Math 115A: Review problems for midterm 1

Sections 1 and 3. Instructor: James Freitag

Due 10/16

Problem 1 Nullity and rank

Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation given by $T(x_1, x_2, x_3, x_4) = (0, x_1, x_2, x_3)$. What is the rank of T? What is the nullity of T?

Problem 2 Basis

Let T be as in the previous problem. Let

$$\beta = ((1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1))$$

be the standard ordered basis of $\mathbb{R}^4$. Compute $[T^i]_\beta^\beta$ for $i = 1, \ldots, 4$, where T^i is the composition of T with itself i-many times.

Problem 3 Compute some linear transformations

Let $\beta = (v_1, v_2, v_3)$ be the standard ordered basis of $\mathbb{R}^3$. Let γ be the ordered basis $((1, 1, 0), (1, 0, 0), (0, 0, 1))$. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation whose matric representation $[T]_\beta^\gamma$ is given by

$$[T]_\beta^\gamma = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Compute $T(4v_1 + 5v_2 + v_3)$.

Problem 4 Give a transformation

Give an example of a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ whose nullspace is the $\{(x, 0, 0) \mid x \in \mathbb{R}\}$ and whose range is $\{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\}$.

Problem 5   A subspace...?

Let $T : V \to W$ be a linear transformation. Fix some arbitrary $w \in W$. Then is $\{v \in V \,|\, T(v) = w\}$ a subspaces of V? Prove this or give a counterexample. If you give a counterexample, what is an example of something you might additionally assume which would change your answer?

Problem 6   Spanning sets

Suppose that A spans V and that A is in the span of B. Prove that B spans V.

Problem 7   Inverse images

Let $T : V \to W$ be linear and suppose $U \leq W$. Show that $T^{-1}(U) = \{v \in V \,|\, T(v) \in U\}$ is a subspace of V. Explain why this shows the nullspace is a subspace.

Problem 8   Exercises from the book

Do the following exercises from book:

- Exercise 3 from section 1.2.

- Exercise 23 from 1.3.

- Exercise 10 from 2.1.