Homework #1 (due 10/11) (100 points)
(Physics 115B, Fall 2017)

(10) 1. The initial state $|\Psi_i\rangle$ of a quantum system is given in an orthonormal basis of three states $|\alpha\rangle$, $|\beta\rangle$, and $|\gamma\rangle$ that form a complete set:

$$\langle\alpha|\Psi_i\rangle = i/(3)^{1/2}, \qquad \langle\beta|\Psi_i\rangle = (2/3)^{1/2}, \qquad \langle\gamma|\Psi_i\rangle = 0$$

Calculate the probability of finding the system in a state $|\Psi_f\rangle$ given in the same basis as

$$\langle\alpha|\Psi_f\rangle = (1+i)/(3)^{1/2}, \quad \langle\beta|\Psi_f\rangle = 1/(6)^{1/2}, \qquad \langle\gamma|\Psi_f\rangle = 1/(6)^{1/2}$$

(20) 2. Griffiths 3.33

(20) 3. Griffiths 3.37

(20) 4. Write the two-state Hamiltonian matrix in a certain basis $|1\rangle$, $|2\rangle$ in a general form as

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

Impose hermiticity of H. Find the eigenvalues and the unitary transformation that diagonalizes the Hamiltonian. Express the eigenstates in terms of old base states.

(10) 5. The neutral K-meson K^0 and its antiparticle K'^0 from a two-state system whose energy matrix is not diagonal, but is given as

$$\begin{pmatrix} mc^2 & c^2\Delta m \\ c^2\Delta m & mc^2 \end{pmatrix}$$

Define a basis $|K_1\rangle$ and $|K_2\rangle$ in which the energy matrix is diagonal. What is the relation between the two bases?

(20) 6. Griffiths 3.38

Homework # 1 Solution

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1.

$|\alpha\rangle, |\beta\rangle, |\gamma\rangle$ are complete $\Longleftrightarrow$ $|\alpha\rangle\langle\alpha| + |\beta\rangle\langle\beta| + |\gamma\rangle\langle\gamma| = \hat{I}$

For any state:

$$|\psi\rangle = \hat{I}\,|\psi\rangle = \Big(|\alpha\rangle\langle\alpha| + |\beta\rangle\langle\beta| + |\gamma\rangle\langle\gamma|\Big)|\psi\rangle$$

$$= \langle\alpha|\psi\rangle\,|\alpha\rangle + \langle\beta|\psi\rangle\,|\beta\rangle + \langle\gamma|\psi\rangle\,|\gamma\rangle$$

$$\doteq \begin{pmatrix} \langle\alpha|\psi\rangle \\ \langle\beta|\psi\rangle \\ \langle\gamma|\psi\rangle \end{pmatrix}$$

Then:

$$|\Psi_i\rangle = \begin{pmatrix} \frac{i}{\sqrt{3}} \\ \sqrt{\frac{2}{3}} \\ 0 \end{pmatrix} \quad ; \quad |\Psi_f\rangle = \begin{pmatrix} \frac{1+i}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}$$

probability is given by:

$$P = |\langle \psi_i | \psi_f \rangle|^2 = \left| \frac{2}{3} - \frac{i}{3} \right|^2 = \frac{5}{9}$$

2. (3.33)

Our purpose: find out matrix representation of $\hat{x}, \hat{p}, \hat{H}$

Using the fact that:

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}} (a^\dagger + a)$$

$$\hat{p} = i\sqrt{\frac{\hbar m\omega}{2}} (a^\dagger - a)$$

Once we know $a^\dagger$, a, we know $\hat{x}$, $\hat{p}$. From the fact:

$$a^\dagger |n\rangle = \sqrt{n+1}\, |n+1\rangle$$

$$a |n\rangle = \sqrt{n}\, |n-1\rangle$$

we have :

$$a^{+} = \begin{pmatrix} 0 & 0 & & & \\ 1 & 0 & \ddots & & \\ & \sqrt{2} & \ddots & 0 & \\ & & \ddots & \sqrt{n} & \ddots \\ & & & & \ddots \end{pmatrix}$$

$$a = \begin{pmatrix} 0 & 1 & & & \\ & 0 & \sqrt{2} & & \\ & & 0 & \ddots & \\ & & & \ddots & \ddots \\ & & & & \ddots \end{pmatrix}$$

