity at θ = 0.

解：将干涉缝看作是三条宽为 w/2 的狭缝的组合，缝间干涉因子为（复数表示）

$$\tilde{N}(\theta) = \sum_i e^{ikL_i} = 1 + e^{ikd} + e^{ik(d-w/4)},$$

干涉因子的模总是大于零的，因此不会出现完全相消的暗斑。选（D）。

Questions 8-9



A plane wave of monochromatic light with wavelength λ in vacuum is incident on two slits of equal width w, as shown schematically above. A pattern is observed on a screen a large distance away. The intensity of the light at θ = 0 is initially I_0.

8. If one slit is blocked the intensity is

(A) $\frac{I_0}{4}$

(B) $\frac{I_0}{2}$

(C) $\frac{I_0}{\sqrt{2}}$

(D) I_0

(E) 2 I_0

解：在θ = 0 处，两波干涉相加。设每个波振幅为 A，则

$$I_0 = (A + A)^2 = 4A^2 \text{。}$$

仅剩一个波时，$I = A^2$。选（A）。

9. When the apparatus is immersed in a medium with index of refraction n, the interference pattern is unchanged if light is used whose wavelength in vacuum is

(A) $\frac{\lambda}{n}$

(B) $\frac{\lambda}{\sqrt{n}}$

(C) λ

(D) $\lambda\sqrt{n}$

(E) λn

解：设介质折射率为 n，则其中单色光的波长为其真空中波长的 1/n，频率不变，从而波速为 c/n。本题中若光在真空中波长为λn，则在介质中为λ，与原来情况相仿。选（E）。

10. The following functions may represent the wave disturbance f(x,t) in a one-dimensional elastic medium in terms of position x, time t, and positive constants A, a, and b. Which function represents a traveling wave moving in the negative x-direction?

(A) $f(x,t) = A\sin(ax + bt)$

(B) $f(x,t) = A\sin(ax - bt)$

(C) $f(x,t) = A\cos ax \cos bt$

(D) $f(x,t) = A\sin ax \sin bt$

(E) $f(x,t) = A\sin ax \cos bt$

解：(C)、(D)、(E)均表示驻波。它们最明显的特点是各点的振幅不同。(A)、(B)表示行波，符合形式 f (x ± vt)。其中（B）沿正方向传播，（A）沿负方向传播。选（A）。

11. Unpolarized light passes through two polarizers whose optical axes are vertical. The intensity of the emerging light is I_0. If a third polarizer is placed between the polarizers so that its axis is at an angle θ with the vertical, the intensity of the emerging light is

(A) zero for all θ

(B) I_0
(C) $I_0\cos\theta$
(D) $I_0\cos^2\theta$
(E) $I_0\cos^4\theta$

解：注意前两个偏振片平行排列，开始时第二个偏振片其实不起作用，所以入射光经过第一个偏振片后强度为 I_0。加入第三个偏振片后，经过第一个偏振片后光矢量要进行两次投影，出射强度为

$$I = A^2 = (A_0\cos\theta\cos\theta)^2 = {A_0}^2\cos^4\theta = I_0\cos^4\theta \text{。}$$

选（E）。

12. Although the sky is blue in the daytime, sunsets are red because
(A) the Sun emits more red light in a forward direction, but more blue light when at a larger angle with the forward direction
(B) the Sun emits more red light than blue light in the evening
(C) the index of refraction of red light in air is greater than that of blue light
(D) there are many ions in the upper atmosphere
(E) red light is less strongly scattered by air molecules than blue light

解：天空为什么是蓝色最早由 Rayleigh 解释。Rayleigh 散射定律说，当散射体的尺度比光的波长小时，散射光强与波长的四次方成正比。因为大气满足以上条件，所以白光中的短波成分（蓝紫色）遭到的散射比短波成分（红黄色）强烈得多，散射光因为短波富集而呈蓝色。日落时光所经大气距离加大，白光中短波成分被更多的散射掉，直射的日光中剩余更多的是长波成分，因此偏红。选（E）。

13. An observer looks through a slit of width 5×10^{-4} meter at two lanterns a distance of 1 kilometer from the slit. The lanterns emit light of wavelength 5×10^{-7} meter. The minimum separation of the lanterns at which the observer can resolve the lantern lights is most nearly
(A) 0.01 m
(B) 0.1 m
(C) 1 m
(D) 10 m
(E) 100 m

解：根据 Rayleigh 判据

$$\theta_{\min} = 1.22\frac{\lambda}{D}\text{，}$$

$$l_{\min} = S\theta_{\min} = 1.22S\frac{\lambda}{D} = 1.22\times10^3\,\frac{5\times10^{-7}}{5\times10^{-4}} = 1.22\ \text{m}\text{，}$$

最短距离应大于上述值。作为近似，选（C）。

14. An antireflection coating with index of refraction 1.25 is put on glass with index of refraction 1.56, as shown above. If the coating is designed for light incident normally on the glass with wavelength in air of 400 Angstroms, which of the following thicknesses of the coating will result in minimum reflection?

(A) 400 A
(B) 800 A
(C) 1600 A
(D) 3200 A
(E) 6400 A
解：由于增透膜的折射率大于空气的，小于玻璃的，所以光在增透膜的两个表面上发生反射时都有半波损，作用抵消。所以厚度 d 满足

$$2n_c d = (k + \frac{1}{2})\lambda,\quad k = 0,1,2,3\ldots。$$

对于本题，

$$d = \frac{k + \frac{1}{2}}{2 \times 1.25} \times 400 = (2k+1) \times 80\mathrm{A}。$$

选（A），对应于 k = 2。

15. In an ordinary hologram, coherent monochromatic light produces a 3-dimentional picture because wave information is recorded for which of the following?
I. Amplitude
II. Phase
III. Wave-front angular frequency
(A) I only
(B) I and II only
(C) I and III only
(D) II and III only
(E) I, II and III
解：所谓全息照相，指记录下全部信息，包括波前的相位和振幅（实际为振幅强度的相对分布）。选（B）。

16. The dispersion law for a certain type of wave motion is $\omega = \left(c^2k^2 + m^2\right)^{\frac{1}{2}}$, where ω is the angular frequency, k is the magnitude of the propagation vector, and c and m are constants. The group velocity of these waves approaches
(A) infinity as k→0 and zero as k→∞
(B) infinity as k→0 and c as k→∞
(C) c as k→0 and zero as k→∞
(D) zero as k→0 and infinity as k→∞
(E) zero as k→0 and c as k→∞
解：由群速度公式

$$v_g = \frac{d\omega}{dk},$$

本题中

$$v_g = \frac{d\omega}{dk} = \frac{c^2k}{\sqrt{c^2k^2 + m^2}},$$

k→∞时，$v_g \to c$。选（E）。请同时牢记相速的公式$v_p = \dfrac{\omega}{k}$。

17. It is necessary to coat a glass lens with a nonreflecting layer. If the wavelength of the light in the coating is λ, the best choice is a layer of material having an index of refraction between those of glass and air and a thickness of

(A) $\dfrac{\lambda}{4}$

(B) $\dfrac{\lambda}{2}$

(C) $\dfrac{\lambda}{\sqrt{2}}$

(D) λ

(E) 1.5λ

解：为发生最大透射，从薄膜前后两表面反射的光波应干涉相消，厚度 d 需满足

$$2d = \left(n + \frac{1}{2}\right)\lambda,$$

$$d = \left(\frac{n}{2} + \frac{1}{4}\right)\lambda。$$

选（A），对应 n = 0。此题未考虑半波损，这只当$1 < n < n_{glass}$时成立。

18. Unpolarized light is incident on two ideal polarizers in series. The polarizers are oriented so that no light emerges through the second polarizer. A third polarizer is now inserted between the first two and its orientation direction is continuously rotated through 180°. The maximum fraction of the incident power transmitted through all three polarizers is

(A) zero

(B) $\dfrac{1}{8}$

(C) $\dfrac{1}{2}$

(D) $\dfrac{1}{\sqrt{2}}$

(E) 1

解：光经过第一个偏振片后为线偏振光，强度为原来的 1/2（不是 1/4，这地方容易出错），

$$I_{out} = \frac{I_0}{2}\sin^2\theta\cos^2\theta,$$

显然θ = 45°时，I_{out}最大，为$\dfrac{I_0}{8}$。选（B）。

19. The screen of a pinhole camera is at a distance D from the pinhole, which has a diameter d. The light has an effective wavelength λ. (λ << D) For which of the following values of d will the image be sharpest?

(A) $\sqrt{\lambda D}$

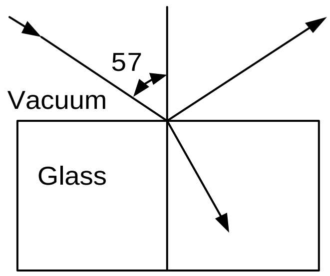
(B) λ

(C) $\frac{\lambda}{10}$

(D) $\frac{\lambda^2}{D}$

(E) $\frac{D^2}{\lambda}$

解：考试现场很可能不知道这个公式，不用慌，从量级上进行一下分析即可。$\lambda \sim 10^{-7}$m 量级，$D \sim 10^{-2}$m 量级，而小孔的直径介于二者之间。选项中（B）（C）（D）答案太小，10^{-7}m 量级的小孔怎么能用来照相。选项（E）答案太大。选(A)。



20. A ray of unpolarized monochromatic light traveling in vacuum is incident on a flat plate of glass at an angle equal to Brewster's angle (57° for this case), as shown above. Which of the following statements is NOT correct?

(A) The tangent of the angle of incidence is equal to the index of refraction of the glass plate.
(B) The angle between the reflected ray and the refracted ray is 90^0.
(C) The refracted ray is partially plane polarized.
(D) The reflected ray is 100% linearly polarized.
(E) The plane of vibration of the electric vector in the reflected ray is parallel to the plane of incidence of the light ray.

解：由 Brewster 角的定义

$$i_B = \tan^{-1}\frac{n_2}{n_1} = \tan^{-1}\frac{n_g}{n_v} = \tan^{-1} n_g \text{,}$$

（A）正确。设 θ 为折射角，由

$$\sin i_B = n_g \sin\theta \text{,}$$

$$\sin\theta = \frac{n_g}{\sin i_B} = \frac{n_g}{n_g / \sqrt{1+n_g{}^2}} = \frac{1}{\sqrt{1+n_g{}^2}} = \cos i_B \text{,}$$

所以

$$\theta = 90^0 - i_B 。$$

（B）正确。

由 Fresnel 公式，不管入射光的偏振态如何，反射光总是线偏振。故 Brewster 角又叫全偏振角，（D）正确。Brewster 角还有一个性质，当入射角 I_B 时，反射光中的 p（与入射面平行的方向）分量为 0。所以反射光的电矢量只有 s（与入射平面垂直）分量。选项（E）认为电矢量在入射平面内，错误。选（E）。

21. The angular separation of the two components of a double star is 8 microadians and the light from the double stat har a wavelength of 5500 angstroms. According to the Rayleigh criterion, the smallest diameter of a telescope mirror that will resolve the double star is most nearly

(A) 5mm

(B) 2cm

(C) 10cm

(D) 50cm

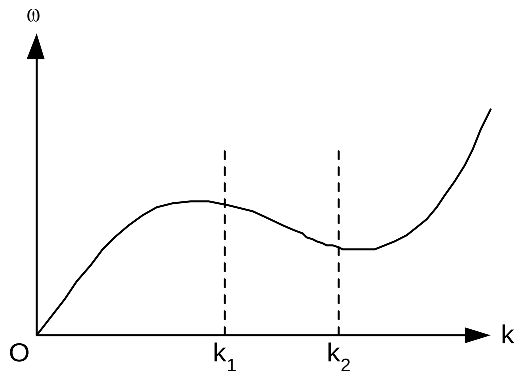
(E) 2m

解：由 Rayleigh 判据

$$\theta = 1.22\frac{\lambda}{D},$$

$$D_{\min} = 1.22\frac{\lambda}{\theta} = 1.22\frac{5500\times10^{-10}}{8\times10^{-6}} = 0.084 \text{ m} 。$$

选（C）。



22. The dispersion curve shown above relates the angular frequency ω to the wave number k. For waves with numbers lying in the range $k_1 < k < k_2$, which of the following is true of the phase velocity and the group velocity?

(A) They are in opposite directions.

(B) They are in the same direction and the phase velocity is larger.

(C) They are in the same direction and the group velocity is larger

(D) The phase velocity is infinite and the group velocity is finite.

(E) They are the same in direction and magnitude.

解：相速度 V_P 和群速度 V_g 的公式为

$$V_P = \frac{\omega}{k}\text{，} V_g = \frac{d\omega}{dk}\text{。}$$

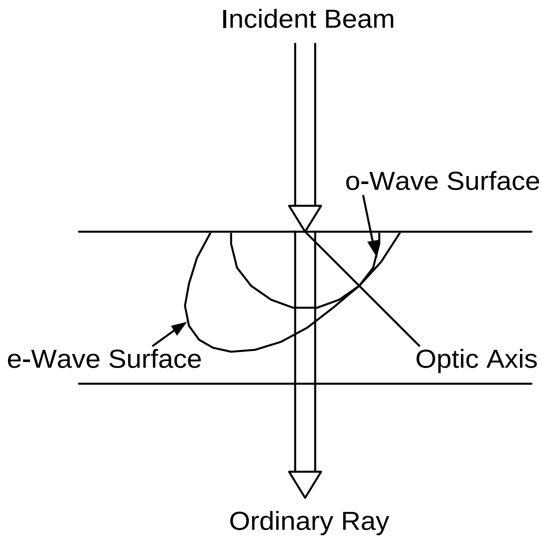
当 $K_1<K<K_2$ 时，ω（k）曲线的切线斜率为负，即

$$V_g = \frac{d\omega}{dk} < 0\text{。}$$

因为ω（k）在第一象限，ω、k 均为正值，所以

$$V_P = \frac{\omega}{k} > 0\text{，}$$

二者方向相反。选（A）。



23. A beam of liaght is incident from above on a slab of doubly rafracting crystal such as calcite whose optic axis is in the plane of the paper as shown above. The beam is split into an ordinary ray(shown) and an extraodinary ray which is displaced. Also shown are the Huygens wave surface for the ordinary and extraordinary waves. Which of the following statements is true?

(A) The slab is a quarter-wave plate.

(B) The ordinary index of refraction n_o is less than the extraordinary index of refraction $n_{e.}$

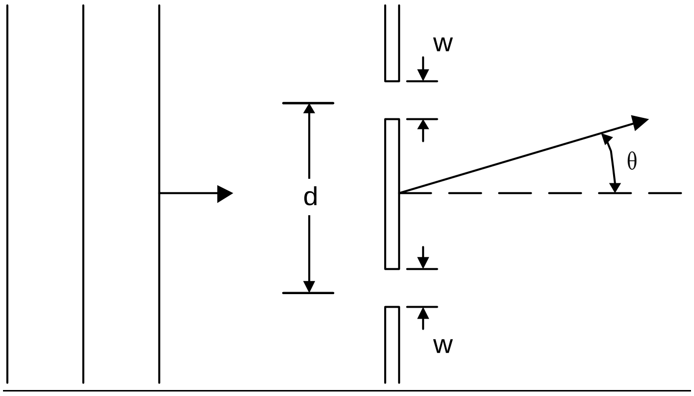
(C) The extraordinary ray is deflected to the left.

(D) The extraordinary wave is unpolarized.

(E) The ordinary ray is polarized with the electric vector in the plane of the paper.

解：e 波的 Huygens 波面为一椭圆，且光轴方向为短轴。所以 $v_e > v_o$ ，从而 e 波的折射率小于 o 波，所以选项（B）不对。另外 e 波和 o 波均为线偏振波，二者振动方向垂直。O 波电矢量的振动方向与主平面（晶体中某条光线与晶体主轴组成的平面，在本题中为纸平面）垂直，e 光电矢量的振动方向在主平面内。选项（D）、（E）不对。由 Huygens 作图法，从入射光束与界面交点向 e 波的 Huygens 波面作切线，切点在光轴左侧，所以 e 波向左偏折。选（C）。

<u>Questions 24-25</u>



A plane wave of monochromatic light with wavelength λ in vacuum is incident on two slits of equal width w, as shown schematically above. A pattern is observed on a screen a large distance away. The intensity of the light at θ = 0 is initially I_0.

24. If one slit is blocked, the intensity at θ = 0 is

(A) $\frac{I_0}{4}$

(B) $\frac{I_0}{2}$

(C) $\frac{I_0}{\sqrt{2}}$

(D) I_0

(E) $2I_0$

解：由对称性分析可得两条狭缝在衍射屏中心产生的分振幅是同相位叠加的，设一条狭缝贡献得分振幅大小为 A_0，则有

$$I_0 = (2A_0)^2 。$$

遮住一条狭缝后，剩余光强为

$$I' = A_0{}^2 = \frac{I_0}{4} 。$$

选（A）。

25. When the apparatus is immersed in a medium with index of refraction n, the interference pattern is uncharged if light is used whose wavelength in vacuum is

(A) $\frac{\lambda}{n}$

(B) $\frac{\lambda}{\sqrt{n}}$

(C) λ

(D) $\lambda\sqrt{n}$

(E) λn

解：在真空中光程等于相应的几何路程长度 l，相应的干涉条件为

$$l = i\lambda',$$

其中 i 为干涉级数。当整个仪器浸入媒介物后，光程为相应的几何路程长度的 n 倍，相应的干涉条件为

$$nl = i\lambda。$$

对比以上两式可以发现，当

$$\lambda' = \frac{\lambda}{n}$$

时，对相同的干涉级次有相同的几何路程 l，即对应相同的干涉图样分布。选（A）。

26. In a hologram a photographic plate contains a wave pattern that produced a three-dimensional picture when illuminated with monochromatic coherent light from a laser. When only half of the photographic plate is illustrated, which of the following is true of the resulting picture?

(A) Only half of the picture is seen.

(B) The picture is still seen, but is less distinct than before.

(C) The picture is still seen, but is smaller than before.

(D) The color of the picture is changed.

(E) The picture is inverted.

解：全息照相原理简介：全息照相以干涉衍射等波动光学规律为基础，过程分记录、再现两步。全息图与普通照相底片不同，它记录的是物体各点的全部光信息，包括振幅和位相，而且图中每一局部都包含了物体各点的光信息。相应它要求光源有很高的时间和空间相干性。由以上简介可以得知，当全息图只剩下一半时，仍然可以完整反映物像，只是相应的细节分辨率有一定的损失。

选（B）。

27. Waves on a string propagate with a speed v and are represented by giving the displacement y as a function x and t. Which of the following is NOT a possible wave? (a, b, and k are constants.)

(A) $y = ae^{-b(x-vt)^2}$

(B) $y = ae^{-(x+vt)^2}\sin k(x+vt)$

(C) $y = a\cos k(x-vt)$

(D) $y = ae^{-bx^2}e^{-b(vt)^2}$

(E) $y = a/\cosh b(x-vt)$

解：波动性的基本要求是存在一个以波速 v 传播的等相面。以上五个 y 方向的位移方程中，显然（C）描述的是以 y 轴为对称轴向两边衰减振荡的振动，没有波动性。选（C）。

28. An electromagnetic disturbance propagates along the y-axis through a medium of index of refraction n. At time t, the equation of the electric field in the medium is represented in MKS units by the following equation.

$$E = E_0 \sin(5\times10^6 \pi y - 5\times10^{14}\pi t + \pi/6)$$

The index of refraction n of the medium is most nearly

(A) 6

(B) 5

(C) 4

(D) 3

(E) 2

解：此媒介中的光速

$$v = \frac{\omega}{k} = \frac{5\times10^{14}}{5\times10^6} = 10^8\left(m/s\right),$$

因此媒介的折射率

$$n = \frac{c}{v} = 3。$$

选（D）。

29. A small object is moved along the optical axis of a spherical concave reflecting surface of radius R. Which of the following best describes the nature of the image for object distance of 3R/4 and R/4, respectively, from the mirror?

	$\frac{3R}{4}$	$\frac{3R}{4}$
(A)	Real and erect	Real and inverted
(B)	Real and inverted	Virtual and erect
(C)	Real and inverted	Virtual and inverted
(D)	Real and erect	Virtual and erect
(E)	Virtual and erect	Virtual and inverted

解：直接利用几何光学中的球面镜成像公式

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R},$$

其中 s、s’分别为物距和像距。s = 3R/4 时，s’ > 0，是倒立的实像；s = R/4 时，s’ < 0，是正立的虚像。选（B）。

30. White light is normally incident on a soap film that has air on both sides. The reflected light is orange. If one assumes that the index of refraction of the film is 1.33 and that 6000 Angstroms is a typical wavelength of orange light in air, a possible thickness of the film is most nearly

(A) 1500 A

(B) 3000 A

(C) 3375 A

(D) 4225 A

(E) 4500 A

解：考虑到前后反射表面存在半波损失，橙色反射光干涉加强的条件为

$$2nd = \left(i + \frac{1}{2}\right)\lambda$$

代入题中给出的数据，当 i = 1 时 d = 3375 A。选（C）。

31. A rocket ship is moving away from Earth at a high speed. A spectral line from a source on the rocket ship is shifted from a wavelength λ to a wavelength 4λ for an observer on Earth. If c is the speed of light and the shift is assumed to be due to the relativistic Doppler effect, the velocity of the rocket ship relative to Earth is most nearly

(A) $\frac{1}{4}c$

(B) $\frac{2}{5}c$

(C) $\frac{1}{2}c$

(D) $\frac{3}{5}c$

(E) $\frac{15}{17}c$

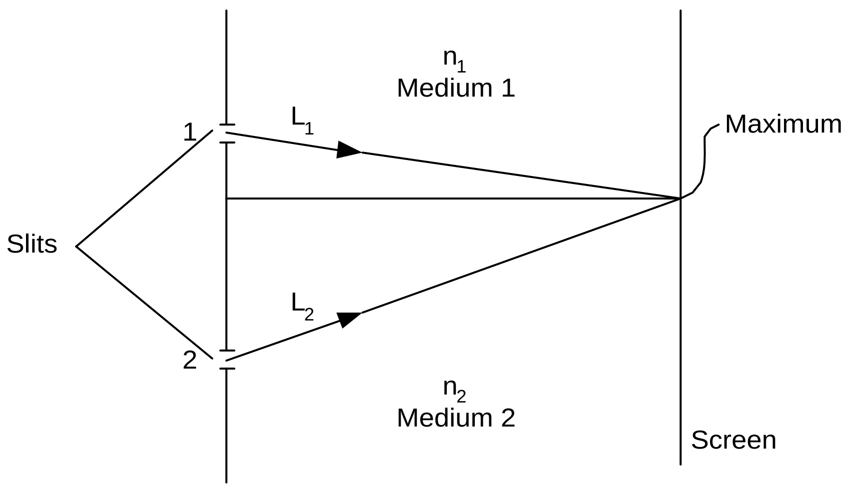
解：利用相对论性的横向 Doppler 效应公式

$$\omega = \frac{\omega_0\sqrt{1-\beta^2}}{1-\beta} = 4\omega_0,$$

解之可得

$$\beta = \frac{15}{17}。$$

选（E）。



32. Coherent monochromatic light passes through two narrow slits and forms an interference pattern on a distant screen. The space between the slits is filled, as shown above, with media having different indices of refraction n_1 and n_2. Light from slit 1 travels a distance L_1, passing only through medium 1 to reach the maximum shown in the figure. Light from slit 2 travels a distance

L_2, passing only through medium 2, to reach the same point. What is a possible relationship between L_1 and L_2?

(A) $L_1 = \frac{n_1}{n_2} L_2$

(B) $L_1 = \sqrt{\frac{n_1}{n_2}} L_2$

(C) $L_1 = \frac{n_2}{n_1} L_2$

(D) $L_1 = \sqrt{\frac{n_2}{n_1}} L_2$

(E) $L_1 = (1 + \frac{n_1}{n_2}) L_2$

解：对荧光屏上的中心极大光强位置，从两条狭缝传播过来的光的光程差为零。

$$\Delta l = n_1 L_1 = n_2 L_2 \text{，}$$

$$L_1 = \frac{n_2}{n_1} L_2 \text{。}$$

选（C）。

33. Radio signals emanating from a distance quasar at s frequency of 30 gigahertz are received with a dish antenna 10 meters in diameter. With what precision can the direction of the quasar be determined?

(A) 10^{-9} radian

(B) 10^{-6} radian

(C) 10^{-3} radian

(D) 10^{-1} radian

(E) The precision cannot be determined from the information given.

解：盘状天线所能分辨的极限角

$$\delta\theta = \frac{\lambda}{D} = \frac{c}{\gamma D} = \frac{3 \times 10^8}{30 \times 10^9 \times 10} = 10^{-3} \text{rad} \text{。}$$

选（C）。

34. The sky appears blue because

(A) red light from the Sun is absorbed by the Earth’s atmosphere

(B) dust particles in the Earth’s atmosphere preferentially scatter red light

(C) molecules in the Earth’s atmosphere preferentially scatter red light

(D) molecules in the Earth’s atmosphere preferentially scatter blue light

(E) oxygen molecules have absorption lines in the red region of the spectrum

解：根据 Rayleigh 散射定律，大气分子中散射强度跟波长的四次方成正比，因此波长较长的红光较容易被四处散射，从而穿过大气到达地面的可见光主要是短波段的，因此看起来呈蓝色。选（C）。

35. When two thin lenses of focal lengths f_1 and f_2 are placed in contact, the focal length f of the combination is given by the relation

(A) $f = f_1 + f_2$

(B) $f = f_1 - f_2$

(C) $f = \dfrac{f_1}{f_2}$

(D) $f = \dfrac{f_2}{f_1}$

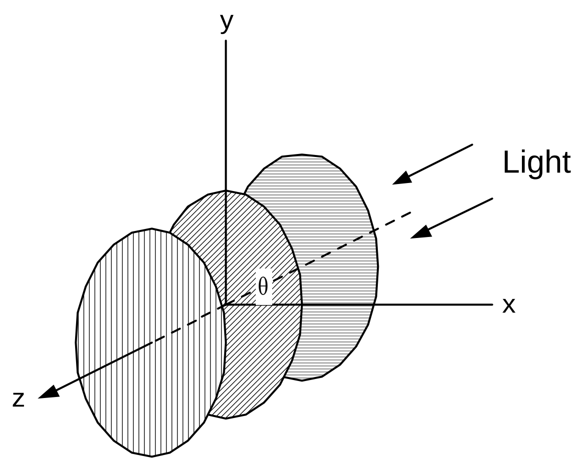
(E) $f = \dfrac{f_1 f_2}{f_1 + f_2}$

解：对密接的薄透镜组，其复合透镜的焦距可由 Gauss 公式求出

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2},$$

$$f = \frac{f_1 f_2}{f_1 + f_2}。$$

选（E）。



36. Unpolarized light of intensity I_0 travelling in the +z direction passes successively through three perfect polarizers. The first of these passes light polarized along the x-axis, the middle one passes light polarized at an angle of θ relative to the x-axis, and the last passes light polarized along the

y-axis. The intensity of light emerging from the third polarizer is

(A) 0

(B) $\frac{1}{2}I_0$

(C) $\frac{1}{2}I_0\sin^2\theta$

(D) $I_0\cos^2\theta$

(E) $\frac{1}{8}I_0\sin^2 2\theta$

解：自然光经过第一个偏振片，光强变为 $\frac{I_0}{2}$，偏振方向沿 x 轴。经过第二个偏振片之后，光强为 $\frac{I_0\cos^2\theta}{2}$，偏振方向转了θ角。经过第三个偏振片之后，偏振方向沿 Y 轴，光强为

$$I=\frac{I_0\cos^2\theta}{2}\cos\left(\frac{\pi}{2}-\theta\right)=\frac{1}{8}I_0\sin^2 2\theta\text{。}$$

选（E）。

37. A transmission diffraction erating gives first-order diffraction for 5000-angstrom light at an angle θ with respect to the original direction of propagation. If the grating diffracts all wavelengths, one would also find which of the following at angle θ?

(A) Second-order diffraction for 1250A

(B) Second-order diffraction for 10,000A

(C) Third-order diffraction for 2500A

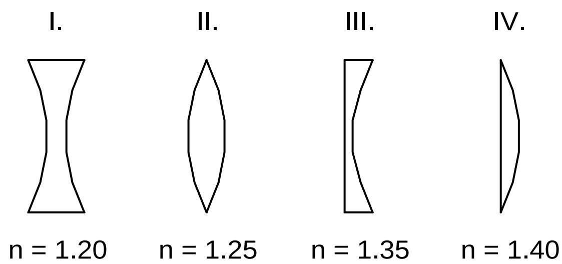
(D) Fourth-order diffraction for 1250A

(E) Fourth-order diffraction for 20,000A

解：根据衍射公式，

$$d\sin\theta=n\lambda\text{。}$$

对固定的θ角，λ = 5000A 时，n = 1；故当λ’ = 1250A 时，n’ = 4 成立。选（D）。



38. To render parallel a converging beam of light propagating in water with index of refraction n = 1.30. Which of the following lenses might be used in the water?

(A) I only

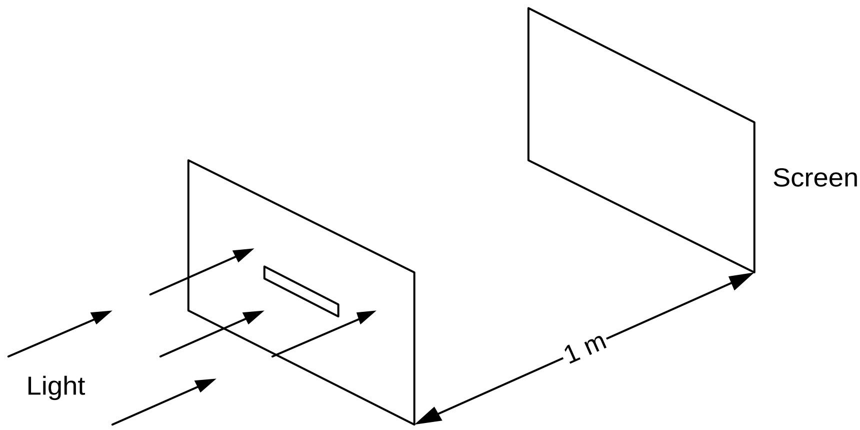
(B) I or III

(C) I or IV
(D) II or III
(E) II or IV
解：图中 I、III 是凹透镜，II、IV 是凸透镜。利用 Gauss 逐步成像的定性计算可得，对于凹透镜，当玻璃折射率大于媒介折射率时，对光线有发散作用；当玻璃折射率小于媒介折射率时，对光线有汇聚作用。对于凸透镜的情况测刚好相反。选（C）。

39. Let $\hat{x}$, $\hat{y}$, and $\hat{z}$ be three orthogonal unit vectors. The expression

$$u(r,t) = 2(\hat{x} - \hat{y})\exp\{(6\hat{z}\cdot\vec{r} + 3t)\}$$

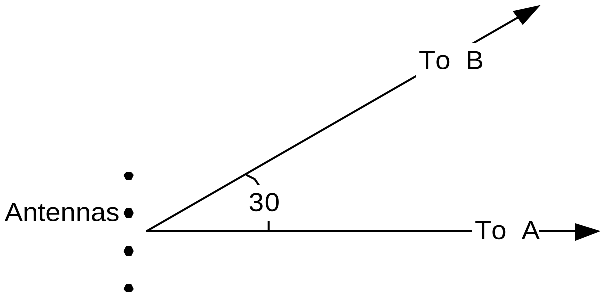
represents a traveling wave that is
(A) transversely polarized and propagating in the negative z direction
(B) transversely polarized and propagating in the positive z direction
(C) circularly polarized and propagating in the negative z direction
(D) circularly polarized and propagating in the positive z direction
(E) longitudinally polarized and propagating in the $\hat{x} - \hat{y}$ direction
解：x 方向和 y 方向的振动反号，位相差为π，因此是线偏振的。对比指数上的传播因子 $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$，知 **k** 是沿–z 方向的。选（A）。



40. A slit of vertical height 1×10^{-4} meter and horizontal width 4×10^{-4} meter is a distance of 1 meter from screen, as shown above. Light of wavelength 5000 A is incident on the slit. At the screen, the pattern consists of a
(A) vertical line of dots
(B) horizontal line of dots
(C) grid of dots with vertical spacing 4 times larger than the horizontal spacing
(D) grid of dots with vertical spacing 4 times smaller than the horizontal spacing
(E) grid of dots with equal horizontal and vertical spacing
解：狭缝衍射的角宽度

$$\Delta\theta \cong \frac{\lambda}{d},$$

因此峰的宽度 d 越小，对应的衍射斑的分布越稀。对于矩形孔的衍射，可看作两个垂直的狭缝衍射图样互相调制的结果。沿宽度方向的扩展比沿长度方向的扩展要稀。选（C）。



View from above

41. The four equally spaced vertical radio antennas shown above transmit equally and in phase at 10^6 hertz. If signals of equal intensity are received at A and B, each 50 kilometers away, a possible spacing between two adjacent antennas is

(A) 100 m
(B) 150 m
(C) 300 m
(D) 350 m
(E) 600 m

解：当

$$d\sin\theta = n\lambda$$

时，从每个天线发射的信号达到 B 时对于到达 A 附加的相因子都是 2π的整数倍，因此叠加的干涉形式没有改变。n = 1 时，得

$$d = \frac{c}{f\sin\theta} = 600\text{m}\text{ 。}$$

选（E）。

42. The speed of sound in an ideal gas is related to the temperature T of the gas. This speed is proportional to

(A) $T^{\frac{1}{4}}$
(B) $T^{\frac{1}{2}}$
(C) T
(D) $T^{\frac{4}{3}}$
(E) T^2

解：波动方程的普遍形式

$$v = \sqrt{\frac{dP}{d\rho}}\text{ 。}$$

把声振动的传播看成是绝热过程，物态方程是

$$\frac{P}{P_0}=\left(\frac{\rho}{\rho_0}\right)^{\gamma},$$

其中 γ 是定压比热和定容比热之比。则

$$\left(\frac{dP}{d\rho}\right)=\gamma\frac{P}{\rho}。$$

设 M 是气体的摩尔质量，则由 Clapeyron 方程

$$PV=\frac{m}{M}RT$$

得

$$\frac{P}{\rho}=\frac{P}{m/V}=\frac{RT}{M}。$$

所以

$$v=\sqrt{\frac{dP}{d\rho}}=\sqrt{\gamma\frac{P}{\rho}}=\sqrt{\gamma\frac{RT}{M}}。$$

选（B）。

43. Two harmonic transverse waves of the same frequency with displacements at right angles to each other can be represented by the equations

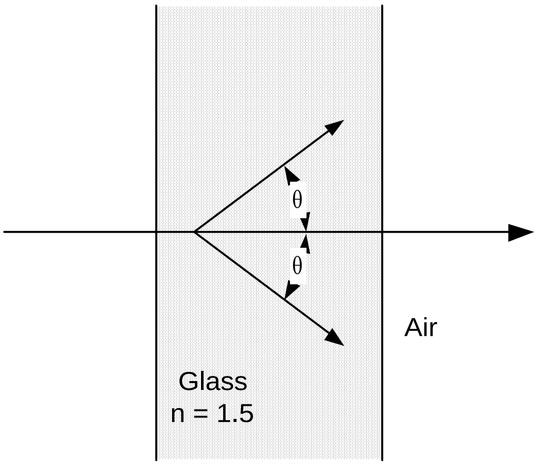
$$y=y_0\sin(\pi t-kx),$$

$$z=z_0\sin(\pi t-kx+\phi),$$

where y_0 and z_0 are nonzero constants.
The equations represent a plane-polarized wave if ϕ equals

(A) $\sqrt{2}$

(B) 3π/2
(C) π/2
(D) π/4
(E) 0
解：同一平面内两个频率相同方向垂直的简谐振动的合成情况有以下几种：当二者相位差为 π 的整数倍时，为线偏振；此外则为椭圆偏振。特例是当二者振幅相同，相位差为 π 半奇数倍时，为圆偏振。选（E）。



44. A Fast charged particle passes perpendicularly through a thin glass sheet of index of refraction 1.5. The cone of Cerenkov light, emitted in the glass at angle θ, is incident on the glass-air interface at the critical angle for total reflection. The speed of the particle is

(A) $\frac{2}{3}c$

(B) $\frac{4}{5}c$

(C) $\sqrt{\frac{5}{9}}c$

(D) $\sqrt{\frac{2}{3}}c$

(E) $\sqrt{\frac{4}{5}}c$

解：由全反射角定义

$$n\sin\theta = 1,$$

$$\sin\theta = \frac{1}{n} = \frac{2}{3}。$$

另一方面

$$\cos\theta = \frac{c}{nv_s} = \sqrt{\frac{5}{9}},$$

$$v_s = \frac{2}{3}\sqrt{\frac{9}{5}}c = \sqrt{\frac{4}{5}}c。$$

选（E）。

第四节 热力学和统计力学

Questions 1-2

The Maxwell distribution of molecular speeds in a gas is given by

$$n(v) = Av^2 e^{-mv^2/2kT}$$

where A is constant.

1. The most probable speed is

(A) $\sqrt{\dfrac{2kT}{m}}$

(B) $\sqrt{\dfrac{3kT}{m}}$

(C) $\sqrt{\dfrac{8kT}{m}}$

(D) $\dfrac{3}{2}kT$

(E) $\sqrt{2\pi mkT}$

解：所谓最可几速度，指出现几率最大的速度，即 n(v)最大。由

$$\frac{dn(v)}{dv}\Big|_{v_p} = 0,$$

得

$$2Av_p e^{-mv_p{}^2/2kT} - \frac{mv}{kT} Av_p{}^3 e^{-mv_p{}^2/2kT} = 0。$$

解得

$$v_p = \sqrt{\frac{2kT}{m}}。$$

选(A)。

2. The root-mean-square speed is

(A) $\sqrt{\dfrac{2kT}{m}}$

(B) $\sqrt{\dfrac{3kT}{m}}$

(C) $\sqrt{\dfrac{8kT}{m}}$

(D) $\dfrac{3}{2}kT$

(E) $\sqrt{2\pi mkT}$

解：由能均分定理

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}kT,$$

均方根速度为

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}}。$$

选(B)。

3. For an ideal gas, the specific heat at constant pressure C_p is greater than the specific heat at constant volume C_v because the

(A) gas does work on its environment when its pressure remains constant while its temperature is increased
(B) heat input per degree increase in temperature is the same in processes for which either the pressure or the volume is kept constant
(C) pressure of the gas remains constant when its temperature remains constant
(D) increase in the gas's internal energy is greater when the pressure remains constant than when the volume remains constant
(E) heat needed is greater when the volume remains constant than when the pressure remains constant

解：理想气体在定压过程中如果温度升高，体积会膨胀，对外做功。为了做这部分功气体要吸额外的热，因此定压比热大于定容比热。选（A）。

4. The number of ways of distributing N distinguishable molecules in K states, with N_1 in state 1, N_2 in state 2, etc., is

(A) $N-\ln N - N - \sum_{i=1}^{k} N_i \ln N_i - N_i$

(B) $(N_1!N_2!...N_k!)^{-1}$

(C) $N_1!(N_2!)^2(N_3!)^3...(N_k!)^k$

(D) $N!/(N_1!N_2!...N_k!)$

(E) $(N_1!N_2!...N_k!)/N!$

解：可能状态数为

$$C_N^{N_1} C_{N-N_1}^{N_2} \cdots C_{N-N_1-\cdots-N_{k-1}}^{N_K},$$

计算方法为

$$\frac{N!}{N_1!(N-N_1)!}\cdot\frac{(N-N_1)!}{N_2!(N-N_1-N_2)!}\cdots\cdot\frac{(N-N_1-\cdots N_{k-1})!}{N_k!(N-N_1-\cdots N_k)!}。$$

答案显然为 $N!/(N_1!N_2!...N_k!)$ 。选(D)。

5. An engine absorbs heat at a temperature of 727 °C and exhausts heat at a temperature of 527 °C, if the engine operates at maximum possible efficiency, for 2000 joules of heat input the amount of work the engine performs is most nearly

(A) 400 J

(B) 1450 J

(C) 1600 J

(D) 2000 J

(E) 2760 J

解：可以证明，Carnot 热机具有最高的效率，且不由工作介质决定。工作在温度为 T_1 和 T_2（$T_1 > T_2$）的热源间的 Carnot 热机效率为

$$\eta = 1 - \frac{T_2}{T_1}\text{。}$$

所以最大做功为

$$W = Q_{input}\eta = (1 - \frac{T_2}{T_1})Q_{input} = (1 - \frac{800}{1000}) \times 2000 = 400\text{J}\text{。}$$

选(A)。

6. In a certain group of N particles, the number of particles with speeds in the range dv is given by dN = kv dv, where k is a constant. The speeds are in the range between zero and a maximum value v_{max}. The average speed of the particle is

(A) $\frac{1}{4}v_{mzx}$

(B) $\frac{1}{3}v_{mzx}$

(C) $\frac{1}{2}v_{mzx}$

(D) $\frac{1}{4}v_{mzx}$

(E) $\frac{1}{4}v_{mzx}$

解：关键是别忘了归一化，或者说除以总粒子数。

$$\bar{v} = \frac{\int_0^{v_{\max}} v \cdot kvdv}{\int_0^{v_{\max}} kvdv} = \frac{2}{3}v_{\max}$$

选（D）。

7. In terms of the Boltzmann constant k, the classical constant-volume specific heat per molecule of helium gas is

(A) $\frac{k}{2}$

(B) k

(C) $\dfrac{3k}{2}$

(D) 2k

(E) 3k

解：由能均分定理，温度为 T 的经典系统，每个自由度的能量为 $\dfrac{1}{2}kT$ 。氦气为单元子分子，只有 3 个自由度，所以每个分子能量为 $\dfrac{3}{2}kT$ ，比热为 $\dfrac{3k}{2}$ 。选(C)。

Question 8-9 refer to the following processes involving systems labeled by numbers 1 through 8.

A bar of iron (1) at 300K is brought into thermal contact with a body (2) at 400K, the two being thermally isolated from all other systems.

An ideal gas (3) is compressed reversibly while in contact with a reservoir (4), the two being thermally isolated from all other systems.

A body of water (5) freezes reversibly.

A container of water (6) is stirred and its temperature increases by 1K.

A chemical reaction takes place in an isolated system (7).

A Carnot engine (8) operates in a cycle.

8. For which of the following systems does the entropy decrease?

(A) 1

(B) 4

(C) 5

(D) 6

(E) 7

解：熵是态函数，两个被可逆过程联系起来的态之间的熵变为

$$\Delta S = S_2 - S_1 = \int_1^2 \frac{dQ}{T} 。$$

先对各个系统逐一分析：

（1）吸收热量，熵增加；（2）放出热量，熵减少。二者组成系统与外界无热交换，也无相互做功，熵不变。

（3）被可逆压缩，条件不足以判断温度的变化情况，无法确定熵值变化情况，从而（4）也无法判断。但是二者组成系统虽然与外界无热交换，但外界对气体做功，所以系统熵增加。

（5）向外放热，体积膨胀对外做功，熵值降低。

（6）由于外界做功，使其温度上升，熵值增加。

（7）会发生熵变，等于生成物的熵值减去反应物的熵值。

（8）Carnot 热机一个循环后回到原来状态，熵不变。

选（C）。

9. For which of the following systems does the entropy increase?

(A) 2

(B) 3

(C) 8

(D) 1 and 2 combined

(E) 3 and 4 combined

解：选（E）。原因见上题分析。

10. Upon freezing at 271.3°C and one atmosphere pressure, liquid bismuth expands and releases heat. It can correctly be inferred from the facts that

(A) bismuth is a metal.

(B) solid bismuth will float on water.

(C) the entropy of the solid is greater than that of the liquid.

(D) the Gibbs free energy of the solid is greater than that of the liquid at the freezing point.

(E) raising the pressure at 271.3°C will cause solid bismuth to melt.

解：铋确实是一种金属，但不可能从题目所给条件推知，所以不能选（A）。同理（B）也推不出来。由于铋凝固时放热，体积膨胀对外做功，所以熵值减小，自由能降低，（C）、（D）不对。选（E）。可以和水在冰点的性质类比。

11. How many ways can two indistinguishable particles obeying Bose-Einstein statistics be arranged among three states?

(A) 3

(B) 6

(C) 9

(D) 12

(E) 15

解：由于二者不可分辨，而且可同处在一个态上，所以可能情况为（态 1，态 1）、（态 2，态 2）、（态 3，态 3）、（态 1，态 2）、（态 2，态 3）、（态 3，态 1）。答案选（B）。

12. The root-mean-square speed of oxygen and nitrogen molecules in air at room temperature is closest to

(A) 100 m/s

(B) 500 m/s

(C) 3000 m/s

(D) 15,000 m/s

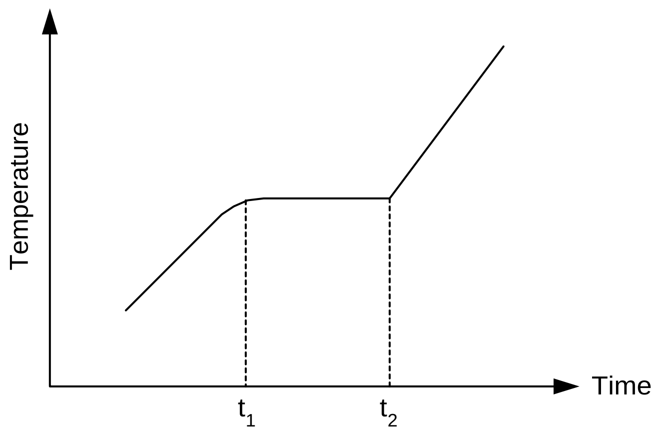
(E) 50,000 m/s

解：因为氧气和氮气都是双原子分子气体，每个分子自由度为 5，所以室温下能量为 $\frac{5}{2}kT$，

$$\frac{5}{2}kT=\frac{1}{2}m\overline{v^2},$$

$$v_{rms}=\sqrt{\overline{v^2}}=\sqrt{\frac{5kT}{m}}=\sqrt{\frac{5RT}{M}}=\sqrt{\frac{5\times8.31\times300}{32\times10^{-3}}}\approx624\text{m/s}。$$

选（B）。



13. Electric power is used to heat and melt 3 kilograms of a certain material. The graph of temperature vs. time for the process is shown above. A current of 10 amperes at a potential difference of 100 volts is used and the time between t_1 and t_2 is approximately 17 minutes. The heat of fusion of the material is most nearly

(A) 80 J/kg

(B) 970×10^2 J/kg

(C) 144×10^3 J/kg

(D) 340×10^3 J/kg

(E) 539×10^3 J/kg

解：相变潜热为

$$T=\frac{Q}{M}=\frac{UIt}{M}=\frac{100\times10\times17\times60}{3}=340\times10^3\,\text{J/kg}\ 。$$

选（D）。

14. For a system in which the number of particles is fixed, the reciprocal of the Kelvin temperature T is given by which of the following derivatives? (Let P=pressure, V=volume, S=entropy, and U=internal energy.)

(A) $\left(\frac{\partial P}{\partial V}\right)_S$

(B) $\left(\frac{\partial P}{\partial S}\right)_V$

(C) $\left(\frac{\partial S}{\partial P}\right)_U$

(D) $\left(\frac{\partial V}{\partial P}\right)_U$

(E) $\left(\frac{\partial S}{\partial U}\right)_V$

解：热力学基本方程

$$dU = TdS - pdV \text{ 。}$$

把 U 看作 S、V 的函数 U = U(S,V)，其全微分为

$$dU = \left(\frac{\partial U}{\partial S}\right)_V dS + \left(\frac{\partial U}{\partial V}\right)_S dV \text{ 。}$$

比较以上两式得

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_V \text{ 。}$$

选（E）。

15. A large isolated system of N weakly interacting particles is in thermal equilibrium. Each particle has only 3 possible nondegenerate states of energies 0, ε, and 3ε. When the system is at an absolute temperature T>>ε/k, where k is Boltzmann's constant, the average energy of each particle is

(A) 0

(B) ε

(C) $\frac{4}{3}\varepsilon$

(D) 2ε

(E) 3ε

解：由 Maxwell 分布，

$$\overline{E} = \frac{0 + \varepsilon e^{-\frac{\varepsilon}{kT}} + 3\varepsilon e^{-\frac{3\varepsilon}{kT}}}{1 + e^{-\frac{\varepsilon}{kT}} + e^{-\frac{3\varepsilon}{kT}}} \text{ 。}$$

当 ε << kT 时，$e^{-\frac{\varepsilon}{kT}} \to 1$，有

$$\overline{E} = \frac{4}{3}\varepsilon \text{ 。}$$

选（C）。

16. In a gas of N diatomic molecules, two possible models for a classical description of a diatomic molecule are:

Which of the following statements about this gas is true?

(A) Model I has a specific heat $c_v = \frac{3}{2}Nk$.

(B) Model II has a smaller specific heat than Model I.

(C) Model I is always correct.

(D) Model II is always correct.

(E) The choice between Models I and II depends on the temperature.

解：在低温下，两个原子间距离这个振动自由度被冻结，此时可取模型 I。当温度较高时，这个自由度被激发，须采用模型 II。答案选（E）。答案(A)不对，模型 I 中两个原子共有 5 个自由度，由能均分定理，$c_v = \frac{5}{2}Nk$。

17. Consider a system of N noninteracting particles confined in a volume V at a temperature such that the particles obey classical Boltzmann statistics. If the temperature is lowered to the point at which quantum effects become important, the pressure of the gas may differ depending on whether the particles are fermions or bosons. Let P_F be the pressure if they are fermions, P_B be the pressure if they are bosons, and P_C be the pressure the particles would exert if quantum effects are ignored. Which of the following is true?

(A) $P_F = P_B = P_C$

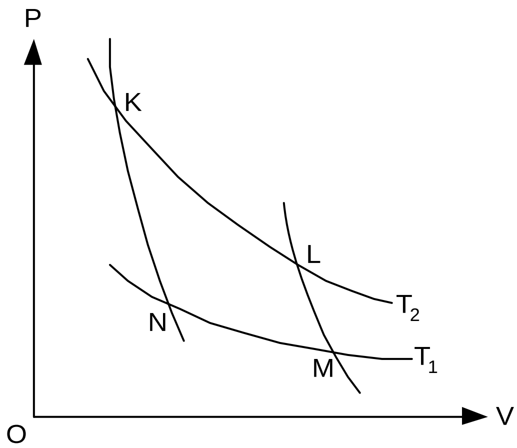
(B) $P_F > P_C > P_B$

(C) $P_F > P_B > P_C$

(D) $P_F < P_B < P_C$

(E) $P_F < P_C < P_B$

解：Fermion 有相互排斥的等效作用，压强变大；而 Boson 有相互吸引的等效作用，压强变小。选（B）。可以联想一下白矮星中子星中惊人的简并压，分别由电子和中子造成，均为 Fermion。而 Bose-Einstein 凝聚则发生在 Boson 之间。



18. In the cycle shown above, KL and NM represent isotherms, while KN and LM represent reversible adiabats. A system is carried through the Carnot cycle KLMN, taking in heat Q_2 form the hot reservoir T_2 and releasing heat Q_1 to the cold reservoir T_1. All of the following statements are true EXCEPT:

(A) $Q_1/T_1=Q_2/T_2$.

(B) The entropy of the hot reservoir decreases.

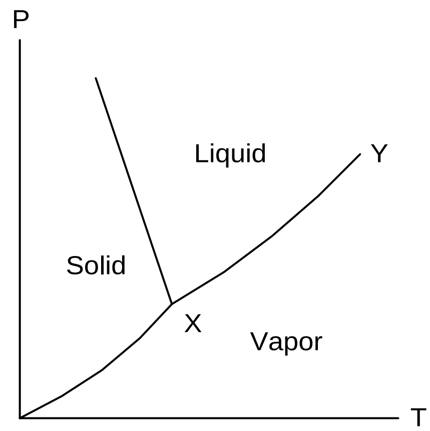
(C) The entropy of the system increases.

(D) The work W done is equal to the net heat absorbed, $Q_2–Q_1$.

(E) The efficiency of the cycle is independent of the working substance.

解：熵是态函数，所以经历一个循环后回到原来状态，熵值不变。选（C）。注意题目中所

指系统指循环系统，不包括热库等。大家可以用本题复习一下 Carnot 循环，其他几个选项均是重要结论。



19. For the substance represented by the diagram above of pressure P versus temperature T, which of the following statement is true?

(A) The substance expands when it freezes.

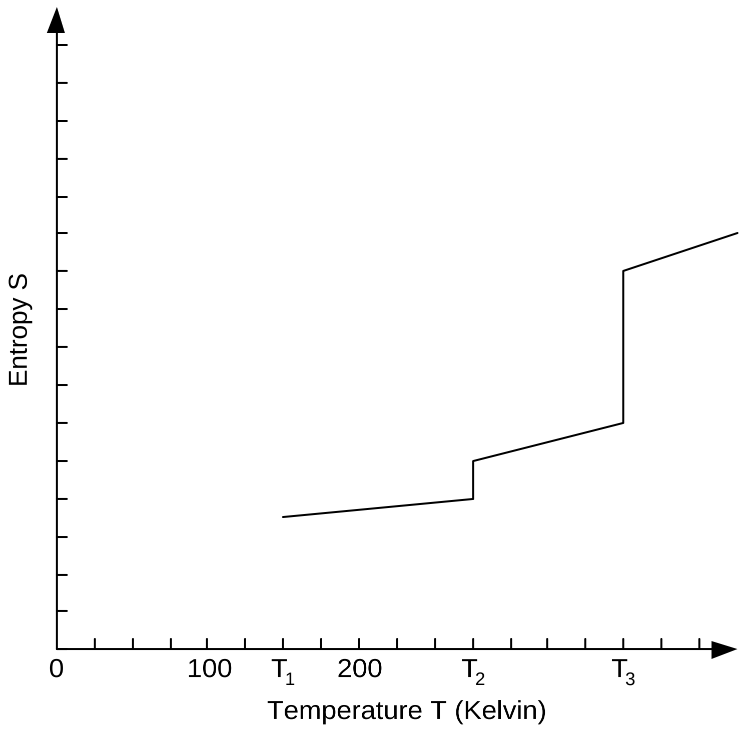
(B) In the vicinity of point X, the vapor is more dense than liquid.

(C) Point X represents the critical point for the substance.

(D) The segment of the graph from X to Y represents the fusion curve for the substance.

(E) At any temperature above that of the triple point, the distinction between liquid and gas disappears.

解：我们在 X 点左侧的范围内做等温直线。从液相沿等温线变为固相时，压强变小，说明发生了膨胀。选（A）。三相点附近气体密度可以大于液体的，（B）不对。X 是三相点，而不是什么相变点，二者概念不同，（C）不对。X 到 Y 一段曲线是汽化曲线，（D）不对。在 Y 点以上，而不是在 X 点以上，气相液相的区别消失，（E）不对。



20. The graph above shows an entropy s versus temperature T curve for isobaric for a certain substance. The S and T scales are linear. Which of the following statement is NOT correct?

(A) The vertical portion of the curve at temperature T_2 represents a first-order phase transition.

(B) The vertical portion of the curve at temperature T_3 represents a second-order phase transition.

(C) The latent heat for the phase change at T_2 is less than the latent heat for the phase change at T_3.

(D) The specific heat at constant pressure in the region between T_1 and T_2 is less than that in the region between T_2 and T_3.

(E) The entropy approaches zero if the temperature is continuously lowered so that T approaches zero.

解：一级相变指相变点两相的化学势连续，但化学势的一阶导存在突变，且存在相变潜热。T_2 处的直线表示相变前后熵值发生了变化，所以为一级相变。而在二级相变中，相变前后熵值不变，没有相变潜热和比容的突变。T_3 处的直线与 T_2 处相同，也为一级相变，只是熵值改变更大，潜热比 T_2 处的相变大，如（C）所述。所以选项（B）不对。由比热容公式

$$c_p = T\left(\frac{\partial s}{\partial T}\right)_p ,$$

而 T_1 到 T_2 一段斜率较小，比容比较小。选项（C）正确。（E）是热力学第三定律的要求，正确。选（B）。

21. A mole of ideal gas initially at temperature T_0 and volume V_0 undergoes a reversible isothermal expansion to volume V_1. If the ratio of specific heats is $c_p/c_v = \gamma$ and if R is the gas constant, the work done is

(A) zero

(B) $RT_0(V_1/V_0)^\gamma$

(C) $RT_0(V_1/V_0 - 1)$

(D) $C_v T_0[1-(V_0/V_1)^{\gamma-1}$

(E) $RT_0 \ln V_1/V_0$

解：等温膨胀，做功为

$$W = \int_{V_0}^{V_1} PdV = \int_{V_0}^{V_1} \frac{RT_0}{V} dV = RT_0 \ln V_1/V_2 ,$$

其中用到了气体方程

$$PV = nRT ,$$

$$P = \frac{RT}{V} 。$$

选（E）。

<u>Questions 22-23</u>

The Maxwell distribution of molecular speeds in a gas is given by

$$n(v) = Av^2 e^{-mv^2/2kT} ,$$

where A is a constant.

