

Math 115A: Problem set 1

Sections 1 and 3. Instructor: James Freitag

Due 10/2

Problem 1 A vector space?

Let $V = \{(x, y) \in \mathbb{R}^2\}$. Consider the standard addition function on the set V and the scalar multiplication given by $c \cdot (x, y) \mapsto (cx, y)$. Prove or disprove that V is a vector space with respect to the standard addition operation and the scalar multiplication given here.

Problem 2 How to not be a subspace

1. Give an example of a subset of \mathbb{R}^3 which is closed under addition, but not under scalar multiplication.
2. Give an example of a subset of \mathbb{R}^3 which is closed under scalar multiplication, but not under addition.

Problem 3 Spanning relations

Work in the vector space \mathbb{R}^3 . Find three vectors, w, v, z with the following properties:

1. $\text{span}(\{w, v\}) = \text{span}(\{v, z\}) = \text{span}(\{w, v, z\})$
2. $\text{span}(\{w, v, z\}) \neq \text{span}(\{w, z\})$

Suppose that w, v, z are *any* three vectors with the above listed properties. Then prove or disprove the following statements:

1. w, v are linearly independent.
2. v, z are linearly independent.
3. w, z are linearly dependent.

Problem 4 Give a basis

Give an example of a basis for $P_3(\mathbb{R})$, the vector space of polynomials over \mathbb{R} of degree at most three. Prove that your set is a basis.

Problem 5 Exercises from the book

Do the following exercises from book:

- 8 from section 1.2.
- 19 from section 1.3.
- 4 from section 1.4.