N1.

$$\Psi(\bar{r},0) = A[\psi_{100} + \sqrt{5}\,\psi_{210}]$$

a) $\langle \Psi \Psi \rangle = \cancel{|A|^2}$

$$= A^2\Big(\underbrace{\langle \psi_{100} | \psi_{100}\rangle}_{=1,\ \text{normal}} + 2\sqrt{5}\underbrace{\langle \psi_{100} | \psi_{210}\rangle}_{0,\ \text{orthogonal}} +$$

$$+ 5\underbrace{\langle \psi_{210} | \psi_{210}\rangle}_{=1,\ \text{normal}}\Big) = 6A^2 \Rightarrow A = \frac{1}{\sqrt{6}}$$

$$\Psi = \frac{1}{\sqrt{6}}[\psi_{100} + \sqrt{5}\,\psi_{210}]$$

b) $L^2 \psi_{nlm} = l(l+1)\hbar^2 \psi_{nlm}$

$\Rightarrow$ L^2 gives $2\hbar^2$ in case of $\psi_{210} \Rightarrow$

$\Rightarrow$ $P_{2\hbar^2} = \left(\frac{\sqrt{5}}{\sqrt{6}}\right)^2 = \frac{5}{6}$

c) $\Psi(\bar{r}, t) = A\left[e^{-iE_1 t}\psi_{100} + e^{-iE_2 t}\sqrt{5}\,\psi_{210}\right]$

d) After the measurement collaps has happened

$$\Psi(\bar{r}, t_0) = \psi_{210}$$

$$\Downarrow$$

$$\Psi(\bar{r}, t) = e^{iE_2 t}\psi_{210}$$

NH.

N2.

$$\Psi(x,y,z) = C(xy + yz + zx)e^{-\alpha r^2}$$

Go to spherical coordinates

$x = r\sin\theta\cos\varphi$

$y = r\sin\theta\sin\varphi$

$z = r\cos\theta$

$\Downarrow$

$$\Psi(r,\theta,\varphi) = r^2 Ce^{-\alpha r^2}\left(\sin^2\theta\sin\varphi\cos\varphi + \sin\theta\cos\theta(\sin\varphi + \cos\varphi)\right)$$

$$= f(r)\left(\frac{1}{2}\sin^2\theta\underset{\underset{\frac{e^{2i\varphi}-e^{-2i\varphi}}{2i}}{\parallel}}{\sin 2\varphi} + \sin\theta\cos\theta\left(\frac{e^{i\varphi}-e^{-i\varphi}}{2i} + \frac{e^{i\varphi}+e^{-i\varphi}}{2}\right)\right) =$$

looking to the TABLE 4.3

$$= f(r)\left(\frac{1}{2}\left(\frac{32\pi}{15}\right)^{1/2}\left(\frac{1}{2i}\left(Y_2^2 - Y_2^{-2}\right)\right) + \right.$$

$$\left. + \left(\frac{1}{2i} + \frac{1}{2}\right)\left(\frac{8\pi}{15}\right)^{1/2} Y_2^1 - \left(\frac{1}{2} - \frac{1}{2i}\right)\left(\frac{8\pi}{15}\right)^{1/2} Y_2^{-1}\right)$$

Since we do not have Y_0^0, the probability to get ~~o~~ $l=0$ is 0.

$6\hbar^2$ $\quad l(l+1)=6 \Rightarrow l=2$

$\Rightarrow$ ~~it is (sum of coefficients before $Y_2^{\pm 2}$ divided by total sum of coefficients)~~

~~$\left|\frac{1}{2}\left(\frac{32\pi}{15}\right)^{\frac{1}{2}}\frac{1}{2i}\right|^2 + \left|\frac{1}{2}\left(\frac{32\pi}{\right.\right.}$~~

$\Rightarrow$ it is 1, because all the Y are $Y_2^{\pm 2, \pm 1}$

$\Rightarrow$ Relative probabilities for different m.