22. The most probable speed is

(A) $\sqrt{\frac{2kT}{m}}$

(B) $\sqrt{\frac{3kT}{m}}$

(C) $\sqrt{\frac{8kT}{\pi m}}$

(D) $\frac{3}{2}kT$

(E) $\sqrt{2\pi mkT}$

解：Maxwell 速度分布律下常用的结论如平均速率、最可几速率、方均根速率应熟记。最可几速率满足条件

$$\frac{dn}{dv} = 0 ,$$

解之即得。

$$2Ave^{-mv^2/2kT} - \frac{mv}{kT} av^2 e^{-mv^2/2kT} = 0 。$$

选（A）。

23. The root-mean-square speed is

(A) $\sqrt{\dfrac{2kT}{m}}$

(B) $\sqrt{\dfrac{3kT}{m}}$

(C) $\sqrt{\dfrac{8kT}{\pi m}}$

(D) $\dfrac{3}{2}kT$

(E) $\sqrt{2\pi mkT}$

解：接上题。方均根速率的计算式为

$$v_{rms} = \sqrt{\frac{\int_0^\infty v^2 n(v)dv}{\int_0^\infty n(v)dv}} = \sqrt{\frac{3kT}{m}}$$

其中积分利用分部积分法，分子分母相约。选（B）。

24. The observed specific heat of the electrons in a metal is much smaller than classical (i.e., nonquantum) statistical mechanics would indicate. The reason for this directly related to

(A) special relativity
(B) The Pauli exclusion principle
(C) the indeterminacy principle
(D) Hund's rule
(E) the principle of least action

解：电子是 Fermion，满足 Pauli 不相容原理，服从 Fermi 统计。相应的比热容计算的结果表明，在一定温度 T 下，只有 Fermi 面附近大致为 k_BT 能量范围内的电子受到热激发，激发能 $\approx$ k_BT。因此量子统计的比热容结果比经典值小很多。选（B）。

25. The results derived from the Bose-Einstein statistics approximate those derived from the Fermi-Dirac statistics under which of the following conditions?

(A) The number of quantum states thermally accessible to each particle is much greater than the number of particles.
(B) The number of quantum states thermally accessible to each particle is approximately equal to the number of particles.
(C) The number of quantum states thermally accessible to each particle is much less than the number of particles.
(D) The particles have spin zero.
(E) The quantum states are degenerate.

解：Fermi-Dirac 统计和 Bose-Einstein 统计为

$$a_l = \frac{\omega_l}{e^{\alpha+\beta\varepsilon_l} \pm 1}$$

由此可以看出，当参数α满足条件

$$e^{\alpha} >> 1$$

时，分母中的±1 就可以忽略，这时两种分布都过渡到 Boltzmann 分布，显然此时 $\alpha_l << \omega_l$，对所有的 l。此条件也称为经典极限条件。选（A）。

26. Copper has a compressibility $\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T = 8\times 10^{-7}\,atm^{-1}$ and coefficient of thermal expansion $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = 6\times 10^{-5}\,K^{-1}$ at 300K. A sample of copper initially at 1 atmosphere is heated 1K at constant volume. What is the final pressure?

(A) –74 atm

(B) 0.987 atm

(C) 1.013 atm

(D) 43 atm

(E) 76 atm

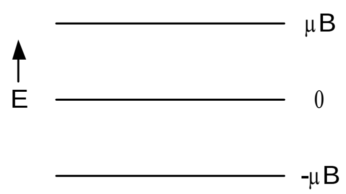
解：利用热力学等式

$$\left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial T}{\partial V}\right)_p = -1$$

从而有

$$\left(\frac{\partial p}{\partial T}\right)_V = -\frac{1}{\left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial T}{\partial V}\right)_p} = \frac{\beta}{\kappa} = 75\text{atm/K}\ 。$$

以下计算即得。选（E）。



27. A set of energy levels for an atom in a magnetic field of magnitude B is shown above. For a collection of such atoms, what fraction is on the lowest state at temperature T?

(A) 1

(B) $e^{-\mu B/kT}$

(C) $e^{\mu B/kT}$

(D) $e^{\mu B/kT}/(e^{\mu B/kT}+e^{-\mu B/kT})$

(E) $e^{\mu B/kT}/(1+e^{\mu B/kT}+e^{-\mu B/kT})$

解：根据 Boltzmann 统计，原子处于能量为 E 的能级上的几率

$$p_E \propto e^{-E/kT},$$

因此在三个能级上的几率比为

$$p_- : p_0 : p_+ = e^{\mu B/kT} : 1 : e^{-\mu B/kT},$$

且总几率和为 1。选（E）。

28. How many ways can two indistinguishable particles obeying Bose-Einstein statistics arranged among three states?

(A) 3
(B) 6
(C) 9
(D) 12
(E) 15

解：两个粒子占据同一个量子态的可能性总数共 3 种，占据不同量子态的可能性总数（考虑到粒子的不可区分性），为

$$\frac{1}{2}C_3^2 = 3$$

种。故一共有 6 种占据方式。选（B）。

29. A photon gas such as results from the “big bang” undergoes an adiabatic expansion during which VT^3 is constant. An ideal gas would obey the same law of adiabatic expansion if its ratio of specific heats $c_p/c_v = \gamma$ were

(A) 1/3
(B) 3/4
(C) 4/3
(D) 3/2
(E) 3

解：理想气体的绝热过程方程为

$$pV^{\gamma} = const,$$

利用状态方程改写形式可得

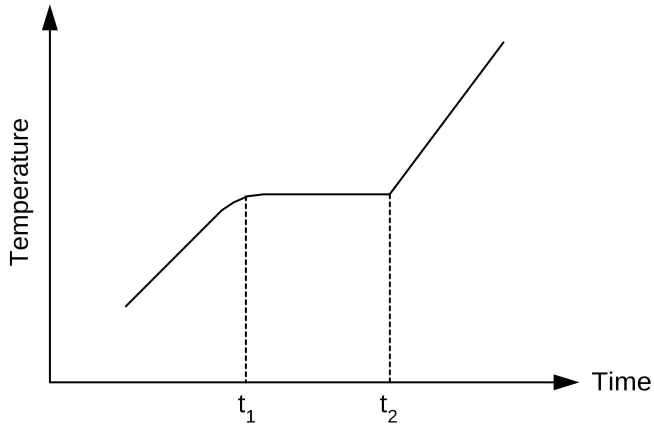
$$VT^{\frac{1}{\gamma-1}} = const,$$

因此有

$$\frac{1}{\gamma-1} = 3,$$

$$\gamma = \frac{4}{3} 。$$

选（C）。



30. Electric power is used to heat and melt 3 kilograms of a certain material. The graph of temperature vs. time for the process is shown above. A current of 10 amperes at a potential difference of 100 volts is used and the time between t_1 and t_2 is approximately 17 minutes. The heat of fusion of the material is most nearly

(A) 80 J/kg

(B) 970×10^2 J/kg

(C) 144×10^3 J/kg

(D) 340×10^3 J/kg

(E) 539×10^3 J/kg

解：t_1 到 t_2 之间被加热物体保持温度不变，进行相变，所吸收的热量全部用来熔解。

$$L = \frac{Q}{m} = \frac{UIt}{m} = \frac{100 \times 10 \times 17 \times 60}{3} = 340 \times 10^3 \text{ J/kg}$$

选（D）。

31. A mixture of one mole of helium (atomic weight = 4) and one mole of argon (atomic weight = 40) is in thermal equilibrium in a container at room temperature. The ratio of the rms speed of a helium atom to the rms speed of an argon atom, v_{He}/v_{Ar}, is most nearly equal to

(A) 1

(B) 0.3

(C) 1

(D) 3

(E) 10

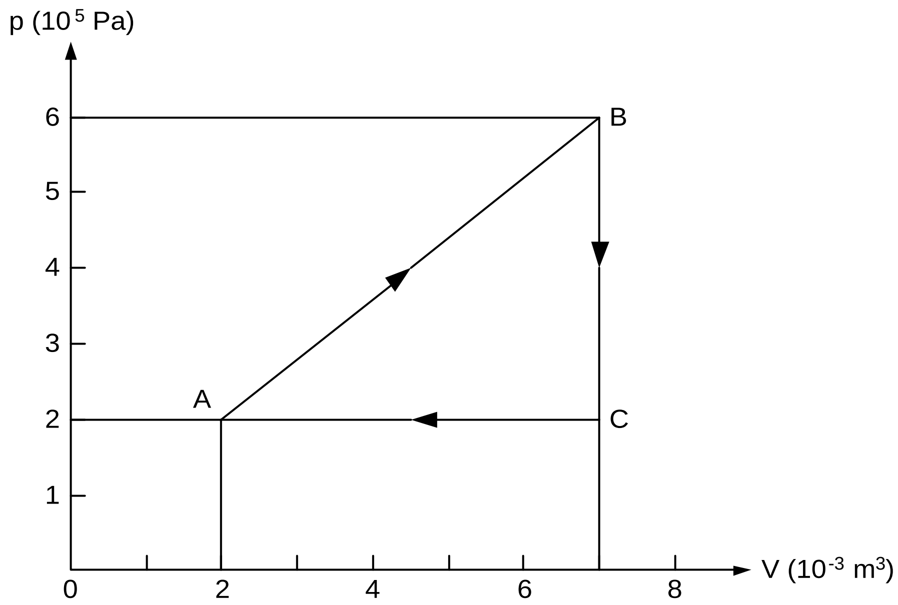
解：根据 Maxwell 速度分布率计算的气体分子方均根速率

$$v_{rms} = \sqrt{\frac{3kT}{m}} 。$$

故两种气体的方均根速率之比

$$\frac{v_{He}}{v_{Ar}} = \sqrt{\frac{m_{Ar}}{m_{He}}} = \sqrt{10} \approx 3 。$$

选（D）。



32. A gas is taken through the cycle A→B→C→A, as shown above. What is the net work done by the gas?

(A) 2000 J

(B) 1000 J

(C) 0 J

(D) –1000 J

(E) –2000 J

解：在 p-V 图上循环曲线所包围的面积就是气体所作的净功，顺时针为正。

$$A = S_{A-B-C-A} = \frac{1}{2}(V_C - V_A)(P_B - P_C) = 2000\text{J}\ 。$$

选（A）。

33. The number of ways of distributing N distinguishable molecules among k states, with N_1 in state 1, N_2 in state 2, etc., is

(A) $N\ln N - N\sum_{i=1}^{k}(N_i \ln N_i - N_i)$

(B) $(N_1!N_2!....N_k!)^{-1}$

(C) $N_1!(N_2!)^2(N_3!)^3....(N_k!)^k$

(D) $N/(N_1!N_2!....N_k!)$

(E) $(N_1!N_2!....N_k!)/N!$

解：N 个可区分分子的总排列数为 N!，其中每个量子态内的 N_i!种排列对应的是统一中分布，总的分布可能性

$$n = N! \Big/ \prod_i N_i!$$ 。

选（D）。

34. For an isotropic two-dimension metal, the Fermi surface is a circle of radius k_F in momentum space. If the energy of an electron of mass m and wave vector k is $\varepsilon = \hbar^2 k^2 / 2m$, then the number of states per unit energy (density of states) at the Fermi energy ε_F is

(A) proportional to $\sqrt{\varepsilon_F}$

(B) proportional to ε_F

(C) proportional to $1/\varepsilon_F$

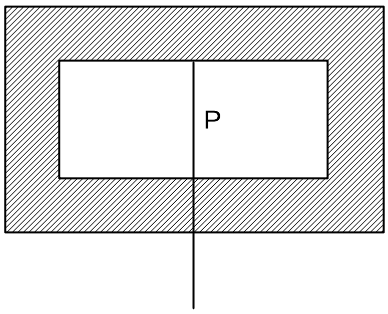
(D) proportional to $1/\sqrt{\varepsilon_F}$

(E) independent of ε_F

解：每个量子态在 k 空间中占据的体积数为 $8\pi^3/V$，能态密度

$$\begin{aligned} N(E) &= \frac{V}{8\pi^3} 4\pi k^2 \frac{dk}{dE} \\ &\propto \varepsilon \frac{d\sqrt{\varepsilon}}{d\varepsilon} = \sqrt{\varepsilon} \end{aligned}$$ 。

选（A）。



35. Gas at standard temperature and pressure is initially in the left-hand compartment of the device shown above. The fight-hand compartment is initially evacuated. The volumes of the two compartments are the same, and the entire device is thermally insulated from its surroundings. If the partition P separating the two compartments is rapidly pulled out sideways, which of the following will be true?

(A) For an ideal gas, the temperature will be lower after the expansion.

(B) For a real gas, the temperature will be the same after the expansion.

(C) The final temperature is determined solely by the specific heat ratio c_P/c_V.

(D) The final pressure must be known before the final temperature can be computed for an ideal gas.

(E) The internal energy will remain constant for any gas, real or ideal.

解：自由扩散运动过程中，气体与外界没有热交换，也没有对外做功。因此，根据热力学第一定律

$$\Delta U = W + Q,$$

气体的内能不变。选（E）。

36. Heat dQ flows from a body at temperature T_1 to a body at temperature T_2. The total change in entropy of the two body is equal to

(A) $dQ(1/T_1 + 1/T_2)$

(B) $dQ/(T_1 + T_2)$

(C) $dQ/(T_1 - T_2)$

(D) $dQ(1/T_1 - 1/T_2)$

(E) dQ/T_2

解：根据熵的定义式计算

$$dS = \frac{-dQ}{T_1} + \frac{dQ}{T_2} = dQ\left(\frac{1}{T_2} - \frac{1}{T_1}\right)。$$

选（D）。

37. A gas consists of N_1 atoms in a state of energy E_1 and N_1 atoms in a state of energy E_2. If the temperature T of the gas is 300 K (kT=1/40 eV), the energy difference $E_1 - E_2$ is 4×10^{-3}eV. The ratio $(N_2-N_1) / (N_1 + N_2)$ is most nearly

(A) 1/10,000

(B) 1/100

(C) 1/12

(D) 1/3

(E) 1/2

解：根据 Boltzmann 分布，

$$N_E \propto e^{-E/kT},$$

因此

$$\frac{N_2 - N_1}{N_2 + N_1} = \frac{e^{-E_2/kT} - e^{-E_1/kT}}{e^{-E_2/kT} + e^{-E_1/kT}} = \frac{e^{\Delta E/kT} - 1}{e^{\Delta E/kT} + 1}。$$

$$= \frac{e^{0.16} - 1}{e^{0.16} + 1} \approx \frac{0.16}{2.16} \sim \frac{1}{100}$$

选（B）。

38. The specific heat of a metal at low temperatures, T < 10 K, varies as $c = \gamma T^m + AT^n$, where the first term arises from the free electrons and the second from the photons. The values of the exponents are

(A) m = 1, n = 1
(B) m = 1, n = 2
(C) m = 1, n = 3
(D) m = 2, n = 2
(E) m = 2, n = 3

解：在低温极限下（$T<<T_D$，$T<<T_F$），根据 Debye T^3 定律，晶格比热（来源于声子）

$$C_V \propto T^3;$$

根据 Fermi 气体模型，电子比热容

$$C_V \propto T 。$$

选（C）。

40. Assuming that all the planes have the same reflection coefficient for sunlight and the same emission coefficient, which of the following relationships would be expected between the planets' average temperatures T in Kelvin and their distance R from the Sun?

(A) $T \propto R^{-2}$
(B) $T \propto R^{-1}$
(C) $T \propto R^{-1/2}$
(D) $T \propto R^{1/2}$
(E) $T \propto R^{2}$

解：假设行星的吸收系数为γ，根据热平衡的条件有

$$\gamma \frac{W}{4\pi R^2} \pi r^2 = \gamma a T^4 4\pi r^2,$$

$$T \propto \left(R^{-2}\right)^{1/4} = R^{-1/2} 。$$

选（C）。

41. A mass m of water at temperature T_1 is mixed with an equal mass of water at temperature T_2 adiabatically. If C_p is the specific heat of water, the total entropy change of the universe equals

(A) $mc_p \ln[(T_1 + T_2)/\sqrt{T_1 T_2}]$

(B) $2mc_p \ln[(T_1 + T_2)/\sqrt{T_1 T_2}]$

(C) $2mc_p$

(D) $mc_p (T_1 - T_2)^2/(2T_1 T_2)$

(E) $mc_p(T_1-T_2)/\sqrt{T_1T_2}$

解：由于两种水的质量一样，比热容也一样，最终达到热平衡时升温与降温相等，末温为

$$T=\frac{T_1+T_2}{2},$$

总的熵变化为

$$\Delta S=mc_p\left(\int_{T_1}^{T}\frac{dT}{T}+\int_{T_2}^{T}\frac{dT}{T}\right)=mc_p\ln\frac{T^2}{T_1T_2}=2mc_p\ln\frac{T}{\sqrt{T_1T_2}}。$$

选（B）。

<u>Questions 42-43</u>

An ideal diatomic gas is initially at temperature T and volume V. The gas is taken through three reversible processes in the following cycle: adiabatic expansion to the volume 2V, constant volume process to the temperature T, isothermal compression to the original volume V.

42. For the complete cycle described above, which of the following is true?

(A) Net thermal energy is transferred from the gas to the surroundings.

(B) The net work done by the gas on the surroundings is positive.

(C) The net work done by the gas on the surroundings is zero.

(D) The internal energy of the gas increases.

(E) The internal energy of the gas decreases.

解：对于本题所涉及的循环，第一个过程绝热膨胀，对外做功，吸热为零，内能下降，温度降低；第二个过程等容升温，吸收热量，做功为零；第三个过程等温压缩，外界对气体做功，气体内能不变，向外放热。在 P—V 图上，代表一个过程的一条线下方的面积等于气体对外或外界对气体做功的绝对值，方向为体积变大则对外做功，反之外界对气体做功。本题中绝热线位于等温线下方，等温压缩中功的绝对值大，所以整个循环外界对气体做功。选项（B）、（C）不对。由于内能为状态量，而系统循环后气体回到原来状态，所以内能不变。（D）、（E）不对。如前所述，外界对气体做功，而气体内能不变，所以要对外放热。选（A）。

43. Which of the following statements about entropy changes in this cycle is true?

(A) The entropy of the gas remains constant during each of the three processes.

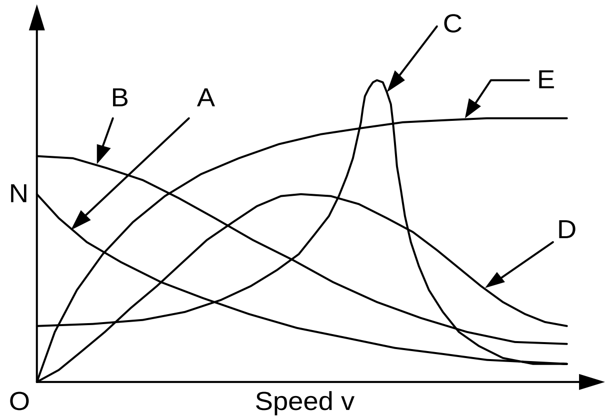
(B) The entropy of the surroundings remains constant during each of the three processes.

(C) The combined entropy of the gas and surroundings remains constant during each of the three processes.

(D) For the complete cycle, the combined entropy of the gas and surroundings increases.

(E) For the complete cycle, the entropy of the gas increases.

解：题目说了可逆过程，总熵变始终为零。选（B）。



44. Which of the curves in the graph above best represents the distribution of speeds of the molecules in an ideal gas at thermal equilibrium?

(A) A

(B) B

(C) C

(D) D

(E) E

解：由 Maxwell 分布率

$$n(v) = Av^2 e^{-\frac{mv^2}{2kT}}\text{。}$$

v = 0 时，$n(v) = 0$，只有（D)、(E）符合。v 的分布应该有一个最可几值，反映在图线上应该有一个极大峰。选（D)。

45. Which of the following describes a liquid of ^{3}He atoms?

(A) Maxwell-Boltzmann statistics only

(B) Bose-Einstein statistics only

(C) Fermi-Dirac statistics only

(D) Bose-Einstein statistics above some critical temperature T_C and Fermi-Dirac statistics below T_C

(E) None of the above, because it is in the liquid state

解：液体原子间相互作用很强，不能应用近独立系统统计。选（E)。

第五节 量子力学

1. The wave function $\psi(x) = A\exp\{-\frac{b^2x^2}{2}\}$, where A and b are real constants, is a normalized eigenfunction of the Schrödinger equation for a particle of mass M and energy E in a one dimensional potential V(x) such that V(x) = 0 at x = 0. Which of the following is correct?

(A) $V=\dfrac{\hbar^2 b^4}{2M}$

(B) $V=\dfrac{\hbar^2 b^4 x^2}{2M}$

(C) $V=\dfrac{\hbar^2 b^6 x^4}{2M}$

(D) $E=\hbar^2 b^2\left(1-b^2x^2\right)$

(E) $E=\dfrac{\hbar^2 b^4}{2M}$

解：Schrödinger 方程

$$-\frac{\hbar^2}{2M}\frac{d^2}{dx^2}\psi=(E-V)\psi\ ,$$

$$-\frac{\hbar^2}{2M}b^2(b^2x^2-1)\exp\left(-\frac{b^2x^2}{2}\right)=(E-V)\exp\left(-\frac{b^2x^2}{2}\right)。$$

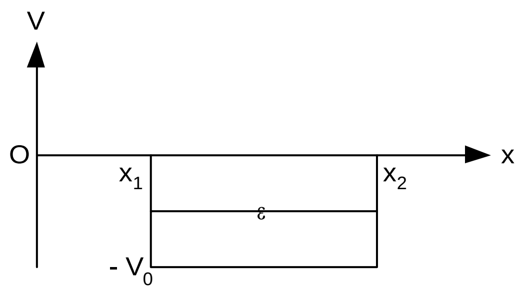
在上式中令 x=0，得

$$E=\frac{\hbar^2 b^2}{2M}。$$

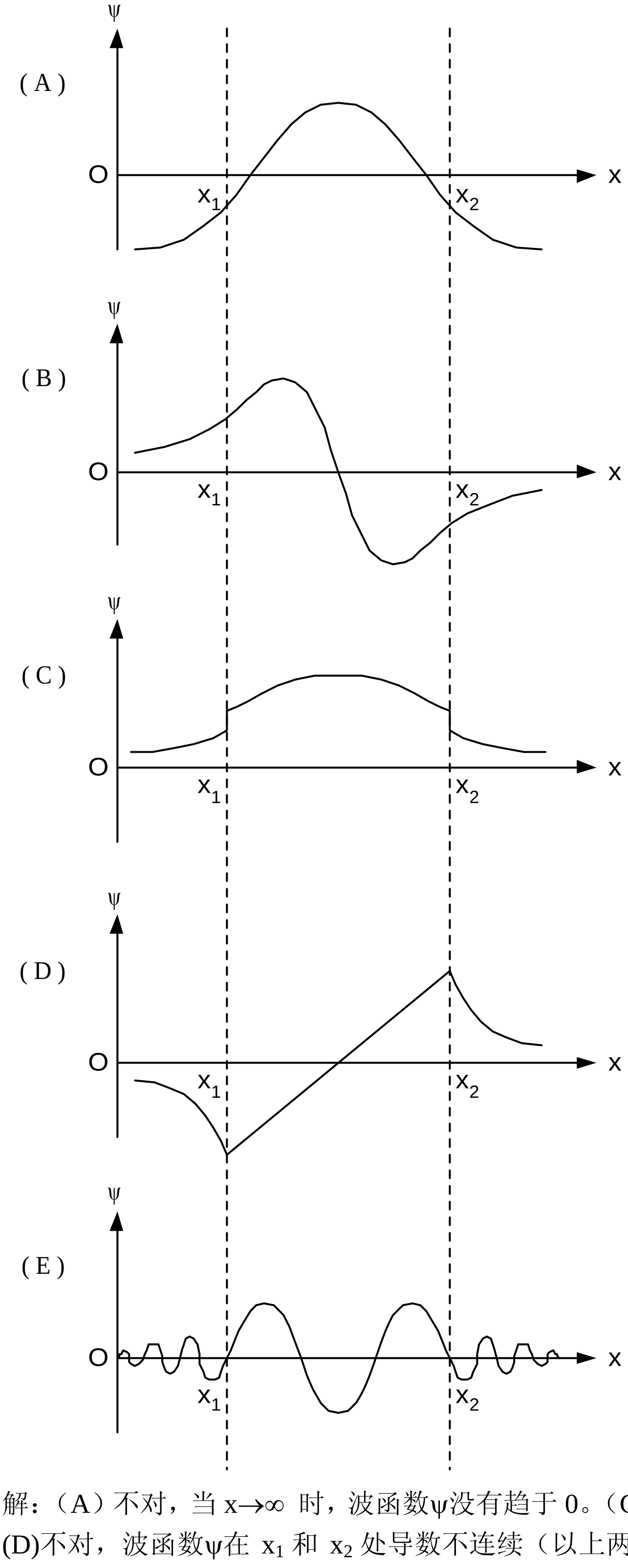
将 E 的表达式代回 Schrödinger 方程，得

$$V=\frac{\hbar^2 b^4}{2M}x^2。$$

选（B）。



2. An attractive one-dimensional square well has depth V_0 as shown above. If there is a bound state at an energy ε<0, which of the following best shows a possible wave function for this state?



解：（A）不对，当 x→∞ 时，波函数ψ没有趋于 0。（C）不对，波函数ψ在 x_1 和 x_2 处不连续。(D)不对，波函数ψ在 x_1 和 x_2 处导数不连续（以上两条对波函数的要求可参阅曾谨言《 量子力学导论》第二版，pp. 53，北京大学出版社）。在 $x > x_2$ 和 $x < x_1$ 处，$V(x) = 0$ 。由 Schrödinger 方程

$$\psi''(x) = -\frac{2m}{\hbar^2}[E - V(x)]\psi(x) = \frac{2m|\varepsilon|}{\hbar^2}\psi(x)$$

所以$\psi(x)$大于零处，$\psi''(x)$大于零；$\psi(x)$小于零处，$\psi''(x)$小于零。其行为正如(B)中所描述，而不应像(E)那样振荡。

3. If an electron were confined to nuclear dimensions, the uncertainty in its momentum would be most nearly
(A) 0.2 eV/c
(B) 200 eV/c
(C) 200 KeV/c
(D) 200 MeV/c
(E) 200 GeV/c
解：测不准关系

$$\Delta x\cdot\Delta p\ge\frac{\hbar}{2}\text{。}$$

取$\Delta x\cdot\Delta p=\hbar$，原子核的尺度为$\Delta x\sim10^{-15}$m，动量不确定度为

$$\Delta p=\frac{\hbar}{\Delta x}=\frac{1.054\times10^{-34}}{10^{-15}}=\frac{1.054\times10^{-19}\times3\times10^{8}}{1.6\times10^{-19}}\ \text{eV/c}=219\,\text{MeV/c}\text{。}$$

选(D)。

4. The total energy in quantum mechanics corresponds to the differential operator given by
(A) $i\hbar\frac{\partial}{\partial t}$
(B) $i\hbar^{2}\frac{\partial^{2}}{\partial t^{2}}$
(C) $-i\hbar\nabla$
(D) $i\hbar\nabla$
(E) $\hbar\nabla\cdot\nabla$
解：Schrödinger 方程

$$i\hbar\frac{\partial}{\partial t}\psi=\hat{H}\psi=E\psi\text{。}$$

选（A）。

5. The third lowest energy level of a one-dimensional quantum mechanical harmonic oscillator of frequency f has an energy of
(A) 0
(B) $\frac{3}{2}hf$
(C) 2hf
(D) $\frac{5}{2}hf$
(E) 3hf

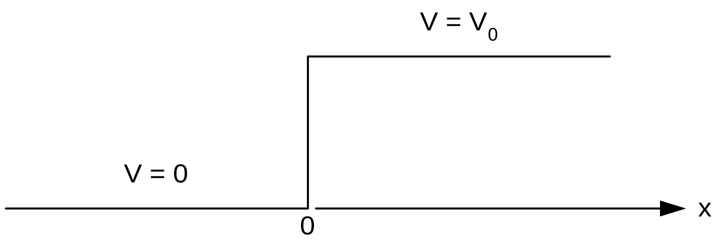
解：由一维谐振子能级公式

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega = \left(n + \frac{1}{2}\right)hf$$

而第三个最低能级对应 n = 2，能量

$$E_2 = \frac{5}{2}hf \text{ 。}$$

选（D）。



$$V(x) = \begin{cases} 0 & \text{for } x < 0 \\ V_0 & \text{for } x > 0 \end{cases}$$

6. The figure above represents a step function in potential energy for electrons moving along the x-direction in a one-dimensional problem. A monochromatic beam of electrons of energy E is incident on the barrier from the left. If $E>V_0$, then which of the following is correct?

(A) At x=0, the sum of the amplitudes of the incident wave and the reflected wave is equal to the amplitude of the transmitted wave.
(B) At x=0, the amplitude of the transmitted wave is zero.
(C) The wave number of the reflected wave is less than that of the incident wave.
(D) The wave number of the reflected wave is equal to that of the transmitted wave.
(E) Electrons pass over the potential barrier without reflection but with reduced speed.

解：设入射平面波为 e^{ikx}，其中 $k = \sqrt{2mE}/\hbar$。在 $x < 0$ 处有入射和反射波，在 $x > 0$ 处仅有透射波。波函数为

$$\psi(x) = \begin{cases} e^{ikx} + \mathrm{R}e^{-ikx}, & x < 0; \\ \mathrm{T}e^{ik'x}, & x > 0 \end{cases}$$

其中 $k' = \sqrt{2m(E - V_0)}/\hbar$。

在 $x = 0$ 处有波函数连续条件

$$1 + R = T \text{ 。}$$

选（A）。

7. If f is frequency and h is Planck’s constant, the zero-point energy of a one-dimensional quantum mechanical harmonic oscillator is

(A) 0

(B) $\frac{1}{3}hf$

(C) $\frac{1}{2}hf$

(D) hf

(E) $\frac{3}{2}hf$

解：由一维谐振子能级公式

$$E_n = \left(n+\frac{1}{2}\right)\hbar\omega = \left(n+\frac{1}{2}\right)hf \text{，}$$

而零点能对应于 n=0，所以

$$E_0 = \frac{1}{2}hf \text{ 。}$$

答案选（C）。

<u>Questions 8-9</u>

Two spin-$\frac{1}{2}$ particles, 1 and 2, have spins in a singlet state with spin wave function

$$\Psi(1,2) = \frac{1}{\sqrt{2}}[\alpha(1)\beta(2) - \alpha(2)\beta(1)],$$

where α and β refer to up and down spins, respectively, along any chosen axis. The spin of particle 1 is measured along the z-axis and found to be up.

8. A simultaneous measurement of the spin of particle 2 along the z-axis would yield which of the following results?

(A) Up with 100% probability

(B) Down with 100% probability

(C) Up with 25% probability, down with 75% probability

(D) Up with 50% probability, down with 50% probability

(E) Up with 75% probability, down with 25% probability

解：对于粒子 2，波函数为$\alpha(2)$（自旋向上）和$\beta(2)$（自旋向下）的等概率叠加。选（D）。

还可以这样看问题。令总自旋$S = S_1 + S_2$，$\psi(1,2)$是（S^2，S_z）的本征态，本征值为（0，0），自旋单态，即总角动量 S 及 z 分量 S_Z 为零。$S_z = s_{1z} + s_{2z}$，所以 s_{2z} 平均值也为零，所以自旋向上向下几率相同。选（D）。

9. A simultaneous measurement of the spin of particle 2 along the x-axis would yield which of the following results

(A) Up with 100% probability

(B) Down with 100% probability

(C) Up with 25% probability, down with 75% probability
(D) Up with 50% probability, down with 50% probability
(E) Up with 75% probability, down with 25% probability
解：由上一问讨论，总角动量为零，则要求角动量的各分量均为零。对于本题，要求

$$S_x = s_{1x} + s_{2x} = 0\text{，}$$

所以 s_{2x} 平均值也应为零，所以自旋 x 分量向上向下的几率应该相同。选（D）。

10. The wave function of a particle is $e^{i(kx-\omega t)}$, where x is position, t is time, and k and ω are positive real numbers. The wave function represents a simultaneous eigenstate of
(A) position and momentum
(B) energy and time
(C) energy and momentum
(D) position and time
(E) energy, momentum, position, and time
解：此为一维平面波，代表一维自由运动粒子。所以能量和动量为守恒量。选(C)。

11. The three operators (L_x, L_y, L_z) for the components of angular momentum commute with the Hamiltonian. Therefore the angular momentum is
(A) equal to zero
(B) equal to energy in magnitude
(C) a unit vector
(D) proportional to sinθ
(E) a constant of motion

解：因为 $[l_\alpha, H] = 0$，其中 α = x, y, z，所以

$$[l_\alpha^2, H] = l_\alpha[l_\alpha, H] + [l_\alpha, H]l_\alpha = 0\text{，}$$
$$[l^2, H] = \sum_\alpha [l_\alpha^2, H] = 0\text{，}$$

角动量为守恒量，或称运动常数。选（E）。

12. Under an exchange of both coordinates and spins, the complete wave function for a system of two electrons must be
(A) antisymmetric
(B) symmetric
(C) additive
(D) incoherent
(E) orthogonal to either independent wave function
解：电子为 Fermion，而 Fermion 要求波函数交换反对称。选(A)。

13. A spinless particle is confined in a cubical box of side L for which the potential is

V = 0, for 0 ≤ x, y, z ≤ L

V = ∞, otherwise.

What is the degeneracy of the third quantum level in the box?

(A) 1

(B) 3

(C) 6

(D) 9

(E) 12

解：为使波函数在盒壁上为零，各方向波矢必须满足

$$k_\alpha L = n\pi,\quad n = 1,2,3...,$$

其中α=x, y, z。因此粒子的能级为

$$E_{lmn} = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2}{2m}\left(k_x{}^2 + k_y{}^2 + k_z{}^2\right) = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2\left(l^2 + m^2 + n^2\right)。$$

第一能级对应（1, m, n）取值为（1, 1, 1），第二能级对应（1, m, n）取值为（1, 1, 2）、（1, 2, 1）、（2, 1, 1），第三能级对应（1, m, n）取值为（1, 2, 2）、（2, 1, 2）、（2, 2, 1）。所以简并度为 3。选（B）。注意这与用周期性边界条件所的情况不同，在那种情况下 n 可以取 0。

14. The hypothesis that an electron possesses spin is qualitatively significant for the explanation of all of the following topics EXCEPT the

(A) structure of the periodic table

(B) specific heat of metals

(C) anomalous Zeeman effect

(D) deflection of a moving electron by a uniform magnetic field

(E) fine structure of atomic spectra

解：（D）涉及的内容为普物甚至是高中课程，显然不需要考虑自旋就可解释。选（D）。（B）比较容易混，关键是自旋的引入使态的数目加倍。

15. Eigenfunctions for a rigid dumbbell rotating about its center have a dependence of the form $\psi(\phi) = Ae^{im\phi}$, where m is a constant. Which of the following values of A will properly normalize the eigenfunction?

(A) $\sqrt{2m}$

(B) 2π

(C) $(2\pi)^2$

(D) $\dfrac{1}{\sqrt{2\pi}}$

(E) $\dfrac{1}{2\pi}$

解：最简单的量子力学题。令

$$\int_0^{2\pi} \psi^* \psi d\phi = \int_0^{2\pi} Ae^{-im\phi} Ae^{im\phi} d\phi = A^2 2\pi = 1,$$

$$A = \frac{1}{\sqrt{2\pi}}。$$

选(D)。

16. The Hamiltonian operator in the Schrödinger equation can be formed from the classical Hamiltonian by substituting

(A) wavelength and frequency for momentum and energy
(B) a differential operator for momentum
(C) transition probability for potential energy
(D) sums over discrete eigenvalues for integrals over continuous variables
(E) Gaussian distributions of observables for exact values

解：只需做如下变换：

$$p \to -i\hbar\nabla,$$

$$\frac{p^2}{2m} \to -\hbar^2\nabla^2。$$

选（B）。

17. If ψ is a normalized solution of the Schrödinger equation and Q is the operator corresponding to a physical observerble x, the quantity ψ*Qψ may be integrated to obtain the

(A) normalization constant for ψ
(B) special overlap of Q with ψ
(C) mean value of x
(D) uncertainty in x
(E) time derivative of x

解：量子力学中，可观测力学量 A 的平均值 $\overline{A} = \int \psi^* \hat{A} \psi d\tau$，其中 $\hat{A}$ 为 A 对应的算符。选（C）。

18. Which of the following is an eigenfunction of the linear momentum operator $-i\hbar\frac{\partial}{\partial x}$ with a positive eigenvalue $\hbar k$; i.e., an eigenfunction that describes a particle that is moving in free space in the direction of positive x with a precise linear momentum?

(A) $\cos kx$
(B) $\sin kx$
(C) e^{-ikx}
(D) e^{ikx}
(E) e^{-kx}

解：自由粒子本征态为量子力学基础知识，选（D）。若不敢肯定，可以现场求导：

$$-i\hbar\frac{\partial}{\partial x}e^{ikx}=-i\hbar\cdot ike^{ikx}=\hbar ke^{ikx}\text{。}$$

19. A system containing two identical particles is described by a wave function of the form

$$\psi=\frac{1}{\sqrt{2}}\left[\psi_{\alpha}(x_1)\psi_{\beta}(x_2)+\psi_{\beta}(x_1)\psi_{\alpha}(x_2)\right]$$

where x_1 and x_2 represent the spatial coordinates of the particles and α and β represent all the quantum numbers, including spin, of the states that they occupy. The particles might be

(A) electrons

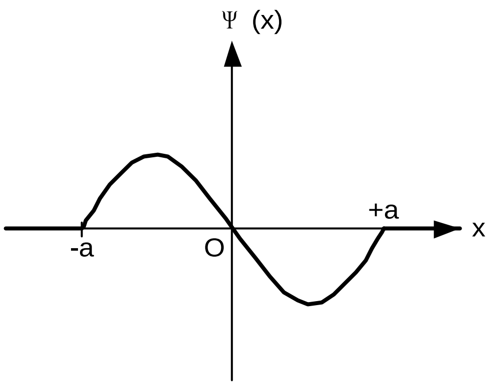
(B) positrons

(C) protons

(D) neutrons

(E) deuterons

解：由交换对称性可知，应为 Boson。(A)、(B)、(C)、(D)分别为电子、正电子、质子、中子，均为典型的 Fermion，交换反对称。(E)为氘核，为 Boson。选（E）。



20. The figure above shows one of the possible energy eigenfunctions ψ(x) for a particle bouncing freely back and forth along the x-axis between impenetrable walls located at x=–a and x=+a. The potential energy equals zero for |x|<a. If the energy of the particle is 2 electron volts when it its in the quantum state associated with this eigenfunction, what is its energy when it is in the quantum state of lowest possible energy?

(A) 0 eV

(B) $\frac{1}{\sqrt{2}}$ eV

(C) $\frac{1}{2}$ eV

(D) 1 eV

(E) 2 eV

解：只有一个节点，说明处于第一激发态，n = 2。由一维无限深势阱的能级公式

$$E_n=\frac{\hbar^2k^2}{2m}=\frac{\hbar^2}{2m}\left(\frac{\pi}{a}\right)^2n^2\text{，}$$

知基态的能量

$$E_0 = \frac{E_1}{4} = \frac{1}{2}eV。$$

选（C）。

21. A particle of mass M is in an infinitely deep square well potential V where

$$V = 0 \text{ for } -a \le x \le a, \text{ and}$$
$$V = \infty \text{ for } x < -a, a < x.$$

A very small perturbing potential V' is superimposed on V such that

$$V' = \varepsilon\left(\frac{a}{2} - |x|\right) \text{ for } \frac{-a}{2} \le x \le \frac{a}{2}, \text{ and}$$
$$V' = 0 \text{ for } x < \frac{-a}{2}, \frac{a}{2} < x.$$

If ψ_0, ψ_1, ψ_2, ψ_3, … are the energy eigenfunctions for a particle in the infinitely deep square well potential, with ψ_0 being the ground state, which of the following statements is correct about the eigenfunction ψ_0' of a particle in the perturbed potential V+V'?

(A) $\psi_0' = a_{00}\psi_0$, $a_{00} \ne 0$

(B) $\psi_0' = \sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} = 0$ for all odd values of n

(C) $\psi_0' = \sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} = 0$ for all even values of n

(D) $\psi_0' = \sum_{n=0}^{\infty} a_{0n}\psi_n$ with $a_{0n} \ne 0$ for all values of n

(E) None of the above

解：V 和 V'均为偶宇称态，则ψ_0'必为偶宇称态。因为一维无限深势阱的全体本征态构成正交完备组，所以ψ_0'可被它们展开。由于第 n 个本征态的宇称为（–）^{n}，所以不应有 n 为奇数的态的成分。选（B）。

<u>Questions 22-23</u>

A particle of charge e and mass m is trapped in a one-dimentional square well of width 2a with impenetrable walls. The walls are located at x = –a and x = a.

22. If the quantum mechanical states are labled by n (n = 1 is the ground state), what is the expectation value of x^2 for very large n ?

(A) 0

(B) $\frac{1}{6}a^2$

(C) $\frac{1}{4}a^2$

(D) $\frac{1}{3}a^2$

(E) $\frac{1}{2}a^2$

解：一维无限深势阱的波函数为 sin 或 cos 函数。由于 n 表示波函数的节点数，当 n 很大时，sin 或 cos 函数的周期很小，以至于各点 ψ 的大小近似相等，即趋于经典情况。则

$$\overline{x^2} = \frac{1}{2a}\int_{-a}^{a} x^2 dx = \frac{1}{3}a^2 \text{。}$$

选（D）。也可进行严格计算，再令 n $\to \infty$，结果相同。

23. A small electric field of strength E_0 is applied in the x direction. What is the change of the energy of the first excited state of this particle due to this electric field? (Assume that the total potential, due to the walls and the electric field, is 0 at x=0.)

(A) $\frac{-eE_0a}{2}$

(B) $\frac{-eE_0a}{3}$

(C) 0

(D) $\frac{eE_0a}{3}$

(E) $\frac{eE_0a}{2}$

解：第一激发态，波函数为

$$\psi_1(x) = \sqrt{\frac{1}{a}}\sin\left(\frac{\pi}{a}x\right) \text{。}$$

电场所产生的微扰可表示为 $H' = -E_0ex$ 。由于微扰 Hamilton 函数为奇函数，

$$E^1 = <\psi_1 | H' | \psi_1> = -E_0 e\int_{-a}^{a} |\psi_1(x)|^2 xdx = 0 \text{。}$$

一级微扰为 0。选（C）。

25. A particle of mass M is infinitely deep quare well potential V where

V = ∞ for x<–a, a<x

V = 0 for –a≤x≤a

What is the eigenfunctions for the two lowest eigenstates?

(A) $\frac{1}{\sqrt{a}}\sin\frac{\pi x}{2a}, \frac{1}{\sqrt{a}}\cos\frac{\pi x}{2a}$

(B) $\frac{1}{\sqrt{a}}\sin\frac{\pi x}{a}, \frac{1}{\sqrt{a}}\cos\frac{2\pi x}{a}$

(C) $\frac{1}{\sqrt{a}}\cos\frac{\pi x}{2a}, \frac{1}{\sqrt{a}}\sin\frac{\pi x}{a}$

(D) $\frac{1}{\sqrt{a}}\cos\frac{\pi x}{a}, \frac{1}{\sqrt{a}}\sin\frac{2\pi x}{a}$

(E) $\frac{1}{\sqrt{a}}\cos\frac{2\pi x}{a}, \frac{1}{\sqrt{a}}\sin\frac{4\pi x}{a}$

解：由 Schrödinger 方程

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi \text{ ,}$$

由于$-a \leqslant x \leqslant a$ 时，$V = 0$。

$$\nabla^2\psi + \frac{2mE}{\hbar^2}\psi = 0 \text{ 。}$$

解为

$$\psi = \sin kx \text{ , } \cos kx \text{ , 其中 } k = \sqrt{\frac{2mE}{\hbar^2}} \text{ 。}$$

由波函数在 $x = \pm a$ 处的连续条件，

$$\psi(\pm a) = 0 \text{ ,}$$

得当$|x| < a$ 时波函数为

$$\psi(x) = \begin{cases} \sqrt{\frac{1}{a}}\cos\left(\frac{n\pi x}{2a}\right), & n = 1,3,5,\ldots \\ \sqrt{\frac{1}{a}}\sin\left(\frac{n\pi x}{2a}\right), & n = 2,4,6,\ldots \end{cases} \text{ 。}$$

选（C）。

26. A particle of energy E moving in one dimension is scattering by a potential barrier of height V_0 ($V_0 > E$) and width L. If $q = \sqrt{8\pi^2 m|E - y_0|/h^2}$, A is constant, and aL >> 1, the transmission coefficient is best approximated by

(A) Ae^{-2qL}

(B) Ae^{2qL}

(C) A sinh qL

(D) A sin qL

(E) $A/[1+(qL)^2]$

解：详细的推导可参阅曾谨言《量子力学导论》，第二版，63 页，北京大学出版社。考试现场显然没时间去推公式，可定性考虑。显然势垒宽度越大，L 越大，透射系数越小。而（B）、（C）随 L 增大，（D）随 L 成振荡关系，均被排除。对于（A）、（E），联想到 α衰变中也发生势垒贯穿，而半衰期随能量减小而迅速变长，透射系数与粒子能量（由 q 代表）的关系应为指数形式。选（A）。

27. A one-dimensional beam of particles each of kinetic energy E travels along the x-axis from left to right. It encounters a potential energy step of height E_0, with $E > E_0$.What is the reflection probability?

(A) $(\frac{E}{E_0})^2$

(B) $\frac{E_0 - \sqrt{E(E-E_0)}}{E_0 + \sqrt{E(E-E_0)}}$

(C) $\frac{2E - E_0 - \sqrt{E(E-E_0)}}{2E - E_0 + \sqrt{E(E-E_0)}}$

(D) $\frac{2E + E_0 - \sqrt{E(E-E_0)}}{2E + E_0 + \sqrt{E(E-E_0)}}$

(E) $\frac{2E - E_0 - 2\sqrt{E(E-E_0)}}{2E - E_0 + 2\sqrt{E(E-E_0)}}$

解：详细的推导可参阅《量子力学导论》，第二版，63 页，北京大学出版社。考试现场可用极限法挑出正确答案。当 $E_0 \to 0$ 时，相当于没势垒，反射几率为 0。几个选项中只有（E）符合此极限。选（E）。

28. A free particle moving in one dimension line has following wave function at time = 0:

$$\psi(X) = d^{-\frac{1}{2}} e^{-|x|/d}$$, where d is a constant.

What is the probability that a measurement of the position of the particle at time = 0 will yield a result between x_1 and x_2 ($x_2 > x_1 > 0$)?

(A) 0

(B) $(x_2 - x_1) / d$

(C) $(x_2 - x_1)^2 / d^2$

(D) $\frac{1}{2}(e^{-x_1/d} - e^{-x_2/d})$

(E) $\frac{1}{2}(e^{-2x_1/d} - e^{-2x_2/d})$

解：几率积分为

$$p(x_2 > x > x_1) = \int_{x_1}^{x_2} |\Psi(x)|^2 dx = \frac{1}{2}\left(-e^{-2x/d}\right)\Big|_{x_1}^{x_2} 。$$

选（E）。

29. Two particles have angular-momentum quantum numbers $l_1 = l_2 = 4$, $m_1 = 3$, and $m_2 = 2$. Which of the following is an allowed value for l corresponding to $L = L_1 + L_2$?

(A) 0
(B) 1
(C) 2
(D) 4
(E) 6

解：根据矢量合成的原理，合角动量的投影

$$m = m_1 + m_2 = 5，$$

而合角动量的大小显然大于它的投影值，l > m = 5。选（E）。

30. A particle of energy E moving in one dimension is scattered by a potential barriers of height $V_0(V_0 > E)$ and width L. If $q = \sqrt{8\pi^2 m|E - V_0|/h^2}$. A is a constant and qL >> 1, the transmission coefficient is best approximated by

(A) Ae^{-2qL}
(B) Ae^{2qL}
(C) A sinh qL
(D) A sin qL
(E) $A / [1+(qL)^2]$

解：一维势垒穿透的隧道效应。在 qL >> 1 的条件下，shqL >> 1，透射系数

$$T \approx \frac{16E(V_0 - E)}{V_0} e^{-2qL} \propto e^{-2qL} 。$$

选（A）。

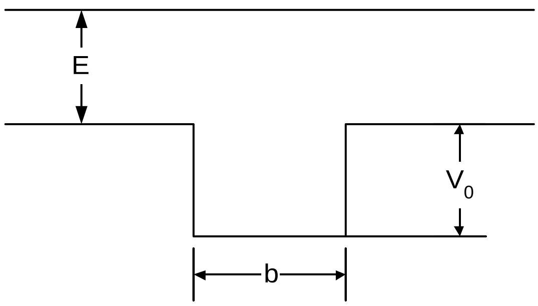
31. The wave function for the lowest energy level of hydrogen is $\Psi = \frac{e^{-r/a_0}}{\sqrt{\pi a_o{}^{3/2}}}$, where a_0 is the Bohr radius. The expectation value $\left\langle \frac{1}{r} \right\rangle$ of reciprocal distance of the electron from the nucleus is

(A) $1 / a_0$
(B) $1.5 / a_0$
(C) $2 / a_0$
(D) a_0
(E) ∞

解：直接积分求平均值

$$\left\langle \frac{1}{r} \right\rangle = \int_0^\infty \frac{1}{r} \Psi^2 \cdot 4\pi r^2 dr = \frac{1}{a_0}$$ 。

选（A）。



32. A wave packet of electrons having a mean energy E is incident from the left on the one-dimensional square well of depth V_0 and width b shown above. Unusually high transmission of the electrons (analogous to the Ramsauer-Townsend effect) will take place when the energy of the electrons most nearly satisfies which of the following conditions?

(A) $\frac{h}{\sqrt{2M(E+V_0)}} = 2b$

(B) $\frac{h}{\sqrt{2M(E+V_0)}} = 4b$

(C) $\frac{h}{\sqrt{2M(E+V_0)}} = 8b$

(D) $\frac{h}{\sqrt{2ME}} = 4b$

(E) $\frac{h}{\sqrt{2ME}} = 2b$

解：形成共振透射（T = 1）的条件是

$$k'b = n\pi ,$$

$$k' = \sqrt{\frac{2m\left(E+V_0\right)}{\hbar^2}}$$ 。

上式中当 n = 1 时即是选项（A）。选（A）。

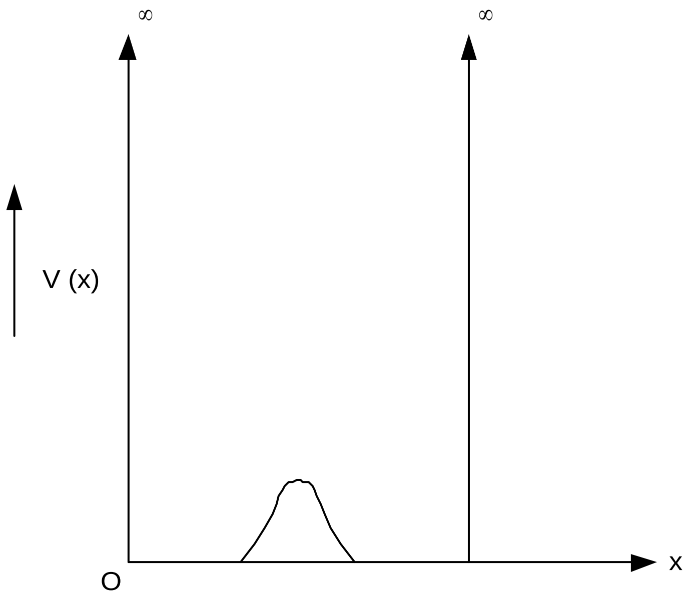
33. The Pauli exclusion principle results from the quantum mechanical fact that no two electrons in an atom can

(A) have the same set of quantum numbers

(B) have the same spatial wave function

(C) have the same spin

(D) interact with each other
(E) be in an excited state simultaneously
解：Pauli 不相容原理：不可能由两个全同的 Fermi 子处于同一个单粒子态。而电子属于 Fermion。选（B）。



34. A one-dimensional square well potential with infinitely high sides is shown above. In the lowest energy state, the wave function is proportional to sin kx. If the potential is altered slightly by introducing a small bulge in the middle as shown, which of the following is true of the ground state?
(A) The energy of the ground state remains unchanged.
(B) The energy of the ground state is increased.
(C) The energy of the ground state is decreased.
(D) The original ground state splits into two states of lower energy.
(E) The original ground state splits into two states of higher energy.
解：微扰的 Hamilton 量始终为正，在以及近似下的修正能量$<\Psi_0'|H'|\Psi>$显然为正，因此基态能级上升。选（B）。

35. X and Y are two stationary states of a particle in a spherically symmetric potential. In which of the following situations will the wave functions of the two states be orthogonal?
I. X and Y correspond to different energies.
II. X and Y correspond to different total orbital angular momenta L.
III. X and Y correspond to the same L but different L_z.
(A) Not neccesarily in any of these situations.
(B) In situation I, but not neccesarily in II or III.
(C) In situation I and II, but not neccesarily in III.
(D) In situation II and III, but not neccesarily in I.
(E) In all three situations.
解：在球对称势场的问题中，不考虑自旋的情况下，完备的量子数为（n, l , l_z)，解为三者的共同本征函数。对于三者中任何不完全相同的两个量子态，都是正交的。选（E）。

36. A particle of mass m moves in a thress-dimensional potential $V(r) = \frac{1}{2}kr^2$. If k is halved, what is the ratio of the new ground-state energy to that of the old ground-state energy?

(A) $1/\sqrt{6}$

(B) 1/2

(C) $1/\sqrt{2}$

(D) $\sqrt{2}$

(E) $\sqrt{6}$

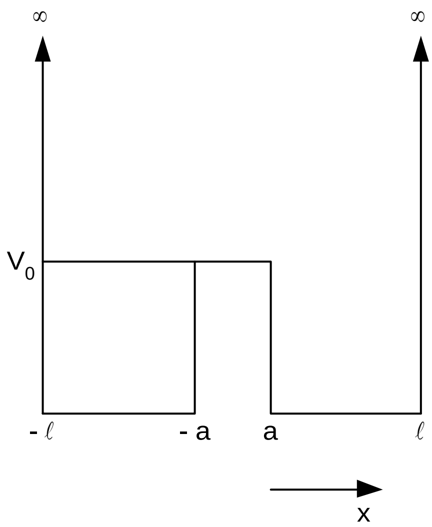
解：一维谐振子的零点能

$$E_0 = \frac{1}{2}\hbar\omega = \frac{\hbar}{2}\sqrt{\frac{k}{\mu}}\text{。}$$

因此前后零点能之比

$$\frac{E_0'}{E_o} = \sqrt{\frac{k_0'}{k_0}} = \frac{1}{\sqrt{2}}\text{。}$$

选（C）。



37. A spinless nonrelativistic particle of mass m is placed in the divided suqare well shoqn above. The potential rises to infinity at x = ± l. Which of the following is NOT true for states ψ(x) of definite energy E?

(A) If $V_0 \to \infty$, there are two states for each allowed energy.

(B) If $V_0 \to 0$, the allowed energies satisfy $\frac{2l}{\pi}\sqrt{\frac{2mE}{\mathrm{h}^2}}$ = integer.

(C) For a general V_0, solutions of definite E have a definite reflection symmetry. ψ(x) = ψ(–x) or ψ(x) = – ψ(–x).
(D) ψ(l) = 0 = ψ(–l).
(E) ψ(a) = 0 = ψ(–a).
解：对一般的 V_0，在 x = ±a 处波函数只需满足连续性及导数连续的边条件。当 $V_0 \to 0$ 时，相当于一个无穷高势垒中的束缚态。而当 $V_0 \to \infty$时，相当于彼此隔断的两个无穷高势垒，分别对应各自的束缚态。选（E）。

38. If S is the total spin quantum number, which of the following lists all possible spin states for three electrons?
(A) Two S = 1/2 doublets and one S = 1 triplet
(B) Two S = 0 singlets, one S = 1/2 doublet, and one S = 3/2 quartet
(C) One S = 0 singlet, one S = 1 triplet, and one S = 3/2 quartet
(D) Two S = 1/2 doublets and one S = 3/2 quartet
(E) Four S = 1/2 doublets
解：三个电子总共可能的自旋取法为：
1、两个同向，一个反向，S = 1/2，为双重态；
2、三个都同向，S = 3/2，为四重态。
选（D）。

39. A quantum system has two eigenstates : $\Psi_1(x)$ with energy E_1 and $\Psi_2(x)$ with energy E_2. These are normalized and orthogonal: that is,

$$\int_{-\infty}^{\infty}\Psi_1^*\Psi_1 dx = 1 = \int_{-\infty}^{\infty}\Psi_2^*\Psi_2 dx$$

and

$$0 = \int_{-\infty}^{\infty}\Psi_1^*\Psi_2 dx$$

If at t = 0 the system was in the state

$$\Psi(x) = \frac{1}{\sqrt{2}}[\Psi_1(x) + \Psi_2(x)],$$

the probability of finding it in this same state $\Psi(x)$ at a later time t is

(A) zero
(B) 1
(C) $\sin(E_1 - E_2)t/h$
(D) $\frac{1}{2}[e^{-iE_1t/h} + e^{-iE_2t/h}]$
(E) $\frac{1}{2}[1+\cos(E_1 - E_2)t/h]$

解：体系的随时间演化的波函数为

$$\Psi(x,t)=\frac{1}{\sqrt{2}}\left(\Psi_1(x)e^{-iE_1t/\hbar}+\Psi_2(x)e^{-iE_2t/\hbar}\right)。$$

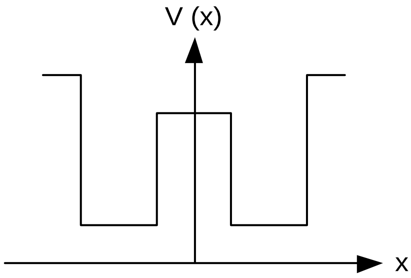
t 时刻粒子处于ψ(x, 0)态的几率是

$$p=\int_{-\infty}^{\infty}\Psi(x,0)^*\Psi(x,t)=\frac{1}{2}[e^{-iE_1t/h}+e^{-iE_2t/h}]。$$

选（D）。

Questions 40-41

The sketch below shows a one-dimensional potential for an electron. The potential is symmetric about the V-axis.



