

Machine Intelligence:: Deep Learning

Week 4

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Outline of the DL Module (tentative)

- Day 1: Jumpstart to DL
 - What is DL
 - Basic Building Blocks
 - Keras
- Day 2: CNN I
 - ImageData
- Day 3: CNN II
 - Tips and Tricks
 - Modern Architectures
 - 1-D Sequential Data
- Day 4: Looking at details
 - Linear Regression
 - Backpropagation
 - Resnet
 - Likelihood principle
- Day 5: Probabilistic Aspects
 - TensorFlow Probability (TFP)
 - Negative Loss Likelihood NLL
 - Count Data
- Day 6: Probabilistic models in the wild
 - Complex Distributions
 - Generative modes with normalizing flows
- Day 7: Uncertainty in DL
 - Bayesian Modeling
- Day 8: Uncertainty cont'd
 - Bayesian Neural Networks
 - Projects

Projects please register (see also website)

<https://docs.google.com/spreadsheets/d/1TZf5hKekzOIBC7J0-EAltGOMTuZyrDhHu3ANve0q6H4/edit#gid=0>

See website for other projects

Learning Objectives for today: looking under the hood

- Get an understanding of
 - Computational Graph
 - Backpropagation in Computational Graph
 - Maximum Likelihood principle for neural networks



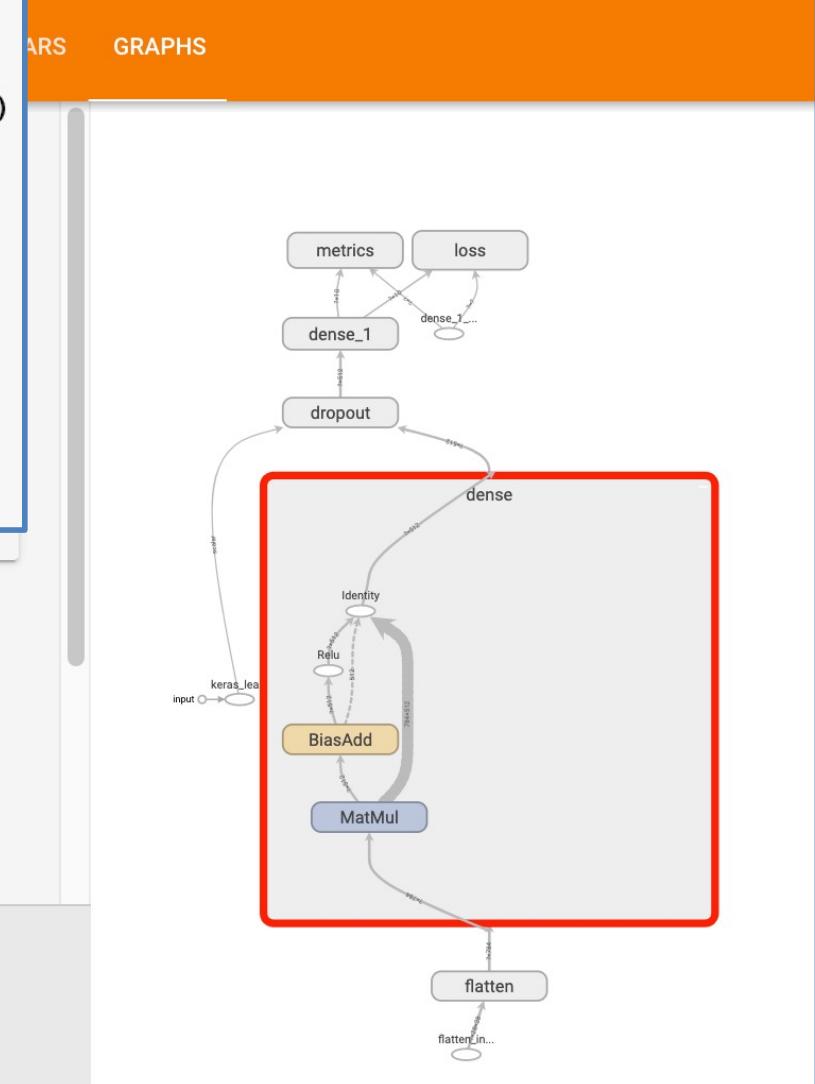
Computational Graph

Looking under the hood of tf / Keras

Keras

```
1 mnist = tf.keras.datasets.mnist
2
3 (x_train, y_train), (x_test, y_test) = mnist.load_data()
4 x_train, x_test = x_train / 255.0, x_test / 255.0
5
6 def create_model():
7     return tf.keras.models.Sequential([
8         tf.keras.layers.Flatten(input_shape=(28, 28)),
9         tf.keras.layers.Dense(512, activation='relu'),
10        tf.keras.layers.Dropout(0.2),
11        tf.keras.layers.Dense(10, activation='softmax')
12    ])
```

TensorFlow



Internal representation (in non-eager mode) is a computational graph.

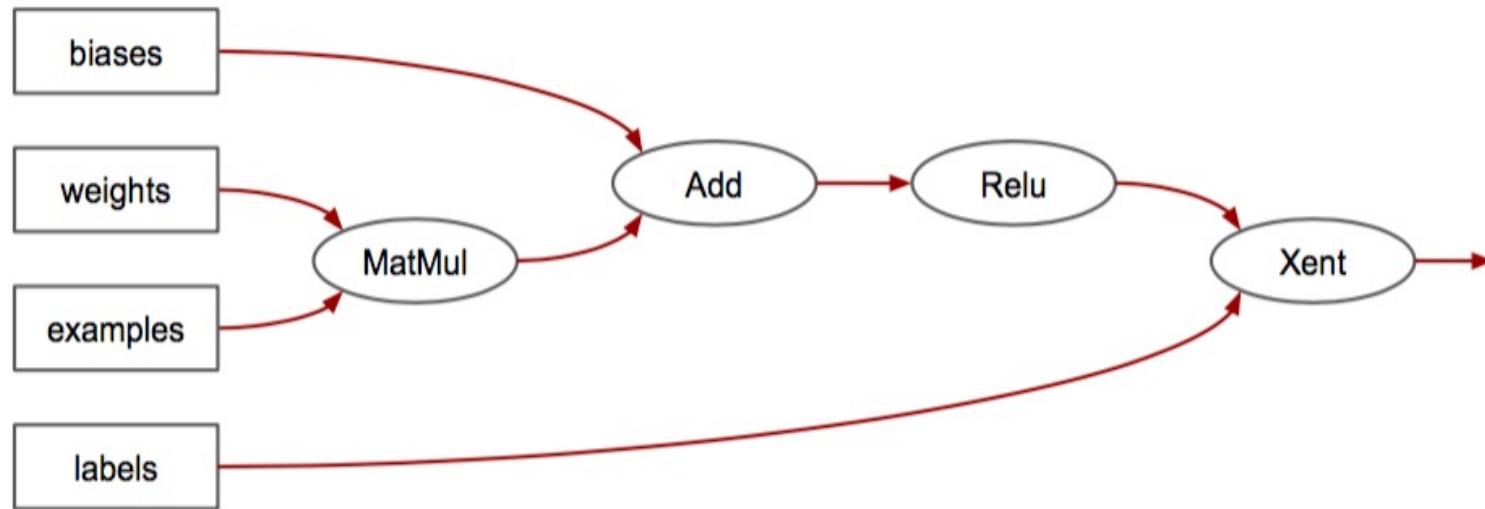
https://github.com/tensorflow/tensorboard/blob/master/docs/get_started.ipynb

Next steps

- Understand the computational graph (theoretical)
- Understand backpropagation in a graph (theoretical)
- Example Linear Regression (the mother of all networks)

Recap

- The computation in TF is done via a computational graph

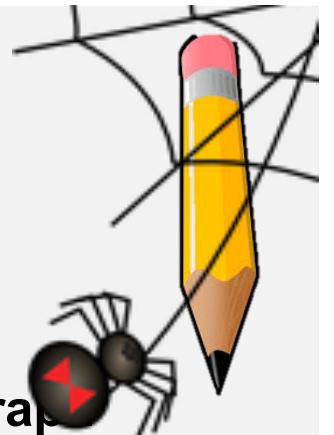


- The nodes are ops
- The edges are the flowing tensors

Recap Matrix Multiplication (scalar and with vector)

$$10 \begin{pmatrix} 3 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} = 120$$

Be the spider who knits a computational graph



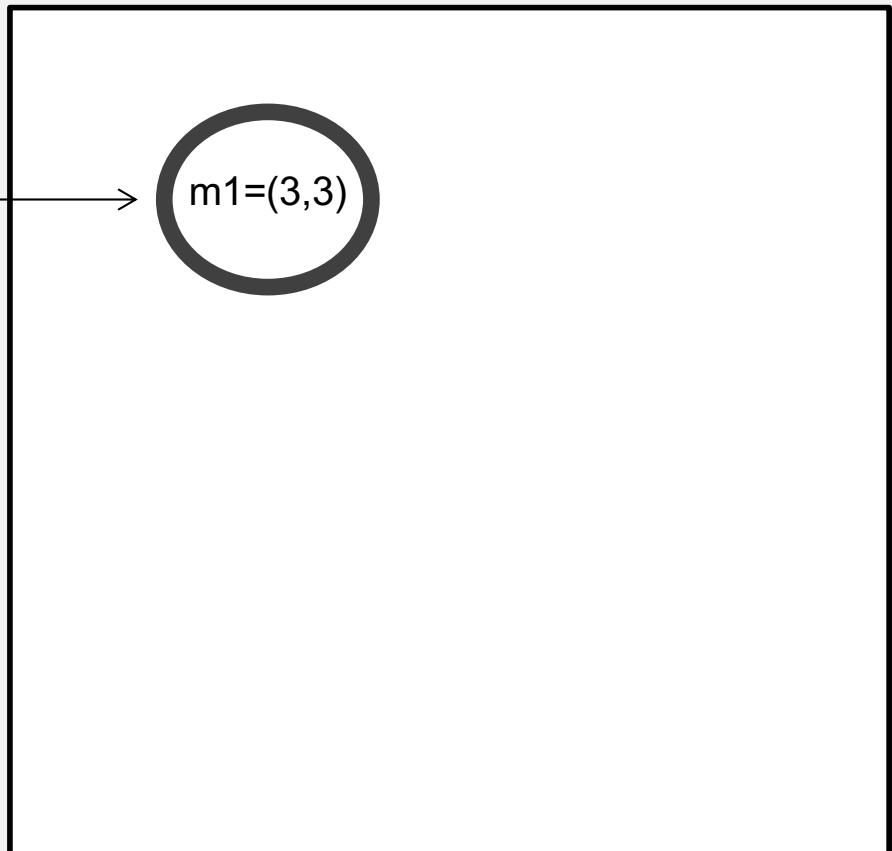
Translate the following TF code in a graph

