105A - Set 3

(Grades are out of 150)

1. (50pt) **Laser Beam in Refractive Medium**
   A well known example of the use of the theory of variations is the **Fermat's principle**: light travels by the path that takes the least amount of time. Consider a medium with an index of refraction given by $n(x, y) = n_0(1 + ky)$. Recall that the speed of light in a medium with index n is given by $v = c/n$, where c is the speed of light. Find the function that describes the path of light in this medium. Determine a specific equation for the path of a laser beam that initially starts at the origin propagating in the x direction, as shown in figure 1. *Hint: Use the second form of the Euler equation (see*
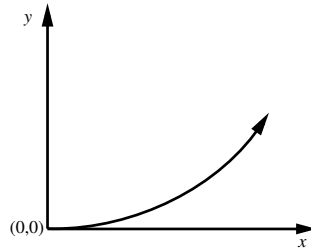
Figure 1: Fermat's principle

reading material online!) and use the plot to find the constant.

2. (50pt) Show that the shortest distance between two points in **three dimensional space** is a straight line.
   Hint 1: Define a family of solution (similar to what we did in class), where your coordinates will depend on the parameter that defines this family of solutions, i.e., $x(\alpha), y(\alpha)$ and $z(\alpha)$.
   Hint 2: You may find this definition useful:

$$dl = d\alpha\sqrt{\left(\frac{dx}{d\alpha}\right)^2 + \left(\frac{dy}{d\alpha}\right)^2 + \left(\frac{dz}{d\alpha}\right)^2} \tag{1}$$

3. (50pt) Show that the geodesic (i.e,. the shortest possible line between two points on a curved surface) on the surface of a circular cylinder is a segment of a helix (see Figure 2 behind the page).
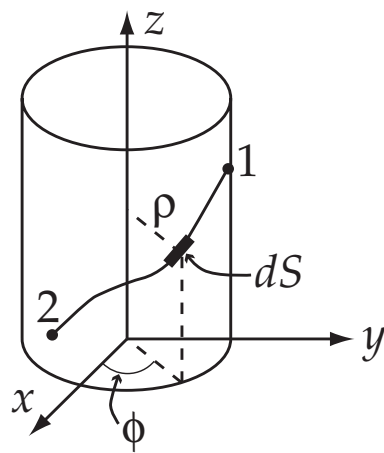
Figure 2: Surface of a cylinder