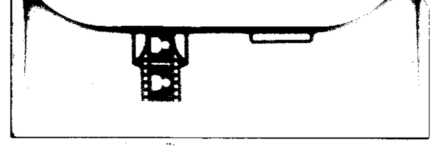
Introduction

This guide is a synthesis of other documents on the website of the Society of Physics Students, my past lecture notes and summaries, and my experience doing the practice/real test papers. I took the test during the fall semester of the senior year and only had two days to fully devote myself to the preparation. I basically went over the materials in this guide and practiced three sample tests. It covers almost all topics in the test. If you fully master them, you should be able to get over 85 questions correct. I hope this document would prove helpful if you are under time constraint. However, it is advisable that you spend roughly a week's time to work through all four available practice tests. You're welcome to improve upon this version of the guide and to type up this document in Latex for online sharing.

Lin Cong
2008.12

EXAM TIPS

* Pick moderate statements. Extreme statements are usually wrong.
* Use Taylor expansion to deal with certain extreme cases
  e.g. $h\nu \ll kT$, $e^{\frac{h\nu}{kT}} \approx 1 + \frac{h\nu}{kT}$
* When knowing L^2 value, be careful to calculate l from $\hbar^2 l(l+1)$, two solutions
* Conservation of momentum (including angular momentum) should be checked before conservation of energy.
* Be careful about dimension of the problem. e.g. in 3D, radial wave
  $P = \int |\psi|^2 d\vec{r} = \int |\psi|^2 4\pi r^2 dr$
* Read underlined words carefully
* Calculate T^4 carefully. it is 4th power!
* Don't think too hard, the questions are easy enough to be solved in 2min
* Use method of elimination
* Dimensional analysis is always useful
* Usually order of magnitude calculation is good enough
* Potential is a SCALAR, be careful
* In general $F = -\nabla(\text{potential energy})$ but in E/M notice V stands for potential, not potential energy, so $F = -\nabla(q \cdot \text{potential})$
* Usually it is convenient to set $h = \hbar = c = \cdots = 1$, but if ans differs from choices, that's a signal that maybe we need to keep them
* When you get stuck, take limits.
* If you haven't realized how important it is, I'll repeat: TAKE LIMITS.
* If some experimenter is involved in the question, it is usually a failed experiment (as GRE is usually set by theorists, according to yosunism.com.)



I CLASSICAL MECHANICS

* A worked example on velocity and acceleration in a curved path in a plane

$\hat{r} = \hat{i}\cos\theta + \hat{j}\sin\theta, \quad \hat{\theta} = -\hat{i}\sin\theta + \hat{j}\cos\theta$

$\vec{v} = \frac{d(R\hat{r})}{dt} = \frac{dR}{dt}\hat{r} + R\frac{d\hat{r}}{dt} = \dot{R}\hat{r} + R\omega\hat{\theta}$

Similarly $\vec{a} = (\ddot{R} - R\omega^2)\hat{r} + (R\dot{\omega} + 2\dot{R}\omega)\hat{\theta}$

The idea is skillfully use $d(AB) = AdB + BdA$. This applies to change of momentum as well.

* Firing rocket

$(V_g - V)\,dM + d(MV) = 0$ M is rocket mass, V is speed, V_g is relative speed of the waste fired out.

* Bernoulli's equation $P + \frac{1}{2}\rho v^2 + \rho g y = \text{const}$ (conservation of energy)

* Torricelli's Theorem: The outlet speed is the free-fall speed. For a barrel with water depth d, an outlet at base has horizontal flow speed $v = \sqrt{2gd}$

* Stokes' Law viscous drag $= 6\pi\eta r_s v$ (Probably won't appear in GRE)

* Poiseuille's Law $\Delta P = \frac{8\mu L Q}{\pi r^4}$ L is length of tube, Q is volume rate

(again, too hard for GRE)

This describes viscous incompressible flow through a constant circular cross-section.

* Kepler's laws ① (If you don't know this, wait a couple years before you take GRE)

② $\frac{dA}{dt} = \frac{1}{2} r^2 \dot{\theta} = \text{constant}$

③ $T = A \Big/ \frac{dA}{dt} = 2\sqrt{\frac{m}{k}}\, R^{\frac{3}{2}}$ i.e $T^2 \propto R^3$

* $F_{\text{coriolis}} = -2m(\vec{\omega} \times \vec{V}_{\text{radial}})$ (unlikely)

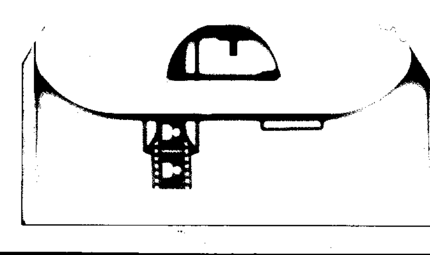
* Diffusion: Fick's law $\vec{J}_x$ (diffusion flux) $= -D\,\nabla_x \phi$

* Frequency of a pendulum of arbitrary shape $\omega = \sqrt{\frac{mgr}{I}}$

* Hamiltonian formulation

$H = \sum_i p_i \dot{q}_i - \mathcal{L} \qquad \dot{p} = -\frac{\partial H}{\partial q} \qquad \dot{q} = \frac{\partial H}{\partial p}$

* Circular orbits exist for almost all potentials. Stable non-circular orbits can occur for the simple harmonic potential and the inverse square rule.



* Orbit questions, $V_{eff}(r) = V(r) + L^2/(2mr^2)$

$V(r) \propto \frac{1}{r}$ for gravitational potential

Total energy of an object $E = \frac{1}{2}mv^2 + V_{eff}$

$E < V_{min}$, a spiral orbit

$E = V_{min}$ circular orbit

$V_{min} < E < 0$, ellipse

$E = 0$ parabolic $E > 0$ hyperbolic

II ELECTROMAGNETISM

* Faraday's laws of electrolysis (unlikely in GRE)
  (a) the mass liberated $\propto$ charge passed through
  (b) mass of different elements liberated $\propto$ atomic weight / valence

  $m = \frac{QA}{FV}$ V = valence, A = atomic weight kg/kmol
  $F = 9.65 \times 10^7$ C/kmol (Faraday)

* Parallel plate capacitor $C = \frac{\epsilon_0 A}{d}$ or $\frac{\epsilon A}{d}$ for dielectric
  Spherical capacitor $C = \frac{4\pi\epsilon_0 ab}{a-b}$

* In charging a capacitor $q = q_0 (1 - e^{-t/RC})$
  discharging $q = q_0 e^{-t/RC}$

* Cyclotron / magnetic bending $r = \frac{mv}{qB}$ (easily derived)

* Torque experienced by a planar coil of N loops, with current I in each loop $\tau = NIAB\sin\theta$, θ is angle btw B and line perpendicular to coil plane
  $\vec{\tau} = \vec{\mu} \times \vec{B}$

* B-field of a long wire $B = \frac{\mu_0 I}{2\pi r}$

  center of a ring wire $\frac{\mu_0 I}{2r}$ (can generalize to arc)

  center, long solenoid $B = \mu_0 n I$, n is turn density

* Conductors do not transmit EM wave, thus $\vec{E}$ vector is reversed upon reflection, B vector is increased by a factor of 2 (by solving ~~propa~~ propagation of EM wave.

