105A - Set 4

(Grades are out of 150)

1. (32pt) A disk of mass M and radius R rolls without slipping down a plane inclined from the horizontal by an angle α, , in the presence of gravity. The disk has a short weightless axle of negligible radius. From this axis is suspended a simple pendulum of length $l < R$ and whose bob has a mass m. Consider that the motion of the pendulum takes place in the plane of the disk, and find the Lagrange's equation of the system.
   Hint 1 : make a sketch of the system.
   Hint 2: remember to take into account the rotational energy of the disk.

2. (30pt) Two blocks, each of mass M, are connected by an extension less, uniform string of length l. One bloc is places on a smooth horizontal surface and the other block hangs over the side in the presence of gravity. The string passing over a frictionless pulley (see Figure 1). The initial conditions given for the system are that the system starts at rest, and at $y(t = 0) = 0$. For the following, **(i)** write the Lagrangian of the system, **(ii)** find the equation of motion **(iii)** and find the solution, using the initial ocniditons.

   (a) (16pt) when the mass of the string is negligible.

   (b) (16pt) when the string has a mass m.

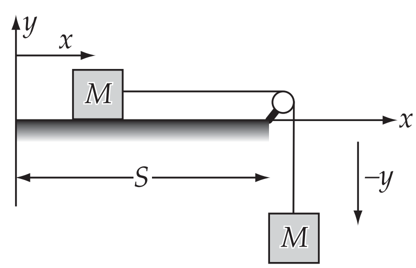

Figure 1: Two masses connected by a string with length l.

3. (30pt) A ladder of mass m and length $2l$ is standing up against a vertical wall with initial angle α relative to the horizontal. There is no friction between the ladder and the wall or the floor. The ladder begins to slide down with zero initial velocity. Denote by$\theta(t)$ the angle the ladder makes with the horizontal after it starts to slide.

   (a) (16pt) Write down Lagrangian equations of motion with two constraints describing the contact of the ladder with the vertical wall and the floor. Use the coordinates x, y and θ.

   (b) (7pt) Find the expression for $\ddot{\theta}$ as a function of θ (and g and l).
   Hint: think, what is the generalized coordinate here?

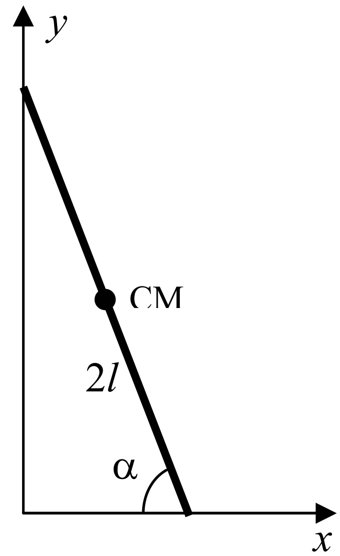

Figure 2: A sliding ladder

(c) (7pt) Find the expression for $\dot{\theta}^2$ as a function of θ (and g and l).
Hint: use energy conservation

(d) **Bonus! + 15pt** Find the angle θ_c when the ladder losses contact with the vertical wall.

4. (28pt) A particle moves without friction in a conservative field of force produced by various mass distribution. In each instance, the force generated by the volume element of the distribution is decided from a potential that is proportional to the mass of the volume element and is a function of the scalar distance frown the volume element. For the following fixed, homogeneous mass distribution, state the conserved quantities in the motion of the particles:

   (a) (4pt) The mass is uniformly distributed in the plane $z = 0$.

   (b) (4pt) The mass is uniformly distributed in the half-plane $z = 0, y > 0$.

   (c) (4pt) The mass is uniformly distributed in a circular cylinder of *infinite* length, with axis along the z axis.

   (d) (4pt) The mass is uniformly distributed in a circular cylinder of *finite* length, with axis along the z axis.

   (e) (4pt) The mass is uniformly distributed in right cylinder of elliptical cross section and infinite length with axis along the z axis.

   (f) (4pt) The mass is uniformly distributed in a dumbbell whose axis is oriented along the z axis.

   (g) (4pt) The mass is in the form of a uniform wire wound in the geometry of an infinite helical solenoid, with axis along the z axis.