(10/2007: Put in a few minor corrections and addenda.)

1. Classical Mechanics - 20%

(such as kinematics, Newton's laws, work and energy, oscillatory motion, rotational motion about a fixed axis, dynamics of systems of particles, central forces and celestial mechanics, three-dimensional particle dynamics, Lagrangian and Hamiltonian formalism, noninertial reference frames, elementary topics in fluid dynamics)

- v in terms of x in uniform acceleration.

$$v^2 = v_0^2 + 2a(x - x_0)$$

  Derivation: $((v + v_0)/2)\Delta t = v_{avg}\Delta t = (x - x_0)$ and $(v - v_0)/\Delta t = a$. Multiply these two equations together.

- Lagrangian. Let T =kinetic energy, U =potential energy.

$$L = L(q, \dot{q}, t) = T - U$$

- Euler-Lagrange equations of motion.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q}$$

- Action. Pick the path with the correct endpoints that minimizes

$$S = \int_{t_0}^{t_1} L(q(t), \dot{q}(t), t)dt$$

- Conjugate momentum.

$$p = \frac{\partial L}{\partial \dot{q}}$$

- Hamiltonian.

$$H = p\dot{q} - L = T + U$$

- Hamiltonian equations of motion.

$$\dot{q} = \frac{\partial H}{\partial p}$$

$$\dot{p} = -\frac{\partial H}{\partial q}$$

- Bernoulli's Equation for fluid flow. (Assume incompressible, nonviscous, laminar flow; this equation holds along flowlines, or if the flow is irrotational, it holds everywhere.) Take g as positive.

$$p + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$$

Derivation: Conservation of energy. The second two terms are energy per unit volume, the first term relates to the fact that when pressure decreases along a flowline, there's a net forward force on the fluid, which increases the energy.

- Fictitious forces from rotating reference frame.

$$\vec{F}_{\text{coriolis}} = -2m\vec{\omega} \times (\vec{v}_{\text{in rotating frame}})$$

$$\vec{F}_{\text{centrifugal}} = -m\vec{\omega} \times (\vec{\omega} \times \vec{r})$$

How to remember Coriolis force: It makes it hard 2 mov straight (remeber ω corresponds to the letter o).

- Torque.

$$\vec{\tau} = \vec{r} \times \vec{F}$$

How to remember the order of $\vec{r}$ and $\vec{F}$: "tariff" or "traffic" or "turf" or "teriffic" or "thrifty" or "trefoil".

- Angular momentum.

$$\vec{L} = \vec{r} \times \vec{p}$$

- Moment of inertia. r is distance from axis.

$$I = \sum_i m_i r_i^2 = \int r^2 dm = \int r^2 \rho \, dV$$

- Frequency of a pendulum of arbitrary shape. I is moment of inertia about the support, r is distance from support to center of mass.

$$\omega = \sqrt{\frac{mgr}{I}}$$

In particular, for a point mass at the end of a massless pendulum of length ℓ, $I = m\ell^2$, $r = \ell$, so

$$\omega = \sqrt{\frac{g}{\ell}}$$

Derivation: $I\ddot{\theta} = \tau = \vec{r} \times \vec{F} = -(r)(mg)(\sin\theta) \approx -rmg\theta$.

2. E&M - 18%

(such as electrostatics, currents and DC circuits, magnetic fields in free space, Lorentz force, induction, Maxwell's equations and their applications, electromagnetic waves, AC circuits, magnetic and electric fields in matter)

- ε and μ (names, $\vec{E}$ and $\vec{D}$, $\vec{H}$ and $\vec{B}$, χ_e, χ_m, n, c).

$$\varepsilon = \text{“permittivity”}$$
$$\mu = \text{“permeability”}$$
$$\vec{D} = \varepsilon \vec{E} \quad \text{“Eeee!”}$$
$$\vec{B} = \mu \vec{H} \quad \text{“Uh!”}$$
$$\varepsilon = \varepsilon_0(1+\chi_e) \quad (\chi_e = \text{“electric susceptibility”})$$
$$\mu = \mu_0(1+\chi_m) \quad (\chi_m = \text{“magnetic susceptibility”})$$
$$n = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \frac{c}{c_{\text{eff}}}$$
$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

