Problem 2 Determining the determinant

Suppose that A, B, C are square matrices. Express

$$det\begin{pmatrix} 0 & C \\ A & B \end{pmatrix}$$

in terms of $det(A), det(B), det(C)$. Prove your formula is correct.

Let $A, B, C \in M_{n\times n}(F)$. Do n row operations swapping rows 1 to n with rows $n+1$ to $2n$, respectively. The resulting matrix $\begin{pmatrix} A & B \\ 0 & C \end{pmatrix}$ has determinant $(-1)^n\left(det\begin{pmatrix} 0 & C \\ A & B \end{pmatrix}\right)$ by row op calculations.

But it has $det\begin{pmatrix} A & B \\ 0 & C \end{pmatrix} = det(A)\cdot det(C)$

↖ we proved this in class, during midterm review.

Now putting everything together,

$$det\begin{pmatrix} 0 & C \\ A & B \end{pmatrix} = (-1)^n det(A)\cdot det(C).$$