40. Which of the following statements correctly describes the ground state of the system with on electron present?
(A) A single electron must be localized in one well.
(B) The ground state will accommodate up to four electrons.
(C) The kinetic energy of the ground state will be one-half its potential energy.
(D) The wave function of the ground state will be antisymmetric with respect to the V-axis.
(E) The wave function of the ground state will be symmetric with respect to the V-axis.
解：因为势垒关于 y 轴对称，

$$V(x)=V(-x),$$

所以波函数有确定的宇称。由于基态波函数与 x 轴没有节点，所以波可能是反对称性的，因为那要求

$$\psi(-x)=-\psi(x),$$

从而左右两边波函数反号，必与 x 轴有节点。所以是关于 y 轴对称的。选（E）。（A）不对，电子甚至还有几率穿透势垒，不会被束缚在一边（或者说由于波函数对称）。（B）不对，由于基态波函数不变号，所以它没有简并态，否则两个简并态波函数之间不可能正交（因为要末乘积始终为正，要末始终为负）。所以只有一个基态波函数，最多能容纳两个自旋相反的电子。

41. A second electron is now added to the system. If the electrons do not interact, which of the following statements is correct?
(A) The second electron must be localized in the well not previously occupied.
(B) In the ground state of the system, each of the two electrons will have the same spatial wave

function.

(C) In the ground state of the system, one electron will be in a spatially symmetric state and one will be in a spatially antisymmetric state.

(D) The second electron will not be bound.

(E) Pair annihilation will occur.

解：如上一问中的分析，基态能级不简并，只有一个基态波函数，所以两个电子必须有相同的空间波函数。但只要二者自旋相反，并不违反 Pauli 不相容原理。答案选（B）。

42. An energy level of a certain isolated atom is split into three components by the hyperfine interaction coupling of the electronic and nuclear angular momenta. The quantum number j, specifying the magnitude of the total electronic angular momentum for the level, has the value j = 3/2. The quantum number i, specifying the magnitude of the nuclear angular momentum, must have the value

(A) 1/2

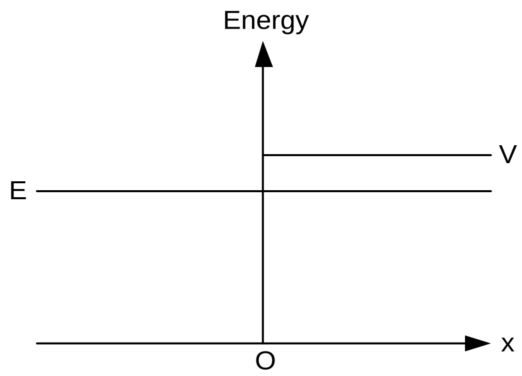
(B) 1

(C) 3/2

(D) 2

(E) 3

解：两个角动量耦合，设其角量子数分别为 p_1、p_2，其中 $p_1 < p_2$，则总角动量量子数 $p = p_1 + p_2$，$p_1 + p_2 - 1$，…，$p_2 - p_1$，共有 $2p_1$ 种可能取值。本题中为 3 分量，总角动量有 3 种取值可能。所以电子角动量 j 和核角动量 i 中较小的一个应为应为 1。而 j = 3/2，所以 i = 1。选（B）。



43. An electron with energy E and momentum $k\hbar$ is incident from the left on a potential step of height V > E at x = 0. For x > 0, the space part of the electron's wave function has the form

(A) e^{ikx}

(B) $e^{-ik'x}$; k' < k

(C) $e^{-\alpha x}$, where α is real and positive

(D) sin kx

(E) identically zero

解：由 Schrödinger 方程

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi\text{，}$$

在 x > 0 处， 因为 V > E，所以

$$\frac{d^2}{dx^2}\psi - \frac{2m}{\hbar^2}(V-E)\psi = 0\text{。}$$

方程的解为

$$\psi = Ae^{-\alpha x}\text{，}$$

其中

$$\alpha = \sqrt{\frac{2m(V-E)}{\hbar^2}}\text{。}$$

选（C）。

44. In a ^3S state of the helium of the possible values of the total electronic angular momentum are

(A) 0 only

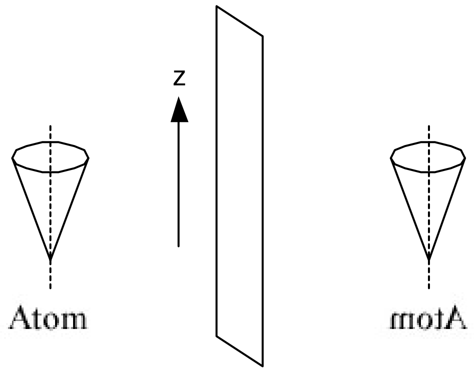
(B) 1 only

(C) 0 and 1 only

(D) 0, 1/2, and 1

(E) 0, 1, and 2

解：S=1，L=0，故 J=1。选（B）。



45. An atom is in a state for which J = 1, m_J = 1 relative to a z axis which is vertical and points upward as shown above. If the atom is observed in a vertical mirror, the image will look like an atom for which m_J

(A) has the value –1

(B) has the value 0

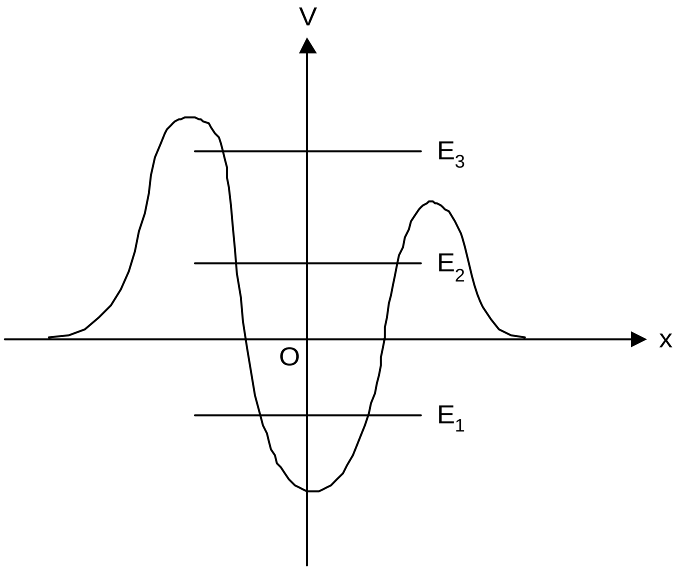
(C) has the value +1

(D) has a well-defined value but not one of the above

(E) does not have a well-defined value

解：宇称变换下角动量反号，m_J 变成–1。选（A）。

Question 46-47 refer to the one-dimensional potential energy function shown below.



46. According to quantum mechanics, which of the energies shown represents a possible permanently bound state for a particle?

(A) E_1 only

(B) E_2 only

(C) E_3 only

(D) E_1 or E_2

(E) E_2 or E_3

解：束缚态的总能量 $E < 0$。图中只有 E_1 满足条件且恰处于势阱的底部。选（A）。

47. Which of the energies shown represents a possible metastable state from which an initially confined particle would eventually tunnel out

(A) E_1 only

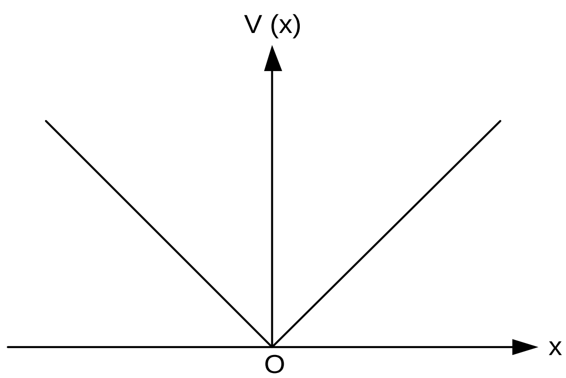
(B) E_2 only

(C) E_3 only

(D) E_1 or E_2

(E) E_2 or E_3

解：E_3 显然粒子不被束缚，可以自由运动到 x 正轴的无穷远处。E_2 是亚稳态，根据量子力学的隧道效应，粒子在其中振动，最终能透射势阱自由运动。选（B）。



48. The graph above shows the potential energy as a function of position x for an infinite triangular well. Which of the following is true of the bound state energy levels of this well?
(A) They are equally spaced for all energies.
(B) They are more closely spaced at higher energies.
(C) They are less closely spaced at higher energies.
(D) They are randomly spaced.
(E) There are no bound state energy levels.
解：线性势的能级越高越密，具体解可参照曾谨言《量子力学》第二版，3.7 节。选（B）。

49. A particle moves in a potential
$V(x) = 0$, for $x < -L$.
$V(x) = -V_0$, for $-L < x < L$.
$V(x) = 0$, for $x > L$.
If the potential between –L and L is shown to $-5V_0$ and the total energy of the particle is negative ($E < 0$), the number of bound states must
(A) stay the same or increase
(B) stay the same or decrease
(C) stay the same
(D) equal to zero
(E) increase
解：一维有限深势阱至少存在一个束缚态。如果 V_0 特别小，势阱中只有一个束缚态，并且深度变为 $5V_0$ 后势阱中仍只有一个束缚态，这种情况下束缚态数目不变。更一般的情况随势阱深度加大束缚态数目增多。选（A）。

第六节 原子物理

1. The energy levels of the hydrogen atom are given in terms of the principal quantum number n and a positive constant A by the expression

(A) $A\left(n+\frac{1}{2}\right)$

(B) $A(1-n^2)$

(C) $A\left(-\frac{1}{4}+\frac{1}{n^2}\right)$

(D) An^2

(E) $-\frac{A}{n^2}$

解：选（E）。类氢原子能级公式是一定要记住的结论。

2. Which of the following values of the atomic number Z corresponds to an atom with valence

+2 ?
(A) 6
(B) 12
(C) 14
(D) 18
(E) 32

解：原子序数为 12 的原子的电子排布式为$1s^2 2s^2 2p^6 3s^2$，当把最外层两个电子失去后为正二价。选（B）。惰性元素的原子序数为 2，10，18，…，所以价态为正二价的原子为 4，12，20，38，…。

3. A muon is a “heavy electron” with a mass about 200 times of a hydrogen atom is E_0, the binding energy of a muon-proton atom is most nearly
(A) E_0
(B) 14 E_0
(C) 200 E_0
(D) 4000 E_0
(E) 40,000 E_0
解：μ子与质子组成原子时，μ子的折合质量为

$$\mu = \frac{M_p m_\mu}{M_p + m_\mu} = \frac{1840 m_e \times 208 m_e}{1840 m_e + 208 m_e} = 187 m_e,$$

而氢原子中电子的折合质量近似为 m_e，且氢原子能级的大小与折合质量成正比。选(C).。

4. The anomalous Zeeman effect results from
(A) a nonuniform applied magnetic field
(B) more than one isotope being present
(C) a magnetic moment arising from electron spin
(D) a nuclear quadrupole moment
(E) forbidden atomic transitions
解：反常 Zeeman 效应是电子有自旋的实验证据之一。由于轨道角动量是$\hbar$的整数倍，因此必须承认电子有$\hbar/2$ 的自旋才能解释谱线分离为偶数条。(A)所说内容发生在 Stern-Gerlach 实验中，也是 SUB 考试的重点之一。

5. In a Compton scattering experiment, a collimated beam of monochromatic x-rays of wavelength 0.7 angstrom impinges on a small cylindrical carbon scatter. The scattered radiation is investigated as a function of the angle between it and the primary beam. The scattered radiation contains
(A) no wavelength other than that of the primary beam
(B) a component shifted in wavelength by an amount that is independent of the wavelength of the primary beam
(C) a component shifted in wavelength by an amount that is independent of the scattering angle
(D) a component shifted in wavelength by an amount that decreases as the scattering angle increases
(E) two components with wavelength shifted up and down by equal amounts

解：由 Compton 散射公式

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta),$$

可见波长的变化量与入射波的波长无关，只是角度的函数。答案选(B)。(D)不对，θ 增大时（从 0°到 180°）cosθ 减小，Δλ变大。其实θ 越大表明碰撞越剧烈，能量损失越大，Δλ越大。这里还需要注意一下 Compton 散射公式的适用条件：入射光子能量比较大，波长在 X 射线范围以下。

6. Stern and Gerlach succeeded in deflecting a beam of silver atoms with an inhomogeneous magnetic field. Which of the following is a generally accepted inference from the results of their measurements?
(A) The technique is useful for precise measurement of magnetic field intensities.
(B) The single deflection observed is principally due to induced magnetic moment in the silver atom.
(C) The two deflections observed are due to the two possible measured values of a component of the magnetic moment of the atom.
(D) The three deflections observed are due to neutral atoms, negatively charged ions, and positively charged ions.
(E) The continuous range of values observed is due to random orientation of the magnetic moments of the atoms.

解：Stern-Gerlach 实验是重点，要好好复习一下。Stern-Gerlach 实验中原子束偏转后分裂为几条线（而不是连续的一片，答案（E）不对），表明原子角动量只取几个特定的方向。如果原子只有轨道角动量，则条纹数为 2l+1，但实验观测中会得到偶数条条纹，说明原子中的电子不仅有轨道角动量，还有其他形式的角动量，即自旋。(B)不对，条纹数为 1 说明原子没有固有磁矩。(D)不对，实验中用的是中性原子，产生偏折由其内禀性质决定。正确答案选（C）。

7. The natural line width of an atomic transistor with a mean lifetime of 5×10^{-9} second is most nearly
(A) 4.1×10^{-15} eV
(B) 1.3×10^{-11} eV
(C) 1.3×10^{-7} eV
(D) 4.1×10^{-2} eV
(E) 13.6 eV

解：由能量—时间测不准关系

$$\Delta E \cdot \tau \geq \frac{\hbar}{2},$$

作为估算，取 $\Delta E \cdot \tau = \hbar$ 。选（C）。

8. The ratio of the spin of the proton to the spin of the electron is most nearly
(A) 1840
(B) 650
(C) 1

(D) 1/650
(E) 1/1840

解：质子和电子的自旋均为$\frac{\hbar}{2}$，答案选（C）。注意自旋为粒子的内禀性质，不同于宏观物体的角动量，是量子化的，且与质量无关。其他几个选项显然为迷惑选项。

9. The ground states of the helium, neon, and argon atoms are all
(A) $^1S_0$
(B) $^2S_{\frac{1}{2}}$
(C) $^3S_1$
(D) $^1P_1$
(E) $^2P_{\frac{1}{2}}$

解：氦、氖、氩为惰性元素，均为满壳层结构，总角量子数 L、总自旋量子数 S 均为 0，从而 J 也为 0，所以基态（$^{2S+1}L_J$）为 $^1S_0$。选（A）。

10. What are the changes in the mass number A and atomic number Z of a nucleus that undergoes electron capture?
(A) A is unchanged; Z decreases by 2.
(B) A is unchanged; Z decreases by 1.
(C) A is unchanged; Z increases by 1.
(D) A decreases by 2; Z increases by 1.
(E) A decreases by 4; Z decreases by 2.

解：电子质量很小，跟原子核相比可忽略，其电量为–1。原子核捕获一个电子后，使一个质子变为中子，质量数不变而原子序数减 1。选(B)。

11. An element that has an atomic energy level diagram similar to lithium (Z=3) has an atomic number equal to
(A) 9
(B) 11
(C) 13
(D) 15
(E) 17

解：原子能级以及原子的许多其他性质均由其最外层电子决定。Li 的电子排布式为$1s^2 2s^1$，而 Na（Z = 11）的电子排布式为$1s^2 2s^2 2p^6 3s^1$，最外层均为一个 s 电子，能级相近。选（B）。

12. The internuclear distance in Angstroms in the hydrogen molecule is most nearly
(A) 0.01 A
(B) 0.1 A
(C) 1 A

(D) 10 A
(E) 100 A
解：两原子间的精确距离为 0.74A。考试现场只要想一想 Bohr 半径约为 0.5A，而两原子之间距离为与这个值数量级相同。选(C)。

13. Which of the following states is possible for an atom with a closed core plus one d-electron (l=2)?

(A) $^3D_{\frac{5}{2}}$

(B) $^4D_{\frac{3}{2}}$

(C) $^2D_{\frac{1}{2}}$

(D) $^2D_{\frac{5}{2}}$

(E) $^3D_{\frac{1}{2}}$

解：原子基态由最外层电子决定。本题中原子只有一个 d 电子，l=2，所以总角量子数 L = 2，总自旋量子数 S = 1/2。J = L+S, L+S–1, …, | L–S |，可取值为 5/2，3/2。因此原子基态可表示为 $^2D_{\frac{5}{2}}$ 或 $^2D_{\frac{3}{2}}$。选（D）。

14. If hyperfine structure is ignored, an atomic state $^2P_{3/2}$ in a weak external magnetic field will be split into a number of states equal to
(A) 2
(B) 3
(C) 4
(D) 5
(E) 6
解：当外加磁场较弱时，为反常 Zeeman 效应，原子将分裂为 2J+1 个态；当外加磁场较强时，为正常 Zeeman 效应（以及 Paschen-Bark 效应），谱线一般分裂为 3 条。本题外加弱磁场，2×3/2+1=4。选（C）。

15. If an electron is in a 4f state, the magnitude of its orbital angular momentum is

(A) $\sqrt{2}\hbar$

(B) $\sqrt{3}\hbar$

(C) $\sqrt{6}\hbar$

(D) $2\sqrt{3}\hbar$

(E) $4\sqrt{5}\hbar$

解：电子处于 4f 态，4 表示其所处能级，f 代表其角动量量子数为 3。所以角动量平方为

$$l^2 = 3(3+1)\hbar^2,$$

$$l = 2\sqrt{3}\hbar \text{。}$$

选(D)。

16. In Compton scattering of a beam of 500-keV x-rays by free electrons initially at rest, which of the following is true of the scattered x-rays?

(A) They are mostly back-scattered.

(B) They are scattered isotropically.

(C) They have less than 500 keV energy.

(D) Thy show a left-right asymmetry.

(E) They are all circularly polarized.

解：由于光子把一部分能量传递给自由静止电子，自身能量下降。或由 Compton 公式

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta) \text{。}$$

可见波长变长，能量降低。选（C）。

17. If a_0 is the radius of the first Bohr orbit in hydrogen, then the radius of the first Bohr orbit of doubly ionized lithium (Z = 3) is

(A) $\frac{a_0}{3}$

(B) $\frac{a_0}{\sqrt{3}}$

(C) a_0

(D) $\sqrt{3}a_0$

(E) $3a_0$

解：类氢原子的基态玻尔半径为

$$a = \frac{\hbar^2}{Z\mu e^2} \text{。}$$

因为电子的折合质量近似不变，所以

$$a = \frac{1}{Z}a_0,$$

选（A）。其实由于核电荷数的增加，类氢原子将电子束缚得更紧，所以基态半径将会变小。

18. A nickel target (Z=28) is bombarded with fast electrons. The minimum electron kinetic energy needed to produce x-rays in the K series is most nearly

(A) 10 eV

(B) 100 eV
(C) 1000 eV
(D) 10,000 eV
(E) 100,000 eV
解：考场上作为估算，对于 K 线系，可用氢原子能级公式估算，

$$E = -\frac{\mu e^4}{2\hbar}\frac{Z^2}{n^2} \approx -13.6\frac{Z^2}{n^2}\ eV\ 。$$

取 Z=28，n=1，约为 10,000eV。选（D）。严格计算可用 Morsley 公式：

$$v_K = 0.248\times 10^{16}(Z-b)^2,\ \ b\sim 1。$$

19. The configuration of the potassium atom in its ground state is $1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$. Which of the following statements about potassium is true?
(A) Its n=3 shell is completely filled.
(B) Its 4s subshell is completely filled.
(C) Its least tightly bound electron has l=4.
(D) Its atomic number is 17.
(E) Its electron charge distribution is spherically symmetrical.
解：（A）不对，还有 3d 轨道。（B）不对，还差一个才满。（C）不对，最外层电子 n=4，角动量由 s 表示为 l=0。（D）不对，显然为 19。（E）正确，因为 s 轨道为球对称，而 p 轨道虽不球对称，但由于被全部占满，所以电荷分布仍为球对称。选（E）。

20. Photons of wavelength λ scatter elastically on free protons initially at rest. The wavelength of the photons scattered at 90° is increased by
(A) λ/137
(B) λ/1836
(C) $h/m_e c$, where h is Planck's constant, m_e the rest mass of an electron, and c the speed of light
(D) $h/m_p c$, where h is Planck's constant, m_p the rest mass of an proton, and c the speed of light
(E) zero
解：可以看作广义的 Compton 散射，

$$\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)\ ,$$

m 为散射物的静止质量。对于本题，θ = 90°，

$$\lambda' - \lambda = \frac{h}{m_p c}\ ,$$

选（D）。请牢记 Compton 散射实验及其公式，否则后果自负。

21. A blackbody at temperature T_1 radiates energy at a power level of 10 milliwatts (mW). The same blackbody, when at a temperature $2T_1$, radiates energy at a power level of
(A) 10 mW
(B) 20 mW
(C) 40 mW

(D) 80 mW

(E) 160 mW

解：由 Stefan 定律，黑体的总辐射能力和它的绝对温度的四次方成正比，

$$P = \sigma T^4 \text{。}$$

选（E）。

22. The Franck-Hertz experiment and related scattering experiments show that

(A) electrons are always scattered elastically from atoms

(B) electrons are never scattered elastically from atoms

(C) electrons of a certain energy range can be scattered in elastically, and the energy lost by electrons is discrete

(D) electrons always lose the same energy when they are scattered inelastically

(E) there is no energy range in which the energy lost by electrons varies continuously

解：即使不知道 Franck-Hertz 试验的具体内容，也可发现选项中只有（C）的叙述是正确的。（A）、(B)、(D)显然不对；（E）选项，当电子能量较低时，无法触及原子内部，此时为连续散射谱。

23. A transition in which one photon is radiated by the electron in a hydrogen atom when the electron's wave function changes from ψ_1 to ψ_2 is forbidden if ψ_1 and ψ_2

(A) have opposite parity

(B) are orthogonal to each other

(C) are zero at the center of the atomic nucleus

(D) are both spherically symmetrical

(E) are associated with different angular momenta

解：由ψ_1和ψ_2 为球对称可知，二者角量子数均为 l=0，而跃迁选择定则要求跃迁前后Δ l= ±1，所以无法跃迁。选（D）。

24. The emission spectrum of an atomic gas in a magnetic field differs from that of the gas in the absence of a magnetic field. Which of the following is true of the phenomenon?

(A) It is called the Stern-Gerlach effect.

(B) It is called the Stark effect.

(C) It is due primarily to the nuclear magnetic moment of the atoms.

(D) The number of emission lines observed for the gas in a magnetic field is always twice the number observed in the absence of a magnetic field.

(E) The number of emission lines observed for the gas in a magnetic field is either greater than or equal to the number observed in the absence of a magnetic field.

解：为 Zeeman 或反常 Zeeman 效应，由于磁场的作用使能级简并解除，产生谱线分裂。但正常 Zeeman 效应一般为三分裂，反常 Zeeman 分裂可以是更多条。故答案(D)不对。选（E）。选项（B）Stark 效应也可产生谱线分裂，但是是由电场造成的。

25. A spectral line is produced by a gas that is sufficiently dense that the mean time between atomic collisions is much shorter than the mean lives of the atomic states responsible for the line. Compared with the same line produced by a low-density gas, the line produced by the

higher-density gas will appear

(A) the same

(B) more highly polarized

(C) broader

(D) shifted toward the blue end of the spectrum

(E) split into a doublet

解：由能量—时间测不准关系，

$$\Delta E \cdot \tau \geq \frac{\hbar}{2},$$

能级的寿命越长，则其展宽越大。在浓度较大的气体中，由于碰撞比较频繁，能级寿命短，从而谱线宽度大。选（C）。

26. Sodium has eleven electrons and the sequence in which energy levels fill in atoms is 1s, 2s, 2p, 3s, 3p, 4s, 3d, etc. What is the ground state of sodium in the usual notation $^{2S+1}L_J$?

(A) $^1S_0$

(B) $^2S_{\frac{1}{2}}$

(C) $^1P_0$

(D) $^2P_{\frac{1}{2}}$

(E) $^3P_{\frac{1}{2}}$

解：核外一共 11 个电子，其电子排布式为 $1s^2 2s^2 2p^6 3s^1$。总自旋角动量 S 和总轨道角动量 L 由最外层 3s 电子决定。S = 1/2，L = 0，J = 1/2，所以基态为 $^2S_{\frac{1}{2}}$。选（B）。

27. Which of the following is NOT compatible width the selection rule that controls electric dipole emission of photons by excited states of atoms?

(A) Δn may have any negative integral value.

(B) $\Delta l=\pm1$

(C) $\Delta m_l=0, \pm1$

(D) $\Delta s=\pm1$

(E) $\Delta j=\pm1$

解：电偶极辐射的跃迁定则为：1.宇称改变；2. $\Delta l=\pm1$；3. $\Delta j=\pm1$, $\Delta m_l=0, \pm1$。关键是光子由角动量，但没有自旋，所以 s 的值无法改变。

28. The positronium “atom” consists of an electron and a positron bound together by their mutual Coulomb attraction and moving about their center of mass, which is located halfway between them. Thus the positronium “atom” is somewhat analogous to a hydrogen atom. The ground-state binding energy of hydrogen is 13.6 electron volts. What is the ground-state binding energy of positronium?

(A) $\left(\frac{1}{2}\right)^2 \times 13.6\,\text{eV}$

(B) $\frac{1}{2}\times 13.6\,\text{eV}$

(C) 13.6 eV

(D) 2 × 13.6 eV

(E) $(2)^2 \times 13.6$ eV

解：利用氢原子能级公式

$$E_n = -\frac{\mu e^4}{2\hbar^2}\frac{1}{n^2}\text{。}$$

本题中电子偶素由于正负电子质量相同，折合质量 $\mu = m_e/2$，而氢原子中 $\mu = m_e$，所以电子偶素基态能量为氢原子基态能级的一半。选(B)。请牢记氢原子（类氢原子）的能级公式。

29. A beam of electrons is accelerated through a potential difference of 25 kilovolts in an x-ray tube. The continuous x-ray spectrum emitted by the target of the tube will have a short wavelength limit of most nearly

(A) 1 A

(B) 0.5 A

(C) 2 A

(D) 25 A

(E) 50 A

解：波长最短的 X 射线对应于获得电子的全部能量，即

$$h\nu_{\max} = h\frac{c}{\lambda_{\min}} = E_e\,,$$

$$\lambda_{\min} = \frac{hc}{E_e} = \frac{6.63\times10^{-34}\times3\times10^{8}}{25\times10^{3}\times1.6\times10^{-19}} = 0.5\times10^{-10}\,\text{m}\text{。}$$

选（B）。

30. The Balmer formula for the wavelength of the spectral lines in the visible spectrum of hydrogen may be written as $1/\lambda = R_H\,(1/2^2 - 1/n^2)$. If the wavelength of the H_α line in the Balmer series is 6563 Angstroms, the wavelength of the series limit is most nearly

(A) 1640 A

(B) 2281 A

(C) 3646 A

(D) 8542 A

(E) 11,813 A

解：H_α线对应于 n=3，而线系限对应于 n=∞，所以

$$\frac{\lambda_\infty}{\lambda_\alpha} = \frac{1/2^2 - 1/3^2}{1/2^2} = \frac{5}{9}\,,$$

$$\lambda_\infty = \frac{5}{9}\lambda_\alpha = \frac{5}{9}\times 6563 = 3646\ \mathrm{A}\ 。$$

选（C）。

31. The following three factors all contribute corrections to the simpless calculation of the energy levels of hydrogen.

I. Interaction between the spin of the electron and the spin of the proton

II. Interaction between the spin of the electron and its orbital motion

III. Relativistic variation of the electron’s mass

The corrections due to these factors compare in which of the following ways?

(A) Those due to I are considerably greater than those due to II and III

(B) Those due to II are considerably greater than those due to I and III.

(C) Those due to III are considerably greater than those due to I and II.

(D) Those due to I and II are considerably and considerably greater than those due to III.

(E) Those due to II and III are considerably and considerably greater than those due to I.

解：对于氢原子，相对论效应和自旋轨道相互作用对能级的影响由相同的数量级。同时考虑二者的作用可得到精细结构项。而核自旋与电子自旋的耦合较弱，从其名字——超精细结构也可想到它对能级改变很小。选（E）。

32. The natural line width of an atomic transition with a mean lifetime of 5×10^{-9} second is most nearly

(A) 4.1×10^{-15} eV

(B) 1.3×10^{-11} eV

(C) 1.3×10^{-7} eV

(D) 4.1×10^{-2} eV

(E) 13.6 eV

解：根据时间-能量的测不准关系：

$$\Delta E\Delta\tau \ge \frac{\hbar}{2},$$

作为数量级的估算得

$$\Delta E \approx \frac{\hbar}{\Delta\tau\cdot e} \sim 10^{-7}\,\mathrm{eV}$$

选（C）。

33. If a weak electric field of magnitude E is applied to an atom in its ground state (Stark effect), what happens to the energy of the atom?

(A) It is changed by an amount proportional to E.

(B) It is changed by an amount proportional to E^2.

(C) It is changed by an amount proportional to E^3.

(D) It is changed by an amount proportional to E^4.

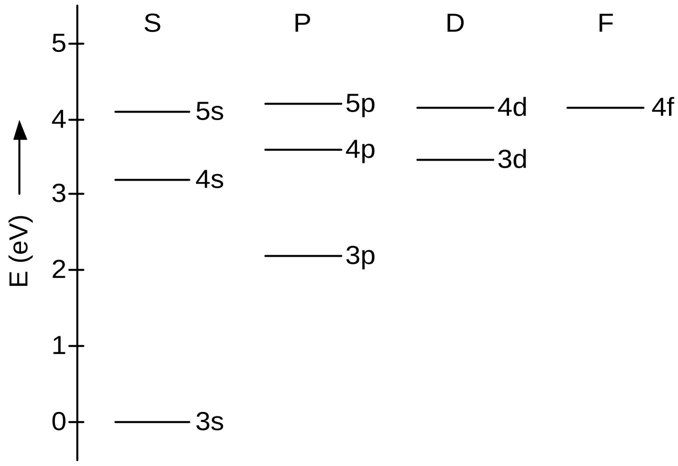
(E) It is not changed.

解：对存在固有电偶极矩的量子态，在弱外电场的作用下，原来对轨道量子数 l 简并的能级分裂，裂矩正比与电场强度，这就是线性 Stark 效应。选（A）。但是，值得注意的是，如碱金属原子的能级由于轨道贯穿和极化等效应的影响，它的能级对 l 是不简并的，这些能级相

应状态的固有电偶极矩为零，因此不存在线性 Stark 效应。在外加电场的作用下，有二级 Stark 效应，或平方 Stark 效应。

Question 34-35

A portion of the energy-level diagram for sodium is shown below.



34. Which of the following optical transitions is NOT allowed?
(A) 5s – 3p
(B) 4p – 3s
(C) 4p – 4s
(D) 4d – 3s
(E) 3d – 3p

解：根据跃迁的量子数选择定则，单电子原子允许的跃迁必须满足：$\Delta l = \pm 1$，多电子原子允许的跃迁必须满足：$\Delta l = 0, \pm 1$。选项（D）中的 $\Delta l = 2$，显然不满足选择定则。选（D）。

35. For sodium the energy levels with the same principal quantum number n and with different angular-momentum quantum numbers l are different, whereas for hydrogen the energy levels depend only on n. The principal reason for this is that
(A) states with different l values have different spin-orbit splittings.
(B) in states with smaller l values, the electron penetrates farther into the electron cloud shielding the nucleus.
(C) the Pauli exclusion principle allows only two electrons to occupy each state.
(D) the relativistic change in mass of the electron lowers its energy.
(E) n is not a good quantum number.

解：原子的内层电子形成稳固的结构，与原子核构成有效电荷数为+1 的原子实。然而，电子对原子实存在极化和贯穿作用：相同主量子数中，角量子数较小的轨道是偏心律较大的椭圆轨道，电子离原子实较近时，极化很强，对能量影响较大。同时，价电子接近原子实的轨道很可能穿入原子实，原子实的有效电荷数就会增加，从而使主量子数 n 表示中出现量子数亏损项 Δ_l。选（B）。

36. Stern and Gerlach succeeded in deflecting a beam of silver atoms with an inhomogeneous magnetic field. Which of the following is a generally accepted inference from the results of their measurements?

(A) The technique is useful for precise measurement of the magnetic field intensities.
(B) The single deflection observed is principally due to an induced magnetic moment in the silver atom.
(C) The two deflections observed are due to the two possible measured values of a component of the magnetic moment of the atom.
(D) The three deflections observed are due to neutral atoms, negatively charged ions and positively charged ions.
(E) The continuous range of values observed is due to random orientation of the magnetic moments of atoms.

解：Stern-Gerlach 实验：原子射线束射入不均匀的磁场区域，偏转后在屏上可观测到几条清晰可见的黑斑。这表明原子磁矩只取几个特定的方向，从而原子角动量的投影是量子化的。应用上没有反过来利用它求磁场的。选（C）。

37. Positronium is a quasi-atomic system consisting of an electron and a positron. In terms of the ionization energy E_0 of the hydrogen atom, what must be the energy of a photon making a transition from the first excited state of positronium to the ground state?

(A) $\frac{3}{2}E_0$
(B) $\frac{3}{4}E_0$
(C) $\frac{1}{2}E_0$
(D) $\frac{3}{8}E_0$
(E) $\frac{1}{8}E_0$

解：正负电子对的能级计算与氢原子类似，其等效质量 m*= m_H* / 2。一些结论

$$a_0' = \frac{4\pi\varepsilon_0\hbar^2}{m*e^2} = 2a_{0H}, E_n' = -\frac{1}{n^2}\frac{e^2}{8\pi\varepsilon_0 a_0} = \frac{1}{2}E_{nH}$$

所以，从 n = 1 到基态的跃迁能量

$$E = E_1{}' - E_2{}' = \frac{E_0 - E_0/4}{2} = \frac{3E_0}{8}\text{。}$$

选（D）。

38. If an electron were confined to nuclear dimensions, the uncertainty in its momentum would be most nearly

(A) 0.2 eV/c
(B) 200 eV/c
(C) 200 KeV/c
(D) 200 MeV/c
(E) 200GeV/c

解：根据测不准原理直接计算

$$\Delta p \approx \frac{\hbar}{\Delta x} \sim \frac{10^{-34}}{10^{-15}}\frac{c}{e} \sim 10^{8}\text{。}$$

选（D）。

39. Which of the following is true of the rare earth elements?
(A) They are magnetic due to the filling of d shells throughout the series.
(B) They are nearly indistinguishable chemically because they differ mainly in the filling of the deeply buried 4f shells.
(C) They are group II elements.
(D) They are inert gas.
(E) They are semiconductors.
解：稀土元素：元素周期表的第六周期中，从 $_{58}$Ce 到 $_{71}$Lu 十四种元素陆续填充 4f 壳层，而 5s、5p 和 6s 保持不变，具有相似的性质，组成稀土元素族。选（B）。

40. In Compton scattering of a beam of 500-keV x-rays by free electrons initially at rest, which of the following is true of the scattered x-rays?
(A) They are mostly back-scattered.
(B) They are scattered isotropically.
(C) They have less than 500 keV energy.
(D) They show a left-right asymmetry.
(E) They are all circularly polarized.
解：Compton 散射指 X 射线被原子散射后发生波长变长的现象，其散射公式应熟记。由于 X 射线与原子中外层电子发生碰撞，把自己能量的一部分转化成了电子的动能，所以 X 射线的能量一定减小了，波长变长。选（C）。

41. In a Rutherford back-scattering experiment, an alpha particle (A = 4) of kinetic energy K = 1 MeV strikes a Si nucleus (A = 28) and is scattered by 180°. The kinetic energy of the scattered alpha particle is most nearly
(A) 0 MeV
(B) 0.2 MeV
(C) 0.56 MeV
(D) 0.75 MeV
(E) 1 MeV
解：Rutherford 散射可以近似应用经典力学模型。

$$m_\alpha v_0 = m_{Si} v_{Si} - m_\alpha v_\alpha$$
$$K = \frac{1}{2} m_\alpha v_\alpha^2 + \frac{1}{2} m_{Si} v_{Si}^2$$

由以上两式联立可以解出 v_{Si}，从而求出 K_{Si} 来。选（C）。

42. A source emits 0.6 watt of visible monochromatic light of wavelength 6×10^{-7} meter. The number of light quanta emitted per second is most nearly
(A) 2×10^{6}

(B) 6×10^{7}
(C) 2×10^{18}
(D) 6×10^{25}
(E) 6×10^{28}
解：功率 P = nhc / λ，由此解得

$$n=\frac{P\lambda}{hc}\sim\frac{10^{-7}}{10^{-33}\times10^{8}}=10^{18}\text{ 。}$$

选（C）。

43. A photon impinges on the surface of a clean copper plate in a vacuum. A retarding potential of 1 volt is necessary to ensure that an emitted photoelectron is thought to rest. If the work function of the copper plate is 4.0 electron volts, the wavelength of the photon is most nearly
(A) 12,000 A
(B) 4000 A
(C) 3000 A
(D) 2500 A
(E) 600 A
解：根据 Einstein 光电效应的理论

$$\frac{hc}{\lambda}=E_k+A=5.0eV\text{ 。}$$

由此可解得入射光子的波长λ ≈ 2500A。选（D）。

44. Oxygen, which has atomic number 8, has the ground state configuration given by which of the following? (The notation nl^k means that there are k electrons with principal quantum number n and angular momentum l).

(A) $1s^2 2s^2 2p^4$

(B) $1s^2 2s^2 3s^2 4s^2$

(C) $1s^2 2s^2 2p^2 3s^2$

(D) $1s^2 2p^2 3d^2 4f^2$

(E) $1s^1 2s^1 3s^1 4s^1 2p^1 3p^1 4p^1 3d^1$

解：O 是第二周期的第六号元素，基态原子核外第三壳层和第四壳层都是空的。基态的电子组态为 $1s^2 2s^2 2p^4$。选（A）。

45. Which of the following atomic transitions CANNOT occur through the emission of electric dipole radiation?

(A) $p_{1/2}\rightarrow d_{3/2}$

(B) $p_{3/2}\rightarrow f_{5/2}$

(C) $p_{1/2} \to s_{1/2}$

(D) $p_{3/2} \to d_{3/2}$

(E) $p_{3/2} \to d_{5/2}$

解：根据电偶极跃迁的选择定则，对单电子原子跃迁，

$$\Delta l = \pm 1，\Delta j = 0,\pm 1；$$

对多电子原子 LS 耦合，

$$\Delta S = 0, \Delta L = 0, \pm 1, \Delta j = 0, \pm 1（0\to 0 \text{ 除外}）。$$

$p_{3/2}\to d_{5/2}$ 的跃迁$\Delta l = 2$，是不允许的。选（B）。

46. The nondegeneracy of the $2p_{1/2}$ and $2p_{3/2}$ levels of the hydrogen atom is an example of
(A) Zeeman splitting
(B) Stark splitting
(C) Fine structure
(D) Hyperfine structure
(E) Lamb shift
解：对氢原子的能级，同时考虑相对论效应和自旋轨道耦合时得到精细结构项。忽略精细结构项时的能级 E_n 只与主量子数 n 有关，对不同的 l 和 j 是简并的。考虑精细结构项时的能级 E_n 只与 n、j 有关，对 l 无精细结构分裂。具有相同的 n、j 值不同的 l 值的能级分裂，是 Lamb 移位。选（D）。

47. The energy of the K x-ray emitted by a lead atom is closest to
(A) 100 eV
(B) 1 keV
(C) 10 keV
(D) 100 keV
(E) 1 MeV
解：根据 Morsley 经验公式，K_α线的频率满足

$$\nu_K = 0.248\times 10^{16}\left(Z-b\right)^2，b\sim 1。$$

对铅原子，原子序数 Z = 82，代入上式后，可求得 K_α线的能量范围。选（D）。

48. In the Bohr model, the radius of an excited hydrogen atom in the n = 100 state is closest to
(A) 100 μm
(B) 10 μm
(C) 5000 $\overset{0}{A}$
(D) 1000 $\overset{0}{A}$
(E) 50 $\overset{0}{A}$

解：氢原子的 Bohr 半径 a_0 = 0.53A。对 n = 100 的激发态，平均半径

$$a_n = na_0 = 53\text{A} 。$$

选（E）。

49. A muon and an antimuon having equal and opposite charges, can form a hydrogenic atom. The masses of the muon and the antimuon are each 207 times the mass of an electron. The magnitude of their charges is the same as the magnitude of the electron's charge. What is the energy of the ground state of this muon-antimuon atom in terms of the ground state energy E_0 of the hydrogen atom?

(A) 207 E_0

(B) 103.5 E_0

(C) E_0

(D) E_0 / 103.5

(E) E_0 / 207

解：正负介子体系的有效质量

$$m_k* = 207m_e / 2 = 103.5m_e ,$$

与氢原子类似

$$E_n' = \frac{1}{n^2} E_0' = \frac{m_\mu^*}{m_e} E_n = 103.5 E_n 。$$

选（B）。

50. In the Stern-Gerlach effect, observed when a beam of atoms is passed through an inhomogeneous magnetic field, which of the following is true?

(A) The atoms must be ionized first.

(B) No deflection is observed for hydrogen atoms.

(C) Only one deflection is observed for hydrogen atoms.

(D) Two distinct deflections are observed for sodium (Z = 11) atoms.

(E) Three distinct deflections are observed for magnesium (Z = 12) atoms.

解：Stern-Gerlach 实验中原子射线束的分裂数为 2j + 1，j 是总角动量量子数。对于基态的 Mg 原子，电子组态为 $1s^2 2s^2 2p^6 3s^2$，最外层电子耦合的原子态为 $^1S_1$，j = 1，因此分裂成三条线。选（E）。

51. Two electrons, e.g., those in an excited state of He, interact by a Coulomb potential. If their spins are parallel, the spatial part of the total wave function must be antisymmetric with respect to exchange. This triplet state is lower in energy than the corresponding singlet (antiparallel spins) because in the triplet state the

(A) magnetic dipole-dipole interaction is weaker

(B) magnetic dipole-dipole interaction is stronger

(C) electrons are on the average closer together

(D) electrons are on the average farther apart

(E) spin-orbit couplings are weaker

解：两个能级的主量子数 n 和角量子数 l 相同，对总角动量量子数 j 的分裂属于精细结构分

裂，是自旋轨道耦合的结果。选（E）。

52. Which of the following properties of the hydrogen atom can be predicted most accurately from the simple Bohr model?

(A) Energy differences between states

(B) Angular momentum of the ground state

(C) Degeneracy of states

(D) Transition probabilities

(E) Selection rules for transitions

解：Bohr 模型计算出一套能级公式。那么人们如何去检验它呢，只有氢原子光谱，从中得出各能级间的能量差。Bohr 模型正是因为正确解释了氢原子光谱而名气斐然的。选（A）。

53. The total energy necessary to remove all three electrons from a lithium atom is most nearly

(A) 2 MeV

(B) 2 KeV

(C) 200 eV

(D) 20 eV

(E) 2 eV

解：类氢原子能级公式

$$E_n = -\frac{\mu Z^2 e^4}{\hbar^2 n^2}。$$

锂，$Z = 3$ ，所以它最后一个电子的电离能为

$$9E_0 = 9\times 13.6 = 122\text{eV}。$$

而由于屏蔽作用以及距离变大，外层的电子的电离能比内层要小，所以第一、 第二电离能为几十个 eV 的数量级。选（C）。

第七节 狭义相对论

1. A positive kaon (K^+) has a rest mass of 494 MeV/c^2, whereas a proton has a rest mass of 938 MeV/c^2. If a kaon has a total energy that is equal to the proton rest energy, the speed of the kaon is most nearly

(A) 0.25c

(B) 0.40c

(C) 0.55c

(D) 0.70c

(E) 0.85c

解：相对论质量变化公式

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}},$$

$$v=\left[1-\left(\frac{m_0}{m}\right)^2\right]c=\left[1-\left(\frac{494}{938}\right)^2\right]c\approx 0.85c\text{ 。}$$

选（E）。

<u>Questions 2-3</u>

In an inertial frame S, a particle has a momentum (P_x, P_y, P_z) = (5, 3, $\sqrt{2}$) MeV/c and a total energy = 10 MeV.

2. The speed u of the particle as measured in a frame S is most nearly

(A) $\frac{3}{8}c$

(B) $\frac{2}{5}c$

(C) $\frac{1}{2}c$

(D) $\frac{3}{5}c$

(E) $\frac{4}{5}c$

解：由相对论公式

$$E^2=m_0{}^2c^4+p^2c^2\text{，}$$

静质量为

$$m_0=\frac{\sqrt{E^2/c^2-p^2}}{c}=\sqrt{100-(25+9+2)}/c=8\text{ MeV/c}^2\text{，}$$

而其运动质量为

$$m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}=10\text{ MeV/c}^2\text{，}$$

由此解得 $v=\frac{3}{5}c$ 。选(D)。

3. Which of the following combinations of momentm p' and energy E' could represent the motion of the particle described above as observed in another inertial frame S' moving with an unspecified velocity v relative to s?

(A) $p'=(0,0,8)\text{MeV/c}$，　　$E'=\sqrt{128}\text{MeV}$

(B) $p'=(8,0,\sqrt{2})\text{MeV/c}$，　　$E'=10\text{MeV}$

(C) $p'=(31,4,6)\text{MeV/c}$，　　$E'=\sqrt{949}\text{MeV}$

(D) $p' = \left(50, -30, \sqrt{200}\right)\text{MeV/c}$, $E' = 100\text{MeV}$

(E) $p' = (100,10,0)\text{MeV/c}$, $E' = 10{,}000\text{MeV}$

解：间隔 $E^2 - p^2c^2 = m_0{}^2c^4$ 为守恒量，与所选参考系无关，可通过这一条验证只有选项

（A） $E'^2 - p'^2c^2 = 64\,\text{MeV}^2$。选(A)。

4. The percentage increase in the energy of a particle whose speed is changed from rest to 0.8c (where c is the speed of light) is closest to

(A) 50%
(B) 67%
(C) 75%
(D) 80%
(E) 88%

解：相对论质量变换公式

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{5}{3}m_0 ,$$

质能关系

$$E = mc^2 = \frac{5}{3}m_0c^2 \text{。}$$

选（B）。

5. Two spaceships, each measuring 100 meters in length in its own rest frame, pass by each other traveling in opposite directions. Instruments on spaceship I determine that the front end of spaceship I requires $\frac{5}{3}\times 10^{-7}$ second to traverse the full length of spaceship II. Given c = 3×10^8 meters per second, what is the magnitude of the relative velocity of the two spaceships?

(A) $\frac{1}{\sqrt{6}}c$

(B) $\frac{1}{2}c$

(C) $\frac{1}{\sqrt{2}}c$

(D) $\frac{2}{\sqrt{5}}c$

(E) $\frac{2}{\sqrt{3}}c$

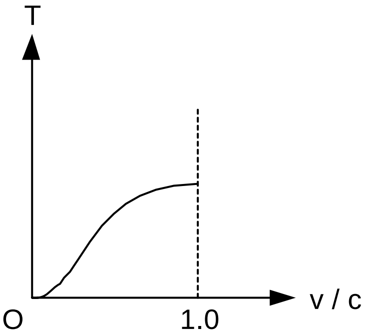
解：由相对论公式

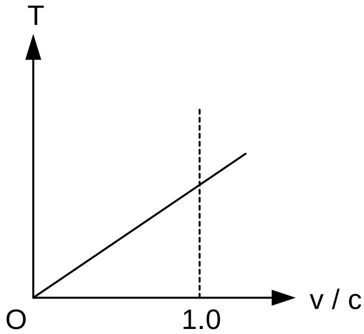
$$l = l_0\sqrt{1-\frac{v^2}{c^2}}$$

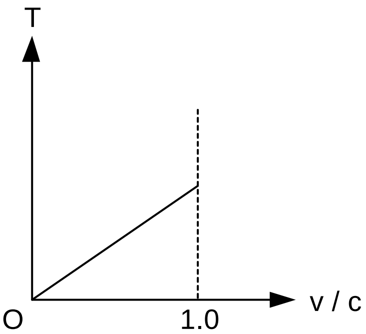
则
$$\frac{l}{v} = \frac{l_0\sqrt{1-\frac{v^2}{c^2}}}{v} = \frac{100\sqrt{1-\frac{v^2}{c^2}}}{v} = t = \frac{5}{3}\times 10^{-7}$$

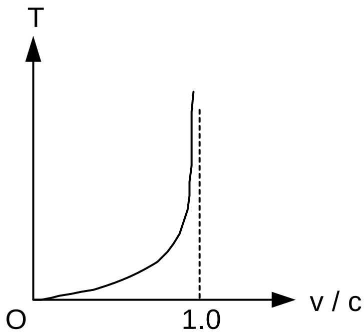
解的 $v = \frac{2}{\sqrt{5}}c$。答案选(D)。

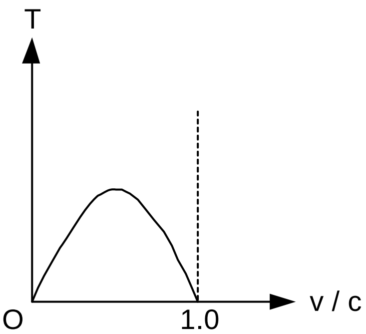
6. An electron has speed v. A plot of the kinetic energy T of the electron versus the ratio $\frac{v}{c}$ would look most like which of the following graphs?

(A)


(B)


(C)


(D)


(E)


解：由相对论质量变换公式

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

和质能关系

$$E_{total} = mc^2,$$

得动能为

$$T = E_{total} - E_s = m_0 c^2 \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right)。$$

v→∞ 时，$\frac{v}{c} \to 1$，而电子的静质量显然不为 0，所以动能 T→∞。选（D）。

7. Two events are each observed by two observers, S and S’, who are in uniform motion with respect to each other. Observer S notes that the two events are separated by 3 seconds, and their locations are 10 meters apart. Observer S’ notes that the two events are separated by 5 seconds. The distance between their locations, as seen by S’, is closest to
(A) 0 m
(B) 2 m
(C) 10 m
(D) 17 m
(E) 10^9 m
解：二事件的时空间隔

$$\Delta S^2 = c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

为协变量，不因参照系的变换而改变。本题中由两事件在 S 和 S’内时空间隔相等得

$$c^2 3^2 - 10^2 = c^2 5^2 - x^2,$$

$$x = 4c = 1.2 \times 10^9 \text{ m}。$$

选（E）。

8. The lifetime of a particular meson at rest is 10^{-8} second and its mass is 10^{-25} gram. If its velocity in the laboratory is 2*10^8 meters per second, how far will it travel in one lifetime, if both distance and lifetime are measured in the laboratory frame?
(A) 10^{-3} m
(B) 2 m
(C) $\sqrt{5}$ m

(D) $\dfrac{6}{\sqrt{5}}$ m

(E) $\dfrac{9}{\sqrt{5}}$ m

解：由相对论公式，在实验室系看这个介子的寿命延长为

$$\tau = \frac{\tau_0}{\sqrt{1-\dfrac{v^2}{c^2}}} = \frac{3}{\sqrt{5}} \times 10^{-8}\,\text{s}\,,$$

所以它可通过的距离为

$$l = v\tau = \frac{6}{\sqrt{5}}\,\text{m}\text{ 。}$$

选(D)。

9. A proton has kinetic energy of 500 GeV (1 GeV = 10^9 eV). The momentum of this proton is most nearly

(A) 22 GeV/c

(B) 30 GeV/c

(C) 250 GeV/c

(D) 500 GeV/c

(E) 707 GeV/c

解：质子的静质量为 938MeV/c，静能约为 1GeV。所以题目中质子的动能远大于静能，总能量近似等于动能。由相对论公式

$$E = \sqrt{{m_0}^2c^4 + P^2c^2} \approx Pc\,,$$

$$P \approx \frac{E}{c} = 500\text{GeV/c}\text{ 。}$$

选（D）。

10. Two observers O and O’ observe two events a and B. the observers have a constant relative speed of 0.8c. In units such that the speed of light is 1, observer O obtained the following coordinates:

Event A: x=3, y=3, z=3, t=3

Event B: x=5, y=3, z=1, t=5

What is the length of the space-time interval between these two events, as measured by O’?

(A) 1

(B) $\sqrt{2}$

(C) 2

(D) 3

(E) $2\sqrt{3}$

解：由狭义相对论，事件的时空距离

$$\Delta S^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2$$

为守恒量，与参考系无关。所以只需要在 O 的参考系中计算即可。选（C）。

11. Which of the following statements most accurately describes how an electromagnetic field behaves under a Lorentz transformation?
(A) The electric field transforms completely into a magnetic field.
(B) If initially there is only an electric field, after the transformation there may be both an electric and a magnetic field.
(C) The electric field is unaltered.
(D) The magnetic field is unaltered.
(E) It cannot be determined unless a gauge transformation is also specified.
解：在相对论下，电磁场被完全统一起来。电磁势φ和 **A** 是同一四维矢量的不同分量，

$$A_\mu = (\mathbf{A}, \frac{i}{c}\varphi),$$

在不同参考系间变换时，可相互转换。选（B）。

12. If a newly discovered particle X moves with a speed equal to the speed of light in vacuum, then which of the following must be true?
(A) The rest mass of X is zero.
(B) The spin of X equals the spin of a photon.
(C) The charge of X is carried on its surface.
(D) X does not spin.
(E) X cannot be detected.
解：由相对论公式

$$E = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} c^2,$$

要使以光速运动的粒子的能量为有限值，则其静质量 m_0 必须为零。选（A）。

<u>Question 13-14</u>

A car of rest length 5 meters passes through a garage of rest length 4 meters. Due to the relativistic Lorentz contraction, the car is only 3 meters long in the garage's rest frame. There are doors on both ends of the garage, which open automatically when the front of the car reaches them and close automatically when the rear passes them. The opening or closing of each doorrequires a negligible amount of time.
13. The velocity of the car in the garage's rest frame is
(A) 0.4c
(B) 0.6c
(C) 0.8c
(D) greater than c
(E) not determinable from the data given

解：由大家熟知的公式，

$$l = l_0\sqrt{1-\frac{v^2}{c^2}}\ ,$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{l}{l_0} = \frac{3}{5}\ ,$$

解得

$$v = 0.8c\ 。$$

选（C）。

14. The length of the garage in the car’s rest frame is
(A) 2.4 m
(B) 4.0 m
(C) 5.0 m
(D) 8.3 m
(E) not determinable from the data given

解：利用上面的公式，且在车库参照系相对汽车参照系速度也为 0.8c，计算得 2.4m。选（A）。

15. Which of the following statements is the best response to the question:

“Was the car ever inside a closed garage?”

(A) No, because the car is longer than the garage in all reference frames.
(B) No, because the Lorentz contraction is not a “real” effect.
(C) Yes, because the car is shorter than the garage in all reference frames.
(D) Yes, because the answer to the question in the garage’s rest frame must apply in all reference frames.
(E) There is no unique answer to the question, as the order of door openings and closings depends on the reference frame.

解：可能有人不敢选择（E），似乎一件事必须有个定论。其实不然，“公说公有理，婆说婆有理”也有可能。在车库参照系中，设后门坐标为 x = 0，且 t = 0 时刻后门开启，则后门关闭的时空坐标为（0，$\frac{3}{0.8c}$），前门开启的时空坐标为（4，$\frac{4}{0.8c}$）。两事件间的时空间隔为

$$\Delta s^2 = c^2\Delta t^2 - \Delta x^2 < 0\ ,$$

为类空事件，即二者之间没有因果联系。因此不同参考系之间说法不一并不矛盾。

16. The measured index of refraction of x-rays in rock salt is less than one. This is consistent with the theory of relativity because
(A) relativity deals with light waves traveling in a vacuum only
(B) x-rays cannot transmit signals
(C) x-ray photons have imaginary mass
(D) the theory of relativity predates the development of solid-state physics
(E) the phase velocity and group velocity are different

解：我们常用的公式 $v=\dfrac{c}{n}$ 中所指速度为相速度 V_p。而相速度超光速是有可能的，因为它不伴随着能量的传播。而群速度 V_g 才表示能量传播的速度，它不能超光速。再记一下两个常用公式

$$V_p=\frac{\omega}{k},\ V_g=\frac{d\omega}{dk}\text{。}$$

17. A Σ^0 particle (mass M_1) decays at rest in the laboratory into a Λ^0 particle (mass M_2) and a massless photon. The energy of the Λ^0 particle is

(A) $\dfrac{M_1}{2}c^2$

(B) $\dfrac{M_1^2+M_2^2}{2M_1}c^2$

(C) $\dfrac{(M_1+M_2)^2}{2M_1}c^2$

(D) $\dfrac{M_1^2-M_2^2}{2M_1}c^2$

(E) $\dfrac{M_1-M_2}{2}c^2$

解：设Λ^0 粒子能量为 E。则光子能量为 M_1c^2-E，动量为 $(M_1c^2-E)/c$。因为反应前系统动量为零，所以Λ^0 粒子动量与光子动量大小相等，也为 $(M_1c^2-E)/c$。代入

$$E=\sqrt{M_2{}^2c^4+p^2c^2}\ ,$$

$$E^2=M_2{}^2c^4+\left(M_1c^2-E\right)^2,$$

解得

$$E=\frac{M_2{}^2+M_1{}^2}{2M_1}c^2\text{。}$$

选（B）。

<u>Questions 18-19</u>

A monoenergetic beam consists of unstable particles with energies 100 times their rest energy. The mean life of the particle is 10^{-10} second in their rest frame. IN the laboratory frame, the length of the beam from the point of production to the detector is 6 meters.