- Typical values of ε, μ.
  - Permittivity: $\varepsilon > \varepsilon_0$ almost always, with $\chi_e > 0$. Field is reduced by polar molecules realigning opposite to the field.
  - Diamagnetics: $\mu < \mu_0$, $\chi_m < 0$. No unpaired electrons. Field is reduced by Lenz's law acting on electron orbits.
  - Paramagnetics: $\mu > \mu_0$, $\chi_m > 0$. Has some unpaired electrons, which align with and increase the magnetic field.
  - Ferromagnetics: $\mu >> \mu_0$, $\chi_m >> 0$ (although of course not linear). Unpaired electrons, domains, and such and such.

  How to remember dia versus para: "In a diamagnet, the magnetic field dies."

- Maxwell's equations.

$$\text{Gauss's Law:} \quad \vec{\nabla}\cdot\vec{D} = \rho_{\text{free}}; \quad \int_{\partial V} \vec{D}\cdot\vec{n}dA = Q_{\text{enclosed}}$$
$$\text{No monopoles:} \quad \vec{\nabla}\cdot\vec{B} = 0; \quad \int_{\partial V} \vec{B}\cdot\vec{n}dA = 0$$
$$\text{Faraday's Law:} \quad \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}; \quad \int_{\partial S} \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\vec{B}_{\text{flux through}}$$
$$\text{Ampère's Law (modified):} \quad \vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}; \quad \int_{\partial S} \vec{H}\cdot d\vec{\ell} = I_{\text{through}} + \frac{d}{dt}D_{\text{flux through}}$$

- Induced voltage from Faraday's Law. Φ =flux of magnetic field through coil, N =number of turns. Sign is determined by Lenz's Law.

$$V = N\frac{d\Phi}{dt}$$

- Coulomb's Law.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

Derivation: Gauss's Law $\to q_1 = 4\pi r^2 D_{\text{produced by 1}} = 4\pi r^2 \varepsilon_0 E_1$, and $F = q_2 E_1$.

- Biot-Savart Law.

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{L}\times\hat{r}}{r^2}$$

- Magnetic field on axis of a circle of current.

$$B = \frac{\mu_0 I}{2}\frac{r^2}{(r^2+z^2)^{3/2}}$$

$$B = \frac{\mu_0 I}{2r}, \qquad \text{at center of loop}$$

Derivation: From Biot-Savart, $B_{\text{perp}} = \frac{\mu_0 I}{4\pi}\frac{(2\pi r)}{z^2+r^2}\cos\theta = \frac{\mu_0 I}{2}\frac{r}{z^2+r^2}\frac{r}{(z^2+r^2)^{1/2}}$

- Magnetic field from infinite straight wire. Direction of $\vec{B}$ from right-hand rule.

$$B = \frac{\mu_0 I}{2\pi R}$$

- Force on a wire from a magnetic field.

$$\vec{F} = I\, d\vec{\ell}\times\vec{B}$$

Derivation: For one charge q moving at velocity $\vec{v}$ in a length of wire $d\vec{\ell}$, $F = q\vec{v}\times\vec{B}$, and $Id\vec{\ell} = q\vec{v}$.

- Boundary conditions for $\vec{E}$, $\vec{D}$, $\vec{H}$, $\vec{B}$ across macroscopic boundaries.
  - The normal component of $\vec{D}$ is continuous (unless there's a surface charge).
  - The normal component of $\vec{B}$ is continuous (always).
  - The tangential component of $\vec{E}$ is continuous (always).
  - The tangential component of $\vec{H}$ is continuous (unless there's a surface current).

- Capacitor – capacitance and energy.

$$Q = CV$$

$$U = \frac{1}{2}CV^2$$

How to remember: "CV" as in curriculum vitae.