Then:

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}} \begin{pmatrix} 0 & 1 & & & \\ 1 & 0 & \sqrt{2} & & \\ & \sqrt{2} & 0 & \ddots & \\ & & \ddots & \ddots & \ddots \\ & & & \ddots & \end{pmatrix}$$

$$\hat{p} = i\sqrt{\frac{\hbar \omega m}{2}} \begin{pmatrix} 0 & -1 & & & \\ 1 & 0 & -\sqrt{2} & & \\ & \sqrt{2} & 0 & \ddots & \\ & & \ddots & \ddots & \ddots \\ & & & \ddots & \end{pmatrix}$$

To get $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2}{2}\hat{x}^2$, compute $\hat{x}^2$, $\hat{p}^2$

$$\hat{x}^2 = \frac{\hbar}{2m\omega}\left(a^+ + a\right)^2 = \frac{\hbar}{2m\omega}\left(a^{+2} + a^2 + a^+a + aa^+\right)$$

$$= \frac{\hbar}{2m\omega}\left(a^{+2} + a^2 + 2a^+a + \hat{I}\right)$$

$$\hat{p}^2 = -\frac{\hbar m\omega}{2}\left(a^+ - a\right)^2 = -\frac{\hbar m\omega}{2}\left(a^{+2} + a^2 - a^+a - aa^+\right)$$

$$= -\frac{\hbar m\omega}{2}\left(a^{+2} + a^2 - 2a^+a - \hat{I}\right)$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2}{2}\hat{x}^2 = \frac{\hbar\omega}{2}\left(2a^+a + \hat{I}\right)$$

$$= \hbar\omega\left(a^+a + \tfrac{1}{2}\hat{I}\right)$$

$$a^+a = \begin{pmatrix} 0 & & & \\ 1 & 0 & & \\ & \sqrt{2} & \ddots & \\ & & \ddots & \ddots \end{pmatrix}\begin{pmatrix} 0 & 1 & & \\ & 0 & \sqrt{2} & \\ & & \ddots & \ddots \\ & & & \ddots \end{pmatrix} = \begin{pmatrix} 0 & & & & \\ & 1 & & & \\ & & 2 & & \\ & & & 3 & \\ & & & & \ddots \end{pmatrix}$$

$$a^+a + \tfrac{1}{2}\hat{I} = \begin{pmatrix} \frac{1}{2} & & & \\ & \frac{3}{2} & & \\ & & \frac{5}{2} & \\ & & & \ddots \end{pmatrix}$$

Then:

$$\hat{H} = \hbar\omega\begin{pmatrix} \frac{1}{2} & & & \\ & \frac{3}{2} & & \\ & & \frac{5}{2} & \\ & & & \ddots \end{pmatrix}$$ which is diagonal.

3. (3.37)

$$\hat{H} = \begin{pmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{pmatrix}$$

First, solve the eigenvalues and the eigenkets.

$$\det \begin{pmatrix} a-\lambda & 0 & b \\ 0 & c-\lambda & 0 \\ b & 0 & a-\lambda \end{pmatrix} = 0$$

$$\Leftrightarrow \quad (\lambda - c)\left[(\lambda-a)^2 - b^2\right] = (\lambda - c)(\lambda - a + b)(\lambda - a - b)$$

$$\lambda_1 = a+b$$

$$\lambda_2 = c$$

$$\lambda_3 = a-b$$

we assume there is no degeneracy. in general.
$a+b \neq c$, $a-b \neq c$,
$a+b \neq a-b$

Eigenket corresponding to λ_1 : $|\lambda_1\rangle = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$\begin{pmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = (a+b) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$x_1 = x_3$$
$$x_2 = 0$$

then:

$$|\lambda_1\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

Eigenket corresponding to λ_2: $|\lambda_2\rangle = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$\begin{pmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = c \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$|\lambda_2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Because eigenkets of a hermitaian operator are orthogonal, we have:

$$|\lambda_3\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

$\lambda_1 = a+b$, $\quad |\lambda_1\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