TensorFlow: Building the graph

```
m1 = tf.constant([[3.0, 3.0]], name='M1')  
m2 = tf.constant([[2.0], [2.0]], name = 'M2')  
product = 10*tf.matmul(m1,m2)
```

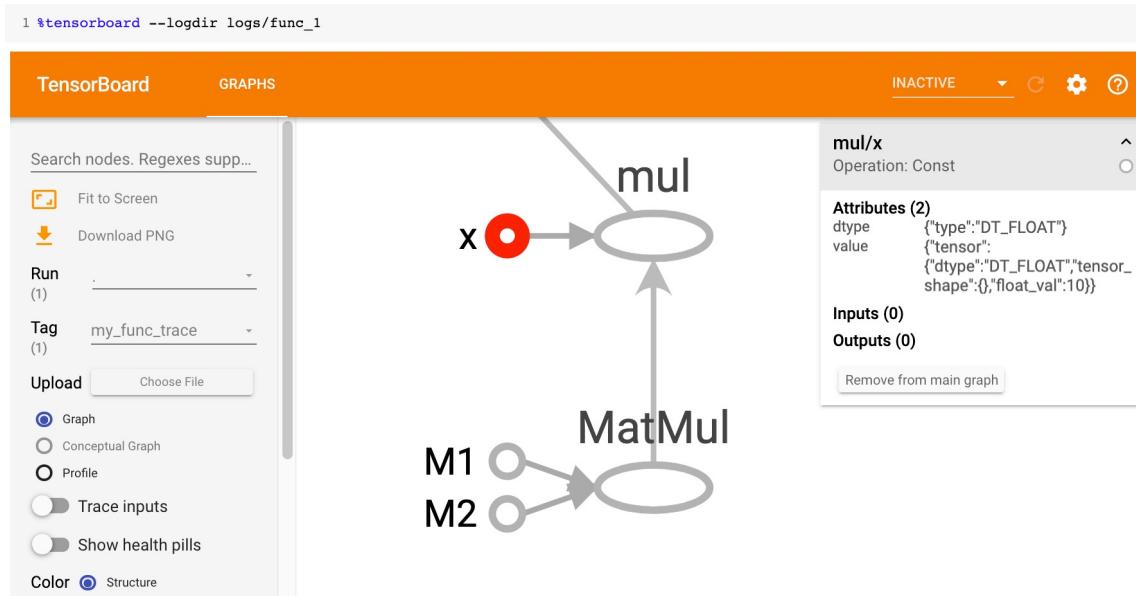
Quite much happen in here!

Finish the computation graph



TensorFlows internal representation

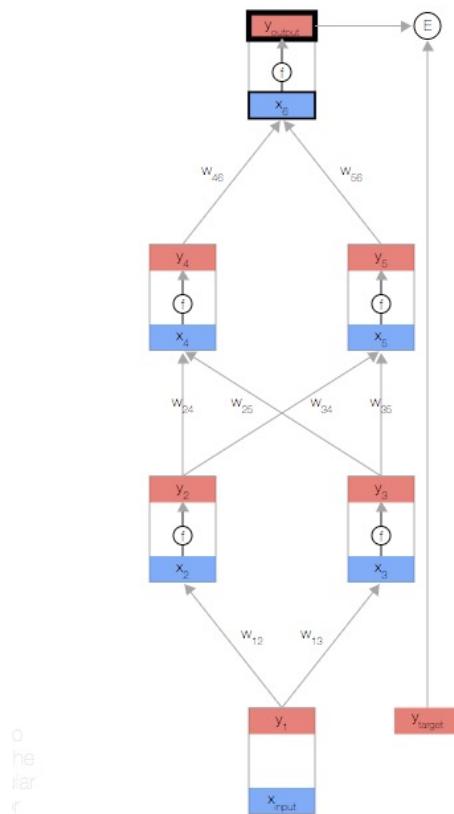
- For fast computation a graph is build
 - Technical detail in tf 2.0 you need to decorate a function with `@tf.function` to build a graph. Otherwise eager execution happens.



The most important benefit of computational graphs is back propagation...

Motivation: The forward and the backward pass

- <https://developers-dot-devsite-v2-prod.appspot.com/machine-learning/crash-course/backprop-scroll>



Scroll until the forward pass and swiftly go over the backward pass.

(The backward pass is described in more details in the next following slides).

On the
far
if

Chain rule recap

- If we have two functions f, g

$$y = f(x) \text{ and}$$

$$z = g(y)$$

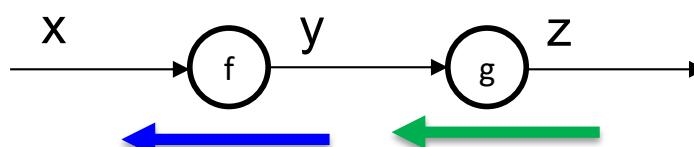
then y and z are dependent variables.



- The chain rule:

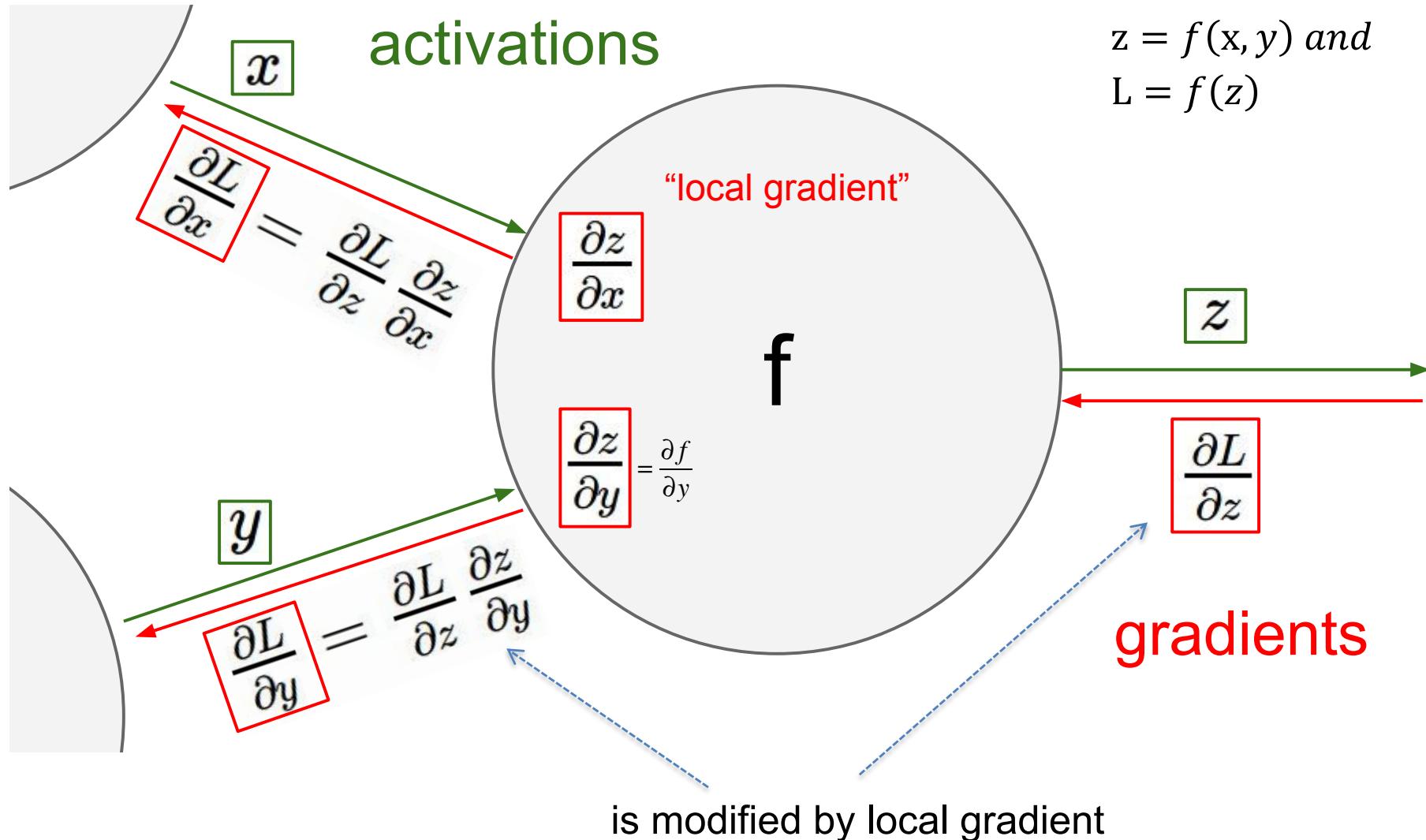
$$\frac{\partial z}{\partial x} = \frac{\partial y}{\partial x} \cdot \frac{\partial z}{\partial y}$$