- Capacitance of a parallel-plate capacitor.

$$C = \frac{\varepsilon A}{d}$$

- Inductor – inductance and energy.

$$V = -L\frac{dI}{dt}$$

$$U = \frac{1}{2}LI^2$$

How to remember: "LI" as in Long Island.

- Inductance of a solenoid. A =cross sectional area, l =length, N =number of turns.

$$L = \frac{\mu N^2 A}{l}$$

Why N^2? If you double the coils, you double the flux and you double the response per unit flux. Another memory trick: the equation has almost the same form as the capacitance of a parallel-plate capacitor.

- Impedance. $(V_0 e^{i\omega t}) = (I_0 e^{i\omega t})Z$, where

$$\text{Resistor:} \quad Z = R$$

$$\text{Capacitor:} \quad Z = \frac{1}{i\omega C}$$

$$\text{Inductor:} \quad Z = i\omega L$$

How to remember: at $\omega = 0$ (DC), capacitor has infinite resistance, and inductor has 0 resistance.

- Cyclotron frequency.

$$\omega = \frac{qB}{m}$$

Derivation: $mv^2/r = qvB \Longrightarrow mv/r = qB \Longrightarrow m\omega = qB$.

- Cherenkov Radiation. When a charged particle is traveling through a medium (with $n > 1$) faster than the speed of light in the medium (i.e. c/n), it emits light in a cone behind it. Why? As the particle passes, the electrons and molecules respond to the field; after the particle is gone, they relax to equilibrium, releasing radiation. The spectrum is continuous, and intensity is proportional to frequency of the photon (This is the reason for the blue glow of nuclear reactors). There is also a gradual high-frequency cutoff, due to the fact that the index of refraction is frequency-dependent and approaches 1 at high frequencies.

- Larmor formula for radiated power. a is acceleration, q is charge.

$$P = \frac{q^2 a^2}{6\pi\varepsilon_0 c^3}$$

3. Optics and Wave Phenomena - 9%

(such as wave properties, superposition, interference, diffraction, geometrical optics, polarization, Doppler effect)

- Phase velocity versus group velocity.

$$v_{phase} = \frac{\omega}{k}$$

$$v_{group} = \frac{d\omega}{dk}$$

- Doppler shift (for waves in a medium). Top signs when moving towards each other.

$$\omega = \omega_0 \frac{1 \pm \frac{v_o}{c}}{1 \mp \frac{v_s}{c}}$$

  How to remember: If source moves towards observer at c, infinite frequency; if observer moves away from source at c, zero frequency (picture the waves).

- Thin lens formula. Figure out signs by ray-tracing.

$$\frac{1}{S_o} + \frac{1}{S_i} = \frac{1}{f}$$

- Rayleigh Criterion. D is the diameter of the beam of light, or of the lens if the light fills the lens. θ is angular resolution. f is the focal length. Δl is the spacial resolution on the image (i.e. the radius of a spot of light on the image).

$$\sin\theta = \frac{\Delta l}{f} \approx 1.22 \frac{\lambda}{D}$$

- Focal length of lens/mirror.

$$\text{MIRROR:} \quad f = \frac{R}{2}$$

$$\text{LENS:} \quad \frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

(sign convention: R_1, R_2 both positive for a regular converging lens.) (replace n by n/n' if the surrounding medium isn't a vaccuum.)

- N-thick-slits diffraction. Let a by the thickness of each slit, d be the distance between adjacent pairs of slits, N be the number of slits.

$$I = I_0 \left(\frac{\sin(ak_0)}{(ak_0)}\right)^2 \left(\frac{\sin(Ndk_0)}{\sin(dk_0)}\right)^2, \text{ where } k_0 = \frac{\pi}{\lambda}\sin\theta = \frac{k}{2}\sin\theta$$

For $\alpha \to 0$ and $N \to \infty$, maxima at $\sin(dk_0) = 0$, i.e. $dk_0 = m\pi$, i.e. $m\lambda = d\sin\theta$.

