Math 115A: Problem set 5

Sections 1 and 3. Instructor: James Freitag

Due 11/6

Problem 1   Similarity

Let $A, B, C \in M_{n \times n}(F)$. Show that A is similar to A. Show that if A is similar to B then B is similar to A. Show that if A is similar to B and B is similar to C then A is similar to C.

Problem 2   Traces of similarity

If $A \in M_{n \times n}(F)$, define the trace $tr(A)$ to be the sum of the diagonal matrices of A.
    Let $A, B \in M_{n \times n}(F)$ be similar matrices. Show that $tr(A) = tr(B)$.

Problem 3   Determinants and area

Let $B = \{(a, b) \in \mathbb{R}^2 \,|\, 0 \le a, b \le 1\}$. Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

be a matrix with nonzero determinant. What is the the image of B under the linear transformation given by A? Prove that the area of $L_A(B) = det(A)$.

Problem 4   Similar eigenvalues

Let A, B be $n \times n$ matrices which are similar. Show that A, B have the same eigenvalues.

Problem 5   Exercises from the book

- Problem 1 from 4.4.

- Problems 6 and 9 from 5.1.