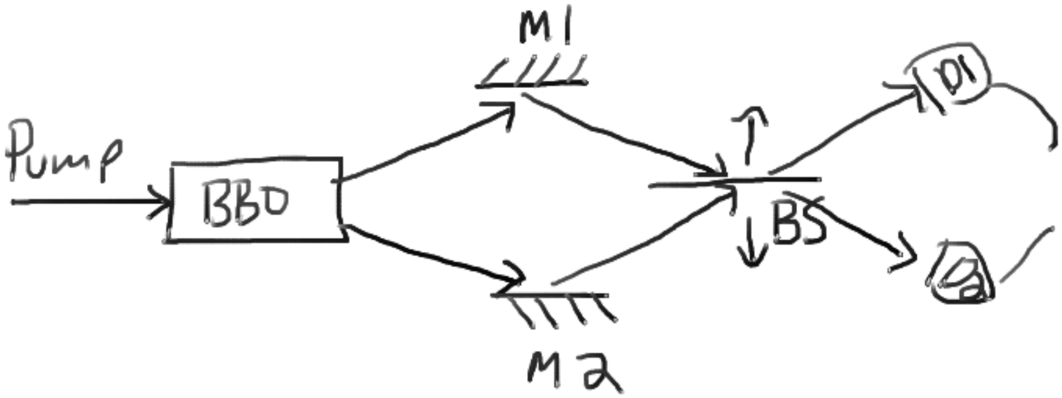
Two-photon interference experiment
ERH

Last week we completed our discussion of entanglement by deriving both the Bell and CHSH inequalities. We found that the correlations present in an entangled state of quantum mechanics are incompatible with any local hidden variable theory – i.e. both can't be right. This week in lab you will do an experiment to see if Nature is compatible with quantum mechanics or with local hidden variable theories.

Today, we leave these topics behind and move on to the last topic of the class: two-photon interference (sometimes referred to as the Hong-Ou-Mandel effect). As you will see, this phenomenon cannot be described by classical electromagnetism. Like the Bell experiment, it is a uniquely quantum effect and it demonstrates, quite clearly, that the concepts of quantum mechanical interference can be farther reaching than our classical intuition would allow.

Two-photon interference is typically studied with an experimental set-up as follows. A pump laser undergoes SPDC in a non-linear crystal. The correlated photons are then directed onto a 50/50 (non-polarizing) beam splitter (BS) and the outputs of the beam splitter ports are directed into two detectors (D1 and D2). The beam splitter, or one of the mirrors, is mounted on a translation stage so that the path length in the two arms can be carefully adjusted:



For this experiment, it is important that the two photons have the same polarization so that they interfere on the beam splitter (BS). Therefore, these experiments either use Type I phase matching or incorporate a half-wave plate in one arm. In this way, you prepare one of the Φ Bell states: $|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|V\rangle_1|V\rangle_2 \pm |H\rangle_1|H\rangle_2)$.

Now, clearly, there are four possible outcomes:

1. Both photons reflect from the beam-splitter
2. Photon 1 transmits and Photon 2 reflects
3. Photon 1 reflects and Photon 2 transmits
4. Both photons transmit through the beam splitter

And these outcomes lead to the following detector signals:

1. Detector 1 and Detector 2 click – i.e. a coincidence count occurs
2. Detector 2 clicks twice – i.e. a coincidence count does NOT occur

3. Detector 1 clicks twice – i.e. a coincidence count does NOT occur
4. Detector 1 and Detector 2 click – i.e. a coincidence count occurs

What happens?

