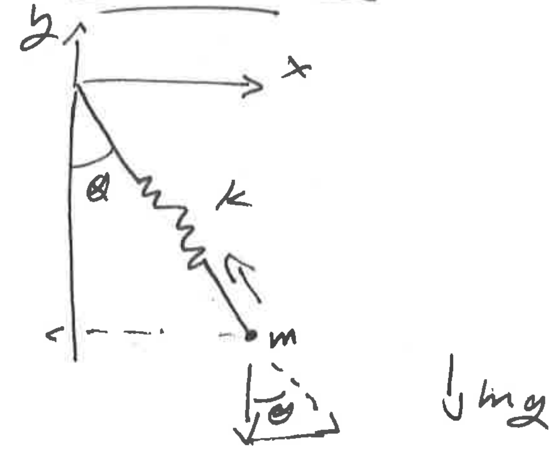
$\mathcal{L}G.5$

Example 3



b = unextended length of spring

find $\mathcal{L}$ and E.o.M.

① What should be the generalized coordinates
How many D.o.f are there (2)

r, θ $\qquad T = \frac{1}{2} m (\dot{\vec{r}})^2$

$\dot{\vec{r}} = \dot{r}\hat{r} + \dot{\theta} r \hat{\theta}$ we saw in week 1

So

$$T = \frac{1}{2} m (\dot{r}^2 + r^2\dot{\theta}^2)$$

$$U = \frac{1}{2} k (r-b)^2 + mgy = \frac{1}{2} k (r-b) - mgr\cos\theta$$

$y = -r\cos\theta$

$$\mathcal{L} = T - U = \frac{1}{2} m (\dot{r}^2 + r^2\dot{\theta}^2) - \frac{1}{2} k (r-b)^2 + mg r\cos\theta$$

$r: \quad \frac{\partial \mathcal{L}}{\partial r} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{r}}\right) = 0 \Rightarrow$

$$mr\dot{\theta}^2 - k(r-b) + mg\cos\theta - (m\ddot{r}) = 0$$

$\theta: \quad -mg\sin\theta - (mr^2\dot{\theta})^{\cdot\cdot} = 0$

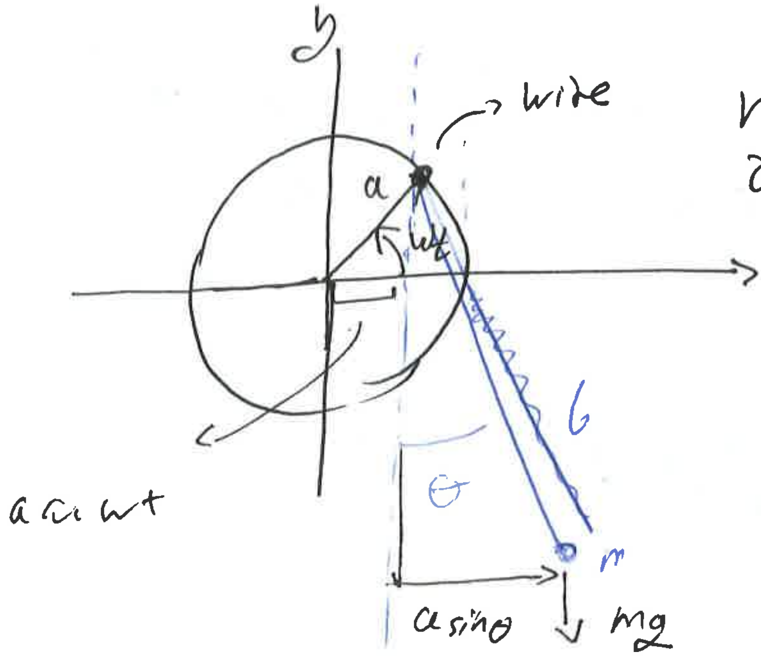
$$m r \dot{\theta}^2 - k(r-l) + mg\cos\theta - m\ddot{r} = 0$$

$$-mg\sin\theta - m r^2\ddot{\theta} - 2m r \dot{r}\dot{\theta} = 0$$

which I can write:

$$\begin{array}{ll} r: & \left[\begin{array}{l} \ddot{r} - r\dot{\theta}^2 + \frac{k}{m}(r-l) - g\cos\theta = 0 \\ \\ \ddot{\theta} + \frac{2}{r}\dot{r}\dot{\theta} + \frac{g}{r}\sin\theta = 0 \end{array} \right] \\ \theta: & \end{array}$$

Example: (7.5 Thornton + Marion)



massless rim radius a, rotating ω

find the E.O.M.

Q. How many degrees of fredom?

A. $\underline{\underline{1}}$

Q. what is the generalize coordinate

A. θ

$$x = a\cos\omega t + b\sin\theta$$
$$y = a\sin\omega t - b\cos\theta$$

$$\dot{x} = -a\omega\sin\omega t + b\dot{\theta}\cos\theta$$
$$\dot{y} = a\omega\cos\omega t + b\dot{\theta}\sin\theta$$

skip
$$\ddot{x} = -a\omega^2\cos\omega t + b(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta)$$
$$\ddot{y} = -a\omega^2\sin\omega t + b(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta)$$

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) \qquad U = mgy.$$

$$\mathcal{L} = T - U = \frac{m}{2}\left[a^2\omega^2 + b^2\dot{\theta}^2 + 2b\dot{\theta}a\omega\sin(\theta - \omega t)\right]$$
$$- mg(a\sin\omega t - b\cos\theta)$$

b^3 So the derivative of the Lagrangian:

$$\frac{\partial \mathcal{L}}{\partial \theta} = mb\dot{\theta}a\omega\cos(\theta-\omega t) - mgb\sin\theta$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = mb^2\dot{\theta} + mba\omega\sin(\theta-\omega t)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) = mb^2\ddot{\theta} + mba\omega(\dot{\theta}-\omega)\cos(\theta-\omega t)$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) = \frac{\partial \mathcal{L}}{\partial \theta}$$

