# Week 3 QM Discussion

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**Problem 1**

When we were solving for hydrogen atom, there was one peculiar moment, which we didn't pay a lot of attention to. When we considered the limit as $\rho \to 0$, we assumed that the term with $1/\rho^2$ to dominate. However, for the case of the spherically symmetric state there is no such term.(Why?)

1) Write the equation for spherically symmetric state in the limit $\rho \to 0$.
2) Look for solution in a form $u = \rho f$, write down the differential equation for $f(\rho)$.
3) Find the solution in the limit we considered.
4) Show that it does not change the solution for the hydrogen atom.

## Problem 2

1) Assume our system is in ground state of hydrogen $|n, l, m>=|1, 0, 0>$. What's the most probable value of $r$? (Hint: First you must figure out the probability that the electron would be found between $r$ and $r + dr$)

2) Assume our system is in state $|\phi(t=0)>=\frac{1}{\sqrt{5}}(|6, 4, 3>+2|7, 2, 1>)$, what's the average energy of the system at time $t$? Does it depend on time? If we measure the energy of the system at time $t$, what's the possible result we can get and what's the probability for each result? If we measure $L_Z$ first and get $3\hbar$, then measure the energy of the system, what's the possible result we can get and what's the probability for each result?

# Problem 1

~~the~~

solve hydrogen atom:

Radial part: $\kappa = \frac{\sqrt{-2mE}}{\hbar}$

$$\frac{1}{\kappa^2}\frac{d^2u}{dr^2} = \left[1 - \frac{me^2}{2\pi\epsilon_0\hbar^2\kappa}\frac{1}{\kappa r} + \frac{l(l+1)}{(\kappa r)^2}\right]u$$

set $\rho = \kappa r$

$$\frac{d^2u}{d\rho^2} = \left[1 - \frac{\rho_0}{\rho} + \frac{l(l+1)}{\rho^2}\right]u$$

consider $\rho \to 0$:

$$\frac{d^2u}{d\rho^2} = \frac{l(l+1)}{\rho^2}u \Rightarrow u \sim c\rho^{l+1}$$

when $\rho \to 0$.

$\to$ But for spherical symmetry state:

$\psi(r,\theta,\varphi) = \psi(r)$

which means $Y_{lm} \sim$ const

$\Rightarrow l = 0$.

1) Then: $$\frac{d^2u}{d\rho^2} = \left[1 - \frac{\rho_0}{\rho}\right] u$$

In the limit $\rho \to 0$:

$$\frac{d^2u}{d\rho^2} = -\frac{\rho_0}{\rho} u$$

2). $u = \rho f$

$$u' = f + \rho f'$$

$$u'' = f' + f' + \rho f''$$

$$= 2f' + \rho f''$$

$$2f' + \rho f'' = -\frac{\rho_0}{\rho} \rho f = -\rho_0 f$$

3) $\rho \to 0$: $2f' + \rho_0 f = 0$

so in the limit:

$$f' = -\frac{\rho_0}{2} f$$

$$f \propto e^{-\frac{\rho_0}{2}\rho}$$

4) $$u(\rho) = e^{-\frac{\rho_0}{2}\rho} \cdot \rho = \rho\left[1 - \frac{\rho_0}{2}\rho + O(\rho^2)\right] \approx \rho$$

in the $l \neq 0$ case,

we have:

$$u \sim c\,\rho^{l+1}$$

when $l \to 0$

$$u \sim c\,\rho \quad .$$

so $u \sim c\,\rho^{l+1}$ holds for all '$l$'.

Problem 2.

1).

ground state wavefunction :

$$\psi = \langle \vec{r} | 100 \rangle = R_{10}(r)\, Y_{00}(\theta, \varphi)$$

$$= 2a^{-\frac{3}{2}} e^{-\frac{r}{a}} \cdot \sqrt{\frac{1}{4\pi}}$$

First, figure out the probability in $(r, r+dr)$.

by definition :

$$1 = \int_0^{+\infty} P(r)\, dr$$

Also, we know :

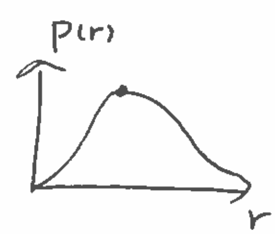
$$1 = \int |\psi|^2 r^2 \sin\theta \, dr\, d\theta\, d\varphi$$

$$= \int |\psi|^2 r^2 \cdot 4\pi \; dr$$

then

$$P(r) = 4\pi r^2 |\psi|^2 = 4\pi \frac{4a^{-3}}{4\pi} e^{-\frac{2r}{a}} r^2$$

$$= 4a^{-3} e^{-\frac{2r}{a}} r^2$$




$$\frac{dP}{dr} = 2r\, e^{-\frac{2r}{a}} + r^2 \left(-\frac{2}{a}\right) e^{-\frac{2r}{a}} = 0$$

$$\Rightarrow r_{max} = a.$$

2).

$$|\phi(t=0)\rangle = \frac{1}{\sqrt{5}}\left(|6,4,3\rangle + 2\,|7,2,1\rangle\right)$$

$$|\phi(t)\rangle = e^{\frac{-i\hat{H}t}{\hbar}}\frac{1}{\sqrt{5}}\left(|6,4,3\rangle + 2\,|7,2,1\rangle\right)$$

$$= \frac{1}{\sqrt{5}}\left(e^{\frac{-iE_6 t}{\hbar}}|6,4,3\rangle + 2\,e^{\frac{-iE_7 t}{\hbar}}|7,2,1\rangle\right)$$

When you measure the energy:

possible result: ~~possibility~~ P:

$E_6$ $\quad\left|\frac{1}{\sqrt{5}}e^{-\frac{iE_6 t}{\hbar}}\right|^2 = \frac{1}{5}$

$E_7$ $\quad\left|\frac{2}{\sqrt{5}}e^{-\frac{iE_7 t}{\hbar}}\right|^2 = \frac{4}{5}$

The average energy:

$$\frac{1}{5}E_6 + \frac{4}{5}E_7$$

$$|\phi\rangle = \frac{1}{\sqrt{5}}\left(|6,4,3\rangle + 2|7,2,1\rangle\right)$$

$\Big\downarrow$ measure $L_z$, get $L_z = 3\hbar$

$$|\tilde{\phi}\rangle = |6,4,3\rangle$$

$\Big\downarrow$

energy has only one result: $E_6$, $P = 1$.