- Backpropagation (flow of the gradient)



$$\frac{\partial z}{\partial x} = \frac{\partial y}{\partial x} * \frac{\partial z}{\partial y}$$

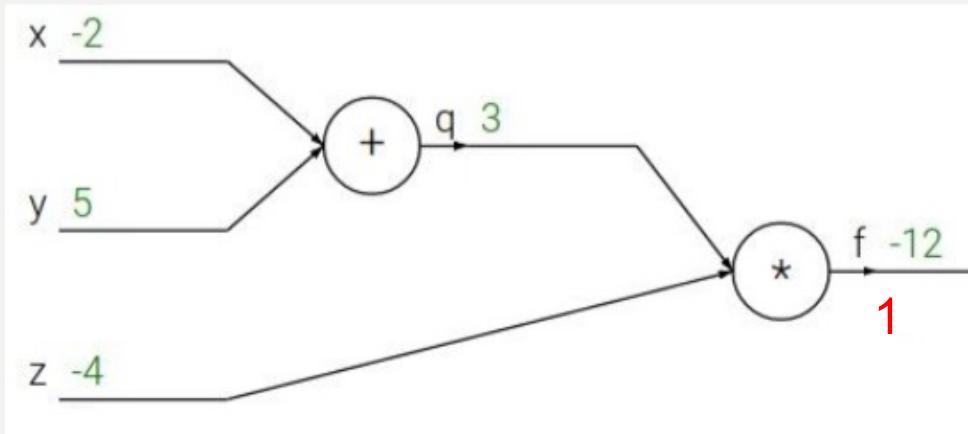
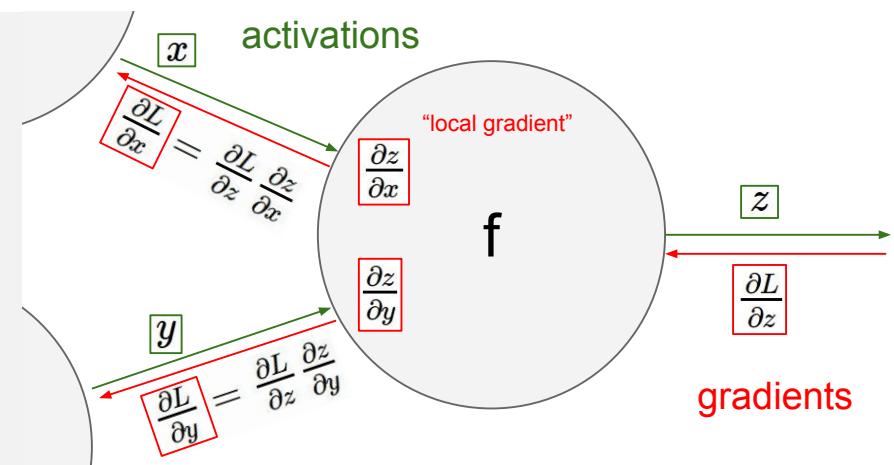
Gradient flow in a computational graph: local junction



Example

$$f(x, y, z) = (x + y)z$$

e.g. $x = -2, y = 5, z = -4$



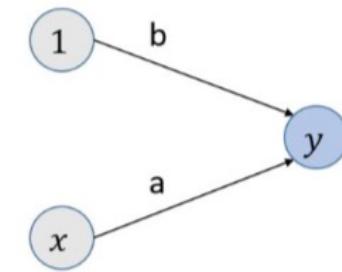
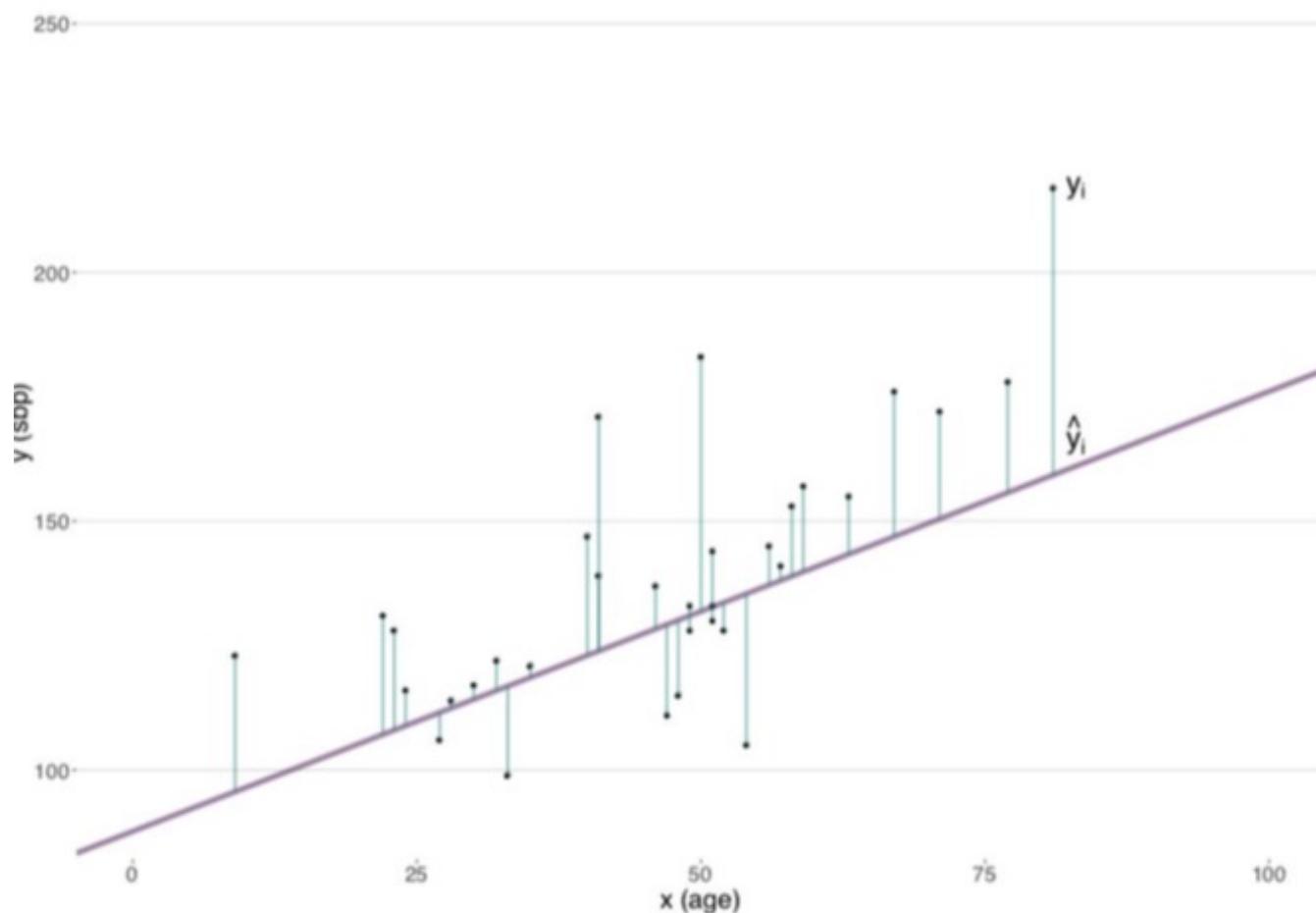
$$\frac{\partial(\alpha + \beta)}{\partial \alpha} = 1 \quad \frac{\partial(\alpha * \beta)}{\partial \alpha} = \beta$$

Task (10min): Calculate the derivatives.
Once by hand, once with
backpropagation (follow the graph)

→ Multiplication do a switch

In depth example: Linear Regression

Example Linear Regression



$$Loss = MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2 = \frac{1}{n} \sum_{i=1}^n (y_i - (a \cdot x_i + b))^2$$

Interlude: Derivations in TF using Tape Mechanism

```
x = tf.Variable(-2.)
y = tf.Variable(5.)
z = tf.Variable(-4.)
```

```
with tf.GradientTape() as tape: #We need to store
    res = (x+y)*z
    print(tape.gradient(res, [z]))
```

https://github.com/tensorchiefs/dl_course_2022/blob/main/notebooks/10_a_gradient_tape.ipynb

