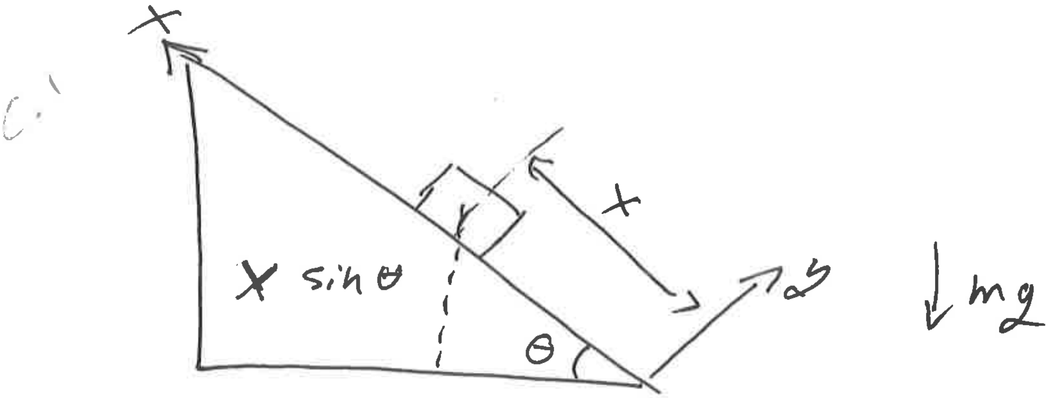

in this coordinate system

(1) $T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2$

(2) $U = mgx \sin\theta + mgy \cos\theta$

(3) $\mathcal{L} = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 - mgx \sin\theta - mgy \cos\theta$

The Constraint Here is:

(4) $G = y = 0$; so Lagrange eqs:

(5) $\frac{\partial \mathcal{L}}{\partial x} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) + \lambda \frac{\partial G}{\partial x} = 0$

(6) $\frac{\partial \mathcal{L}}{\partial y} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{y}}\right) + \lambda \frac{\partial G}{\partial y} = 0$

and from (4)

(7) $\ddot{G} = \ddot{y} = 0$

from eq (5) we have:

(8) $-mg \sin\theta - m\ddot{x} = 0$

(9) $\ddot{x} = -g \sin\theta$

and from (6) we have

(10) $-mg\cos\theta - m\ddot{y} + \lambda = 0$

plugging in (7); $\ddot{y} = 0$ we get

(11) $\lambda = mg\cos\theta$

(12) $a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = g\sin\theta$ ✓

(13) $F_x = \lambda \frac{\partial G}{\partial x}$

The Constraint

(14) $f_x = 0$ as expected

and

(15) $F_y = \lambda \frac{\partial G}{\partial y}$

(16) $f_y = mg\cos\theta$ normal force