$\lambda_2 = c$, $\quad |\lambda_2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\lambda_3 = a-b$, $\quad |\lambda_3\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

(a).

$$|\varphi(0)\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = |\lambda_2\rangle$$

$$|\varphi(t)\rangle = e^{-\frac{i\hat{H}t}{\hbar}} |\varphi(0)\rangle = e^{-\frac{i\hat{H}t}{\hbar}} |\lambda_2\rangle = e^{-\frac{i\lambda_2 t}{\hbar}} |\lambda_2\rangle$$

$$= e^{-\frac{ict}{\hbar}} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

(b)

$$|\varphi(0)\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}|\lambda_1\rangle + \frac{1}{\sqrt{2}}|\lambda_2\rangle$$

$$|\varphi(t)\rangle = e^{-\frac{i\hat{H}t}{\hbar}} \frac{1}{\sqrt{2}} |\lambda_1\rangle + e^{-\frac{i\hat{H}t}{\hbar}} \frac{1}{\sqrt{2}} |\lambda_3\rangle$$

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} e^{\frac{-i\lambda_1 t}{\hbar}} |\lambda_1\rangle + \frac{1}{\sqrt{2}} e^{\frac{-i\lambda_3 t}{\hbar}} |\lambda_3\rangle$$

$$= \frac{1}{\sqrt{2}} e^{-\frac{i(a+b)t}{\hbar}} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} + \frac{1}{\sqrt{2}} e^{\frac{-i(a-b)t}{\hbar}} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$= e^{-\frac{iat}{\hbar}} \begin{pmatrix} \cos\left(\frac{bt}{\hbar}\right) \\ 0 \\ -i\sin\left(\frac{bt}{\hbar}\right) \end{pmatrix}$$

4.

$$\hat{H} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$$

due to $\hat{H}^\dagger = \hat{H}$, we have:

H_{11}, H_{22} are real

$$H_{12} = H_{21}^*$$

To get the eigenvalues and eigenstates:

$$0 = \det \begin{pmatrix} H_{11} - \lambda & H_{12} \\ H_{21} & H_{22} - \lambda \end{pmatrix}$$

$$= (\lambda - H_{11})(\lambda - H_{22}) - |H_{12}|^2$$

$\Rightarrow$

$$\lambda_\pm = \frac{H_{11} + H_{22} \pm \sqrt{(H_{11}+H_{22})^2 - 4\left(H_{11}H_{22} - |H_{12}|^2\right)}}{2}$$

$$= \frac{H_{11} + H_{22} \pm \sqrt{(H_{11}-H_{22})^2 + 4|H_{12}|^2}}{2}$$

1° when $\lambda = \lambda_+$

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \lambda_+ \begin{pmatrix} a \\ b \end{pmatrix}$$

$$H_{12}\, b = (\lambda_+ - H_{11})\, a$$

$$|\lambda_+\rangle = A \begin{pmatrix} H_{12} \\ \lambda_+ - H_{11} \end{pmatrix}$$

'A' is the normalization constant.

$$= \frac{1}{\sqrt{|H_{12}|^2 + |\lambda_+ - H_{11}|^2}} \begin{pmatrix} H_{12} \\ \lambda_+ - H_{11} \end{pmatrix}$$

2° when $\lambda = \lambda_-$

we have $\lambda_+ \to \lambda_-$ in 1°,

$$|\lambda_-\rangle = \frac{1}{\sqrt{|H_{12}|^2 + |\lambda_- - H_{11}|^2}} \begin{pmatrix} H_{12} \\ \lambda_- - H_{11} \end{pmatrix}$$