- Polarization and intensity. If light is unpolarized, and goes through a polarizer, its intensity is halved. If light is linearly polarized at angle θ, and goes through a linear polarizer at angle ϕ, its intensity is multiplied by $\cos^2(\theta - \phi)$.

- Telescope. There are two lenses – first, the light passes through a weak "objective" lens with a long focal length, then through a powerful "eyepiece" lens with a short focal lenght. The lenses have the same focal point.

4. Thermodynamics and Statistical Mechanics - 10%

(such as the laws of thermodynamics, thermodynamic processes, equations of state, ideal gases, kinetic theory, ensembles, statistical concepts and calculation of thermodynamic quantities, thermal expansion and heat transfer)

- Carnot cycle. Start in top-left of P-V diagram and go around clockwise. Isothermal expansion at T_H (heat enters), adiabatic expansion, isothermal compression at T_C (heat exits), adiabatic compression. Efficiency is $(T_H - T_C)/T_H$. When volume is expanding, system does work $\int P dV$, so clockwise orientation of loop implies that the system does net positive work.

- Boltzmann distribution.

$$Z = \sum_s e^{-E_s/kT}$$

$$P(s) = e^{-E_s/kT}/Z$$

- Gibbs distribution.

$$\mathcal{Z} = \sum_s e^{(N_s\mu - E_s)/kT}$$

$$P(s) = e^{(N_s\mu - E_s)/kT}/\mathcal{Z}$$

- Relation between entropy and heat. Heat added (reversibly) to the system is $\int TdS$. How to remember: an adiabatic change is when S is constant, and also when no heat is exchanged, so $dQ \propto dS$.

- Statistical definition of entropy – approximate and exact.

$$\text{Approx:} \quad S = k_B \ln \Omega,$$

where Ω is number of possible microscopic configurations consistant with the given macroscopic parameters.

$$\text{Exact:} \quad S = -k_B \sum_s p_s \ln(p_s),$$

where p_s is the probability of state s. Derive former from latter: assume there are Ω equally-probable states, and zero probability for all the others. Then $S = -k_B\Omega(1/\Omega)\ln(1/\Omega) = k_B \ln \Omega$.

- Equipartition theorem. A system has average energy $(1/2)kT$ for each quadratic term in the Hamiltonian (as long as kT is large compared to the gap between successive quantum levels.) In particular, each translational or rotational degree of freedom gives $(1/2)kT$, and each vibrational degree of freedom gives kT (the Hamiltonian for a 1-dimensional simple harmonic oscillator, $H = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\omega^2x^2$, has two quadratic terms.)

- Wien's Law. For a blackbody, λ_{max} is the wavelength radiated with highest intensity. T is the blackbody temperature.

$$\lambda_{max}T \approx 3 \times 10^{-3} m \cdot K$$

(don't need to memorize the RHS constant.)

5. Quantum Mechanics - 12%

(such as fundamental concepts, solutions of the Schrödinger equation (including square wells, harmonic oscillators, and hydrogenic atoms), spin, angular momentum, wave function symmetry, elementary perturbation theory)

- Time-Independent Perturbation Theory: 2nd order for energy, 1st order for wave function.
  Notation: If $H = H_0 + \lambda\Delta H$, then $E_n = E_n^{(0)} + \lambda E_n^{(1)} + \cdots$, and $|n\rangle = |n^{(0)}\rangle + \lambda|n^{(1)}\rangle + \cdots$.

$$E_n^{(1)} = \langle n^{(0)}|\Delta H|n^{(0)}\rangle$$

$$|n^{(1)}\rangle = \sum_{k\neq n} \frac{\langle k^{(0)}|\Delta H|n^{(0)}\rangle}{E_n^{(0)} - E_k^{(0)}}|k^{(0)}\rangle$$

$$E_n^{(2)} = \sum_{k\neq n} \frac{|\langle k^{(0)}|\Delta H|n^{(0)}\rangle|^2}{E_n^{(0)} - E_k^{(0)}}$$

  (In the latter two equations, deal with degeneracies by picking correct basis (so ΔH is diagonal in the degenerate subspace), then summing just over those k nondegenerate with n.)

