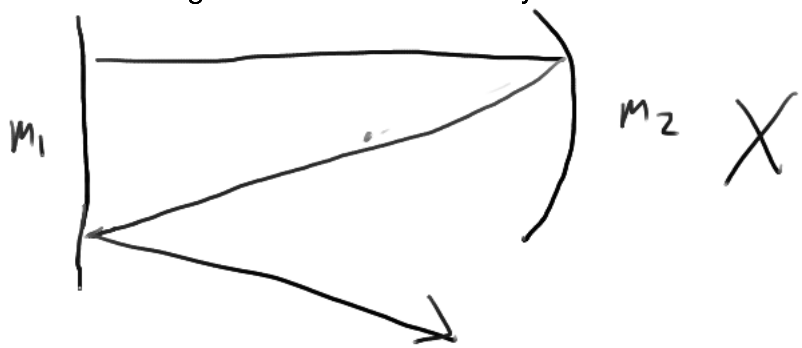
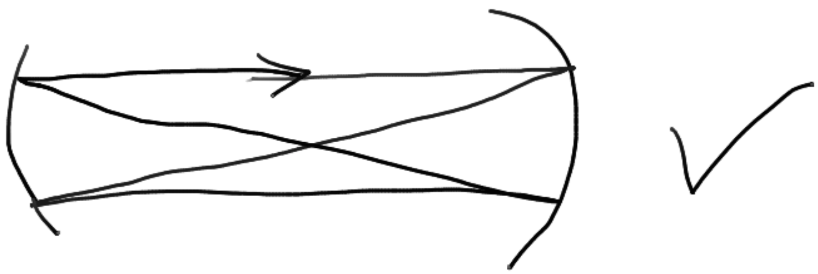
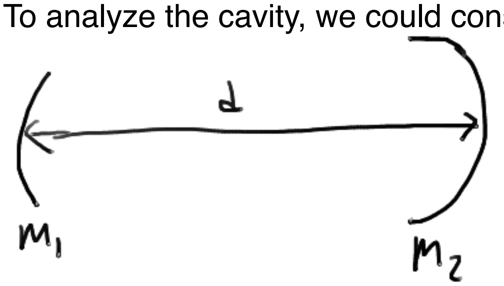
An optical cavity is an arrangement of two or more mirrors and possibly some lenses. We'll work out the resonance conditions of a cavity later (n Lambda/2 = Length), but before you worry about things like cavity finesse you need to know if the cavity will bounce the light back and forth stably:

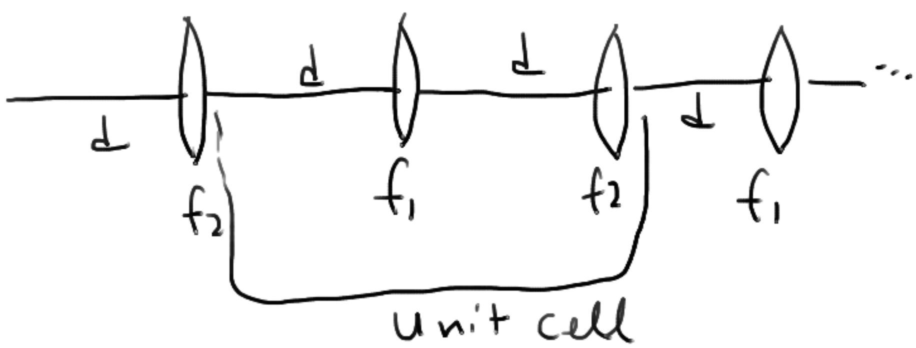




Now these are just examples of possible unstable/stable cavities. Both of the geometries shown can be made stable or unstable depending on the specific parameters you choose.

To analyze the cavity, we could construct the equivalent lens system as:





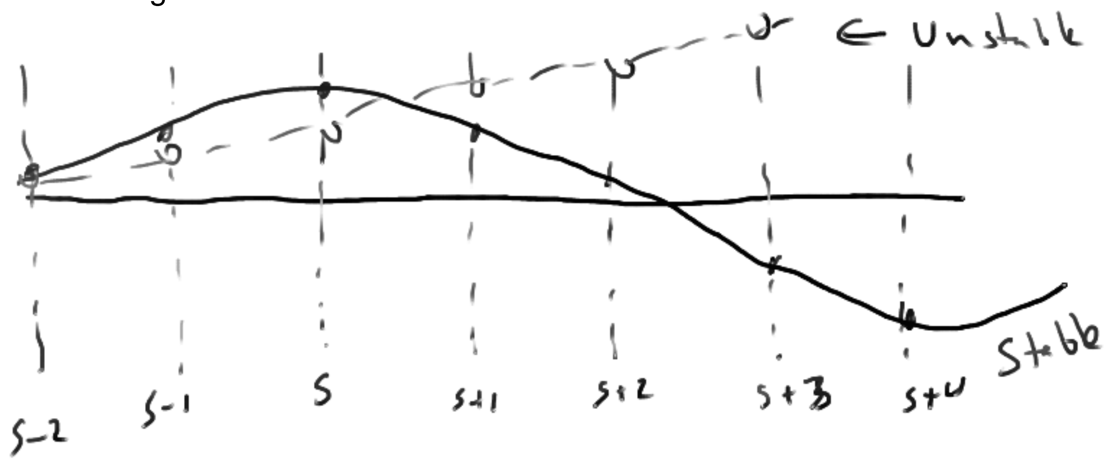
Here f2 = R2/2 and f1 = R1/2. I like the lens system just because it's easier for ME to think about. You may prefer to just use the curved mirror T matrix. Thus, we have the total T matrix as:

$$T=\begin{pmatrix}1&0\\-\frac{1}{f_2}&1\end{pmatrix}\begin{pmatrix}1&d\\0&1\end{pmatrix}\begin{pmatrix}1&0\\-\frac{1}{f_1}&1\end{pmatrix}\begin{pmatrix}1&d\\0&1\end{pmatrix}$$

$$T=\begin{pmatrix}1&d\\-\frac{1}{f_2}&1-\frac{d}{f_2}\end{pmatrix}\begin{pmatrix}1&d\\-\frac{1}{f_1}&1-\frac{d}{f_1}\end{pmatrix}$$

$$T=\begin{pmatrix}1-\frac{d}{f_1}&d+d\left(1-\frac{d}{f_1}\right)\\-\frac{1}{f_2}-\frac{1}{f_1}\left(1-\frac{d}{f_2}\right)&-\frac{d}{f_2}+\left(1-\frac{d}{f_2}\right)\left(1-\frac{d}{f_1}\right)\end{pmatrix}$$

So, you could just apply this matrix to some intial {r,r'} and see if the solution is stable (i.e. the r value oscillates between some maximum excursion values OR if it grows, in which case it will eventually miss a mirror). However, with just a little bit of maths we can do even better. Imagine we look at the position of the ray at several planes of the succeeding unit cells.



We can relate the {r,r'} at different planes via their ABCD matrix that connects them:

(1) $r_{s+1} = A\, r_s + B\, r'_s$

(2) $r'_{s+1} = C\, r_s + D\, r'_s$

We can combine the two to make the difference equation:

(1) yields --> $r'_{s+1} = 1/B(r_{s+2} - A\, r_{s+1})$

Comparing this with (2) yields --> $1/B(r_{s+2} - A\, r_{s+1}) = C\, r_s + D\, r'_s = C\, r_s + D/B(r_{s+1} - Ar_s)$

Or, finally: $r_{s+2} - A\, r_{s+1} - D(r_{s+1} - Ar_s) - BC\, r_s = 0$

$r_{s+2} - A\, r_{s+1} - Dr_{s+1} + (AD - BC)\, r_s = 0$

$r_{s+2} - 2(A + D)/2\, r_{s+1} + r_s = 0$

Now, we guess the solution of the form:

$rs = r_o\, e^{is\theta}$

which gives the characteristic equation:

$$r_0 e^{i\theta s}\Big(\underbrace{e^{2i\theta} - 2\left(\tfrac{A+D}{2}\right)e^{i\theta} + 1}_{\text{must be zero}}\Big) = 0$$

$$\uparrow$$
NOT zero

Quadratic eqn:

$$e^{i\theta} = \frac{A+D}{2} \pm i\sqrt{1 - \left(\frac{A+D}{2}\right)^2}$$

As long as, the square root is real the solutions are complex conjugates, so you have a general solution of the form:

$$r_s = r_0 e^{i\theta s} + r_0^* e^{-i\theta s}$$

$$= r_{max} \sin(s\theta + \alpha)$$

which is oscillatory and stable. Thus, the condition for stability is:

$$-1 \le \left(\frac{A+D}{2}\right) \le 1$$

which we can write as:

$$0 \le \frac{A+D+2}{4} \le 1$$

$$\frac{A+D+2}{4} = \frac{1}{4}\left(1 - \frac{d}{f_1} - \frac{d}{f_2} + \left(1 - \frac{d}{f_1}\right)\left(1 - \frac{d}{f_2}\right) + 2\right)$$

$$= \frac{1}{4}\left(4 - \frac{2d}{f_1} - \frac{2d}{f_2} + \frac{d^2}{f_1 f_2}\right)$$

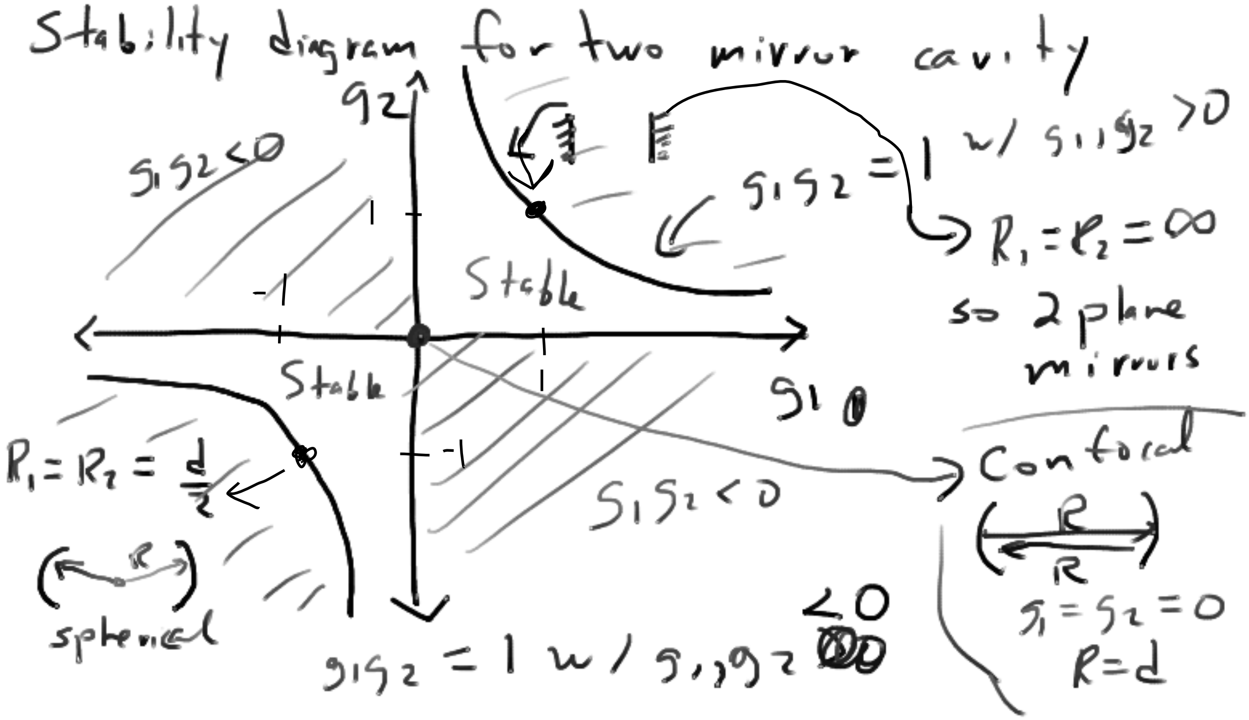
$$= \left(1 - \frac{d}{2f_1}\right)\left(1 - \frac{d}{2f_2}\right)$$