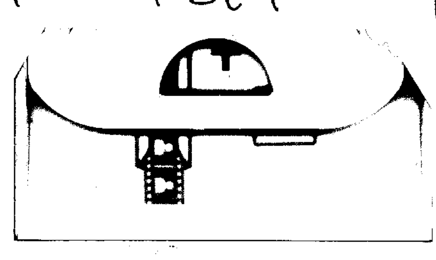
* $B = \mu H = \mu_0 (H + M) = \mu_0 (H + \chi_m H)$
  Diamagnetic $\longleftrightarrow$ χ_m very small & -ve constant
  Paramagnetic $\longleftrightarrow$ χ_m small and positive, inversely proportional to the absolute temp.

  Ferromagnetic $\longleftrightarrow$ χ_m positive, can be greater than 1
  M is no longer proportional to H

* For solenoid and toroid $H = nI$ n is no. density

* Self inductance $\mathcal{E} = -L\frac{di}{dt}$ L is in henrie ($1H = 1 V\cdot s/A = 1 J/A^2 = 1 Wb/A$
  $N\Phi = LI$ flux linkage   inductance of solenoid $L = \frac{\mu N^2 A}{l}$

* Induced e.m.f. $|\mathcal{E}_s| = M\left|\frac{\Delta i_p}{\Delta t}\right| = N_s\left|\frac{\Delta\Phi}{\Delta t}\right|$
  primary



* Time constant for R-L circuit $t = L/R$ for RC is $t = RC$
frequency for L-C circuit $w_0 = \frac{1}{\sqrt{LC}}$

* $X_L = 2\pi f L$ inductive reactance $X_C = 1/2\pi f C$ capactive reactance
Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$ for series
$\frac{1}{Z} = \left[\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2 \right]^{1/2}$ for parallel
Current is maximized at resonance $X_L = wL = X_C = \frac{1}{wC}$
(many questions on this)

* Larmor formula for radiation $P = \frac{\mu_0 q^2 a^2}{6\pi c} \propto q^2 a^2$
q = charge a = acceleration
Notice energy per unit area decreases as distance increases
inverse-square relation

* Mean Drift speed $\vec{V}_D = \frac{\vec{J}}{ne} = \frac{I}{nqV}$ n = no. of atoms per volume j = current density $= \frac{i}{A}$

* Impedance of capacitor $Z = \frac{1}{jwC}$, of inductor $Z = jwL$

* magnetic field on axis of a circle of current
$B = \frac{\mu_0 I}{2} \frac{r^2}{(r^2 + z^2)^{3/2}}$

* Bremstrahlung: electromagnetic radiation produced by the deceleration of a charged particle.

* For incident wave reflecting off a plane, just set up a boundary value problem
$E_1^{\perp} - E_2^{\perp} = \sigma_f$ $E_1^{\parallel} = E_2^{\parallel}$ and remember Poynting vector
$\vec{S} \propto \vec{E} \times \vec{B}$ points in the direction of propagation
$E_0 + E_0^{\text{reflected}} = E_0^{\text{transmitted}}$

* Lenz's law: The idea is the system responds in a way to restore or at least attempt to restore to original state. See 0177 Q42

* Impedance matching to maximize power transfer or to prevent terminal-end reflection $Z_{load} = Z^*_{source}$
$I(X_g) + I(X_L) = IR$, $j^2 = -1$
generator impedance $= R_g + jX_g$ load impedance $= R_L + jX_L$
$Z = R + j\left(wL - \frac{1}{wC}\right)$

* Propogation vector $\vec{k}$, $\tilde{E}(\vec{r},t) = \tilde{E}_0 \, e^{i(\vec{k}\cdot\vec{r} - \omega t)} \, \hat{n}$

$\tilde{B}(\vec{r},t) = \frac{1}{c} \, |\tilde{E}(\vec{r},t)| \, c(\hat{k}\times\hat{n}) = \frac{1}{c} \hat{k}\times\tilde{E}$

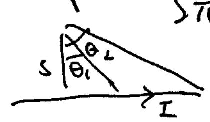
* No electric field in a constant potential enclosure implies constant V inside
* Hall effect $R_H = \frac{1}{(p-n)e}$ p for positive, n for negative
  can be used to test the nature of charge carrier
* Lorentz Force (of course) $\vec{F} = q(\vec{E} * \vec{v}\times\vec{B})$
* $\nabla\cdot(\nabla\times\vec{H}) = 0$, $\nabla\times(\nabla f) = 0$
* one usually have cycloid motion whenever the electric and magnetic fields are perpendicular
* Faraday's law $emf = \vec{E}\cdot d\vec{l} = -\frac{d\Phi}{dt}$
* Visible spectrum
  Radio 10^3 buildings ; Microwave 10^{-2} ; infrared 10^{-5} ; visible 10^{-6} 700nm–400nm
  ultraviolet 10^{-8} molecules ; X-ray 10^{-10} Atoms ; Gamma ray 10^{-12} Nuclei.

* $\vec{D} = \epsilon_0\vec{E} + \vec{P} = \epsilon_0\vec{E} + \epsilon_0\chi_e\vec{E} = \epsilon_0(1+\chi_e)\vec{E} = \epsilon\vec{E}$
  $\epsilon_r = 1+\chi_e = \frac{\epsilon}{\epsilon_0}$ dialectric constant
  $\left.\begin{array}{l}\sigma_b = \vec{P}\cdot\vec{n} \\ \rho_b = -\vec{\nabla}\cdot\vec{P}\end{array}\right\}$ bound charges. ~~currents~~
  $\nabla\times D = \nabla\times P$ not necessarily zero.
* $\vec{B} = \begin{cases} \mu_0 n I \hat{z}, & \text{inside the solenoid, } n = \text{density per length} \\ 0, & \text{outside solenoid.} \end{cases}$

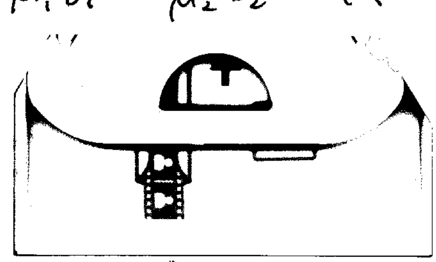
$\vec{B}(\vec{r}) = \begin{cases} \frac{\mu_0 N I}{2\pi s}\hat{\phi}, & \text{for points inside coil} \\ 0, & \text{outside} \end{cases}$

Toroid ~~$\mu_0 NI$~~

* Force between two wires $f = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$ Force perlength
* $B = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)$