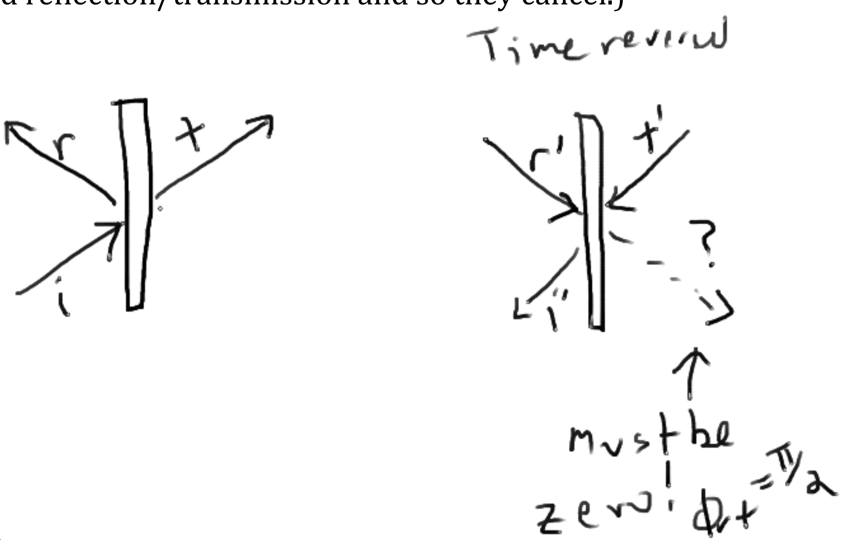
Naively, given a 50/50 beam splitter and ignoring interference effects, we would expect that these all occur with equal probability.

However, even in classical electromagnetism, we know that interference exists, so we can do a bit better than this. Let's see what classical electromagnetism says should happen. Suppose that the electric field of the down converted photons in the crystal is identical and given as: E_o. Then the electric field at detector 1 is:

$$E_1 = E_o e^{ikL_1}\sqrt{R}e^{ikL_D} + \ E_o e^{ikL_2}\sqrt{T}e^{ikL_D}$$

where L_i and L_D are the path lengths from the crystal to BS and from BS to D1, respectively. For a 50/50 beam splitter the power reflection and transmission coefficients are $R = T = \frac{1}{2}$, however, we are dealing with the electric field so must take the square root of ½. Further, we must also include the $\frac{\pi}{2}$ phase shift of the reflected wave relative to the transmitted wave, thus: $\sqrt{R} = \frac{i}{\sqrt{2}}$ and $\sqrt{T} = \frac{1}{\sqrt{2}}$.

(NOTE: You are probably used to saying that a reflected wave gets a 0 or π phase shift *relative* to the incoming beam. Here we are talking about the phase shift between the transmitted and reflected beams. You can see that the phase between these two beams must by $\frac{\pi}{2}$ by time reversal symmetry. If you run the problem backwards, you would expect there to be a reflected wave coming from the back interface when the transmitted light runs "backwards" into it. However, that ray clearly does not exist in the un-time-reversed situation. It is straightforward to show that in the time reversed problem you can make this ray disappear by having the transmitted portion of the "reversed" reflected ray destructively interfere with this ray – thus the $\frac{\pi}{2}$ phase shift is required by time reversal symmetry. Basically, if t' is $\frac{\pi}{2}$ relative to r' to start with then, then it picks up another $\frac{\pi}{2}$ during the time reversed reflection/transmission and so they cancel.)



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And we have the electric field at the first detector as:

$$E_1 = \frac{E_o}{\sqrt{2}} e^{ikL_D}(ie^{ikL_1} + e^{ikL_2})$$

Likewise, the electric field at the second detector is:

$$E_2 = \frac{E_o}{\sqrt{2}} e^{ikL_D}(ie^{ikL_2} + e^{ikL_1})$$

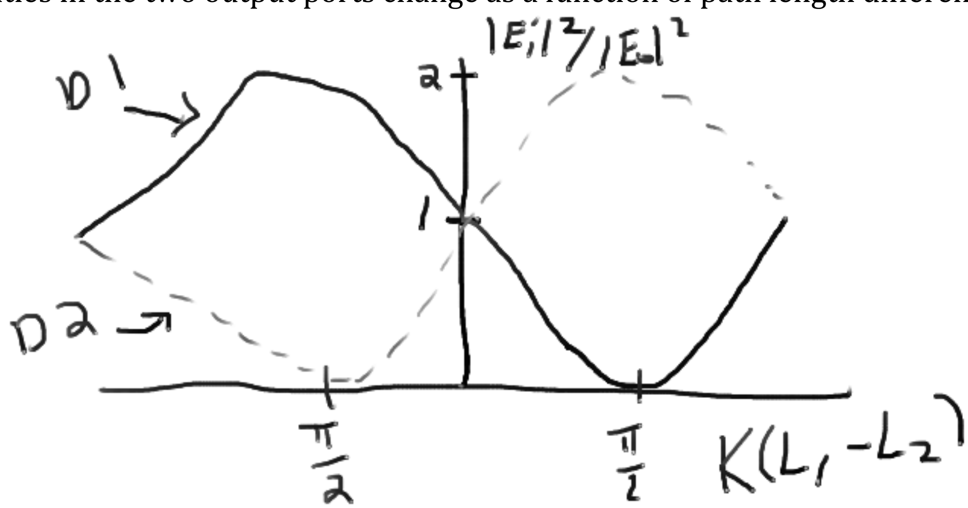
Now the photon count rate is proportional to the intensity of the field, which we calculate to be:

$$|E_1|^2 = \frac{|E_o|^2}{2}(2 + i(e^{ik(L_1-L_2)} - e^{-ik(L_1-L_2)}) = |E_o|^2(1 - \sin k(L_1 - L_2))$$

and

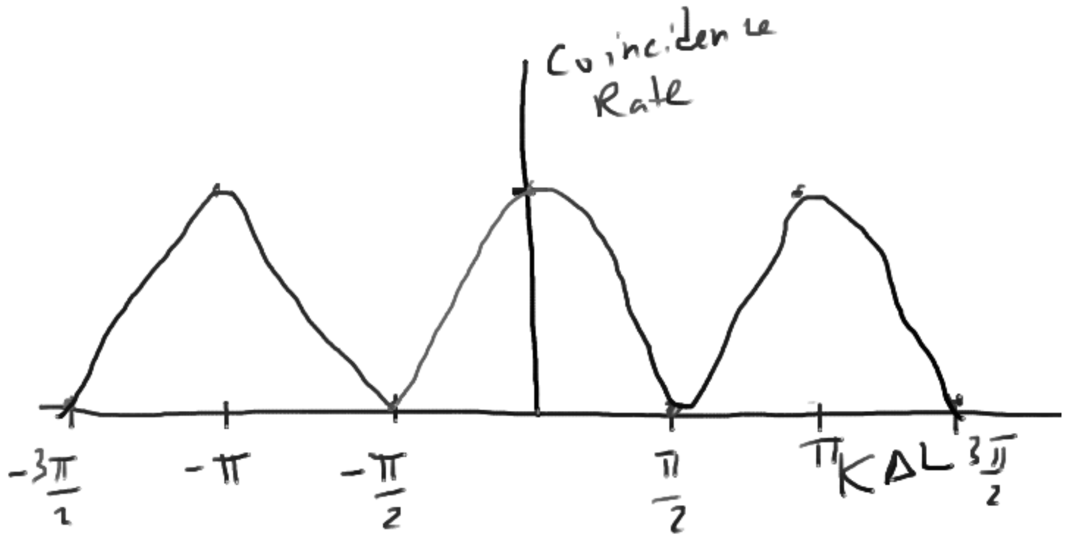
$$|E_2|^2 = |E_o|^2(1 + \sin k(L_1 - L_2))$$

Thus, we see that we have built an interferometer and we expect that the intensities in the two output ports change as a function of path length difference as:



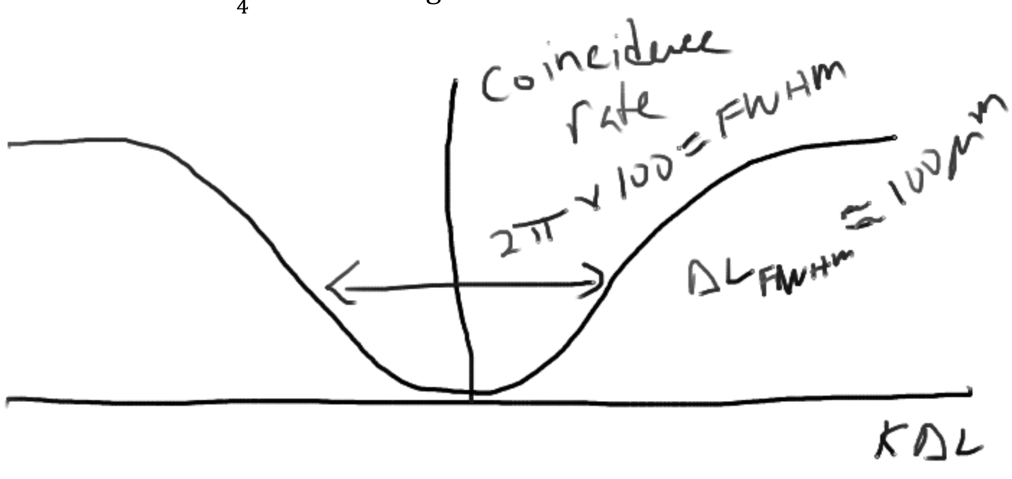
Thus, we appear to have done a bit better than our equal probability for each of the four outcomes. We see that electromagnetism would predict that for equal path lengths we get equal amplitude in both arms – i.e. process 2 and 3 are happening and we should therefore see a maximum in coincidence counts. Further, if we increase or decrease the path length difference we see that the photons choose to go preferentially into one arm over the other, so we should see a decrease in the path length. We also see that as the path length $\Delta L = L_1 - L_2$ moves by only a fraction of a wavelength -- $\frac{\pi}{2} = k\Delta L \rightarrow \Delta L = \frac{\lambda}{4}$ -- the photons change from equal rates in both arms (maximum in coincidence counts) to a going completely into one arm or the other (minimum in coincidence counts). And, finally, we see that if we keep increasing the path length difference the output oscillates back and forth between these situations. This can be summarized in a graph of the coincidence counts observed by our detectors:

$$|E_1|^2|E_2|^2 = |E_o|^4(1 - \sin k(L_1 - L_2))\,(1 + \sin k(L_1 - L_2)) = |E_o|^4(\cos k(L_1 - L_2))^2$$



Q: This is all seems very reasonable, but does it agree with the experiment data?
A: Not even remotely…

Experiment shows a MINIMUM in coincidence rate at zero path length. Further, this dip only appears once – there are no oscillations! And, perhaps hardest of all to reconcile with classical electromagnetism, the dip typically has a width much, much wider than $\Delta L = \frac{\lambda}{4}$ -- often as high as $\Delta L = 100\ \lambda$!



How can this be? How can our classical electromagnetism be so bad here?

Well, we've really pushed the theory beyond what it was intended for. Clearly, it's a bit odd to talk about a single photon as a plane wave of constant phase! In fact, for a single photon you can't really even define a phase of the electromagnetic wave.

To derive this from first principles requires quantized electromagnetic fields, which we will brush on later, but even without the heavy math we can see the essence of what is going on here. Because the photons are indistinguishable, Process 1 (both reflect) and Process 4 (both transmit) are fundamentally indistinguishable from one another. Therefore, following the Feynman rules you must add the probability **amplitudes** for these processes before we take the absolute value to find the probability. Now, the probabilities for the four processes are just (assuming equal path lengths):