- Heisenberg Uncertainty Principle.

$$(\sigma_A)(\sigma_B) \geq \left|\frac{1}{2}\langle[A, B]\rangle\right|$$

$$(\sigma_p)(\sigma_x) \geq \frac{\hbar}{2}$$

$$(\sigma_E)(\sigma_t) \geq \frac{\hbar}{2}$$

- de Broglie Wavelength.

$$\lambda = \frac{h}{p} = \frac{2\pi\hbar}{p}$$

- Pauli spin matrices. $\vec{S} = \frac{\hbar}{2}\vec{\sigma}$, where

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

  How to remember: The minus signs in σ_y and σ_z go near the (geographical) center of the matrix.

- Schrodinger Equation.

$$\text{Hamiltonian:} \quad H = -\frac{\hbar^2}{2m}\nabla^2 + V = \frac{p^2}{2m} + V$$

$$\text{Time-dependent S.E.:} \quad i\hbar\frac{\partial\Psi}{\partial t} = H\Psi, \quad \Psi(t) = e^{-iHt/\hbar}\Psi(0)$$

$$\text{Time-independent S.E.:} \quad H\psi_n = E_n\psi_n, \quad \psi_n(t) = e^{-iE_nt/\hbar}\psi_n(0)$$

- Position and momentum.

$$\hat{p} = -i\hbar\nabla$$

$$[\hat{x}, \hat{p}] = i\hbar$$

$$[f(x), \hat{p}] = i\hbar \frac{df}{dx}$$

6. Atomic Physics - 10%

(such as properties of electrons, Bohr model, energy quantization, atomic structure, atomic spectra, selection rules, black-body radiation, x-rays, atoms in electric and magnetic fields)

- Hydrogen electron quantum numbers.
  - $n = 1, 2, 3, \ldots$ – principle quantum number, controls radial wavefunction (along with ℓ) and energy.
  - $\ell = 0, 1, \ldots, n-1$ – orbital quantum number, controls radial wavefunction (along with n) and θ wavefunction (along with m_ℓ). If $\vec{L} = \vec{r} \times \vec{p}$ (a vector of operators), then $L^2\psi = \hbar^2\ell(\ell+1)\psi$.
  - $m_\ell = -\ell, -\ell+1, \ldots, \ell-1, \ell$ – magnetic quantum number, controls θ wavefunction (along with ℓ) and ϕ wavefunction. Defined by $L_z\psi = \hbar m_\ell \psi$.
  - $s = (1/2)$ – electron spin. If $\vec{S} = (S_x, S_y, S_z)$ is the vector of operators defined by $\hbar/2$ times the pauli matrices, then $S^2|\psi\rangle = \hbar^2 s(s+1)|\psi\rangle$.
  - $m_s = \pm(1/2)$ – z-component of electron spin. $S_z|\psi\rangle = \hbar m_s|\psi\rangle$.
  - $j = |\ell - s|, \ldots, (\ell + s) = |\ell \pm (1/2)|$ – total angular momentum quantum number. If $\vec{J} = \vec{L} + \vec{S}$, then $J^2|\psi\rangle = \hbar^2 j(j+1)|\psi\rangle$.
  - $m_j = -j, -j+1, \ldots, j-1, j$ – z-component of total angular momentum. Note that $m_j = m_\ell + m_s$, and $J_z|\psi\rangle = \hbar m_j|\psi\rangle$.
- Electric dipole transition. Selection rules are as follows:

$$\Delta\ell = \pm 1 \quad (\neq 0)$$

$$\Delta m_\ell = 0, \pm 1$$

$$\Delta j = 0, \pm 1$$

$$\Delta m_s = 0$$

- Sources of line-splitting in the spectrum.
  - Zeeman Effect: When you apply a uniform external magnetic field, each transition energy $E_{n_1\ell_1 \to n_2\ell_2}$ is split into three equally-spaced lines, due to whether m_ℓ increases by one, decreases by one, or stays the same in the transition.

