# Week 9 QM Discussion

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Office Hours: Tuesday 10am-12pm, Tutoring Center.

**Warm up Problem**

(a) A 2-fermion system has a wave function $\Psi(1,2)$. What condition must it satisfy if the particle are identical?

(b) How does this imply the elementary statement of the Pauli exclusion principle that no two electrons in an atom can have identical quantum numbers?

(c) The first excited state of Mg has the configuration (3s,3p) for the valence electrons. Which values of $L$ and $S$ are possible? What is the form of the spatial part of their wave functions using the single-particle functions $\psi_s(\vec{r})$ and $\psi_p(\vec{r})$? Which will have the lowest energy, and why?

**White Dwarf**

White dwarf is a star which does not collapse due to the pressure of the degenerate electron gas, i.e., due to the Pauli exclusion principle. When the mass of this star is not very large, the pressure is also not so large and average energy of the electron is much smaller than its rest energy, i.e., the electron can be considered as non-relativistic objects. However, as mass grows, approaching the Chandrasekhar limit (when white dwarf collapses to the neutron star) the average energy of the electrons can be considered as ultra-relativistic, i.e., with rest energy much smaller than kinetic energy. Then their energy depends on their momentum in the form $E = c|\vec{p}|$. (Simply speaking, our system now is electron gas with dispersion relation $E = c|\vec{p}|$ instead of $E = \frac{p^2}{2m}$.)

(a) Obtain the Fermi energy in terms of the density of electrons.

(b) What's the density of state?

(c) What's the total energy in terms of total volume and the number of electrons?

(a). $\psi(1,2) = -\psi(2,1)$

(b) have the same quantum number:

$$\psi(1,1) = -\psi(1,1)$$

$$\Rightarrow \psi(1,1) = 0$$

such a state does not exist.

(c) $(3s, 3p)$

$l_1 = 0$, $l_2 = 1$ $\qquad \vec{L} = \vec{l_1} + \vec{l_2}$

$L$ can be $1$

$s_1 = \frac{1}{2}$, $s_2 = \frac{1}{2}$ $\qquad S$ can be $0, 1$

$$\phi_{L=1,\, S=0}(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}}\left(\psi_s(\vec{r}_1)\,\psi_p(\vec{r}_2) + \psi_s(\vec{r}_2)\,\psi_p(\vec{r}_1)\right)$$

due to spin part is antisymmetric

$$\phi_{L=1,\, S=1}(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}}\left(\psi_s(\vec{r}_1)\,\psi_p(\vec{r}_2) - \psi_s(\vec{r}_2)\,\psi_p(\vec{r}_1)\right)$$

due to the fact that spin part is symmetric.

$\phi_{L=1,S=1}$ will have the lower energy.

Reason: From class we know that:

$$\langle(\Delta x)^2\rangle_+ = \langle(\Delta x)^2\rangle_d - 2\left|\langle x\rangle_{ab}\right|^2$$

$$\langle(\Delta x)^2\rangle_- = \langle(\Delta x)^2\rangle_d + 2\left|\langle x\rangle_{ab}\right|^2$$

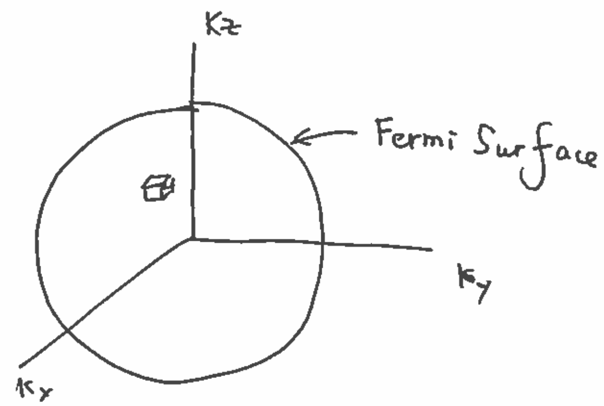
so for antisymmetric wave function, the electrons ~~tren t~~ trend to get far away from each other. (exchange force)

we know you need put in energy to get electrons closer. so $\phi_{L=1,S=1}$ will have the lower energy.

# White Dwarf:

(a) Similar to the case we learned in class:

$$N = 2 \cdot \frac{\frac{4}{3}\pi k_F^2}{\frac{(2\pi)^3}{L_x L_y L_z}} \Rightarrow k_F = (3\pi^2 n)^{1/3}$$




$$E_F = c\hbar k_F = c\hbar \left(3\pi^2 n\right)^{\frac{1}{3}}$$

$$n = \frac{1}{3\pi^2 (c\hbar)^3} E^3$$

(b). Density of ~~the~~ state: $\rho(E) = \frac{dn}{dE} = \frac{E^2}{\pi^2 (c\hbar)^3}$

(c) $$E_{tot} = \int_0^{E_F} E \cdot V \ \rho(E)\, dE$$

$$= \frac{V}{\pi^2 (c\hbar)^3} \int_0^{E_F} E^3 dE = \frac{V}{\pi^2 (c\hbar)^3} \frac{E_F^4}{4}$$