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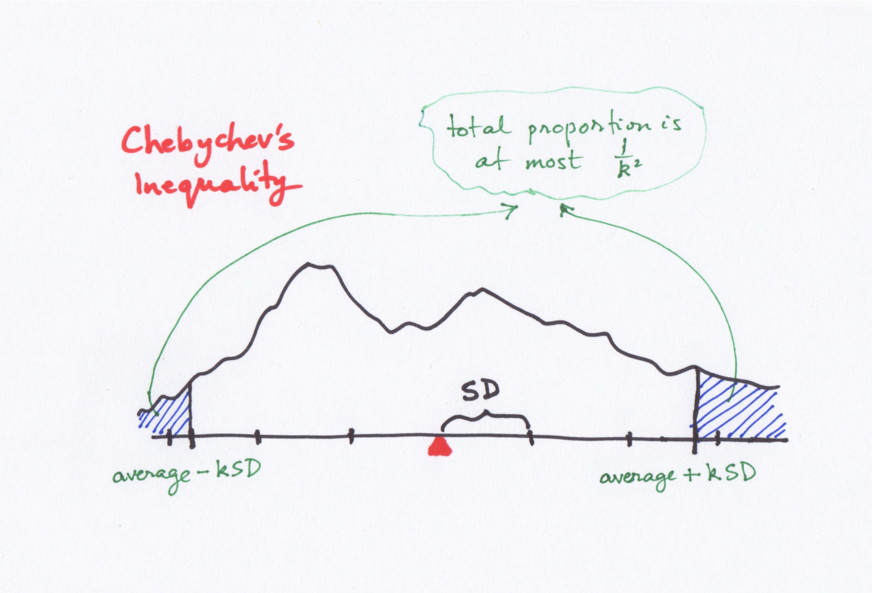
(Tschebyscheff!!)

Chebychev's inequality



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- **At most 25%** of the list is **outside** the interval **average $\pm$ 2$\times$SD**.
- **At least 75%** of the list is **in** the interval **average $\pm$ 2$\times$SD**.

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No matter what the list, the vast majority of entries will be in the range **average $\pm$ a few SDs.**

Precise statement

No matter what the list, a proportion of at least $1 - 1/k^2$ of the entries will be in the range **average $\pm$ $k\times$SD.**

A bound on one tail

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The proportion in the right tail is at most $1/k^2$.

But you **cannot say** that the proportion in one tail is at most **half** of $1/k^2$, unless you know that the two tails are equal.

Typical use

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What percent are more than 80 years old?

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Both bounds are correct. But Chebychev's bound is much sharper, because it uses the SD, not just the average.