1. Both photons reflect from the beam-splitter → $A_{rr} = E_o\sqrt{R}\sqrt{R} = -\frac{E_o}{2}$
2. Photon 1 transmits and Photon 2 reflects → $A_{tr} = E_o\sqrt{T}\sqrt{R} = \frac{iE_o}{2}$
3. Photon 1 reflects and Photon 2 transmits → $A_{rt} = E_o\sqrt{R}\sqrt{T} = \frac{iE_o}{2}$
4. Both photons transmit through the beam splitter→ $A_{tt} = E_o\sqrt{T}\sqrt{T} = \frac{E_o}{2}$

Thus, the probability for a coincidence count on the two detectors is:

$$P_c = |A_{rr} + A_{tt}|^2 = 0!$$

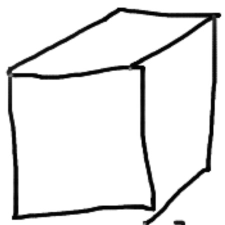
As a result, we never see a coincidence counts! The two photons always "choose" the same port! (Note, people often say here that this is because photons are bosons and they "like" to be together. While it is true this result would not hold if photons were fermions, I'm not sure this is a good statement to make. This is just what happens for bosons.)

The essential difference between quantum mechanics and classical electromagnetism here is that in classical electromagnetism we treat the two detector outcomes as separate events. That is, we allow for interference at each detector, but not between the two detectors. We are implicitly assigning more reality to the situation than quantum mechanics allows. Quantum mechanics says the different paths are indistinguishable and therefore they interfere!

A (bit) more proper derivation of all of this:

Quantizing the electromagnetic field is a tad beyond the scope of our class, but the results are actually very intuitive. Consider a rectangular volume V, the total energy (Hamiltonian) of the electromagnetic field in the box is given as:

$$H = \int_V d\vec{r}\, u = \int_V d\vec{r}\, \frac{\epsilon_0}{2}\left(|\vec{E}|^2 + c^2|\vec{B}|^2\right)$$



$V = L^3$

Now, let's look at this Hamiltonian for a minute. It depends on the square of two canonically conjugate variables. Remind you of anything?

That's right it's just a harmonic oscillator! It turns out the electromagnetic field in the box can be described as an infinite collection of **uncoupled** harmonic oscillators. These harmonic oscillators have energy: $E = \left(n_k + \frac{1}{2}\right)\hbar\omega_k$, where n_k is the principle quantum number of the kth mode and is the number of photons in that mode. The frequency of each mode is simply given by $\omega_k = ck$, where k can be found from the boundary conditions as $j\left(\frac{\lambda}{2}\right) = L \rightarrow k = \frac{j\pi}{L}$ and $j = [1, \infty)$. That is, the electromagnetic field of the box is described by a set of $n's$ which tell you the number of photons in each mode of the box. And the modes of the box are given in exactly the same way as the particle in a box. (NOTE: Technically, this is a three dimensional problem and therefore n and k are 3-vectors.)

As a result the wavefunction of the electromagnetic field can be written as: $|\psi\rangle = |n_1, n_2, n_3, \dots\rangle$. Thus, a state of the electromagnetic field with two photons in the j = 1 mode, a single photon in the j = 3 mode, and all other modes empty is written as $|\psi\rangle = |2,0,1,0,\dots\rangle$. Likewise, the vacuum state, where all modes are empty is written as: $|\psi\rangle = |0\rangle$. (By the way, what is the energy of the vacuum state? What does that mean? Are there any experimental consequences of this? Casimir effect.)

