105A - Set 5

(Grades are out of 150)

1. (36pt) Two blocks connected by a spring of spring constant k are free to slide frictionlessly along a horizontal surface, as shown in Fig. 1. The unstretched length of the spring is a.

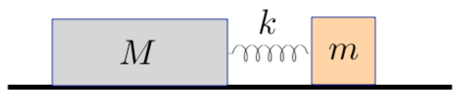

Figure 1: Two masses connected by a spring sliding horizontally along a frictionless surface.

   (a) (12pt) Identify a set of generalized coordinates and write the Lagrangian.

   (b) (12pt) Find the equations of motion.

   (c) (12pt) Find a complete solution to the equations of motion.

2. (30pt) In the previous problem you had to find the Lagrangian and equation of motion for two blocks connected by a spring of spring constant k are free to slide frictionlessly along a horizontal surface, as shown in Fig. 1. After we've learned about conserved quantities two students decided to reexamine this problem. Student A assigned x_1 with mass M and x_2 with mass m and wrote the Lagrangian as

$$L = \frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 - \frac{k}{2}(x_2 - x_1)^2 \ . \tag{1}$$

He then decided to move to the center of mass, where the definition of the center of mass (basically moving the origin of the coordinate system to the center of mass) $Mx_1 + mx_2 = 0$ so $x_1 = -mx_2/M$. Plugging this into the Lagrangian he wrote:

$$L = \frac{1}{2}M\dot{x}_1^2 + \frac{1}{2}m\dot{x}_2^2 - \frac{k}{2}\left(1 + \frac{m}{M}\right)^2 x_2^2 \ . \tag{2}$$

He then said that since

$$\frac{\partial L}{\partial x_1} = 0 \ . \tag{3}$$

the conjugate momentum $p_{x_1} = M\dot{x}_1$ is constant. Student B said she thinks he is wrong. Is she correct? If so what were her arguments? If not, what was her mistake?

3. (40pt) A particle of mass m is attracted to a force centered with a forced magnitude k/r^2. Use plane polar coordinates.

   (a) (20pt) Write the Lagrangian of the system.

   (b) (20pt) Find the momenta. What are the conserved quantities ?

4. (44pt) A particle of mass m moves under the influence of gravity along a helix $z = k\theta, r = \text{Const}$, where k is constant and z is vertical (see Figure 1).
(i) Choose a coordinate system.
(ii) How many degrees of freedom the system has?
(iii) Write the Lagrangian
(iv) Find all the conserved quantities.
(v) Find the equations of motion.

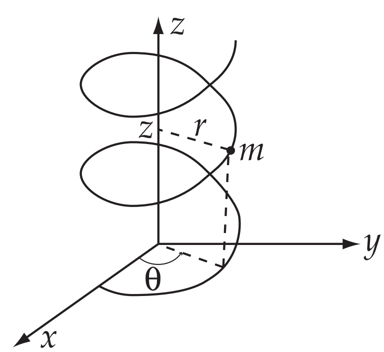

Figure 2: mass on a helix