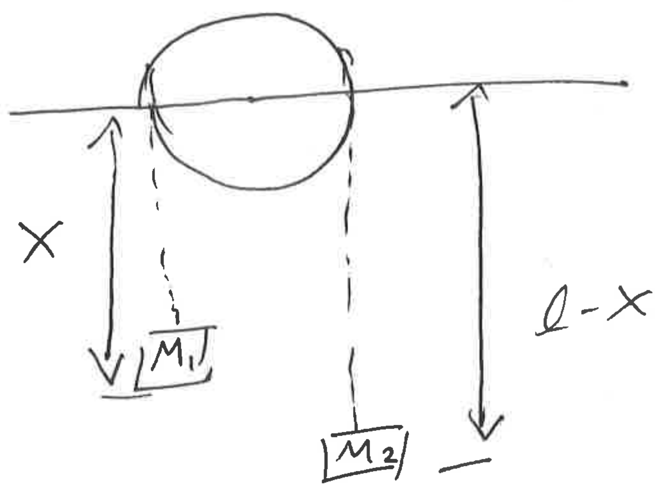
§6.4

Atwood's machine




pulley is frictionless and massless

the length of the rope between them is $l$.

Q what is the independent coordinate?

A. $x$

① $\mathcal{L} = ?$ ② $\ddot{x} = ?$

$$U = -M_1 g x - M_2 g (l-x)$$

$$T = \frac{1}{2}(M_1 + M_2)\dot{x}^2$$

$$\mathcal{L} = T - U = \frac{1}{2}(M_1 + M_2)\dot{x}^2 + M_1 g x + M_2 g (l-x)$$

$$\frac{\partial \mathcal{L}}{\partial x} = (M_1 - M_2) g$$

$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = (M_1 + M_2)\dot{x} \qquad \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = (M_1 + M_2)\ddot{x}$$

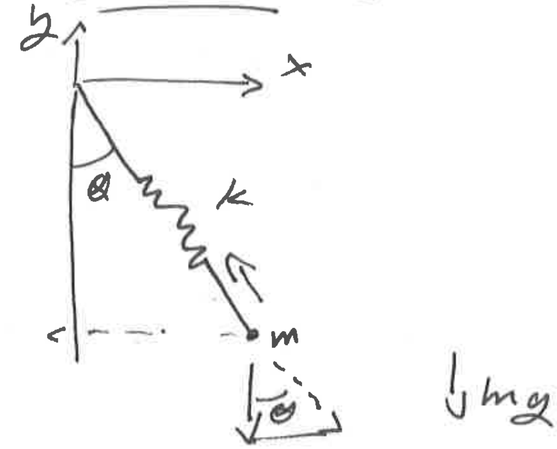
So $$\frac{\partial \mathcal{L}}{\partial x} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right)$$

$$(M_1 + M_2)\ddot{x} = (M_1 - M_2)g$$

$$\boxed{\ddot{x} = \frac{M_1 - M_2}{M_1 + M_2}\, g}$$

$\mathcal{L}$G.5 ~~Thell spherical~~ Example 3

$b$ = unextended length of spring




find $\mathcal{L}$ and E.o.M.

① what should be the generalized coordinates
How many D.o.f are there (2)

$r, \theta$ $\qquad T = \frac{1}{2} m (\dot{\vec{r}})^2$

$$\dot{\vec{r}} = \dot{r}\hat{r} + \dot{\theta} r \hat{\theta}$$ we saw in week 1

So
$$T = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2)$$

$$U = \frac{1}{2} k (r-b)^2 + mgy = \frac{1}{2} k (r-b) - mgr\cos\theta$$
$y = -r\cos\theta$

$$\mathcal{L} = T - U = \frac{1}{2} m (\dot{r}^2 + r^2\dot{\theta}^2) - \frac{1}{2} k (r-b)^2 + mgr\cos\theta$$

$$r: \quad \frac{\partial \mathcal{L}}{\partial r} - \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{r}}\right) = 0 \Rightarrow \cancel{m\dot{\theta}^2 - k(r-b) + mg\cos\theta = m\ddot{r}}$$

$$mr\dot{\theta}^2 - k(r-b) + mg\cos\theta - (m\ddot{r}) = 0$$

$$\theta: \quad -mg\sin\theta - (\ddot{mr^2\dot{\theta}}) = 0$$

$$m r \dot{\theta}^2 - k(r-b) + mg\cos\theta - m\ddot{r} = 0$$

$$-mg\sin\theta - m r^2 \ddot{\theta} - 2m r \dot{r}\dot{\theta} = 0$$

which I can write:

$$\begin{array}{ll} r: & \\ \theta: & \end{array} \left[ \begin{array}{l} \ddot{r} - r\dot{\theta}^2 + \frac{k}{m}(r-b) - g\cos\theta = 0 \\ \\ \ddot{\theta} + \frac{2}{r}\dot{r}\dot{\theta} + \frac{g}{r}\sin\theta = 0 \end{array} \right]$$