  - Anomalous Zeeman Effect: In Zeeman effect, the contribution of electron spin to total angular momentum means that it isn't always three lines and they are not always equally spaced.
  - Stark Effect: When you apply a uniform electric field, it induces a dipole moment and interacts with it, thereby splitting and shifting lines.

- Bohr model. Assume classical orbiting with angular momentum a multiple of $\hbar$. Z is an integer representing nuclear charge. m_{red} is (reduced) electron mass $= m_1 m_2/(m_1 + m_2)$. n is orbit.

$$\text{Radius:} \quad a_n = \frac{4\pi\varepsilon_0\hbar^2}{m_{\text{red}}e^2 Z}n^2 \approx (0.529\,\text{Å})\left(\frac{m_{\text{elec}}}{m_{\text{red}}} \cdot \frac{n^2}{Z}\right)$$

$$\text{Energy:} \quad E_n = \frac{Z^2 m_{\text{red}} e^4}{8\varepsilon_0^2 h^2}\left(\frac{-1}{n^2}\right) \approx (-13.6\text{ eV})\left(\frac{Z^2}{n^2} \cdot \frac{m_{\text{red}}}{m_{\text{elec}}}\right)$$

  How to remember: decreasing the effective mass m_{red} increases a and decreases $|E_n|$. If you double Z, you half a, which together quadruples the coulomb potential energy between the nucleus and electron.

- X-ray spectrum from electrons fired at atoms. "Auger transition" is when incoming particle knocks out inner-shell electron, and vacancy gets filled by outer-shell electron, creating a spike in the spectrum. "Bremsstrahlung" is the continuous spectrum of light released from the deceleration of electrons. Put them together to get the full spectrum (a continuous spectrum with a few spikes on top).

- Term symbol – General definition. For an atom with a particular configuration of electrons, the electrons combine to have some specific total spin S, total orbital angular momentum L, and total momentum J. The electrons within filled subshells have no contribution. Pick the letter associated with L (S,P,D,F,G,H,... for $L = 0, 1, 2, \ldots$), then write

$$^{2S+1}L_J$$

  Obviously, J is between $L + S$ and $|L - S|$.
  Shell memorization mnemonic: Some Physicists are Destined to Flunk the GRE.

- Term symbol for the ground state. Use the following prescription. In the partially-filled subshell, start by putting electrons with $m_s = 1/2$, starting with the maximum possible m_ℓ and proceeding downwards. Next, add electrons with $m_s = -1/2$, in the same order. Then it turns out that $S = \sum m_s$, $L = \sum m_\ell$, and J is $|L - S|$ if at most half the subshell is filled, or $L + S$ if at least half the subshell is filled.

- Intrinsic magnetic moment, gyromagnetic ratio. For an electron orbit: $\vec{\mu} = (-e/2m_e)\vec{L}$ where $\vec{L}$ is the angular momentum. Derivation: $\mu = IA = [(-ev)/(2\pi r)][\pi r^2] = (-e/(2m_e))(m_e vr) = (-e/2m_e)L$. But for the intrinsic angular momentum of a particle, quantum electrodynamics gives a correction by a factor of $g \approx 2$ ("Landé g-factor"), i.e. $\vec{\mu} = (-eg/2m_e)\vec{S}$. The "gyromagnetic ratio" is $\gamma = \mu/S = (-eg/2m_e)$.

7. Special Relativity - 6%

(such as introductory concepts, time dilation, length contraction, simultaneity, energy and momentum, four-vectors and Lorentz transformation, velocity addition)

- Lorentz factor.

