Name: ______________________

Student ID number: ______________________

105A - Midterm 2

Please read the following very carefully

- This is a closed book exam. You may **not** use a calculator. All other electronic devices should **not** be around!
- You have 50 minutes to complete the exam.
- Grades are out of 150
- Answer all **three** questions, i.e., (1), (2), and (3).
- Make sure to write your name at the top of each page of this exam. Use the space provided on the exam pages to do your work. You may use the back of the pages also, but please mark clearly which problem you are working on (and also state underneath that problem that you have done work on the back of the page).
- Partial credit will be given. Show as much work/justification as possible (diagrams where appropriate). If you can not figure out how to complete a particular computation, a written statement of the concepts involved and qualitative comments on what you think the answer should be may be assigned partial credit.
- Mistakes in grading: If you find a mistake in the grading of your exam, alert the instructor within one week of the exams being returned DO NOT write on the returned graded exam you may make a note of the problems you thought were misgraded on a separate page.

(Grades are out of 150)

A damped linear oscillator, with a restoring acceleration of $-\omega_0^2 x$, and friction acceleration of $-2\beta v$, where v is the velocity and $\beta > 0$ is set originally in its equilibrium position and given a velocity $v_0 = \frac{a}{\tau\omega_0^2}$. The oscillator is subjected to a forcing function given by:

$$\frac{f(t)}{m} = a\frac{t}{\tau} \ . \tag{1}$$

The equation of motion of this oscillator is:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = a\frac{t}{\tau} \ . \tag{2}$$

1. (40pt) Find the particular solution in terms of a, β, ω_0 and τ.

2. (70pt) Find the homogenous solution for $x_0(t)$ (i.e., solve: $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0$). And from that find the general solution $x(t) = x_p(t) + x_0(t)$. Find all the parameters in terms of β, ω_0, a and τ.

you may continue your answer here.

3. (40pt) Studying for the midterm two students solved a similar problem. Student A said that if $\beta = \omega_0$ then the system will be in resonance However, student B disagreed with student A. Who is right? Student A or student B? If student A is correct, show how the system can enter a resonance, if student B is correct, explain what will happen when $\beta = \omega_0$. Explain your answer.