105A - Set 1

(Grades are out of 150)

1. (30pt) A particle moves in the x-y plan (center $\mathcal{O}$) with constant angular velocity ω counter- clockwise. The particle's position is given by

$$\mathbf{r}(t) = 2b\cos(\omega t)\hat{\mathbf{x}} + b\sin(\omega t)\hat{\mathbf{y}} \ , \tag{1}$$

   where the convention is that bold face represents a vector, i.e., instead of writing $\vec{r}$ we write $\mathbf{r}$.

   (a) (3pt) Find the particle's velocity.

   (b) (6pt) Find the particle's velocity magnitude (speed).

   (c) (6pt) Find the particle's acceleration. Express your answer in terms of ω and $\mathbf{r}$. What are the magnitude and direction of the acceleration ($\mathbf{a}$)?

   (d) (9pt) What is the angle between $\mathbf{v}$ and $\mathbf{a}$ at time $t = \pi/(2\omega)$?

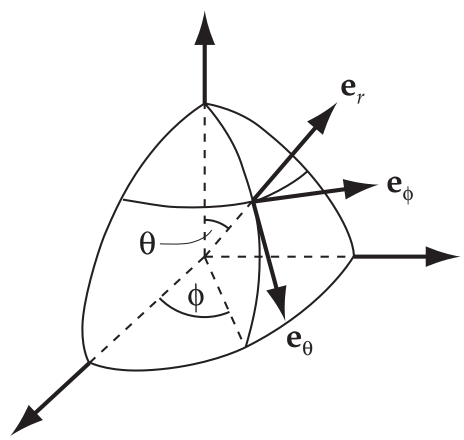

Figure 1: Spherical coordinates

2. (30pt) Find the components of the acceleration vector $\mathbf{a} = \ddot{\mathbf{r}}$ in spherical coordinates (see Figure 1). Be as detailed as possible.

3. (30pt) **Work and kinetic energy**
   Consider the definition of Work:

$$W_{12} = \int_1^2 \mathbf{F} \cdot d\mathbf{s} \tag{2}$$

   Show that for a constant mass the

$$W_{12} = \frac{m}{2}(v_2^2 - v_1^2) = T_2 - T_1 \ , \tag{3}$$

   where T_1 (T_2) is the kinetic energy of the system at state "1" ("2").

4. (30pt) A particle with mass m moves in a medium under the influence of a retarding force equal to $mk(v^3 + a^2 v)$, where k and a are constant. Show that for any value of the initial speed the particle will never move a grater distance than $\pi/(2ka)$ and that the particle comes to rest only at $t \to \infty$.

5. (30pt) A particle of mass m is sliding down an inclined plane under the influence of gravity. If the motion is resisted by a force $f = kmv^2$, show that the time required to move a distance d after starting from rest is

$$t = \frac{\cosh^{-1}(e^{kd})}{\sqrt{kg \sin\theta}} \ , \tag{4}$$

where θ is the angle of the inclination of the plane.