Let us first cancel out $\left(\frac{8\pi}{15}\right)^{\frac{1}{2}}$. Then we get

$$\frac{1}{2i}\left(Y_2^2 - Y_2^{-2}\right) +$$

$$+\left(\frac{1}{2i}+\frac{1}{2}\right)Y_2^1 + \left(\frac{1}{2i}-\frac{1}{2}\right)Y_2^{-1}$$

Then squared it gives us

2	$\cancel{\frac{1}{32}}$ $\frac{1}{4}$
1	$\frac{1}{2}$
0	0
-1	$\frac{1}{2}$
-2	$\frac{1}{4}$

Before normalisation

And

m	p
2	$\frac{1}{6}$
1	$\cancel{\frac{2}{8}}$ $\frac{1}{3}$
0	0
-1	$\cancel{\frac{2}{8}}$ $\frac{1}{3}$
2	$\cancel{\frac{1}{10}}$ $\frac{1}{6}$

N3.

Basis

$$Y_2^2 \quad Y_2^1 \quad Y_2^0 \quad Y_2^{-1} \quad Y_2^{-2}$$

$$\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix} \quad \begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix} \quad \begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix} \quad \begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix} \quad \begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}$$

$$L_z Y_l^m = m Y_l^m \Rightarrow$$

$$\Rightarrow L_z = \begin{pmatrix}2 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & -1 & 0\\ 0 & 0 & 0 & 0 & -2\end{pmatrix} \hbar$$

$$L_+ Y_\ell^m = \sqrt{\ell(\ell+1) - m(m+1)}\ Y_\ell^{m+1}$$

$$L_+ = \begin{pmatrix} 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & \sqrt{6} & 0 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \hbar$$

$$L_- Y_\ell^m = \sqrt{\ell(\ell+1) - m(m-1)}\ Y_\ell^{m-1}$$

$$L_- = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{6} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{6} & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix} \hbar$$

$$L_+ = L_x + iL_y$$

$$L_- = L_x - iL_y$$

$$\Rightarrow L_x = \frac{1}{2}(L_+ + L_-)$$

$$L_y = \frac{1}{2i}(L_+ - L_-)$$

$$L_x = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & \frac{\sqrt{6}}{2} & 0 & 0 \\ 0 & \frac{\sqrt{6}}{2} & 0 & \frac{\sqrt{6}}{2} & 0 \\ 0 & 0 & \frac{\sqrt{6}}{2} & 0 & 1 \\ 0 & 0 & 1 & 0 & \end{pmatrix} \hbar$$

$$L_y = \begin{pmatrix} 0 & -i & 0 & 0 & 0 \\ i & 0 & \frac{\sqrt{6}}{2i} & 0 & 0 \\ 0 & -\frac{\sqrt{6}}{2i} & 0 & \frac{\sqrt{6}}{2i} & 0 \\ 0 & 0 & -\frac{\sqrt{6}}{2i} & 0 & -i \\ 0 & 0 & 0 & i & 0 \end{pmatrix} \hbar$$

$$L^2 Y_l^m = l(l+1) Y_l^m \Rightarrow$$

$$\Rightarrow L^2 = \begin{pmatrix} 6 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 6 \end{pmatrix} \hbar^2$$

N 4.

~~1.~~ Y_1^{+1} Y_1^{0} Y_1^{-1}

$$L_+ = \begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} \hbar$$

$$L_- = \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix} \hbar$$

$$L_x = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \hbar$$

Eigenvalues 1, 0, -1

~~Eigenvalues~~ $\begin{pmatrix} 1 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 1 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$$a + \frac{b}{\sqrt{2}} = 0$$

$$\frac{a}{\sqrt{2}} + b + \frac{c}{\sqrt{2}} = 0$$

$$\frac{b}{\sqrt{2}} + c = 0 \qquad \Rightarrow a = c$$

$$\Rightarrow \begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{1}{2} \\ -\frac{\sqrt{2}}{2} \\ \frac{1}{2} \end{pmatrix} \Rightarrow \lambda = -1$$

