Math 115 A Lec 1 HW 1

Reader's Soln Wenjun Cai

(f)

#3) ① Find the correct w, v, z:

pt 1 eg. $\vec{w} = \langle 1,1,1 \rangle$, $\vec{v} = \langle 1,2,3 \rangle$, $\vec{z} = \langle 2,2,2 \rangle$.

generally. $\vec{w}, \vec{z}$ dependent, $\vec{w}, \vec{v}$ indep.

pt 1 ② pf of 1°: if w, v are dep.

then $\underbrace{\text{span}(\{w,v\}) = \text{span}(\{v,z\})}_{[1]} \Rightarrow v, z$ dep.

$\Rightarrow \text{span}(\{w,v,z\}) = \text{span}(\{w,z\})$ —×— with [2].

pt 1 ③ pf of 2°: if v, z are dep

then similarily [1] $\Rightarrow w, v$ dep

$\Rightarrow$ ... —×— with [2].

pt 2. ④ pf of 3°: If w, z are indep.

w, v indep by 1° $\Rightarrow \dim(\text{span}(\{w,v\})) = 2 = \dim(\text{span}\{w,v,z\})$.

∵ w, z indep ∴ v is dep with either w or z

But by 1° and 2°. v is indep with both —×—

∴ w, z are dependent.

(f)

pt 1 #4) $S = \{1, x, x^2, x^3\}$.

pt 2. ① S spans $P_3(\mathbb{R})$: $\forall\, P_3(\mathbb{R}) = a_3x^3 + a_2x^2 + a_1x + a_0$, it can be written as the linear combination of S elements in

pt 2. ② elements in S are lin. indep:

$b_0 + b_1x + b_2x^2 + b_3x^3 = 0 \iff b_0 = b_1 = b_2 = b_3 = 0$ ✓

∴ S is a basis for $P_3(\mathbb{R})$

☆ $\boxed{\text{"0" Case}}$

#3) If $w = z = 0$. $v \neq 0 \Rightarrow$ still true but w, v dep. z, v dep

$\Rightarrow$ Statement 1, 2 are false

My Criteria: ① if student doesn't consider "0" case but prove independence, ✓

② if student give contra-example ("0" case), ✓.

③ if discuss both, ✓.