Stability Condition:

$$0 \leq \left(1 - \frac{d}{2f_1}\right)\left(1 - \frac{d}{2f_2}\right) \leq 1$$

$$0 \leq g_1 g_2 \leq 1$$

gi = 1 - d/Ri.



Anywhere on the boundary lines is a “conditionally” stable cavity -- requires perfect alignment, so you usually want to be somewhere well inside the stable region.
Note you can work this out for ANY cavity system starting from the -1 <= (A+D)/2 <=1 condition!

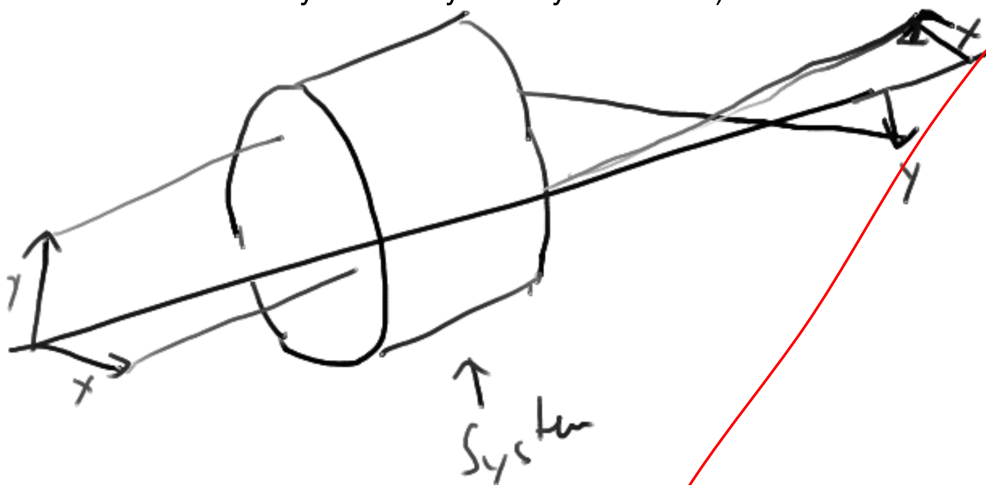
All this means that after some number of roundtrips (usually 1 or 2) the light has come back to EXACTLY the same place as before (spatially). Clearly this a prerequisite for any cavity to work. We’ll revisit this more later.

Problems:
1. Analyze some Verdeyen cavities.

Imperfect Imaging (Astigmatism, Chroma, ?)

We are now in a position to discuss the performance (or lack thereof) of an imaging system.

Astigmatism
Astigmatism arises when the focus of a system differs in two transverse directions (i.e. there's an fx and an fy -- axial symmetry is broken.)



Can correct with cylindrical lenses.

Spherical Aberrations, Coma
Perfect lens has a paraboloid shape, however, hard to grind. Using spherical lenses can lead to curvature of the wavefronts at the focus. Use parabolic lenses if you care (coma can still happen. Off axis images are distorted by coma due to lens imperfections.)

Chroma (or chromatic aberrations)
The index of refraction is a function of wavelength thus different wavelengths can be focussed differently. Using lens doublets made of different material you can negate this effect -- achromat (or achromatic lens).

All of these things can be treated in the ABCD framework easily.

If you ever look carefully at the output of a cavity you'll see exactly this. (A real laser beam can be described as a combination of these Hermite-Gaussian basis functions.)

Problem: Align a cavity and see this?

ABCD Law for Gaussian Beams

If we had done the standard derivation of the Hermite gaussian by solving that awful differential equation then at one point we would have found in convenient to define a complex function:

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i\frac{\lambda_0}{\pi n w^2(z)}$$

In this parameter is everything you care about for a Gaussian beam. And, as such, people use it to describe their beams -- it's called the "q" parameter.

It turns out there is an extremely useful relationship for the q parameter at one z location and the q parameter at another z location if you know the ABCD matrix that connects the two spots (obviously for free space you can just change z in q). It is:

q2 = (Aq1+ B)/(Cq1 + D) -- no formal proof has been shown as far as Verdeyen knows.

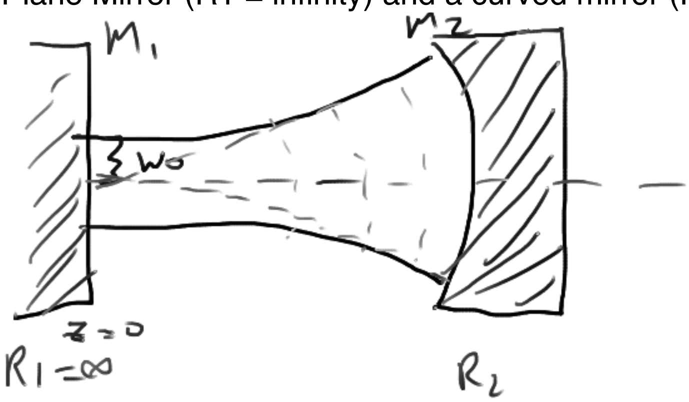
Since it's usually easier to work with 1/q, we rearrange:

1/q2 = (C + D(1/q1))/(A + B(1/q1)).
An example?