Forward and Backwardpass for linear regression 10/11

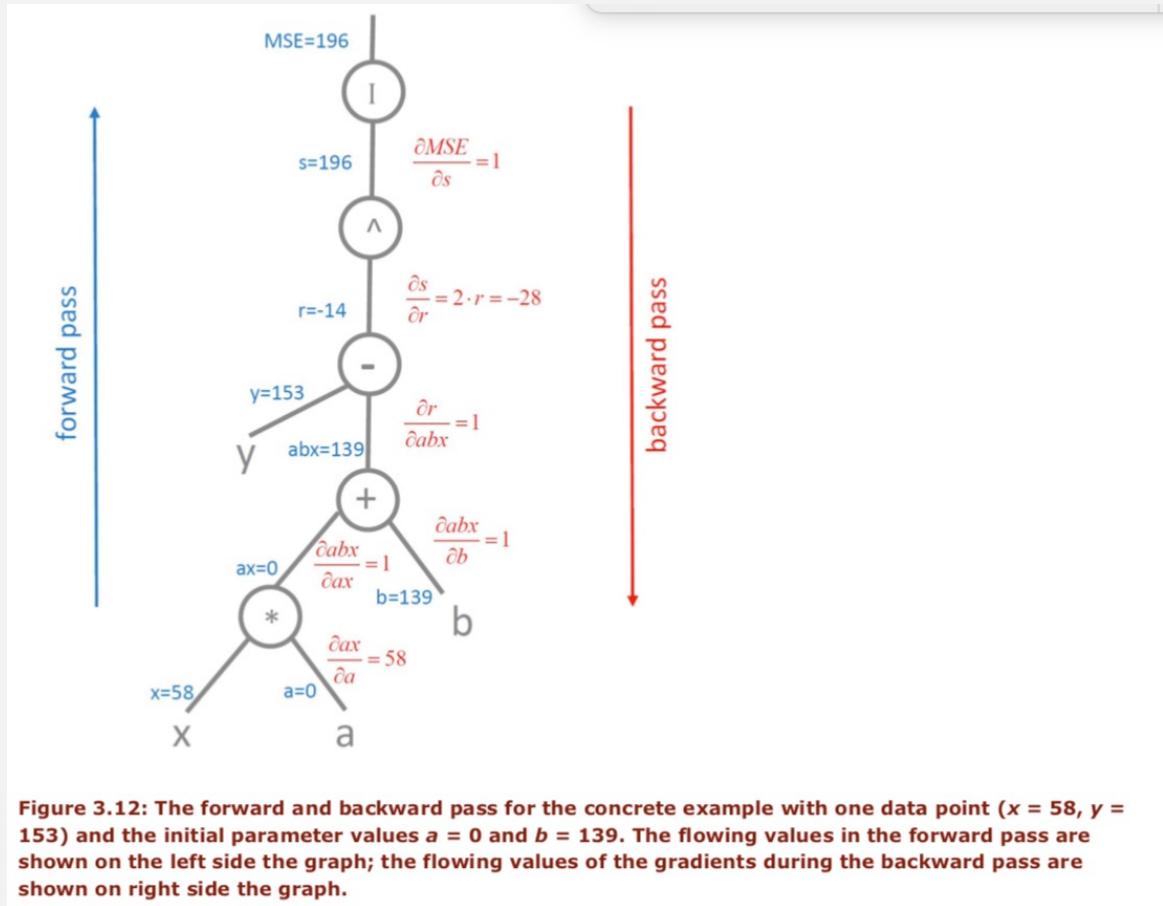


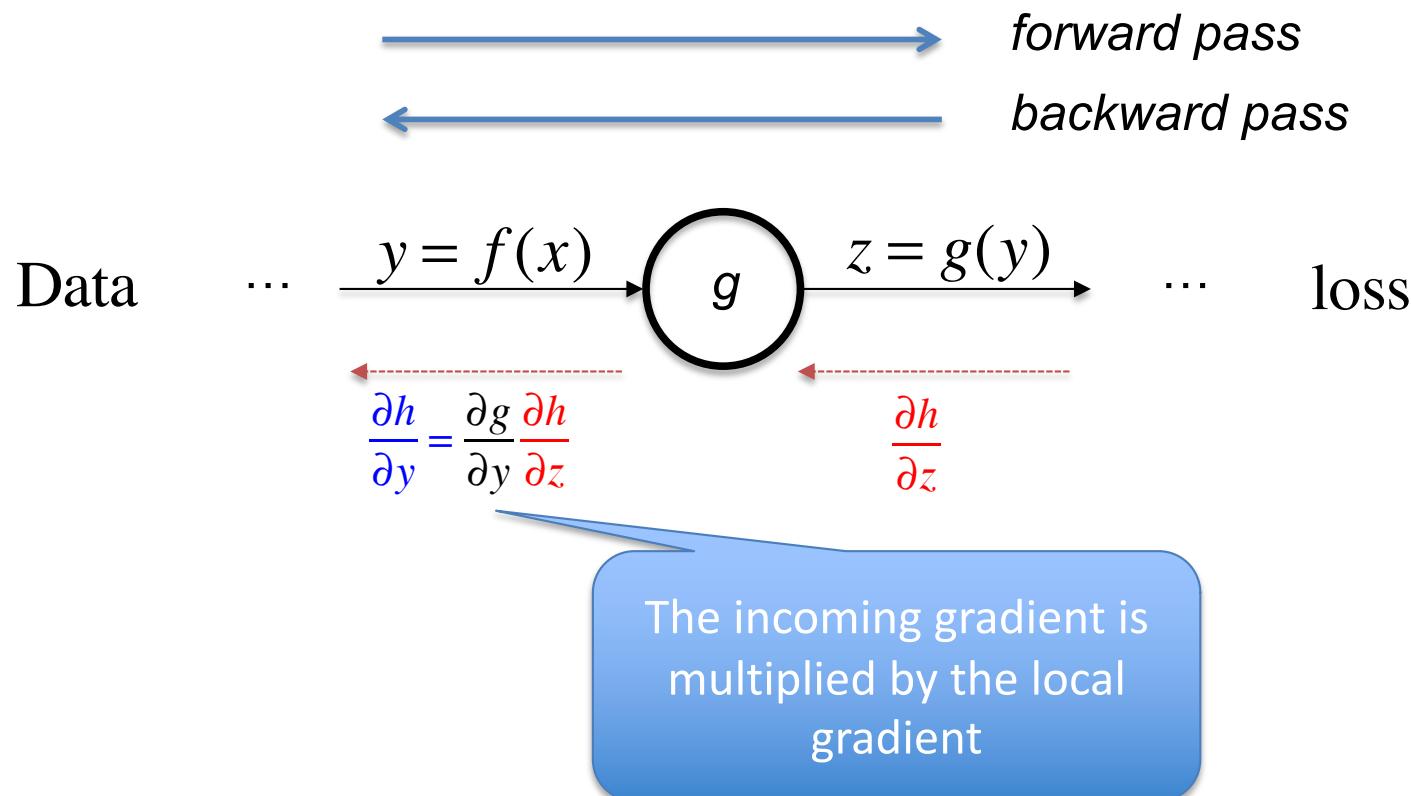
Figure 3.12: The forward and backward pass for the concrete example with one data point ($x = 58, y = 153$) and the initial parameter values $a = 0$ and $b = 139$. The flowing values in the forward pass are shown on the left side the graph; the flowing values of the gradients during the backward pass are shown on right side the graph.

Do exercise 10 (works with Tape Gradient)

If you have time you can skim over 11 (Works with the static graph)

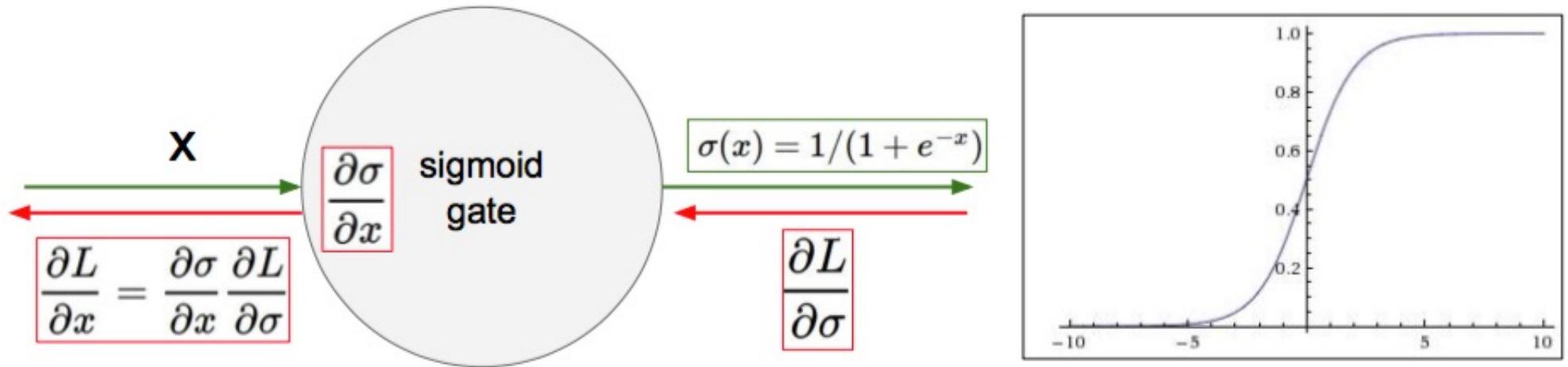
Further References / Summary

- For a more in depth treatment have a look at
 - Lecture 4 of <http://cs231n.stanford.edu/>
 - Slides http://cs231n.stanford.edu/slides/winter1516_lecture4.pdf
- Gradient flow is important for learning: remember!



Consequences of Backprop

Backpropagation through sigmoid



What happens when $x = -10$?

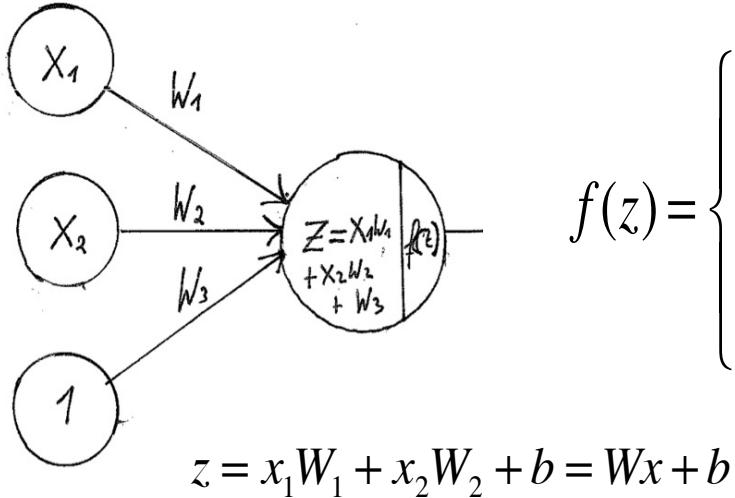
What happens when $x = 0$?