* Mutual inductance of two loops $M_{21} = \frac{\mu_0}{4\pi}\oint\oint\frac{d\vec{l}_1\cdot d\vec{l}_2}{r}$
* Radiation pressure $P = \frac{I}{c}$ ($\frac{2I}{c}$ for perfect reflector)
* $\nabla\cdot\vec{D} = \rho_f$ $\nabla\times\vec{H} = J_f + \frac{\partial D}{\partial t}$, $\nabla\cdot B = 0$ $\nabla\times E = -\frac{\partial B}{\partial t}$
  Boundary conditions
  $\epsilon_1\vec{E}_1^{\perp} - \epsilon_2\vec{E}_2^{\perp} = \sigma_f$ $\vec{B}_1^{\perp} - \vec{B}_2^{\perp} = 0$
  $\vec{E}_1^{\parallel} - \vec{E}_2^{\parallel} = 0$ $\frac{1}{\mu_1}\vec{B}_1^{\parallel} - \frac{1}{\mu_2}\vec{B}_2^{\parallel} = \vec{K}_f\times\hat{n}$



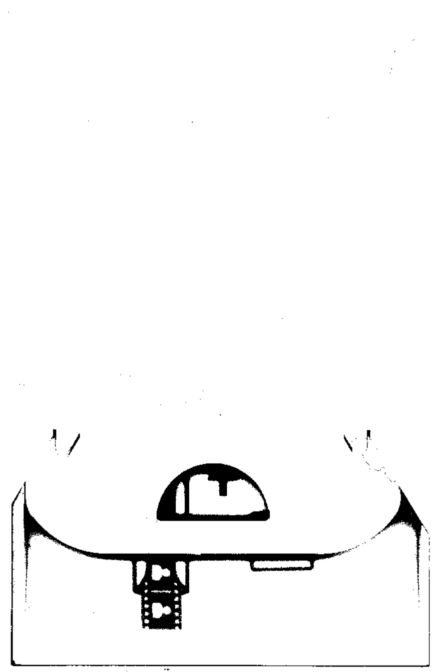
* Biot-Savart law: $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}' \times \hat{\mathscr{r}}}{\mathscr{r}^2}$

* B-field at center of a ring current $\vec{B} = \frac{\mu_0 I}{2r}$

* $H = \frac{1}{\mu_0} B - M$ $\quad \vec{J}_b = \nabla \times \vec{M}$ $\quad \vec{K}_b = \vec{M} \times \hat{n}$

  $\vec{B} = \mu \vec{H}$ $\quad \mu = \mu_0 (1 + \chi_m)$

* Practically H is more important than D, though they're on equal footing theoretically.



III OPTICS AND WAVE PHENOMENA

* Speed of propogation for waves

  Transverse on string $v = \sqrt{\frac{T}{\rho}}$

  Longitudinal in liquid $v = \sqrt{\frac{B}{\rho}}$ B is Bulk modulus $= \left|\frac{\Delta P}{\Delta V/V}\right|$

  in solid $v = \sqrt{\frac{Y}{\rho}}$ Y is Young's modulus

  in gases $v = \sqrt{\frac{\gamma P}{\rho}}$

* For open pipe  fundamental frequency is $\frac{v}{2L}$

  v is speed of sound. For closed pipe it is $(2n-1)\frac{\lambda}{4} = L$

  The idea is $\lambda \cdot f = v$ wavelength · frequency = speed

* Speed of sound in air $[(\gamma RT)/m]^{1/2} \propto \sqrt{T}$
* Resonant frequency of a rectangular drum $f_{mn} = \frac{v}{2}\sqrt{\left(\frac{m}{L_x}\right)^2 + \left(\frac{n}{L_y}\right)^2}$
* Doppler Effect $f' = \frac{v}{v + v_{s,r}} f$ v is velocity in medium

  $v_{s,r}$ is source velocity w.r.t. medium

  In general $\frac{f_{listener}}{v \pm v_{lis}} = \frac{f_{source}}{v \pm v_{source}}$ The +, - sign can be easily determined by examining if the frequency received is higher or lower.

  Relativistic Doppler Effect see section VII

* lens optics $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ sign convention real image +

  converging lens +

  Concave mirror +

  concave 

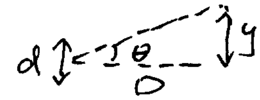
  Now if a real image is input and lies to the right of the lens, take it as -ve for left → right process, or +ve for right → left process

* Len's maker's equation $\frac{1}{f} = \frac{(n'-1)}{~~n~~}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ in medium n'

  R_1 positive → convex, negative → concave

  R_2 positive → concave, negative → convex

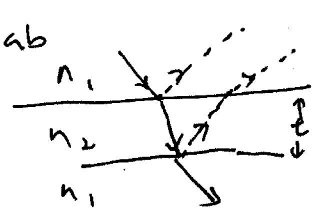
* Young's double slit $d \sin\theta = m\lambda$ maxima

  $\frac{y}{D} \cdot d = m D \lambda$ for $d \ll D$, θ small

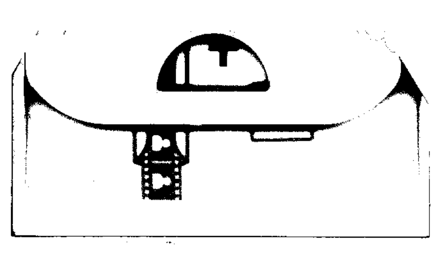
  $d \sin\theta = (m + \frac{1}{2})\lambda$ minima

* Slab

  $\frac{2n_2 t}{n_1 \lambda_1} = m + \frac{1}{2}$ max

  $\frac{2n_2 t}{n_1 \lambda_1} = m + 1$ min



* Diffraction grating $d\sin\theta = m\lambda$ ~~Intensity minima~~ ϕ

If incident at angle θ_i: $d(\sin\theta_m + \sin\theta_i) = m\lambda$

The overall result is interference pattern modulated by single slit diffraction envelope.

Intensity of interference $I = I_0 \dfrac{\sin^2[N\frac{\phi}{2}]}{\sin^2[\frac{\phi}{2}]}$, $\phi = \frac{2\pi}{\lambda} d\sin\theta$

minima occurs at $\frac{N\phi}{2} = \pi, \dots, n\pi$, $\frac{n}{N}$ not integer

maxima occurs at $\frac{\phi}{2} = 0, \pi, 2\pi, \dots$

single-slit envelope $I = I_0 \dfrac{\sin^2[\frac{\phi'}{2}]}{[\frac{\phi'}{2}]^2}$ $\phi' = \frac{2\pi}{\lambda} w\sin\theta$ w = width

Overall $I = I_0 \dfrac{\sin^2[\frac{\phi'}{2}]}{[\frac{\phi'}{2}]^2} \dfrac{\sin^2[N\frac{\phi}{2}]}{\sin^2[\frac{\phi}{2}]}$

e.g. maxima will be missing when interference is max and single-slit is min

* Bragg's law of reflection

$m\lambda = 2d\sin\theta$, careful θ is glancing angle, not angle of incidence

* Brewster's angle $\tan\theta = \frac{n_2}{n_1}$ n_1 n_2

* Diffraction again (more background info)

The light diffracted by a grating is found by summing the light diffracted from each of the elements, and is essentially a convolution of diffraction and interference patterns.

