problem set 1, <u>integrales</u>

Q 1.

The integral to solve is:

$$(1)\quad I = \int \frac{dx}{x\,(x^2+a^2)}$$

have: $u = \dfrac{x^2}{a^2+x^2}$; $du = 2x\dfrac{a^2}{a^2+x^2}dx$

so $dx = \dfrac{(a^2+x^2)^2}{2xa^2}du$

$$(2)\quad I = \int \frac{(a^2+x^2)^2}{2xa^2}\frac{1}{x(x^2+a^2)}du =$$

$$= \int \frac{a^2+x^2}{2\underbrace{x^2a^2}_{u}}du = \int \frac{du}{2a^2u} = \frac{1}{2a^2}\ln u$$

so

$$(3)\quad \boxed{I = \frac{1}{2a^2}\ln\frac{x^2}{a^2+x^2}}$$

Q. 5 The first integral was

$$(4)\quad I = \int \frac{dv}{\frac{g}{k}\sin\theta - v^2} = \int \frac{dv}{a^2 - v^2} = \int \frac{dv}{(a-v)(a+v)}$$

$$a^2 = \frac{g}{k}\sin\theta$$

have $u = \dfrac{a-v}{a+v}$ $\quad du = \left(\dfrac{-1}{a+v} - \dfrac{a-v}{(a+v)^2}\right)dv =$

$$= \frac{-a-v-a+v}{(a+v)^2} = \frac{-2a}{(a+v)^2}\,dv$$

pluging this to (4)

$$(5)\quad I = \int \frac{(a+v)^2\,du}{-2a\,(a-v)(a+v)} = \frac{1}{-2a}\int \underbrace{\frac{a+v}{a-v}}_{u^{-1}}\,du = \frac{-1}{2a}\int \frac{du}{u}$$

$$= -\frac{1}{2a}\ln u = -\frac{1}{2a}\ln\left(\frac{a-v}{a+v}\right) \equiv \frac{1}{2a}\tanh^{-1}\left(\frac{\text{[illegible]}}{\text{[illegible]}}\right)$$

Def of $\tanh^{-1}$

so

$$(6)\quad \boxed{I = \tanh^{-1}\left(\frac{v}{\sqrt{g/k\,\sin\theta}}\right)}$$

Q. 5 the second integral was

(7) $I = \int \tanh(\sqrt{gk\sin\theta}\, t)\,dt = \int \tanh(at)\,dt$

remember that

$$\tanh x = \frac{e^{2x}-1}{e^{2x}+1} \quad ; \quad \cosh x = \frac{e^{x}+e^{-x}}{2}$$

(8) $$I = \int \frac{e^{2at}-1}{e^{2at}+1}\,dt \underset{x=at}{=} \frac{1}{a}\int \frac{e^{2x}-1}{e^{2x}+1}\,dx$$

set: $\frac{1}{2}(e^{x}+e^{-x}) = u$; $\frac{1}{2}(e^{x}-e^{-x})\,dx = du$

So

$$I = \frac{1}{a}\int \frac{e^{2x}-1}{e^{2x}+1}\,\frac{2\,du}{e^{x}-e^{-x}} = \frac{1}{a}\int \frac{e^{-x}}{e^{-x}}\,\frac{e^{2x}-1}{e^{2x}+1}\,\frac{2\,du}{e^{x}-e^{-x}}$$

$$= \frac{1}{a}\int \frac{\cancel{e^{x}-e^{-x}}}{\underbrace{e^{x}+e^{-x}}_{u}}\,\frac{2\,du}{\cancel{e^{x}-e^{-x}}} = \frac{1}{a}\int\frac{du}{u} = \frac{1}{a}\ln u$$

$$= \frac{1}{a}\ln\left(\frac{e^{x}+e^{-x}}{2}\right) = \frac{1}{a}\ln(\cosh x) = \boxed{\frac{1}{a}\ln\left[\cosh(\sqrt{gk\sin\theta}\,t)\right]}$$

$x = \sqrt{gk\sin\theta}\,t$