In summary:

$$\lambda_+ = \frac{H_{11} + H_{22} + \sqrt{(H_{11} - H_{22})^2 + 4|H_{12}|^2}}{2} \qquad |\lambda_+\rangle = \frac{1}{\sqrt{|H_{12}|^2 + |\lambda_+ - H_{11}|^2}} \begin{pmatrix} H_{12} \\ \lambda_+ - H_{11} \end{pmatrix}$$

$$\lambda_- = \frac{H_{11} + H_{22} - \sqrt{(H_{11} - H_{22})^2 + 4|H_{12}|^2}}{2} \qquad |\lambda_-\rangle = \frac{1}{\sqrt{|H_{12}|^2 + |\lambda_- - H_{11}|^2}} \begin{pmatrix} H_{12} \\ \lambda_- - H_{11} \end{pmatrix}$$

The unitary transformation that diagonalizes the Hamiltonian:

$$U = \begin{pmatrix} |+\rangle & |-\rangle \end{pmatrix} = \begin{pmatrix} \dfrac{H_{12}}{\sqrt{|H_{12}|^2 + |\lambda_+ - H_{11}|^2}} & \dfrac{H_{12}}{\sqrt{|H_{12}|^2 + |\lambda_- - H_{11}|^2}} \\[2em] \dfrac{\lambda_+ - H_{11}}{\sqrt{|H_{12}|^2 + |\lambda_+ - H_{11}|^2}} & \dfrac{\lambda - H_{11}}{\sqrt{|H_{12}|^2 + |\lambda_- - H_{12}|^2}} \end{pmatrix}$$

and:

$$U^{\dagger} \hat{H} U = \begin{pmatrix} \lambda_+ & \\ & \lambda_- \end{pmatrix}$$

5.

Use the result we got in 4.

Let:
$$H_{11} = H_{22} = mc^2$$
$$H_{12} = H_{21} = \Delta m c^2$$

Then:

$$\lambda_+ = (m + \Delta m) c^2 \qquad |\lambda_+\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\lambda_- = (m - \Delta m) c^2 \qquad |\lambda_-\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

The relation between the two basis:

$$|\lambda_+\rangle = U^{\bullet} |1\rangle$$
$$|\lambda_-\rangle = U^{\bullet} |2\rangle$$

where $|1\rangle$, $|2\rangle$ are original bases.

$$U = \begin{pmatrix} |\lambda_+\rangle & |\lambda_-\rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \quad \text{here } U^{+} = U$$

6. (3.38)

(a).

1° $\hat{H} = \hbar\omega \begin{pmatrix} 1 & & \\ & 2 & \\ & & 2 \end{pmatrix}$

$\lambda_1 = \hbar\omega$ $\quad |\lambda_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\lambda_2 = 2\hbar\omega$ $\quad |\lambda_2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\lambda_3 = 2\hbar\omega$ $\quad |\lambda_3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

Remark: due to $\lambda_2 = \lambda_3$, the subspace spanned by $|\lambda_2\rangle$ and $|\lambda_3\rangle$ are degenerate. So we can choose arbitary linear ~~comba~~ combination of $|\lambda_2\rangle$ and $|\lambda_3\rangle$ as our eigenvectors as long as they are orthogonal to each other.

2° $$\hat{A} = \begin{pmatrix} 0 & \lambda & 0 \\ \lambda & 0 & 0 \\ 0 & 0 & 2\lambda \end{pmatrix} = \begin{pmatrix} 0 & \lambda \\ \lambda & 0 \end{pmatrix} \oplus (2\lambda)$$

Then the eigenvalues:

$$0 = \det \begin{pmatrix} -\mu & \lambda \\ \lambda & -\mu \end{pmatrix} = \mu^2 - \lambda^2$$

$$\mu = \pm \lambda$$

$\mu_1 = \lambda$ $$\begin{pmatrix} 0 & \lambda \\ \lambda & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \lambda \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\Rightarrow \quad |\mu_1\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\mu_2 = -\lambda$ orthogonal to $|\mu_1\rangle$

$$|\mu_2\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

For total space:

$\mu_1 = \lambda$ $$|\mu_1\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$\mu_2 = -\lambda$ $$|\mu_2\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix}$$

$\mu_3 = 2\lambda$ $\qquad |\mu_3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$3^\circ \quad \hat{B} = \begin{pmatrix} 2\mu & 0 & 0 \\ 0 & 0 & \mu \\ 0 & \mu & 0 \end{pmatrix}$

