105A - Set 2

(grades out of 150pt)

1. (40pt) A particle is under the influence of a force $F = -kx + kx^3/\alpha^2$ where k and α are constant and $k > 0$. Determine the potential energy $U(x)$, sketch it and find the possible particle motions. What happens when $E = k\alpha^2/4$? Sketch $U(x)$.

2. (40pt) Two masses $m_1 = 100$ g and $m_2 = 200$ g slide freely in a horizontal frictionless track and are connected by a spring whose force constant is $k = 0.5$ N m^{-1}. Find the frequency of oscillatory motion for this system.

3. (40pt) Given the equation of motion of a damped oscillations:

$$m\ddot{x} + b\dot{x} + kx = 0 \tag{1}$$

   and $b < 0$, i.e, the damping resistance is negative. Find the solution for $x(t)$ and discuss the three oscillatory solutions.

4. (30pt) A simple pendulum is made from a wooden block of mass M suspended from a string of length l. Initially the pendulum is at rest. A bullet of mass m is fired horizontally with velocity v_0 and gets imbedded inside the block at $t = 0$. Write down an equation for subsequent motion of the block $\theta(t)$ assuming the amplitude of oscillations is small. You can also assume $m << M$. Then solve for $\theta(t)$.

5. **Bonus Question + 20pt**
   In my colloquium I have discussed the dynamical evolution of hierarchical triple systems and showed that the orbits undergoes eccentricity and inclination oscillations. Adding tidal friction between the two inner objects (just like the tides between the earth and the moon - they convert orbital energy to heat), and general relativity (not important for the question) I have calculated the evolution of triple stars. In Figure 1 I am showing two different examples of the evolution of hierarchical triple systems. The left column shows a system that resulted in merger while the right column is of an inner binary that shirnked its separation to a stable tight configuration. Both had different initial conditions that resulted in these different fates.
   Look at the figure and the oscillations of the eccentricity of each system [eccentricity is a value between 1 and 0 that measures how elliptical an orbit is. If its $e = 0$ the orbit is circular and if $e = 1$ its a radial orbit - we will learn more about eccentricity later in the course, but its easy to understand it already now]. Can you relate these two examples to the subject learned in class? How would you classify those two examples in relation to what we discussed in class?

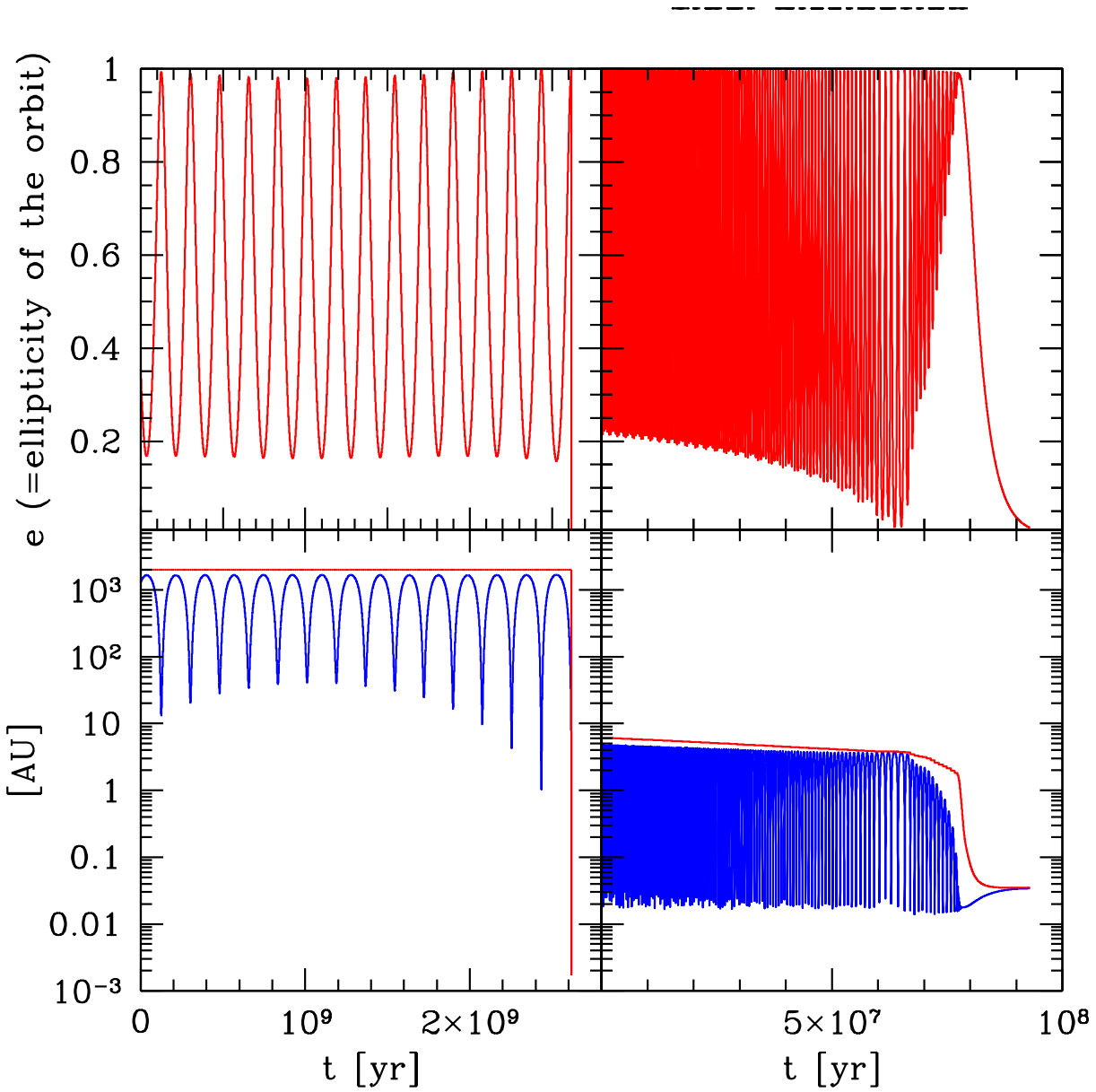

Figure 1: **The evolution of hierarchical stellar systems**. The left column shows a system that resulted in merger while the right column is of an inner binary that shirnked its separation to a stable tight configuration. In the top panel we show the eccentricity (a value between 1 and 0 which measures how elliptical an orbit is. if its 0 its circular and if $e = 1$ its a radial orbit) for the inner binary (red lines). Bottom panel shown the inner binary separation (red lines) and the pericenter distance, closest approach in an orbit (blue lines). Figure adopted (and somewhat reconfigured) from Naoz and Fabrycky (2014).