18. What fraction of the particles in the beam reach the detector?

(A) e^{-200}

(B) e^{-2}

(C) 0.5

(D) e^{-1}

(E) 1.0

解：由相对论公式

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}},$$

对于本题，

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 100,$$

v 很接近于光速 c。粒子的寿命在实验室中观测为

$$\tau = \frac{\tau}{\sqrt{1-\frac{v^2}{c^2}}} = 10^{-8}\text{s}。$$

作为估算，取 v = c，则粒子束长度为

$$l = \tau c = 3\text{m}。$$

选（C）。

19. If the particles have rest mass m, their momentum is most nearly

(A) mc

(B) 10 mc

(C) 70 mc

(D) 100 mc

(E) 10^4 mc

解：由于总能量远大于粒子的静止能量，则

$$E = m'c^2 = \sqrt{m_0{}^2c^4 + P^2c^2} \approx Pc,$$

$$P = \frac{E}{c} = 100mc。$$

选（D）。

<u>Questions 20-21</u>

An ion which charge +q travels in the +x direction relative to a right-handed coordinates system. A magnetic field of intensity B webers per square meter is applied in the +y direction.

20. For the ion to be undeflected if its velocity is v = 0.01c, there must also be an electric field E of magnitude in volts per meter and direction equal to

(A) B, in the –y direction

(B) vB, in the –y direction
(C) B, in the +z direction
(D) vB, in the +z direction
(E) vB, in the –z direction
解：电场力应与 Lorentz 力平衡。

$$\vec{E} = -\frac{\vec{F}}{q} = -\frac{q\vec{v}\times\vec{B}}{q} = -vB\vec{k}$$

选（E）。

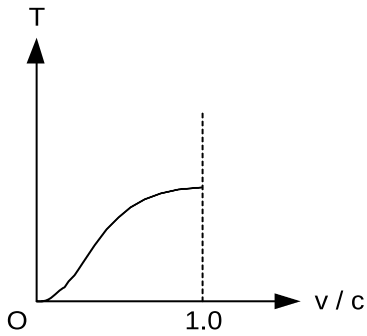
21. For the ion to be undeflected if its velocity is v = 0.99c, the magnitude in volts per meter of the required electric field E is equal to
(A) B
(B) vB
(C) r $\frac{B}{\sqrt{1-\frac{v^2}{c^2}}}$
(D) $\frac{vB}{\sqrt{1-\frac{v^2}{c^2}}}$
(E) $\frac{vB}{\left(1-\frac{v^2}{c^2}\right)}$
解：与上题类似，带电粒子受的总电场力

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v}\times\mathbf{B}\text{。}$$

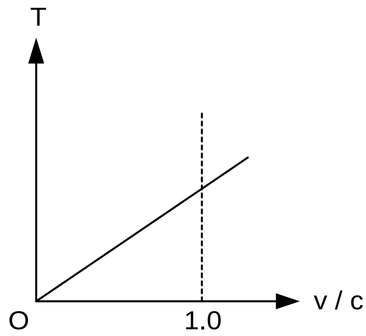
选（B）。

22. An electron has speed v. A plot of the kinetic energy T of the electron versus the ratio $\frac{v}{c}$ would look most like which of the following graphs?

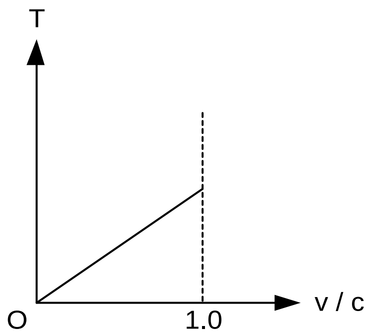
(A)



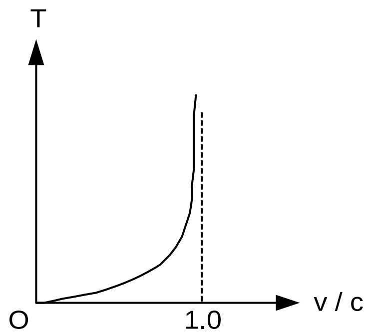
(B)



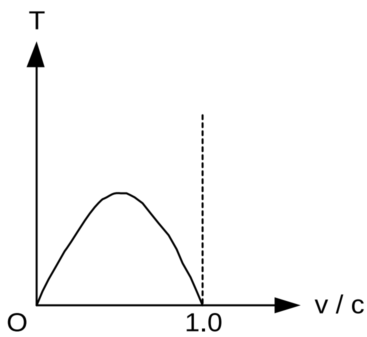
(C)



(D)



(E)



解：根据相对论的能量关系

$$T = mc^2 - m_0c^2 = m_0c^2\left(\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - 1\right),$$

定性上看，当β=0 时，T=0；当β→1 时，T→∞。选（D）。

23. Two events occur at the same place but at different times as measured in an inertial reference frame (IRF) attached to a rocket. The second event occurs t’ seconds later than the first. The rocket is moving at a relativistic speed v with respect to a laboratory IRF. What is the distance between the two events, as measured in the laboratory IRF? ($\gamma = 1/\sqrt{1-v^2/c^2}$)

(A) vt’

(B) $vt'\gamma$
(C) vt' / γ
(D) v^2t' / c
(E) vt' / γ^2

解：较为简单的解法是利用这两个事件的时空间隔的相对论不变性

$$c^2t^2 - r^2 = c^2t'^2 - r'^2 ,$$

$$c^2(\gamma t')^2 - r^2 = c^2t'^2 。$$

或者可以直接利用 Lorentz 变换式

$$x' = \gamma(x - vt)$$

进行坐标变换。选（B）。

24. A Σ^0 particle (mass M_1) decays at rest in the laboratory into a Λ^0 particle (mass M_2) and a massless photon. The total energy of the Λ^0 particle is

(A) $\frac{M_1}{2}c^2$

(B) $\frac{{M_1}^2 + {M_2}^2}{2M_1}c^2$

(C) $\frac{(M_1 + M_2)^2}{2M_1}c^2$

(D) $\frac{{M_1}^2 - {M_2}^2}{2M_1}c^2$

(E) $\frac{M_1 - M_2}{2}c^2$

解：利用核反应前后的动量-能量守恒关系

$$p_\gamma + p_2 = 0$$
$$M_1c^2 = p_\gamma c + \sqrt{M_2^2c^4 + p_2^2c^2}$$

解得

$$p_2 = \frac{m_1^2 - m_2^2}{2m_1}c ,$$

从而

$$E_2 = \sqrt{M_2^2c^4 + p_2^2c^2} 。$$

选（C）。

25. Our galaxy is about 10^5 light-years across. (1 light-year ≅ 10^{16} meters) The most energetic particles known have an energy of about 10^{19} eV. How long would it take a person ($mc^2 = 10^9$ eV) of that energy to traverse the galaxy from on the proton?

(A) $\frac{1}{3}\times 10^3 s$

(B) $\frac{1}{3}\times 10^6 s$

(C) $\frac{1}{3}\times 10^9 s$

(D) $\frac{1}{3}\times 10^{13} s$

(E) $\frac{1}{3}\times 10^{23} s$

解：粒子的 E >> mc^2，故它的速度很接近于光速 c。其

$$\gamma = \frac{E}{E_0} = 10^{10}\text{。}$$

在粒子的静止系中看来

$$t = \frac{l'}{v} \cong \frac{l}{\gamma c} = \frac{10^5 \times 10^{16}}{10^{10}\times 3\times 10^8} = \frac{1}{3}\times 10^3 (s)\text{。}$$

选（A）。

26. If the total energy of a particle is exactly twice its rest energy, the speed of the particle is

(A) $\frac{1}{2}c$

(B) $\frac{\sqrt{2}}{2}c$

(C) $\frac{3}{4}c$

(D) $\frac{\sqrt{3}}{2}c$

(E) $\sqrt{2}c$

解：利用相对论性的能量公式

$$E = \frac{E_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = 2E_0$$

不难解得

$$\frac{v}{c} = \frac{\sqrt{3}}{2}\text{。}$$

选（D）。

27. A proton has kinetic energy of 500 GeV (1GeV = 10^9 eV). The momentum of this proton is most nearly

(A) 22 GeV/c

(B) 30 GeV/c

(C) 250 GeV/c

(D) 500 GeV/c

(E) 707 GeV/c

解：在 E >> E_0 的条件下，质子的动量

$$p=\frac{\sqrt{E^2-E_0^2}}{c}\approx\frac{E}{c}=500\left(GeV/c\right)。$$

选（D）。SUB 考试中这样的近似计算很常用，应该熟记。

<u>Questions 28-29</u>

In an inertial frame S, a particle has a momentum (p_x, p_y, p_z) = (5, 3, $\sqrt{2}$) MeV/c and a total energy E = 10 MeV.

28. The speed v of the particle as measured in frame S is most nearly

(A) $\frac{3}{8}c$

(B) $\frac{2}{5}c$

(C) $\frac{1}{2}c$

(D) $\frac{3}{5}c$

(E) $\frac{4}{5}c$

解：质点的总动量

$$p=\sqrt{p_x^2+p_y^2+p_z^2}=6\text{MeV/c}。$$

因此

$$v=\frac{p}{m}=\frac{pc^2}{E}=\frac{3}{5}c。$$

选（D）。

29. Which of the following combinations of momentum p and energy E could represent the motion of the particle described above, as observed in another inertial frame S, moving with an unspecified velocity v, relative to S ?

(A) $p'=(0,0,8)MeV/c$

(B) $p' = (8,0,\sqrt{2})MeV/c$

(C) $p' = (31,4,6)MeV/c$

(D) $p' = (50,-30,\sqrt{200})MeV/c$

(E) $p' = (100,10,0)MeV/c$

解：在不同的惯性参照系中，质点的静能

$$E_0 = m_0c^2 = \sqrt{E^2 - p^2c^2}$$

应该不变，等于在 S 系中的值 8 MeV。选（B）。

30. A particle at rest in a laboratory lives for a time τ, as measured in the laboratory, before it decays. How long does this particle live according to observers in a reference frame moving at 0.6c with respect to the laboratory?

(A) 6τ

(B) 0.8τ

(C) τ

(D) $\dfrac{1}{0.8}\tau$

(E) $\dfrac{1}{0.6}\tau$

解：根据相对论中的时钟缩短效应

$$\tau' = \gamma\tau = \frac{\tau}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{\tau}{0.8}\text{。}$$

选（D）。

31. An explosion occurs at the center of a rocket shio moving at a relativistic speed v with respect to a laboratory inertial reference frame. The length of the rocket, as measured on the rocket, is L. Let t_F and t_R be the respective times at which light from the explosion reaches the front and rear of the rocket ship, where t_F and t_R are measured in the laboratory. If $\beta = v/c$ and $\gamma = \sqrt{1-\beta^2}$, what is $t_F - t_R$?

(A) 0

(B) $\dfrac{L\beta\gamma}{c}$

(C) $\dfrac{L\beta\gamma^2}{c}$

(D) $\dfrac{L\beta^2\gamma^2}{c}$

(E) $\dfrac{L\beta\gamma^3}{c}$

解：在实验室参照系中看来，火箭的长度为 L’ = L/γ，光讯号的速度为 c 不变。

$$t_F = \frac{L'}{2(c-v)},$$

$$t_R = \frac{L'}{2(c+v)},$$

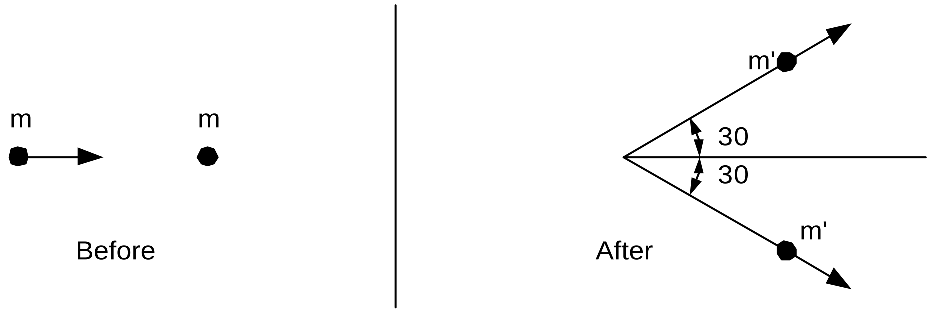
故

$$t_F - t_R = \frac{L}{2\gamma}\left(\frac{1}{c-v} - \frac{1}{c+v}\right) = \frac{L\beta\gamma}{c} 。$$

选（B）。

Questions 32-33

A particle with rest mass m and momentum mc/2 collides with a particle of the same rest mass that is initially at rest. After the collision, the original two particles have disappeared. Two other particles, each with rest mass m’, are observed to leave the region of the collision at equal angles of 30° with respect to the direction of the original moving particle, as shown below.



32. What is the speed of the original moving particle?

(A) $c/5$

(B) $c/3$

(C) $c/\sqrt{7}$

(D) $c/\sqrt{5}$

(E) $c/2$

解：由相对论公式

$$E = \sqrt{m_0^{\ 2}c^4 + p^2c^2},$$

$$m_v = \frac{E}{c^2} = \sqrt{m^2 + \frac{p^2}{c^2}} = \frac{\sqrt{5}}{2}m,$$

而

$$m_v = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}},$$

由此解得

$$v = c/\sqrt{5}。$$

选（D）。

33. What is the momentum of each of the two particles produced by the collision?

(A) $mc/5$

(B) $mc/2\sqrt{3}$

(C) $mc/\sqrt{5}$

(D) $mc/2$

(E) $mc/\sqrt{3}$

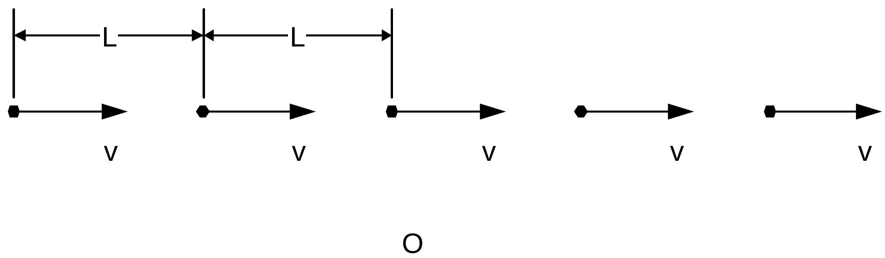
解：设新产成粒子的动量为 P'，由碰撞前后水平动量守恒，有

$$2P'\cos 30^0 = mc/2,$$

所以

$$P' = \frac{mc}{4\cos 30^0} = mc/2\sqrt{3}。$$

选（B）。



34. A steady stream of identical particle, some of which are shown in the figure above, passes an observer O. As measured by O, the particles move with the same relativistic speed v and are spread along a straight line at equal intervals L. Let $\gamma = 1/\sqrt{1-v^2/c^2}$. The number of particles that pass O in unit time is nearly

(A) $\frac{v}{L}$

(B) $\frac{L}{v}$

(C) $\frac{L}{v\gamma}$

(D) $\dfrac{v}{L\gamma}$

(E) $\dfrac{v\gamma}{L}$

解：选（A）。此题与相对论根本无关，不要上当。

第八节 实验方法

1. The resolving time of a counter is 2.5×10^{-6} second and the dead time of the recording device is 3×10^{-4} second. If the recording number of incident particles is N = 5×10^{3} per second, the number of recorded particles is closest to

(A) 1×10^{3} pulses s^{-1}

(B) 2×10^{3} pulses s^{-1}

(C) 3×10^{3} pulses s^{-1}

(D) 4×10^{3} pulses s^{-1}

(E) 5×10^{3} pulses s^{-1}

解：计数器的分辨时间很短，由于受到记录设备的限制，最大计数频率为

$$f = \frac{1}{3\times10^{-4}} = 3.3\times10^{3}\ pluse\cdot s^{-1},$$

低于散射粒子的出现频率。选（C）。

2. The initial intensity I_0 of a beam of photons of a certain energy is reduced to $\frac{1}{2}I_0$ as the beam traverses a thin sheet of lead of thickness d. If the beam traverses a sheet of lead of thickness 3d, its intensity is reduced to

(A) $\dfrac{1}{3}I_0$

(B) $\dfrac{1}{4}I_0$

(C) $\dfrac{1}{6}I_0$

(D) $\dfrac{1}{8}I_0$

(E) $\dfrac{1}{9}I_0$

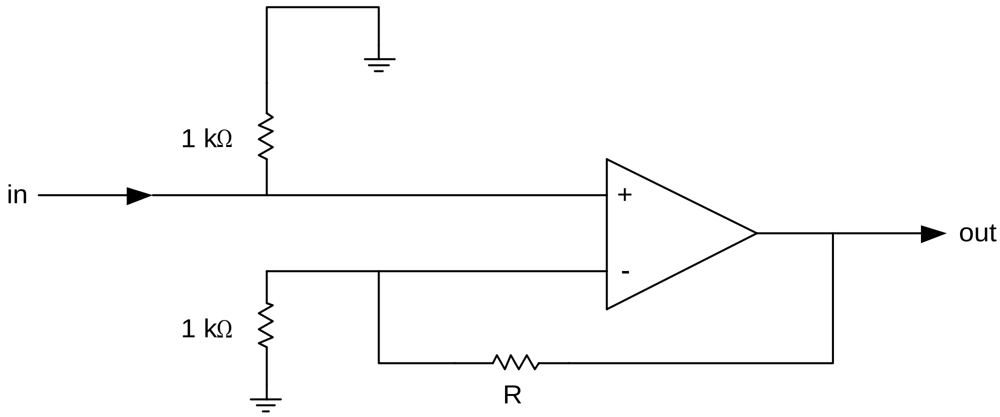
解：由散射公式

$$I = I_0 e^{-ks}$$

得

$$I(3d) = I_0 e^{-3kd} = I_0\left(e^{-kd}\right)^3 = \left(\frac{1}{2}\right)^3 I_0 = \frac{1}{8}I_0。$$

选(D)。



3. For which of the following values of resistor R will the gain of the operational amplifier shown above be closest to 100?

(A) 10 kΩ

(B) 20 kΩ

(C) 100 kΩ

(D) 200 kΩ

(E) 500 kΩ

解：理想放大器放大倍数为 ∞ ，为保证输出电压为有限值，要求

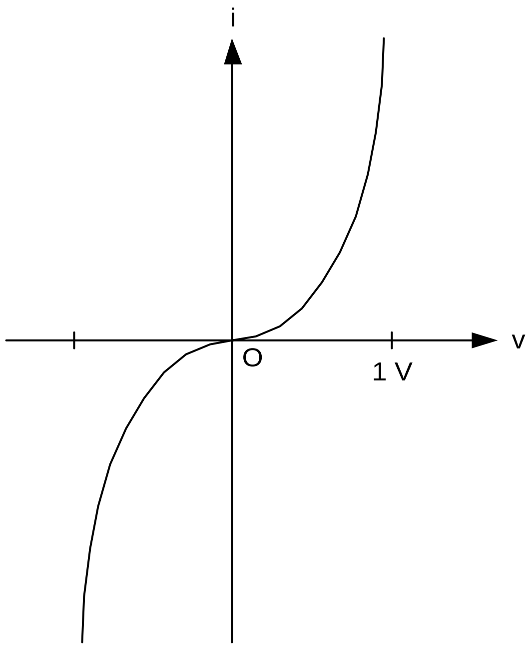
$$V_{+} = V_{-},$$

$$V_{in} = V_{+} = V_{-} = \frac{1}{1+R} V_{out} 。$$

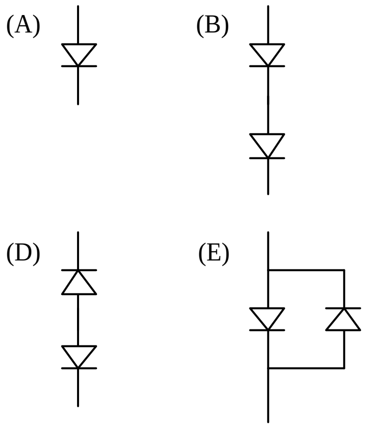
放大倍数为

$$A = \frac{V_{out}}{V_{in}} = 1 + R 。$$

选（C）。



4. Which of the following circuits will have the v – i characteristic shown above? (All the diodes are silicon.)



解：答案选（E）。(A)、(B)、(C)等同于普通的二极管，而答案(E)中两个二极管反接，几乎不导通。

5. The time a laser light beam takes to travel from the Earth to the Moon and back is most nearly

(A) 2 microseconds

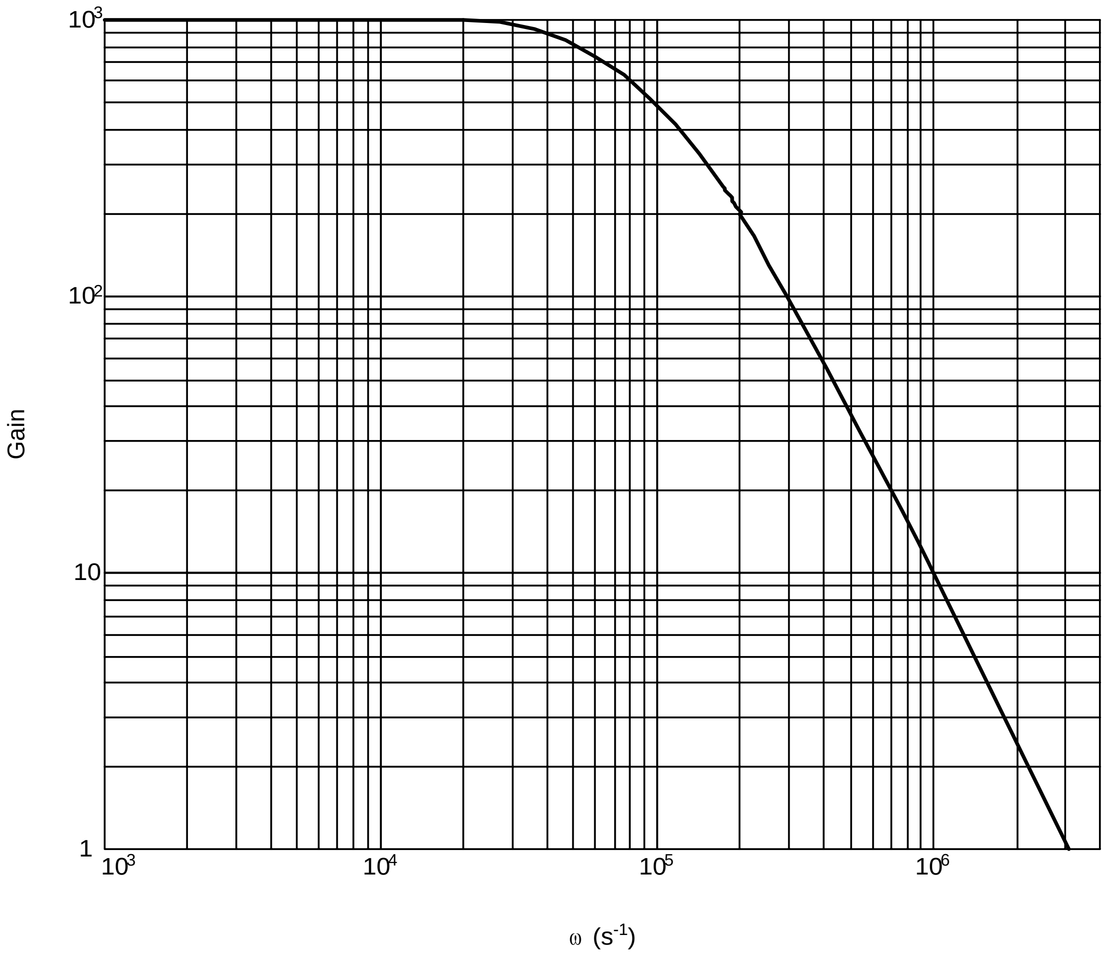
(B) 2 milliseconds

(C) 2 seconds

(D) 2 minutes

(E) 2 hours

解：地月间距平均为 38 万公里，真空中光速为 30 万公里每秒，均为基本常识。选（C）。



6. The gain of an amplifier is plotted *versus* angular frequency ω in the diagram above. If K and a are positive constants, the frequency dependence of the gain near $\omega=3\times10^{-5}$ second^{-1} is most accurately expressed by

(A) $Ke^{-a\omega}$

(B) $K\omega^2$

(C) $K\omega$

(D) $K\omega^{-1}$

(E) $K\omega^{-2}$

解：在$\omega=3\times10^5\text{second}^{-1}$处，约为斜率为-2 的直线。注意到此图为双对数曲线，所以

$$\ln \text{gain} = -2\ln\omega + c\ ,$$

$$\text{gain} = e^c\omega^{-2} = k\omega^{-2}\ 。$$

选(E)。

7. A proton beam is incident on a scatterer 0.1 centimeter thick. The scatterer contains 10^{20} target

nuclei per cubic centimeter. In passing through the scatterer, one proton per incident million is scattered. The scattering cross section is

(A) 10^{-29} cm^2

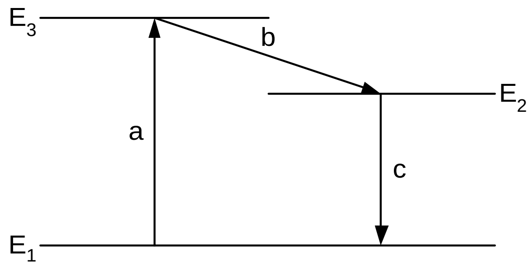
(B) 10^{-27} cm^2

(C) 10^{-25} cm^2

(D) 10^{-23} cm^2

(E) 10^{-21} cm^2

解：散射几率 P=1/10^6，为无量纲量。散射截面σ的量纲为米 ^{2}，靶核浓度 n 量纲为米$^{-3}$，厚度 l 量纲为米。所以为凑得一个无量纲数，显然公式为 $P = n \cdot l \cdot \sigma$（其实公式本身很简单，这里只是举一个量纲分析的例子，不一定很恰当。由于 P 为无量纲数，似乎 P 可以等于 $(n \cdot l \cdot \sigma)^n$，这时需要用物理思想判断 n 的取值）。选（C）。



8. The diagram above shows the levels and transitions of a ruby laser. All of the following statements about the laser are correct EXCEPT:

(A) The laser works because of population inversion.

(B) E_2 is the energy of a metastable state.

(C) Transition a involves absorption of radiation.

(D) Transition b involves stimulated emission.

(E) The laser must be optically pumped by radiation of frequency $f = \dfrac{E_3 - E_1}{h}$.

解：受激辐射指在满足频率要求的外来光子的激励下高能级的原子向低能级跃迁。而众所周知，激光器中粒子通过受激吸收从基态 E_0 跃迁到激发态 E_3 上，粒子从激发态 E_3 通过碰撞以无辐射跃迁的方式转移到亚稳态 E_2。由于 E_2 能级寿命较长，在 E_2 上粒子积累，实现粒子数反转。选（D）。

9. Measurement of the degree of a vacuum by a thermocouple gauge is based primarily on the

(A) decrease in thermal conductivity of a gas with decreasing pressure

(B) increase in thermal conductivity of a gas with decreasing pressure

(C) decrease in the electrical conductivity of a gas with decreasing pressure

(D) increase in the electrical conductivity of a gas with decreasing pressure

(E) pressure dependence of the Thomson

解：热电偶真空计简介：它是通常用来测量低真空的真空计，可测范围为 10^{-1}~10^{-3} Torr。其中有一根细金属丝以恒定功率加热到约 200℃。由于气体热导律随压强变化（压强越低热导率越小），所以热丝的温度成为压强的函数，经过校准定标后，可以由测量热丝温度的热电偶的指示测定压强。

选（A）。

10. Of the following quantities, measurement of which would normally be the most practical way to determine the magnitude of a magnetic field B of approximately 1 tesla (weber per meter squared)?
(A) Deflection of a laser beam
(B) Curvature of the path of a charged particle of known velocity
(C) Torque produced on a permanent magnet
(D) Eddy-current heating of a moving metallic object
(E) Hall voltage across a small probe
解：由题意，应当是易于操作的。（A）、（B）、（C）、（D）的效应都很难在实验上操作，而且不能测量 1T 这样量级的磁场。Hall 效应

$$V_H = R_H \frac{IB}{d},$$

其中 R_H 为 Hall 系数。一般 I 的数量级在 A，d 的数量级在 cm，Hall 电势差的数量级在 T 左右。选（E）。

11. A 10-bit analog-to-digit converter converts voltages to digital form with an accuracy of approximately 1 part in
(A) 10
(B) 10^2
(C) 10^3
(D) 10^4
(E) 10^{10}
解：数模转换卡中采用二进制计数，n 位转换卡的转换精度为（最大量程为 V_0）

$$\Delta V = \frac{1}{2^n} V_0,$$

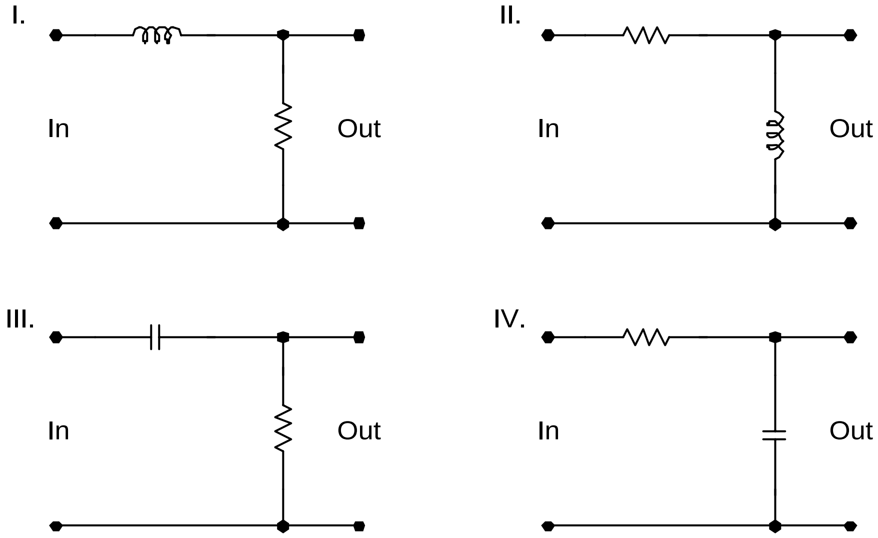
对 10 位的卡，

$$\frac{\Delta V}{V_0} = \frac{1}{2^{10}} \sim 1000\text{。}$$

选（C）。

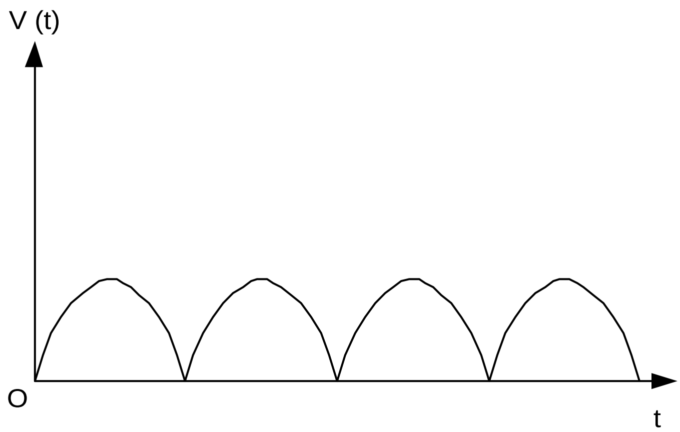
12. A certain experiment requires the maintenance of a vacuum of 10^{-2} atmospheres in a closed-off system. Which of the following is the simplest pumping arrangement that will meet this requirement?
(A) A mechanical pump only
(B) A mechanical pump and a oil-diffusion pump only
(C) A mechanical pump, a oil-diffusion pump, and a dry-ice trap
(D) A mechanical pump, a oil-diffusion pump, and a liquid-air trap
(E) A mechanical pump, a oil-diffusion pump, and a liquid-helium trap
解：一般真空获得中，机械泵的极限压强约为 10^{-3}Torr（10^{-5}atm），扩散泵的极限压强约为 10^{-6}Torr（10^{-8}atm）。它们都属于外排型真空泵。选（A）。

13. Which of the following circuits are high-pass filters?

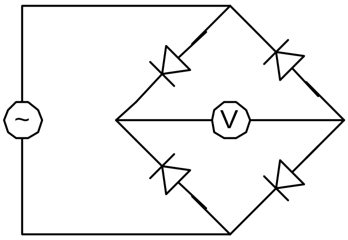


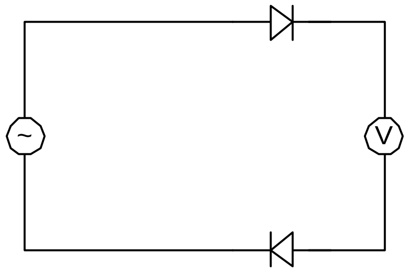
(A) I and II
(B) I and III
(C) I and IV
(D) II and III
(E) II and IV

解：在滤波电路中电感的作用是通低频，阻高频；电容的作用是通高频，阻低频。电阻则没有频率特性。II 和 III 的电路当频率很高时，输出信号近似等于输入。选（D）。



14. Which of the following circuits employing diodes constitutes a full-wave rectifier in which a sinusoidal input signal is converted to an output V(t), as shown in the diagram above?

(A)


(D)


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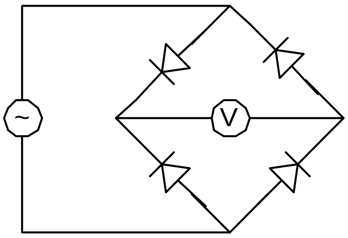

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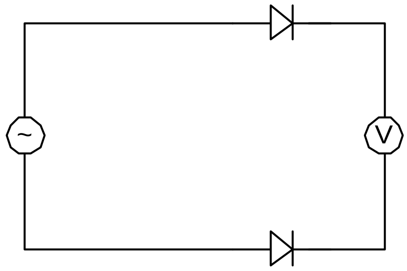

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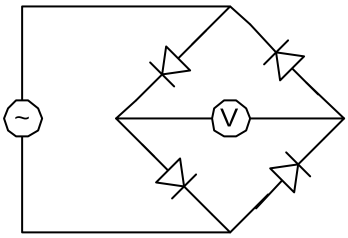


解：（B）、（E）不形成通路，（C）是短路，（D）是半波整流电路，只有（A）是全波整流电路。选（A）。

15. Which of the following measure the charge of an electron independently of its mass?

(A) Millikan oil-drop experiment
(B) Thomson experiment
(C) Franck-Hertz experiment
(D) Cyclotron resonance
(E) Compton effect

解：（A）是一个宏观实验，带电体是油滴，所以在测电荷电量时显然不涉及电子的质量。选（A）。

16. All of the following pure liquids or mixtures at 1 atmosphere pressure are correctly paired with the temperatures of the constant temperature baths they may be used to produce EXCEPT

(A) helium, 4K
(B) nitrogen, 77K
(C) hydrogen, 90K
(D) dry ice and acetone, 195K
(E) ice and water, 273K

解：单质 H_2 的沸点为 20.28K<90K。选（C）。

17. High-energy gamma rays can be produced by back-scattering laser light from a beam of very high-energy electrons. This method depends on the properties of

(A) Rayleigh scattering
(B) Thompson scattering
(C) Bragg scattering
(D) Raman scattering
(E) Compton scattering

解：为光子和电子间的碰撞，应属于 Compton 散射。只是常见的 Compton 散射是光子碰撞静止电子，因此光子能量减少，波长变长。本题中光子与高能电子碰撞，获得能量。选（E）。

18. An experimenter measures the counting rate from a radioactive source to be 10,150 counts in 100 minutes. Without changing any of the conditions, the experimenter counts for one minute. There is a probability of about 15 percent that the number of counts recorded will be fewer than

(A) 50
(B) 70
(C) 90
(D) 100
(E) 110

解：Poisson 分布。计数平均值为 N 时，某次特定的计数落在 $(N-\sqrt{N}, N+\sqrt{N})$ 区间里的几率为 68% ≈ 70%，所以单次计数 $< N-\sqrt{N}$ 的几率为

$$\frac{1-70\%}{2}=15\%\text{。}$$

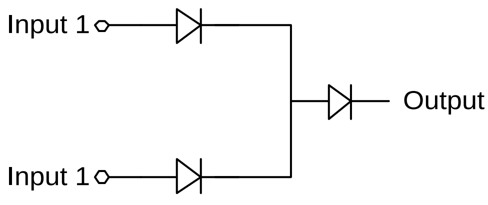
对于本题，1 分钟计数的最可几值为

$$\frac{10150}{100}\approx 100\text{，}$$

因此计数小于

$$100-\sqrt{100}=90$$

的几率约为 15%。选（C）。



19. If logical 0 is 0 volts and logical 1 is +1 volt, the circuit shown above is a logic circuit commonly known as

(A) an OR gate
(B) an AND gate
(C) a 2-bit adder
(D) a flip-flop
(E) a fanout

解：真值表如下

input1	input2	output
0	0	0

1	0	1
0	1	1
1	1	1

或门。答案选（A）。

20. An experimenter measures 9934 counts during one hour from a radioactive sample. From this number the counting rate of the sample can be estimated with a standard deviation of most nearly
(A) 100
(B) 200
(C) 300
(D) 400
(E) 500
解：Poisson 分布。标准差

$$\sqrt{9934} \approx 100 。$$

选（A）。

21. A charged particle traverses a proportional counter. About 10^4 times as many electrons are collected as are formed by the particle in traversing the counter. This is the result of
(A) stimulated emission
(B) photoelectron production
(C) ionization by collisions
(D) magnetic resonance
(E) the Auger effect
解：一个带电粒子通过正比计数器的时候，打中计数器里的气体原子，将其电离。电离后的正负粒子在电场作用下加速，进一步电离更多的原子，形成一个放电脉冲。选（C）。

22. A technique would most likely increase the signal-to-noise ratio of a photomultiplier tube is to
(A) operate the tube at a lower temperature
(B) operate the tube at higher voltage per dynode than usually specified
(C) use a radioactive source to saturate the noise level
(D) use a retarding potential on the first dynode
(E) use a photocathode of lower work function
解：降温是通常的方法。选（A）。

23. Which of the following is NOT an attribute of a typical photomultiplier tube?
(A) It operates by producing an avalanche of ions in a gas.
(B) It can produce electrical output pulses less than 10^{-6} second long.
(C) It can produce 10^3 or more electrons per incident photon.
(D) It is affected by a ambient magnetic fields greater than a few gauss.
(E) It detects photoelectrons ejected by light from a photocathode.
解：光电倍增管里要抽真空，不能有气体，（A）不对。选（A）。

24. Work hardening of a solid is a result of the
(A) tangling of dislocation lines

(B) annealing of point defects
(C) breaking of superfluous bonds
(D) generation of Frenkel pairs
(E) softening of the phonon spectrum
解：Work hardening：加工硬化，指金属由于发生了很大的形变从而超过其弹性限度后，将其硬化的过程。它的主要原因是金属中的缺陷错位在压力下聚集相互影响。选（A）。

25. Which of the following is most commonly used to provide a stable reference voltage of about 12 volts in an electronic circuit?
(A) Zener diode
(B) Tunnel diode
(C) Thynstor
(D) Vanstor
(E) Field-effect transistor
解：Zener 二极管可以用来稳压。选（A）。

26. Which of the following is most useful for measuring temperatures of about 3,000K?
(A) Optical pyrometer
(B) Carbon resistor
(C) Gas-bulb thermometer
(D) Mercury thermometer
(E) Thermocouple
解：Optical pyrometer 叫光学高温计，当然可用来测高温。选（A）。

27. A counter near a long-lived radioactive source measures an average of 100 counts per minute. The probability that more than 110 counts will be recorded in a given one-minute interval is most nearly?
(A) zero
(B) 0.001
(C) 0.025
(D) 0.15
(E) 0.5
解：Poisson 分布，15%。见 18 题。选（D）。

第九章 特殊专题

1. Except for mass, the properties of the muon most closely resemble the properties of the
(A) electron
(B) graviton
(C) photon
(D) pion
(E) proton
解：选（A）。μ子和电子只是同位旋不同。

2. Suppose that ${}_{Z}^{A}X$ decays by natural radioactivity in two stages to ${}_{Z-1}^{A-4}Y$. The two stages would most likely be which of the following?

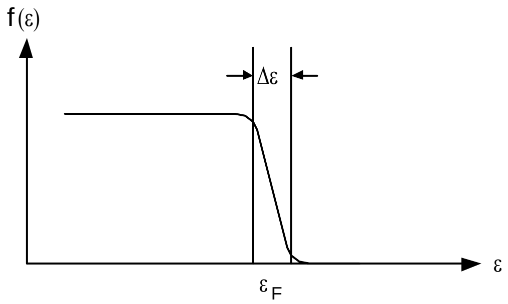
	First Stage	Second Stage
(A)	β^- emission with an antineutrino	α emission
(B)	β^- emission	α emission with a neutrino
(C)	β^- emission	γ emission
(D)	Emission of a deuteron	Emission of two neutrons
(E)	α emission	γ emission

解：选（A）。（B）错，α衰变不能出中微子。
antineutrino：反中微子
deuteron：氘核

3. The longest wavelength x-ray that can undergo Bragg diffraction in a crystal for a given family of planes of spacing d is
(A) $\dfrac{d}{4}$
(B) $\dfrac{d}{2}$
(C) d
(D) 2d
(E) 4d
解：由 Bragg 公式

$$2d\sin\theta = n\lambda ,$$

当θ=90°时（垂直入射），$\lambda_{\max} = 2d$ 。选（D）。



4. The Fermi distribution function f(ε) for electrons in a metal is shown above. Which of the following statements is true for this metal?

(A) The Fermi energy ε_F is of the order of kT.

(B) The spread in energies Δε is independent of the temperature.

(C) The higher the density of elections, the lower the Fermi level.

(D) The distribution results from the assumption that any number of elections can occupy a given quantum state.

(E) Only those elections which ~ kT of the Fermi level can be excited thermally.

解：费米能级ε_F 主要由电子决定。例如绝对零度下

$$\varepsilon_F = \frac{\hbar^2}{2m}\left(3n\pi^2\right)^{\frac{2}{3}},$$

其中 n 为电子浓度。费米能级ε_F比 kT 的量级要大很多，(A)不对。由上述关系，显然 n 越大费米能级ε_F 越大，或者说电子浓度越大时，电子需要占据的能态越高，(C)不对。费米能级的由来就是因为任一能级上只能占据有限个粒子，(D)不对。能级的展开Δε由热运动造成，只有费米能级ε_F以下 kT 左右的电子可被激发。选（E）。

5. Solid argon is held together by which of the following bonding mechanism?

(A) Ionic bond only

(B) Covalent bond only

(C) Partly covalent and partly ionic bond

(D) Metallic bond

(E) van der Waals bond

解：氩为惰性气体。满壳层结构，所以不可能是离子键或共价键。惰性气体均为分子晶体，为分子键，或称 van der Waals 键。选(E)。

6. A metal surface emits photoelectrons with maximum energy 1.0 electron volt (eV) when illuminated by 2.0-eV photons. When the surface is illuminated by 4.0-eV photons, the maximum energy of the photoelectrons is

(A) 0 eV

(B) 2.0 eV

(C) 3.0 eV

(D) 4.0 eV

(E) 5.0 eV

解：光电子的最大出射能量等于入射光子能量减去脱出功 W。根据第一次的数据，脱出功

$$W = 2.0 - 1.0 = 1.0 \text{ eV},$$

则第二次最大能量为 3.0 eV。选（C）。

7. Conduction electrons is an elemental semiconductor such as germanium and silicon can be produced by all of the following means EXCEPT

(A) thermal excitation
(B) optical excitation
(C) electron injection in a junction
(D) doping with a group III element
(E) doping with a group V element

解：锗硅均为四价，所以应引入五价的施主（经常为磷、砷）才能提高电子浓度。选(D)。

8. A necessary and sufficient condition that a force field F be conservative is

(A) $\nabla F = 0$
(B) $\nabla \cdot F = 0$
(C) $\nabla\nabla F = 0$
(D) $\nabla \cdot \nabla F = 0$
(E) $\nabla \times F = 0$

解：保守场的必须满足环量积分为 0，即

$$\oint \mathbf{F} \cdot d\mathbf{l} = 0\text{。}$$

由 Stokes 公式

$$\oint \mathbf{F} \cdot d\mathbf{l} = \oiint (\nabla \times \mathbf{F}) \cdot d\mathbf{s}$$

所以 $\nabla \times \mathbf{F} = 0$ 可以保证 $\oint \mathbf{F} \cdot d\mathbf{l} = 0$。选(E)。

9. Which of the following decays is possible in vacuum?

(A) $\pi^+ \to \mu^- + \nu_\mu$

(B) $\pi^- \to \pi^+ + e^- + e^-$

(C) $\pi^0 \to e^- + p$

(D) $p \to n + e^+ + \nu_e$

(E) $n \to p + e^- + \bar{\nu}_e$

解：（D）和（E）均可能发生。但（D）不能在真空中发生，原因是质子为稳定粒子（虽然大统一理论要求质子也能衰变，但是至少现在没有迹象证明这种假设）。自由中子为不稳定粒子，其寿命约为 917 秒。其他几个衰变的正确反应式为：

$$\pi^+ \to \mu^+ + \nu_\mu\text{，}$$

$$\pi^- \to \mu^- + \bar{\nu}_\mu\text{，}$$

$$\pi^0 \to \gamma + \gamma 。$$

选（E）。

10. The electrical conductivity of a relatively pure semiconductor increases with increasing temperature primarily because

(A) the scattering of the charge carriers decreases

(B) the density of the charge carriers increases

(C) the density of the material decreases due to volume expansion

(D) the electric field penetrates further into the material

(E) the lattice vibrations increase in amplitude

解：本征半导体（指未掺杂，没有施主或受主存在）载流子主要为热激发，载流子浓度由温度决定。温度越高，热激发能量越高，载流子浓度越大。具体公式为

$$n = p = \sqrt{N_+ N_-}\, e^{-E_g/2kT} ,$$

其中 E_g 为能隙宽度，N_+、N_-为随温度上升而上升的量。由此可见本征半导体的导电性能随温度升高而增强。选(B)。

11. Electrons of kinetic energy 20 keV are brought to rest by colliding with a solid. Which of the following is true of the shortest wavelength photon that may be emitted?

(A) It has an energy of 20 keV.

(B) It has an energy equal to the ionization energy of an atom in the solid.

(C) It must have a wavelength given by Bragg’s law for diffraction.

(D) It is in the infrared range.

(E) It is one of the characteristic x-rays of the solid.

解：当用电子轰击固体时，能量较低时只产生连续光谱；能量高到一定程度，产生的辐射光谱，在连续谱上还叠加由线性光谱（特征辐射）。连续光谱波长连续变化，存在一最短波长，对应于电子动能全部转化为 X 射线能量：

$$h\frac{c}{\lambda_{\min}} = eU 。$$

选(A)。

12. Which of the following quantities is the most suitable to measure in order to determine whether a semiconductor is n-type or p-type?

(A) Resistivity.

(B) Hall coefficient.

(C) Diffusion coefficient.

(D) Carrier lifetime.

(E) Tunneling rate.

解：Hall 系数为

$$R_H = \frac{1}{nq}$$

其中 n 为载流子浓度，q 为载流子所带电量。n 型半导体霍尔系数为负，p 型为正。选（B）。

13. Each of the persons named below is noted for contributions to our knowledge of electricity and magnetism. In which of the following are they placed in chronological order of their scientific contributions?

(A) A. Volta, B. Franklin, J. C. Maxwell, M. Faraday, H. Hertz
(B) B. Franklin, A. Volta, M. Faraday, J. C. Maxwell, H. Hertz
(C) A. Volta, B. Franklin, M. Faraday, H. Hertz, J. C. Maxwell
(D) B. Franklin, M. Faraday, A. Volta, H. Hertz, J. C. Maxwell
(E) M. Faraday, A. Volta, H. Hertz, B. Franklin, J. C. Maxwell

解：Franklin 必为老大，因为此君对电只有感性上的认识，比如雨夜放风筝玩。Volta 做出了第一个实用的电池，也数远古贡献。Faraday，电磁感应定律，十九世纪上半叶。Maxwell 和 Hertz 比较容易混，记住 Maxwell 预言了电磁波的存在，而 Hertz 用实验证实。选（B）。

14. The energies of alpha particles emitted by naturally occurring radioactive nuclei typically range from

(A) 1 eV to 10 eV
(B) 100 eV to 1keV
(C) 10 keV to 100 keV
(D) 1 MeV to 10 MeV
(E) 100 MeV to 1000 MeV

解：早期的核试验中经常用α衰变放出的α粒子去轰击其他物质，可见其能量不会很小。答案选（D）。作为常识应记住这个答案。

15. Which of the following is NOT true for the Debye theory of the specific heat of solids?

(A) The number of vibrational modes is 3N where N is the number of atoms in the solid.
(B) The vibrational modes are all assumed to have the same frequency.
(C) Well above the Debye temperature, the classical equipartition theorem holds.
(D) The specific heat is proportional to T^3 at low temperatures.
(E) The lattice modes are assumed to correspond to sound waves in the solid.

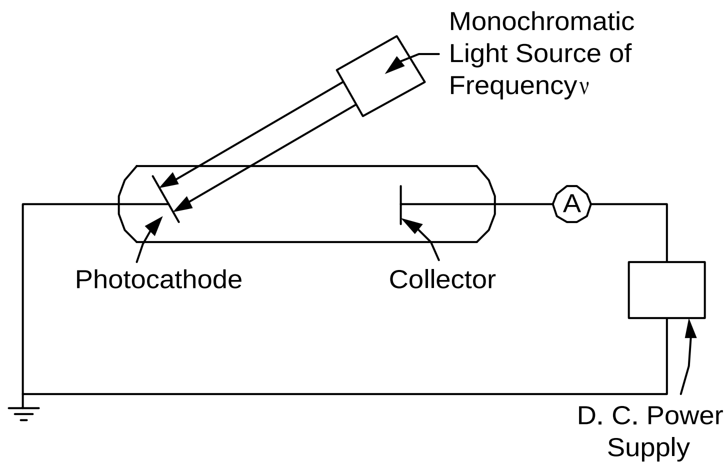
解：选项（B）为 Einstein 模型的假设，所有振动模式频率相等。而 Debye 模型要求ω从 0 连续变大到一截至频率，即 Debye 频率。答案选（B）。其他几个为 Debye 模型的假设或结论，可以记忆一下。

16. Which of the following statements concerning the electrical conductivities at room temperature of a pure copper sample and a pure silicon sample is NOT true?

(A) The conductivity of the copper sample is many orders of magnitude greater than that of the silicon sample.
(B) If the temperature of the copper sample is increased, its conductivity will decrease.
(C) If the temperature of the silicon sample is increased, its conductivity will increase.
(D) The addition of an impurity in the copper sample always decreases its conductivity.
(E) The addition of an impurity in the silicon sample always decreases its conductivity.

解：对于导体（本题中为铅），随着温度上升电子与离子实的碰撞逐渐剧烈，电子自由程下降，导电性能降低；对于本征半导体(本题中为硅)，温度上升使载流子浓度上升，导电能力

升高。而掺杂（施主或受主）往往能使半导体的导电性大大增加。选（E）。



Question 17-18 refer to the following apparatus used to study the photoelectric effect.

In this apparatus, the photocathode and the collector are made from the same material. The potential V of the collector, measured relative to ground, initially zero and is then increased or decreased monotonically. The effect is described by Einstein's photoelectric equation

$$|eV| = h\nu - W$$

17. When the photoelectric equation is satisfied and applicable to this situation, V is the
(A) negative value at which the current stops
(B) negative value at which the current starts
(C) positive value at which the current stops
(D) positive value at which the current starts
(E) voltage induced when the light is on

解：收集极加负电压，排斥电子。只有能量足够高的电子才能被收集。光电子出射方向各异，水平出射最易达到收集极。当收集极恰好接收不到电子时，说明|eV|是出射光电子的最大动能。选（A）。

18. The photoelectric equation is derived under the assumption that
(A) electrons are restricted to orbits of angular momentum $n\hbar$, where n is an integer
(B) electrons are associated with waves of wavelength $\lambda = h/p$, where p is momentum
(C) light is emitted only when electrons jump between orbits
(D) light is absorbed in quanta of energy $E = h\nu$
(E) light behaves like a wave

解：熟知的常识。选（D）。

19. The quantity W in the photoelectric equation is the
(A) energy difference between the two lowest electron orbits in the atoms of the photocathode
(B) total light energy absorbed by the photocathode during the measurement
(C) minimum energy a photon must have in order to be absorbed by the photocathode

(D) minimum energy required to free an electron from its binding to the cathode material
(E) average energy of all electrons in the photocathode
解：W 表示脱出功。选（D）。

20. Which of the following nuclei has the largest binding energy per nucleon? (Consider the most abundant isotope of each element.)
(A) Helium
(B) Carbon
(C) Iron
(D) Uranium
(E) Plutonium
解：核子的平均结合能和原子序数 Z 的关系是：两头小，中间大，在 Z=55 左右达到极值。大家可以思考一下为什么为获取能量，Z 小时为核聚变，Z 大时为核裂变。铁的原子序数 Z=56，平均结合能最大。选（C）。不要误以为铀是核燃料就选择（D）。

21. The Hall effect is used in solid-state physics to measure
(A) ratio of charge to mass
(B) magnetic susceptibility
(C) the sign of the charge carriers
(D) the width of the gap between the conduction and valence bands
(E) Fermi energy
解：Hall 系数

$$R_H = \frac{1}{nq},$$

其中 n 为载流子浓度，q 为载流子电量。当载流子带负电时，霍尔系数为负；相反则为正。选（C）。此外 Hall 系数还可用来测量载流子浓度。

22. One feature common to both the Debye theory and the Einstein theory of the specific heat of a crystal composed of N identical atoms is that the
(A) average energy of each atom is 3kT
(B) vibrational energy of the crystal is equivalent to the energy of 3N
(C) crystal is assumed to be continuous for all elastic waves
(D) speed of the longitudinal elastic waves is less than the speed of the transverse elastic waves
(E) upper cutoff frequency of the elastic waves is the same
解：Einstein 和 Debye 模型的共同点是：晶格振动等效为 3N 个谐振子（因为自由度为 3N）；不同点是：Einstein 很粗糙地假设所有谐振子频率相同，Debye 假设ω从 0 增大到一个截至频率ω_D。答案选（B）。

23. The mean kinetic energy of electrons in metals at room temperature is usually many times the thermal energy kT. Which of the following can best be used to explain this fact?
(A) the energy-time uncertainty relation
(B) The Pauli exclusion principle
(C) The degeneracy of the energy levels

(D) The Born approximation
(E) The wave particle duality

解：Pauli 不相容原理限制一个能级上可允许的电子数目，从而迫使电子向更高能级上排布。使平均动能往往远大于 kT。选（B）。

24. When a narrow beam of monoenergetic electrons impinges on the surface of a single metal crystal at an angle of 30 degrees with the plane of the surface, first-order reflection is observed. If the spacing of the reflecting crystal planes is known from x-ray measurements to be 3 angstroms, the speed of the electrons is most nearly

(A) 1.4×10^{-4} m/s
(B) 2.4 m/s
(C) 5.0×10^{3} m/s
(D) 2.4×10^{6} m/s
(E) 4.5×10^{9} m/s

解：由 Bragg 公式

$$2d\sin\theta = n\lambda\,,$$

本题中 n = 1，θ = 30°。所以速度为

$$v=\frac{P}{m}=\frac{h}{\lambda m}=\frac{h}{2d\lambda\sin\theta}\,。$$

代入数字计算，选(D)。

25. ${}^{13}_{5}B \rightarrow {}^{7}_{3}Li + {}^{4}_{2}He + Q$

An unstable boron nucleus decays at rest into a lithium nucleus and a helium nuclues.with total kinetic energy Q as shown above. Which of the following is true?

(A) Each decay product has half the kinetic energy.
(B) Each decay product has the same speed
(C) The decay products tend to go in the same direction.
(D) The lithium nucleus has more momentum than the helium nucleus.
(E) The helium nuclueus has more kinetic energy than the lithium nucleus.

解：反应前后动量守恒。由于反应前重子静止，动量为 0。所以反应后锂原子和氦原子动量大小相等，方向相反。质量大的动能小。选（B）。

26. Which of the following properties of a nucleus can be determined from studies of the hyperfine structure of optical spectra

(A) Spin
(B) Mass defect
(C) Binding energy
(D) Charge radius
(E) Atomic number

解：产生超精细结构的原因是，核外电子的总角动量与核的自旋耦合，组成新的总角动量。选（A）。

27. Why is β^{-} decay more common than β^{+} decay among naturally occurring radioactive

elements?

(A) The binding of positions in nuclei is greater than that of electrons.

(B) Electrons are more easily excited by γ rays than are positrons.

(C) The binding energy per nucleon is approximately constant for heavy nuclei.

