Math 115A: Problem set 4

Sections 1 and 3. Instructor: James Freitag

Due 10/30

Problem 1 Lagrange map

For any two real numbers a, b, there is a polynomial $f_{a,b}$ of degree at most 1 such that $f_{a,b}(0) = a$, $f_{a,b}(1) = b$. Define the map $T : \mathbb{R}^2 \to P_1(\mathbb{R})$ given by $(a, b) \mapsto f_{a,b}$. Prove that T is a linear map.

Problem 2 Matrix for Lagrange

Consider the map T from the previous problem. Let α be the standard ordered basis of $\mathbb{R}^2$, $\{(1,0),(0,1)\}$. Let $\beta = (1, x)$. Compute $[T]_\alpha^\beta$.

Problem 3 A map in the other direction

Let α and β be as in the previous problem. Let $S : P_1(\mathbb{R}) \to \mathbb{R}^2$ be given by $S(f) = (f(0), f(1))$. Show that S is a linear map. Compute $[S]_\beta^\alpha$.

Problem 4 The inverse map

Show that $[S]_\beta^\alpha [T]_\alpha^\beta = [T]_\alpha^\beta [S]_\beta^\alpha = id_{2\times 2}$. Explain why this implies that T is the inverse of S.

Problem 5 Exercises from the book

Do the following exercises from book:

- Problems 1 and 9 from section 2.4.
- Problems 3 and 5 from section 2.5.