ELL793 Report
Assignment 1: Camera Calibration

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1 Aim

To find intrinsic and extrinsic camera calibration parameters of a mobile phone's camera. Intrinsic calibration parameters include focal length, skew, radial distortion parameters, other distortion parameters, camera's optic center. Extrinsic calibration parameters include Rotation, Translation (scale) computation for every photo taken by the mobile phone.

2 Setup

1. An 8x11 checkerboard with 25mm sides was printed out on an A4 sheet.

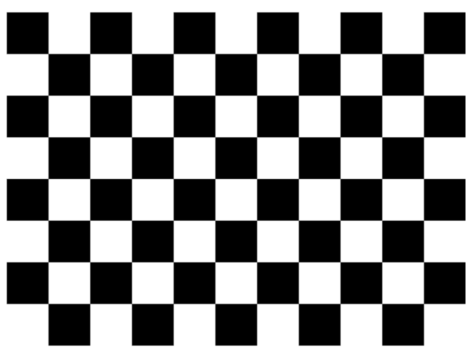

Figure 1: 8x11 Checkerboard

2. The checkerboard was taped on a corner wall so that it formed orthogonal planes. Some points for calibration were marked by pink dots.

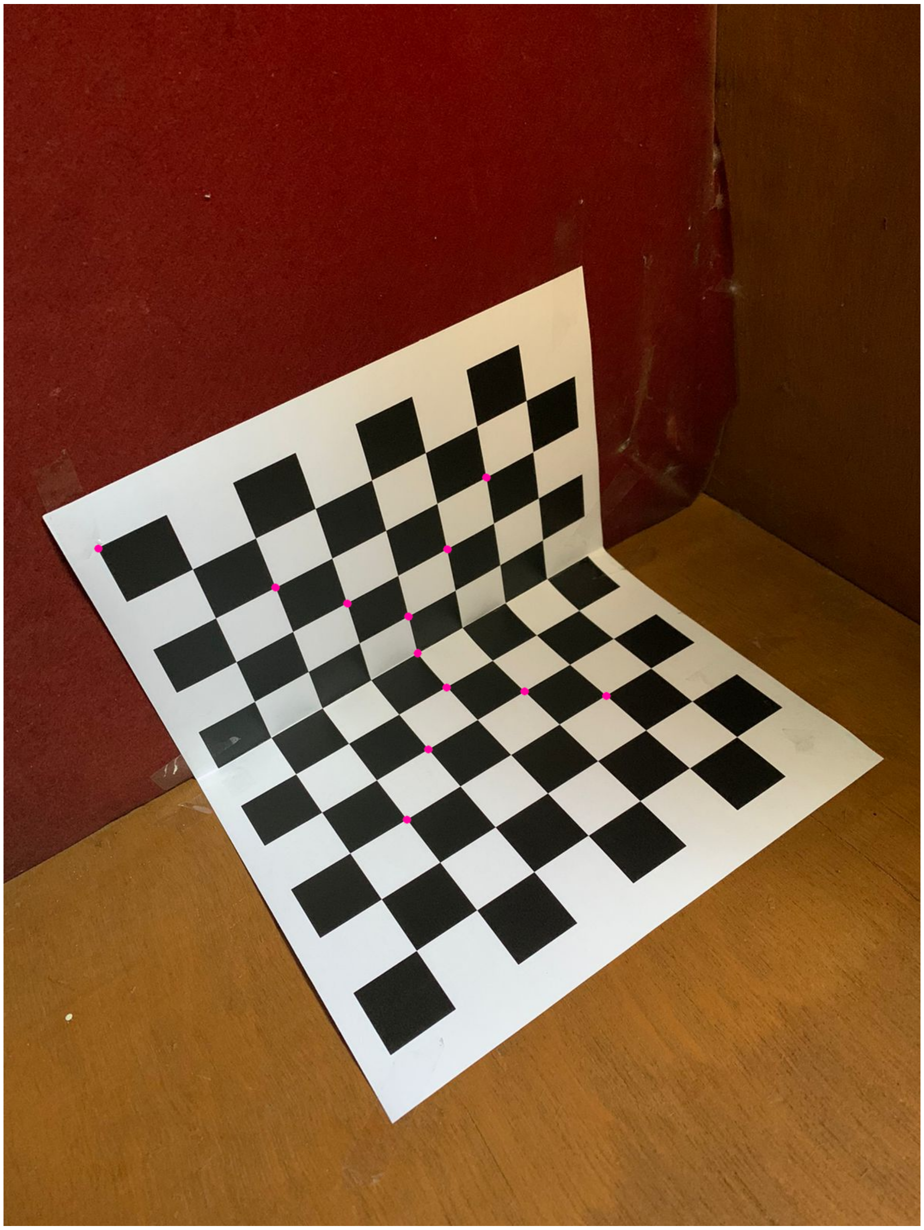

Figure 2: Checkerboard on orthogonal walls with points marked in pink

3 Dataset

12 points were marked on the Checkerboard. With a corner point as the origin of the World Coordinate frame, the 3D coordinates of other points were measured. The side of a square, which is 25mm, is considered 1 unit. The coordinates of the same points in the 2D Image were also measured.