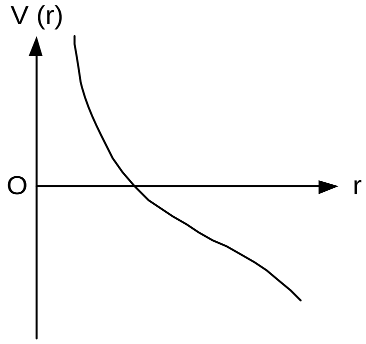
(D) Positrons cannot exist in ordinary matter.

(E) Alpha (α) decay leaves nuclei neutron-rich.

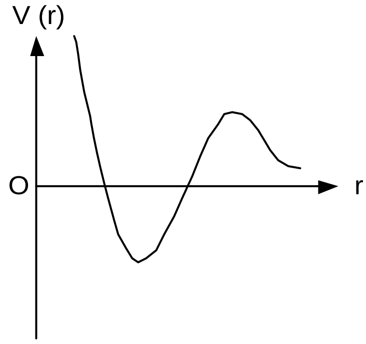
解：β 衰变本质上是中子与质子间的转变。丰中子核发生β^-衰变，使一个中子变为质子，增加一个核电荷数。相反贫中子核发生β^+衰变，质子变为中子。由于自然界中丰中子核占多数，另外α衰变使核中的中子过剩，倾向于发生β^-衰变。选（E）。

28. A meson consists of a quark and an antiquark bound by the strong interaction potential V(r), where r is the distance between them. Which of the following curves most closely represents this potential?

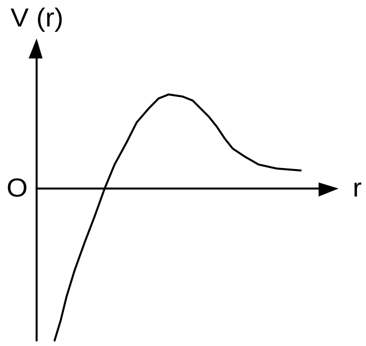
(A)



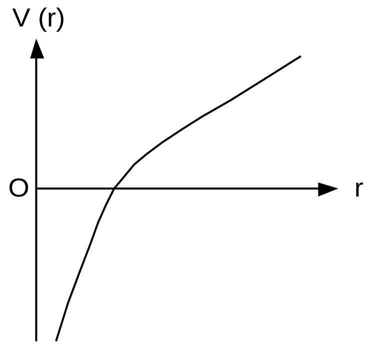
(B)



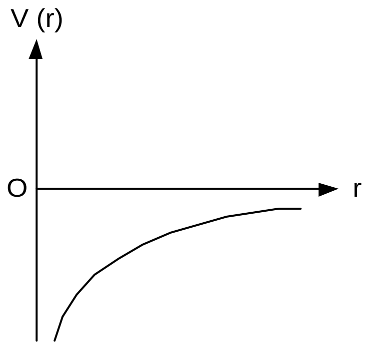
(C)



(D)



(E)



解：V(r)满足：⑴ $r \to \infty$ 时，$V(r) \to 0$。由此排除（A)、（D)。⑵ r 比较大时为吸引力， r 小时为排斥力，阻碍夸克与反夸克无限靠近。所以 r 较小时 V(r)为正，而且当 $r \to 0$ 时，V(r) $\to \infty$ 。排除（C)、（E)。选（B)。

29. The particle decay $\Lambda \to p + \pi^-$ must be a weak interaction because

(A) the π^- is a lepton.

(B) the Λ has spin zero

(C) no neutrino is produced in the decay

(D) it does not conserve angular momentum

(E) it does not conserve strangeness

解：核子和π 介子等强子统称为普通强子。普通强子和光子、电子、μ子、τ子、中微子以及他们的反粒子和起来称为普通粒子。奇异子，例如Λ^0，K^0，$K^{\pm}$，Σ^0，$\Sigma^{\pm}$ 等，有确定的奇异数 S。在强相互作用中和电磁相互作用中 S 守恒，在弱相互作用中 S 不守恒。本题中 Λ 的奇异数为 S = –1，而 p、π^-，为普通粒子，S 不守恒。选（E）。

30. A necessary and sufficient condition that a force field F be conservative is

(A) $\nabla F = 0$

(B) $\nabla \cdot F = 0$

(C) $\nabla\nabla F = 0$

(D) $\nabla \cdot \nabla F = 0$

(E) $\nabla \times F = 0$

解： $F = \nabla\varphi \Leftrightarrow \nabla \times F = \nabla \times \nabla\varphi = 0$，有势场即无旋场。选（E）。

31. A metal surface emits photoelectrons with maximum energy 1.0 electron volts (eV) when illuminated by 2.0-eV photons. When the surface is illuminated by 4.0-eV photons, the maximum energy of the photoelectrons is

(A) 0eV

(B) 2.0eV

(C) 3.0eV

(D) 4.0eV

(E) 5.0eV

解：根据 Einstein 的光电理论，金属的脱出功

$$A = h\nu - E_k = 1.0\text{eV}，$$

当光子能量为 4.0eV 时，

$$E_k = h\nu - A = 3.0\text{eV}。$$

选（C）。

32. A beam of particles is incident on a thin target of thickness t. If the cross section per nucleus for a scattering of the particles by the nuclei of the of the target is σ, and the number of nuclei per unit volume is n, the fraction of particles scattered is

(A) σ / nt

(B) σt

(C) ntσ

(D) nt / σ

(E) nσ

解：核反应截面的物理含义是表示一个入射粒子与靶上一个靶核发生核反应的几率，具有面积的量纲。定义为

$$\sigma = \frac{n_r}{n_i N t}\text{。}$$

因此发生核反应的几率

$$\frac{n_r}{n_i} = nt\sigma\text{。}$$

选（C）。

33. From a knowledge of the sizes of atoms and nuclei, which of the following is the most reasonable approximation for the ratio of the density of nuclear matter to the density of water?

(A) 1

(B) 10^3

(C) 10^6

(D) 10^{10}

(E) 10^{14}

解：原子核的质量 m ~ 10^{-27} kg，线度 r ~ 10^{-15} m，水的密度 $\rho_0 = 10^3$ kg/m^3，因此估算数量级有

$$\frac{\rho}{\rho_0} = \frac{m}{\frac{4}{3}\pi r^3 \rho_0} \approx \frac{10^{-27}}{10^{-45}\cdot 10^3} \sim 10^{-14}\text{。}$$

选（E）。

34. What are the changes in the mass number A and atomic number Z of a numbers that undergoes electron capture?

(A) A is unchanged; Z decreases by 2.

(B) A is unchanged; Z decreases by 1.

(C) A is unchanged; Z increases by 1.

(D) A decreases by 2; Z increases by 1.

(E) A decreases by 4; Z decreases by 2.

解：电子吸收的一般反应式为

$${}_Z^AX + e \rightarrow {}_{Z-1}^{A}Y\text{。}$$

选（B）。注意，一些主要的核反应衰变形式例如α衰变、β衰变（β^-衰变、β^+衰变和轨道电子俘获 EC）、γ衰变的反应式应该掌握。

35. A hypothetical boson has a nonzero value for all of the following quantities. Which quantity is the same for the boson and its antiparticle? (Ignore the existence of weak interactions.)

(A) Strangeness

(B) Charm
(C) Intrinsic parity
(D) Magnetic moment
(E) Electric charge

解：选（C）。关于正反粒子，它们的质量、寿命、自旋磁矩的 g 因子完全相同。由于由一对正反粒子组成的纯中性体系具有确定的宇称，推出正反 Fermion 有相反的内禀宇称，正反 Boson 有相同的内禀宇称。请记住该结论。

36. Which of the following could NOT be expected to contribute substantially to the resistivity of a metal crystal as its temperature approaches absolute zero

(A) Phonons
(B) Dislocations
(C) Atomic impurities
(D) Grain boundaries
(E) Lattice vacancies

解：金属晶体中存在的杂质与缺陷等因素将破坏周期势场，引起电子的散射，对金属的电阻是有贡献的。而平均声子数随温度的下降指数衰减，在低温极限下，只有晶格的低频振动即长声学波才对散射有贡献。且随着温度的降低，有贡献的晶格振动模式不断减少，可以忽略。选（A）。

37. According to the nuclear shell model, the spin and parity, respectively, of lithium-7 (Z = 3, N = 4) should be

(A) 0, even
(B) 1/2, even
(C) 1, odd
(D) 3/2, even
(E) 3/2, odd

解：根据原子核的强自旋轨道耦合壳层模型，Li–7 是奇偶核，未配对的核子是第三个质子，处于 $1p_{3/2}$ 态，因此自旋为奇。选（E）。

38. The lines in the characteristic emission spectra of elements in galaxies at great distance from Earth are shifted to longer wavelengths relative to those observed from these elements on Earth. The principal cause of this shift in wavelength is commonly thought to be

(A) scattering of the shorter wavelength compoments out of the light from the galaxies, similar to the effect observed in the reddening of the light from the Sun when it is near the horizon
(B) multiple scattering of the photons during their trip to Earth
(C) an increase in the velocity of the photons during their trip to Earth.
(D) a gravitational effect as the radiation escapes
(E) a Doppler effect caused by the galaxies reddening from the Earth

解：遥远星系发出的光谱谱线都是红移的，这是运动光源的 Doppler 效应引起的。根据 Hubble 膨胀宇宙的观点，所有的星系都在离开地球而膨胀。选（E）。

39. In the nuclear reaction $\alpha + {}^{40}Ar \rightarrow p + X$, the nucleus X is

(A) ^{41}K
(B) ^{42}K
(C) ^{43}K
(D) ^{44}K
(E) ^{41}Ar
解：由反应的质量数和电荷数守恒计算可得，

$$A_X = 4 + 40 - 1 = 43,$$

$$Z_X = 2 + 18 - 1 = 19。$$

选（C）。

40. A radioactive nucleus A has a mean life of 10 seconds. The nucleus decays into its daughter B. The mean life of B is 10 years. Starting with 10^{22} nuclei A at t = 0, the rate of decay of B at t = 1 year is most nearly
(A) 10^{22} per year
(B) 10^{21} per year
(C) 10^{20} per year
(D) 10^{19} per year
(E) zero
解：由于$\tau_A << t < \tau_B$，近似可以认为在 t 时刻内所有的 A 核都衰变成了 B 核，而大部分 B 核还未衰变。$n_B \sim 10^{22}$。根据原子核的衰变规律

$$\frac{dn}{dt} = -\frac{n}{\tau} \approx \frac{10^{22}}{10} = 10^{21} \text{ / year}。$$

选（B）。

41. Which of the following reactions or decays is allowed by the laws of nature?

(A) $p \to \pi^+ + \pi^0$

(B) $\pi^- + p \to K^+ + \Sigma^-$

(C) $\pi^- \to e^- + \gamma$

(D) $p \to \Sigma^+ + \pi^0$

(E) $p + p \to n + p + \pi^+ + \pi^- + \pi^0$

解：（A）反应前后重子数不守恒，（C）反应前后轻子数不守恒，（D）反应前后奇异数不守恒（注意强相互作用中末态奇异粒子必须协同产生，才能满足奇异数守恒），（E）反应前后电荷数不守恒。选（B）。

42. For a simple inorganic lattice such as NaCl, first-order Bragg scattering of x-rays might reasonably take place at close to 30° for x-rays of wavelength most nearly equal to

(A) 0.3 A
(B) 3.0 A
(C) 30 A
(D) 300 A
(E) 3000 A
解：典型晶体的晶格常数 d~A。根据 Bragg 衍射条件，

$$2d\sin\theta = n\lambda,$$

n = 1，θ = 30°时有

$$\lambda = d \text{。}$$

因此λ ~ A。选（B）。

43. Which of the following is NOT characteristic of any superconductor?
(A) The resistivity vanishes below the transition temperature.
(B) A sufficiently large magnetic field can destroy the superconducting state.
(C) A gap exists in the allowable energy levels of the material.
(D) A magnetic field is excluded from the superconductor.
(E) The superconductor is paramagnetic.
解：超导体的基本特性有：在临界温度以下，超导体呈现零电阻性和完全抗磁性。超导态可以被大于临界磁场的外加磁场破坏。超导体一般不是铁磁体。选（C）。

44. If only the volume and surface energies are considered, which of the following expressions best approximates the binding energy of a large nucleus containing A nucleons (a and b are positive constants)?

(A) $aA - bA^{-1/3}$

(B) $aA - bA^{-2/3}$

(C) $aA - bA^{2/3}$

(D) $aA + bA^{1/3}$

(E) $aA + bA^{2/3}$

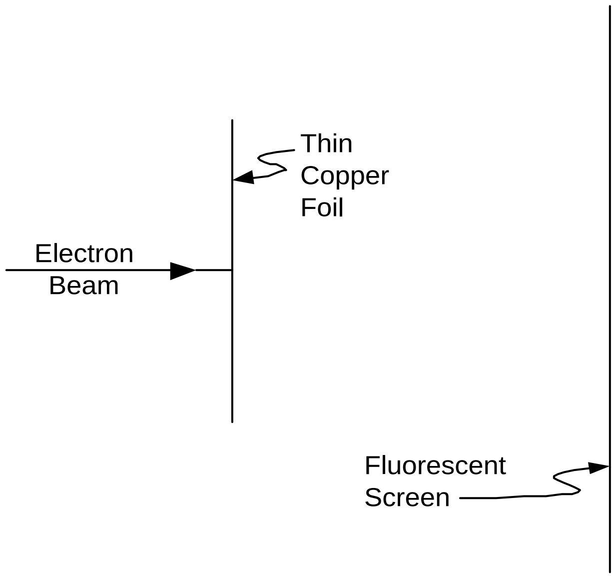
解：体积能正比于核子个数 A，处于表面的核子能量比内部的要低，而表面的总原子数正比于 $A^{2/3}$，因此总能量应具有

$$aA - bA^{2/3}$$

的形式。选（C）。

45. The ratio of the nuclear radius to the atomic radius of an element near the middle of the periodic table is most nearly
(A) 10^{-2}
(B) 10^{-5}
(C) 10^{-8}

(D) 10^{-11}
(E) 10^{-14}
解：作为常识，原子半径的数量极为 10^{-10} 米，而原子核半径为 10^{-15} 米。选（B）。



46. In order to observe a ring diffraction pattern on the screen shown above, which of the following conditions must be met?
(A) The electron beam must be polarized.
(B) The electron beam must be approximately monoenergetic.
(C) The copper foil must be a single crystal specimen.
(D) The copper foil must be uniform thickness.
(E) The electron beam must strike the foil at normal incidence.
解：晶体结构的实验衍射方法主要有以下几种。
Laue 法：用一束连续波长的 X 射线或中子辐射束照射固定的单晶上。衍射图像是一组亮斑点。
旋转晶体法：单晶围绕一个固定轴在单色 X 射线束中旋转，改变 θ 角使不同原子面满足布拉格方程。当θ 角满足条件时，入射 X 射线束就从衍射晶面反射。
Debye 粉末法：单色 X 射线束照射在细粉末样品或细晶粒多晶样品上，微晶取向接近连续变化。衍射图像是一系列同心圆环。
本题类似于第三种情况，电子束能量单一化，就保证其波长近似相等。选（B）。选项（D）、（E）并不是必要的。

47. Materials that are good electrical conductors also tend to be good thermal conductors because
(A) they have highly elastic lattice structures
(B) they have energy gaps between the allowed electron energy bands
(C) impurities aid both processes
(D) surface states are important in both processes
(E) conduction electrons contribute to both processes
解：导体可看作由固定于晶格上的离子实和自由传导电子组成。传导电子做近自由运动，联系着导体的不同部分，在导电的同时还承担着热传导的重任。电子在温度较高的地方通过与离子实碰撞，获得较高的能量；在温度较低的地方，把一部分能量交给离子实。定量的说，

金属导体的电导率为

$$\sigma = \frac{ne^2\tau}{m},$$

而热导率为

$$\kappa = \frac{1}{3}Cvl = \frac{1}{3}nc\overline{v^2}\tau,$$

其中 τ 为弛豫时间，c 为平均一个粒子的热容。Frantz 定律更直接地表现了二者间的关系

$$\frac{\kappa}{\sigma T} = \frac{3}{2}\left(\frac{k_B}{e}\right)^2 = 1.11\times10^{-8}\ \mathrm{W\cdot\Omega/K^2}\text{。}$$

选（E）。

48. Which of the following statements about the lattice vibrations of a crystal of NaCl is NOT true?

(A) The acoustic modes for long wavelengths correspond to elastic waves.

(B) In the acoustic mode, the Na and Cl ions tend to move in the same direction.

(C) For each value of the propagation vector k, there is one transverse acoustic mode and one longitudinal acoustic mode.

(D) In the optical mode, the Na and Cl ions move in opposite directions.

(E) The optical mode has an oscillatory dipole moment.

解：一个纵波两个横波。选（C）。

49. Which of the following would NOT be expected to contribute substantially to the resistivity of a metal crystal as its temperature approaches absolute zero?

(A) Phonons

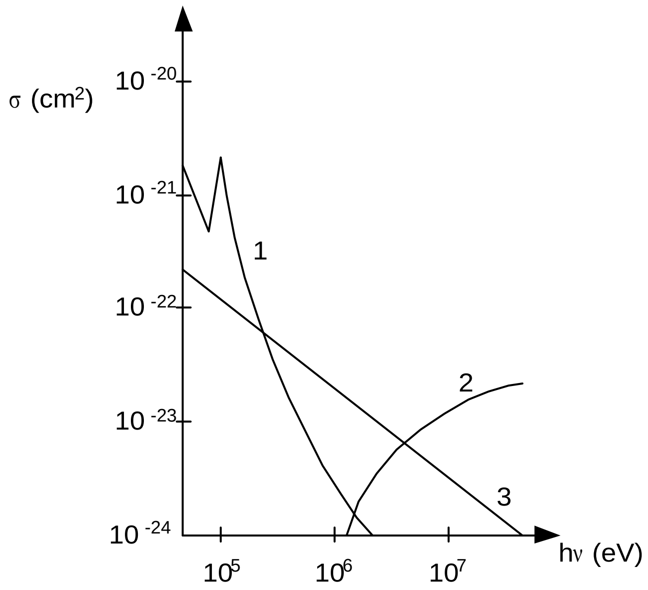
(B) Dislocations

(C) Atomic impurities

(D) Grain boundaries

(E) Lattice vacancies

解：声子比热~T^3，当 T 趋于 0 时趋于 0。选（A）。



50. The figure above shows the photon interaction cross section for lead in the energy range where the Compton, photoelectric, and pair production processes all play a role. What is the correct identification of these cross sections?

(A) 1=photoelectric, 2=Compton, 3=pair production

(B) 1=photoelectric, 2=pair production, 3=Compton

(C) 1=Compton, 2=pair production, 3=photoelectric

(D) 1=Compton, 2=photoelectric, 3=pair production

(E) 1=pair production, 2=photoelectric, 3=Compton

解：光电效应有一个截止频率，只有 1 能反映出“截止”的特性（不光滑），因此 1 是光电效应。Pair production 是光子产生正负电子对的过程，显然能量越高越能产生电子对，即σ随 hν增大而增大，2 是 pair production。选（B）

第三部分 模拟练习题

模拟练习题一

1. The wave function of a particle is $e^{i(kx-\omega t)}$, where x is distance, t is time, and k and ω are positive real numbers. The x-component of the momentum of the particle is

(A) 0
(B) $\hbar\omega$
(C) $\hbar k$
(D) $\frac{\hbar\omega}{c}$
(E) $\frac{\hbar k}{\omega}$

2. The longest wavelength x-ray that can undergo Bragg diffraction in a crystal for a given family of planes of spacing d is

(A) $\frac{d}{4}$
(B) $\frac{d}{2}$
(C) d
(D) 2 d
(E) 4 d

3. The ratio of energies of the K characteristic x-rays of carbon (Z = 6) to those of magnesium (Z = 19) is most nearly

(A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) 1
(D) 2
(E) 4

<u>Questions 4-5</u>

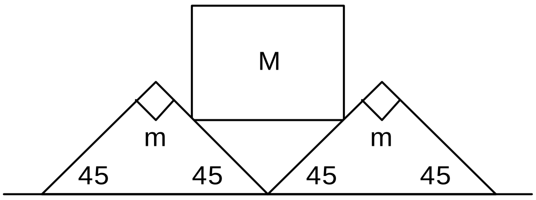
The magnitude of the Earth's gravitational force on a point mass is F(r), where r is the distance from the Earth's center to the point mass. Assure the Earth is homogeneous sphere of radius R.

4. What is $\dfrac{F(R)}{F(2R)}$

(A) 32
(B) 8
(C) 4
(D) 2
(E) 1

5. Suppose there is a very small shaft in the Earth such that the point mass can be placed at a radius of R/2. What is $\dfrac{F(R)}{F\left(\dfrac{R}{2}\right)}$

(A) 8
(B) 4
(C) 2
(D) $\dfrac{1}{2}$
(E) $\dfrac{1}{4}$



6. Two wedges, each of mass m, are placed next to each other on a flat floor. A cube of mass M is balanced on the wedges as shown above. Assume on friction between the cube and the wedges, but a coefficient of static friction μ < 1 between the wedges and the floor. What is the largest M that can be balanced as shown without motion of the wedges?

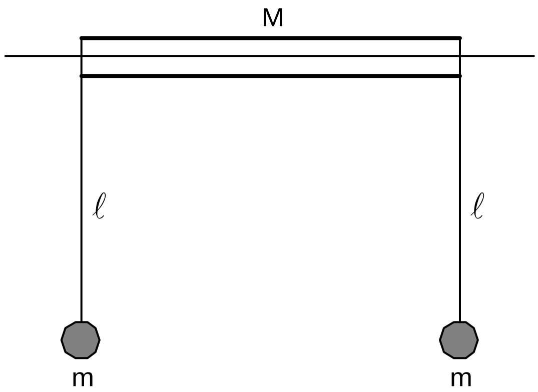
(A) $\dfrac{m}{\sqrt{2}}$

(B) $\dfrac{\mu m}{\sqrt{2}}$

(C) $\dfrac{\mu m}{1-\mu}$

(D) $\dfrac{2\mu m}{1-\mu}$

(E) All M will balance.



7. A cylindrical tube of mass M can slide on a horizontal wire. Two identical pendulums, each of mass m and length l, hang from the ends of the tube, as shown above. For small oscillations of the pendulums in the plane of the paper, the eigenfrequencies of the normal modes of oscillation of this system are 0, $\sqrt{\dfrac{g(M+2m)}{lM}}$, and

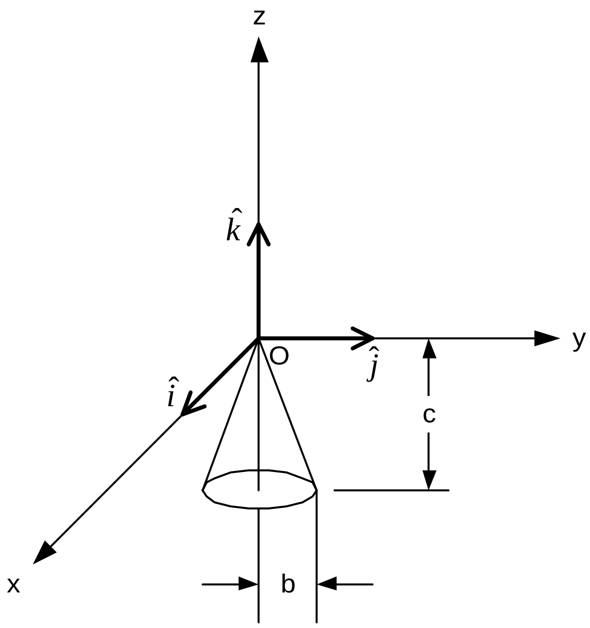
(A) $\sqrt{\dfrac{g}{l}}$

(B) $\sqrt{\dfrac{g}{l}\dfrac{M+m}{M}}$

(C) $\sqrt{\dfrac{g}{l}\dfrac{m}{M}}$

(D) $\sqrt{\dfrac{g}{l}\dfrac{m}{M+m}}$

(E) $\sqrt{\dfrac{g}{l}\dfrac{m}{M+2m}}$



8. A solid cone hangs from a frictionless pivot at the origin O, as shown above. If $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit vectors, and a, b, and c are positive constants, which of the following forces **F** applied to the rim of the cone at a point P results in a torque τ on the cone with a negative component τ_z?

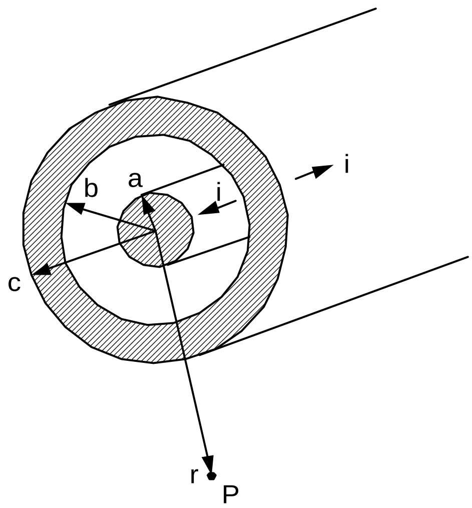
(A) $F = a\hat{k}$, P is (0, b, –c)

(B) $F = -a\hat{k}$, P is (0, –b, –c)

(C) $F = a\hat{j}$, P is (–b, 0, –c)

(D) $F = a\hat{j}$, P is (b, 0, –c)

(E) $F = -a\hat{k}$, P is (–b, 0, –c)



9. A coaxial cable having radii a, b and c carries equal and opposite currents of magnitude i on the inner and outer conductors. What is the magnitude of the magnetic induction at point P outside of the cable at distance r from the axis?

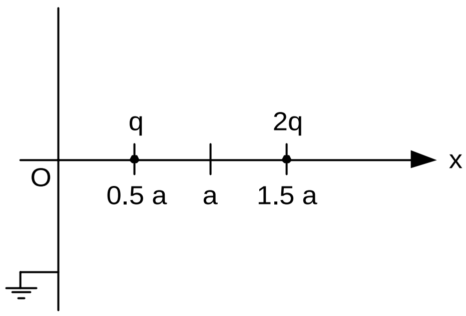
(A) Zero

(B) $\dfrac{\mu_0 ir}{2\pi a^2}$

(C) $\dfrac{\mu_0 i}{2\pi r}$

(D) $\dfrac{\mu_0 i}{2\pi r}\dfrac{c^2 - r^2}{c^2 - b^2}$

(E) $\dfrac{\mu_0 i}{2\pi r}\dfrac{r^2 - b^2}{c^2 - b^2}$



10. Two positive charges of q and 2q coulombs are located on the x-axis at x = 0.5a and 1.5a, respectively, as shown above. There is an infinite, grounded conducting plane at x = 0. What is the magnitude of the net force on the charge q?

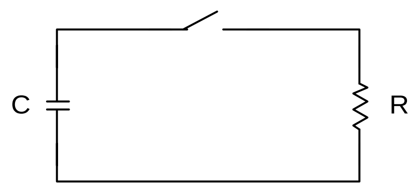
(A) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{a^2}$

(B) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{3q^2}{2a^2}$

(C) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{2q^2}{a^2}$

(D) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{3q^2}{a^2}$

(E) $\dfrac{1}{4\pi\varepsilon_0}\dfrac{7q^2}{2a^2}$



11. The capacitor in the circuit shown above is initially charged. After closing the switch, how much time elapses until one-half of the capacitor's initial stored energy is dissipated?

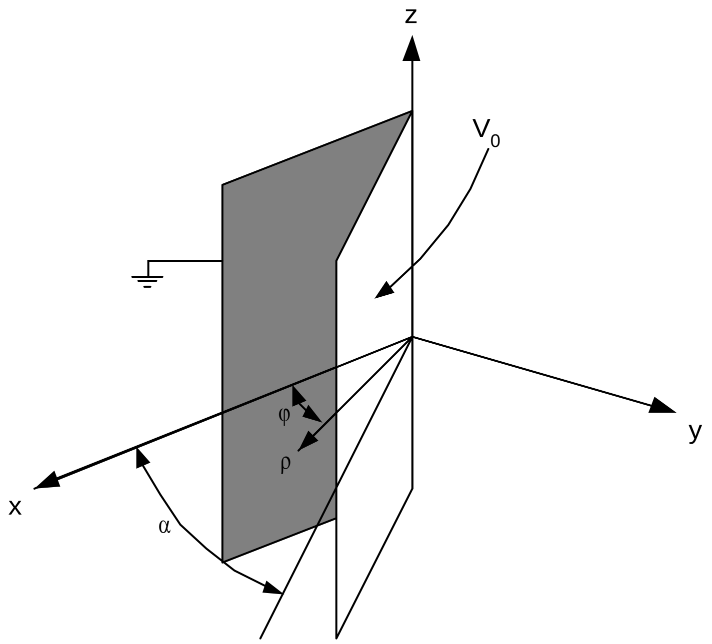
(A) RC

(B) $\dfrac{RC}{2}$

(C) $\dfrac{RC}{4}$

(D) 2RC ln(2)

(E) $\dfrac{RC\ln(2)}{2}$



12. Two large conducting planes from a wedge of angle α as shown in the diagram above. The plates are insulated from each other; one has a potential V_0 and the other is grounded. Assuming that the plates are large enough so that the potential difference between them is independent of the cylindrical coordinates z and ρ, the potential anywhere between the plates as a function of the angle φ is

(A) $\dfrac{V_0}{\alpha}$

(B) $\dfrac{V_0\varphi}{\alpha}$

(C) $\dfrac{V_0\alpha}{\varphi}$

(D) $\dfrac{V_0\varphi^2}{\alpha}$

(E) $\dfrac{V_0\alpha}{\varphi^2}$

13. Listed below are Maxwell's equations of electromagnetism. If magnetic monopoles exist, which of those equations would be INCORRECT?

I. Curl **H** = **J** + $\dfrac{\partial D}{\partial t}$

II. Curl **E** = $-\dfrac{\partial B}{\partial t}$

III. div **D** = ρ

IV. div **B** = 0

(A) IV only

(B) I and II

(C) I and III

(D) II and IV

(E) III and IV

14. The total energy of a blackbody radiation source is collected for one minute and used to heat water. The temperature of the water increases from 20.0°C to 20.5°C. If the absolute temperature of the blackbody were doubled and the experiment repeated, which of the following statements would be most nearly correct?

(A) The temperature of the water would increase form 20°C to a final temperature of 21°C.

(B) The temperature of the water would increase form 20°C to a final temperature of 24°C.

(C) The temperature of the water would increase form 20°C to a final temperature of 28°C.

(D) The temperature of the water would increase form 20°C to a final temperature of 36°C.

(E) The water would boil within the one-minute period.

15. A classical model of a diatomic molecule is a springy dumbbell, as shown above, where the dumbbell is free to rotate about axes perpendicular to the spring. In the limit of high temperature, what is the specific heat per mole at constant volume?

(A) $\dfrac{3}{2}R$

(B) $\dfrac{5}{2}R$

(C) $\frac{7}{2}R$

(D) $\frac{9}{2}R$

(E) $\frac{11}{2}R$

16. An engine absorbs heat at a temperature of 727℃ and exhausts heat at a temperature of 527℃. If the engine operates at maximum possible efficiency, for 2000 joules of heat input the amount of work the engine performs is most nearly

(A) 400 J

(B) 1450 J

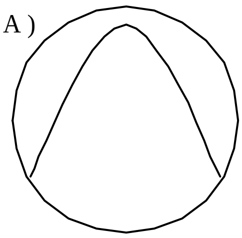
(C) 1600 J

(D) 2000 J

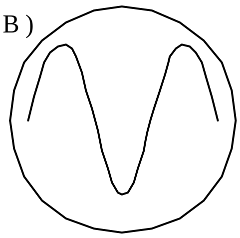
(E) 2760 J

17. The outputs of two electrical oscillators are compared on an oscilloscope screen. The oscilloscope spot is initially at the center of the screen. Oscillator Y is connected to the vertical terminals of the oscilloscope and oscillator X to the horizontal terminals. Which of the following patterns could appear on the oscilloscope screen, if the frequency of oscillator Y is twice that of oscillator X?

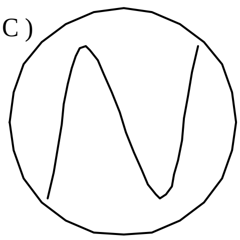
(A)



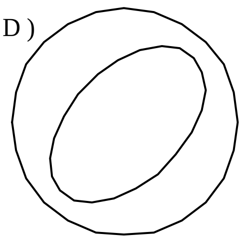
(B)



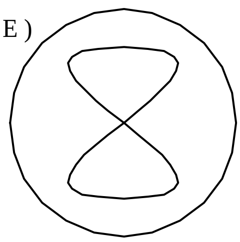
(C)



(D)



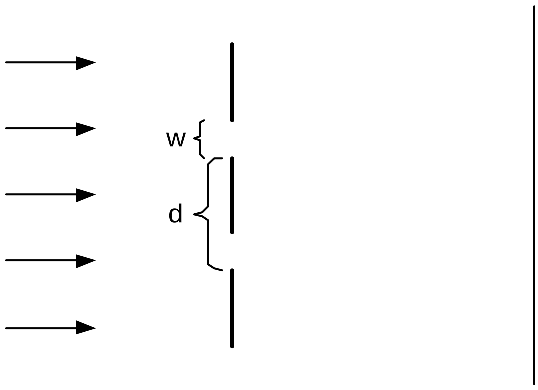
(E)



18. In transmitting high frequency signals on a coaxial cable, it is important that the cable be

terminated at an end with its characteristic impedance in order to avoid
(A) leakage of the signal out of the cable
(B) overhearing of the cable
(C) reflecting of signals from the terminated end of the cable
(D) attenuation of the signal propagating in the cable
(E) production of image currents in the outer conductor

19. Which of the following is most nearly the mass of the Earth? (The radius of the Earth is about 6.4×10^6 meters)
(A) 6×10^{24} kg
(B) 6×10^{27} kg
(C) 6×10^{30} kg
(D) 6×10^{33} kg
(E) 6×10^{36} kg



20. In a double-slit interference experiment, d is the distance between the centers of the slits and w is the width of each slit, as shown in the figure above. For incident plane waves, an interference maximum on a distant screen will be “missing” when

(A) $d = \sqrt{2}w$

(B) $d = \sqrt{3}w$

(C) 2d = w
(D) 2d = 3w
(E) 3d = 2w

21. A soap film with index of refraction greater than air is formed on a circular wire frame that is held in a vertical plane. The film is viewed by reflected light from a white-light source. Bands of color are observed at the lower parts of the soap film, but the areas near the top appears black. A correct explanation for this phenomenon would involve which of the following?
I. The top of the soap film absorbs all of the light incident on it; none is transmitted.
II. The thickness of the top part of the soap film has become much less than a wavelength of visible light.
III. There is a phase change of 180° for all wavelength of light reflected from the front surface

of the soap film.

IV. There is no phase change from any wavelength of light reflected from the back surface of the soap film.

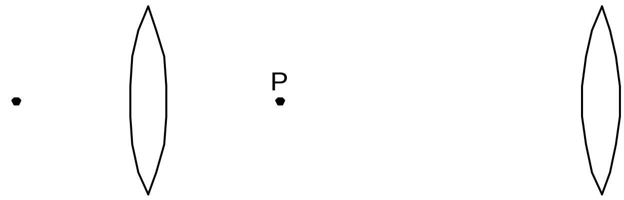
(A) I only

(B) II and III only

(C) III and IV only

(D) I, II and III

(E) II, III and IV



22. A simple telescope consists of two convex lenses, the objective and the eyepiece, which have a common focal point P, as shown in the figure above. If the focal length of the objective is 1.0 meter and the angular magnification of the telescope is 10, what is the optical path length between objective and eyepiece?

(A) 0.1 m

(B) 0.9 m

(C) 1.0 m

(D) 1.1 m

(E) 10 m

23. The Fermi temperature of Cu is about 80,000 K. Which of the following is most nearly equal to the average speed of a conduction electron in Cu?

(A) 2×10^{-2} m/s

(B) 2 m/s

(C) 2×10^{2} m/s

(D) 2×10^{4} m/s

(E) 2×10^{6} m/s

24. Solid argon is held together by which of the following bonding mechanism?

(A) Ionic bond only

(B) Covalent bond only

(C) Partly covalent and partly ionic bond

(D) Metallic bond

(E) van der Waals bond

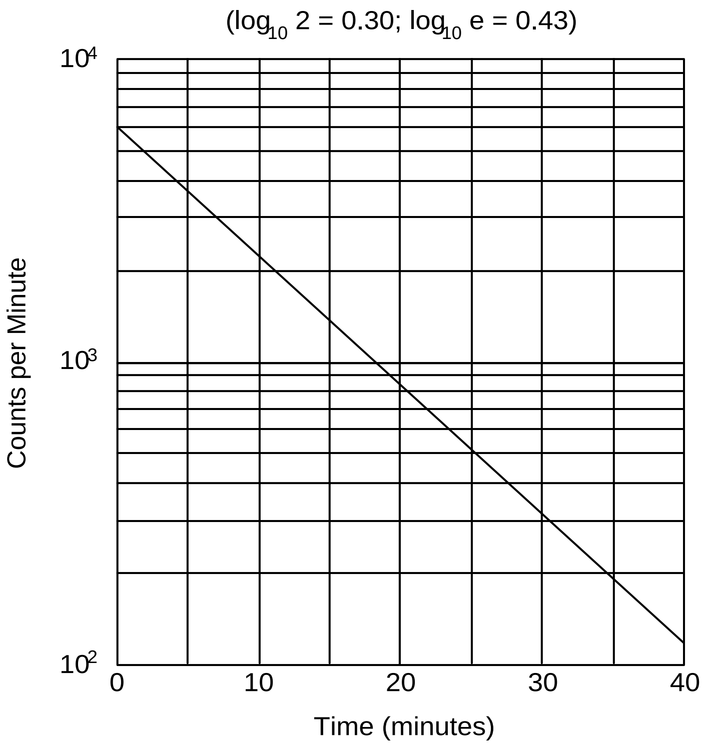
25. In experiments located deep underground, the two types of cosmic rays that most commonly reach the experimental apparatus are

(A) alpha particles and neutrons

(B) protons and electrons

(C) iron nuclei and carbon nuclei

(D) muons and neutrinos
(E) positrons and electrons



26. A radioactive nucleus decays, with the activity shown in the graph above. The half-life of the nucleus is
(A) 2 min
(B) 7 min
(C) 11 min
(D) 18 min
(E) 23 min

27. If a freely moving electron is localized in space to within Δx_0 of x_0, its wave function can be described by a wave packet $\psi(x,t) = \int_{-\infty}^{\infty} e^{i(kx-\omega t)} f(k)dk$, where f(k) is peaked around a central value k_0. Which of the following is most nearly the width of the peak in k?

(A) $\Delta k = \dfrac{1}{x_0}$

(B) $\Delta k = \dfrac{1}{\Delta x_0}$

(C) $\Delta k = \dfrac{\Delta x_0}{x_0^{\ 2}}$

(D) $\Delta k = \left(\frac{\Delta x_0}{x_0}\right) k_0$

(E) $\Delta k = \sqrt{{k_0}^2 + \left(\frac{1}{x_0}\right)^2}$

28. A system is known to be in the normalized state described by the wave function

$\psi(\theta,\varphi) = \frac{1}{\sqrt{30}}(5{Y_4}^3 + {Y_6}^3 - 2{Y_6}^0)$, where the $Y_l^m(\theta,\varphi)$ are the spherical harmonics.

The probability of finding the system in a state with azimuthal orbital quantum number m = 3 is

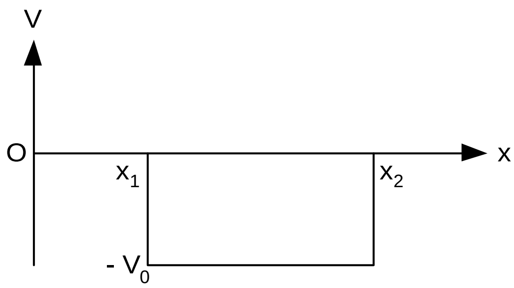
(A) 0

(B) $\frac{1}{15}$

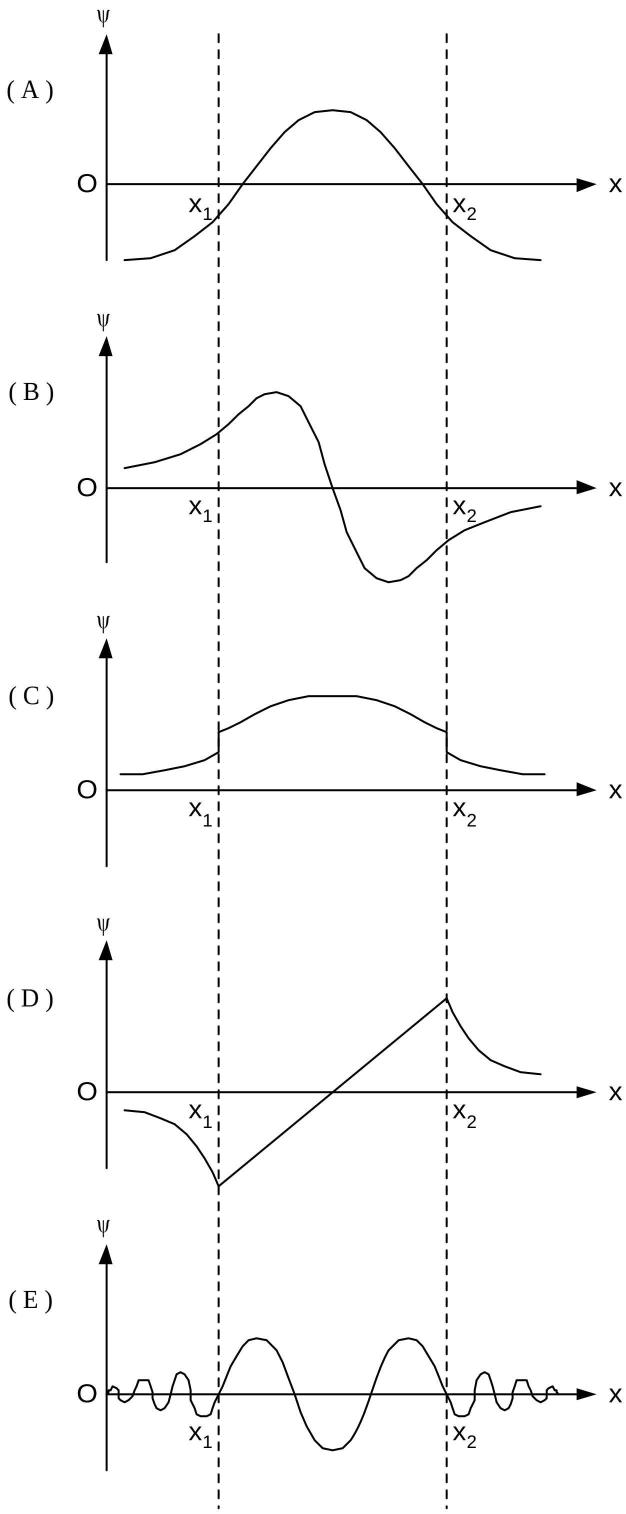
(C) $\frac{1}{6}$

(D) $\frac{1}{3}$

(E) $\frac{13}{15}$



29. An attractive, one-dimensional square well has depth V_0 as shown above. Which of the following bast shows a possible wave function for a bound state?



30. Given that the binding energy of the hydrogen atom ground state is E_0 = 13.6 eV, the binding energy of the n = 2 state of positronium (positron-electron system) is

(A) $8E_0$

(B) $4E_0$

(C) E_0

(D) $\dfrac{E_0}{4}$

(E) $\dfrac{E_0}{8}$

31. In a ^3S state of the helium atom, the possible values of the total electronic angular momentum quantum number are

(A) 0 only

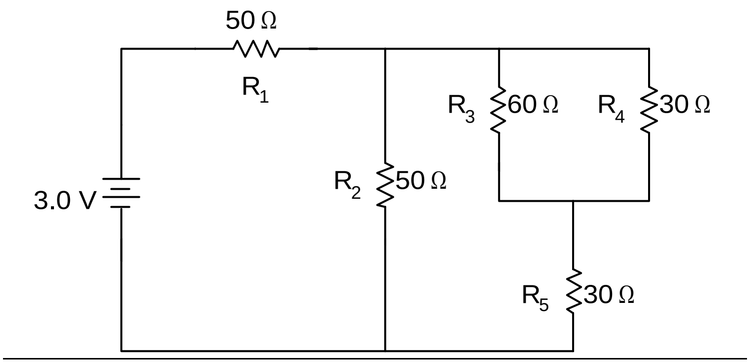
(B) 1 only

(C) 0 and 1 only

(D) 0, $\dfrac{1}{2}$, and 1

(E) 0, 1, and 2

<u>Question 32-33</u>



In the circuit shown above, the resistances are given in ohms and the battery is assumed ideal with emf equal to 3.0 volts.

32. The resistor that dissipates the most power is

(A) R_1

(B) R_2

(C) R_3

(D) R_4

(E) R_5

33. The voltage across resistor R_4 is

(A) 0.4 V

(B) 0.6 V

(C) 1.2 V

(D) 1.5 V

(E) 3.0 V

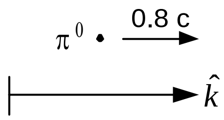
34. A conducting cavity is driven as an electromagnetic resonator. If perfect conductivity is assumed, the transverse and normal field components must obey which of the following conditions at the inner cavity walls?

(A) $E_n = 0$, $B_n = 0$

(B) $E_n = 0$, $B_t = 0$
(C) $E_t = 0$, $B_t = 0$
(D) $E_t = 0$, $B_n = 0$
(E) None of the above

35. Light of wavelength 5200 angstroms is incident normally on a transmission diffraction grating with 2000 lines per centimeter. The first-order diffraction maximum is at an angle, with respect to the incident beam, that is most nearly
(A) 3°
(B) 6°
(C) 9°
(D) 12°
(E) 15°

36. A plane-polarized electromagnetic wave is incident normally on a flat, perfectly conducting surface. Upon reflection at the surface, which of the following is true?
(A) Both the electric vector and magnetic vector are reversed.
(B) Neither the electric vector nor the magnetic vector is reversed.
(C) The electric vector is reversed; the magnetic vector is not.
(D) The magnetic vector is reversed; the electric vector is not.
(E) The directions of the electric and magnetic vectors are interchanged.



37. A π^0 meson (rest-mass energy 135 MeV) is moving with velocity $0.8c\hat{k}$ in the laboratory rest frame when it decays into two photons, γ_1 and γ_2. In the π^0 rest frame, γ_1 is emitted forward and γ_2 is emitted backward relative to the π^0 direction of flight. The velocity of γ_2 in the laboratory rest frame is

(A) $-0.1c\hat{k}$

(B) $-0.2c\hat{k}$

(A) $+0.8c\hat{k}$

(B) $+1.0c\hat{k}$

(C) $+1.8c\hat{k}$

38. Tau leptons are observed to have an average half-life of Δt_1 in the frame S_1 in which the leptons are at rest. In an inertial frame S_2, which is moving at a speed v_{12} relative to S_1, the leptons are observed to have an average half-life of Δt_2. In another inertial reference frame S_3. which is

moving at a speed v_{13} relative to S_1 and v_{23} relative to S_2, the leptons have an observed half-life of Δt_3. Which of the following is a correct relationship among two of the half-lives, Δt_1, Δt_2, and Δt_3?

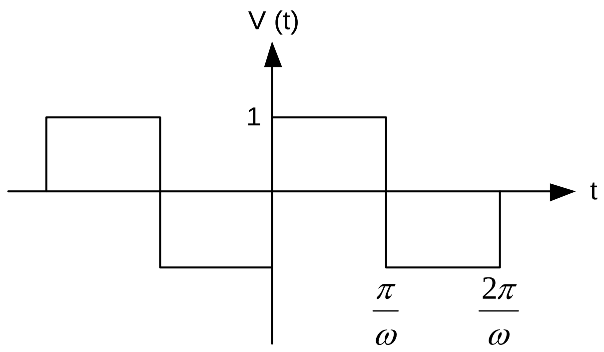
(A) $\Delta t_2 = \Delta t_1\sqrt{1-(v_{12})^2/c^2}$

(B) $\Delta t_1 = \Delta t_3\sqrt{1-(v_{13})^2/c^2}$

(C) $\Delta t_2 = \Delta t_3\sqrt{1-(v_{23})^2/c^2}$

(D) $\Delta t_3 = \Delta t_2\sqrt{1-(v_{23})^2/c^2}$

(E) $\Delta t_1 = \Delta t_2\sqrt{1-(v_{23})^2/c^2}$



39. If n is an integer ranging from 1 to infinity, ω is an angular frequency, and t is time, then the Fourier series for a square wave, as shown above, is given by which of the following?

(A) $V(t) = \frac{4}{\pi}\sum_{1}^{\infty}\frac{1}{n}\sin(n\omega t)$

(B) $V(t) = \frac{4}{\pi}\sum_{0}^{\infty}\frac{1}{(2n+1)}\sin((2n+1)\omega t)$

(C) $V(t) = \frac{4}{\pi}\sum_{1}^{\infty}\frac{1}{n}\cos(n\omega t)$

(D) $V(t) = \frac{4}{\pi}\sum_{0}^{\infty}\frac{1}{(2n+1)}\sin((2n+1)\omega t)$

(E) $V(t) = \frac{4}{\pi}\sum_{1}^{\infty}\frac{1}{n}\sin(n\omega t)$

40. A rigid cylinder rolls at constant speed without slipping on top of a horizontal plane surface. The acceleration of a point on the circumference of the cylinder at the moment when the point touches the plane is

(A) directed forward
(B) directed backward
(C) directed up
(D) directed down
(E) zero

Question 41-42
A cylinder with moment of inertia 4 kg·m^2 about a fixed axis initially rotates at 80 radians per second about this axis. A constant torque is applied to slow it down to 40 radians per second.
41. The kinetic energy lost by the cylinder is
(A) 80J
(B) 800J
(C) 4000J
(D) 9600 J
(E) 19,200 J

42. If the cylinder takes 10 seconds to reach 40 radians per second, the magnitude of the applied torque is
(A) 80 N·m
(B) 40 N·m
(C) 32 N·m
(D) 16 N·m
(E) 8 N·m

43. If $\frac{\partial L}{\partial q_n} = 0$, where L is the Lagrangian for a conservative system without constraints and q_n is a generalized coordinate, then the generalized momentum p_n is
(A) an ignorable coordinate
(B) constant
(C) undefined
(D) equal to $\frac{d}{dt}\left(\frac{\partial L}{\partial q_n}\right)$
(E) equal to the Hamiltonian fir the system

44. A particle of mass m on the Earth's surface is confined to move on the parabolic curve y = ax^2, where y is up. Which of the following is a Lagrangian for the particle?

(A) $L = \frac{1}{2}m\dot{y}^2\left(1+\frac{1}{4ay}\right) - mgy$

(B) $L = \frac{1}{2}m\dot{y}^2\left(1-\frac{1}{4ay}\right) - mgy$

(C) $L = \frac{1}{2} m\dot{x}^2 \left(1 + \frac{1}{4ax}\right) - mgx$

(D) $L = \frac{1}{2} m\dot{x}^2 \left(1 + 4a^2x^2\right) + mgx$

(E) $L = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} m\dot{y}^2 + mgy$

45. A ball is dropped from a height h. As it bounces off the floor, its speed is 80 percent of what it was just before it hit the floor. The ball will then rise to a height of most nearly

(A) 0.94 h

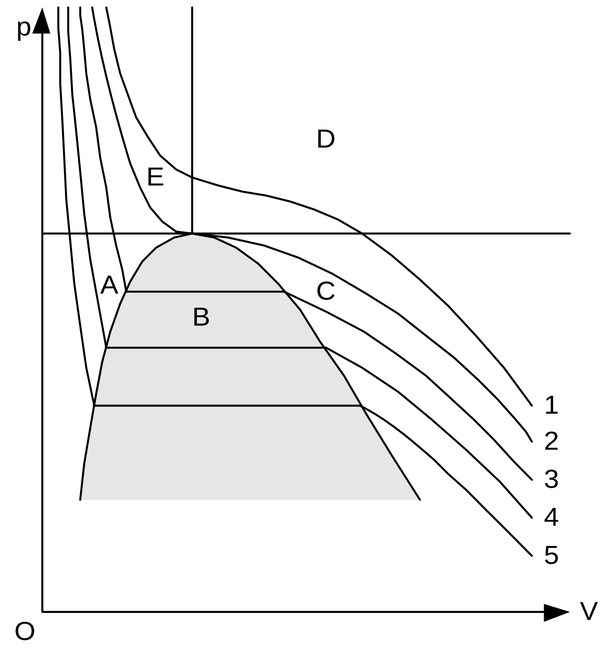
(B) 0.80 h

(C) 0.75 h

(D) 0.64 h

(E) 0.50 h

<u>Question 46-47</u>



Isotherms and coexistence curves are shown in the pV diagram above for a liquid-gas system. The dashed lines are the boundaries of the labeled regions.

46. Which numbered curve is the critical isotherm?

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

47. In which region are the liquid and the vapor in equilibrium with each other?

(A) A

(B) B
(C) C
(D) D
(E) E

48. The magnitude of the force F on an object can be determined by measuring both the mass m of an object and the magnitude of its acceleration a, where F = ma. Assume that these measurements are uncorrelated and normally distributed. If the standard deviations of the measurements of the mass and the acceleration are σ_m and σ_a, respectively, then σ_F/F is

(A) $\left(\frac{\sigma_m}{m}\right)^2+\left(\frac{\sigma_a}{a}\right)^2$

(B) $\left(\frac{\sigma_m}{m}+\frac{\sigma_a}{a}\right)^{\frac{1}{2}}$

(C) $\left[\left(\frac{\sigma_m}{m}\right)^2+\left(\frac{\sigma_a}{a}\right)^2\right]^{\frac{1}{2}}$

(D) $\frac{\sigma_m\sigma_a}{ma}$

(E) $\frac{\sigma_m}{m}+\frac{\sigma_a}{a}$

49. Two horizontal scintillation counters are located near the Earth's surface. One is 3.0 meters directly above the others. Of the following, which is the largest scintillator resolving time that can be used to distinguish downward-going relativistic muons from upward-going relativistic muons using the relative time of the scintillator signals?
(A) 1 picosecond
(B) 1 nanosecond
(C) 1 microsecond
(D) 1 millisecond
(E) 1 second

50. The state of a quantum mechanical system is described by a wave function ψ. Consider two physical observables that have discrete eigenvalues: observable A with eigenvalues $\{\alpha\}$, and observable B with eigenvalues $\{\beta\}$. Under what circumstances can all wave functions be expanded in a set of basis states, each of which is a simultaneous eigenfunction of both A and B?
(A) Only if the values $\{\alpha\}$ and $\{\beta\}$ are nondegenerate
(B) Only if A and B commute
(C) Only if A commutes with the Hamiltonian of the system
(D) Only if B commutes with the Hamiltonian of the system
(E) Under all circumstances

Questions 51-53

A particle of mass m is confined to an infinitely deep square-well potential:

$$V(x) = \infty, x \le 0, x \ge a$$
$$V(x) = 0, 0 < x < a.$$

The normalized eigenfunctions, labeled by the quantum number n, are $\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$.

51. For any state n, the expectation value of the momentum of the particle is

(A) 0

(B) $\frac{\hbar n\pi}{a}$

(C) $\frac{2\hbar n\pi}{a}$

(D) $\frac{\hbar n\pi}{a}(\cos n\pi - 1)$

(E) $\frac{-i\hbar n\pi}{a}(\cos n\pi - 1)$

52. The eigenfunctions satisfy the condition $\int_0^a \psi_n{}^*(x)\psi_l(x)dx = \delta_{nl}$, $\delta_{nl} = 1$, if $n = l$, otherwise $\delta_{nl} = 0$. This is a statement that the eigenfunctions are

(A) solutions to the Schrödinger equation

(B) orthonormal

(C) bounded

(D) linearly dependent

(E) symmetric

53. A measurement of energy E will always satisfy which of the following relationships?

(A) $E \le \frac{\pi^2\hbar^2}{8ma^2}$

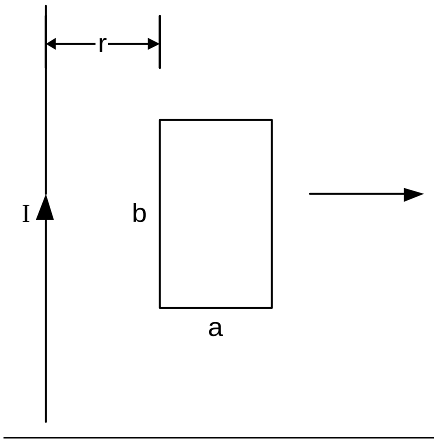
(B) $E \ge \frac{\pi^2\hbar^2}{2ma^2}$

(C) $E = \frac{\pi^2\hbar^2}{8ma^2}$

(D) $E = \frac{n^2\pi^2\hbar^2}{8ma^2}$

(E) $E = \frac{\pi^2\hbar^2}{2ma^2}$

Questions 54-55



A rectangular loop of wire with dimensions above is coplanar with a long wire carrying current I. The distance between the wire and the left side of the loop is r. The loop is pulled to the right as indicated.

54. What are the directions of the induced current in the loop and the magnetic forces on the left and the right sides of the loop as the loop is pulled?