~~compta~~ compared with 2°.

in 2°: $\lambda \longrightarrow \mu$

$(1\ 2\ 3) \longrightarrow (3\ 2\ 1)$

we get result for 3°:

$\nu_1 = 2\mu \qquad |\nu_1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\nu_2 = -\mu \qquad |\nu_2\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$

$\nu_3 = \mu \qquad |\nu_3\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

(b)

$$\langle \hat{H} \rangle = \begin{pmatrix} c_1^* & c_2^* & c_3^* \end{pmatrix} \begin{pmatrix} \hbar\omega & & \\ & 2\hbar\omega & \\ & & 2\hbar\omega \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

$$= \begin{pmatrix} c_1^* & c_2^* & c_3^* \end{pmatrix} \begin{pmatrix} \hbar\omega\, c_1 \\ 2\hbar\omega\, c_2 \\ 2\hbar\omega\, c_3 \end{pmatrix}$$

$$= \hbar\omega\left(|c_1|^2 + 2|c_2|^2 + 2|c_3|^2\right)$$

$$\langle \hat{A} \rangle = \begin{pmatrix} c_1^* & c_2^* & c_3^* \end{pmatrix} \begin{pmatrix} 0 & \lambda & 0 \\ \lambda & 0 & 0 \\ 0 & 0 & 2\lambda \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

$$= \lambda\left(c_1^* c_2 + c_2^* c_1 + 2|c_3|^2\right)$$

$$\langle \hat{B} \rangle = \begin{pmatrix} c_1^* & c_2^* & c_3^* \end{pmatrix} \begin{pmatrix} 2\mu & 0 & 0 \\ 0 & 0 & \mu \\ 0 & \mu & 0 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

$$= \mu\left(2|c_1|^2 + c_2^* c_3 + c_3^* c_2\right)$$

(c)

$$|\psi(0)\rangle = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}, \quad |\psi(t)\rangle = e^{-\frac{i\hat{H}t}{\hbar}} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

Then:

$$|\psi(t)\rangle = \begin{pmatrix} e^{-i\omega t} c_1 \\ e^{-2i\omega t} c_2 \\ e^{-2i\omega t} c_3 \end{pmatrix}$$

When we measure $\hat{H}$: we might get:

$$\lambda_1 = \hbar\omega \qquad P_{\lambda_1} = |\langle\lambda_1|\psi(t)\rangle|^2 = |c_1|^2$$

$$\lambda_2 = \lambda_3 = 2\hbar\omega \qquad P_{\lambda=2\hbar\omega} = |\langle\lambda_2|\psi(t)\rangle|^2 + |\langle\lambda_3|\psi(t)\rangle|^2$$

$$= |c_2|^2 + |c_3|^2$$

When we measure $\hat{A}$, we might get:

$$\mu_1 = \lambda \qquad P_{\mu_1} = |\langle\mu_1|\psi\rangle|^2 = \left| \frac{e^{-i\omega t} c_1}{\sqrt{2}} + \frac{e^{-2i\omega t} c_2}{\sqrt{2}} \right|^2$$

$$= \frac{1}{2} \left| c_1 + e^{-i\omega t} c_2 \right|^2$$

$\mu_2 = -\lambda \qquad P_{\mu_2} = |\langle \mu_2 | \psi \rangle|^2$

$= \frac{1}{2} \left| c_1 - e^{-i\omega t} c_2 \right|^2$

$\mu_3 = 2\lambda \qquad P_{\mu_3} = |\langle \mu_3 | \psi \rangle|^2 = |c_3|^2$

When we measure $\hat{B}$: we might get:

$\nu_1 = 2\mu \qquad P_{\nu_1} = |\langle \nu_1 | \psi(t) \rangle|^2$

$= |c_1|^2$

$\nu_2 = -\mu \qquad P_{\nu_2} = |\langle \nu_2 | \psi(t) \rangle|^2$

$= \frac{1}{2} |c_2 - c_3|^2$

$\nu_3 = \mu \qquad P_{\nu_3} = \frac{1}{2} |c_2 + c_3|^2$