Week 7 QM Discussion

Ji Zou, jzeeucla@physics.ucla.edu
Office Hours: Tuesday 10am-12pm, Tutoring Center.

Two Identical Particles with Spin

A particular one-dimensional potential well has the following bound state single-particle energy eigenfunctions:

$$\phi_a(x), \phi_b(x), \phi_c(x)..., where E_a < E_b < E_c...$$

Two non-interacting particles are placed in the well. For each of the cases(a),(b),(c) listed below write down:

the two lowest total energies available to the two-particle system, the degeneracy of the two energy levels, the possible two-particle wave functions associated with each of the levels. (Use ϕ to express the spatial part and a ket $|S, m_s\rangle$ to express the spin part. S is the total spin.)

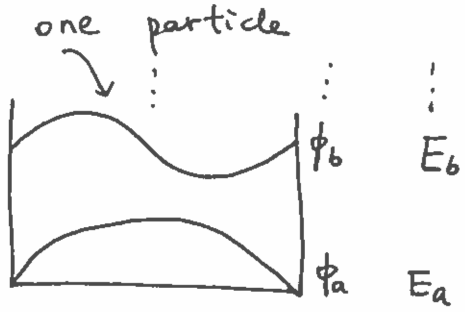
(a) Two distinguishable spin-1/2 particles.

(b) Two identical spin-1/2 particles.

(c) Two identical spin-0 particles.

Solution:

Beforewestart:



$$\hat{H}(x)\,\phi_a = E_a\,\phi_a$$

$$\hat{H}(x)\,\phi_b = E_b\,\phi_b$$

$$\vdots$$

Now two particles:

$$\hat{H}_{total} \equiv \hat{H}_t = \hat{H}(x_1) + \hat{H}(x_2)$$

We can construct the eigenkets of this Hamiltonian easily using ϕ_i: For example:

$$\hat{H}_t\,\phi_a(x_1)\,\phi_a(x_2) = \hat{H}(x_1)\,\phi_a(x_1)\,\phi_a(x_2) + \hat{H}(x_2)\,\phi_a(x_1)\,\phi_a(x_2)$$

$$= (E_a + E_a)\,\phi_a(x_1)\,\phi_a(x_2)$$

so you can see:

$\phi_a(x_1)\,\phi_a(x_2)$ is the eigenfunction of H_t.

Remember the total wave function:

$$\psi = (\text{spatial part})\cdot(\text{spin part})$$

(a). Two distinguishable spin $1/2$ particle.

means we don't need symmetrize or antisymmetrize wave functions.

The ground state should be $\phi_a(x_1)\phi_a(x_2) \rightarrow 2E_a$.

spatial part of

Then:

ground state: $2E_0$

$$\phi_a(x_1)\phi_a(x_2)\,|00\rangle$$

$$\phi_a(x_1)\phi_a(x_2)\,|1,1\rangle$$

$$\phi_a(x_1)\phi_a(x_2)\,|1,0\rangle$$

$$\phi_a(x_1)\phi_a(x_2)\,|1,-1\rangle$$

degeneracies: 4

First excited state: $\phi_a(x_1)\phi_b(x_2)$ or $\phi_a(x_2)\phi_b(x_1) \rightarrow E_a + E_b$

spin part: $|0,0\rangle$ or $|1,m\rangle$

so:

degeneracies: $2 \times 4 = 8$

(b). Two identical spin 1/2 particles: $\psi = (\text{spatial})(\text{spin})$ must be antisymmetrized.

Note that $|0.0\rangle$ is antisymmetrized.

$|1,\pm1\rangle$ and $|1,0\rangle$ is symmetrized.

→ Ground State:

If we choose spatial part as:

$\phi_a(x_1)\,\phi_a(x_2)$ ← (only choice for ground state)

So: spin part must be:

$|0,0\rangle$

So:

ground state: $2E_a \longrightarrow \underbrace{\phi_a(x_1)\,\phi_a(x_2)\,|0,0\rangle}_{\text{antisymmetrized}}$.

→ First Excited State:

If we choose spatial part as:

$$\frac{1}{\sqrt{2}}\left(\phi_a(x_1)\phi_b(x_2) + \phi_a(x_2)\phi_b(x_1)\right)$$ symmetrized.

we need choose : $|0\,0\rangle$ as spin part.

If we choose :

$$\frac{1}{\sqrt{2}}\left(\phi_a(x_1)\phi_b(x_2) - \phi_a(x_2)\phi_b(x_1)\right)$$

we have to choose : $|1, m\rangle$ $(m=0, \pm 1)$ as spin part.

so :

degeneracies : 4.

(c). spin part can be only : $|0\,0\rangle$ which is symmetrized.

Ground state : $\phi_a(x_1)\phi_a(x_2)$ $\longrightarrow$ Degene... : 1

First excited states: $\frac{1}{\sqrt{2}}\left(\phi_a(x_1)\phi_b(x_2) + \phi_a(x_2)\phi_b(x_1)\right)$ $\rightarrow$ Degen: 1

(No $\frac{1}{\sqrt{2}}\left(\phi_a(x_1)\phi_b(x_2) - (x_2 \leftrightarrow x_1)\right)$

because total wave function must be symmetrized)