Math 115A: Sample midterm

Sections 1 and 3. Instructor: James Freitag

Problem 1 Bases and linear transformations.

Let $\beta = ((1,0),(0,1))$ be the standard ordered basis for $\mathbb{R}^2$. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that $T(3,4) = (2,3)$ and $T(1,1) = (0,3)$. Calculate $[T]^{\beta}_{\beta}$.

Problem 2 How to span a space

Let $T : V \to W$ be a linear transformation. Show that the nullity of T is zero if and only if T is injective.

Problem 3 Rank and nullity

Let U, V, W be vector spaces such that $dim(U) = 6, dim(V) = 4, dim(W) = 5$ and let $T : U \to V$ and $S : V \to W$ be linear. Let $R = S \circ T$ be the composition. Prove that R is not surjective.

Problem 4 Nullity and rank

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

What are the rank and nullity of T?