Cavities

So, we now have seen how to find out whether a cavity is stable or not. And we also know how to describe a real laser beam being affected by curved mirrors, lenses, and so on; but, how do we put them together? I mean with ray-tracing we can show a cavity should be stable, but what does the light inside of it really look like? Answer: Use our gaussian beam analysis... but first let's look at a simple case where we can intuit the answer. Later we'll do it generally/formally.

Plane Mirror (R1 = infinity) and a curved mirror (R2) for a cavity of length L.



As we'll see the beam/laser must fit n*lambda/2 inside a cavity so the radiation has to be a constant phase front on each mirror. Since R1 is flat, the beam must have a waist there. Necessarily, in this simple cavity this will be the only focus and is also called the cavity waist. The curvature R2 also matches the field curvature at M2. So, we have the beam waist, trivially, in the cavity as:

$$w^2(z) = w_0^2\left(1+\left(\frac{z}{z_0}\right)^2\right)$$

but what is the value of wo? Enforce the boundary condition at the other end of the cavity :

$$R(z) = z\left(1+\left(\frac{z_0}{z}\right)^2\right)$$

$$R(L) = R_2 = L\left(1+\left(\frac{z_0}{L}\right)^2\right)$$

$$z_0 = L\sqrt{\frac{R_2}{L}-1} = \sqrt{LR_2}\sqrt{1-\frac{L}{R_2}}$$

$$\text{Also, } z_0 = \frac{\pi w_0^2}{\lambda}$$

(So note first off for a stable resonator to use a Gaussian beam 0 <= L/R2 <= 1.)
Within these limits the cavity waist is at the flat mirror and has a waist of:

$$w_0^2 = \frac{\lambda}{\pi}\sqrt{LR_2}\sqrt{1-\frac{L}{R_2}}$$

$$\text{And } w^2(z) = w_0^2\left(1+\left(\frac{z}{z_0}\right)^2\right)$$

(Similar eqn for $R(z)$).

What about if two curved mirrors? Little more complicated. Three mirrors and a lens? Etc. Need a formal/generic treatment --> q parameter plus ABCD matrix.

<u>ABCD Laws and cavities for Gaussian beams:</u>

<u>Important Realization:</u>
The gaussian beam inside the cavity must reproduce itself in both shape and in phase after a round trip through the system. (A field that does this is called a cavity mode.)

Therefore after one round trip we have:

$$q(z_1 + \text{round trip}) = q(z_1)$$

Or, if we know the ABCD matrix of one roundtrip (unit cell) then:

$$q(z_1) = \frac{A q(z_1) + B}{C q(z_1) + D}$$

OR 1/q(z1) = (C + D(1/q(z1)))/(A + B(1/q(z1))).

(Two equations, two unknowns -- q is complex).
Solve for 1/q:

$$\frac{1}{q(z_1)} = \frac{C + D\left(\frac{1}{q(z_1)}\right)}{A + B\left(\frac{1}{q(z_1)}\right)}$$

$$B\left(\frac{1}{q(z_1)}\right)^2 + (A - D)\frac{1}{q(z_1)} - C = 0$$

$$\therefore \frac{1}{q} = -\frac{(A-D)}{2B} \pm \frac{1}{B}\sqrt{\left(\frac{A-D}{2}\right)^2 + BC}$$

Now, AD-BC = 1, thus we can write:

$$\sqrt{\frac{A^2+D^2}{4} - \underbrace{\frac{AD}{2}}_{+\frac{AD}{2} - \frac{AD}{2}} + BC}$$

$$= \sqrt{\frac{A^2+D^2+2AD}{4} - AD + BC}$$

$$= \sqrt{\left(\frac{A+D}{2}\right)^2 - 1}$$

$$= i\sqrt{1 - \left(\frac{A+D}{2}\right)^2}$$

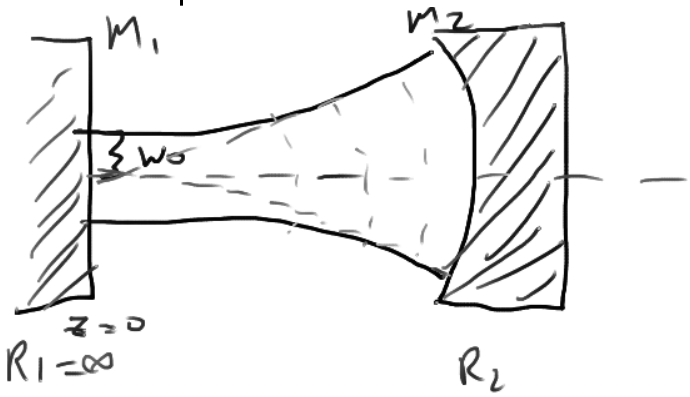
And:

$$\frac{1}{q} = -\frac{A-D}{2B} - i\,\frac{\sqrt{1-\left(\frac{A+D}{2}\right)^2}}{B}$$