What happens when $x = 10$?

Gradients are killed, when not in active region! Slow learning!

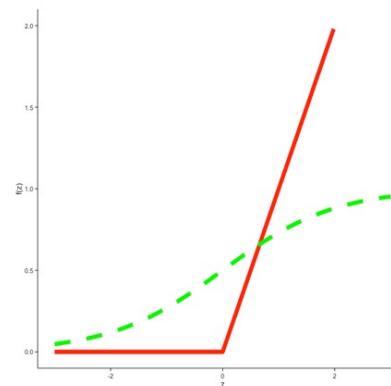
Different activations in inner layers

N-D log regression



$$f(z) = \begin{cases} \frac{\exp(z)}{1+\exp(z)} \\ \max(0, z) \end{cases}$$

Activation function a.k.a.
Nonlinearity $f(z)$



Motivation:
Green:
logistic regression.
Red:
ReLU faster
convergence

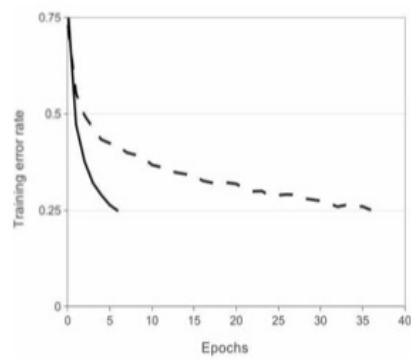


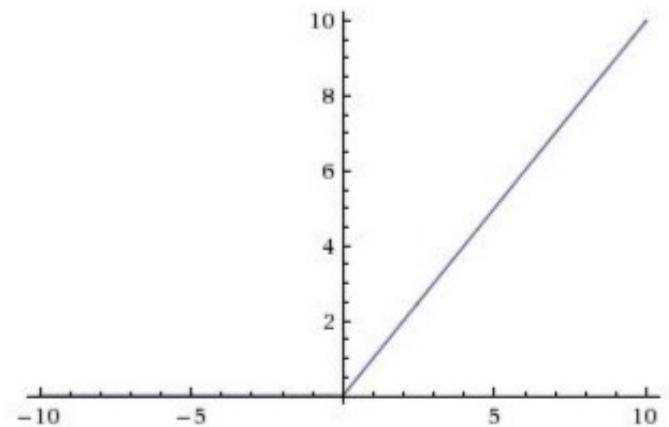
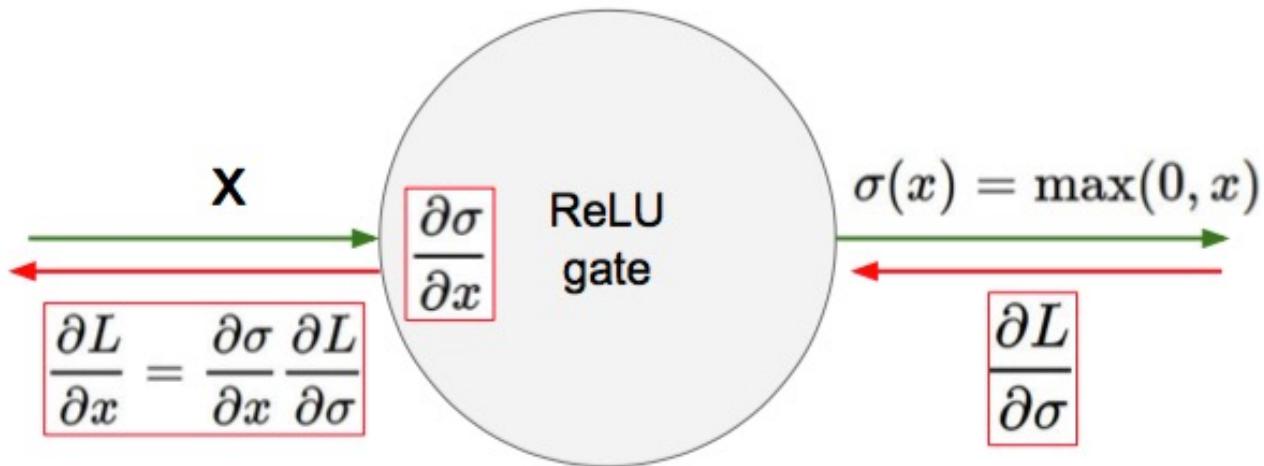
Figure 1: A four-layer convolutional neural network with ReLUs (solid line) reaches a 25% training error rate on CIFAR-10 six times faster than an equivalent network with tanh neurons

Source:
Alexnet
Krizhevsky et al 2012

There are other alternatives besides sigmoid and ReLU.

Currently ReLU is standard

Backpropagation through ReLU



What happens when $x = -10$?

What happens when $x = 0$?

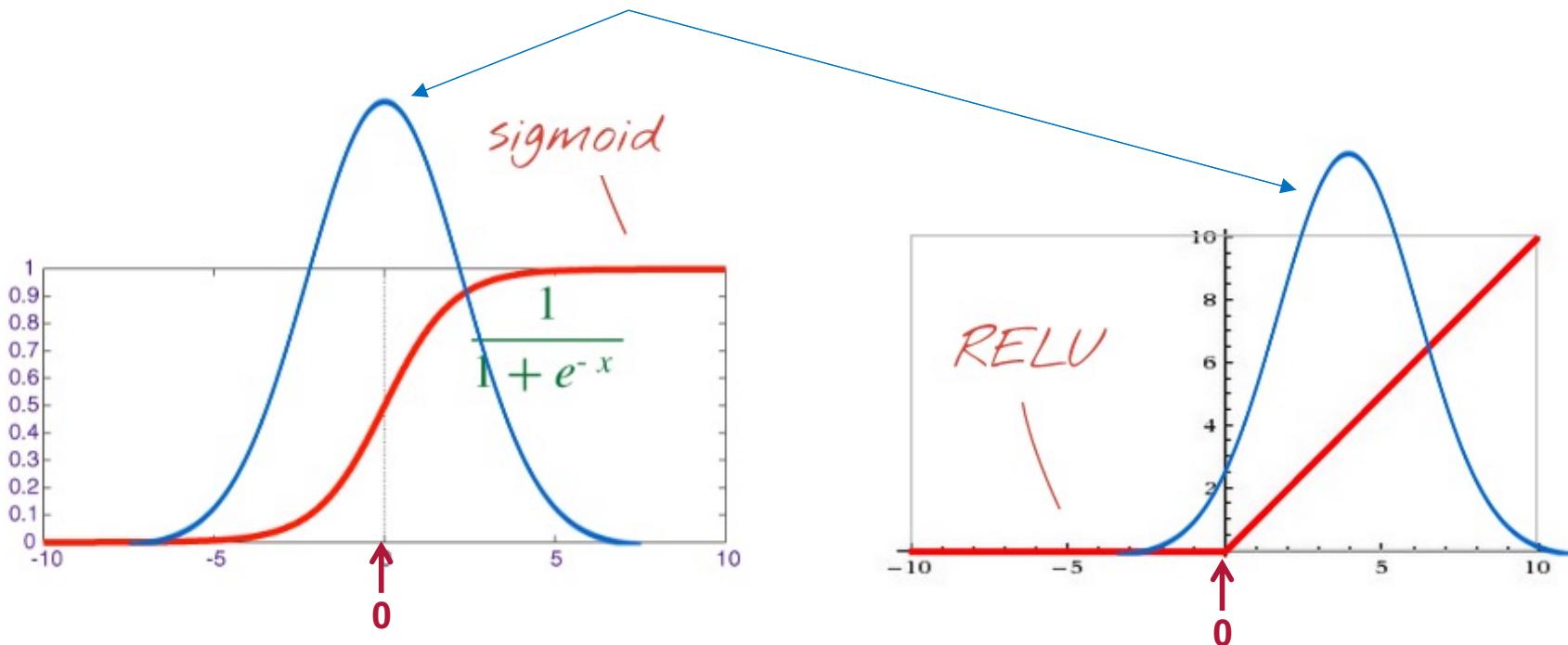
What happens when $x = 10$?

Gradients are killed, only when $x < 0$

Recap: Batch Normalization is beneficial in many NN

After BN the input to the activation function is in the sweet spot

Observed distributions of signal after BN before going into the activation layer.