Fresnel diffraction (near-field)
Fraunhofer diffraction (far-field)

increasing planar nature of outgoing diffracted waves.

* Diffraction-limited imaging

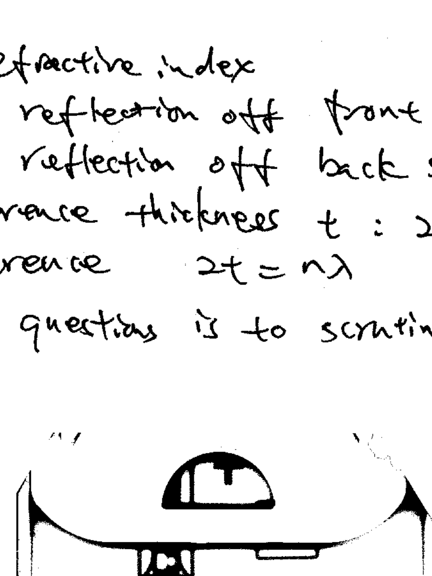
$d = 1.22\lambda N$ $N = \frac{\text{focal length}}{\text{diameter}}$ (call F number)

angular resolution is $\sin\theta = 1.22\frac{\lambda}{D}$, D is lens aperture

* Thin-film theory

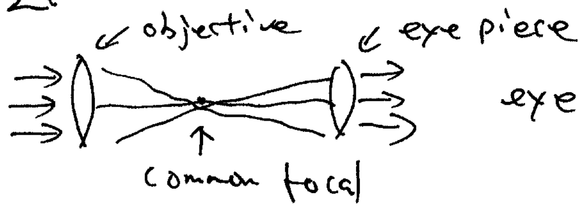
Say, film has higher refractive index
then phase change for reflection off front surface
no phase change for reflection off back surface
constructive interference thickness t : $2t = (n + \frac{1}{2})\lambda$
destructive interference $2t = n\lambda$

* The key idea for many questions is to scrutinize path difference. (optical)



* convex   concave

* Telescope



angular magnification $= \left| \frac{f_{objective}}{f_{eye}} \right|$

magnifying power = max angular magnification $= \frac{\text{image size with lens}}{\text{image size without lens}}$

* Microscopy

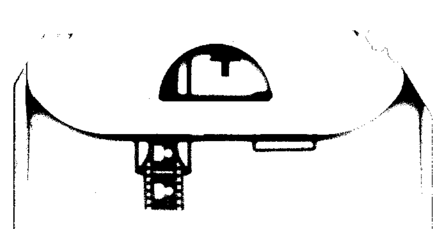
magnifying power $= \frac{\beta}{\alpha}$

* In Michelson interferometer a change of distance $\frac{\lambda}{2}$ of the optical path btw the mirrors generally results in a change of λ of optical path of light ray, thus potentially giving a cycle of bright → dark → bright fringes

* mirror with curvature $f \approx \frac{R}{2}$

* Beats beat frequency is $f_1 - f_2$

$\sin(2\pi f_1 t) + \sin(2\pi f_2 t) = 2\cos(2\pi \frac{f_1 - f_2}{2} t) \sin(2\pi \frac{f_1 + f_2}{2} t)$



IV THERMODYNAMICS AND STATISTICAL MECHANICS

* Heat Transfer
  conduction: rate $H = \frac{\Delta Q}{\Delta t} = -kA\,\frac{T_2 - T_1}{L}$, $\frac{dQ}{dt} = -kA\,\frac{dT}{dx}$, A is area, k is a constant

  Convection (Probably not in GRE), $H = \frac{\Delta Q}{\Delta t} = hA(T_s - T_\infty)$
  T_s = surface temp. h = convective heat-transfer coefficient.
  There are both natural & forced convections.

* Radiation Power $= \epsilon \sigma A T^4$ $\sigma = 5.67 \times 10^{-8}\,W/(m^2 \cdot K^4)$ Stefan-Boltzmann constant.
  ϵ = emissivity, $\epsilon \in [0,1]$, Net loss $= \epsilon \sigma A\,(T_{emission}^4 - T_{absorption}^4)$

* Wien's displacement law: The absolute temperature of a blackbody and the peak wavelength of its radiation are inversely proportional
  $\lambda_m \cdot T = 2898\ \mu m \cdot K$ (need to memorize)

* $PV = nRT = NkT$ $N = \frac{nR}{k}$ = no. of molecures

* Kinetic Theory of gas
  $P = \frac{1}{3} \rho V_{rms}^2$ $V_{rms} = \sqrt{\frac{3kT}{m}}$, $\bar{V} = \left(\frac{8kT}{\pi m}\right)^{\frac{1}{2}}$, $V_{most\ probable} = V_m = \left(\frac{2kT}{m}\right)^{\frac{1}{2}}$

* Maxwell–Boltzmann distribution (less likely to be in GRE)
  no. of molecules with energy E & $E + dE$
  $N(E)\,dE = \frac{2N}{\sqrt{\pi}\,(kT)^{3/2}} \sqrt{E}\, e^{-E/kT}\, dE$ $f(\vec{v})\,d^3\vec{v} = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}\, d^3\vec{v}$
  $P(v) = \sqrt{\frac{2}{\pi}} \left(\frac{m}{kT}\right)^{3/2} v^2 e^{-mv^2/2kT}$ from this can derive V_m

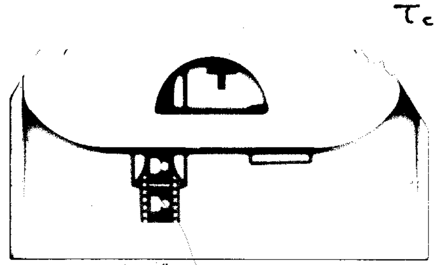
* Mean free path of a gas molecule of radius b $l = \frac{1}{4\pi\sqrt{2}\, b^2 (N/V)}$

* van der Waals equation of state (unlikely to be in GRE)
  $(P + an^2/V^2)(V - bn) = nRT$

* Adiabatic process PV^γ = const
  For an ideal gas to expand adiabatically from (P_1, V_1) to (P_2, V_2), work done by the gas is $W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$ (derived from $\int_{V_1}^{V_2} P\,dV$) notice (P_2, V_2) is final state

* The greatest possible thermal efficiency of an engine operating between two heat reservoirs is that of a Carnot engine, one that operates in the Carnot cycle
  Max efficiency is $\eta^* = 1 - \frac{T_{cold}}{T_{hot}}$
  For the case of refrigerator $K = \frac{Q_{cold}}{W}$ $K_{carnot} = \left(\frac{T_{hot}}{T_{cold}} - 1\right)^{-1}$

  Carnot = adiabatic + isothermall $dS = 0 \Rightarrow$ (entropy constant)
  Otto = adiabatic + isobaric $\eta = 1 - \frac{T_d - T_a}{T_c - T_b}$



* Dalton's Law $P = P_1 + P_2 = (n_1 + n_2)\,[(RT)/V]$

* The critical isotherm is the line that just touches the critical liquid-vapor region $\left(\frac{dP}{dV}\right)_c = 0$ $\left(\frac{d^2P}{dV^2}\right)_c = 0$, c = critical point

equilibrium region is where pressure and chemical potential for the two states of matter equal, usually a pressure const region in P-V diagram.