Point	**2D Coordinates**		**3D Coordinates**		
	x	y	X	Y	Z
0 (Origin)	124	887	0	0	0
1	450	815	3	3	0
2	542	750	5	4	0
3	556	624	5	3	2
4	632	980	2	6	0
5	530	798	4	4	0
6	356	836	2	2	0
7	581	886	3	5	0
8	682	700	5	5	2
9	528	532	5	2	3
10	789	694	5	6	3
11	580	705	5	4	1

Table 1: Dataset created corresponding to points marked in Figure 2.

Points 6-11 were used for calculating the intrinsic and extrinsic parameters of the camera and points 0-5 were used to measure the calibration error.

4 Procedure

1. The data points were normalized such that the centroid of 2D and 3D points are at the origin and the average Euclidean distance of 2D and 3D points from the origin is $\sqrt{2}$ and $\sqrt{3}$. The transformation matrices T and U that achieve this for 2D and 3D respectively are:

$$T = \begin{bmatrix} \frac{1}{d_2} & 0 & \frac{-x_c}{d_2} \\ 0 & \frac{1}{d_2} & \frac{-y_c}{d_2} \\ 0 & 0 & 1 \end{bmatrix}$$

   Here, $p_c = (x_c, y_c, 1)$ is the centroid of 2D coordinates and

$$d_2 = \frac{\Sigma_n \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2}}{n\sqrt{2}}$$

.

$$U = \begin{bmatrix} \frac{1}{d_3} & 0 & 0 & \frac{-X_c}{d_3} \\ 0 & \frac{1}{d_3} & 0 & \frac{-Y_c}{d_3} \\ 0 & 0 & \frac{1}{d_3} & \frac{-Z_c}{d_3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Here, $P_c = (X_c, Y_c, Z_c, 1)$ is the centroid of 3D coordinates and

$$d_3 = \frac{\Sigma_n \sqrt{(X_i - X_c)^2 + (Y_i - Y_c)^2 + (Z_i - Z_c)^2}}{n\sqrt{3}}$$

.

Why normalize? If the units of 3D World coordinates is γ times the units of 2D Image coordinates, then any change δ in Image coordinates will scale with γ and, the variance will scale with γ^2. Thus, if γ is large, then DLT would face numerical instability due to a higher condition number of the matrix as norm scales with γ^2. Thus, normalization is essential to ensure numerical stability and accuracy.

2. The normalized projection matrix $\hat{M}$ is estimated using the DLT method. Solving the following system of equations:

$$x_i^\star = \frac{\hat{m}_1 \cdot X^\star}{\hat{m}_3 \cdot Z^\star}$$

$$y_i^\star = \frac{\hat{m}_2 \cdot Y^\star}{\hat{m}_3 \cdot Z^\star}$$

where,

$$\begin{bmatrix} x^\star \\ y^\star \\ 1 \end{bmatrix} = T \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

and

$$\begin{bmatrix} X^\star \\ Y^\star \\ Z^\star \\ 1 \end{bmatrix} = U \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Therefore we get:

$$\hat{M} = \begin{bmatrix} \hat{m}_1 \\ \hat{m}_2 \\ \hat{m}_3 \end{bmatrix}$$

$$M = T^{-1} \hat{M} U$$

3. After computing M, the intrinsic and extrinsic parameters can be computed by reducing:
$$K = \begin{bmatrix} \alpha & -\alpha cot(\theta) & x_0 \\ 0 & \beta cosec(\theta) & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$M = K[R|t]$$
Writing $M = [A|b]$ where A is 3x3 and b is 3x1, we can write the equations as:
$$\rho \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \alpha r_1 + -\alpha cot(\theta) r_2 + x_0 r_3 \\ \beta cosec(\theta) r2 + y_0 r_3 \\ r_3 \end{bmatrix}$$
which can be easily solved to give the intrinsic and extrinsic parameters.

4. The radial distortion is reduced by reducing the reprojection error. Ideally, we should have:
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$
Due to distortion, it is more closely approximated by $p' = [x', y', 1]$, where:
$$x' = x/\lambda$$
$$y' = y/\lambda$$
λ is a polynomial function of the squared distance between the image center and the image point p in normalized image coordinates.
$$d^2 = \hat{x}^2 + \hat{y}^2 = ||K^{-1}p||^2 - 1$$
$$\lambda = 1 + \Sigma_{p=1}^{q} K_p d^{2p}$$
where $q <= 3$. For our study, we have kept $q = 2$ so that:
$$\lambda = 1 + K_1 d^2 + K_2 d^4$$
Hence, we solve:
$$x' - x/\lambda = 0$$
$$y' - y/\lambda = 0$$
to get values of K_1 and K_2.

5 Results

Parameter	Value
α	1170.25 pixels
β	-1166.12 pixels
θ	88.92°
x_0	572.58 pixels
y_0	671.27 pixels

Table 2: Intrinsic parameters determined by camera calibration.

