

① Complex numbers

These are useful abstract quantities and are very useful in solving problems in physics, Engineering etc.

The idea is that we expand numbers from 1D to 2D by introducing an imaginary unit

$$i = \sqrt{-1}$$

So each number can be described as $a + ib$; $x + iy$ etc

What we consider regular numbers such as 1, 2, 7, 107 etc are the

Real part of the complex number.

So if $z = a + ib$ is the complex number

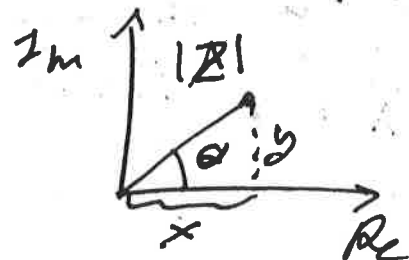
$$x = a = \text{Re}(z) \quad \text{Real part}$$

$$y = b = \text{Im}(z) \quad \text{imaginary part}$$

$$z = x + iy = |z| [\cos \theta + i \sin \theta] = |z| e^{i\theta} \quad \text{Euler formula}$$

$$\text{Where } |z| = \sqrt{x^2 + y^2}$$

$$\theta = \arctan\left(\frac{y}{x}\right)$$



For example if we have

$z = 3 - 4i$ it means that

$$\operatorname{Re}(z) = 3$$

$$\operatorname{Im}(z) = -4$$

$$|z| = \sqrt{3^2 + 4^2} = 5$$

$$\theta = \arctan\left(\frac{-4}{3}\right) = -0.9273 \text{ rad} = -53.13^\circ$$

$$\text{So } z = 5e^{-i53.13^\circ}$$

So what's the connection to oscillators?

So: in our case we have:

$$x = A \cos(\omega t + \phi)$$

So

$$x = \operatorname{Re} [A e^{i\omega t + \phi}] = \operatorname{Re} [A (\cos(\omega t + \phi) + i \sin(\omega t + \phi))] = A \cos(\omega t + \phi) \quad \text{or}$$

it's much simpler to solve differential eqs with the complex exponential.

it's easier to solve it

So

$$\ddot{x} = -\frac{k}{m} x$$

this should be k/m !

now we will guess

$$x = A e^{i\omega t} \quad (\text{take Re @ end})$$

3) $\dot{x} = i\omega A e^{i\omega t} = i\omega x$

$$\boxed{\ddot{x} = -\omega^2 x} \quad \omega^2 = \frac{k}{m}$$

Well that is easy but when things will
become more complicated it will really help!