* $C_V = \frac{dE}{dT} = 3R$ in the Dulong-Petit law

* Laws of Thermodynamics (wiki)

0th : If two thermodynamic systems are each in thermal equilibrium with a third, then they are in thermal equilibrium with each other.

1st : $\Delta U = Q - W$ $\boxed{\text{heat flows from hot to cold}}$

2nd : Entropy increases / heat cannot be completely converted into work

3rd : $T \to 0$, $S \to$ constant minimum.

4th : Many versions, one is Onsager reciprocal relations ↓ 1968 Nobel Prize in chemistry.

5th : your call.

* Partition function $Z = \sum_i e^{-\beta E_i} = \int dE\, w(E)\, e^{-\beta E}$

$$= \int dE\, e^{-\beta A(E)}$$ ← Helmholtz free energy

$P(E_i) = \frac{e^{-\beta E_i}}{Z}$, entropy $= k \ln w(E) = -k \sum P(\psi_j) \ln P(\psi_j)$

* Equipartition Thm : ① classical canonical
② quadratical dependence
$\Rightarrow \langle \alpha x^2 \rangle = \frac{1}{2} kT$ for each degree of freedom

e.g. HO has 2 degrees of freedom

* Internal energy $dU = TdS - PdV$

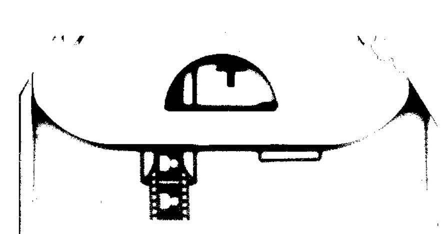
Enthalpy $H = U + PV$ $dH = TdS + VdP$ isobaric

Helmholtz $F = U - TS$ $dF = -SdT - PdV$ isothermal

Gibbs Free energy $dG = -SdT + VdP$

$G = U - TS + PV$

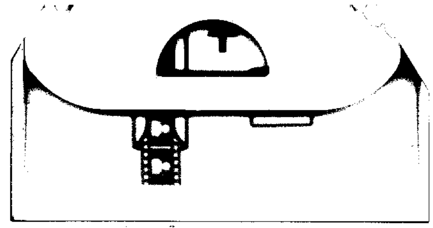
* $C_V = \left(\frac{\partial U}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V$ $C_P = \left(\frac{\partial U}{\partial T}\right)_P + P\left(\frac{\partial V}{\partial T}\right)_P = T\left(\frac{\partial S}{\partial T}\right)_P = \left(\frac{\partial H}{\partial T}\right)_P$



* $\bar{E} = -\frac{\partial}{\partial \beta} \ln Z$ $F = -kT \ln Z$ $S = k \ln Z + \bar{E}/T$, $dS = \int \frac{dQ}{T}$

$U = F + TS = F - T\left(\frac{\partial F}{\partial T}\right)_V = -T^2 \left(\frac{\partial}{\partial T}\right)_V \left(\frac{F}{T}\right)$ Gibbs-Helmholtz equation.

* Availability of a system $A = U + P_0 V - T_0 S$
  In natural change, A cannot increase
* Ideal gas $PV = nRT = N k_B T$
* Diatomic Gas $U = \frac{5}{2} kT$
* Maxwell relations : simple result based on multivar. calculus
* For ideal gas in adiabatic process $W = \Delta U = \frac{3}{2} N \Delta T$
* Clockwise enclosed area in a P-V diagram is the work done by gas in a cycle.
* chemical potential $\mu(T,V,N) = \left(\frac{\partial F}{\partial N}\right)_{T,V}$
  at equilibrium μ is uniform, F achieves minimum
* $P_{boson} \propto T^{5/2}$, $P_{classical} \propto T$, $P_{fermion} \propto T_F \approx$ big
  $T_{classical} \gg T_{boson}$
* A thermodynamic system in maximal probability state is stable
* Both debye and Einstein assume 3N independent Harmonic oscillators for lattice. Einstein took a constant frequency.



V QUANTOM MECHANICS

* Uncertainty Principle $\Delta x \Delta p \geq \frac{h}{4\pi} = \frac{\hbar}{2}$ $\Delta E \Delta t \geq \frac{\hbar}{2}$
* Schrödinger equation $i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V\psi$ (1D case, can be extended)
* $[AB, C] = [A, C]B + A[B, C]$
* De Broglie $\lambda = h/p = \frac{hc}{E}$ $\left(= \frac{h}{\sqrt{2mk_BT}}\right.$ for thermal wavelength$\left.\right)$
* 1-dimensional problem has no degenerate states
* Heisenberg's uncertainty $\Delta A \Delta B \geq \frac{1}{2}|\langle [A,B]\rangle|$, in particular $\Delta x \Delta p \geq \frac{\hbar}{2}$, $\Delta E \Delta t \geq \frac{\hbar}{2}$
* Infinite square well $\psi_n = \frac{\sqrt{2}}{\sqrt{a}} \sin\frac{n\pi x}{a}$ $E_n = \frac{\pi^2 n^2 \hbar^2}{2ma^2}$ $n \geq 1$ $E_n \propto n^2$

  Delta-function well $V = -\alpha\delta(x)$

  Only 1 bound state, many scattering states

  $\psi(x) = \sqrt{\frac{m\alpha}{\hbar}}\, e^{-m\alpha|x|/\hbar^2}$; $E = -\frac{m\alpha^2}{2\hbar^2}$

  shallow, narrow well, there are always at least one bound state
* Selection rule $\Delta l = \pm 1$, $\Delta m_l = \pm 1$ or 0 $\Delta j = \pm 1$ or 0 (total momen)

  No selection rule for spin

  "Electric dipole radiation" $\Leftrightarrow \Delta l = 0$

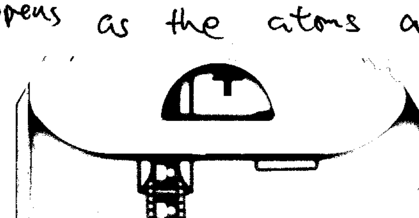
  magnetic dipole or electric quadrupole transitions are "forbidden" (but do actually occur)
* Stimulated & spontaneous emission rate $\propto |p|^2$