	Induced Current	Force on Left Side	Force on Right Side
(A)	Counterclockwise	To the left	To the right
(B)	Counterclockwise	To the left	To the left
(C)	Counterclockwise	To the right	To the left
(D)	Clockwise	To the right	To the left
(E)	Clockwise	To the left	To the right

55. What is the magnitude of the net force on the loop when the induced current is i?

(A) $\frac{\mu_0 iI}{2\pi}\ln\left(\frac{r+a}{r}\right)$

(B) $\frac{\mu_0 iI}{2\pi}\ln\left(\frac{r}{r+a}\right)$

(C) $\frac{\mu_0 iI}{2\pi}\frac{b}{a}$

(D) $\frac{\mu_0 iI}{2\pi}\frac{ab}{r(r+a)}$

(E) $\frac{\mu_0 iI}{2\pi}\frac{r(r+a)}{ab}$

56. If v is frequency and h is Plank's constant, the ground state energy of a one-dimensional quantum mechanical harmonic oscillator is

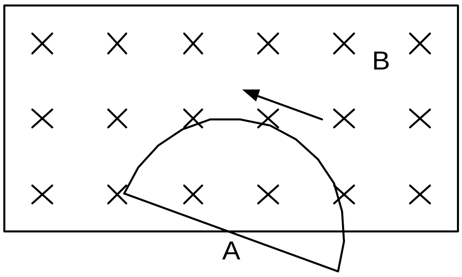
(A) 0

(B) $\frac{1}{3}h\nu$

(C) $\frac{1}{2}h\nu$

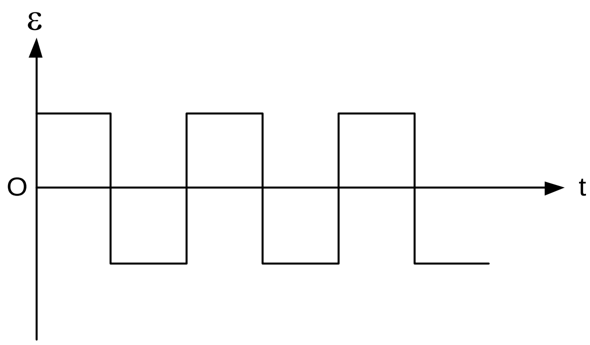
(D) $h\nu$

(E) $\frac{3}{2}h\nu$

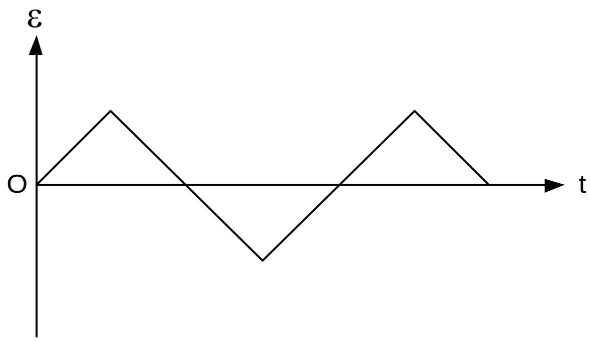


57. A uniform and constant magnetic field **B** is directed perpendicularly into the plane of the page everywhere within a rectangular region as shown above. A wire circuit in the shape of a semicircle is uniformly rotated counterclockwise in the plane of the page about an axis **A**. The axis **A** is perpendicular to the page at the edge of the field and directed through the center of the straight-line portion of the circuit. Which of the following graphs best approximates the emf ε induced in the circuit as a function of time t?

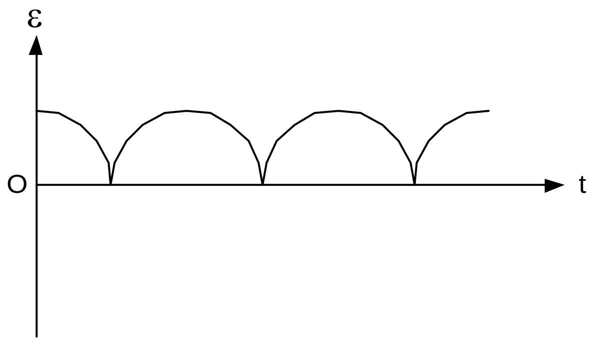
(A)



(B)



(C)



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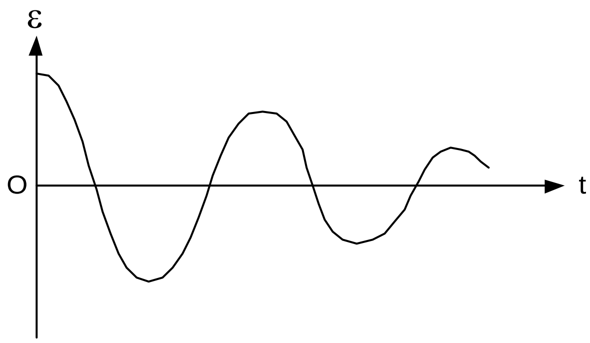
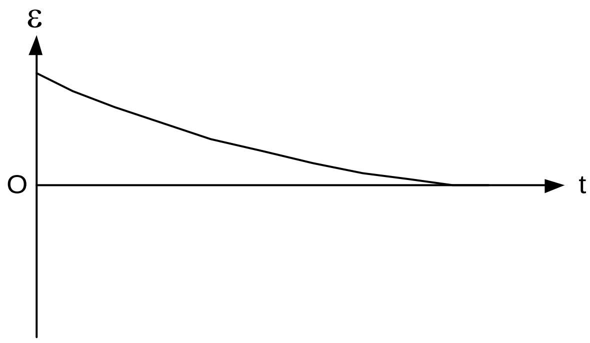


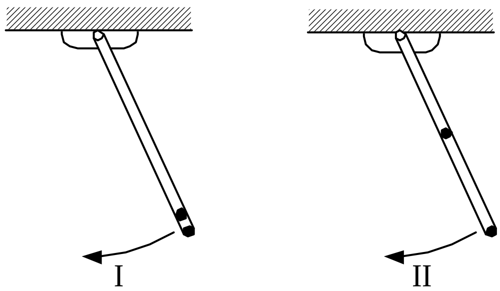
(E)



58. The ground state configuration of a neutral sodium atom (Z = 11) is
(A) $1s^2 2s^2 2p^5 3s^2$
(B) $1s^2 2s^3 2p^6$
(C) $1s^2 2s^2 2p^6 3s$
(D) $1s^2 2s^2 2p^6 3p$
(E) $1s^2 2s^2 2p^5$

59. The ground state of the helium atom is a spin
(A) singlet
(B) doublet
(C) triplet
(D) quartet
(E) quintuplet

60. An electron in a metal has an effective mass $m^* = 0.1m_e$. If this metal is placed in a magnetic field of magnitude 1 tesla, the cyclotron resonance frequency, ω_c, is most nearly
(A) 930 rad/s
(B) 8.5×10^6 rad/s
(C) 2.8×10^{11} rad/s
(D) 1.8×10^{12} rad/s
(E) 7.7×10^{20} rad/s



61. A long, straight, and massless rod pivots about one end in a vertical plane. In configuration I, shown above, two small identical masses are attached to the free end; in configuration II, one mass is moved to the center of the rod. What is the ratio of the frequency of small oscillations of configuration II to that of configuration I?
(A) $(6/5)^{\frac{1}{2}}$
(B) $(3/2)^{\frac{1}{2}}$
(C) 6/5
(D) 3/2
(E) 5/3

62. A mole of ideal gas initially at temperature T_0 and volume V_0 undergoes a reversible

isothermal expansion to volume V_1. If the ratio of specific heats is $c_p/c_v = \gamma$ and if R is the gas constant, the work done by the gas is

(A) zero

(B) $RT_0 (V_1 / V_0)^{\gamma}$

(C) $RT_0 (V_1 / V_0 - 1)$

(D) $C_vT_0 [1 - (V_0 / V_1) \gamma^{-1}]$

(E) $RT_0 \ln (V_1 / V_0)$

63. Which of the following is true if the arrangement of an isolated thermodynamic system is of maximal probability?

(A) Spontaneous change to a lower probability occurs.

(B) The entropy is a minimum.

(C) Boltzmann's constant approaches zero.

(D) No spontaneous change occurs.

(E) The entropy is zero.

64. If an electric field is given in a certain region by $E_x = 0$, $E_y = 0$, $E_z = k\,z$, where k is a nonzero constant, which of the following is true?

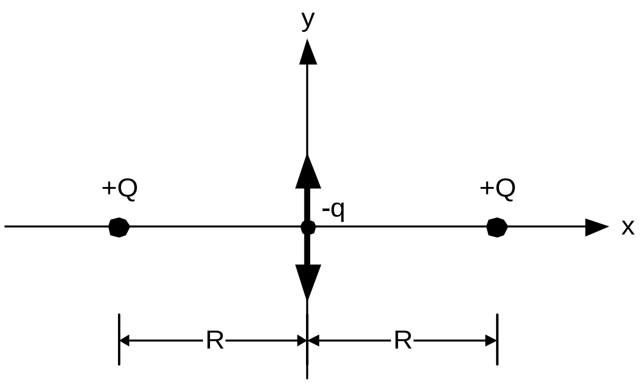
(A) There is a time-varying magnetic field.

(B) There is charge density in the region.

(C) The electric field cannot be constant in time.

(D) The electric field is impossible under any circumstances.

(E) None of the above.



65. Two point charges with the same charge +Q are fixed along the x-axis and are a distance 2R apart as shown. A small particle with mass m and charge –q is placed at the midpoint between them. What is the angular frequency ω of small oscillations of this particle along the y-direction?

(A) $\dfrac{Qq}{2\pi\varepsilon_0 mR^2}$

(B) $\dfrac{Qq}{4\pi\varepsilon_0 mR^2}$

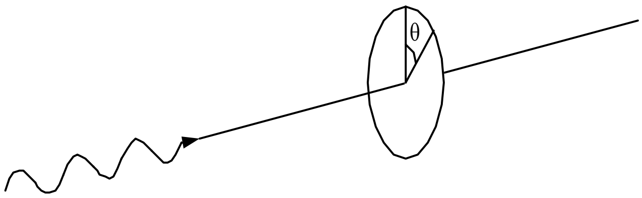
(C) $\dfrac{Qq}{2\pi\varepsilon_0 mR^3}$

(D) $\left(\dfrac{Qq}{4\pi\varepsilon_0 mR^2}\right)^{\frac{1}{2}}$

(E) $\left(\dfrac{Qq}{2\pi\varepsilon_0 mR^3}\right)^{\frac{1}{2}}$

66. A thin uniform steel chain is 10 meters long with a mass density of 2 kilometers per meter. One end of the chain is attached to a horizontal axle having a radius that is small compared to the length of the chain. If the chain initially hangs vertically, the work required to slowly wind it up on to the axle is closest to

(A) 100 J
(B) 200 J
(C) 1,000 J
(D) 2,000 J
(E) 10,000 J



67. A steady beam of light is normally incident on a piece of polaroid. As the polariod is rotated around the beam axis, the transmitted intensity varies as A + B cos2θ, where θ is the angle of rotation, and A and B are constants with A > B > 0. Which of the following may be correctly concluded about the incident light?

(A) The light is completely unpolarized.
(B) The light is completely plane polarized.
(C) The light is partly plane polarized and partly unpolarized.
(D) The light is partly circularly polarized and partly unpolarized.
(E) The light is completely circularly polarized.

68. The angular separation of the two components of a double star is 8 microradians, and the light from the double star has a wavelength of 5500 angstroms. The smallest diameter of a telescope mirror that will resolve the double star is most nearly

(A) 1 mm
(B) 1 cm
(C) 10 cm
(D) 1 m
(E) 100 m

69. A fast charged particle passes perpendicularly through a thin glass sheet of index of refraction 1.5. The particle emits light in the glass. The minimum speed of the particle is

(A) $\frac{1}{3}c$

(B) $\frac{4}{9}c$

(C) $\frac{5}{9}c$

(D) $\frac{2}{3}c$

(E) c

70. A monoenergetic beam consists of unstable particles with total energies 100 times their rest energy. If the particles have rest mass m, their momentum is most nearly

(A) mc

(B) 10 mc

(C) 70 mc

(D) 100 mc

(E) 10^4 mc

<u>Question 71-73</u>

A system in thermal equilibrium at temperature T consists of a large number N_0 of subsystems, each of which can exist only in two states of energy E_1 and E_2, where $E_1 - E_2 = c > 0$. In the expressions that follow, k is the Boltzmann constant.

71. For a system at temperature T, the average number of subsystems in the states of energy E_1 is given by

(A) $\frac{N_0}{2}$

(B) $\frac{N_0}{1+e^{-\varepsilon/kT}}$

(C) $N_0 e^{-\varepsilon/kT}$

(D) $\frac{N_0}{1+e^{\varepsilon/kT}}$

(E) $\frac{N_0 e^{\varepsilon/kT}}{2}$

72. The internal energy of this system at any temperature T is given by $E_1 N_0 + \frac{N_0}{1+e^{\varepsilon/kT}}$. The heat capacity of the system is given by which of the following expressions?

(A) $N_0 k\left(\frac{\varepsilon}{kT}\right)^2 \frac{e^{\varepsilon/kT}}{\left(1+e^{\varepsilon/kT}\right)^2}$

(B) $N_0 k\left(\frac{\varepsilon}{kT}\right)^2 \frac{1}{\left(1+e^{\varepsilon/kT}\right)^2}$

(C) $N_0 k\left(\frac{\varepsilon}{kT}\right)^2 e^{-\varepsilon/kT}$

(D) $N_0 k\left(\frac{\varepsilon}{kT}\right)^2$

(E) $\frac{3}{2}N_0 k$

73. Which of the following is true of the entropy of the system?
(A) It increases without limit with T from zero at T = 0.
(B) It decreases with increasing T.
(C) It increases from zero at T = 0 to N_0k ln2 at arbitrarily high temperature.
(D) It is given by $N_0 k\left[\frac{5}{2}\ln T - \ln p + constant\right]$.
(E) It cannot be calculated from the information given.

74. Two circular hoops, X and Y, are hanging on nails in a wall. The mass of X is four times that of Y, and the diameter of X is also four times that of Y. If the period of small oscillations of X is T, the period of small oscillations of Y is
(A) T
(B) T / 2
(C) T / 4
(D) T / 8
(E) T / 16

$${}^{235}_{92}U \rightarrow {}^{231}_{90}Th + {}^{4}_{2}He$$

75. A uranium nucleus decays at rest into a thorium nucleus and a helium nucleus, as shown above. Which of the following is true?
(A) Each decay product has the same kinetic energy.
(B) Each decay product has the same speed.
(C) The decay products tend to go in the same direction.
(D) The thorium nucleus has more momentum than the helium nucleus.
(E) The helium nucleus has more kinetic energy than the thorium nucleus.

76. The configuration of three electrons 1s2p3p has which of the following as the value of its

maximum possible total angular momentum quantum number?

(A) $\frac{7}{2}$

(B) 3

(C) $\frac{5}{2}$

(D) 2

(E) $\frac{3}{2}$

77. Consider a heavy nucleus with spin $\frac{1}{2}$. The magnitude of the ratio of the intrinsic magnetic moment of this nucleus of that of an electron is

(A) zero, because the nucleus has no intrinsic magnetic moment

(B) greater than 1, because the nucleus contains many protons

(C) greater than 1, because the nucleus is so much larger in diameter than the electron

(D) less than 1,because of the strong interactions among the nucleons in a nucleus

(E) less than 1, because the nucleus has a mass much larger than that of the electron

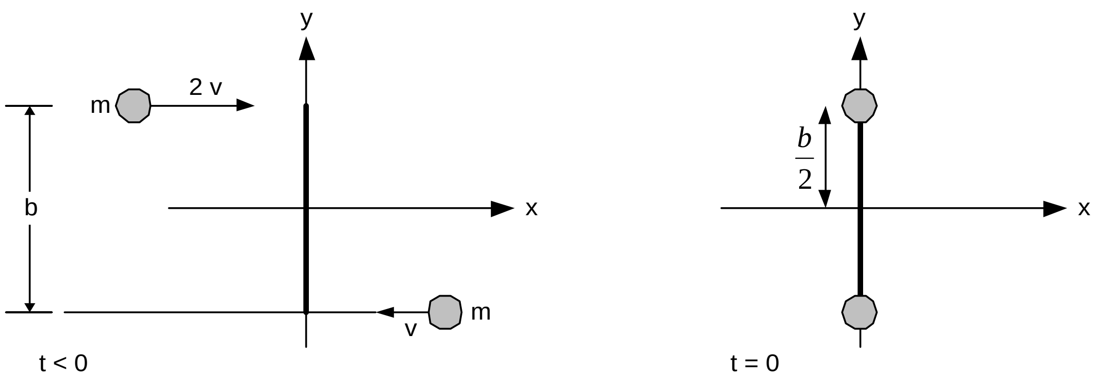


Figure I

Figure II

78. One ice skater of mass m moves with speed 2v to the right, while another of the same mass m moves with speed v toward the left, as shown in Figure I. Their paths are separated by a distance b. At t = 0, when they are both at x = 0, they grasp a pole of length b and negligible mass. For t > 0, consider the system as a rigid body of two masses m separated by distance b, as shown in Figure II. Which of the following is the correct formula for the motion after t = 0 of the skater initially at y = b / 2?

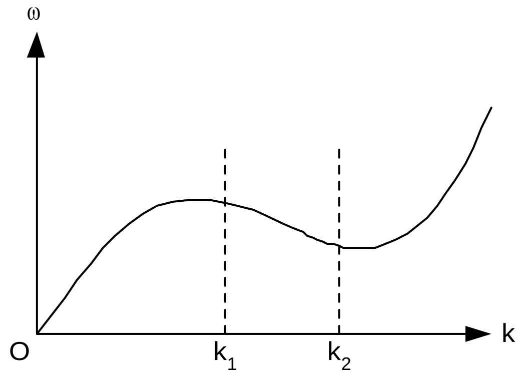
(A) x = 2vt, y = b/2

(B) x = vt + 0.5b sin(3vt/b), y = 0.5b cos(3vt/b)

(C) x = 0.5vt + 0.5b sin(3vt/b), y = 0.5b cos(3vt/b)

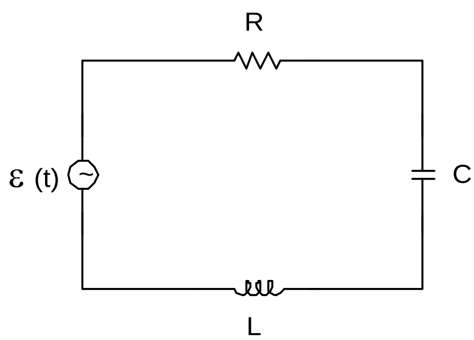
(D) x = vt + 0.5b sin(6vt/b), y = 0.5b cos(6vt/b)

(E) x = 0.5vt + 0.5b sin(6vt/b), y = 0.5b cos(6vt/b)



79. The dispersion curve shown above relates the angular frequency ω to the wave function k. For waves with wave numbers lying in the range $k_1 < k < k_2$, which of the following is true of the phase velocity and the group velocity?
(A) They are in opposite directions.
(B) They are in the same direction and the phase velocity is larger.
(C) They are in the same direction and the group velocity is larger.
(D) The phase velocity is infinite and the group velocity is finite.
(E) They are the same in direction and magnitude.

80. A beam of electrons is accelerated through a potential difference of 25 kilovolts in an x-ray tube. The continuous x-ray spectrum emitted by the target of the tube will have a short wavelength limit of most nearly
(A) 0.1 A
(B) 0.5 A
(C) 2 A
(D) 25 A
(E) 50 A



81. In the RCL circuit shown above, the applied voltage is $\varepsilon(t) = \varepsilon_m \cos\omega t$.

For a constant ε_m, at what angular frequency ω does the current have its maximum steady-state amplitude after the transients have died out?

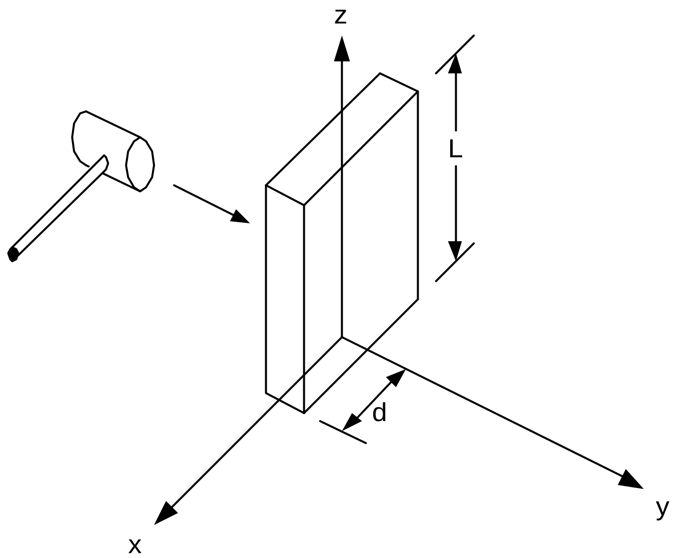
(A) $\dfrac{1}{RC}$

(B) $\dfrac{2L}{R}$

(C) $\dfrac{1}{\sqrt{LC}}$

(D) $\sqrt{\dfrac{1}{LC} - \left(\dfrac{R}{2L}\right)^2}$

(E) $\sqrt{\left(\dfrac{1}{RC}\right)^2 - \left(\dfrac{L}{R}\right)^2}$



82. A thin plate of mass M, length L, and width 2d is mounted vertically on a frictionless axle along the z-axis, as shown above. Initially the object is at rest. It is then tapped with a hammer to provide a torque τ, which produces an angular impulse **H** about the z-axis of magnitude $H = \int \tau dt$. What is the angular speed ω of the plate about the z-axis after the tap?

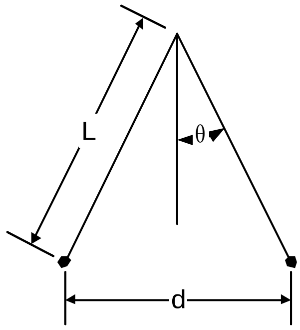
(A) $\dfrac{H}{2Md^2}$

(B) $\dfrac{H}{Md^2}$

(C) $\dfrac{2H}{Md^2}$

(D) $\dfrac{3H}{Md^2}$

(E) $\dfrac{4H}{Md^2}$



83. Two pith balls of equal mass M and equal charge q are suspended from the same point on long massless threads of length L as shown in the figure above. If k is the Coulomb's law constant, then for small values of θ, the distance d between the charged pith balls at equilibrium is

(A) $\left(\frac{2kq^2L}{Mg}\right)^{\frac{1}{3}}$

(B) $\left(\frac{kq^2L}{Mg}\right)^{\frac{1}{3}}$

(C) $\left(\frac{2kq^2L}{Mg}\right)^{\frac{1}{2}}$

(D) $\left(\frac{kq^2L}{Mg}\right)^{\frac{1}{2}}$

(E) $\frac{L}{4}$

84. An electron oscillates back and forth along the + and – x-axes, consequently emitting electromagnetic radiation. Which of the following statements concerning the radiation is NOT true?

(A) The total rate of radiation of energy into all directions is proportional to the square of the electron's acceleration.

(B) The total rate of radiation of energy into all directions is proportional to the square of the electron's charge.

(C) Far from the electron, the rate at which radiated energy crossed a perpendicular unit area decreases as the inverse square of the distance from the electron.

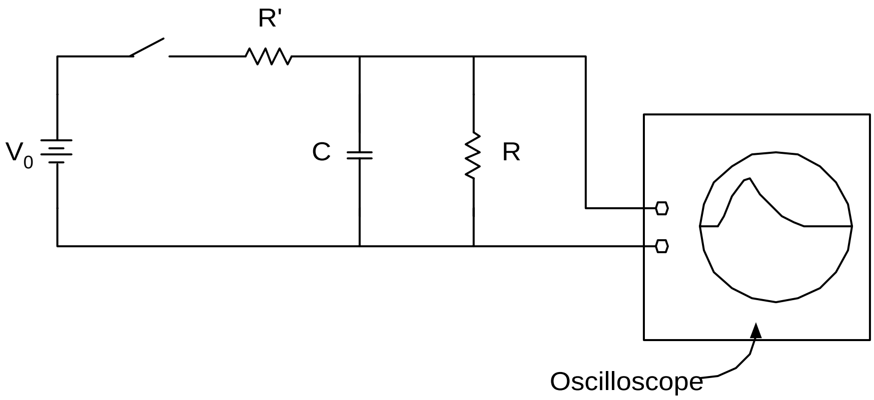
(D) Far from the electron, the rate at which radiated energy crossed a perpendicular unit area is an maximum when the unit area is located on the + or – x-axes.

(E) Far from the electron, the radiated energy is carried equally by the transverse electric and the transverse magnetic fields.

85. A free electron (rest mass m_e = 0.5 MeV/c^2) has a total energy of 1.5 MeV. Its momentum p in units of MeV/c is about

(A) 0.86

(B) 1.0
(C) 1.4
(D) 1.5
(E) 2.0



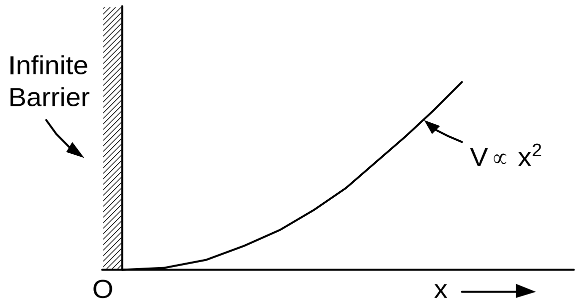
86. The circuit shown above is used to measure the size of the capacitance C. The y-coordinate of the spot on the oscilloscope screen is proportional to the potential difference across R, and the x-coordinate of the spot is swept at a constant speed *s*. The switch is closed and then opened. One can then calculate C from the shape and the size of the curve on the screen plus a knowledge of which of the following?
(A) V_0 and R
(B) *s* and R
(C) *s* and V_0
(D) R and R'
(E) The sensitivity of the oscilloscope

87. A particle of mass M moves in a circular orbit of radius *r* around a fixed point under the influence of an attractive force $F = \frac{K}{r^3}$, where K is a constant. If the potential energy of the particle is zero at an infinite distance from the force center, the total energy of the particle in the circular orbit is
(A) $-\frac{K}{r^2}$
(B) $-\frac{K}{2r^2}$
(C) 0
(D) $\frac{K}{2r^2}$
(E) $\frac{K}{r^2}$

88. A parallel-plate capacitor is connected to a battery. V_0 is the potential difference between the plates, Q_0 the charge on the positive plate, E_0 the magnitude of the electric field, and D_0 the magni-tude of the displacement vector. The original vacuum between the plates in filled with a

dielectric and then the battery is disconnected. If the corresponding electrical parameters for the final state of the capacitor are denoted by a subscript *f*, which of the following is true?

(A) $V_f > V_0$
(B) $V_f < V_0$
(C) $Q_f = Q_0$
(D) $E_f > E_0$
(E) $D_f > D_0$



89. The energy levels for the one-dimensional harmonic oscillator are $h\gamma\left(n+\frac{1}{2}\right), n = 0,1,2\ldots$

How will the energy levels for the potential shown in the graph above differ from those for the harmonic oscillator?

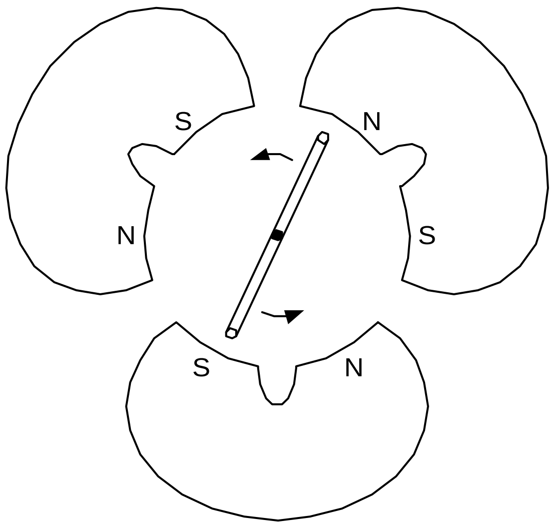
(A) The term $\frac{1}{2}$ will be changed to $\frac{3}{2}$.
(B) The energy of each level will be doubled.
(C) The energy of each level will be halved.
(D) Only those for even values of n will be present.
(E) Only those for odd values of n will be present.

90. The spacing of the rotational energy levels for the hydrogen molecule H_2 is most nearly

(A) 10^{-8} eV
(B) 10^{-3} eV
(C) 10 eV
(D) 10 MeV
(E) 100 MeV

91. The particle decay $\Lambda \rightarrow p + \pi^-$ must be a weak interaction because

(A) the π^- is a lepton
(B) the Λ has spin zero
(C) no neutrino is produced in the decay
(D) it does not conserve angular momentum
(E) it does not conserve strangeness



92. A flat coil of wire is rotated at a frequency of 10 hertz in the magnetic field produced by three pairs of magnets as shown above. The axis of rotation of the coil lies in the plane of the coil and is perpendicular to the field lines. What is the frequency of the alternating voltage in the coil?

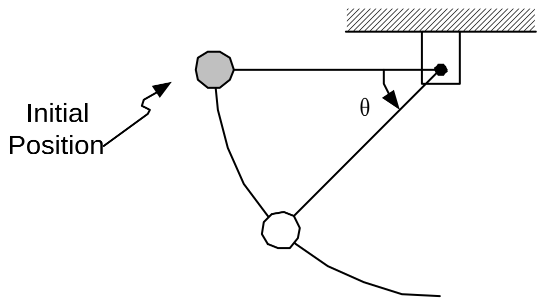
(A) $\frac{10}{6}$ Hz

(B) $\frac{10}{3}$ Hz

(C) 10 Hz

(D) 30 Hz

(E) 60 Hz



93. The figure above shows a small mass connected to a string, which is attached to a vertical post. If the mass is released when the string is horizontal as shown, the magnitude of the total acceleration of the mass as a function of the angle θ is

(A) g sinθ

(B) 2g cosθ

(C) 2g sinθ

(D) $g\sqrt{3\cos^2\theta+1}$

(E) $g\sqrt{3\sin^2\theta+1}$

94. Which of the following is a Lorentz transformation? (Assume a system of units such that the velocity of light is 1)

(A) x’ = 4x
y’ = y
z’ = z
t’ = .25t

(B) x’ = x – .75t
y’ = y
z’ = z
t’ = t

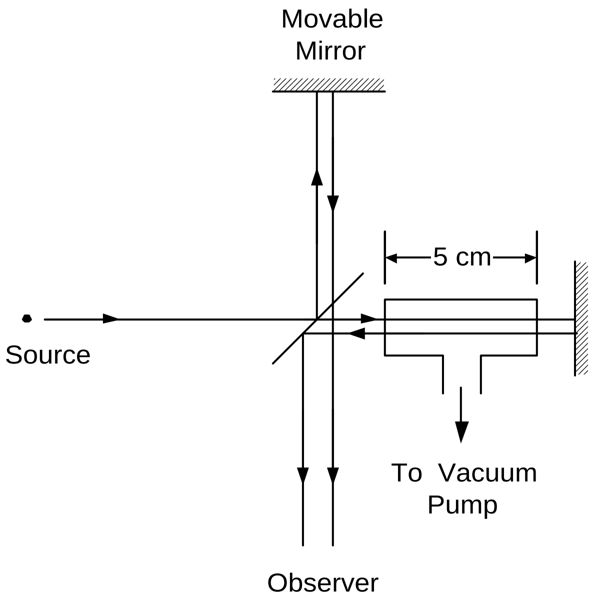
(C) x’ = 1.25x – .75t
y’ = y
z’ = z
t’ = 1.25t – .75x

(D) x’ = 1.25x – .75t
y’ = y
z’ = z
t’ = .75t – 1.25x

(E) None of the above

95. A beam of 10^{12} protons per second is incident on a target containing 10^{20} nuclei per square centimeter. At an angle of 10 degrees, there are 10^{2} protons per second elastically scattered into a detector that subtends a solid angle of 10^{-4} steradians. What is the differential elastic scattering cross section, in units of square centimeters per steradian?

(A) 10^{-24}
(B) 10^{-25}
(C) 10^{-26}
(D) 10^{-27}
(E) 10^{-28}



96. A gas-filled cell of length 5 centimeters is inserted in one arm of a Michelson interferometer, as shown in the figure above. The interferometer is illuminated by light of wavelength 500 nanometers. As the gas is evacuated from the cell, 40 fringes cross a point in the field of view. The refractive index of this gas is most nearly

(A) 1.02
(B) 1.002
(C) 1.0002
(D) 1.00002
(E) 0.98

97. Lattice forces affect the motion of electrons in a metallic crystal, so that the relationship between the energy E and wave number k is not the classical equation $E = \hbar^2 k^2 / 2m$, when m is the electron mass. Instead, it is possible to use an effective mass m* given by which of the following?

(A) $m^* = \frac{1}{2}\hbar^2 k\left(\frac{dk}{dE}\right)$

(B) $m^* = \frac{\hbar^2 k}{\left(\frac{dk}{dE}\right)}$

(C) $m^* = \hbar^2 k\left(\frac{d^2 k}{dE^2}\right)^{\frac{1}{3}}$

(D) $m^* = \frac{\hbar^2}{\left(\frac{d^2 E}{dk^2}\right)}$

(E) $m^* = \frac{1}{2}\hbar^2 m^2 \left(\frac{d^2E}{dk^2}\right)$

98. The matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ has three eigenvalues λ_i defined by $Av_i = \lambda_i v_i$.Which of the following statements is NOT true?

(A) $\lambda_1 + \lambda_2 + \lambda_3 = 0$

(B) λ_1, λ_2, λ_3 are all real numbers.

(C) $\lambda_2\lambda_3 = +1$ for some pair of roots.

(D) $\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = 0$

(E) $\lambda_i^3 = +1$, i = 1, 2, 3

99. In perturbation theory, what is the first order correction to the energy of a hydrogen atom (Bohr radius a_0) in its ground state due to the presence of a static electric field E?

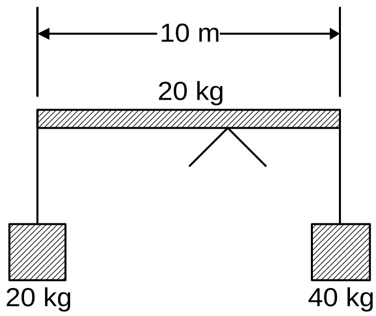
(A) Zero

(B) eEa_0

(C) $3eEa_0$

(D) $\frac{8e^2Ea_0{}^3}{3}$

(E) $\frac{8e^2E^2a_0{}^3}{3}$



100. A uniform rod of length 10 meters and mass 20 kilograms is balance on a fulcrum with a 40-kilogram on one end of the rod and a 20-kilogram mass on the other end, as shown above. How far is the fulcrum located from the center of the rod?

(A) 0 m

(B) 1 m

(C) 1.25 m

(D) 1.5 m

(E) 2 m

参考答案

CDACC　DACAE　EBDCC　AACAD　EDEED
BBEBE　BAADB　CABBC　DDBAD　BBCBB
ABBED　CACAD　AEDBE　CCCDD　BACBE
AECAB　CDADC　BCEEB　EDECC　CDBAC

参考解答

1. 解：选（C）。平面波函数。

2. 解：选（D）。根据 Bragg 方程 $2d\sin\theta = n\lambda$ 。

3. 解：选（A）。根据 Morseley 定律 $\sqrt{\nu_K} = \alpha(Z-\sigma)$ 。

4. 解：在球外，F ∝ r^{-2}。选（C）。

5. 解：在球内，F ∝ ρv / r^2 ∝ r。选（C）。

6. 解：利用对称性对下面的楔块进行受力分析。选（D）。

7. 解：选（A）。三种本征状态分别对应于不振动，两摆球相反相位及相同相位的振动。显然反相振动时悬点静止，与单摆相同。

8. 解：力矩定义。选（C）。

9. 解：选（A）。根据对称性利用 Ampere 环路定理易得。

10. 解：选（D）。利用电像法求出感应电荷的作用力。

11. 解：RC 电路的放电过程，最典型最简单的暂态过程，时间常数为 τ = RC。电容的电荷

$$q = q_0 e^{-\frac{1}{RC}t},$$

而电容中储存的能量

$$E = \frac{q^2}{2C},$$

当

$$e^{-\frac{2}{RC}t}=\frac{1}{2}$$

时，$E = E_0 / 2$。选（E）。

12. 解：根据题意，两点间的电势差与 z 和 ρ 无关，两极板间的电场沿切向均匀分布。

$$\Delta V \propto \Delta\varphi\,,$$

$$V\left(0\le\varphi\le\alpha\right)=\frac{V_0}{\alpha}\varphi\text{。}$$

选（B）。

13. 解：假设磁单极子存在，与电荷和电流相对应的是方程中应该出现磁荷项ρ_B和磁流密度项 J_B。根据电磁场对称性 Maxwell 方程组应改写

$$\begin{aligned}&curl\mathbf{H}=\mathbf{J}_B-\frac{\partial\mathbf{B}}{\partial t}\\&div\mathbf{B}=\rho_B\end{aligned}\text{。}$$

选（D）。

14. 解：根据 Stefan-Boltzmann 黑体辐射定律，绝对温度为 T 的黑体，其辐射能量

$$u=\alpha T^4\propto T^4$$

因此当黑体的绝对温度上升一倍，其总的辐射功率为原来的 16 倍。相同条件下，相同质量的水的温升相应的也应该是初始的 16 倍。选（C）。

15. 解：双原子分子总的平动自由度是 3 个（由质心位置决定），转动自由度是 2 个，振动自由度是 1 个。根据经典的能均分定理

$$c_v=\frac{3+2+1\times2}{2}R=\frac{7}{2}R$$

选（C）。

16. 解：根据 Carnot 定理，最大功率就是理想 Carnot 热机的功率

$$\eta_{\max}=1-\frac{T_m}{T_M}=1-\frac{800}{1000}=20\%$$

因此最大功率

$$W=Q\eta=400\text{J}\text{。}$$

选（A）。

17. 解：Lissajou 图形：当两个互相垂直的简谐振动频率不同时，合成的轨迹与频率之比和两者的相位都有关系，图形一般较为复杂，很难用数学式子表达。当两者的频率成整数比时，

轨迹是闭合的，运动是周期性的，形成 Lissajou 图形。
两个方向的频率比可以简单的由两方向上最大的切点数 n 之比求出：

$$\frac{f_X}{f_Y}=\frac{n_Y}{n_X}\text{。}$$

选（A）。

18. 解：为了防止信号反射降低传输率。选（C）。

19. 解：利用重力加速度 g 来估算

$$g=\frac{GM}{R^2},$$

$$M=\frac{gR^2}{G}\sim\frac{10\times10^{12}}{10^{-11}}=10^{-24}\text{。}$$

选（A）。

20. 解：多缝衍射的振幅分布公式为

$$A_\theta=a_0\frac{\sin\alpha}{\alpha}\frac{\sin N\beta}{\beta},$$

其中 sinNβ/β是缝间干涉因子，决定了主极强缝值的位置。sinα/α是单缝衍射因子，对主极强起调制的作用。d > w，当 2d = 3w 时，满足

$$\sin\theta=\frac{\lambda}{w}=\frac{3\lambda}{2d}$$

条件的主极强比如±3、±6 等会消失，形成“缺级”。选（D）。

21. 解：完整的解释是：肥皂膜前表面的反射有半波损失，后表面的反射没有半波损失。在肥皂膜的顶端，厚度很薄与波长相比可以忽略不计。因此由于前后表面附加的 180° 相位差，干涉总是相消的，出射光看起来是暗的。选（E）。

22. 解：这是最简单的光学望远镜，透射式的，物镜与目镜共焦，总的放大倍数

$$m=\frac{f_0}{f_e}=10,$$

$$l=f_0+f_e=1.0+\frac{1.0}{10}=1.1\text{m}\text{。}$$

选（D）。

23. 解：在非相对论情形下估算，结果不对的话再用相对论的有关公式计算。注意由于平均动能

$$\overline{E}=\frac{3E_F}{5},$$

故$\bar{v} \sim v_F$，两者的数量级一致。

$$\frac{1}{2}mv_F^2 = k_B T \text{，}$$

$$\bar{v} \sim v_F = \sqrt{\frac{2k_B T}{m}} \sim 10^6 \,\text{m/s}\text{。}$$

选（E）。

24. 解：Ar 原子最外层 8 个电子，形成满壳层结构，因此结合不是靠离子键或共价键，也不是金属键。是 van der Waals 分子键，所有惰性气体都是靠分子键结合的。
选（E）。

25. 解：μ介子和中微子几乎很难反应掉，而且很难被吸收，在宇宙射线中到达地底深处的一般是这两种粒子。选（D）。

26. 解：根据核衰变的规律 $n = n_0 e^{-t/\tau}$，写成微分形式可得计数率随时间的变化

$$\frac{dn}{dt} = \left(\frac{dn}{dt}\right)\Big|_{t=0} e^{-\frac{t}{\tau}}$$

当 t = τ 时，计数率为初始时候的 1/e。选（B）。

27. 解：测不准关系。选（B）。

28. 解：Ψ 的三个基态中，Y_4^3 和 Y_6^3 态的磁量子数都是 m = 3，总共可能观测到的几率是

$$p_{m=3} = \frac{5^2 + 1^2}{30} = \frac{13}{15}\text{。}$$

选（E）。

29. 解：有限深方势阱中的束缚态在势阱边界上应该满足波函数及其导数连续的条件，在势阱外，波函数的形式是 $e^{-ik|x|}$，随坐标的扩张指数衰减的。由排除法可得答案。选（B）。

30. 解：正负电子对是 SUB 中的常考点。根据等效质量的对应关系

$$m^* = \frac{m_H{}^*}{2}\text{，}$$

得

$$E_n = \frac{1}{2}E_{nH} = -\frac{1}{2}\frac{m_e Z^2 e^4}{2\hbar^2 (4\pi\varepsilon_0)^2} \cdot \frac{1}{n^2} = \frac{1}{2n^2}E_0$$

将 n = 2 代入上式即得。选（E）。

31. 解：显然自旋量子数 s = 1（由 2s + 1 = 3 推出），S 态的轨道量子数 l = 0。根据 LS 耦合

的法则，总量子数 j 的可取值只能为 1。选（B）。

注：LS 耦合下 $j=|l-s|,|l-s|+1,\ldots,|l+s|$。

32. 解：进行分析，电路中的总电流全部流经 R_1，R_1 的阻值又很大，故它的功率一定比 R_2、R_4、R_5 大。对于 R_3，由于经过了两次分流，猜测其上的功率应比 R_1 的小。
一般的计算结果如下：

$$\begin{aligned}
&V_1=2V, P_1=I_1^2R_1=0.08W\\
&V_2=1V, P_2=I_2^2R_2=0.02W\\
&V_3=0.4V, P_3=I_3^2R_3=0.0027W\\
&V_4=0.4V, P_4=I_4^2R_4=0.0053W\\
&V_5=0.6V, P_5=I_5^2R_5=0.012W
\end{aligned}$$ 。

选（A）。

33. 解：详细的计算可参照上题。选（A）。

34. 解：波导管壁上的边界连接条件为

$$\nabla\cdot\mathbf{E}=0,\ \ \nabla\times\mathbf{B}=0,$$

电磁波的边界条件为

$$E_t=0,\ \ B_n=0。$$

选（D）。

35. 解：第一级衍射亮纹满足的条件为

$$d\sin\theta=\frac{1}{n}\sin\theta=\lambda。$$

作为小角近似，可以取

$$\theta\approx\sin\theta=n\lambda,$$

再换算为角度制。选（B）。

36. 解：反射后电矢量方向反转，磁矢量方向不变。选（C）。

37. 解：在π^0介子的固有参照系中来看，向后的光子的速度为–c。根据相对论的速度合成公式

$$v_2=\frac{u_\pi+v_2'}{1+u_\pi v_2'/c^2}=-c。$$

选（A）。

38. 解：只需利用相对论中的时钟缩短效应，注意分清各个物理量的含义即可。

$$\Delta t_i = \frac{\Delta t_1}{\sqrt{1-\left(\frac{v_{1i}}{c}\right)^2}},\quad i=2,3。$$

选（B）。

39. 解：首先应做定性判断，由于 V(t)是时间 t 的奇函数，故展开式中不含余弦项，排除（C）、（D）、（E）。而沿时间轴ωt 平移距离为π后，所得结果应与原来的函数只差一个负号，故排除（A）。一般的，

$$a_n = \frac{2}{T}\int_0^T V(t)\sin n\omega t dt,$$

$$b_n = \frac{2}{T}\int_0^T V(t)\cos n\omega t dt。$$

选（B）。

40. 解：圆柱体的质心做匀速直线运动，是惯性参考系。在质心系中，最下放点的加速度是向心加速度，向上。也就是该点在地面参考系的加速度。选（C）。

41. 解：动能损失

$$\Delta E = \frac{1}{2}I\left(\omega_0^2-\omega^2\right) = \frac{1}{2}\times 4\times\left(80^2-40^2\right) = 9600\mathrm{J}。$$

选（D）。

42. 解：力矩大小可直接计算

$$\begin{aligned} M &= I\beta = I\frac{\Delta\omega}{\Delta t} \\ &= 4\times\frac{80-40}{10} = 16\mathrm{N}\cdot\mathrm{m} \end{aligned}$$

选（D）。

43. 解：选（B）。Langrange 方程的结果。

44. 解：粒子的 Lagrangian

$$\begin{aligned} L &= T-V \\ &= \frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)-mgy \\ &= \frac{1}{2}m\dot{x}^2\left(1+\left(\frac{dy}{dx}\right)^2\right)-mgy。 \\ &= \frac{1}{2}m\dot{y}^2\left(1+\frac{1}{4ay}\right)-mgy \end{aligned}$$

选（A）。

45. 解：速度为碰前的 80%，则动能为碰前的 64%。

$$mgH=\frac{1}{2}mv^2=64\%\left(\frac{1}{2}mv_0^2\right)=64\%mgH_0。$$

选（D）。

46. 解：ABCD 四块区域的交点为临界点，经过临界点的曲线是临界等温线。选（B）。

47. 解：在临界线以上的区域气液两相是无法区分的。A 区主要是液态，C 区主要是气态。之间的 B 区物质等压吸热膨胀（或反之），处于相变过程，对应两相共存。选（B）。

48. 解：由于 m、a 的测量互不关联，首先将 F 的表达式利用对数求导

$$\frac{\delta F}{F}=\frac{\delta m}{m}+\frac{\delta a}{a},$$

然后将各部分误差合成

$$\left(\frac{\sigma_F}{F}\right)^2=\left(\frac{\sigma_m}{m}\right)^2+\left(\frac{\sigma_a}{a}\right)^2。$$

选（C）。

49. 解：两个探测器间可联系的最短时间

$$\tau=\frac{d}{c}=\frac{3}{3\times10^8}=10^{-8}(s)。$$

选（B）。

50. 解：算符对易就有共同本征态。选（B）。

51. 解：动量的平均值

$$\langle p_x\rangle=\int_0^a-i\hbar\nabla\Psi_n dx\propto\left(\cos\frac{n\pi x}{a}\right)\Big|_0^a=0$$

选（A）。

52. 解：说明 $\Psi_n(x)$是一组正交归一的基函数。选（B）。

53. 解：第 n 条能级上的能量观测值（即本征值）

$$E_n=\frac{\hbar^2k^2}{2m}=\frac{n^2\pi^2\hbar^2}{2ma^2},$$

n = 1 时能量最小。选（B）。

54. 解：线圈中的磁力线垂直纸面向里，且总磁通量不断减小。根据 Lentz 定理不难判断出回路中的感应电流方向是顺时针的。然后根据 Ampere 力公式

$$\mathbf{F} = I\mathbf{l} \times \mathbf{B}$$

不难判断出线圈各段所受磁场力的方向。选（E）。

55. 解：接上问，线圈的上下两段不受磁场力，合力由左右两段受力决定

$$\begin{aligned} F &= ib\left(B_L - B_R\right) = ib \cdot \frac{\mu_0 I}{2\pi}\left(\frac{1}{r} - \frac{1}{r+a}\right) \\ &= \frac{\mu_0 iI}{2\pi}\frac{ab}{r(r+a)} \end{aligned}\text{。}$$

选（D）。

56. 解：一维谐振子的能级公式为

$$E_n = \left(n + \frac{1}{2}\right)h\nu\text{，}$$

基态对应 n = 0。选（C）。

57. 解：因为磁场中线圈的面积随时间的变化率为一个常量，而磁场的大小一定，故总的磁通量随时间的变化率大小是一定的。因此感生电动势的大小也是一个常量，进一步判断其方向可知，感生电动势是一个方波的形式。选（A）。

58. 解：第 11 号元素是第三周期的第一个元素，第三壳层电子只填了一个最低能态的，其余内部是满壳层。其电子组态为 $1s^2 2s^2 2p^6 3s$。选（C）。

59. 解：He 原子核 Z = A = 1，是奇奇核。质子和中子的自旋都为 1/2，轨道角动量都是零，耦合出来的结果是单重态。选（A）。

60. 解：根据经典的电子回转模型，向心力由磁场力提供

$$\omega = \frac{eB}{m^*} \sim \frac{10^{-19} \times 1}{0.1 \times 10^{-30}} = 10^{12}\text{。}$$

选（D）。

61. 解：利用复摆的简谐振动周期计算公式

$$T = 2\pi\sqrt{\frac{I}{2mgl_c}}\text{，}$$

图 I 的 $I_I = 2ml^2$；图 II 的 $I_{II} = 5ml^2 / 4$。因此

$$\frac{f_{II}}{f_I} = \sqrt{\frac{I_I l_{cII}}{I_{II} l_{cI}}} = \sqrt{\frac{2 \times \frac{3}{4} l}{\frac{5}{4} \times l}} = \sqrt{\frac{6}{5}}\text{。}$$

选（A）。

62. 解：等温过程中理想气体的内能不变，气体做的功

$$A=-\int_{V_0}^{V_1}pdV=-RT_0\int_{V_0}^{V_1}\frac{1}{V}dV=\mathrm{RT_0}\ln\left(\mathrm{V_1/V_0}\right)。$$

选（E）。

63. 解：系统的熵达到极大，是最可几状态。选（D）。

64. 解：由

$$\rho=\varepsilon_0\nabla\cdot\vec{E}=k\varepsilon_0$$

知电荷密度不为零。在该区域内这样的一个电荷分布就能决定一个这样的静电场。选（B）。

65. 解：最简单的办法是写出−q 粒子在 y 方向发生小位移时电势能的展开式

$$V(y)=-2\frac{Qq}{4\pi\varepsilon_0(R+x)}=-\frac{Qq}{4\pi\varepsilon_0 R}\left(1-\frac{x}{R}+\frac{x^2}{R^2}\right),$$

利用其中二次项的系数求振动圆频率

$$\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{Qq}{2\pi\varepsilon_0 mR^3}}。$$

选（E）。

66. 解：以悬点为重力势能零点，末态的总势能为零。由机械能守恒定律可知，所需的能量等于重力势能的增值

$$E=-mgH=2\times10\times10\times\frac{10}{2}=1000\mathrm{J}。$$

选（C）。

67. 解：自然光经过偏振片后的出射光强

$$I=\frac{I_0}{2},$$

平面偏振光经过偏振片后得出射光强

$$I=I_0\cos^2\theta。$$

题中所给出的

$$I=(A-B)+2B\cos^2\theta,$$

可以看作是两部分的叠加。选（C）。

68. 解：望远镜的极限角分辨率

$$\delta\theta=\frac{\lambda}{D}。$$

因此最小的镜面直径

$$D = \frac{\lambda}{\delta\theta} = 0.1(m)\text{。}$$

选（C）。

69. 解：电荷的最小速度是该媒质中的光速，

$$v_{\min} = \frac{c}{n} = \frac{2}{3}c\text{。}$$

选（D）。

70. 解：在 E >> E_0 的条件下，很方便的近似有

$$p = \sqrt{m^2 - m_0^2}c \approx \frac{E}{c} = mc\text{。}$$

选（D）。

71. 解：根据 Boltzmann 分布，分布在能级 E 上的几率

$$p \propto e^{-E/kT}\text{，}$$

因此

$$N_1 = N_0\frac{p_1}{p_1 + p_2} = N_0\frac{e^{-E_1/kT}}{e^{-E_1/kT} + e^{-E_2/kT}} = N_0\frac{1}{1 + e^{-\varepsilon/kT}}\text{。}$$

选（B）。

72. 解：系统的比热

$$C = \frac{dU}{dt} = N_0 k\left(\frac{\varepsilon}{kT}\right)^2 \frac{e^{\varepsilon/kT}}{\left(1 + e^{\varepsilon/kT}\right)^2}\text{。}$$

选（A）。

73. 解：T=0 时所有子系统均只能处在低能级，S=0。T=∞时高能级和低能级的出现几率都是 1/2，按 Boltzmann 关系知 $S = N_0 k\ln 2$。选（C）。

74. 解：Y 的转动惯量

$$\mathrm{I_Y} = \mathrm{m_Y R_Y^2} = \frac{1}{4}\mathrm{m_x}\left(\frac{R_X}{\$}\right)^2 = \frac{1}{64}I_X\text{，}$$

因此根据周期公式

$$T_Y = 2\pi\sqrt{\frac{I_Y}{m_Y g R_{CY}}} = 2\pi\sqrt{\frac{I_x/64}{\frac{m_x}{4}g\frac{R_X}{4}}} = T\sqrt{\frac{4\times 4}{64}} = T/2\text{。}$$

选（B）。

75. 解：根据能量-动量守恒，反映后生成粒子的总动量为零，因此

$$p_{He} = p_{Th}\,,$$

$$E_{He} = \frac{p_{He}^2}{2m_{He}} > E_{Th} = \frac{p_{Th}^2}{2m_{Th}}\,。$$

选（E）。

76. 解：三个电子的自旋分别可取±1/2，它们的轨道角动量量子数分别为 0, 1, 1。因此耦合的最大总角动量量子数

$$J = \frac{1}{2}\times 3 + 0 + 1 + 1 = \frac{7}{2}\,。$$

选（A）。

77. 解：自旋磁矩$\mu_S = -g_S\mu_B$，其中 g_S 是 Lande 因子。电子的 Bohr 磁子

$$\mu_B = \frac{e}{2m_e}$$

对于重原子核，自旋磁矩的表达有类似形式，因为 $m_n >> m_e$，所以核的磁矩比电子的要小很多。选（E）。

78. 解：体系的动量守恒，质心系是惯性参考系。在质心系中，相遇前后的过程满足角动量守恒，列出有关方程如下：

$$2mv - mv = 2mv_c\,,$$

$$v_c = \frac{1}{2}v\,;$$

$$2\times m\cdot\frac{3}{2}v\frac{b}{2} = m\left(\frac{b}{2}\right)^2\omega\,,$$

$$\omega = \frac{3v}{b}\,。$$

t > 0 时上面一个滑冰者做一个附加在 $v_c = v/2$ 匀速直线运动上的匀速圆周运动。选（C）。

79. 解：相速和群速的计算公式是

$$\begin{aligned} v_p &= \frac{\omega}{k}(>0) \\ v_G &= \frac{d\omega}{dk}(<0,\ \ k_1 < k < k_2) \end{aligned}\,。$$

选（A）。

80. 解：出射 x 射线的最大能量约等于入射电子的全部能量

$$\frac{hc}{\lambda_m} = eV\text{，}$$

$$\lambda_m = \frac{hc}{eV} = 0.5\text{A}\text{。}$$

选（B）。

81. 解：串联 RCL 电路的总阻抗

$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)\text{。}$$

显然当

$$\omega^2 = \frac{1}{LC}$$

时虚部为零，总阻抗最小，总电流最大。选（C）。

82. 解：薄板相对于 z 轴的转动惯量与细长杆的转动惯量类似，积分得

$$I = \frac{1}{12}M(2d)^2 = \frac{1}{3}Md^2\text{，}$$

根据冲量矩定理直接计算可得

$$\omega = \frac{H}{I}\text{。}$$

选（D）。

83. 解：受力平衡时拉力、重力、Coulomb 斥力三者平衡，有

$$\tan\theta \cong \theta = \frac{\frac{kq^2}{(d)^2}}{Mg} = \frac{d}{2L}\text{，}$$

$$d = \left(\frac{2kq^2L}{Mg}\right)^{\frac{1}{3}}\text{。}$$

选（A）。

84. 解：非相对论性粒子辐射的 Lamor 公式

$$P = \frac{1}{6\pi\varepsilon_0}\cdot\frac{e^2a^2}{c^3}\text{。}$$

功率的角分布在直线运动的情况下，是垂直速度或加速度的方向最大，$\theta = \pi/2$。选（D）。

85. 解：根据相对论性的能量-动量公式

$$E = \sqrt{E_0^2 + p^2c^2}\text{，}$$

$$p=\frac{\sqrt{E^2-E_0^2}}{c}\approx 1.4\text{MeV/c}\text{。}$$

选（C）。

86. 解：电容

$$C=\frac{Q}{V},$$

其中 Q 通过 R 上的电流积分求出，正比于示波器上充电曲线下的面积，必须知道扫描速度 s 和电阻 R。V 即充电结束时 R 上的电压，是示波器显示电压的峰值。选（B）。

87. 解：对半径为 r 的圆周运动

$$m\frac{v^2}{r}=\frac{K}{r^3},$$

$$V(r)=-\int_r^\infty\frac{K}{r^3}dr=-\frac{K}{2r^2}=-T,$$

$$T=\frac{1}{2}mv^2=\frac{K}{2r^2}\text{。}$$

选（C）。

88. 解：平行板电容器的电容

$$C_f=\varepsilon C_0,$$

由于电源的电动势不变，因此充电完成后电容的电压就是 V_0，电场强度

$$E_f=E_0\text{。}$$

而电量

$$Q_f=C_fV_0=\varepsilon Q_0,$$

因此极板间的电位移矢量

$$D_f=\sigma_f=\varepsilon\sigma_0>\sigma_0=D_0\text{。}$$

选（E）。

89. 解：一维谐振子波函数的宇称为$(-1)^n$。图中给出的势能 x 轴负半轴为无穷高势垒，因此波函数在 x = 0 点必须为零，因此只能取宇称为奇的谐振子波函数解。因此只有 n 为奇数的能级存在。选（E）。

90. 解：双原子分子的转动能级可以由刚性分子模型计算，量子力学给出分子转动能量为

$$E_r=\frac{h^2}{8\pi^2I}J(J+1),$$

其中 J 为转动量子数，$I=\mu r^2$ 为分子绕质心的转动惯量，可以由原子的质量和核间距估算。

一般分子内部运动的能级中，电子运动能量 $E_e \sim 10\text{eV}$，振动能量 $E_v \sim 0.1\text{eV}$，转动能量 $E_r \sim 0.001\text{eV}$。选（B）。

91. 解：反应前粒子的宇称为+，反应后的总宇称为–，宇称不守恒。因此必为弱相互作用。选（E）。

92. 解：根据图中磁极分布的对称性，线圈每旋转一圈其中的感生电动势应该变化了三个周期。因此

$$f = 10 \times 3 = 30\text{Hz} \text{。}$$

选（D）。

93. 解：小球的加速度分两部分，切向加速度和法向加速度。

$$\begin{aligned} a_\tau &= g\cos\theta \\ a_n &= \frac{v^2}{R} = \frac{2gR\sin\theta}{R} = 2g\sin\theta \end{aligned},$$

总加速度是以上两个分量的合成。选（E）。

94. 解：Lorentz 变换的一般公式为

$$\begin{aligned} x' &= \frac{x - vt}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma(x - \beta ct) \\ y' &= y \\ z' &= z \\ t' &= \frac{t - vx/c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \gamma(t - \beta x/c) \end{aligned} \quad ,\ \gamma \ge 1,\ \beta \le 1\text{。}$$

对照以上四式，选（C）。

95. 解：进入到探测器立体角里的质子对应的入射截面是

$$\frac{10^2}{10^{12}} = 10^{-10}\,\text{cm}^2,$$

微分截面是

$$\frac{10^{-10}}{10^{-4}} = 10^{-6}\,\text{cm}^2/\text{steradian} \text{。}$$

它由 10^{20} 个核的贡献组成，每个核的贡献是

$$\frac{10^{-6}}{10^{20}} = 10^{-26}\,\text{cm}^2/\text{steradian} \text{。}$$

选（C）。

96. 解：抽去气体后，总的光程差减小了 40 倍波长。

$$\Delta l = 40\lambda = 2(n-1)\times 5\text{cm},$$

解得 n = 1.0002。选（C）。

97. 解：晶体中 Bloch 波函数波包的速度

$$\mathbf{v}_k = \frac{2\pi(\nabla_{\mathbf{k}}E)}{h},$$

根据准经典运动公式

$$\frac{dv}{dt} = \frac{d}{dt}\left(\frac{1}{\hbar}\frac{dE(k)}{dk}\right) = \frac{1}{\hbar}\frac{dk}{dt}\frac{d^2E(k)}{dt^2} = \frac{F}{\hbar^2}\frac{d^2E(k)}{dt^2} = \frac{F}{m^*},$$

$$m^* = \frac{\hbar^2}{\left(\dfrac{d^2E}{dk^2}\right)}。$$

选（D）。

98. 解：求解本征值

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 1 & 0 & -\lambda \end{vmatrix} = 0,$$

解得：$\lambda^3 = 1$，λ为三次单位根。选（B）。

99. 解：微扰的 Hamilton 量为

$$H' = -e\mathbf{E}\cdot\mathbf{r} = -eEr\cos\theta,$$

氢原子基态的波函数只含径向部分，因此一级微扰修正中关于角度部分的积分必为零。
选（A）。

100. 解：平衡时相对支点的总力矩为零，

$$20gx + 20g(5+x) = 40g(5-x),$$

解得

$$x = 1.25\text{m}。$$

选（C）。

模拟练习题二

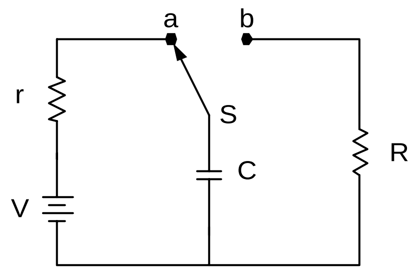


Figure 1

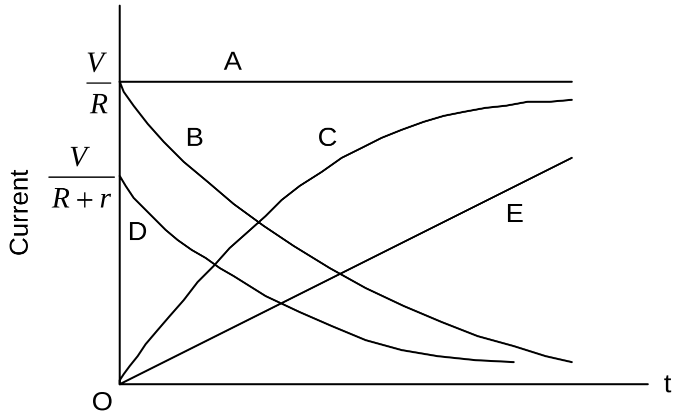


Figure 2

1. The capacitor shown in Figure 1 above is charged by connecting switch S to contact a. If switch S is thrown to contact b at time t = 0, which of the curves in Figure 2 above represents the magnitude of the current

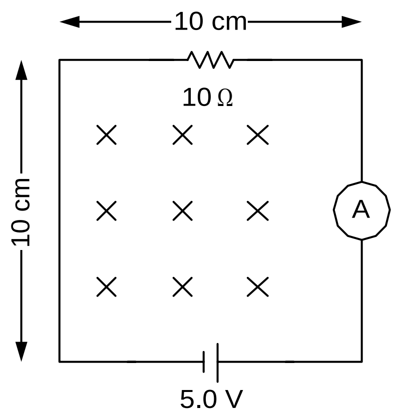
(A) A

(B) B

(C) C

(D) D

(E) E



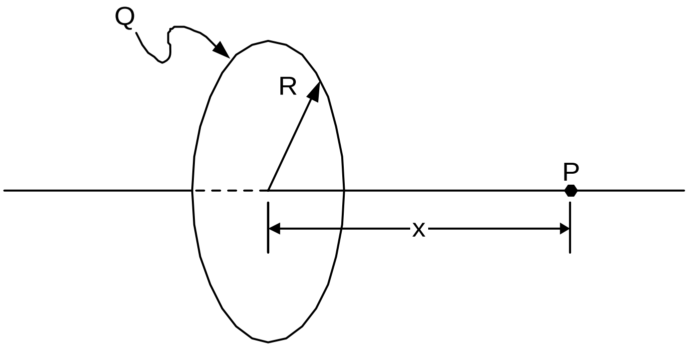
2. The circuit shown above is in a uniform magnetic field <u>that is into</u> the page and is decreasing in magnitude at the rate of 150 tesla/second. The ammeter reads

(A) 15 A

(B) 0.35 A

(C) 0.50 A
(D) 0.65 A
(E) 0.80 A

<u>Questions 3-4</u> refer to a thin ring of radius R, as shown below, which has a charge Q uniformly spread out on it.



