# EXERCISES FOR LANDSBERG'S TENORS LECTURES

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Refer to the notes for definitions and terminology.

(1) As a trilinear map, $M_{\langle \mathbf{n} \rangle}(X,Y,Z) = \mathrm{trace}(XYZ)$.

(2) $M_{\langle \mathbf{n} \rangle}^{\boxtimes N} = M_{\langle n^N \rangle}$.

(3) $\underline{\mathbf{R}}(T^{\boxtimes N}) \leq \underline{\mathbf{R}}(T)^N$. More generally, let $M_{\langle \ell, \mathbf{m}, \mathbf{n} \rangle}$ denote the rectangular matrix multiplication tensor. Show $M_{\langle \ell, \mathbf{m}, \mathbf{n} \rangle} \boxtimes M_{\langle \ell', \mathbf{m}', \mathbf{n}' \rangle} = M_{\langle \ell\ell', \mathbf{mm}', \mathbf{nn}' \rangle}$.

(4) Show that the exponent of matrix multiplication $\omega = 2$ iff $\underset{\sim}{\mathbf{R}}(M_{\langle \mathbf{n} \rangle}) = n$ for any $n \geq 2$ iff $\underset{\sim}{\mathbf{R}}(M_{\langle \mathbf{n} \rangle}) = n \forall n \geq 2$

(5) If $T \trianglerighteq S$, then $\underline{\mathbf{R}}(T) \geq \underline{\mathbf{R}}(S)$.

(6) Strassen's equations are, for all $\alpha, \alpha_1, \alpha_2 \in A^*$,
$$[T(\alpha_1)T(\alpha)^{cof}, T(\alpha_2)T(\alpha)^{cof}] = 0$$
where $[X,Y] = XY - YX$ is commutator. These are equations of degree $2m = 1 + (m-1) + 1 + (m-1)$. Show that there are equivalent equations of degree $m+1$.

(7) Show that the tensor $a_1 \otimes b_2 \otimes c_2 + a_2 \otimes b_1 \otimes c_2 + a_2 \otimes b_2 \otimes c_1 + a_1 \otimes b_1 \otimes c_3 + a_1 \otimes b_3 \otimes c_1 + a_3 \otimes b_1 \otimes c_1$ is a sum $x' + x''$ of tangent vector and second derivative of a curve at a point of the Segre.

(8) Show that the tensor $a_2 \otimes b_1 \otimes c_2 + a_2 \otimes b_2 \otimes c_1 + a_1 \otimes b_1 \otimes c_3 + a_1 \otimes b_3 \otimes c_1 + a_3 \otimes b_1 \otimes c_1$ is the sum of two tangent vectors at two distinct points that lie on a line on Segre

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