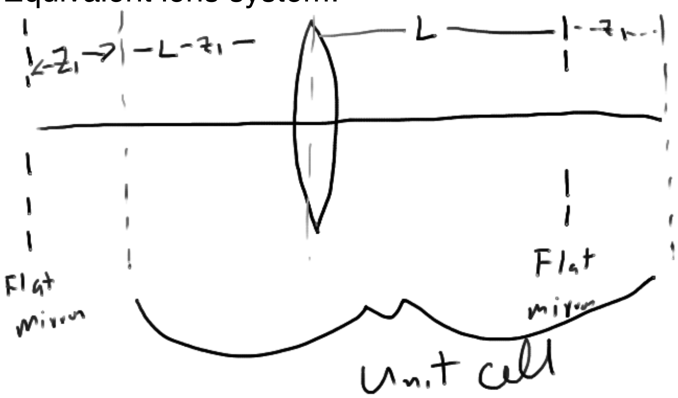
$$\& \quad \frac{1}{q(z)} = \frac{1}{R(z)} - i\,\frac{\lambda}{\pi n w(z)^2}$$

$$\underset{@\, z_1!}{\underset{\uparrow}{R(z_1)}} = \frac{-2B}{A-D} \quad \& \quad w(z_1^2) = \frac{B\lambda}{\pi n\sqrt{1-\left(\frac{A+D}{2}\right)^2}}$$

So, all you need to know is the ABCD matrix of a cavity and you're done! Recall our earlier example:



Equivalent lens system:



$$T = \begin{pmatrix} 1 & L+z_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} 1 & L-z_1 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & L+z_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & L-z_1 \\ -\frac{1}{f} & 1-\left(\frac{L-z_1}{f}\right) \end{pmatrix}$$

$$= \begin{pmatrix} 1-\left(\frac{L+z_1}{f}\right) & L-z_1 + (L+z_1)\left(1-\left(\frac{L-z_1}{f}\right)\right) \\ -\frac{1}{f} & 1-\left(\frac{L-z_1}{f}\right) \end{pmatrix}$$

Now,

$$R(z_1) = \frac{-2B}{A-D} = \frac{-2\left(L - z_1 + (L+z_1)\left(1 - \left(\frac{L-z_1}{f}\right)\right)\right)}{\cancel{1} - \left(\frac{L+z_1}{f}\right) - \left[\cancel{1} - \left(\frac{L-z_1}{f}\right)\right]}$$

$$R(z_1) = \frac{-2\left(2L - (L+z_1)\left(\frac{L-z_1}{f}\right)\right)}{-2z_1/f}$$

$$R(z_1) = \frac{f\left(2L - (L+z_1)\left(\frac{L-z_1}{f}\right)\right)}{z_1}$$

$$z_1 \to 0 \qquad R(0) = \infty \quad \checkmark$$

$$z_1 \to L \qquad R(L) = \frac{f}{L}\left(2L - 2L\cdot 0\right)$$

$$= 2f = R_1 \quad \checkmark$$

Recall: $\boxed{f_1 = \frac{R_1}{2}}$

So, we have an expression for R everywhere in the cavity!
Likewise,

$$W(z_1)^2 = \frac{\lambda}{n\pi} \frac{B}{\sqrt{1 - \left(\frac{A+D}{2}\right)^2}}$$

$$\frac{A+D}{2} = \frac{2 - \frac{2L}{f}}{2} = 1 - \frac{L}{f}$$

$$w(z_1)^2 = \frac{\lambda}{n\pi} \frac{\left(2L - (L+z_1)\left(\frac{L-z_1}{f}\right)\right)}{\sqrt{1 - \left(1 - \frac{L}{f}\right)^2}}$$

$$w(z)^2 = \frac{\lambda}{n\pi} \frac{\left(2L - \frac{L^2}{f} + \frac{z_1^2}{f}\right)}{\sqrt{\left(1 - \left(1 - \frac{L}{f}\right)^2\right)}}$$

$$w(z)^2 = \frac{\lambda}{n\pi} \cdot \frac{\left(2L - \frac{L^2}{f} + \frac{z_1^2}{f}\right)}{\sqrt{\frac{2L}{f} - \frac{L^2}{f^2}}}$$

Now, clearly z1 = 0 is the minimum waist as we expected. And it has a value of:

$$w(0)^2 = \frac{\lambda}{n\pi} \frac{2L - 2L^2/R}{\sqrt{\left(\frac{4L}{R}\right)\left(1-\frac{L}{R}\right)}} = \frac{\lambda}{n\pi} \frac{2L\left(1-\frac{L}{R}\right)}{\sqrt{\frac{4L}{R}\left(1-\frac{L}{R}\right)}}$$

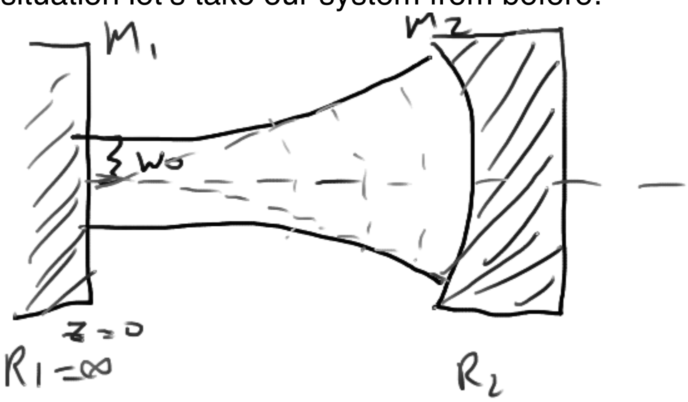
$$w(0)^2 = \frac{\lambda}{n\pi} \sqrt{LR} \sqrt{1-\frac{L}{R}} \quad \checkmark$$

This works for any cavity! Just find the ABCD matrix and you're done!