  $p = q\langle \psi_b | z | \psi_a \rangle$

  life time of excited state $\tau = \frac{1}{A_1 + A_2 + \cdots}$ A_i are spontaneous emission rates
* Time-independent 1st order perturbation

  $E_n^1 = E_n^0 + \langle \psi_n^0 | H' | \psi_n^0 \rangle$ $\psi_n^1 = \psi_n^0 + \sum_{m \neq n} \frac{\langle \psi_m^0 | H' | \psi_n^0 \rangle}{E_n^0 - E_m^0} \psi_m^0$
* Quantum approximation of rotational energy $E_{rot} = \frac{\hbar^2 l(l+1)}{2I}$
* $E_F = kT_F \simeq \frac{1}{2}mv^2$
* Differential cross-section $\frac{d\sigma}{d\Omega} = \frac{\text{Scattered flux / unit of solid angle}}{\text{Incident flux / unit of surface}}$
* Intrinsic magnetic moment $\vec{\mu} = \gamma \vec{S}$, $\gamma = \frac{eg}{2m}$ (g is Lande g-factor) If $m \nearrow$, $\mu \downarrow$
* total cross section $\sigma = \int D(\theta)\, d\Omega$, $D(\theta) = \frac{d\sigma}{d\Omega}$
* Stark effect is electrical analog to the Zeeman effect
* Born-Oppenheimer approximation : a reiterative idea
* In Stern-Gerlach experiment, a beam of neutral silver atoms are sent through an inhomogeneous magnetic field.

  classically, nothing happens as the atoms are neutral



with Larmor precession, the beam would be deflected into a smear. But it actually deflects into $2S+1$ beams, thus corroborating with the fact electrons are of spin $\frac{1}{2}$.

* Know the basic spherical harmonics

$$Y_0^0 = \frac{1}{2}\sqrt{\frac{1}{\pi}} \quad Y_1^{-1} = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\sin\theta\, e^{-i\varphi} \quad Y_1^0 = \frac{1}{2}\sqrt{\frac{3}{\pi}}\cos\theta \quad Y_1^1(\theta,\varphi) = -\frac{1}{2}\sqrt{\frac{3}{2\pi}}\sin\theta\, e^{i\varphi}$$

* Probability density current

$$\vec{j} = \frac{\hbar}{2mi}(\psi^*\nabla\psi - \psi\nabla\psi^*) = Re\left(\psi^* \frac{\hbar}{im}\nabla\psi\right)$$

* Laser operates by going from lower state to high state (population inversion), then falls back on a metastable-state in-btw. (not all the way down due to selection rule)
* $\langle p\rangle = \int \Psi^*\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\Psi\, dx \quad [\hat{x},\hat{p}] = i\hbar, \quad [\hat{f}(x),\hat{p}] = i\hbar\frac{\partial f}{\partial x}$
* $m\frac{d^2\langle x\rangle}{dt^2} = \frac{d\langle p\rangle}{dt} = \left\langle -\frac{\partial V}{\partial x}\right\rangle$ Ehrenfest's Thm: Expectation values obey classical laws.
* If $V(x)$ is even, $\psi(x)$ can always be taken to be either even or odd
* $\langle H\rangle = \sum_{n=1}^{\infty}|C_n|^2 E_n \qquad \delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx}\, dk$
* Tunneling show exponential decay
* The ground state of even potential is even and has no nodes
* In stationary states, all expectation values are independent of t
* Harmonic Oscillators

$$H = \hbar\omega\left(a_-a_+ - \frac{1}{2}\right) = \hbar\omega\left(a_+a_- + \frac{1}{2}\right) \qquad a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$$

$$[a_-, a_+] = 1 \qquad [x,p] = i\hbar$$

$$a_+a_-\psi_n = n\psi_n, \quad a_-a_+\psi_n = (n+1)\psi_n, \quad a_+\psi_n = \sqrt{n+1}\,\psi_{n+1}$$

$$a_-\psi_n = \sqrt{n}\,\psi_{n-1}, \quad \psi_n = \frac{1}{\sqrt{n!}}(a_+)^n\psi_0, \quad x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-); \; p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-)$$

* $\Phi(p,t) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty} e^{-ipx/\hbar}\,\Psi(x,t)\, dx$

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty} e^{ipx/\hbar}\,\Phi(p,t)\, dp$$

* $\frac{d}{dt}\langle Q\rangle = \frac{i}{\hbar}\langle[\hat{H},\hat{Q}]\rangle + \left\langle\frac{\partial\hat{Q}}{\partial t}\right\rangle$
* Virial Thm, in stationary state $2\langle T\rangle = \left\langle x\frac{dV}{dx}\right\rangle$
* Hydrogen Atom revisited

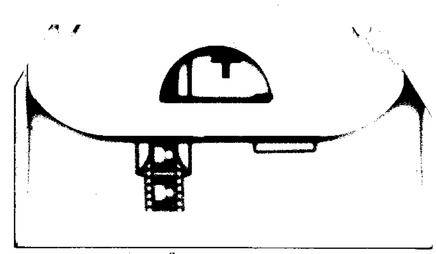
$E_n \propto$ reduced mass

$\propto Z^2$

$\propto \frac{1}{n^2}$

$$E_n = -\left[\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2\right]\frac{1}{n^2} = \frac{E_1}{n^2}$$

$$E_n(Z) = Z^2 E_n \quad a(Z) = \frac{a}{Z} \qquad R(Z) = Z^2 R$$



Bohr radius $a = \frac{4\pi\epsilon_0 \hbar^2}{me^2} = 0.529 \times 10^{-10}\,m$

$\psi_{100}(r,\theta,\phi) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$

* $[L_x, L_y] = i\hbar L_z$ $\quad [L^2, \vec{L}] = 0 \quad L_\pm = L_x \pm i L_y, \; [L^2, L_\pm] = 0$

$L^2 f_l^m = \hbar^2 l(l+1) f_l^m \qquad L_z f_l^m = \hbar m f_l^m$

$L_\pm f_l^m = \hbar\sqrt{(l \mp m)(l \pm m + 1)} = \hbar\sqrt{l(l+1) - m(m \pm 1)}$

$[L_z, x] = i\hbar y \quad [L_z, P_x] = i\hbar P_y \quad [L_z, y] = -i\hbar x, \quad [L_z, P_y] = -i\hbar P_x$

* $L_z = \frac{\hbar}{i}\frac{\partial}{\partial\phi}$ $\qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

* $S^2 = \frac{3}{4}\hbar^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Pauli matrices $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \vec{S} = \frac{\hbar}{2}\vec{\sigma}$

$\chi_+^{(x)} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$ eigenvalue $+\frac{\hbar}{2}$ $\quad \chi_-^{(x)} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$ eigenvalue $-\frac{\hbar}{2}$