$$\begin{pmatrix} -1 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -1 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -1 \end{pmatrix}$$

$$-a + \frac{b}{\sqrt{2}} = 0$$

$$\frac{a}{\sqrt{2}} - b + \frac{c}{\sqrt{2}} = 0$$

$$\frac{b}{\sqrt{2}} - c = 0$$

$$c = a$$

$$\begin{pmatrix} \quad \end{pmatrix} \begin{pmatrix} 1/2 \\ \sqrt{2}/2 \\ 1/2 \end{pmatrix} \Rightarrow \lambda = 1$$

~~Ox~~

$$\begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$\lambda = 0$

$|0\rangle \leftarrow$ ~~state~~ state

such that $L_x |0\rangle = 0$

$$\psi = \frac{1}{\sqrt{26}} \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}$$

$$\left| \langle \psi | 0 \rangle \right|^2 = \left| \frac{1}{\sqrt{26}} \frac{1}{\sqrt{2}} (-2) \right|^2 = \frac{1}{13}$$

$$\Downarrow$$

$$p = \frac{1}{13}$$

N 5 (4.19)

a) $[L_z; x] = [xp_y - yp_x, x] =$

$= [xp_y; x] - [yp_x, x] = 0 - y[p_x, x] = i\hbar y$

$[L_z, p_x] = [xp_y - yp_x, p_x] = [xp_y, p_x] -$

$- [yp_x, p_x] = p_y[x, p_x] - 0 = i\hbar p_y$

$[L_z, z] = [xp_y - yp_x, z] = [xp_y, z] - [yp_x, z] = 0$

б) $[L_z, L_x] = [L_z, yp_z - zp_y] =$

$= [L_z, yp_z] - [L_z, zp_y] =$

$= [L_z, y]p_z - [L_z, p_y]z =$

$= -i\hbar x p_z + i\hbar p_x z = i\hbar L_y$

c) $[L_z, r^2] = [L_z, x^2] + [L_z, y^2] + [L_z, z^2]$

$= i\hbar y x + x i\hbar y + (-i\hbar x) y + y(-i\hbar x) = \underline{0}$

d) $[L_z, p^2] = [L_z, p_x^2] + [L_z, p_y^2] + [L_z, p_z^2] =$

$= i\hbar p_y p_x + p_x i\hbar p_y + (-i\hbar p_x) p_y + p_y(-i\hbar p_x) = \underline{0}$

d) L commute with p^2 and

L commute with $r^2 \Rightarrow L$ commute with $V(\sqrt{r^2})$

$\Rightarrow L$ commute with $H = \frac{p^2}{2m} + V(\sqrt{r^2})$

N 4.20

a) Eq. 3.71 $\Rightarrow \frac{d\langle L_x\rangle}{dt} = \frac{i}{\hbar}\langle [H, L_x] \rangle$

$\Rightarrow [H, L_x] = \underbrace{\frac{1}{2m}[p^2, L_x]}_{0} + [V, L_x]$

$[V, L_x] = [V, y p_z - z p_y] = y[V, p_z] - z[V, p_y]$

$[V, p_z] = i\hbar \frac{\partial V}{\partial z}$

$\Rightarrow [H, L_x] = y\, i\hbar \frac{\partial V}{\partial z} - z\, i\hbar \frac{\partial V}{\partial y} = i\hbar [\bar{r} \times (\nabla V)]_x$

$\Rightarrow \cancel{\,[H\,}$

$\Rightarrow \frac{d\langle \bar{L}\rangle}{dt} = \langle [\bar{r} \times (-\nabla \hat{V})] \rangle = \langle \bar{N} \rangle$

b) ~~Spherical~~ $V(\bar{r}) = V(r) \Rightarrow$

$\Rightarrow [L, H] = 0 \Rightarrow \frac{d\langle L\rangle}{dt} = 0$