From here we also get that camera's optic center is at [-6.73, -0.13, 11.07], the focal length is [1170.25($= f_x$), -1166.32($= f_y$)] and the skew value is -21.97

Parameter	Value
R	$\begin{bmatrix} 0.168 & 0.831 & 0.529 \\ 0.638 & -0.501 & 0.584 \\ 0.751 & 0.239 & -0.615 \end{bmatrix}$
t	$\begin{bmatrix} -4.607 \\ -2.235 \\ 11.904 \end{bmatrix}$

Table 3: Extrinsic parameters determined by camera calibration.

Parameter	Value
K_1	-2.099e-05
K_2	9.811e-04

Table 4: Distortion parameters determined by reprojection minimization.

Set	RMSE (distortion not removed)	RMSE (distortion removed)
Calibration Points	0.0867434	0.0867375
Test Points	2.372674	2.371977

Table 5: The RMSE error between true and projected coordinates

6 Visualisation

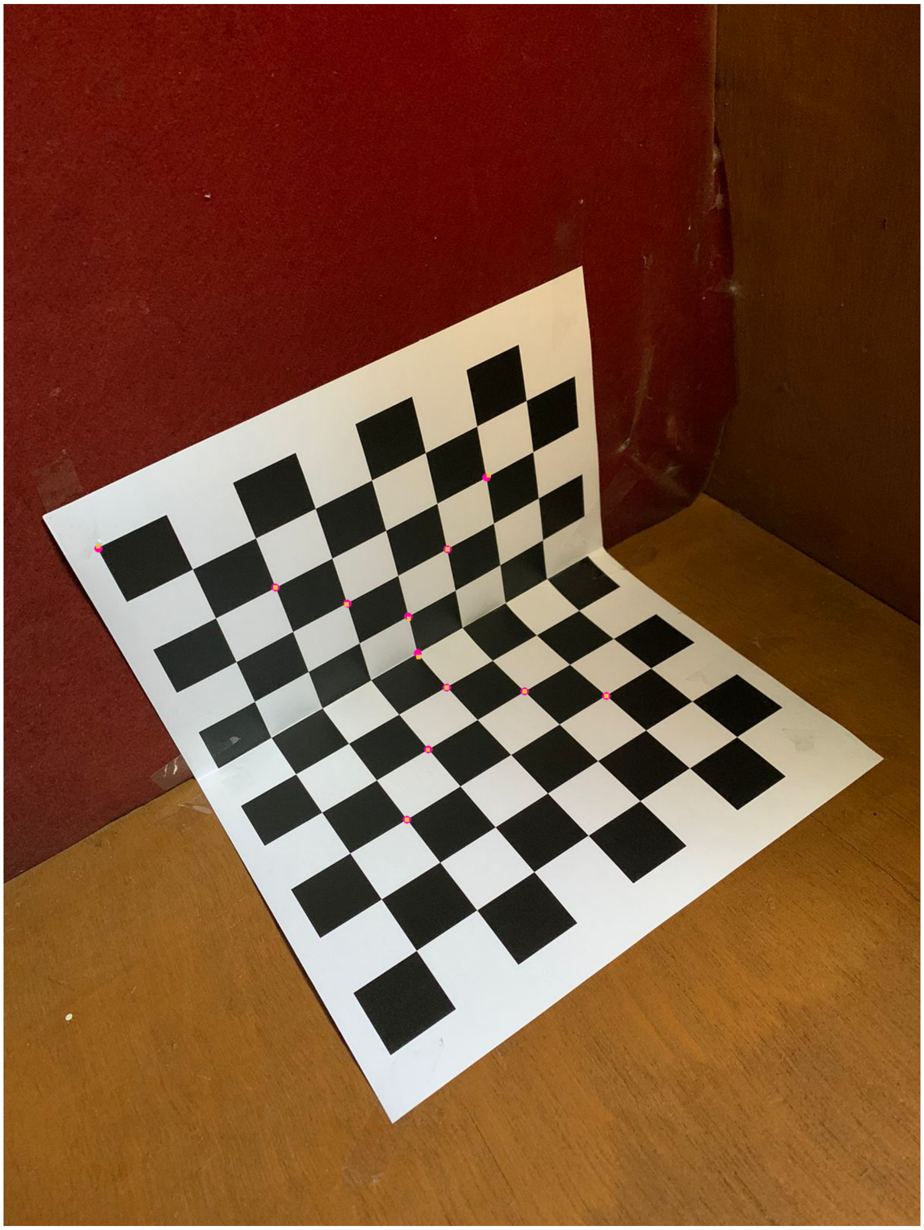

Figure 3: Checkerboard on with original points marked in pink, and recover points marked in yellow.

References

[1] David Forsyth, Jean Ponce, *Computer Vision (A Modern Approach)*.

[2] http://www.cs.cmu.edu/ 16385/lectures/lecture10.pdf

[3] https://ori.codes/artificial-intelligence/camera-calibration/camera-distortions/