3. The electric potential at a point P, which is located on the axis of symmetry a distance x from the center of the ring, is given by

(A) $\dfrac{Q}{4\pi\varepsilon_0 x}$

(B) $\dfrac{Q}{4\pi\varepsilon_0 \sqrt{R^2 + x^2}}$

(C) $\dfrac{Qx}{4\pi\varepsilon_0 (R^2 + x^2)}$

(D) $\dfrac{Qx}{4\pi\varepsilon_0 (R^2 + x^2)^{\frac{3}{2}}}$

(E) $\dfrac{QR}{4\pi\varepsilon_0 (R^2 + x^2)}$

4. A small particle of mass m and charge –q is place at point P and released. If R >> x, the particle will undergo oscillations along the axis of symmetry with an angular frequency that is equal to

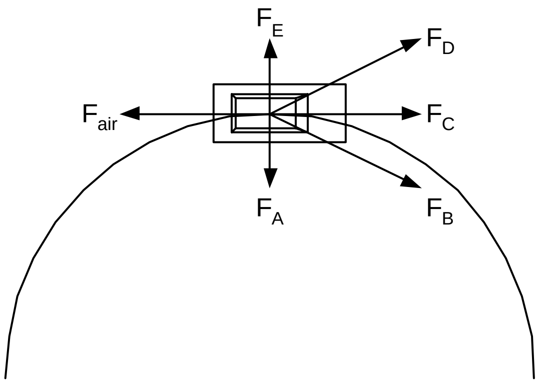
(A) $\sqrt{\dfrac{qQ}{4\pi\varepsilon_0 m R^3}}$

(B) $\sqrt{\dfrac{qQx}{4\pi\varepsilon_0 m R^4}}$

(C) $\dfrac{qQ}{4\pi\varepsilon_0 m R^3}$

(D) $\dfrac{qQx}{4\pi\varepsilon_0 mR^4}$

(E) $\sqrt{\dfrac{qQx}{4\pi\varepsilon_0 m}}\sqrt{\dfrac{1}{R^2 - x^2}}$



5. A car travels with constant speed on a circular road on level ground. In the diagram above, F_{air} is the force of air resistance on the car. Which of the other forces shown best represents the horizontal force of the road on the car's tires?

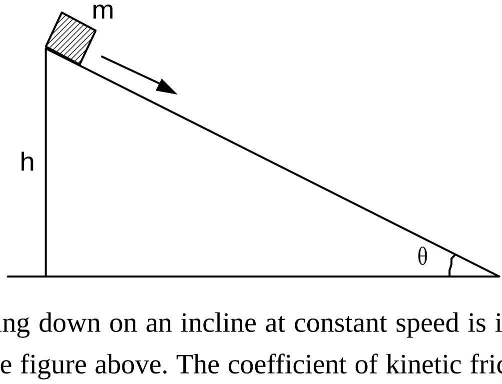
(A) F_A

(B) F_B

(C) F_C

(D) F_D

(E) F_E



6. A block of mass m sliding down on an incline at constant speed is initially at a height h above the ground, as shown in the figure above. The coefficient of kinetic friction between the mass and the incline is μ. If the mass continue to slide down the incline at a constant speed, how much energy is dissipated by friction by the time the mass reaches the bottom of the incline?

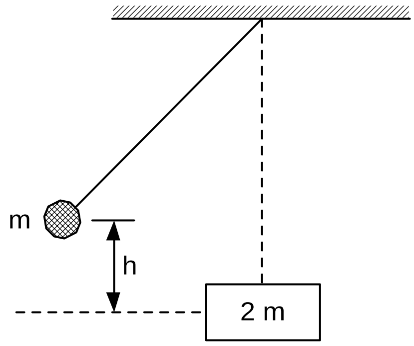
(A) mgh / μ

(B) mgh

(C) μmgh / sinθ

(D) mgh sinθ

(E) 0



7. As shown above, a ball of mass m, suspended on the end of a wire, is released from height h and collides elastically, when it is at its lowest point, with a block of mass 2m at rest on a frictionless surface. After the collision, the ball rises to a final height equal to
(A) 1/9 h
(B) 1/8 h
(C) 1/3 h
(D) 1/2 h
(E) 2/3 h

8. A particle of mass m undergoes harmonic oscillation with period T_0. A force f proportional to the speed v of the particle, f = –bv, is introduced. If the particle continues to oscillate, the period with f acting is
(A) larger than T_0
(B) smaller than T_0
(C) independent of b
(D) dependent largely on b
(E) constantly changing

9. In the spectrum of hydrogen, what is the ratio of the longest wavelength in the Lyman series n_f = 1, to the longest wavelength in the Balmer series (n_f = 2) ?
(A) 5/27
(B) 1/3
(C) 4/9
(D) 3/2
(E) 3

10. Internal conversion is the process whereby an excited nucleus transfers its energy directly to one of the most tightly bound atomic electrons, causing the electron to be ejected from the atom and leaving the atom in an excited state. The most probable process after an internal conversion electron is ejected from an atom with a high atomic number is that the
(A) atom returns to its ground state through inelastic collisions with other atoms.
(B) atom emits one or several x-rays
(C) nucleus emits a γ-ray
(D) nucleus emits an electron
(E) nucleus emits a position

11. A beam of neutral hydrogen atoms in their ground state is moving into the plane of this page and passes through a region of a strong inhomogeneous magnetic field that is directed upward in the plane of the page. After the beam passes through this field, a detector would find that it has been

(A) deflected upward

(B) deflected to the right

(C) undeviated

(D) split vertically into two beams

(E) split horizontally into three beams

12. The ground-state energy of positronium is most nearly equal to

(A) – 27.2 eV

(B) – 13.6 eV

(C) – 6.8 eV

(D) – 3.4 eV

(E) 13.6 eV

13. A 100-watt electric heating element is placed in a pan containing one liter of water. Although the heating element is on far a long time, the water, though close to boiling, does not boil. When the heating element is removed, approximately how long will take the water to cool by 1°C? (Assume that the specific heat for water is 4.2 kilojoules/kilogram °C.)

(A) 20 s

(B) 40 s

(C) 60 s

(D) 130 s

(E) 200 s

14. Two identical 1.0-kilogram blocks of copper metal, one initially at a temperature T_1 = 0°C and the other initially at a temperature T_2 = 100°C, are enclosed in a perfectly insulating container. The two blocks are placed in contact, they come to equilibrium at a final temperature T_f. The amount of heat exchanged between the two blocks in this process is equal to which of the following? (The specific heat of copper metal is equal to 0.1 kilocalories/kilogram °K.)

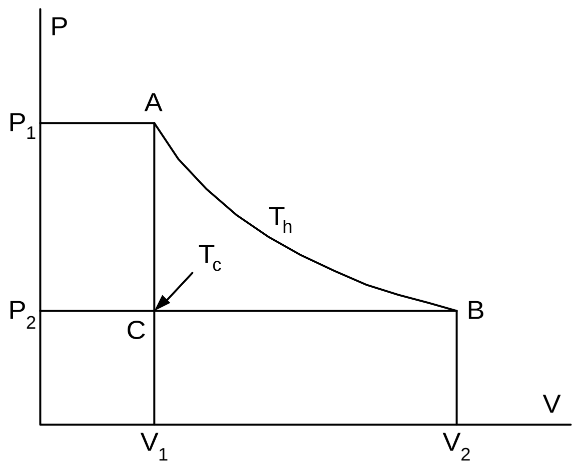
(A) 50 kcal

(B) 25 kcal

(C) 10 kcal

(D) 5 kcal

(E) 1 kcal



15. Suppose one mole of an ideal gas undergoes the reversible cycle ABCA shown in the P/V diagram above, where AB is an isotherm. The molar heat capacities are C_p at constant pressure and C_v at constant volume. The net heat added to the gas during the cycle is equal to

(A) RT_hV_2/V_1

(B) $-C_p(T_h-T_c)$

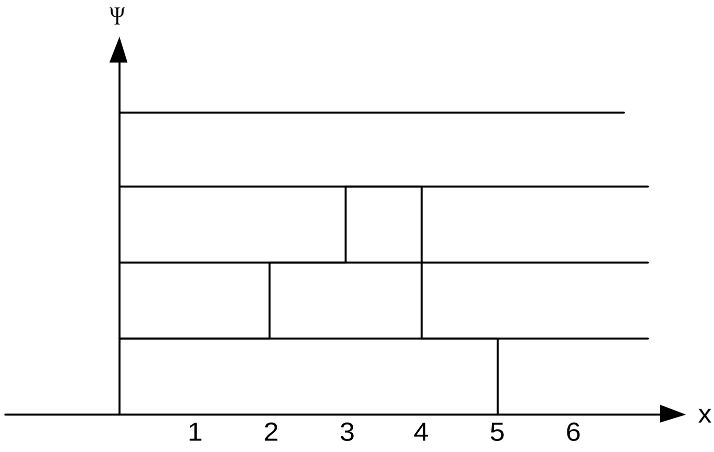
(C) $C_v(T_k-T_c)$

(D) $RT_h \ln V_2/V_1 - C_p(T_h-T_c)$

(E) $RT_h \ln V_2/V_1 - R(T_h-T_c)$

16. The mean free path for the molecules of a gas is approximately given by $\dfrac{1}{\eta\sigma}$, where η is the number density and σ is the collision cross section. The mean free path for air molecules at room conditions is approximately

(A) 10^{-4} m
(B) 10^{-7} m
(C) 10^{-10} m
(D) 10^{-13} m
(E) 10^{-16} m



17. The wave function for a particle constrained to move in one dimension is shown in the graph above ($\Psi = 0$ for $x \leqslant 0$ and $x \geqslant 5$). What is the probability that the particle would be found between $x = 2$ and $x = 4$?

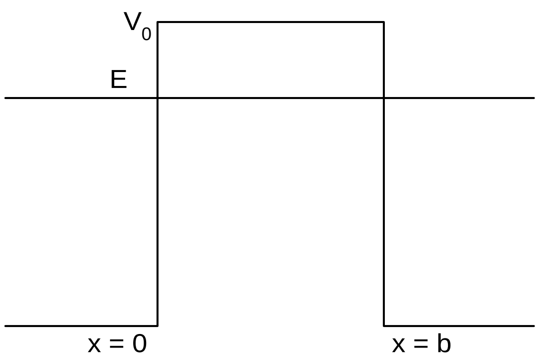
(A) 17/64

(B) 1/2

(C) 5/8

(D) $\sqrt{5/8}$

(E) 13/16



18. Consider a potential of the form

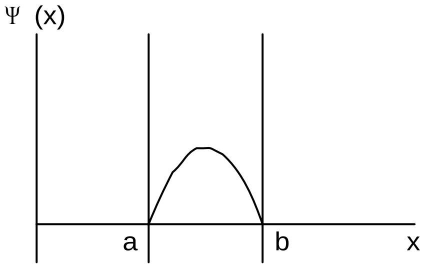
$V(x) = 0, x \leqslant a$

$V(x) = V_0, a<x<b$

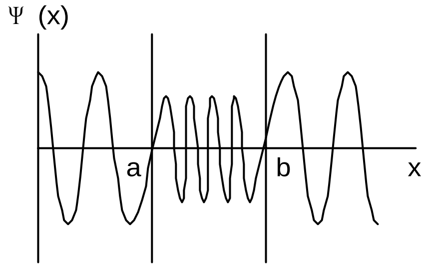
$V(x) = 0, x \geqslant b$

As shown in the figure above. Which of the following wave functions is possible for a particle incident from the left with energy $E < V_0$?

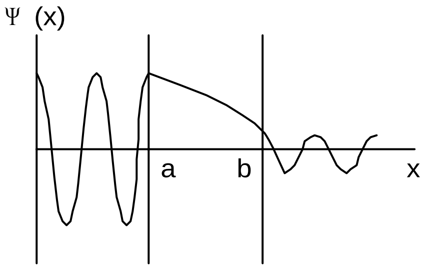
(A)



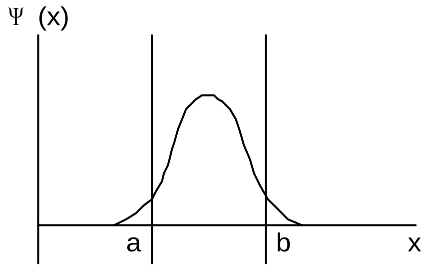
(B)



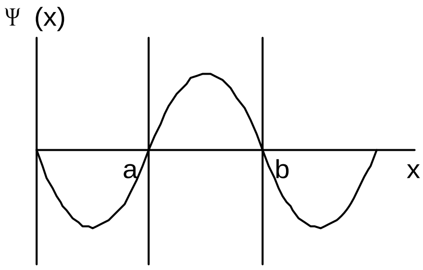
(C)



(D)



(E)



19. When alpha particles are directed onto atoms in a thin metal foil, some make very close collisions with the nuclei of the atoms and are scattered at large angles. If an alpha particle with an initial kinetic energy of 5 MeV happens to be scattered through an angle of 180°, which of the following must have been its distance of closest approach to the scattering nucleus? (Assume that the metal foil is made of sliver, with Z = 50.)

(A) $1.22\times 50^{\frac{1}{3}}$ fm

(B) 2.9×10^{-14} m

(C) 1.0×10^{-12} m

(D) 3.0×10^{-4} m

(E) 1.7×10^{-7} m

20. A helium atom, mass 4u, travels with nonrelativistic speed **v** normal to the surface of a certain material, makes an elastic collision with an (essentially free) surface atom, and leaves in the opposite direction with speed 0.6v. The atom on the surface must be an atom of

(A) hydrogen, mass 1u
(B) helium, mass 4u
(C) carbon, mass 12u
(D) oxygen, mass 16u
(E) silicon, mass 28u

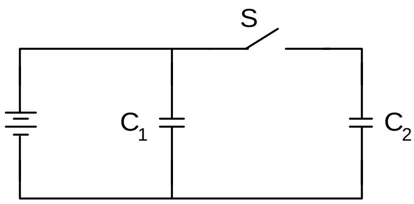
21. The period of a physical pendulum is $2\pi\sqrt{I / mgd}$, where I is the moment of inertia about the pivot point and d is the distance from the pivot to the center of mass. A circular hoop hangs from a nail on a barn wall. The mass of the hoop is 3 kilograms and its radius is 20 centimeters. If it is displaced slightly by a passing breeze, what is the period of the resulting oscillation?
(A) 0.63 s
(B) 1.0 s
(C) 1.3 s
(D) 1.8 s
(E) 2.1 s

22. The curvature of Mars is such that its surface drops a vertical distance of 2.0 meters for every 3600 meters tangent to the surface. In addtion, the gravitational acceleration near its surface is 0.4 times that near the surface of Earth. What is the speed a golf ball would need to orbit Mars near the surface, ignoring the effects of resistance?
(A) 0.9 km/s
(B) 1.8 km/s
(C) 3.6 km/s
(D) 4.5 km/s
(E) 5.4 km/s

23. Suppose that the gravitational force law between two massive objects were $\mathbf{F}_{12} = \hat{\mathbf{r}}_{12} G m_1 m_2 / r_{12}^{2+\varepsilon}$, where ε is a small positive number. Which of the following statements would be FALSE?
(A) The total mechanical energy of the planet-Sun system would be conserved..
(B) The angular momentum of a single planet moving about the Sun would be conserved.
(C) The periods of planets in circular orbit would be proportional to the (3+ε)/2 power of their respective orbital radii.
(D) A single planet could move in stationary noncircular elliptical orbit about the Sun.
(E) A single planet could move in a stationary circular orbit about the Sun.

24. Two identical conducting spheres, A and B, carry equal charge. They are initially separated by a distance much larger than their diameters and the force between them is F. A third identical conducting sphere, C, is uncharged. Sphere C is first touched to A, then to B, and then removed. After that the force between A and B is equal to
(A) 0
(B) F / 8

(C) F / 4
(D) 3F / 8
(E) F / 2

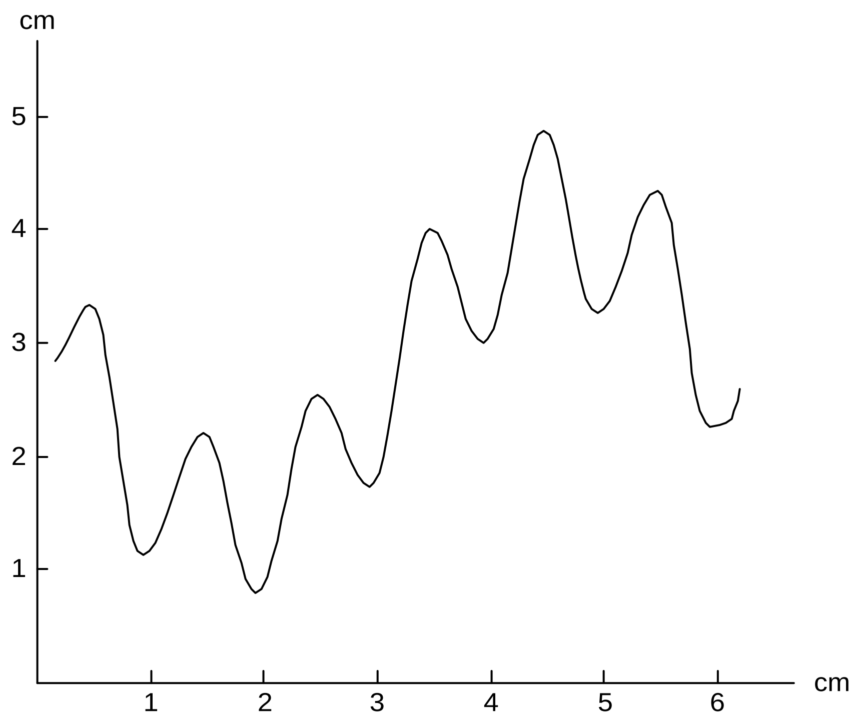


25. Two real capacitors of equal capacitance ($C_1 = C_2$) are shown in the figure above. Initially, while the switch S is open, one of the capacitors is uncharged and the other carries charge Q_0. The energy stored in the charged capacitor is U_0. Sometime after the switch is closed, the capacitors C_1 and C_2 carry charges Q_1 and Q_2, respectively; the voltages across the capacitors are V_1 and V_2; and the energies stored in the capacitors are U_1 and U_2. Which of the following statements is INCORRECT?
(A) $Q_0 = (Q_1 + Q_2)/2$
(B) $Q_1 = Q_2$
(C) $V_1 = V_2$
(D) $U_1 = U_2$
(E) $U_0 = U_1 + U_2$

26. A series RLC circuit is used in a radio to tune an FM station broadcasting at 103.7 MHz. The resistance in the circuit is 10 ohms and the inductance is 2.0 microhenries. What is the best estimate of the capacitance that should be used?
(A) 200 pF
(B) 59 pF
(C) 1 pF
(D) 0.2 pF
(E) 0.02 pF

27. In laboratory experiment, graphs are employed to determine how one measured variable depends on another. These graphs generally fall into three categories: linear, semilog (logarithmic versus linear), and log-log. Which type of graph listed in the third column below would NOT be the best for plotting data to test relationship given in the first and second columns?

	Relation	Variables Plotted	Type of Graph
(A)	$dN/dt \propto e^{-2t}$	Activity vs. time for a radioactive isotope	Semilog
(B)	$eV_s = hf - W$	Stopping potential vs. frequency for the photoelectric effect	Linear
(C)	$s \propto t^2$	Distance vs. time for an object undergoing constant acceleration	Log-Log
(D)	$V_{out}/V_{in} \propto 1/\omega$	Gain vs. frequency for a low-pass filter	Linear
(E)	$P \propto T^4$	Power radiated vs. temperature for blackbody radiation	Log-Log



28. The figure above represents the trace on the screen of a cathode ray oscilloscope. The screen is graduated in centimeters. The spot on the screen moves horizontally with a constant speed of 0.5 centimeter/millisecond, and the vertical scale is 2 volts/centimeters. Which of the following are most nearly the observed amplitude and frequency of these two oscillations?

	Oscillation 1	Oscillation 2
(A)	5 V, 250 Hz	2.5 V, 1000 Hz
(B)	1.5 V, 250 Hz	3 V, 1500 Hz
(C)	5 V, 6 Hz	2 V, 2 Hz
(D)	2.5 V, 83 Hz	1.25 V, 500 Hz
(E)	6.14 V, 98 Hz	1.35 V, 257 Hz

29. The characteristic distance at which quantum gravitational effects are significent, the Plank length, can be determinedfrom a suitable combination of the physical constants G, h, and c. Which of the following correctly gives the plank length?

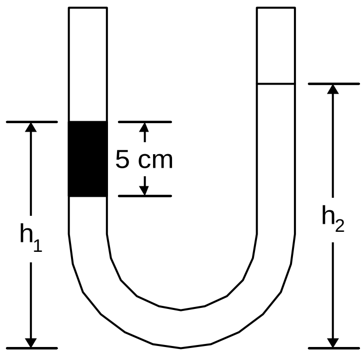
(A) Ghc

(B) Gh^2c^2

(C) G^2hc

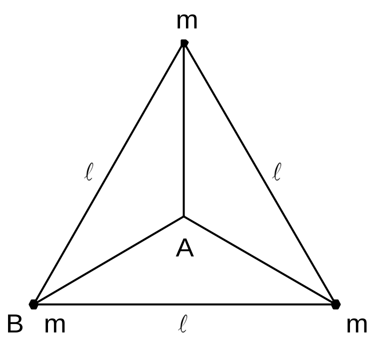
(D) $G^{\frac{1}{2}}h^2c$

(E) Gh / c



30. An open-ended U-tube of uniform cross-sectional area contains water (density 1.0 gram / centimeter^3) standing initially 20 centimeters from the bottom in each arm. An immiscible liquid of density 4.0 grams / centimeter^3 is added to one arm until a layer 5 centimeters high forms, as shown in the figure above. What is the ratio h_2 / h_1 of the heights of the liquid in the arms?
(A) 3/1
(B) 5/2
(C) 2/1
(D) 3/2
(E) 1/1

31. A sphere of mass m is released from rest in a viscous medium. In addition to the gravitational force of magnitude mg, the sphere experiences a retarding force of magnitude bv, where v is the speed of the sphere and b is a constant. Assume that the floating force is negligible. Which of the following statements about the sphere is correct?
(A) Its kinetic energy decreases due to the retarding force.
(B) Its kinetic energy increases to a maximum, then decreases to a final terminal speed.
(C) Its speed increases to a maximum, then decreases back to a final terminal speed.
(D) Its speed increases monotonically, approaching a terminal speed that depends on b but not on m.
(E) Its speed increases monotonically, approaching a terminal speed that depends on both b and m.



32. Three equal masses m are rigidly connected to each other by massless rods of length l forming an equilateral triangle, as shown above. The assembly is to be given an angular velocity ω about an axis perpendicular to the triangle. For fixed ω, the ratio of the kinetic energy of the assembly for an axis through B compared with that for an axis through A is equal to

(A) 3
(B) 2
(C) 1
(D) 1/2
(E) 1/3

33. A diatomic molecule is initially in the state $\Psi(\theta,\Phi) = (5Y_1^1 + 3Y_5^1 + 2Y_5^{-1})/(38)^{\frac{1}{2}}$, where Y_l^m is a sperical harmonic. If measurements are made of the total angular mementum quantum number l and of the azimuthal angular momentum quantum number m, what is the probability of obtaining the result l = 5 ?
(A) 36 / 1444
(B) 9 / 38
(C) 13 / 38
(D) $5/(38)^{1/2}$
(E) 34 / 38

34. When the beta decay of ^{60}Co nucleus observed as low temperature in a magnetic field that aligns the spins of the nuclei, it is found that the electrons are emitted preferentially in a direction opposite to the ^{60}Co spin direction. Which of the following invariances is violated by this decay?
(A) Gauge invariance
(B) Time invariance
(C) Translation invariance
(D) Reflection invariance
(E) Rotation invariance

35. The wave function for identical fermions is antisymmetric under particle interchange. Which of the following is a consequence of this property?
(A) Pauli exclusion principle
(B) Bohr correspondence principle
(C) Heisenberg uncertainty principle
(D) Bose-Einstein condensation
(E) Fermi's golden rule

36. A lump of clay whose mass is 4 kilograms is traveling at three-fifths the speed of light when it collides head-on with an identical lump going the opposite direction at the same speed. If the two lumps stick together and no energy is radiated away, what is the mass of the composite lump?
(A) 4 kg
(B) 6.4 kg
(C) 8 kg
(D) 10 kg
(E) 13.3 kg

37. An atom moving at speed 0.3c emits an electron along the same direction with speed 0.6c in the internal rest frame of the atom. The speed of the electron in the lab frame is equal to
(A) 25c
(B) 0.51c
(C) 0.66c
(D) 0.76c
(E) 0.90c

38. What is the speed of a particle having a momentum of 5MeV/c and a total relativistic energy of 10 MeV?
(A) c
(B) 0.75c
(C) $\frac{1}{\sqrt{3}}c$
(D) $\frac{1}{2}c$
(E) $\frac{1}{4}c$

39. Which of the following atoms has the lowest ionization potential?
(A) ${}^{2}_{4}He$
(B) ${}^{7}_{14}N$
(C) ${}^{8}_{16}O$
(D) ${}^{18}_{39}Ar$
(E) ${}^{55}_{132}Cs$

40. If singly ionized helium atom in an n = 4 state emits a photon of wavelength 470 nanometers, which of the following gives the approximate final energy level E_f of the atom, and the n value n_f of this final state?

	E_f (eV)	n_f
(A)	–6.0	3
(B)	–6.0	2
(C)	–14	2
(D)	–14	1
(E)	–52	1

41. A 3p electron is found in the ${}^{2}P_{\frac{3}{2}}$ energy level of a hydrogen atom. Which of the following is

true about the electron in this state?

(A) It is allowed to make an electric dipole transition to the $^2S_{\frac{1}{2}}$ level.

(B) It is allowed to make an electric dipole transition to the $^2P_{\frac{1}{2}}$ level.

(C) It has quantum numbers l = 3, j = 3/2, s = 1/2.

(D) It as quantum number n = 3, l = 1, s = 3/2.

(E) It has exactly the same energy level as in the $^2D_{\frac{3}{2}}$ level.

42. Light of wavelength 500 nanometers is incident on sodium, with work function 2.28 electron volts. What is the maximum kinetic energy of the ejected photoelectrons?

(A) 0.03 eV

(B) 0.2 eV

(C) 0.6 eV

(D) 1.3 eV

(E) 2.0 eV

43. The line integral of u = y **i** –x **j** + 2 **k** around a circle of radius R in the x-y plane with center at the origin is equal to

(A) 0

(B) $2\pi R$

(C) $2\pi R^2$

(D) πR^2

(E) $3R^2$

44. A particle of unit mass undergoes one-dimensional motion such that its velocity varies according to

$$v(x) = \beta x^{-n}$$

where β and n are constants and x is the position of the particle. What is the acceleration of the particle as a function of x ?

(A) $-n\beta^2 x^{-2n-1}$

(B) $-n\beta^2 x^{-n-1}$

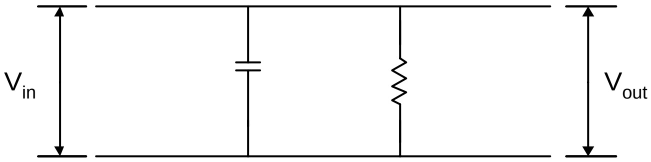
(C) $-n\beta^2 x^{-n}$

(D) $-\beta x^{-n-1}$

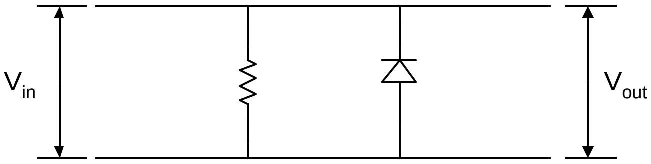
(E) $-\beta x^{-2n-1}$

45. The circuits below consist of two-element combinations of capacitors, diodes and resistors. V_{in} represents an ac-voltage with variable frequency. It is desired to build a circuit for which $V_{out} = V_{in}$ at high frequencies and $V_{out} = 0$ at low frequencies. Which of the following circuits will perform the task?

(A)



(B)



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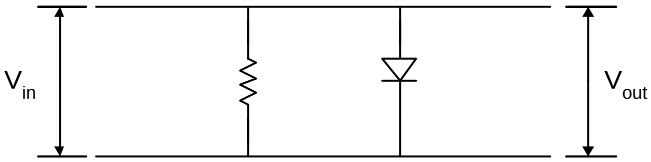

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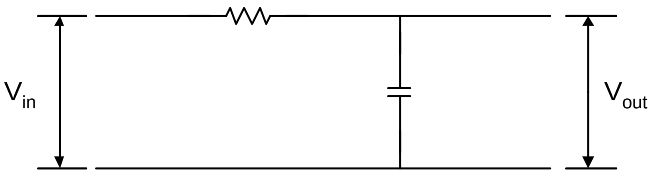
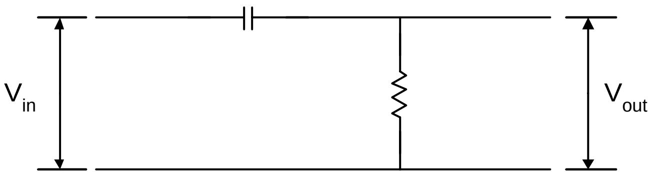
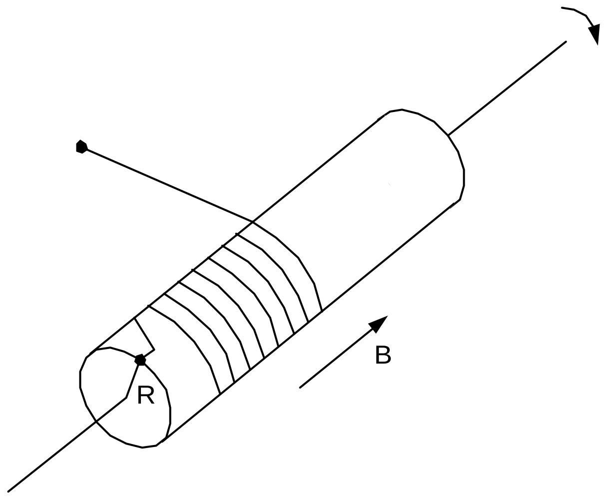


(E)



46. Two electrons, e.g., those in an excited state of He, interact by a Coulomb potential. If their spins are parallel, the spatial part of the total wave function must be antisymmetric with respect to exchange. This triplet state is lower in energy than the corresponding singlet state (antiparallel spins) because in the triplet state the

(A) magnetic dipole-dipole interaction is weaker
(B) magnetic dipole-dipole interaction is stronger
(C) electrons are on the average closer together
(D) electrons are on the average farther apart
(E) spin-orbit couplings are weaker



47. A wire is being wound around a rotating wooden cylinder of radius R. One end of the wire is connected to the axis of the cylinder, as shown in the figure above. The cylinder is placed in a uniform magnetic field of magnitude B parallel to its axis and rotates at N revolutions per second. What is the potential difference between the open ends of the wire?

(A) 0

(B) $2\pi NBR$

(C) πNBR^2

(D) BR^2 / N

(E) πNBR^3

48. The half-life of a π^+ meson at rest is 2.5×10^{-8} second. A beam of π^+ mesons is generated at a point 15 meters from a detector. Only 1/2 of the π^+ mesons live to reach the detector. The speed of the π^+ mesons is

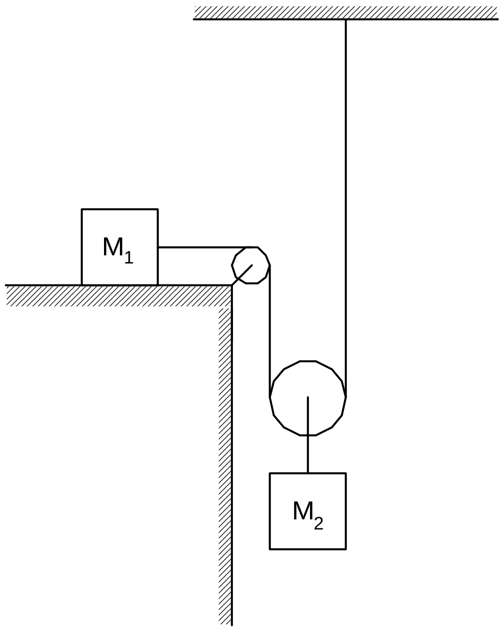
(A) $\frac{1}{2}c$

(B) $\sqrt{\frac{2}{5}}c$

(C) $\frac{2}{\sqrt{5}}c$

(D) c

(E) $2c$



49. If all friction and the masses of the pulleys and ropes in the system shown above are negligible, the acceleration of mass M_1 is

(A) $\frac{M_2}{2M_1}g$

(B) $\frac{M_2}{M_1}g$

(C) $\frac{M_2}{M_1+M_2}g$

(D) $\frac{M_2}{4M_1+M_2}g$

(E) $\frac{2M_2}{4M_1+M_2}g$

50. In inertial frame S, two events occur at the same instant in time and 3c-minutes apart in space. In inertial frame S', the same events occur at 5c-minutes apart. What is the time interval between the events in S'?

(A) 0 min
(B) 2 min
(C) 4 min
(D) 6 min
(E) 8 min

51. The solution to the Schrodinger equation for a particle bound in a one-dimensional infinitely

deep potential well, indexed by quantum number n, indicates that in the middle of the well the probability density vanishes for

(A) the ground state (n = 1) only

(B) states of even n (n = 2, 4, …)

(C) states of odd n (n = 1, 3, …)

(D) all states (n = 1, 2, 3, …)

(E) all states except the ground state

52. At a given instant of time, a rigid rotator is in the state $\Psi(\theta,\phi) = \sqrt{3/4\pi}\sin\theta\sin\phi$, where θ is the polar angle relative to the z-axis and ϕ is the azimuthal angle. Measurement will find which of the following possible values of the z-component of the angular momentum l_z ?

(A) 0

(B) $\hbar/2, -\hbar/2$

(C) $\hbar, -\hbar$

(D) $2\hbar, -2\hbar$

(E) $\hbar, 0, \hbar$

53. Positronium is the bound state of an electron and a positron. Consider only the states of zero orbital angular momentum (l = 0). The most probable decay product of any such state of positronium with spin zero (singlet) is

(A) 0 photons

(B) 1 photon

(C) 2 photons

(D) 3 photons

(E) 4 photons

<u>Questions 54-55</u> concern a plane electromagnetic wave that is a superposition of two independent orthogonal plane waves and can be written as the real part of

$$\mathbf{E} = \hat{\mathbf{x}}E_1 \exp(i[kz - \omega t]) + \hat{\mathbf{y}}E_2 \exp(i[kz - \omega t + \pi])$$

where k, ω, E_1 and E_2 are real.

54. If $E_2 = E_1$, the tip of the electric field vector will describe a trajectory that, as viewed along the z-axis from positive z and looking toward the origin, is a

(A) line at 45° to the +x-axis

(B) line at 135° to the +x-axis

(C) clockwise circle

(D) counterclockwise circle

(E) random path

55. If the plane wave is split and recombined on a screen after the two portions, which are polarized in the x- and y-directions, have traveled an optical path difference of 2π / k, the observed average intensity will be proportional to

(A) $E_1^2 + E_2^2$

(B) $E_1^2 - E_2^2$

(C) $(E_1 + E_2)^2$

(D) $(E_1 - E_2)^2$

(E) 0

56. A light source is at the bottom of a pool of water. The index of refraction of water is 1.33. At what minimum angle of incidence will a ray be totally reflected at the surface?
(A) 0°
(B) 25°
(C) 50°
(D) 75°
(E) 90°

57. Which of the following is most nearly equal to the lifetime of an excited atomic state that decays by an allowed electric dipole optical transition in spontaneous emission?
(A) 10^{-20} s
(B) 10^{-8} s
(C) 10^{-4} s
(D) 1 s
(E) 10^{4} s

58. A collimated laser beam emerging from a commercial HeNe laser has a diameter of about 1 millimeter. In order to convert this beam into a well-collimated beam of diameter 10 millimeters, two convex lenses are to be used. The first lens is of focal length 1.5 centimeters and is to be mounted at the output of the laser. What is the focal length, f, of the second lens and how far from the first lens should it be placed?

	f	Distance
(A)	4.5 cm	6.0 cm
(B)	10 cm	10 cm
(C)	10 cm	11.5 cm
(D)	15 cm	15 cm
(E)	15 cm	16.5 cm

59. The approximate number of photons in a femtosecond (10^{-15} s) pulse of 600 nanometers wavelength light from a 10-kilowatt peak-power dye laser is
(A) 10^{3}

(B) 10^7
(C) 10^{11}
(D) 10^{15}
(E) 10^{18}

60. The Lyman alpha spectral line of hydrogen (λ = 122 nanometers) differs by 1.8×10^{-12} meter in spectra taken at opposite ends of the Sun's equator. What is the speed of a particle on the equator due to the Sun's rotation, in kilometers per second?
(A) 22
(B) 2.2
(C) 22
(D) 220
(E) 2200

61. A sphere of radius R carries charge density proportional to the square of the distance from the center $\rho = Ar^2$, where A is a positive constant. At a distance of R/2 from the center, the magnitude of the electric field is

(A) $A/4\pi\varepsilon_0$

(B) $AR^2/40\varepsilon_0$

(C) $AR^2/24\varepsilon_0$

(D) $AR^2/5\varepsilon_0$

(E) $AR^2/3\varepsilon_0$

62. Two capacitors of capacitor 1.0 microfarad and 2.0 microfarads are each charged by being connected across a 5.0-volt battery. They are disconnected from the battery and then connected to each other with resistive wires so that plates of opposite charge are connected together. What will be the magnitude of the final voltage across the 2.0-microfarad capacitor?
(A) 0 V
(B) 0.6 V
(C) 1.7 V
(D) 3.3 V
(E) 5.0 V

63. According to the Standard Model of elementary particles, which of the following is NOT a composite object?
(A) Muon
(B) Pi-meson
(C) Neutron

(D) Deuteron
(E) Alpha particle

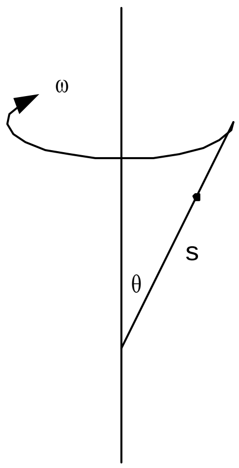
64. The binding energy of a heavy necleus is about 7 million electron volts per nucleon, where the binding energy of a medium-weight nucleus is about 8 million electron volts per nucleon. Therefore, the total kinetic energy liberated when a heavy nucleus ubdergoes symmetric fission is most nearly
(A) 1876 MeV
(B) 938 MeV
(C) 200 MeV
(D) 20 MeV
(E) 2 MeV

65. A man of mass m on an initially stationary boat gets off the boat by leaping to the left in an exactly horizontal direction. Immediately after the leap, the boat, of mass M, is observed to be moving to the right at speed v. How much work did the man do during the leap (both on his own body and on the boat)?
(A) $\frac{1}{2}Mv^2$
(B) $\frac{1}{2}mv^2$
(C) $\frac{1}{2}(M-m)v^2$
(D) $\frac{1}{2}(M+\frac{M^2}{m})v^2$
(E) $\frac{1}{2}(\frac{Mm}{M+m})v^2$

66. When it is about the same distance from the Sun as is Jupiter, a spacecraft on a mission to the outer planets has a speed that is 1.5 times the speed of Jupiter in its orbit. Which of the following describes the orbit of the spacecraft about the Sun?
(A) Spiral
(B) Circle
(C) Ellipse
(D) Parabola
(E) Hyperbola

67. A black hole is an object whose gravitational field is so strong that even light cannot escape. To what approximate radius would Earth (mass = 5.98×10^{24} kilograms) have to be compressed in order to become a black hole?
(A) 1 nm
(B) 1 μm
(C) 1 cm

(D) 100 m
(E) 10 km



68. A bead is constrained to slide on a frictionless rod that is fixed at an angle θ with a vertical axis and is rotating with angular frequency ω about the axis, as shown above. Taking the distance s along the rod as the variable, the Lagrangian for the bead is equal to

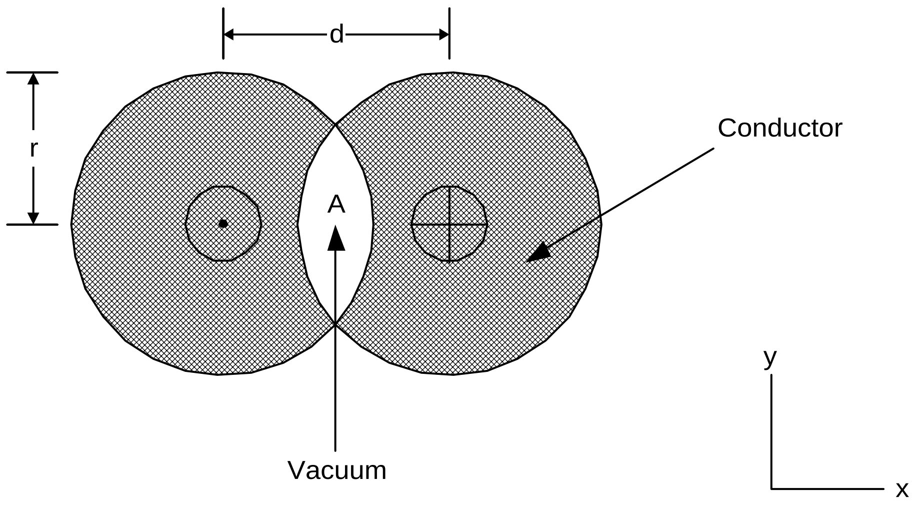
(A) $\frac{1}{2}m\dot{s}^2 - mgs\cos\theta$

(B) $\frac{1}{2}m\dot{s}^2 + \frac{1}{2}m(\omega s)^2 - mgs$

(C) $\frac{1}{2}m\dot{s}^2 + \frac{1}{2}m(\omega s\cos\theta)^2 + mgs\cos\theta$

(D) $\frac{1}{2}m(\dot{s}\sin\theta)^2 - mgs\cos\theta$

(E) $\frac{1}{2}m\dot{s}^2 + \frac{1}{2}m(\omega s\cos\theta)^2 - mgs\cos\theta$



69. Two long conductors are arranged as shown above to form overlapping cylinders, each of radius r, whose centers are separated by a distance d. Current of density J flows into the plane of

the page along the shaded part of one conductor and an equal current flows out of the plane of the page along the shaded portion of the pther, as shown. What are the magnitude and direction of the magnetic field at point A ?

(A) $(\mu_0 / 2\pi)\pi dJ$, in the +y-direction

(B) $(\mu_0 / 2\pi)d^2 J / r$, in the +y-direction

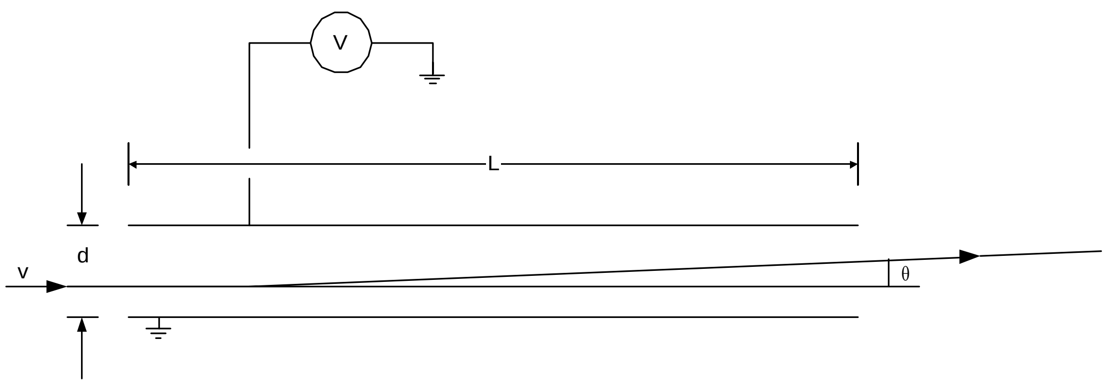
(C) $(\mu_0 / 2\pi)4d^2 J / r$, in the –y-direction

(D) $(\mu_0 / 2\pi)Jr^2 / d$, in the –y-direction

(E) There is no magnetic field at A.

70. A charged particle, A, moving at a speed much less than c, decelerates uniformly. A second particle, B, has one-half the mass, twice the charge, three times the velocity, and four times the acceleration of particle A. According to classical electrodynamics, the ratio P_B/P_A of the powers radiated is

(A) 16
(B) 32
(C) 48
(D) 64
(E) 72



71. The figure above shows the trajectory of a particle that is deflected as it moves through the uniform electric field between parallel plates. There is potential difference V and distance d between the plates, and they have length L. The particle (mass m, charge q) has nonrelativistic speed v before it enters the field, and its direction at this time is perpendicular to the field. For small deflections, which of the following expressions is the best approximation of the deflection angle θ?

(A) $\arctan((L/d)(Vq/mv^2))$

(B) $\arctan((L/d)(Vq/mv^2)^2)$

(C) $\arctan((L/d)^2(Vq/mv^2))$

(D) $\arctan((L/d)(2Vq/mv^2)^{1/2})$

(E) $\arctan((L/d)^{1/2}(2Vq/mv^2))$

72. In a voltage amplifier, which of the following is NOT usually a result of introducing negative feedback?
(A) Increased amplification
(B) Increased bandwidth
(C) Increased stability
(D) Decreased distortion
(E) Decreased voltage gain

73. The adiabatic expansion of an ideal gas is described by the equation $PV^{\gamma} = C$, where γ and C are constants. The work done by the gas in expanding adiabatically from the state (V_i, P_i) to (V_f, P_f) is equal to

(A) $P_f V_f$

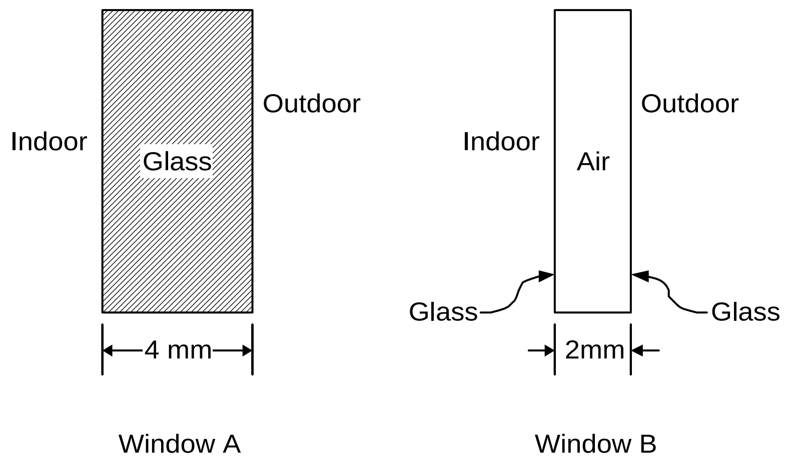
(B) $\frac{P_i + P_f}{2}(V_f - V_i)$

(C) $\frac{P_f V_f - P_i V_i}{1-\gamma}$

(D) $\frac{P_i(V_f^{1-\gamma} - V_i^{1-\gamma})}{1-\gamma}$

(E) $\frac{P_f(V_f^{1-\gamma} - V_i^{1-\gamma})}{1-\gamma}$

74. A body of mass m with specific heat C at temperature 500 K is brought into contact with an identical body at temperature 100 K, and the two are isolated from their surroundings. The change in entropy of the system is equal to
(A) (4/3) mC
(B) mC ln(9/5)
(C) mC ln(3)
(D) – mC ln(5/3)
(E) 0



75. Window A is a pane of glass 4 millimeters thick, as shown above. Window B is a sandwich consisting of two extremely thin layers of glass seperated by an air gap 2 millimeters thick, as shown above. If the thermal conductivities of glass and air are 0.8 watt/meter °C and 0.025 watt/meter °C, respectively, then the ratio of the heat flow through window A to the heat flow through window B is

(A) 2
(B) 4
(C) 8
(D) 16
(E) 32

76. A gaussian wave packet travels through free space. Which of the following statements about the wave packet are correct for all such wave packets?

I. The average momentum of the wave packet is zero.
II. The width of the wave packet increases with time, as $t \to \infty$
III. The amplitude of the wave packet remains constant with time.
IV. The narrower the wave packet is in momentum space, the wider it is in coordinate space.

(A) I and III only
(B) II and IV only
(C) I, II, and IV only
(D) II, III, and IV only
(E) I, II, III, and IV

77. Two ions, 1 and 2, at fixed separation, with spin angular momentum operators S_1 and S_2, have the interaction Hamiltonian $H = -JS_1 \cdot S_2$, where $J > 0$. The values of S_1^2 and S_2^2 are fixed at $S_1(S_1+1)$ and $S_2(S_2+1)$, respectively. Which of the following is the energy of the ground state of the system?

(A) 0

(B) $-JS_1S_2$

(C) $-J[S_1(S_1+1)-S_2(S_2+1)]$

(D) $-\frac{J}{2}[(S_1+S_2)(S_1+S_2+1)-S_1(S_1+1)-S_2(S_2+1)]$

(E) $-\frac{J}{2}\left[\frac{S_1(S_1+1)+S_2(S_2+1)}{(S_1+S_2)(S_1+S_2+1)}\right]$

78. In an n-type semiconductors, which of the following is true of impurity atoms?

(A) They accept electrons from the filled valence band into empty energy levels just above the valence band.

(B) They accept electrons from the filled valence band into empty energy levels just below the valence band.

(C) They accept electrons from the conduction band into empty energy levels just below the conduction band.

(D) They donate electrons to the filled valence band from donor levels just above the conduction band.

(E) They donate electrons to the conductor band from filled donor levels just below the conduction band.