When using BN consider the following:

- Using a higher learning rate might work better
- Use less regularization, e.g. reduce dropout probability
- In the linear transformation the biases can be dropped (step 2 takes care of the shift)
- In case of ReLu only the shift β in steps 2 need to be learned (α can be dropped)

Image credits: Martin Gorner:

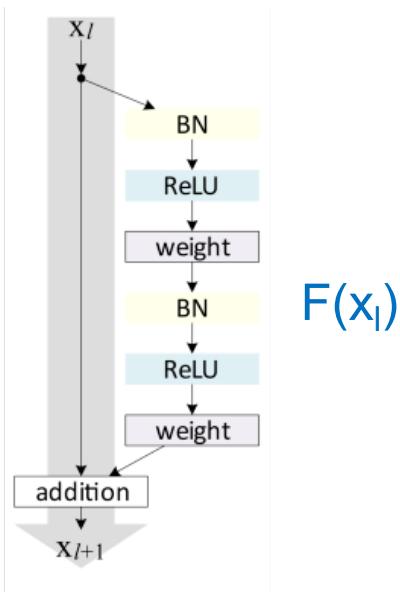
https://docs.google.com/presentation/d/e/2PACX-1vRouj_3cYsmLrNNI3Uq5gv5-hYp_QFdeoan2GlxKglZRSejozruAbVV0IMXB0PsINB7Jw92vJo2EAM/pub?slide=id.g187d73109b_1_2921

"ResNet" from Microsoft 2015 winner of imageNet

152
layers

ResNet basic design (VGG-style)

- add shortcut connections every two
- all 3x3 conv (almost)

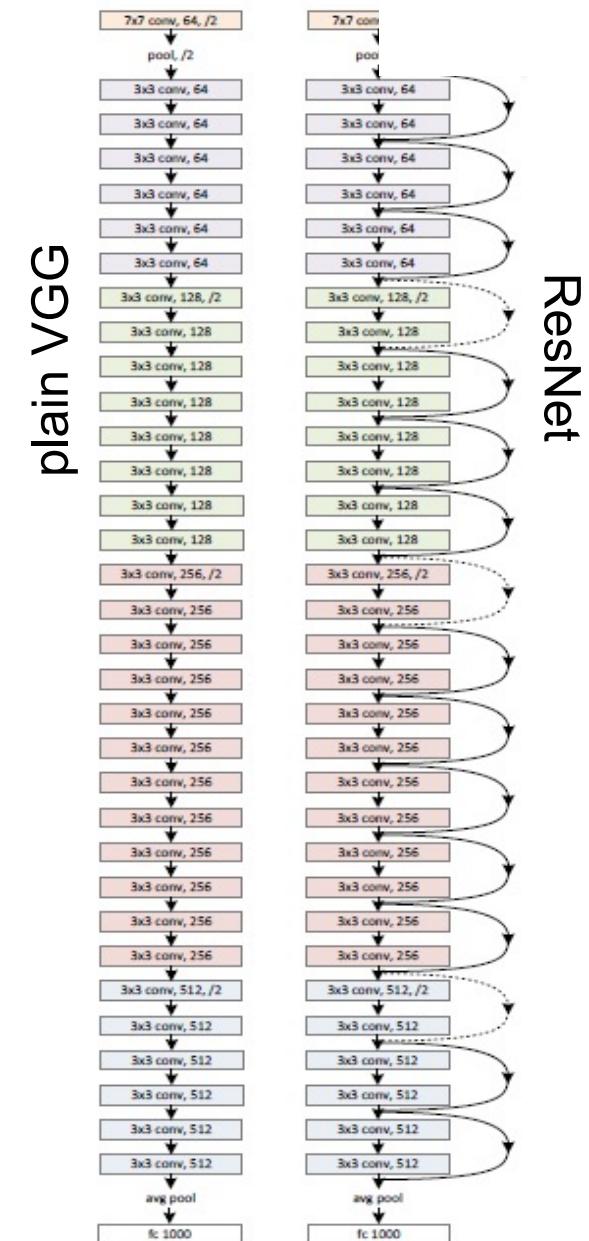


$$H(x_I) = x_{I+1} = x_I + F(x_I)$$

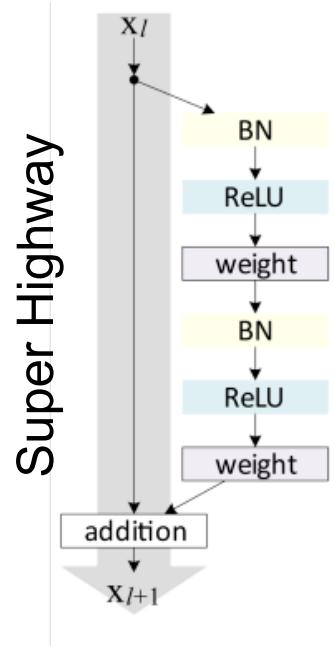
$F(x)$ is called "residual" since it only learns the "delta" which is needed to add to x to get $H(x)$

152 layers:
Why does this train at all?

This deep architecture
could still be trained, since
the gradients can skip
layers which diminish the
gradient!



Closer Look



$$\frac{\partial(\alpha + \beta)}{\partial \alpha} = 1$$

→ 'Gradient Super Highways'

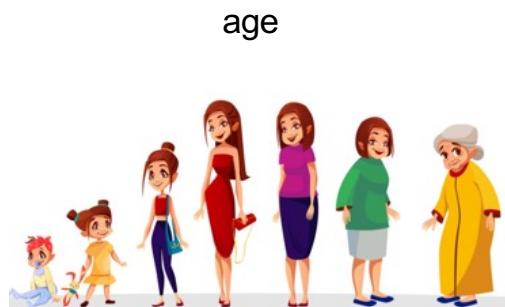
What comes in (on the right)
does go out (on the left)

Similar to LTMS (just in case
you know)



Building Loss Functions with Maximum Likelihood

Simple regression via a NN: no probabilistic model in mind

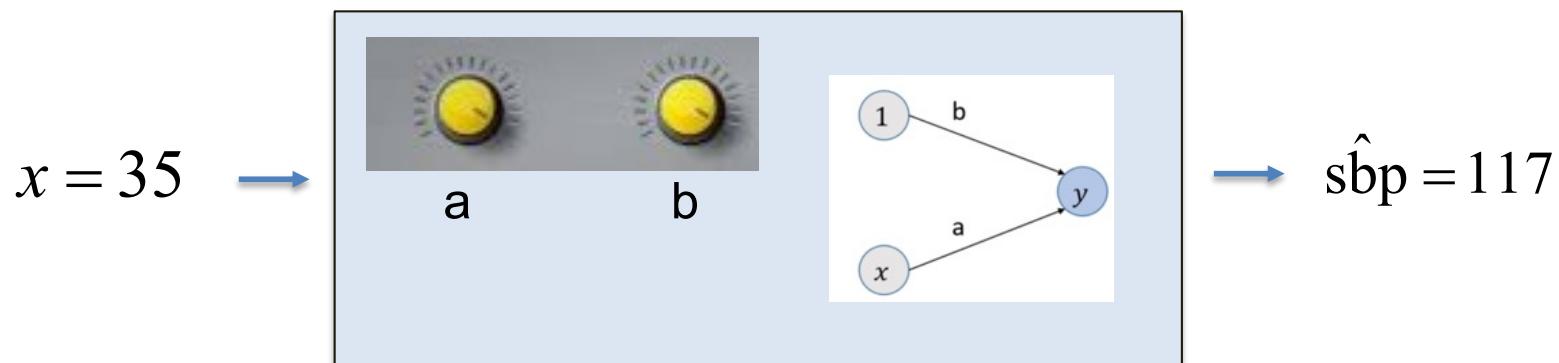


Input x

Systolic blood pressure

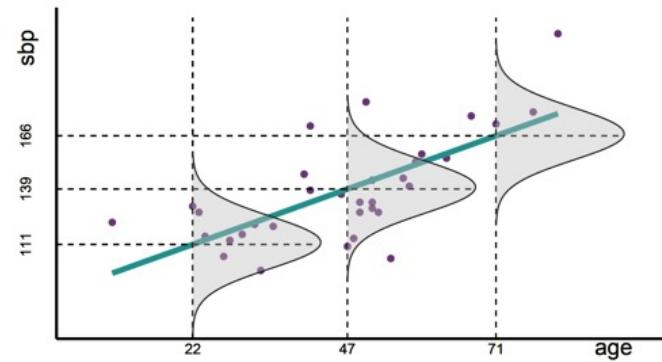
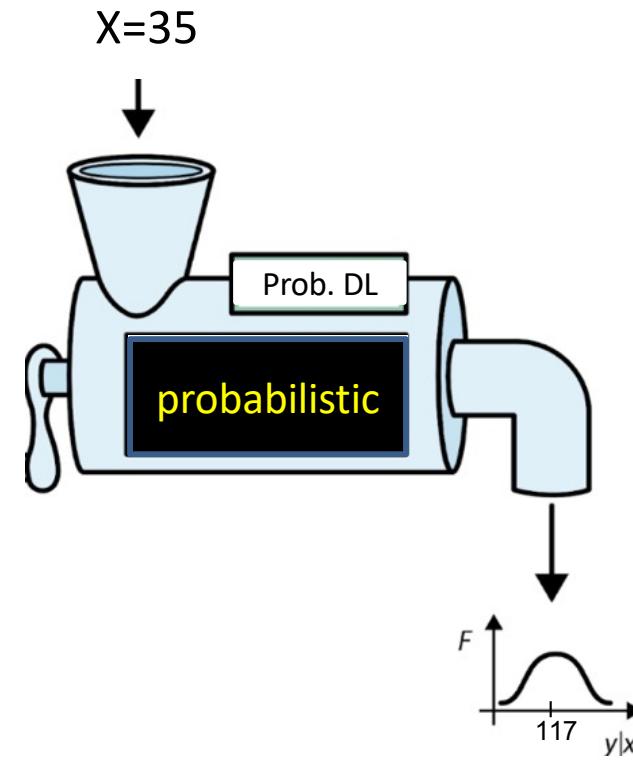
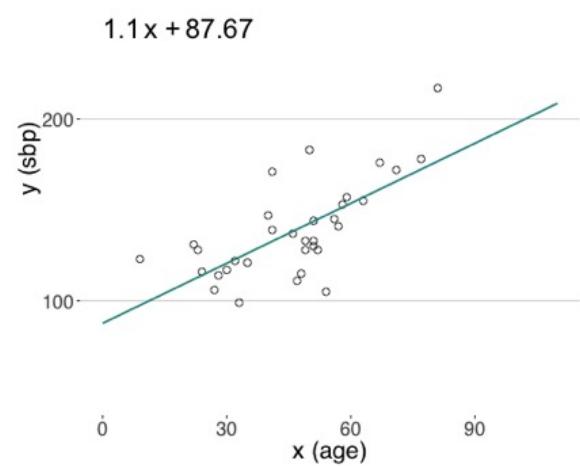
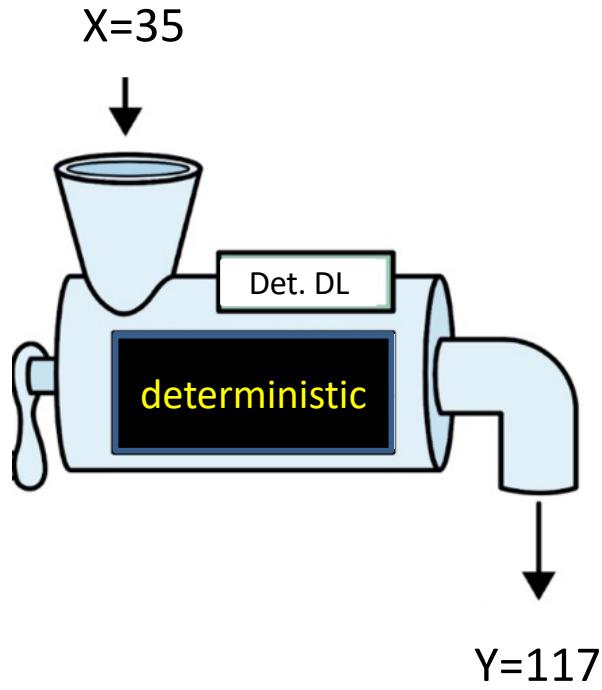