Thus, we can represent the state of the field **at the beam splitter** in our two-photon interferometer as:
$|\psi\rangle = e^{ikL_1}e^{ikL_2}a_1^\star a_2^\star|0\rangle$
(here the e^ikL's only operator on the respective photon mode)
where $a_i^\star$ is called the photon creation operator. It is nothing more than the raising ladder operator you already learned about when you did the harmonic oscillator in quantum mechanics. (NOTE: Here the label's 1 and 2 refer to the modes propagating in the arm, and not the box modes. It turns out you can take the limit where the box goes to infinity and you still get the harmonic oscillator solution for the quantized EM field.) Now, the effect of the beam splitter is that it either transmits the photon

leaving it in its original arm or it reflects the photon into the other arm. Thus, the effect of the beam splitter in the four distinct processes can be written as:

1. Both photons reflect from the beam-splitter → $\sqrt{R}\sqrt{R}$
2. Photon 1 transmits and Photon 2 reflects → $\frac{\sqrt{T}\sqrt{R}}{\sqrt{n_2}}\ a_2^{\star}a_1$
3. Photon 1 reflects and Photon 2 transmits → $\frac{\sqrt{R}\sqrt{T}}{\sqrt{n_1}}\ a_1^{\star}a_2$
4. Both photons transmit through the beam splitter → $\sqrt{T}\sqrt{T}(a_2^{\star}a_1)(a_1^{\star}a_2)$

For 1, the photons stay in their original arms, but their probability amplitudes are modified by the reflection coefficients. For arms 2 and 3, the reflected photon is "destroyed" from the original arm and "created" in the other arm. For 4, both photons are destroyed in their original arm and created in the other arm.

Thus, the wavefunction after the beam splitter is given as:

$$|\psi^f\rangle = \left(\sqrt{R}\sqrt{R} + \frac{\sqrt{T}\sqrt{R}}{\sqrt{n_2}}\ a_2^{\star}a_1 + \frac{\sqrt{R}\sqrt{T}}{\sqrt{n_1}}\ a_1^{\star}a_2 + \sqrt{T}\sqrt{T}(a_2^{\star}a_1a_1^{\star}a_2)\right)|\psi^i\rangle$$

$$|\psi^f\rangle = \sqrt{R}\sqrt{T}e^{ikL_1}e^{ikL_2}\left(a_1^{\star 2} + a_2^{\star 2}\right)|0\rangle + (\sqrt{R}\sqrt{R} + \sqrt{T}\sqrt{T})e^{ikL_1}e^{ikL_2}a_1^{\star}a_2^{\star}|0\rangle$$

And for a 50/50 BS, we have:

$$|\psi^f\rangle = \sqrt{R}\sqrt{T}e^{ikL_1}e^{ikL_2}\left(a_1^{\star 2} + a_2^{\star 2}\right)|0\rangle$$

and we see that the final state is two photons paired in either arm 1 or arm 2, but never one photon in each arm! Therefore, we never have a coincidence count. The process of both photons reflecting and both photons transmitting destructively interfere! This is clearly not encapsulated in classical electromagnetism in any way (since it treats the detectors independently) and can be only explained with quantum mechanics!

How big is a photon and what sets the size of the dip?

The width of the "coincidence dip" you'll measure is set by the size of the photon wavepacket. By the uncertainty relation, a photon of perfectly defined energy (i.e. a momentum eigenstate) has infinite extent, but the wavepacket can be smaller if the energy less well defined (as $\Delta x \Delta k \sim 1$). (Just bread-and-butter fourier wavepacket stuff.)Thus, the narrower band laser we use the larger the dip width.

In our experiment, we use a filter to only allow a smaller wavelength range to participate, which increases the wavepacket size – sometimes called the coherence length. If you remove the filter the dip gets narrower! Try it.

Final Paper Requirements

1. Pick either the Bell experiment or the two-photon interference experiment as your topic.
2. Write a normal journal style paper:
    a. Title, Author, Abstract
    b. Introduction. Explain why this is relevant and its importance. Give a brief outline of the structure of your paper and result. Also explain what you will be testing theoretically (e.g. derive the CHSH inequality)
    c. Experimental section: Explain the experimental apparatus
    d. Results section: Explain data taking protocol and show experimental data
    e. Conclusion: Based on data and your error estimate, state the outcome of your experiment (e.g. violated Bell's inequality by 3 standard deviations, etc.) Interpret
    f. Summary
3. Due Friday of finals week (NO LATE ACCEPTANCES).