79. For an ideal diatomic gas equilibrium, the ratio of the molar heat capacity at constant volume at very high temperature to that at very low temperature is equal to

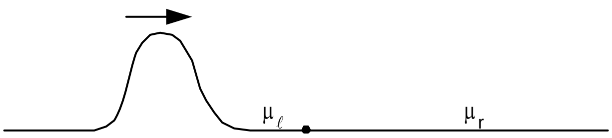
(A) 1

(B) 5/3

(C) 2

(D) 7/3

(E) 4



80. A string consists of two parts attached at x = 0. The right part of the string (x > 0) has mass μ_r per unit length and the left part of the string (x < 0) has mass μ_l per unit length. The string tension is T. If a wave of unit amplitude travels along the left part of the string, as shown in the figure above, what is the amplitude of the wave that is transmitted to the right part of the string?

(A) 1

(B) $\frac{2}{1+\sqrt{\mu_l/\mu_r}}$

(C) $\frac{2\sqrt{\mu_l/\mu_r}}{1+\sqrt{\mu_l/\mu_r}}$

(D) $\frac{\sqrt{\mu_l/\mu_r}-1}{\sqrt{\mu_l/\mu_r}+1}$

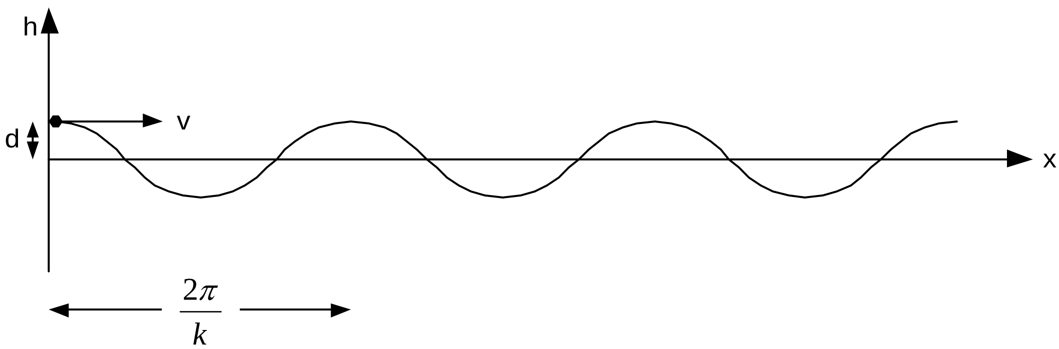
(E) 0

81. A piano tuner who wishes to tune the note D_2 corresponding to a frequency of 73.416 hertz has tuned A_4 to a frequency of 440.000 hertz. Which harmonic of D_2 counting the fundamental as the first harmonic will give the lowest number of beats per second, and approximately how many beats will this be when the two notes are tuned properly?

	Harmony	Number of Beats
(A)	8	5
(B)	6	0.5
(C)	5	0.1
(D)	3	0.372
(E)	2	4.5

82. An excited atom of argon has a single vacancy in its K shell (1s shell). If in the process of losing the excitation energy the atom undergoes an Auger transition, which of the following occurs?

(A) A radiative transition with the emission of a characteristic K x-ray photon
(B) A nonradiative transition in which second K electron is ejected from the atom
(C) A nonradiative transition in which an L- or M-shell electron is ejected from the atom
(D) A process in which a K-shell electron is captured by the nucleus of the atom
(E) None of the above



83. Consider a particle moving without friction on a rippled surface, as shown above. Gravity acts down in the negative h direction. The elevation height of the surface is given by h(x) = d cos(kx). If the particle starts at x = 0 with a speed v in the x direction, for what values of v will the particle stay on the surface at all time?

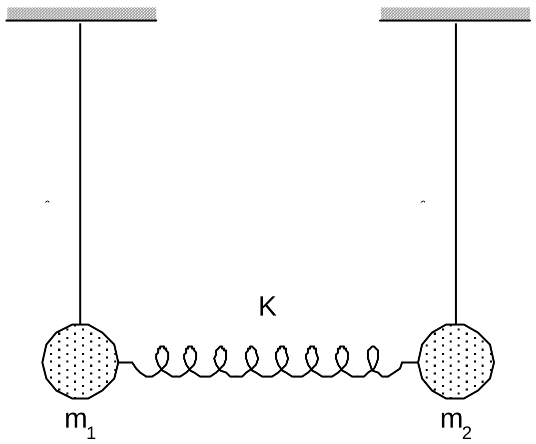
(A) $v \le \sqrt{gd}$

(B) $v \le \sqrt{\frac{g}{k}}$

(C) $v \le \sqrt{gkd^3}$

(D) $v \le \sqrt{\frac{g}{k^2 d}}$

(E) $v > 0$



84. Two pendulums are attached to a massless spring, as shown above. The arms of the pendulums are of identical length l, but the pendulum balls have unequal masses m_1 and m_2. The initial distance between the masses is the equilibrium length of the spring, which has spring constant K. What is the highest normal mode frequency of the system?

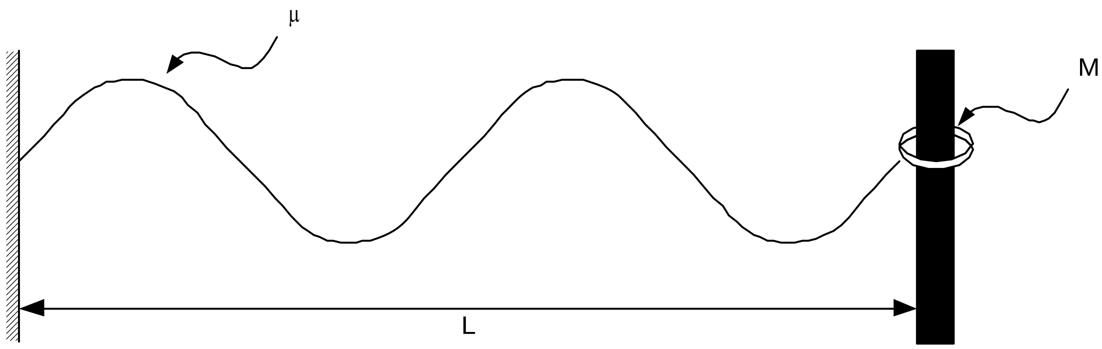
(A) $\sqrt{g/l}$

(B) $\sqrt{\dfrac{K}{m_1 + m_2}}$

(C) $\sqrt{\dfrac{K}{m_1} + \dfrac{K}{m_2}}$

(D) $\sqrt{\dfrac{g}{l} + \dfrac{K}{m_1} + \dfrac{K}{m_2}}$

(E) $\sqrt{\dfrac{2g}{l} + \dfrac{K}{m_1 + m_2}}$



85. Small amplitude standing waves of wavelength λ occur on a string with tension T, mass per unit length μ, and length L. One end of the string is fixed and the other end is attached to a ring of mass M that slides on a frictionless rod, as shown in the figure above. When gravity is neglected,

which of the following conditions correctely determines the wavelength? (You might want to consider the limiting cases M $\rightarrow$ 0 and M $\rightarrow$ ∞)

(A) $\mu / M = \frac{2\pi}{\lambda} \cot \frac{2\pi L}{\lambda}$

(B) $\mu / M = \frac{2\pi}{\lambda} \tan \frac{2\pi L}{\lambda}$

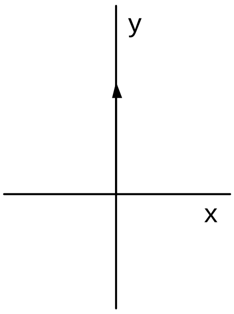
(C) $\mu / M = \frac{2\pi}{\lambda} \sin \frac{2\pi L}{\lambda}$

(D) $\lambda = 2L / n, n = 1,2,3...$

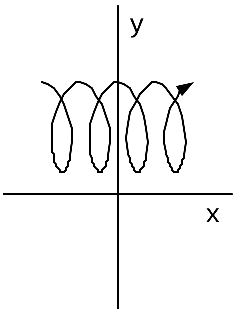
(E) $\lambda = 2L / (n + \frac{1}{2}), n = 1,2,3...$

86. A positively charged particle is moving in the xy-plane in a region where there is a non-zero uniform magnetic field B in the +z-direction. Which of the following is a possible trajectory for the particle?

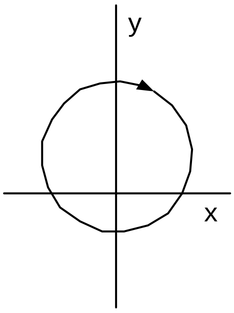
(A)



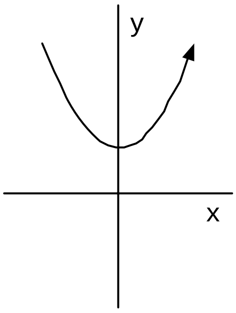
(B)



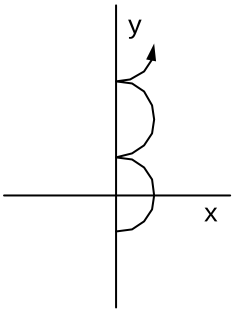
(C)

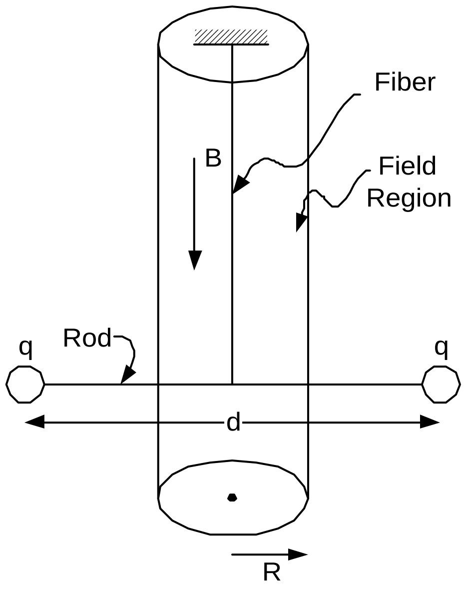


(D)



(E)





87. Two small pith balls, each carrying a charge q, are attached to the ends of a light rod of length d, which is suspended from the ceiling by a thin torsion-free fiber, as shown in the figure above. There is a uniform magnetic field B, pointing straight down, in the cylindrical region of radius R around the fiber. The system is initially at rest. If the magnetic field is turned off, which of the following describes what happens to the system?

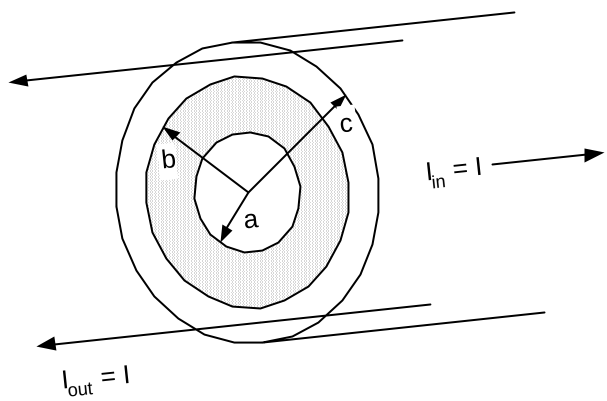
(A) It rotates with angular momentum qBR^2

(B) It rotates with angular momentum $\frac{1}{4}qBd^2$

(C) It rotates with angular momentum $\frac{1}{2}qBRd$

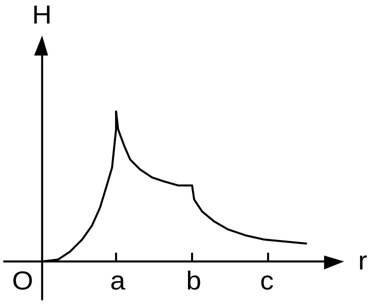
(D) It does not rotate because to do so would violate conservation of angular momentum.

(E) It does not move because magnetic forces do no work.

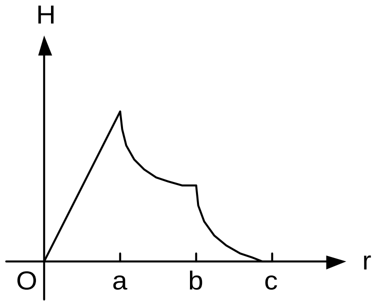


88. A coaxial cable has the cross section shown in the figure above. The shaded region is insulated. The regions in which r<a and b<r<c are conducting. A uniform dc current density of total current I flows along the inner part of the cable (r<a) and turns along the outer part of the cable (b<r<c) in the directions shown. The radial dependence of the magnitude of the magnetic field, H, is shown by which of the following?

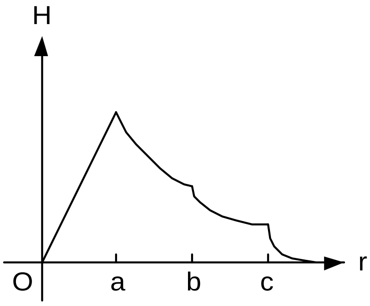
(A)



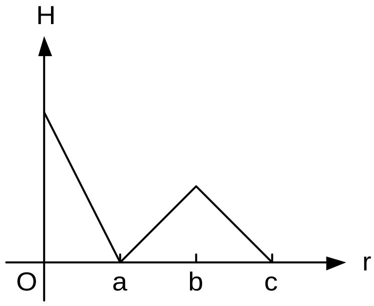
(B)



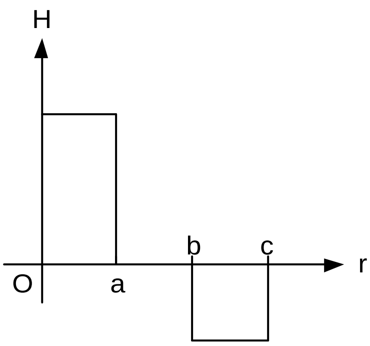
(C)

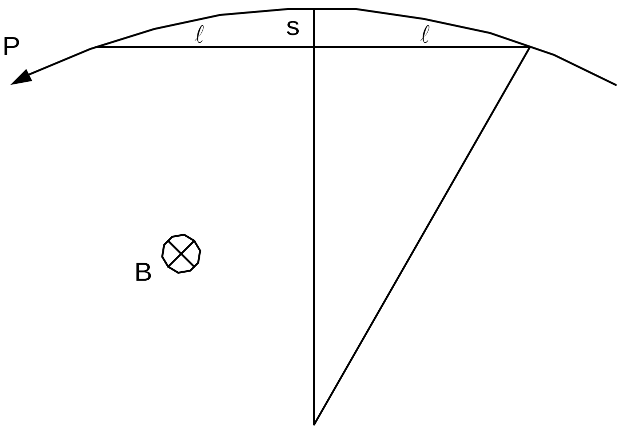


(D)



(E)





89. A particle with charge q and momentum p is moving in the horizontal plane under the action of a uniform vertical magnetic field of magnitude B. Measurements are made of the particle's trajectory to determine the "sagitta" s and half-chord length l, as shown in the figure above. Which of the following expressions gives the particle's momentum in terms of q, B, s, and l?

(A) $qBs^2/2l$

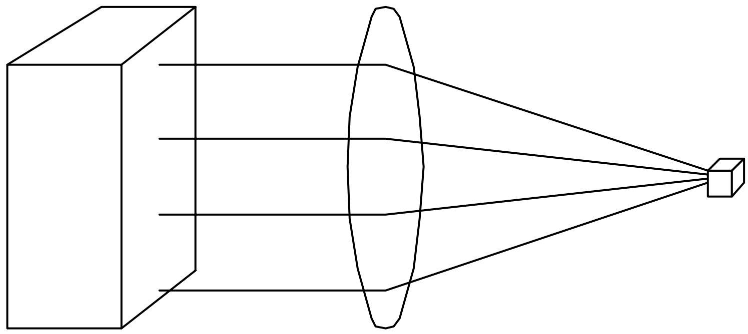
(B) qBs^2/l

(C) qBl / s

(D) $qBl^2 / 2s$

(E) $qBl^2 / 8s$

90. Two solenoids whose diameters are small compared to their lengths have identical cross-sectional areas and have the save self-inductance. Solenoid X is wound with 200 turns of wire per centimeter and is 10 centimeters long. Solenoid Y is wound with 100 turns per centimeter. How long is solenoid Y?
(A) 20 cm
(B) 40 cm
(C) $10\sqrt{2}$ cm
(D) 5 cm
(E) 2.5 cm



91. An experimenter needs to heat a small sample to 900 K, but the only available oven has a maximum temperature of 600 K. Could the experimenter heat the sample to 900 K by using a large lens to concentrate the radiation from the oven onto the sample, as shown above?
(A) Yes, if the volume of the oven is at least 3/2 the volume of the sample.
(B) Yes, if the area of the front of the oven is at least 3/2 the area of the front of the sample.
(C) Yes, if the sample is placed at the focal point of the lens.
(D) No, because it would violate conservation of energy.
(E) No, because it would violate the second law of thermodynamics.

92. A particle of mass m moves in a one-dimensional potential $V(x) = -ax^2 + bx^4$, where a and b are positive constants. The angular frequency of small oscillations about the minima of the potential is equal to

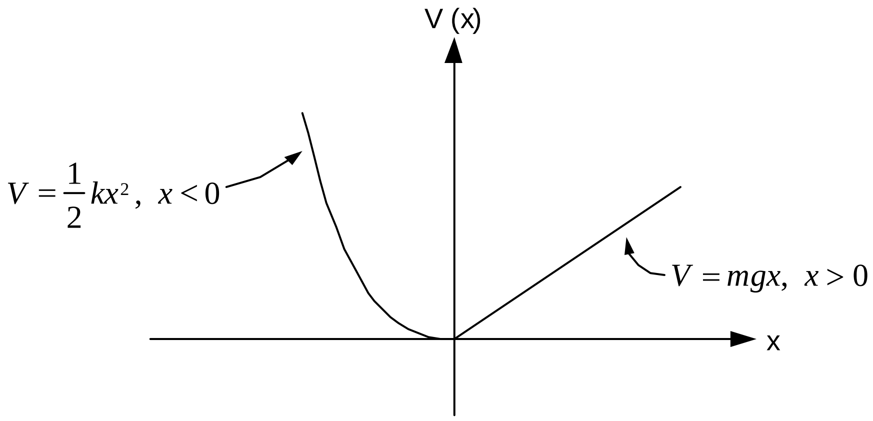
(A) $\pi(a / 2b)^{1/2}$

(B) $\pi(a / m)^{1/2}$

(C) $(a/mb)^{1/2}$

(D) $2(a/m)^{1/2}$

(E) $(a/2m)^{1/2}$



93. A particle of mass m moves in the potential shown above. The period of the motion when the particle has energy E is

(A) $\sqrt{k/m}$

(B) $2\pi\sqrt{m/k}$

(C) $2\sqrt{2E/mg^2}$

(D) $\pi\sqrt{m/k} + 2\sqrt{2E/mg^2}$

(E) $2\pi\sqrt{m/k} + 4\sqrt{2E/mg^2}$

94. A system of N weakly interacting subsystems, each with two internal quantum states with energies 0 and ε. The internal energy for this system at absolute temperature T is equal to

(A) Nε

(B) $\frac{3}{2}NkT$

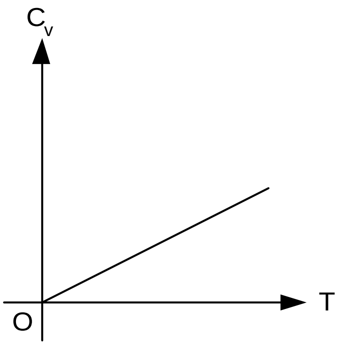
(C) $N\varepsilon\, e^{-\varepsilon/kT}$

(D) $\frac{N\varepsilon}{(e^{\varepsilon/kT}+1)}$

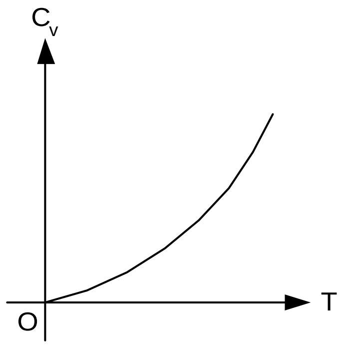
(E) $\frac{N\varepsilon}{(1-e^{-\varepsilon/kT})}$

95. Which of the following curves is characteristic of the specific heat C_v of a metal such as lead, tin, or aluminum in the temperature region where it becomes superconducting?

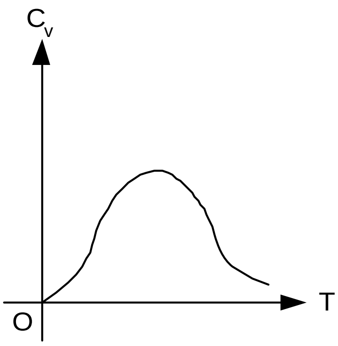
(A)



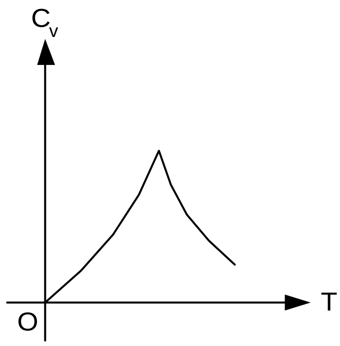
(B)



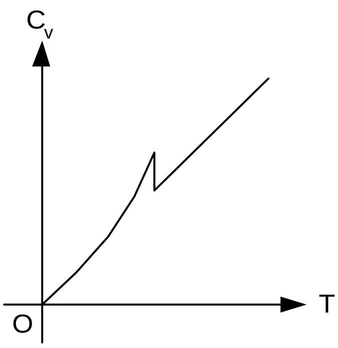
(C)



(D)



(E)



96. Which of the following reasons explains why a photon cannot decay to an electron and a positron ($\gamma \to e^{+} + e^{-}$) in free space?

(A) Linear momentum and energy are not both conserved.
(B) Linear momentum and angular momentum are not both conserved.
(C) Angular momentum and parity are not both conserved.
(D) Parity and strangeness are not both conserved.
(E) Charge and lepton number are not both conserved.

97. A particle of mass m has the wave function

$$\Psi(x,t) = e^{i\omega t}[\alpha\cos(kx) + \beta\sin(kx)],$$

where α and β are complex constants and ω and k are real constants. The probability current density is equal to which of the following (Note: α^* denotes the complex conjugate of α, and $|\alpha|^2 = \alpha^* \alpha$.)

(A) 0

(B) $\hbar k / m$

(C) $\frac{\hbar k}{2m}(|\alpha|^2 + |\beta|^2)$

(D) $\frac{\hbar k}{2m}(|\alpha|^2 - |\beta|^2)$

(E) $\frac{\hbar k}{2mi}(\alpha^* \beta - \beta^* \alpha)$

98. A particle of mass m is acted on by a harmonic force with potential energy function $V(x) = m\omega^2 x^2 / 2$ (a one-dimensional simple harmonic oscillator). If there is a wall at x = 0 so that $V = \infty$ for x < 0, then the energy levels are equal to

(A) 0, $\hbar\omega$, $2\hbar\omega$, …

(B) 0, $\frac{\hbar\omega}{2}$, $\hbar\omega$, …

(C) $\frac{\hbar\omega}{2}$, $\frac{3\hbar\omega}{2}$, $\frac{5\hbar\omega}{2}$, …

(D) $\frac{3\hbar\omega}{2}$, $\frac{7\hbar\omega}{2}$, $\frac{11\hbar\omega}{2}$, …

(E) 0, $\frac{3\hbar\omega}{2}$, $\frac{5\hbar\omega}{2}$, …

99. The electronic energy levels of atoms of a certain gas are given by $E_n = E_1 n^2$, where n = 1, 2, 3,… Assume that transitions are between all levels. If one wanted to construct a laser from this gas by pumping the n = 1 $\rightarrow$ n = 3 transition, which energy level or levels would have to be metastable?

(A) n = 1 only

(B) n = 2 only

(C) n = 1 and n = 3 only

(D) n = 1, n = 2, and n = 3

(E) none

100. The operator $\hat{a} = \sqrt{\frac{m\omega_0}{2\hbar}}(\hat{x} + i\frac{\hat{p}}{m\omega_0})$, when operating on a harmonic energy eigenstate Ψ_n with energy E_n, produces another energy eigenstate whose energy is $E_n - \hbar\omega_0$. Which of the following is true?

I. $\hat{a}$ commutes with the Hamiltonian

II. $\hat{a}$ is a Hermitian operator and therefore an observable

III. The adjoint operator $\hat{a}^+ \neq \hat{a}$

(A) I only
(B) II only
(C) III only
(D) I and II only
(E) I and III only

参考答案

BBBAB BAAAB DCBDE BECBD CCDDE
CDDEC EBCDA DDDEA ABCAE DCCEC
BCCBA CBEBB BCACD ECEAD AACBD
BDEDC BCDDB BABDB EDDDE AEDBC

参考解答

1. 解：t=0 时开关刚扳到 b，电容 C 两端电压不能突变，仍保持为先前电压 V，由 Ohm 定律知此时电流为 V/R。这以后电流随时间指数衰减逐渐到零。选（B）。

2. 解：电路中由变化磁场产生的感应电动势为

$$150 \times 0.1 \times 0.1 = 1.5\ \text{V},$$

方向由 Lentz 定律判定与电源电动势相反。故电流为

$$\frac{5-1.5}{10} = 0.35\ \text{A}。$$

选（B）。

3. 解：圆环上任取一个带电量为 dq 的小段，在 P 点产生的电势为

$$dV = \frac{dq}{4\pi\varepsilon_0\sqrt{R^2+x^2}},$$

故整个圆环在 P 点产生的电势为

$$V = \frac{Q}{4\pi\varepsilon_0\sqrt{R^2+x^2}}。$$

选（B）。

4. 解：简单地从物理上考虑一下便知道，带电量为–q 的粒子只有在环心处才不受力，因此 x=0 是振动的唯一可能的平衡位置。把势能在此点展开：

$$E=\frac{-Qq}{4\pi\varepsilon_0\sqrt{R^2+x^2}}=\frac{-Qq}{4\pi\varepsilon_0 R}\left(1+\frac{x^2}{R^2}\right)^{-\frac{1}{2}}=\frac{-Qq}{4\pi\varepsilon_0 R}\left(1-\frac{x^2}{2R^2}+\cdots\right),$$

另一方面，振动能量的 x^2 项应具有 $\frac{1}{2}m\omega^2x^2$ 的形式，比较得

$$\omega=\sqrt{\frac{qQ}{4\pi\varepsilon_0 mR^3}}\text{。}$$

选（A）。
此题更有技巧的做法是判定平衡位置是 x=0 后，从量纲分析将正确选项局限在（A）和（B）。进一步在（B）选项中代入 x=0 得到ω=0，将其排除。

5. 解：汽车作匀速率圆周运动，切向合力应为零，法向合力应向圆心以提供向心加速度。选（B）。

6. 解：物块下滑过程中速度保持不变，动能也保持不变，物块的总机械能中只有重力势能损失了 mgh，这部分势能通过摩擦转化成了热能。选（B）。

7. 解：设碰撞前一刹那小球速度为 v，碰撞后小球速度和方块速度分别变为 v_1、v_2。二体弹性碰撞系统动量守恒，相对速度不变：

$$\begin{cases}mv=mv_1+2mv_2\\ v=v_2-v_1\end{cases}$$

解出 $v_1=-\frac{1}{3}v$，即能量变为碰撞前的 1/9。选（A）。

8. 解：阻尼振动的一般公式为

$$x=Ae^{-\mu t}\sin(\sqrt{\omega^2-\mu^2}t+\alpha)$$

其中 $\mu=\frac{b}{2m}$，积分常数 A 与μ分别表示初振幅和初位相。由于 $\sqrt{\omega^2-\mu^2}<\omega$，有阻尼时振动周期比自由振动时的周期长。选（A）。
阻尼振动公式为 SUB 常见考点，建议熟记。

9. 解：某一线系中最长的波长对应该线系最小的能量差。由氢原子能级公式知

$$\frac{\lambda_{Lyman}}{\lambda_{Balmer}}=\frac{E_{Balmer}}{E_{Lyman}}=\frac{\frac{1}{2^2}-\frac{1}{3^2}}{\frac{1}{1^2}-\frac{1}{2^2}}=\frac{5}{27}\text{。}$$

选（A）。

10. 解：内层电子被电离后，原子可以通过发射 X 射线或者 Auger 电子回到基态。所谓发射 Auger 电子是指一个外壳层的电子进入内壳层的空穴，将释放的能量传递给另一个壳层的电

子使其电离的过程。一般来说，轻元素发射 Auger 电子的几率较大，重元素发射 X 射线的几率较大。题中指明大原子序数，故选（B）。

11. 解：此题就是 Stern-Gerlach 实验。氢原子基态 J=1/2，沿磁场变化方向分裂成两束。选（D）。

12. 解：氢原子能级公式

$$E_n = -\frac{2\pi^2 me^4}{h^2 n^2}\text{。}$$

电子偶的折合质量约为氢原子体系（电子—质子）折合质量的 1/2，故基态能量为–13.6/2 = –6.8 eV。选（C）。
positronium：电子偶（一个正电子和一个负电子形成的束缚态体系）。
注意 positron：正电子。

13. 解：100W 的电热装置恰好能维持锅里的水恒温，也就是说水每秒钟散热 100J。要让水温下降 1°C，需向外界散热

$$4.2\times10^3\ \text{J/kg °C} \times 1\ \text{kg} \times 1\ \text{°C} = 4200\ \text{J，}$$

共需时间

$$4200 / 100 = 42\ \text{s。}$$

选（B）。

14. 解：因为两个铜块全同，所以末温为 50°C。交换热量为

$$0.1\ \text{kcal/kg K} \times 1.0\ \text{kg} \times 50\ \text{K} = 5\ \text{kcal。}$$

选（D）。

15. 解：气体吸热等于气体对外界做的功，即曲线 ABCA 包的面积。等温线 AB 的方程为

$$pV = RT_h\text{。}$$

气体吸热为

$$\int_{V_1}^{V_2}(p-p_2)dV = \int_{V_1}^{V_2}\frac{RT_h}{V}dV - p_2(V_2-V_1) = RT_h\ln\frac{V_2}{V_1} - R(T_h-T_c)\text{。}$$

选（E）。

16. 解：平均碰撞频率 Z 的严格公式为

$$Z = \sqrt{2}\pi d^2 n\bar{v}\text{。}$$

气体中单个分子连续两次碰撞之间平均通过的距离 $\bar{\lambda}$，叫分子的平均自由程。显然平均自由程

$$\bar{\lambda} = \frac{\bar{v}}{Z} = \frac{1}{\sqrt{2}n\pi d^2}\text{。}$$

平均速率可近似取作方均根速率

$$\overline{v}=\left(\frac{3v_0 p}{\mu}\right)^{1/2},$$

其中μ为气体的摩尔质量，v_0 为气体的摩尔体积。将上式代入平均碰撞频率的表达式，得

$$Z=\sqrt{2}\pi d^2 n\left(\frac{3v_0 p}{\mu}\right)^{1/2}=\sqrt{2}\pi d^2 N_A\left(\frac{3p}{\mu v_0}\right)^{1/2}。$$

以 0°C 及 1atm 下的氮气为例，$\mu = 28\times10^{-3}$ kg·mol^{-1}，$v_0 = 2.24\times10^{-2}$ m^3·mol^{-1}，p = 101325 N·m^{-1}，作为估计，取 N_2 的有效直径 $d\approx4\times10^{-10}$ m，将这些数据代入，可得出氮气分子的平均碰撞频率 Z 约为 9.4×10^9 s^{-1}，平均自由程 $\overline{\lambda}\approx5.3\times10^{-8}$ m 。选（B）。

17. 解：列出$|\Psi|^2$的值如下：

$\lvert\Psi\rvert^2$		1	1	4	9	1	0
x	0	1	2	3	4	5	6

容易算出总几率为 1+1+4+9+1 = 16，2≤x≤4 的几率为 4+9 = 13，故所求相对几率为 13/16。选（E）。

18. 解：隧道效应。穿透一测几率幅应小于入射一侧几率幅。选（C）。

19. 解：银原子质量远大于α粒子质量，在碰撞过程中可认为不动。在散射角是 180°的情况下，α粒子达到最近点时其动能完全转化为势能，即

$$\frac{2Ze^2}{4\pi\varepsilon_0 r}=E,$$

$$r=\frac{2Ze^2}{4\pi\varepsilon_0 E}=\frac{2\times50\times1.6\times10^{-19}}{5\times10^6}\times9\times10^9\approx3\times10^{-14}\ \text{m}。$$

选（B）。

20. 解：设表面原子质量为 m。二体弹性碰撞满足动量守恒，并且不改变相对速度。

$$\begin{cases}4v=-4\times0.6v+mv'\\ v=0.6v+v'\end{cases},$$

解出 m = 16 u。选（D）。

21. 解：均匀圆环绕其圆心的转动惯量为 mR^2，绕其圆周上一点的转动惯量可用平行轴定理算出：

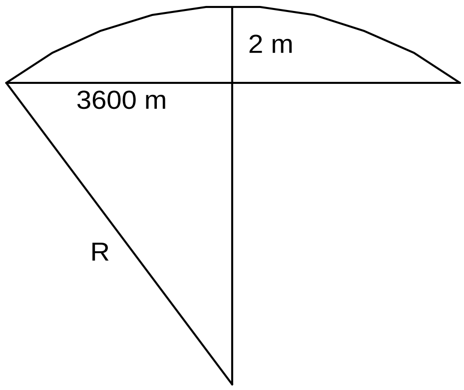
$$I=mR^2+mR^2=2mR^2。$$

振动周期为

$$T=2\pi\sqrt{\frac{I}{mgR}}=2\pi\sqrt{\frac{2mR^2}{mgR}}=2\pi\sqrt{\frac{2R}{g}}=2\times3.1\times\sqrt{\frac{2\times0.2}{10}}\approx1.2\ \text{s}。$$

选（C）。
barn：谷仓。

22. 解：如图所示，



$$2 = R - \sqrt{R^2 - 3600^2} \approx R - R(1 - \frac{3600^2}{2R^2}) = \frac{3600^2}{2R}，$$

$$R = \frac{3600^2}{4} = 1800^2。$$

第一宇宙速度

$$v = \sqrt{gR} = \sqrt{0.4 \times 10} \times 1800 = 3600 \text{ m/s}。$$

选（C）。

23. 解：系统能量守恒与力的具体形式无关，（A）对。有心力场一定有角动量守恒，（B）对。圆周运动动力学方程为

$$\frac{GM}{R^{2+\varepsilon}} = \frac{v^2}{R},$$

$$v \propto R^{\frac{-1-\varepsilon}{2}},$$

$$T \propto \frac{R}{v} \propto R^{\frac{3+\varepsilon}{2}},$$

故（C）对。只有当力是平方反比形式，才会有椭圆轨道。（D）不对。可以证明，形如 $F \propto R^n$ 的力在 n>–3 时相应的圆轨道是稳定的（见胡慧玲《理论力学基础教程》pp.111 ~ pp.113），故（E）对。
选（D）。

24. 解：设初始时 A、B 各带电量 Q。A 和 C 接触后二者各带电 Q/2，B 和 C 接触后二者各带电 3Q/4。最后 A、B 间作用力为

$$\frac{1}{r^2}\frac{Q}{2}\frac{3Q}{4}=\frac{3}{8}\frac{Q^2}{r}=\frac{3}{8}F\text{。}$$

选（D）。

25. 解：容易知道 $Q_0=Q_1=Q_2$，$V_1=V_2$，$U_1=U_2$。（E）不对，$U_0=U_1=U_2$。选（E）。

26. 解：RLC 电路的谐振频率为

$$\omega=2\pi f=\frac{1}{\sqrt{LC}}\text{，}$$

$$C=\frac{1}{L(2\pi f)^2}=\frac{1}{2\times10^{-6}\times(2\times3.14\times103.7\times10^6)^2}\approx10^{-12}\ \text{F}\text{。}$$

选（C）。

27. 解：（D）是错的。增益和频率不呈线性关系，应该用 Semilog 或者 Log-Log 作图。习惯上作这种增益—频率特性曲线都要将频率取对数，因为考虑的频率区间往往很大（到几 K 或几 MHz），不取对数会画不下。
activity：放射性。
radioactive isotope：放射性同位素。
gain：增益
low-pass filter：低通滤波器。

28. 解：相邻峰谷在纵坐标上大约相差 1.5cm，对应电压是 3V，说明电压振幅大约为 1.5V。相邻的峰—峰或者谷—谷在横坐标上大约相差 1cm，对应频率为 1 / 2ms = 500Hz。由这两个结论即可判定（D）是正确答案。Oscillation 1 对应于低频振动，横坐标位于 2cm 到 5cm 的区间为该振动的一个 1/4 周期。

29. 解：选（E）。量纲分析。

30. 解：列出压强平衡条件

$$4\times5=1\times(h_2-h_1+5)\text{。}$$

管中液体总长度为 45cm，即

$$h_1+h_2=45\text{。}$$

联立求解得 h_1=15cm，h_2=30cm。h_2 / h_1 = 2 / 1，选（C）。
immiscible：不能混合的。

31. 解：球从它被释放那一刻起逐渐加速，直到它最终达到收尾速度。这个过程中球的速度和动能始终保持单调增加，故（A）、（B）、（C）均错。收尾速度由球的受力平衡条件决定，

$$mg=bv\text{，}$$

$$v=\frac{mg}{b}\text{。}$$

可见收尾速度同时依赖于 v 和 b，选（E）。

32. 解：转动动能正比于转动惯量。系统绕 B 轴的转动惯量可用平行轴定理求出，

$$\frac{E_B}{E_A}=\frac{I_B}{I_A}=\frac{3mx^2+3mx^2}{3mx^2}=2\text{，}$$

其中 x 是等边三角形顶点到中心的距离。选（B）。
equilateral triangle：等边三角形。

33. 解：发现 l=5 的概率为

$$\frac{9}{38}+\frac{4}{38}=\frac{13}{38}\text{。}$$

选（C）。

34. 解：选（D）。在这一反应中，宇称守恒被破坏了。

35. 解：选（A）。Pauli 不相容原理是全同 fermion 体系波函数反对称的结果。

36. 解：由相对论能量公式

$$E=\frac{m}{\sqrt{1-\frac{v^2}{c^2}}}c^2$$

和能量守恒即可算出混合粘土块的质量

$$m=\frac{4}{\sqrt{1-\left(\frac{3}{5}\right)^2}}\times 2=10\ \text{kg}\text{。}$$

选（D）。

37. 解：由相对论速度合成公式

$$v=\frac{v_1+v_2}{\sqrt{1+\frac{v_1v_2}{c^2}}}=\frac{0.3+0.6}{\sqrt{1+0.3\times 0.6}}=0.76c\text{。}$$

选（D）。

38. 解：相对论能量和动量分别为

$$E=mc^2\text{，}\quad p=mv\text{。}$$

二式相比得

$$v=\frac{p}{E}=\frac{5}{10}=\frac{1}{2}c\text{。}$$

选（D）。

39. 解：一般来说原子序数越大的原子电离能越小，因为最外层电子离原子核越远。选（E）。

40. 解：470nm 对应的能级差为

$$E=\frac{hc}{\lambda}=\frac{6.63\times10^{-34}\times3\times10^{8}}{470\times10^{-9}}=4.2\times10^{-19}\,\mathrm{J}=2.6\mathrm{eV}\text{。}$$

类氢原子能级公式为

$$E_n=-\frac{2\pi^2mZ^2e^4}{h^2n^2},$$

故 He 原子（Z=2）基态能量为

$$E_1=-13.6\times2^2\,\mathrm{eV}\text{。}$$

于是

$$2.6=13.6\times4\times\left(\frac{1}{n_f}-\frac{1}{4^2}\right),$$

$$n_f=3\text{。}$$

选（A）。

41. 解：（A）符合电偶极跃迁的选择定则，正确。（B）违反$\Delta l=\pm1$ 的选择定则。该 3p 电子的正确量子数为 n=3，l=1，s=1/2，j=3/2。

42. 解：500nm 光子的能量为

$$E=\frac{hc}{\lambda}=\frac{6.63\times10^{-34}\times3\times10^{8}}{500\times10^{-9}}=4\times10^{-19}\,\mathrm{J}=2.5\mathrm{eV}\text{。}$$

光电子的最大动能为

$$E=2.5-2.28=0.2\mathrm{eV}\text{。}$$

选（B）。

43. 解：由 Stokes 公式，

$$\oint udl=\iint\left(\frac{\partial u_y}{\partial x}-\frac{\partial u_x}{\partial y}\right)dxdy=\iint(-1-1)dxdy=-2\pi R^2\text{。}$$

选（C）。因为线积分方向的不同，结果可以相差一个符号。

44. 解：$a=\frac{dv}{dt}=\frac{dv}{dx}\frac{dx}{dt}=\frac{dv}{dx}v=-n\beta x^{-n-1}\cdot\beta x^{-n}=-n\beta^2x^{-2n-1}$。选（A）。

45. 解：题目要求一个高通滤波器，（E）符合要求。高频成分很容易穿过电容，交流电压大部分落于电感之上；低频成分无法通过电容，大部分直流电压落在电容上，从电感输出的电压几乎为零。

46. 解：三重态波函数的自旋部分是对称的，空间部分是反对称的，因此两个电子的间距较大，Coulomb 势能较低。选（D）。

47. 解：每秒钟增加的总磁通量为

$$\Psi = NB\pi R^2,$$

此即产生的感应电动势。选（C）。

48. 解：只有一半的π^+粒子抵达探测器，说明在相对π^+粒子静止的参考系里，π^+粒子刚好经历了一个半衰期。根据相对论的尺缩效应，

$$15\sqrt{1-\frac{v^2}{c^2}} = 2.5\times10^{-8}v = 7.5\frac{v}{c},$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{2}\frac{v}{c},$$

$$v = \frac{2}{\sqrt{5}}c\text{。}$$

选（C）。

49. 解：设绳中张力为 T，则方程可列作：

$$\begin{cases} M_2 g - 2T = M_2 a_2 \\ T = M_1 a_1 \\ a_2 = \dfrac{a_1}{2} \end{cases},$$

解得

$$a_1 = \frac{2M_2}{4M_1 + M_2} g\text{。}$$

选（E）。

50. 解：间隔不变性：

$$3 = \sqrt{5^2 - t'^2},$$

$$t' = 4\,\text{min}\text{。}$$

选（C）。

51. 解：一维无限深方势阱的解是

$$E_n = \frac{\hbar^2\pi^2 n^2}{2ma^2}，\quad n = 1,2,3,\cdots$$

$$\psi_n(x) = \sqrt{\frac{2}{n}}\sin\left(\frac{n\pi}{a}x\right) \text{。}$$

n=奇数的态是偶宇称态，n=偶数的态是奇宇称态。奇宇称态在势阱中心处有节点。选（B）。

52. 解：l=1，m=±1 的球谐函数为

$$Y_{1,\pm 1} = \mp\sqrt{3/8\pi}\sin\theta\exp(\pm i\phi),$$

由此可见题给波函数可分解为

$$\Psi(\theta,\phi) = \frac{1}{\sqrt{2}}(Y_{1,1} + Y_{1,-1}),$$

这说明题给波函数只含有 m=±1 的成分。选（C）。

53. 解：电子偶 l=0，s=0，故 j=0。光子的角动量是 1，为满足角动量守恒必须辐射偶数个光子。一般说来辐射的光子越少，反应发生的几率越大，因此辐射两个光子的几率大于辐射四个光子的几率。选（C）。

54. 解：**E** 的 x 分量和 y 分量相差π相位，振动方向相反，**E** 的方向在 x-y 平面的二、四象限内。选（B）。

55. 解：x 和 y 方向的两束偏振光经历 2π / k 光程差，相位差了 2π，跟没差一样。**E** 的方向仍在二、四象限。电磁波的平均强度与振幅平方成正比，

$$E^2 = E_1^2 + E_2^2 \text{。}$$

选（A）。

56. 解：临界角

$$\theta = \sin^{-1}\frac{1}{1.33} = 50°$$

选（C）。

57. 解：选（B）。常识。

58. 解：两个凸透镜的焦点应该重合。为了得到 10 倍的放大率，第二个凸透镜的焦距应该是第一个的 10 倍，

$$f_2 = 10 f_1 = 15\text{cm},$$

两个镜子相距

$$d = f_1 + f_2 = 16.5\text{cm}\ 。$$

选（E）。

59. 解：$N = \dfrac{Pt}{hc/\lambda} = \dfrac{10^4 \times 10^{-15}}{6.63\times10^{-34} \times 3\times10^8 / 600\times10^{-9}} \approx 10^7$。选（B）。

60. 解：相对论 Doppler 效应

$$\nu' = \sqrt{\frac{c+v}{c-v}}\nu \approx \left(1+\frac{v}{c}\right)\nu\ ,$$

其中 v 是太阳赤道上的速度。赤道两端的速度差为 2v，故

$$\Delta\nu = \frac{2v}{c}\nu\ 。$$

因为

$$\lambda = \frac{c}{\nu}\ ,$$

所以

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta\nu}{\nu} = \frac{2v}{c}\ ,$$

$$v = \frac{\Delta\lambda}{\lambda}\frac{c}{2} = \frac{1.8\times10^{-12}}{122\times10^{-9}}\times\frac{3\times10^8}{2} = 2200\text{m/s} = 2.2\text{km/s}$$

选（B）。

61. 解：此题显然要用 Gauss 定理。半径为 R/2 的球面内的总电荷为

$$Q = \int_0^{R/2}\rho 4\pi r^2 dr = \int_0^{R/2} 4\pi A r^4 dr = \frac{4\pi A}{5}\left(\frac{R}{2}\right)^5 = \frac{\pi A R^5}{40}\ ,$$

$$E = \frac{Q}{4\pi\left(\dfrac{R}{2}\right)^2\varepsilon_0} = \frac{AR^2}{40\varepsilon_0}\ 。$$

选（B）。

62. 解：开始两个电容分别带有电量 5mC 和 10mC，反接后每对相互连接的极板共带电 10–5=5mC，这 5mC 电量按电容比例分在两个极板上，其中 1mF 的电容分得 5/3mC，其两端电压为

$$U = \frac{Q}{C} = \frac{5/3}{1} = 1.7\text{V}\ 。$$

选（A）。

63. 解：在标准模型中只有μ子是基本粒子。π子是由更基本的夸克组成的。选（A）。

64. 解：每个核子在裂变过程中释放出 1MeV 的能量，一般的重核大约有 200 多个核子，要释放出 200 多 MeV 的能量。选（C）。

65. 解：由动量守恒知人的速度为

$$u = \frac{M}{m} v\,,$$

人做的总功

$$W = \frac{1}{2} m u^2 + \frac{1}{2} M v^2 = \frac{1}{2}\left(M + \frac{M^2}{m}\right) v^2\,。$$

选（D）。

66. 解：木星的动能和势能近似有 V=–2T 的关系（位力定理），卫星的速度是木星的 1.5 倍，其动能是自身势能绝对值的 $1.5^2/2=1.125$ 倍，因此卫星的 E=T+V>0，为双曲线轨道。选（E）。

67. 解：光子不能脱离的条件是黑洞的第二宇宙速度达到光速，即

$$c = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}\,,$$

$$R = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{\left(3 \times 10^8\right)^2} \approx 0.01\mathrm{m} = 1\mathrm{cm}\,。$$

选（C）。

68. 解：L = T – V，动能和势能分别为

$$T = \frac{1}{2} m\left(\dot{s}^2 + (\omega s \sin\theta)^2\right),$$

$$V = mgs\cos\theta\,。$$

选（E）。

69. 解：左边的电流在 A 点产生的磁场用 Ampere 环路定理计算得

$$\mu_0 J \pi \left(\frac{d}{2}\right)^2 = B_1 \cdot 2\pi \frac{d}{2}\,,$$

$$B_1 = \frac{\mu_0 J d}{4}\,。$$

A 处磁场是该改值的两倍，

$$B = 2B_1 = \frac{\mu_0 J d}{2}\,。$$

选（A）。

70. 解：非相对论加速运动的 Larmor 辐射公式

$$P=\frac{1}{6\pi\varepsilon_0}\frac{e^2a^2}{c^2},$$

故

$$\frac{P_B}{P_A}=2^2\times4^2=64。$$

选（D）。

71. 解：粒子在板间飞行时间为

$$t=\frac{L}{v},$$

在竖直方向飞过距离

$$y=\frac{1}{2}\frac{Eq}{m}t^2=\frac{VqL^2}{2dmv^2}。$$

由几何关系容易知道偏转角

$$\theta=\arctan\left(\frac{y}{\mathrm{L}/2}\right)=\arctan\left(\frac{VqL}{dmv^2}\right)。$$

选（A）。

72. 解：增大的放大倍数是正反馈。选（A）。

73. 解：$W=\int pdV=\int\frac{C}{V^\gamma}dV=\frac{1}{1-\gamma}C(V_f^{1-\gamma}-V_i^{1-\gamma})=\frac{P_fV_f-P_iV_i}{1-\gamma}$，选（C）。

74. 解：末态两个物体都达到 300 K，在接触传热的过程中每个物体

$$dQ=TdS=mCdT,$$

$$\Delta S=mC\ln\frac{T_f}{T_i}。$$

系统总熵变为

$$\Delta S=mC\ln\frac{300}{500}+mC\ln\frac{300}{100}=mC\ln\frac{9}{5}。$$

选（B）。

75. 解：根据 Fourier 传热律，

$$P=-K\frac{\Delta T}{\Delta x}\propto\frac{K}{\Delta x},$$

$$\frac{P_1}{P_2}=\frac{K_1}{K_2}\frac{\Delta x_2}{\Delta x_1}=\frac{0.8}{0.025}\frac{2}{4}=16\text{。}$$

选（D）。

76. 解：选（B）。Gauss 波包随时间扩散，II 对。IV 就是测不准原理。

77. 解：Hamilton 可表为

$$H=-JS_1\cdot S_2=-\frac{J}{2}\left[(S_1+S_2)^2-S_1{}^2-S_2{}^2\right],$$

基态是能量最低的状态，上式中$(S_1+S_2)^2$的本征值取最大值(S_1+S_2) (S_1+S_2+1)时，H 最低，

$$E=-\frac{J}{2}\left[(S_1+S_2)(S_1+S_2+1)-S_1(S_1+1)-S_2(S_2+1)\right]\text{。}$$

选（D）。

78. 解：电子从施主能级填进导带。选（E）。

79. 解：极低温时分子振动、转动自由度均被冻结，$C_V=\frac{3}{2}R$；极高温时$C_V=\frac{7}{2}R$。二者之比 7/3，选（D）。

80. 解：如果$\mu_l=\mu_r$，则无反射，透射振幅为 1，由此排除（D）、（E）。如果$\mu_r=\infty$，则无透射，即透射振幅为 0，由此排除（A）、（B）。（C）是正确选项。此题如果按连接条件老老实实计算显然不可能在 5 分钟内做出。

81. 解：$73.416\times 6=440.496$，与 A_4 相差 $0.496\cong 0.5$Hz，此即拍频。选（B）。

82. 解：Auger 电子是辐射出去的外壳层电子。选（C）。

83. 解：x 处的曲率半径为

$$\rho=\left|\frac{\left(1+y'^2\right)^{3/2}}{y''}\right|=\left|\frac{\left(1+\left(kd\sin kx\right)^2\right)^{3/2}}{k^2d\cos kx}\right|\text{。}$$

质点运动到 x 位置时的速度为

$$u=\sqrt{v^2+2gd\left(1-\cos kx\right)}\text{。}$$

质点不脱离曲面的条件是重力能提供足够的法向加速度：

$$g\left|\frac{1}{\sqrt{1+y'^2}}\right|\geq\frac{u^2}{\rho},$$

$$g\frac{1}{\sqrt{1+(kd\sin kx)^2}}\geq\frac{2gd(1-\cos kx)+v^2}{\dfrac{\left(1+\left(kd\sin kx\right)^2\right)^{3/2}}{k^2d\cos kx}}=\frac{\left(2gd(1-\cos kx)+v^2\right)k^2d\cos kx}{\left(1+(kd\sin kx)^2\right)^{3/2}},$$

$$1+(kd\sin kx)^2 \ge \left(2d(1-\cos kx)+\frac{v^2}{g}\right)k^2 d\cos kx\text{，}$$

$$\cos^2 kx-\left(2+\frac{v^2}{gd}\right)\cos kx+\left(1+\frac{1}{k^2d^2}\right)\ge 0\text{。}$$

上面的不等式必须在 $0 \le \cos kx \le 1$ 成立，因为使上式左方二次函数取极值的 $\cos kx > 1$，所以只需让上式在 $\cos kx = 0$ 和 $\cos kx = 1$ 的两个端点分别成立。代入 $\cos kx = 1$，得

$$1-\left(1+\frac{v^2}{gd}\right)+\left(1+\frac{1}{k^2d^2}\right)\ge 0\text{，}$$

$$v \le \sqrt{\frac{g}{k^2d}}\text{。}$$

选（D）。

84. 解：两个球的系统有两个自由度，故有两个简正模。一个是两球相对距离保持不变，作为一个整体振动，其频率就是一般的摆长为 l 的单摆的振动频率 $\sqrt{\frac{g}{l}}$。另一个简正模是两球质心不动，两球相对质心做方向相反的振动。左边的球与质心间距是二球间距的 $\frac{m_2}{m_1+m_2}$，故其感受到的有效倔强系数为

$$K_1=\frac{m_1+m_2}{m_2}K\text{。}$$

振动频率为

$$f=\sqrt{\frac{g}{l}+\frac{K_1}{m_1}}=\sqrt{\frac{g}{l}+\frac{K}{m_1}+\frac{K}{m_2}}\text{。}$$

选（D）。

85. 解：选（B）。此题只需检验 m=0 和 m=∞的极限情况。m=0 相当于末端自由，

$$L=\left(n+\frac{1}{4}\right)\lambda,\left(n+\frac{3}{4}\right)\lambda\text{，}$$

（B）左右两边都等于∞。m=∞相当于末端固定，

$$L=n\lambda,\left(n+\frac{1}{2}\right)\lambda\text{，}$$

（B）左右两边都等于 0。其余四个选项都可用相同的方法排除。

86. 解：粒子运动轨迹是一个匀速直线运动和一个匀速圆周运动的叠加，为螺旋线。选（B）。

87. 解：撤去外磁场的过程中会产生感应电场，小球感受到的电场是

$$E=\frac{\frac{dB}{dt}\pi R^2}{\pi d}=\frac{dB}{dt}\frac{R^2}{d},$$

两球受到的总冲量矩为

$$I=\int dtEqd=\int dt\frac{dB}{dt}R^2q=qBR^2,$$

此即系统最终的角动量。选（A）。
pith：木髓。

88. 解：由 Ampere 环路定理，r<a 的部分磁场为

$$B\cdot 2\pi r=I\frac{r^2}{a^2},$$

故 $B\propto r$。仍从环路定理可知 r>c 部分 B=0。由这两点判定（B）正确。

89. 解：粒子在外磁场中做匀速圆周运动，方程为

$$qvB=m\frac{v^2}{r},$$

故其动量为

$$p=mv=qBr。$$

由射影定理知

$$r=\frac{l^2}{2s},$$

代入动量表达式即得

$$p=\frac{qBl^2}{2s}。$$

选（D）。

90. 解：自感公式为

$$L=\mu_0 NV。$$

选（B）。

91. 解：不可能，因为热不能从低温物体传向高温物体，违反热二律。选（E）。

92. 解：振动的平衡位置为

$$x=\sqrt{\frac{a}{2b}},$$

势能的二次项系数决定振动频率，

$$\omega=\sqrt{\frac{U''}{m}}\Bigg|_{x=\sqrt{\frac{a}{2b}}}=\sqrt{\frac{-2a+12b\left(\frac{a}{2b}\right)}{m}}=2\sqrt{\frac{a}{m}}\text{。}$$

选（D）。

93. 解：物体在 x<0 位置做简谐振动，一来一回共花时间

$$\pi\sqrt{m/k}\,,$$

在右边做匀加速运动，一来一回共花时间

$$2v/g=2\sqrt{\frac{2E}{m}}\Big/g\,,$$

相加即得运动周期。选（D）。

94. 解：系统总内能按统计权重计算为

$$E=N\frac{\varepsilon e^{-\varepsilon/kT}}{1+e^{-\varepsilon/kT}}=\frac{N\varepsilon}{e^{\varepsilon/kT}+1}\,,$$

选（D）。

95. 解：超导是二级相变，在临界温度处 C_v 突变。选（E）。

96. 解：列出动量、能量守恒的方程（c=1）：

$$\begin{cases}p=p_{+}+p_{-}\\ p=\sqrt{{p_{+}}^{2}+m^{2}}+\sqrt{{p_{-}}^{2}+m^{2}}\end{cases},$$

显然无解。选（A）。

97. 解：

$$\begin{aligned}j&=-\frac{i\hbar}{2m}\left(\psi^*\frac{d}{dx}\psi-c.c.\right)\\&=-\frac{i\hbar}{2m}\left((\alpha^*\cos kx+\beta^*\sin kx)(-k\alpha\sin kx+k\beta\cos kx)-c.c.\right)\\&=-\frac{i\hbar k}{2m}\left(\alpha^*\beta\sin^2 kx-\beta^*\alpha\cos^2 kx-c.c.\right)\\&=\frac{\hbar k}{2mi}(\alpha^*\beta-\beta^*\alpha)\end{aligned}$$

选（E）。

98. 解：所有原先奇宇称的态保留下来，这些态的能级是

$$E=\left(n+\frac{1}{2}\right)\hbar\omega,\quad \text{n为奇数}。$$

选（D）。

99. 解：亚稳态应该介于 n=3 和 n=1 之间，选（B）。

100. 解：$\hat{a}$ 不是 Hermitian 算符，也不和 H 对易。只有 III 对。选（C）。