Output y



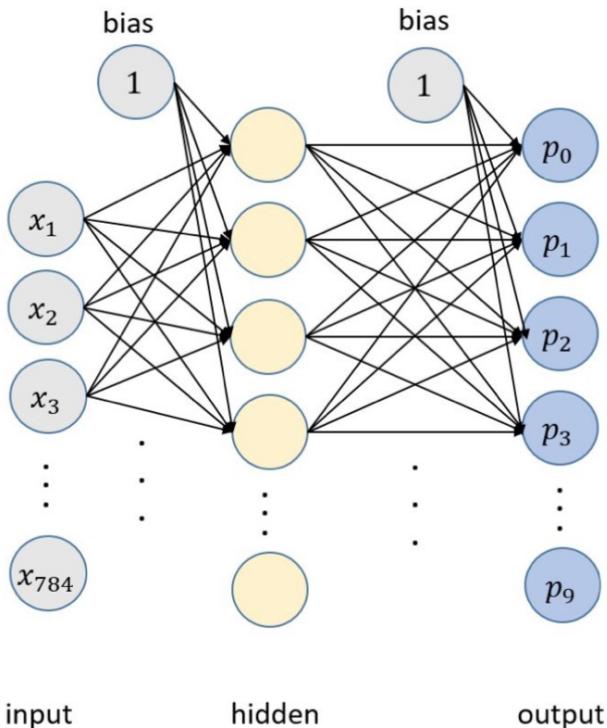
One input x (age) \rightarrow one predicted outcome (sbp)

Traditional versus probabilistic regression DL models



Describes the spread of the data

Recap Classification: Softmax Activation



$p_0, p_1 \dots p_9$ are probabilities for the classes 0 to 9.

Activation of last layer z_i incoming

$$p_i = \frac{e^{z_i}}{\sum_{j=0}^9 e^{z_j}}$$

Makes outcome positive

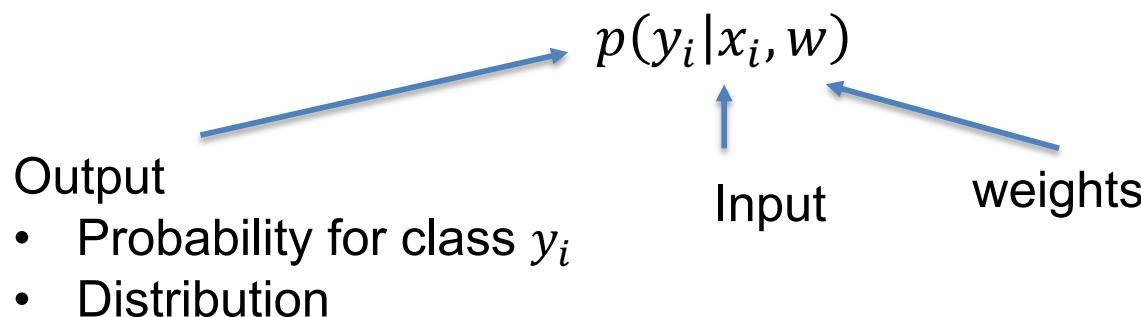
Ensures that p_i 's sum up to one

This activation is called softmax

Figure 2.12: A fully connected NN with 2 hidden layers. For the MNIST example, the input layer has 784 values for the 28 x 28 pixels and the output layer out of 10 nodes for the 10 classes.

Neural networks are probabilistic models

- The output of a neural network, can be understood as a probability*
 - Classification
 - Probability of class 1...,K
 - Regression
 - Probability Distribution
- Output of a neural network for training example i



*More on probabilistic interpretation next lecture

Maximum Likelihood



Tune the parameters weights of the network, so that observed data (training data) is most likely.

Practically: Minimize Negative Log-Likelihood of the CPD

$$\hat{w} = \operatorname{argmin} \sum_{i=1}^N -\log(p(y_i | x_i, w))$$

Maximum Likelihood (one of the most beautiful ideas in statistics)



Likelihood / “probability“
(often known)

$M(\theta)$ \longrightarrow Data

Ronald Fisher in 1913
Also used before by
Gauss, Laplace

Tune the parameter(s) θ of the model M
so that (observed) data is most likely

What's the likelihood of the data for lin. regression...

Motivating Example of MaxLike

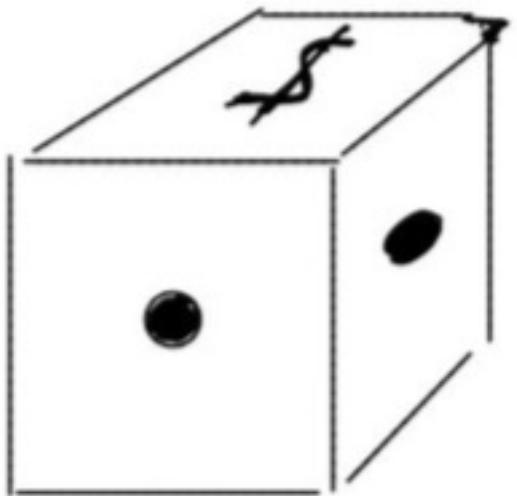


Figure 4.2 A die with one side showing a dollar sign and the others a dot.

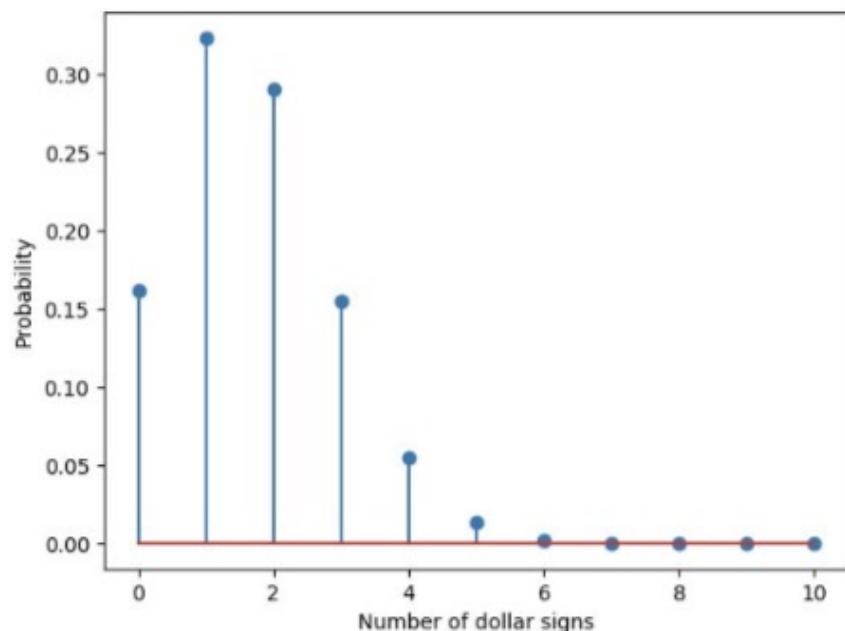
Question: What is probability to see one \$-signs in 10 throws?

A see Blackboard:

$$k \sim \text{binom}(n=10, p = 1/6)$$

Solution

```
from scipy.stats import binom  
ndollar = np.asarray(np.linspace(0,10,11), dtype='int')  
pdollar_sign = binom.pmf(k=ndollar, n=10, p=1/6)  
plt.stem(ndollar, pdollar_sign)  
plt.xlabel('Number of dollar signs')  
plt.ylabel('Probability')
```



Exercise 12_maxlike



Now you don't know how many dollar signs are on the die.

You throw the die 10 times and get k=2 dollar signs.

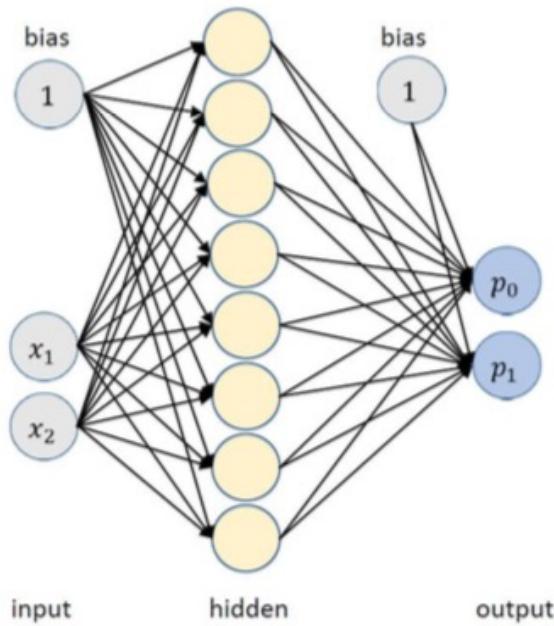
What is your best guess?