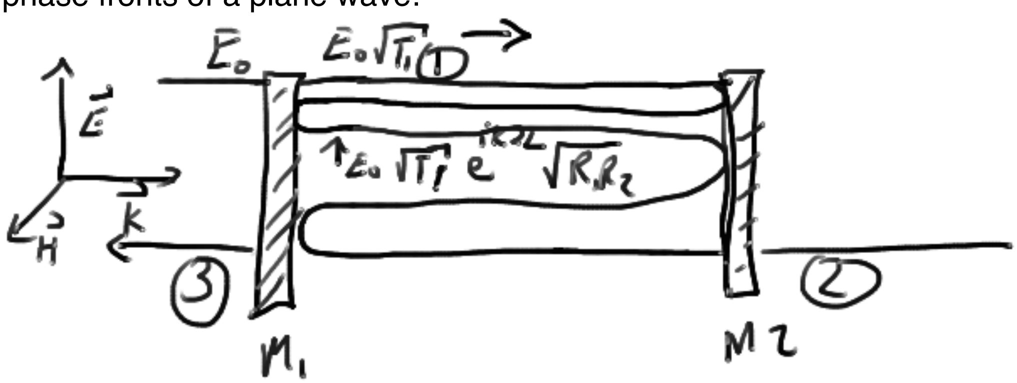
PROBLEMS:
1. Calculate mode volume? Basically assign Verdeyen 5.4 as a homework problem.
2. Do a bunch of different cavities like in verdeyen chp 5 homework problems.
3. Actual numbers + Mode matching problem

Resonant Cavities

So far we've just been concerned with the stability of the cavity as determined by ray tracing, but for light to appreciably propagate inside of a cavity it must also be resonant. That is, there must be an integer number of half-wavelengths that fit in it. To analyze the situation let's take our system from before:



where the phase fronts must match the mirror surfaces and turn them into plane mirrors and plane waves. It's much easier to see resonance effects for the simple evolving phase fronts of a plane wave:



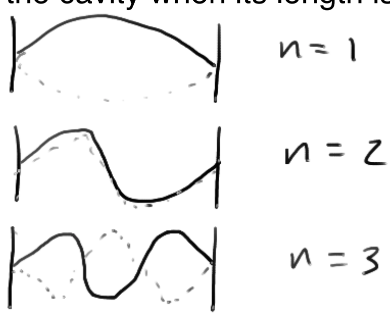
M1 has POWER reflectivity, transmission, and loss of R_1, T_1, and L_1. Wit $T_1 = 1 - R_1 - L_1$. Same for M2.

Now, the E-field at position 1, just inside the first mirror is:

$$E_1 = E_0\sqrt{T_1}\left(1 + \sqrt{R_1R_2}\,e^{2ikL} + \left(\sqrt{R_1R_2}\right)^2 e^{4ikL} + \ldots\right)$$
$$= E_0\sqrt{T_1}\sum_{j=0}^{\infty}\left(\sqrt{R_1R_2}\right)^j e^{2ijkL}$$

Now, unless the exponent of the e is an integer multiple of 2π, then the ∑ = 0 --> No light in the cavity. (Basically, all the reflected rays destructively interfere with each other).

If, however, 2kL = 2πn --> L = nλ/2 then the sum is non-zero. That is, energy can be in the cavity when its length is an integer multiple of a half-wavelength.



Notice how wave starts over exactly the same!

The cavity is then said to have a resonance at: f = nc/(2L), where n denotes the longitudinal mode number.

The n and n+1 modes are split by an amount equal to: c/(2L). This is called the cavity “Free Spectral Range”: FSR = c/(2L).

The name comes about because it’s how far you can tune a laser or cavity before another half-wavelength can fit in the cavity. So, it’s sort of like the spectral range that is free of complications and useful for analyzing a laser (or continuously tuning a laser), etc.

Now, suppose 2kL = 2πn --> exp(2ijkL) = 1, thus:

$$\sum_{j=0}^{\infty} \left(\sqrt{R_1 R_2}\right)^j = \frac{1}{1-\sqrt{R_1 R_2}}$$

Thus, @ (1):

$$E_1 = \frac{E_0\sqrt{T_1}}{1-\sqrt{R_1 R_2}} \Rightarrow \overset{\text{Power:}}{} \frac{P_1}{P_0} = \frac{T_1}{\left(1-\sqrt{R_1 R_2}\right)^2}$$

$$\boxed{\begin{array}{l}\text{If } L_i = 0 \;\&\; R_1 = R_2 = R \;\; (\text{i.e. } T_1 = 1-R): \\ \qquad \dfrac{P_1}{P_0} = \dfrac{1}{1-R} \qquad (\therefore P_1 \gg P_0!)\end{array}}$$

For example if R = 0.98 then the power is 50X higher inside the cavity than the power that's being put into the cavity! How is this possible? Does it violate energy conservation? No, it's power not energy. The light bounces around inside the cavity for awhile and gets stored up. You can see this if you look at the power transmitted through the cavity:

$$P_2 = T_2\frac{P_1}{P_0} = 1 \quad \left(\begin{array}{l}\text{for no losses \&} \\ \text{identical mirrors.}\end{array}\right).$$

Now, let's remove the on-resonance approximation:

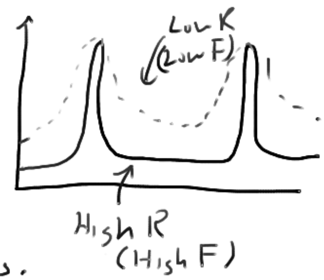
$$\sum_{j=0}^{\infty} \left(\sqrt{R_1 R_2}\, e^{2ikL}\right)^j = \frac{1}{1-\sqrt{R_1 R_2}\, e^{2ikL}}.$$

And the power transmitted through the cavity is:

$$\frac{P_3}{P_0} = \frac{T_2 P_1}{P_0} \xrightarrow{\text{No losses, Identical mirrors}} = \frac{T^2}{1 - 2R\cos(2KL) + R^2}$$

$$\frac{P_3}{P_0} = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2(KL)}$$

closer R is to 1 more peaked cavity transmission is.



