Physics 1BH, Midterm #1 "cheat sheet" (4 pages)
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This will be handed out with the exam.

A few things are deliberately **not** on here so you memorize them forever. These are things physicists and engineers are expected to know without looking up. (The values below to 3 digits, eg. $\pi = 3.14$.

- elementary charge e in both Coulombs and esu
- electron and proton masses
- permittivity of free space
- All metric prefixes from femto to Peta
- Stokes's theorem and the Divergence (Gauss's) Theorem
- Taylor expansion of $(1+\epsilon)^p$ to the first term with ϵ in it.
- dV in Cartesian, cylindrical and spherical coordinates, including the limits of integration

$$\boxed{\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2 \hat{\mathbf{r}}_{21}}{r_{21}^2}}$$

Integrating this force, we find that the *potential energy* of a system of charges (the work necessary to bring them in from infinity) equals

$$U = \frac{1}{2}\sum_{j=1}^{N}\sum_{k\neq j}\frac{1}{4\pi\epsilon_0}\frac{q_j q_k}{r_{jk}}. \qquad (1.58)$$

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int\frac{\rho(x', y', z')\hat{\mathbf{r}}\,dx'\,dy'\,dz'}{r^2} \quad \text{or} \quad \frac{1}{4\pi\epsilon_0}\sum_{j=1}^{N}\frac{q_j\hat{\mathbf{r}}_j}{r_j^2}.$$

- The *energy density* of an electric field is $\epsilon_0 E^2/2$, so the total energy in a system equals

$$U = \frac{\epsilon_0}{2}\int E^2\,dv. \qquad (1.64)$$

Gauss's law gives the fields for a sphere, line, and sheet of charge as

$$E_{\text{sphere}} = \frac{Q}{4\pi\epsilon_0 r^2}, \qquad E_{\text{line}} = \frac{\lambda}{2\pi\epsilon_0 r}, \qquad E_{\text{sheet}} = \frac{\sigma}{2\epsilon_0}. \qquad (1.62)$$

More generally, the discontinuity in the normal component of $\mathbf{E}$ across a sheet is $\Delta E_{\perp} = \sigma/\epsilon_0$. Gauss's law is always valid, although it is useful for calculating the electric field only in cases where there is sufficient symmetry.

- For an electrostatic field, the line integral $\int_{P_1}^{P_2} \mathbf{E} \cdot d\mathbf{s}$ is independent of the path from P_1 to P_2. This allows us to define uniquely the *electric potential difference*:

$$\phi_{21} = -\int_{P_1}^{P_2} \mathbf{E} \cdot d\mathbf{s}. \tag{2.99}$$

Relative to infinity, the potential due to a charge distribution is (depending on whether the distribution is continuous or discrete)

$$\phi(x, y, z) = \int \frac{\rho(x', y', z')\, dx'\, dy'\, dz'}{4\pi\epsilon_0 r} \qquad \text{or} \qquad \sum \frac{q_i}{4\pi\epsilon_0 r}. \tag{2.100}$$

$$\mathbf{E} = -\nabla\phi. \qquad \nabla^2\phi = -\frac{\rho}{\epsilon_0}.$$

- A *dipole* consists of two charges $\pm q$ located a distance ℓ apart. The dipole moment is $p \equiv q\ell$. At large distances, the potential and field due to a dipole are

$$\begin{aligned} \phi(r, \theta) &= \frac{p\cos\theta}{4\pi\epsilon_0 r^2}, \\ \mathbf{E}(r, \theta) &= \frac{p}{4\pi\epsilon_0 r^3}\big(2\cos\theta\, \hat{\mathbf{r}} + \sin\theta\, \hat{\boldsymbol{\theta}}\big). \end{aligned} \tag{2.104}$$

- In Cartesian coordinates, the *curl* of a vector function (written as curl $\mathbf{F}$ or $\nabla \times \mathbf{F}$) is

$$\text{curl}\,\mathbf{F} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix}. \tag{2.111}$$

$$\int \frac{dx}{x^2+r^2} = \frac{1}{r}\tan^{-1}\left(\frac{x}{r}\right)$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \ln\left(x+\sqrt{x^2-1}\right)$$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(\sqrt{x^2+a^2}+x\right)$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2(a^2+x^2)^{1/2}}$$

$$\int \ln x \, dx = x\ln x - x$$

$$\int x^n \ln\left(\frac{a}{x}\right) dx = \frac{x^{n+1}}{(n+1)^2} + \frac{x^{n+1}}{n+1}\ln\left(\frac{a}{x}\right)$$

$$\int x e^{-x}\, dx = -(x+1)e^{-x}$$

$$\int x^2 e^{-x}\, dx = -(x^2+2x+2)e^{-x}$$

$$\int \sin^3 x \, dx = -\cos x + \frac{\cos^3 x}{3}$$

$$\int \cos^3 x \, dx = \sin x - \frac{\sin^3 x}{3}$$

$$\int \frac{dx}{\cos x} = \ln\left(\frac{1+\sin x}{\cos x}\right)$$

$$\int \frac{dx}{\sin x} = \ln\left(\frac{1-\cos x}{\sin x}\right)$$

Vector operators

Cartesian coordinates

$$d\mathbf{s} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$$

$$\nabla = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$$

$$\nabla f = \frac{\partial f}{\partial x}\,\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\,\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\,\hat{\mathbf{z}}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Cylindrical coordinates

$$d\mathbf{s} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + dz\,\hat{\mathbf{z}}$$

$$\nabla = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$$

$$\nabla f = \frac{\partial f}{\partial r}\,\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\,\hat{\boldsymbol{\theta}} + \frac{\partial f}{\partial z}\,\hat{\mathbf{z}}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\theta}{\partial \theta} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \theta} - \frac{\partial A_\theta}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial (rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\hat{\mathbf{z}}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical coordinates

$$d\mathbf{s} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$$

$$\nabla = \hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\boldsymbol{\theta}}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

$$\nabla f = \frac{\partial f}{\partial r}\,\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\,\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\,\hat{\boldsymbol{\phi}}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial (A_\theta \sin\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \mathbf{A} = \frac{1}{r\sin\theta}\left(\frac{\partial (A_\phi \sin\theta)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi}\right)\hat{\mathbf{r}} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial (rA_\phi)}{\partial r}\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left(\frac{\partial (rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\hat{\boldsymbol{\phi}}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$