$\langle S_x^2 \rangle = \langle S_y^2 \rangle = \langle S_z^2 \rangle = \frac{\hbar^2}{4}$

* Clebsch-Gordan coefficients

$|s\,m\rangle = \sum_{m_1+m_2=m} C^{s_1 s_2 s}_{m_1 m_2 m} |s_1 m_1\rangle |s_2 m_2\rangle$

$|s_1 m_1\rangle |s_2 m_2\rangle = \sum_s C^{s_1 s_2 s}_{m_1 m_2 m}$

* Continuity equation $\nabla \cdot \vec{J} = -\frac{\partial}{\partial t}|\Psi|^2, \quad \int_S \vec{J}\cdot d\vec{a} = -\frac{d}{dt}\int_V |\Psi|^2 d^3\vec{r}$

* ${}^{2S+1}L_J$ $\quad$ S = spin (number), L = orbital (a letter) $\quad$ J = ~~total~~ (number)

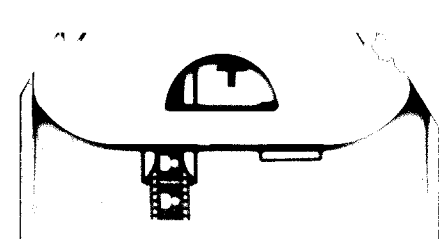
Hund's rule ① State with highest spin will have lowest energy given Pauli principle satisfied; ② For given spin and antisymmetrization highest L have lowest energy; ③ no than than half filled, lowest level has $J = |L - S|$, if more than half-filled $J = L + S$

* Fermi Gas $k_F = (3\rho\pi^2)^{\frac{1}{3}}$ $\quad \rho = \frac{Nq}{V}$ $\quad$ Fermi velocity $V_F = \sqrt{\frac{2E_F}{m}}$

$P = \frac{(3\pi^2)^{\frac{2}{3}}\hbar^2}{5m}\rho^{5/3}$ degeneracy pressure

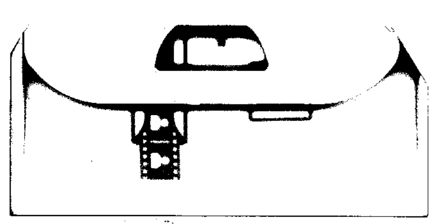
* $n(\epsilon) = \begin{cases} e^{-(\epsilon-\mu)/k_BT} & \text{Classical} \\ \frac{1}{e^{(\epsilon-\mu)/k_BT}+1} & \text{Fermion} \\ \frac{1}{e^{(\epsilon-\mu)/k_BT}-1} & \text{Boson} \end{cases}$

Black body $\rho(\omega) = \frac{\hbar\omega^3}{\pi^2 c^3 (e^{\hbar\omega/k_BT} - 1)}$



* fine structure → Spin-orbit coupling
  relativistic correction $\alpha = \frac{1}{137.036}$
  Then Lamb shift electric field
  Then Hyperfine structure due to magnetic interaction
  between electrons and protons
  spin-spin coupling (21cm line)

* Fine structure breaks degeneracy in l but still have j
* Fermi's golden rule is a way to calculate the transition rate (probability of transition per unit time) from one energy eigenstate of a quantum system into a continuum of energy eigenstates, due to a perturbation
* Full shell & close to full shell config are more difficult to ionize.
* Larmor precession $\vec{T} = \gamma \vec{J} \times \vec{B}$ $\omega = \gamma B$



VI ATOMIC PHYSISS

* $\Delta E = hf = \hbar\omega = \frac{hc}{\lambda}$, $hc = 12.4$ KeV·Å $= 1240$ eV·nm , de Broglie wavelength $\lambda = \frac{h}{mv}$
* Emission due to transition from level n to leveal a

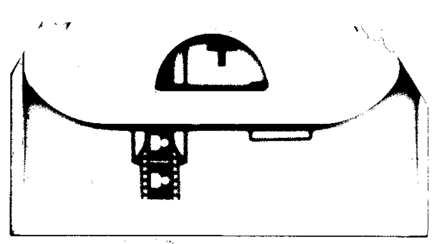
  $\frac{1}{\lambda} = R\left(\frac{1}{a^2} - \frac{1}{n^2}\right)$ $a=1$ Lyman series, $a=2$ Balmer series

  $R = 1.097 \times 10^7\ m^{-1}$ (not given in the table) , $E_n = -\frac{13.6\,eV}{n^2}$
* Hydrogen model extended , Z = no. of protons, quantities scale as

  $E \sim Z^2$, $\lambda \sim \frac{1}{Z^2}$.

  reduced-mass correction to emission formula is $\frac{1}{\lambda} = \frac{R_\infty Z^2}{1+(m/M)}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$

  m = ~~weight~~ mass of electron, M = mass of proton $m/M = 1/1836$
* Bohr postulate $L = mvr = \frac{nh}{2\pi} = n\hbar$
* Zeeman effect: splitting of a spectral line into several components in the presence of a static magnetic field
* k series refers to the inner most shell (K, L, M, N) so transition to inner-most shell $E = -13.6\underbrace{(Z-1)^2}_{\text{shielding}}\left(1 - \frac{1}{n_i^2}\right)$ eV
* Frank-Hertz Expt: Electrons of a certain energy range can be scattered inelastically, and the energy lost by electrons is discrete.



VII SPECIAL RELATIVITY

* $E^2 = (pc)^2 + (m_0 c^2)^2$ for photon (massless particle) $E = pc = h\nu$
* Relativistic Doppler Effect $\lambda = \sqrt{\frac{1+\beta}{1-\beta}}\,\lambda_0$ $\quad f = \sqrt{\frac{1-\beta}{1+\beta}}\, f_0$. $\beta = \frac{v}{c}$

  Sign can be determined by whether it is moving away/closer
* space-time interval $\quad \Delta s = \left((c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2\right)^{\frac{1}{2}}$
* Lorentz transformation

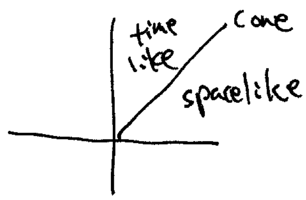
  $ct' = \gamma(ct - \beta x)$, $x' = \gamma(x - \beta(ct))$, $y' = y$, $z' = z$
* Relativistic addition of velocities

  $u_x' = \dfrac{u_x + v}{\frac{u_x v}{c^2} + 1}$ $\quad u_y' = \dfrac{u_y}{\gamma(1 + u_x v/c^2)}$, $\quad u_z' = \dfrac{u_z}{\gamma(1 + u_x v/c^2)}$ $\quad \gamma = \dfrac{1}{\sqrt{1-\beta^2}}$
* Lorentz-transformation of EM , $\vec{E}_{\parallel}' = \vec{E}_{\parallel}$, $\vec{E}_{\perp}' = \gamma\,[\vec{E}_{\perp} + (\vec{v} \times \vec{B})]$, $\vec{B}_{\parallel}' = \vec{B}_{\parallel}$