The cavity transmission falls to 1/2 when:

$$\frac{4R}{(1-R)^2}\sin^2\left(\frac{2\pi\,\delta f L}{c}\right) = 1$$

$\left(KL \rightarrow \frac{2\pi\,\delta f L}{c} \right.$ b/c around each resonance is the same as $\left. KL = 0. \right)$

$$\delta f = \frac{c}{2\pi L}\arcsin\left(\frac{1-R}{2\sqrt{R}}\right)$$

The Finesse of a cavity is defined as the cavity FSR divided by the trasmission FWHM (more useful that Q):

$$\boxed{F = \frac{FSR}{2\,\delta f} = \frac{\pi}{2\arcsin\left(\frac{1-R}{2\sqrt{R}}\right)}}$$

This is the correct expression for the Finesse, but it’s hugely annoying. Luckily, we usually care about good mirrors where R ~ 1. Then:

$$\arcsin(x) \approx 0 + \left.\frac{1}{\sqrt{1-x^2}}\right|_{x=0} x + \ldots$$

$$x = \frac{1-R}{2\sqrt{R}} \simeq 0 \text{ for good mirrors.}$$

$$\text{So, } F = \frac{\pi\sqrt{R}}{1-R}$$

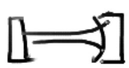
So, for R = 0.98 mirrors F = 156 and power build up is about 50X inside of the cavity.

<u>Non-plane wave effects</u>
So, we worked all of this out for plane waves which have a phase factor of exp(i(kx-wt)), however, we know real beams aren't plane waves and the gaussian beams have the form:

$$\vec{E}(x,y,z) = \vec{E}_{m,p}\, H_m\left(\frac{\sqrt{2}\,x}{w(z)}\right) H_p\left(\frac{\sqrt{2}\,y}{w(z)}\right)$$
$$\times \frac{w_0}{w(z)} \exp\left(-\frac{x^2+y^2}{w^2(z)}\right)$$
$$\times \exp\left(-i\left(kz-(1+m+p)\tan^{-1}\left(\frac{z}{z_0}\right)\right)\right)$$
$$\times \exp\left(\frac{-ikr^2}{2R(z)}\right)$$

So, we have the requirement that after a round trip the phase has accumulated 2π. Thus,

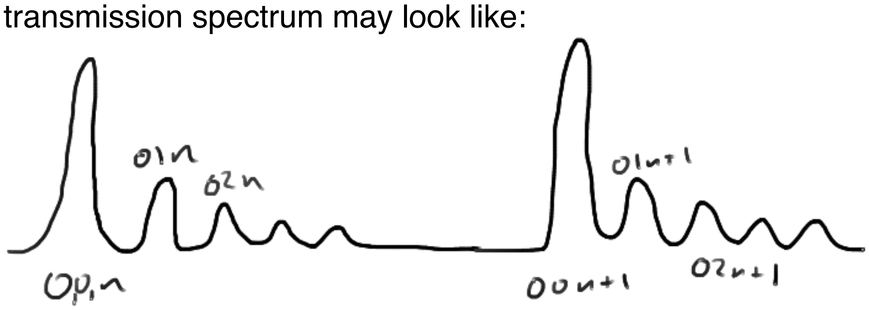
$$n\pi = kL-(1+m+p)\tan^{-1}\left(\frac{L}{z_0}\right)$$

Arithmetic for 

$$f_{0_{n,m,p}} = \frac{c}{2L}\left(n + \frac{1+m+p}{\pi}\tan^{-1}\left(\frac{\sqrt{L/R_2}}{\sqrt{1-L/R_2}}\right)\right)$$

Here R_2 is radius of curvature.

So, the different TEM modes have different resonant frequencies. So cavity transmission spectrum may look like:

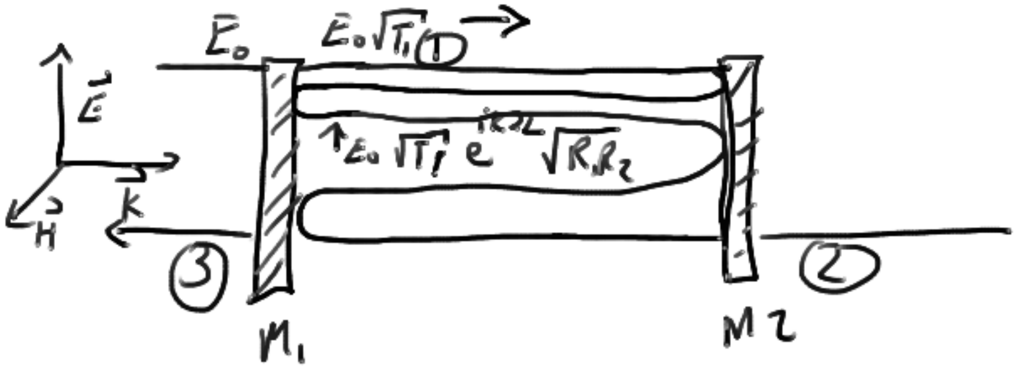


For confocal cavity modes are degenerate.

PROBLEMS:
1. Work out power reflected from the cavity. Why does the PDH lock use this light instead of the light trasmitted through the cavity.
2. A cavity is said to be impedance matched when the amount of light going into the cavity equals the amount of light going out of a cavity (T + losses). Build a doubling cavity where the doubling efficiency is 30%. What are the mirror specs? Why should you impedance match a cavity? (maybe ask that first.)

