PROBLEM SET #5

Physics 1BH

Winter 2016, Prof. Saltzberg

DUE: Tuesday February 16, *in class* **EXTENDED to 2/19 11:30am in my office***
READING: Section 6.1, our class notes on the Biot-Savart law, Chapter 5, G.1-G.3

You are encouraged to work with others, but try as much as you can on your own. At a minimum, check your answers with someone else.

1. PM 6.31 [three wires]

2. PM 6.52 [right angled wire]. Hint #1: once you solve the x-axis contribution, the contribution from the y-axis is the same result with $y \leftrightarrow x$. Hint #2 when solving from the current along the x-axis, you need to keep track of three horizontal values. For example, define x to be the position of ``the point in the xy plane'', x' to be the position of the current element along the x-axis and $x'' = x'-x$. Remember when changing the integral from x' to x'', to change its limits.

2b. Take the limits as $x \gg y$ and $y \gg x$ and explain why they make sense.

3. In class we showed time dilation by using a "light clock" with the path of a light beam perpendicular to the relative velocity of two inertial frames. In class, we derived length contraction using the Lorentz transformations. Show instead you can find length contraction from considering a light clock oriented so the light travels parallel to the direction of motion, i.e. without using the full Lorentz Transformation equations. Hint: find the time it takes in each frame assuming the clock has length L' and L respectively. Then find the times it takes in each frame for the light to make a complete round-trip, which you know are related by a factor γ.

4. Particles called muons are produced in cosmic-ray collisions in the upper atmosphere. Muons at rest decay with a half-life of 1.5 microseconds. For simplicity say they are produced 10 km above sea level and head straight down with a speed 99.5% the speed of light (which are reasonable values). Without special relativity, what fraction of the muons would survive to reach sea level? Now using special relativity, what fraction survive to reach a detector resting at sea level in the detector's inertial frame? Re-evaluate this situation in the muon's rest frame, where a muon half-life is 1.5 microseconds, without a factor γ. Now what fraction survive to hit the detector on the ground?

5. Consider inertial frames F and F' with a relative velocity along the x axis. In Galilean relativity, $\Delta t = \Delta t'$ between events is always true. In special relativity we saw this was not true except in special cases. Show that a new variable (curiously called ``the interval" and written as Δs^2) we will define as $\Delta s^2 = c^2 \Delta t^2 - \Delta x^2$ is the same value in all inertial frames with relative velocities in the x direction. Do this in

two ways a) using the Lorentz transformations and b) using Einstein's postulate that the speed of light is the same in all reference frames. FYI, you can see from part b that this is true in three dimensions for the variable as it is usually defined: $\Delta s^2 = c^2\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$.)

6. PM 5.15 [Gauss's law for a moving charge]

7. PM 5.27 [Equal velocities]. Compare the electric force direction and magnitude on such a test charge q_0 in this case with the magnetic field you would have calculated in the frame of the wire.

8. PM 5.19 [Reversing the motion] Indicate which part of your diagram corresponds to radiated electric radiation. This is known as "bremsstrahlung" which means "braking radiation". Radiation must happen because the proton is a charged particle that undergoes acceleration. Hint: Don't worry about what the fields are doing at very far distances (they connect like a dipole field but don't worry about it.)

Note that the electric field drops as 1/r. You might worry if that is consistent with conservation of energy since it expands outward on a shell whose surface area is increase with r^2. But the energy density (and thus the energy carried) of the electric field in the region where it is transverse goes as electric field squared, so $1/r^2$.