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \geq 1$$

$$\gamma \approx 1 + \frac{1}{2}\frac{v^2}{c^2}, \qquad \frac{1}{\gamma} \approx 1 - \frac{1}{2}\frac{v^2}{c^2}$$

- Lorentz transformations. Let $\beta = \frac{v}{c}$.

$$ct' = \gamma\,(ct - \beta x)$$

$$x' = \gamma\,(x - \beta(ct))$$

$$y' = y$$

$$z' = z$$

- Relativistic addition of velocities. According to a guy moving at speed v in the x-direction, a ball is thrown with velocity $\vec{u}$. Rest frame velocity of the ball is $\vec{u}'$, where:

$$u'_x = \frac{u_x + v}{1 + u_x v/c^2}$$

$$u'_y = \frac{u_y}{\gamma(1 + u_x v/c^2)}$$

$$u'_z = \frac{u_z}{\gamma(1 + u_x v/c^2)}$$

  Derivation: plug in $x = u_x t$ and $y = u_y t$, Lorentz transform, and compute x'/t' or y'/t'.

- Length contraction.

$$L' = \frac{L_{\text{proper}}}{\gamma}$$

- Time dilation.

$$T' = (T_{\text{proper}})\gamma$$

- Doppler shift (for light).

$$\omega = \omega_0\sqrt{\frac{1 - v/c}{1 + v/c}}$$

  (choose sign of v so that shift is in correct direction.)

- Relativistic momentum, energy.

$$p = mv\gamma$$
$$E = \sqrt{(mc^2)^2 + (pc)^2} = mc^2\gamma$$

- Schwartzchild radius. You get a black hole if:

$$R = \frac{2GM}{c^2}$$

Rough derivation: the total energy of a particle of mass m is $mc^2 - GMm/R$, and the particle cannot escape if this is less than zero. This gives $R = GM/c^2$, off only by a factor of two.

8. Laboratory Methods - 6%

(such as data and error analysis, electronics, instrumentation, radiation detection, counting statistics, interaction of charged particles with matter, lasers and optical interferometers, dimensional analysis, fundamental applications of probability and statistics)

- Accuracy versus Precision. Accuracy: Close to reality. Precision: Repeatable. How to remember: "Precise can be repeated thrice, accurate is on track."

- Error Propagation. Given a function $f(x_1, \ldots, x_n)$ where the uncertainty (standard deviation) of x_i is Δx_i, and the errors are uncorrelated,

$$\Delta f = \left(\sum_{i=1}^{n} [(\Delta x_i)(\partial_i f(x_1, \ldots, x_n))]^2 \right)^{1/2}$$

Derivation: since Δx_i is small, $\Delta x_i \partial_i f$ is the standard deviation in f from the mis-measurement of x_i. Standard deviation squared equals variance, and the variance of a sum is the sum of the variances.

- Joules to eV conversion.

$$1 \text{ J} \approx 6 \times 10^{18} \text{ eV}$$

- Room temperature in eV.

$$(k_B)(300 \text{ K}) \approx .02 \text{ eV}$$

- Visible light in m, Hz, and eV.

$$\lambda = 700 - 400 \text{ nm}, \quad \omega = 2500 - 4500 \text{ THz} = 2.5 - 4.5 \times 10^{15} \text{ Hz}, \quad E = 2 - 3 \text{ eV}$$

9. Specialized Topics - 9%

Nuclear and Particle physics (e.g., nuclear properties, radioactive decay, fission and fusion, reactions, fundamental properties of elementary particles), Condensed Matter (e.g., crystal structure, x-ray diffraction, thermal properties, electron theory of metals, semiconductors, superconductors), Miscellaneous (e.g., astrophysics, mathematical methods, computer applications)

- Conservation of baryon/lepton number.
  - Baryon number. Each quark has baryon number 1/3.
  - Lepton number. Three types: electron number, muon number, tau number. Each separately is conserved.
- Binding energy. A positive quantity, such that

$$\text{(binding energy)}/c^2 = \text{(total mass of constituent nucleons)} - \text{(mass of nucleus)}$$