↳ rearranging

$$\ddot{\theta} = \frac{\omega^2 a}{b}\cos(\theta-\omega t) - \frac{g}{b}\sin\theta$$

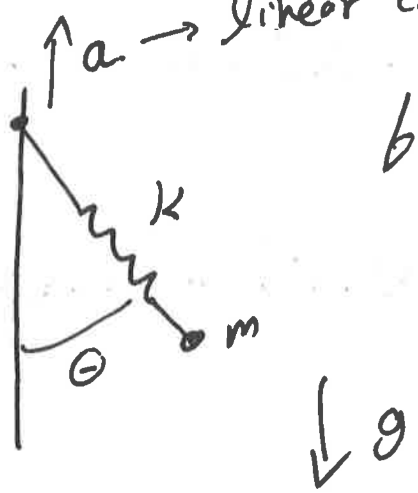
Q. what is this?

A. simple pendulum with Forced
so we expect a simple

what do you expect that
will happen @ $\dot{\theta} = \omega$?

Chesschase.

H.6



b unextended length.

① find coordinates and write the lagrangian and the ~~eqs~~ different momenta ~~and the Hamiltonian~~.

$$x = l \sin\theta \qquad \dot{x} = \dot{l}\sin\theta + l\dot{\theta}\cos\theta$$

$$y = \frac{1}{2}at^2 - l\cos\theta \qquad \dot{y} = at - \dot{l}\cos\theta + l\dot{\theta}\sin\theta$$

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$$

$$U = mgy + \frac{1}{2}k(l-b)^2$$

$$\mathcal{L} = T - U = \frac{1}{2}m\big(\dot{l}^2 + l^2\dot{\theta}^2 + a^2t^2 + 2at(l\dot{\theta}\sin\theta - \dot{l}\cos\theta)\big) + mg\left(l\cos\theta - \frac{at^2}{2}\right) - \frac{k}{2}(l-b)^2$$

We can find the momenta

$$p_l = \frac{\partial \mathcal{L}}{\partial \dot{l}} = m\dot{l} - mat\cos\theta \rightarrow \dot{l} = \frac{p_l}{m} + at\cos\theta$$

$$p_\theta = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = ml^2\dot{\theta} + matl\sin\theta$$

$$\dot{\theta} = \frac{p_\theta}{ml^2} - \frac{at\sin\theta}{l}$$

Now

$$\mathcal{H} = \sum p\dot{q} - \mathcal{L} = p_l\dot{l} + p_\theta\dot{\theta} - \mathcal{L}.$$

So

$$\mathcal{H} = \frac{p_l^2}{2m} + \frac{p_\theta^2}{2ml^2} - \frac{at}{l}p_\theta\sin\theta +$$

$$+ atp_l\cos\theta + \frac{1}{2}k(l-b)^2 + \frac{1}{2}mgat^2 - mgl\cos\theta$$

<u>Fix E.O.M.</u>

We'll use $\mathcal{L}$.

$$l: \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{l}}\right) = \frac{d}{dt}\left(m\dot{l} - amt\cos\theta\right) =$$

$$ml\dot{\theta}^2 + mat\dot{\theta}\,\overset{\cos}{\cancel{\sin}}\theta + mg\cos\theta - k(l-b)$$

$$\ddot{l} - l\dot{\theta}^2 - (a+g)\cos\theta + \frac{k}{m}(l-b) = 0$$

4.7

θ: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) \frac{d}{dt}\left[ml^2\dot\theta + matl\sin\theta\right] =$

$= mat\dot l \sin\theta - mgl\sin\theta + mgl\dot\theta\cos\theta$

$m2l\dot l\dot\theta + ml^2\ddot\theta + mal\sin\theta + ma\dot l t\sin\theta$

$+ matl\cos\theta\,\dot\theta = mat\dot l\sin\theta - mgl\sin\theta$

$+ matl\dot\theta\cos\theta$

$$\boxed{\ddot\theta + \frac{2}{l}\dot l\dot\theta + \frac{a+g}{l}\sin\theta = 0}$$

Now take $\theta << 1$ $\sin\theta \approx \theta$

$\cos\theta \approx 1 - \frac{\theta^2}{2}$

Lets start with $\dot\theta$

$\ddot\theta + \frac{2}{l}\dot l\dot\theta + \frac{a+g}{l}\theta = 0$

$\dot l < 0$ $\dot\theta < 0$

$\ddot\theta + \frac{a+g}{l}\theta = 0$

$l \to$ a func of time

looking @ $\ddot{\ell}$

$$\ddot{l} - l\dot{\theta}^2 - (a+g)\left(1-\frac{\theta^2}{2}\right)$$

$$+ \frac{k}{m}(l-b) = 0$$

So $$\boxed{\ddot{l} + \frac{k}{m} l = a + g + \frac{k}{m} b}$$

The sol: $l = l_{homogen} + l_{particular}$

$$\ddot{l} + \frac{k}{m} l = 0$$

$$l_0 = A\cos\sqrt{\frac{k}{m}}\, t + B\sin\cos\sqrt{\frac{k}{m}}$$

$$l_{partic} \underbrace{=}_{\text{guess @ the order of sol}} C'$$

$$\ddot{l}_p = 0 \qquad C' = a + g + \frac{k}{m} b$$

$$l = A\cos\sqrt{\frac{k}{m}}\, t + B\sin\sqrt{\frac{k}{m}}\, t + a + g + \frac{k}{m} b.$$