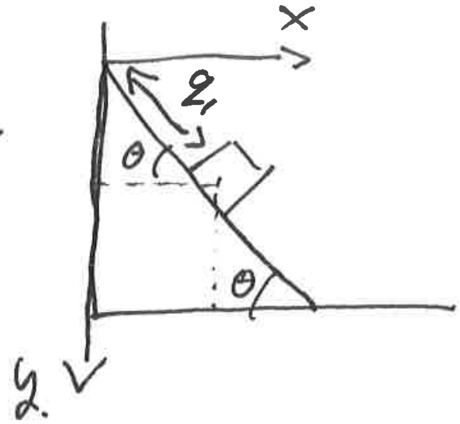
$\mathcal{L}.5.1$ smoother choice:

Note x-y!



(1) $x = q_2$

(2) $y = q_1 \sin\theta$

(3) $T = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 = \frac{1}{2}m\dot{q}_2^{\,2} + \frac{1}{2}m\dot{q}_1^{\,2}\sin^2\theta$

(4) $U = -mgy = -mgq_1\sin\theta$

So

(5) $\mathcal{L} = T - U = \frac{1}{2}m\dot{q}_2^{\,2} + \frac{1}{2}m\dot{q}_1^{\,2}\sin^2\theta + mgq_1\sin\theta$

q_1: $\frac{\partial \mathcal{L}}{\partial q_1} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_1}\right) = 0$

(6) $mg\sin\theta - m\ddot{q}_1\sin^2\theta = 0$

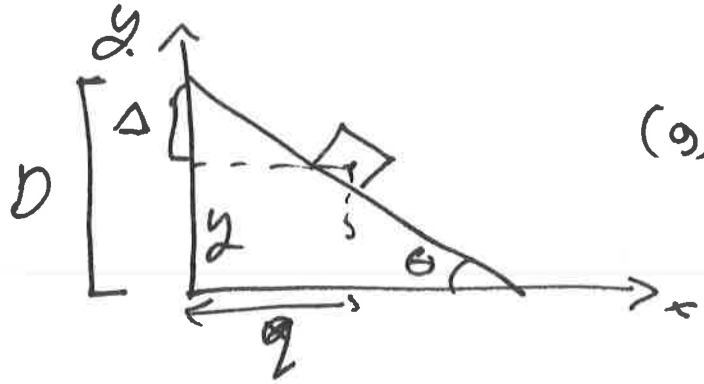
(7) $\ddot{q}_1 = \frac{g}{\sin\theta}$

(8) q_2: $m\ddot{q}_2 = 0$

This is ok but Not good enough because the connection to acceleration is Complicated.

$\mathcal{L}$.5.2

instead The BEST way is to see that the system has only ONE D.O.F.



(9) $\Delta = q \tan\theta$

(10) $x = q$

(11) $y = D - q\tan\theta$

So

(12) $T = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} m\dot{y}^2 = \frac{1}{2} m\dot{q}^2 + \frac{1}{2} m\dot{q}^2 \tan^2\theta$

(13) $U = mgy = mg(D - q\tan\theta)$

(14) $\mathcal{L} = \frac{1}{2} m\dot{q}^2 + \frac{1}{2} m\dot{q}^2\tan^2\theta - mg(D - q\tan\theta)$

(15) $\frac{\partial\mathcal{L}}{\partial q} - \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{q}}\right) = mg\tan\theta - m\ddot{q} - m\ddot{q}\tan^2\theta \stackrel{!}{=} 0$

(16) $\boxed{\ddot{q} = g\frac{\tan\theta}{1+\tan^2\theta} = g\sin\theta\cos\theta}$

(17) $a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{\ddot{q}^2(1+\tan^2\theta)} = g\sin\theta\cos\theta\sqrt{\frac{1}{\cos^2\theta}} = g\sin\theta$