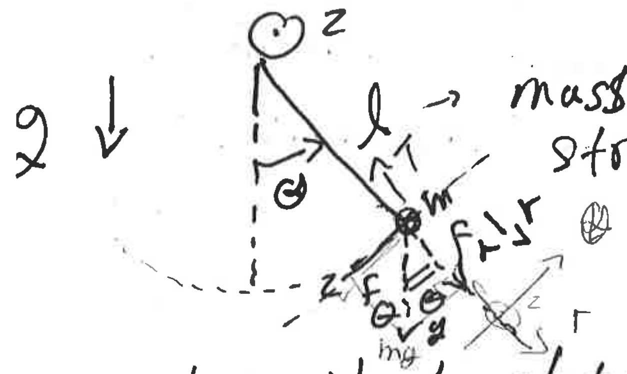
3) The simple pendulum

show example.



my coordinate choose is then the r, θ, z z along out of the page.

lets think about the forces

$\hat{r}$ $f_r = mg\cos\theta$

$\hat{\theta}$ $f_\theta = -mg\sin\theta$

why minus? because θ is positive to the count side.

$\hat{z}$ $f_z = 0$

f_r is balanced by tension. T on the string

what is the torque

$$\bar{\tau} = \bar{r} \times \bar{F}$$

$$\bar{\tau} = \begin{pmatrix} \hat{r}, & \hat{\theta}, & \hat{z} \\ l, & 0, & 0 \end{pmatrix} \times (mg\cos\theta, -mg\sin\theta, 0) =$$

$$= l\hat{r} \times (mg\cos\theta\,\hat{r} - mg\sin\theta\,\hat{\theta})$$

$$\begin{vmatrix} \hat{r} & \theta & \hat{z} \\ r & 0 & 0 \\ mg\cos\theta & -mg\sin\theta & 0 \end{vmatrix} = \hat{r}(0) - \hat{\theta}(0) - rmg\sin\theta\,\hat{z}$$

$(\hat{r} \times \hat{\theta}) = \hat{z}$

no torque on the r direction.

So $\boxed{\bar{\tau} = -mgl\sin\theta\,\hat{z}}$

Now we also know that torque can be written as the moment of inertia times the angular acceleration

$$\bar{\tau} = I\bar{\alpha}$$

moment of inertia (I) — angular acceleration ($\bar{\alpha}$)

$$I\bar{\alpha} = I\frac{d^2\theta}{dt^2}\hat{z}$$

this real

$$\bar{\tau} = \frac{d\bar{L}}{dt} = \frac{d(I\bar{\omega})}{dt} = I\frac{d\bar{\omega}}{dt} = I\bar{\alpha}$$

L is along the z axis in this case.

So we get

$$I = \int r^2\,dm = l^2 m$$

$$I\frac{d^2\theta}{dt^2} = -mgl\sin\theta$$

$$\ddot{\theta} = \frac{d^2\theta}{dt^2} = -\frac{mgl}{I}\sin\theta$$

phase diagram.

its more complicated than simple Harmonic pendulum. So simple pendulum is different.

) if $\theta \ll 1$ than we get that its more like SHM.

$\theta \ll 1 \quad \sin\theta \approx \theta - \frac{1}{3!}\theta^3 + \dots$

$$\ddot{\theta} \approx -\frac{mgl}{I}\theta$$

$$\theta = A\cos(\omega t + \phi)$$ A is the max angle

$$\omega = \sqrt{\frac{mgl}{I}}$$

for our case wher we have a massless string. $I = \int r^2 dm$ so

for us $I = l^2 m.$

$$\omega = \sqrt{\frac{mgl}{l^2 m}} = \sqrt{\frac{g}{l}}.$$ does not depend of the mass

in diffcent plant. it ~~will~~ the freqwenq will be largar

$f = \frac{\omega}{2\pi} \quad T = \frac{1}{f} = 2\pi\sqrt{\frac{l}{g}}$