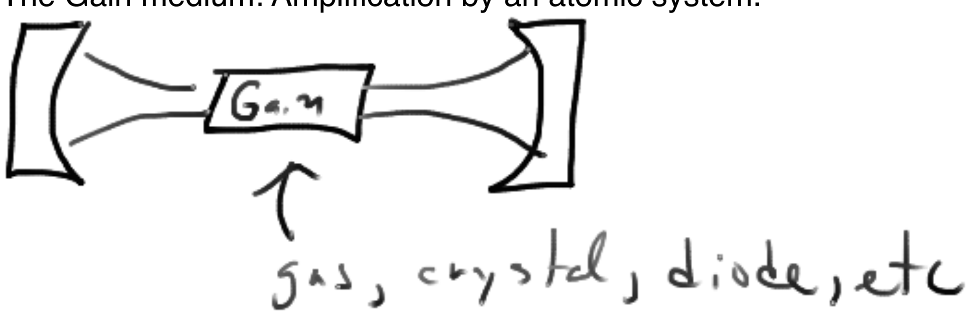
3. Mode matching.
4. Work out cavity resonance expression for Hermite-Gaussians in a cavity with two curved mirrors. What does it look like in the confocal geometry?

Lasers
Consider our cavity from before:

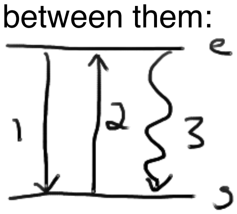


Suppose now that we insert some Gain medium in the middle of it. Such that the Power out of the Gain medium is Pout = G*Pin. Thus, if sqrt(G)*sqrt(R1*R2)>=1 then after each round trip the amplitude has increased. This runaway process is a laser.

The Gain medium: Amplification by an atomic system.



Consider two levels of the Gain medium. There are three things that can happen between them:



1. Stimulated emission. The stimulated photon is identical to the stimulating photon -- same exact wavefunction.
2. Absorption.
3. Spontaneous emission.

In traversing a small part dz of the gain medium the light intensity changes by dI, such that:

$$\frac{dI_\nu}{dz} = +hf\,\frac{B_{21} I_\nu}{c}\, g(\nu)\, N_2 \qquad (\text{Stim. Emission})$$

$$- hf\,\frac{B_{12} I_\nu}{c}\, g(\nu)\, N_1 \qquad (\text{absorption})$$

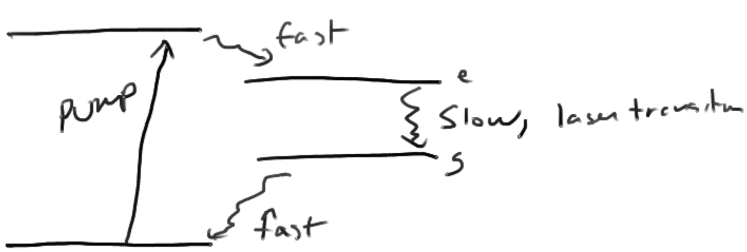
$$+ hf\, A_{21}\,\Delta\nu\, g(\nu) \underset{\substack{\uparrow \\ \text{so proper} \\ \text{pol.}}}{\left(\frac{1}{2}\right)} \underset{\substack{\uparrow \\ \text{solid} \\ \text{angle}}}{\frac{d\Omega}{4\pi}} \times N_2$$

g(v) should be g(f) and is the lineshape function. It's integral is normalized to 1. With g1B12 = g2B21 and A21/B21 = 8π^2hf^3/c^3:

$$\frac{dI_f}{dz} = \left\{\frac{A_{21}\,\lambda^2}{8\pi n^2}\, g(\nu)\right\}\left(N_2 - \frac{g_2}{g_1} N_1\right) I_f$$

$$\boxed{\frac{dI_f}{dz} \equiv \gamma(f)\, I_f}$$

So for the intensity to grow the gamma>1. The only way for that to happen is if N2< g2/g1 N1! (for g2=g1) then this means there must be an inversion (N2> N1). Making an inversion of population is thermodynamically a negative temperature --> So you can't do it easily. We won't worry about how you do it, but usually it's like this:

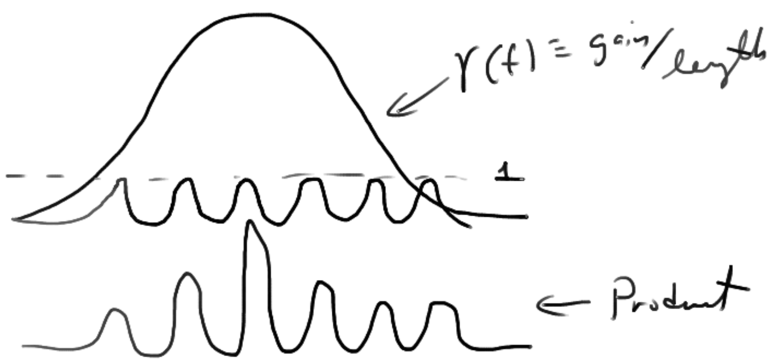


Suppose the gain medium is homogeneously broadened and can be described by a Lorentzian lineshape. Then,

$$\gamma(f) = A_{21}\left(\frac{\lambda^2}{8\pi n^2}\right)(N_2 - N_1)\frac{(\Delta f/2)^2}{(f_0-f)^2+\left(\frac{\Delta f}{2}\right)^2}$$

$$\gamma(f) = \gamma(f_0)\frac{(\Delta f/2)^2}{(f_0-f)^2+\left(\frac{\Delta f}{2}\right)^2}$$

gamma can be greater than 1 over a wide range of frequencies (say GHz for Doppler broadening), how does a laser choose it's wavelength?



So, which cavity mode does the laser choose to operate at? The one with the largest gain. After a few round trips the laser intensity can just build indefinitely. Eventually, the stimulating field is so large that the atoms are stimulated as fast as they are pumped up. This leads to an equilibrium and the gain is said to be saturated. At this point all of the gain is gobbled up by the mode with the fewest losses -- gain competition. And the other modes die out. (For heterogeneous broadening laser can run multimode.)

Gain Saturation and Threshold:
1. Assign a problem like section 8.3 in Verdeyen.

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