# 105A - Set 3.5 Bonus

**Due Monday Feb. 13 in Class**

(Grades are out of 150)

1. (60pt) Consider the motion of a skier of mass $m$ down a track with a constant slope $\theta$. The friction of the skies is negligible, but the motion is subject to air resistance. The resistance force is given by $f = -kv^2$. The initial velocity of the skier is equal to zero.

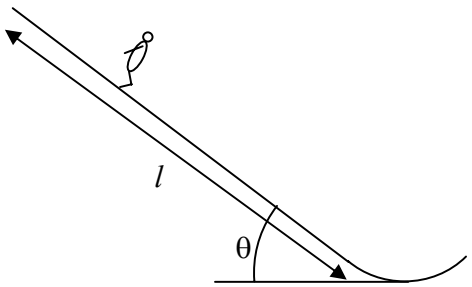


*Figure 1: A skier.*

   (a) (15pt) What is the velocity of the skier as a function of time?

   (b) (15pt) What is the distance $l$ traveled by the skier as a function of time?

   (c) (15pt) How far does he travel by the time his velocity reaches approximately 90% of its maximum possible value?

   (d) (15pt) The coefficient of air resistance is approximately given by $k = \rho A/2$, where $\rho$ is the density of air and $A$ is the cross-sectional area of the skier. Taking $\rho = 1$ kg m$^{-3}$, $A = 1$ m$^2$, $m = 100$ kg and $\theta = 45^\circ$ ,estimate the length $l$ needed to achieve 90% of the maximum velocity. How does it compare with the length of a typical ski jump track of about 100 m?

2. (50pt) Find the shortest distance between two points that are located on a spherical surface with radius $R$.
   *Hint1: use spherical coordinates.*
   *Hint2: Guess the solution by knowing that the shortest distance between points in the sphere is a great circle, which I can write as:*

$$\frac{\cos\psi}{\sin\psi} = A\cos\theta + B\sin\theta \quad \textit{with} \quad A^2 + B^2 < 1 \tag{1}$$

3. (40pt) A small mass $m$ can slide without friction on the inside of a conical surface with opening angle $\alpha$. Use cylindrical coordinates $\rho, \theta$ and $z$ to describe the position of the mass with $z$ =0 at the apex of the cone. Gravity points down.

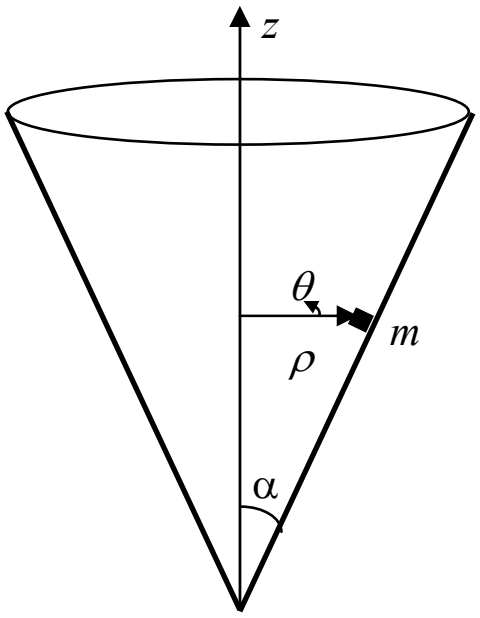


*Figure 2: Conical surface*

(a) (12pt) Find the Lagrangian for the mass $m$ in terms of coordinates $\rho$ and $\theta$.

(b) (14pt) Find the equation of motions using the Euler Lagrange differential equations for $\rho$ and $\theta$.

(c) (14pt) Find a general solution for the problem.
*Hint: remember that $\alpha$ is some constant, and simplify your equation so it will look familiar*