Work Through Exercise:

Work through the code until you reach the first exercise. In the exercise it is your task to determine the probability to observe two-times a dollar sign in ten dice throws, if you consider a die that has dollar signs on 0, 1, 2, 3, 4, 5, or all 6 faces.

https://github.com/tensorchiefs/dl_book/blob/master/chapter_04/nb_ch04_01.ipynb

ML principle for binary classification



x_i, y_i Training data $i = 1, \dots N$

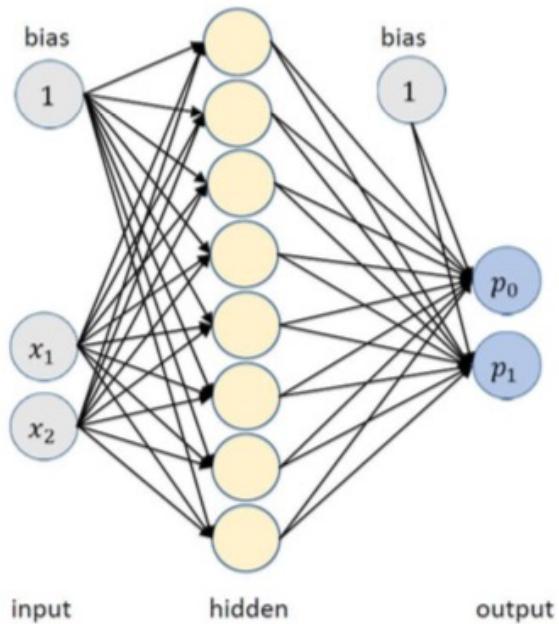
$p_0(x_i)$ is probability for $y_i = 0$

$p_1(x_i)$ is probability for $y_i = 1$

Question:

What is probability for the training set of say 5 examples? The first 3 are of class 0, last two of class 1?

ML principle for binary classification



x_i, y_i Training data $i = 1, \dots N$

$p_0(x_i)$ is probability for $y_i = 0$

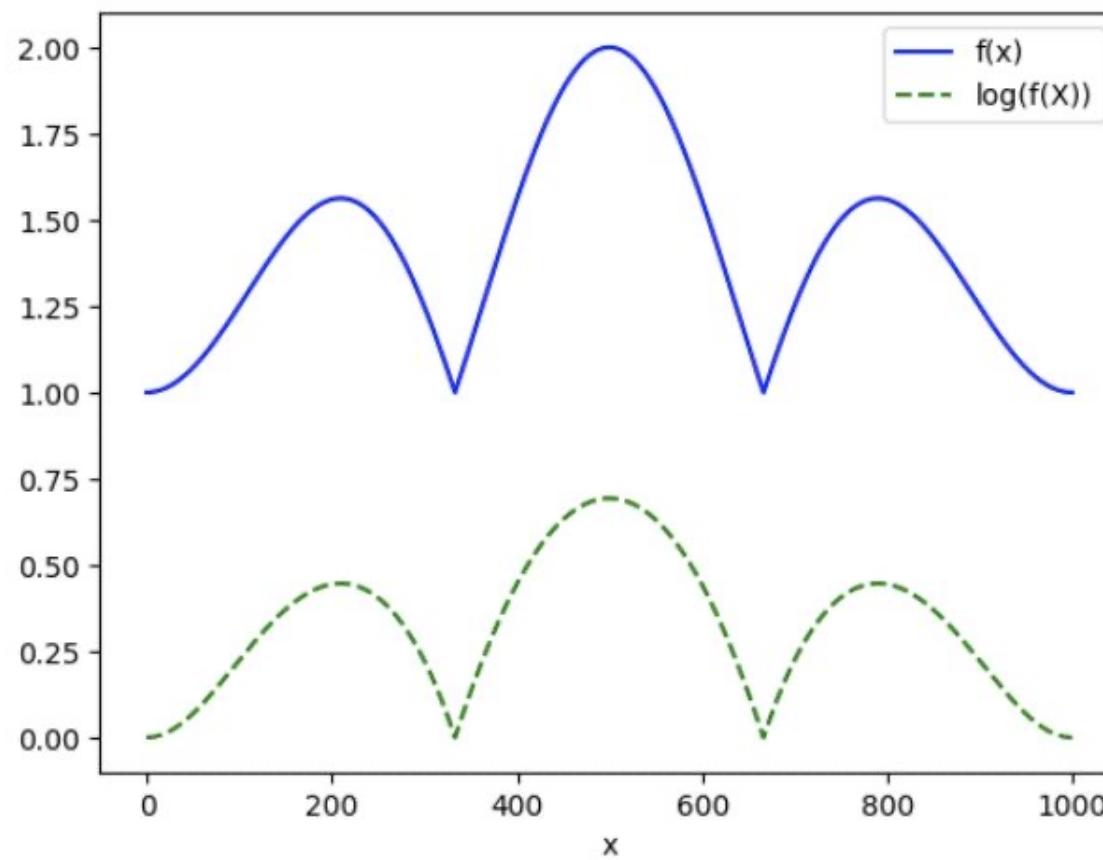
$p_1(x_i)$ is probability for $y_i = 1$

Answer:

What is probability for the training set of say 5 examples? The first 3 are of class 0, last two of class 1?

$$Pr(Training) = p_0(x_1) \cdot p_0(x_2) \cdot p_0(x_3) \cdot p_1(x_4) \cdot p_1(x_5) = \prod_{j=1}^3 p_0(x_j) \cdot \prod_{j=4}^5 p_1(x_j)$$

Taking the log



To determine the maximal value, taking log is also ok.

Negative Log-Likelihood (NLL)

- Likelihood of training data

$$Pr(Training) = \prod_{j \text{ for } y=0} p_0(x_j) \cdot \prod_{j \text{ for } y=1} p_1(x_j)$$

- LogLike

$$\log(Pr(Training)) = \sum_{j \text{ for } y=0} \log(p_0(x_j)) + \sum_{j \text{ for } y=1} \log(p_1(x_j))$$

- Crossentropy / NNL negative log likelihood (per example divided by n)

$$crossentropy = -\frac{1}{n} (\sum_{j \text{ for } y=0} \log(p_0(x_j)) + \sum_{j \text{ for } y=1} \log(p_1(x_j)))$$

More than 2 classes

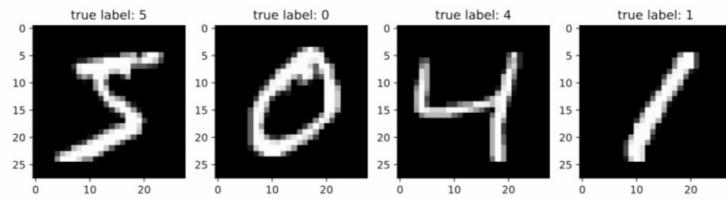
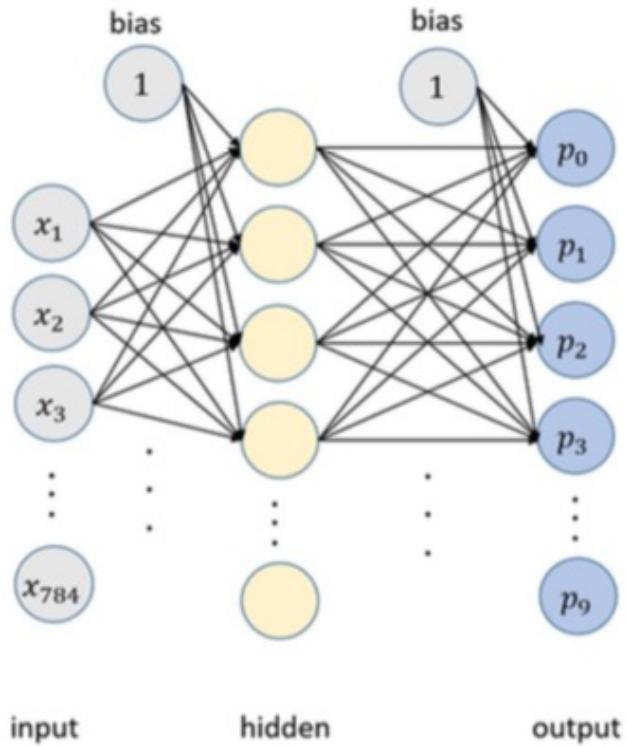


Figure 2.11 The first four digits of the MNIST data set—the standard data set used for benchmarking NN for images classification

$$\text{crossentropy} = -\frac{1}{n} \left(\sum_{j \text{ for } y=0} \log(p_0(x_j)) + \sum_{j \text{ for } y=1} \log(p_1(x_j)) + \dots + \sum_{j \text{ for } y=K-1} \log(p_{k-1}(x_j)) \right)$$

$$\text{crossentropy} = -\frac{1}{n} \sum_{i=1}^n \text{true } p_i \cdot \log(p_i)$$

Excercise



Task:

- Calculate (with calculator or numpy) the expected cross-entropy for the MNIST example if you just guess, each class with $p=1/10$.
- Load MNIST and make a small CNN without training and calculate the loss
- See NB [12b_mnist_loglike](#)