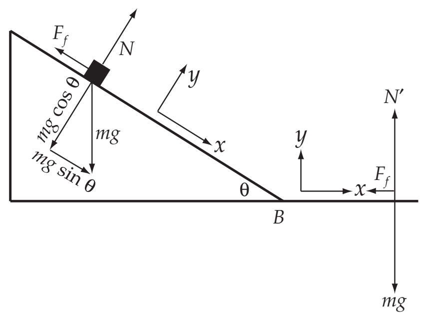
2-24. A skier weighing 90 kg starts from rest down a hill inclined at 17°. He skis 100 m down the hill and then coasts for 70 m along level snow until he stops. Find the coefficient of kinetic friction between the skis and the snow. What velocity does the skier have at the bottom of the hill?

2-24.



d = length of incline
s = distance skier travels along level ground

While on the plane:

$$\sum F_y = N - mg\cos\theta = m\ddot{y} = 0 \qquad \text{so } N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_f\,; \qquad F_f = \mu N = \mu mg\cos\theta$$

$$mg\sin\theta - \mu mg\cos\theta = m\ddot{x}$$

So the acceleration down the plane is:

$$a_1 = g\left(\sin\theta - \mu\cos\theta\right) = \text{ constant}$$

While on level ground: $N' = mg\,;\ F_f = -\mu mg$

So $\sum F_x = m\ddot{x}$ becomes $-\mu mg = m\ddot{x}$

The acceleration while on level ground is

$$a_2 = -\mu g = \text{ constant}$$

For motion with constant acceleration, we can get the velocity and position by simple integration:

$$\ddot{x} = a$$

$$v = \dot{x} = at + v_0 \tag{1}$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2 \tag{2}$$

Solving (1) for t and substituting into (2) gives:

$$\frac{v - v_0}{a} = t$$

$$x - x_0 = \frac{v_0\left(v - v_0\right)}{a} + \frac{1}{2}\cdot\frac{\left(v - v_0\right)^2}{a}$$

or

$$2a\left(x - x_0\right) = v^2 - v_0^2$$

Using this equation with the initial and final points being the top and bottom of the incline respectively, we get:

$$2a_1 d = V_B^2 \qquad V_B = \text{speed at bottom of incline}$$

Using the same equation for motion along the ground:

$$2a_2 s = -V_B^2 \tag{3}$$

Thus

$$a_1 d = -a_2 s \qquad a_1 = g\left(\sin\theta - \mu\cos\theta\right) \qquad a_2 = -\mu g$$

So

$$gd\left(\sin\theta - \mu\cos\theta\right) = \mu g s$$

Solving for μ gives

$$\mu = \frac{d\sin\theta}{d\cos\theta + s}$$

Substituting $\theta = 17°$, d = 100 m, s = 70 m gives

$$\boxed{\mu = 0.18}$$

Substituting this value into (3):

$$-2\mu g s = -V_B^2$$

$$V_B = \sqrt{2\mu g s}$$

$$\boxed{V_B = 15.6 \text{ m/sec}}$$