  For example in x direction

  $E_x' = E_x$ $\quad E_y' = \gamma(E_y - vB_z)$ $\quad E_z' = \gamma(E_z + vB_y)$ $\quad \vec{B}_{\perp}' = \gamma\,[\vec{B}_{\perp} - \frac{1}{c^2}(\vec{v} \times \vec{E})]$

  $B_x' = B_x$ $\quad B_y' = \gamma(B_y + E_z v/c^2)$ $\quad B_z' = \gamma(B_z - vE_y/c^2)$
* $E = \gamma m c^2$, $P = \gamma m v$,
* In every closed system, the total relativistic energy and momentum are conserved
* Space like (separation), i.e. can happen at same time $c^2\Delta t^2 - \Delta x^2 < 0$



* Transverse Doppler shift : $f = \dfrac{f'}{\sqrt{1-\beta^2}}$ or $f = f'\sqrt{1-\beta^2}$

VIII. LABORATORY METHODS

* If measures are independent (or intervals in a poisson process are indep.), both expected value and variance increase linearly with horizon (time in this case), so longer time can improve uncertainty (which is usually defined as $\frac{\sigma}{R} \propto \frac{1}{\sqrt{t}}$
* In Poisson distribution $\sigma = \sqrt{\bar{x}}$ sd ≈ square-root of the average

* Error Analysis. Estimating uncertainties

  If you are sure the value is close to 26mm than to 25 or 27mm

  Then record best estimate 36 ± 0.5 mm.

* Propagation of uncertainties

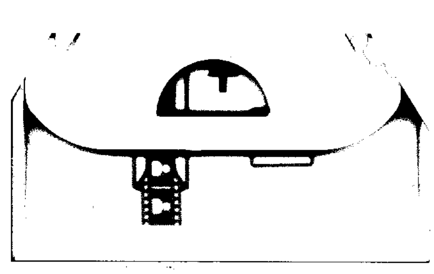
  for sum of random and independent variables

  $$\delta x = \left[(\delta x_1)^2 + (\delta x_2)^2 + (\delta x_3)^2 + (\delta x_4)^2 + \cdots \right]^{\frac{1}{2}}$$

  If multiplication or divisions are involved, use fractional uncertainty

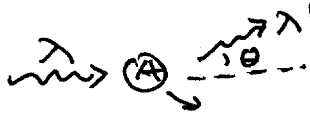
  $$\frac{\delta q}{|q|} = \left[\left(\frac{\delta x_1}{x_1}\right)^2 + \left(\frac{\delta x_2}{x_2}\right)^2 + \cdots \right]^{1/2}$$

* Experimental uncertainties that can be revealed by repeating the measurements are called random errors; those that cannot be revealed in this way are called systematic errors.



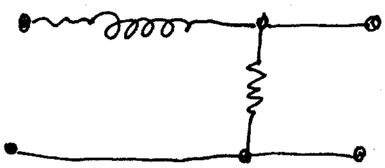
IX SPECIALIZED TOPICS

* Photoelectric effect E_{photon} = Work function + KE_{max}

* Compton Scattering 

$\lambda' - \lambda = \frac{h}{m_A c}(1-\cos\theta)$
  $\frac{h}{m_A c}$ is compton wavelength

* X-Ray Bragg reflection $n\lambda = 2d\sin\theta$ (compare to diffraction grating $n\lambda = d\sin\theta$)
* $1.602\times10^{-19} J = (1.602\times10^{-19} C)(1V) = 1eV$
* In solid-state physics effective mass $m^* = \hbar^2 / \frac{d^2E}{dk^2}$
* Electronic filters



In Out

High-pass means $\omega \to \infty$, $V_{in} = V_{out}$
usually look at $I = \frac{V_{in}}{Z}$ $Z = R + i(X_L - X_C)$
$X_L = \omega L$ $X_C = \frac{1}{\omega C}$

* Band spectra is a term that refers to using EM waves to probe molecules
* Solid state: primitive cell = $\frac{\text{unit cell}}{\text{\# of lattice points in a Bravais lattice}}$
  simple cubic → 1 point Body centered → 2 points
  face-centered → 4 points

* Resistivity of undoped semiconductor varies as $1/T$
* Nuclear physics: binding energy is a form of potential energy, convention is to take it as positive. It's the energy needed to separate into ~~separ~~ different constituents. It is usually subtracted from other energy to tally total energy.
* Pair production refers to the creation of an elementary particle and its antiparticle. Usually need high energy (at least the total rest mass)
* At low energies, photoelectric-effect dominates Compton Scattering
* Radioactivity: Beta decay $X^A_Z = X'^{A}_{Z+1} + \beta^0_{-1} + \bar{\nu}$
  Alpha: $X^A_Z \to X'^{A-4}_{Z-2} + He_2^4$ Gamma $X^A_Z \to X^A_Z + \gamma$
  Deuteron Decay (not natural) $X^A_Z \to X^{A-2}_{Z-1} + H_1^2$
  Radioactivity usually follow Poisson
* Coaxial cable terminated at an end with characteristic impedance in order to avoid reflection of signals from the terminated end of cable.
* Human eyes can only see things in motion up to ~25 Hz
* In magnetic field, e are more likely to be emitted in a direction opposite to the spin direction of the decaying atom.

* Op-amp (operational amplifiers) : If you only have two days to prepare for GRE, this is not worth the effort, maximum one question on this. It is ~~interest~~ interesting read nonetheless. Recommend "The Art of Electronics".

* The specific heat of a superconductor jumps to a lower value at the critical temperature (resistivity jumps too)

* Elementary particles.

Quarks	u up	c charm	t top	g gluon
	d down	s strange	b bottom	γ photon
Leptons	ν_e e neutrino	ν_μ μ neutrino	ν_τ τ neutrino	W w boson
	e electron	μ muon	τ tau	Z z boson
	I	II	III	← Generations
	Fermions			bosons

Force carriers

Hadron (bound state of quarks)

Baryons
3 quarks form fermion
(Baryon no. = 1)
e.g. N (nucleon)
Δ
Λ
Σ
Ξ
Ω
usually spin $\frac{1}{2}$

Meson
quark-antiquark pair form boson
Baryon no. $B = 0$
e.g. π, K (kaon)

* Family no. is preserved
* lepton no. conserved
(# of leptons − # of antileptons)
* Strangeness is conserved except for weak interactions
$(S = -(N_s - N_{\bar{s}}))$ ← strange anti quark
* Baryon no. conserved
$B = \frac{N_q - N_{\bar{q}}}{3}$ ← anti quark

* Internal conversion is a radioactive decay where an excited nucleus ~~interacts~~ interacts with an electron in one of the lower electron shells, causing the electron to be emitted from the atom. It is not Beta decay