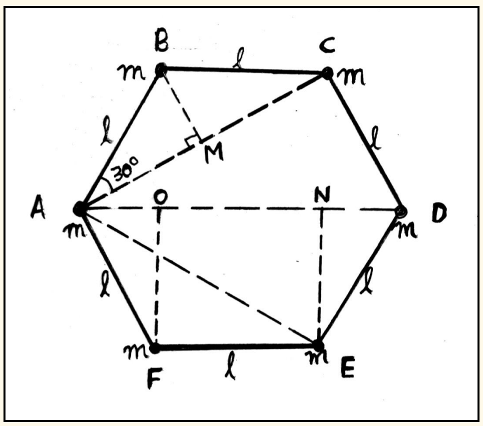
Nationwide Free Mock Test
JEE MAIN Solutions

PHYSICS

1. (c) 336 m/sec

$$f = \frac{v}{4(l + 0.6r)}$$
$$v = 4f(l + 0.6r)$$
$$= 4 \cdot 500[0.15 + 0.6 \cdot 0.03]$$
$$= 336 m/sec$$

2. (a)



AE = AC = 2lcos θ = $\sqrt{3}l$

AD = 2 l

AB = AF= l

Force on A due to B,

F_{AB} = $\frac{Gm^2}{l^2}$ along AB

F_{AC} = $\frac{Gm^2}{3l^2}$ along AC

F_{AD} = $\frac{Gm^2}{4l^2}$ along AD

F_{AE} = $\frac{Gm^2}{3l^2}$ along AE

F_{AF} = $\frac{Gm^2}{l^2}$ along AF

Resultant due to F_{AB} and F_{AF} is

F_{R1} = $\frac{Gm^2}{l^2}$ along AD

Resultant due to F_{AC} and F_{AE} is

F_{R2} = $\frac{Gm^2}{\sqrt{3}l^2}$ along AD

Net Force Along AD,

F_R = $\frac{Gm^2}{l^2}$ [$\frac{5}{4}$ + $\frac{1}{\sqrt{3}}$]

3. (b)

$$E = CB$$
$$= (3 \times 10^8) \cdot (3 \times 10^{-6})$$
$$= 9 \times 10^2$$

4. (d)

When C strikes A,

$\frac{1}{2}$ m v_0^2 = $\frac{1}{2}mu^2$ + $\frac{1}{2}kx_0^2$ (u is the velocity at the instant)

$kx_0^2 = m(v_0^2 - u^2)$ --------- (i)

$\frac{1}{2} \times 2mu^2 = \frac{1}{2} kx_0^2$ (When A and B blocks attains equal velocity)

$\therefore \frac{1}{2}kx_0^2 = mu^2$ ------------- (ii)

From eqns (i) and (ii),

$kx_0^2 = mv_0^2 - mu^2 = mv_0^2 - \frac{kx_0^2}{2}$

$k = \frac{2mv_0^2}{3x_0^2}$

5. (b)

$$f' = 20 \times \frac{320+20}{320-20}$$
$$= 22.66 Hz$$

6. (a)

The radius of the n 'th orbit of a hydrogen like atom with atomic number Z is $r_n = \frac{r_0 \mathrm{n}^2}{\mathrm{Z}}$

So for hydrogen atom, $\alpha = \frac{r_0 \times 2^2}{1} = 4r_0$.

Now for the third excited state of the given atom with Z=4, we have to find the radius of the 4 th orbit.

Therefore $r_4 = \frac{r_0 \times 4^2}{4} = 4r_0$ which gives $r_4 = \alpha$.

7. (b)

In constant pressure ,

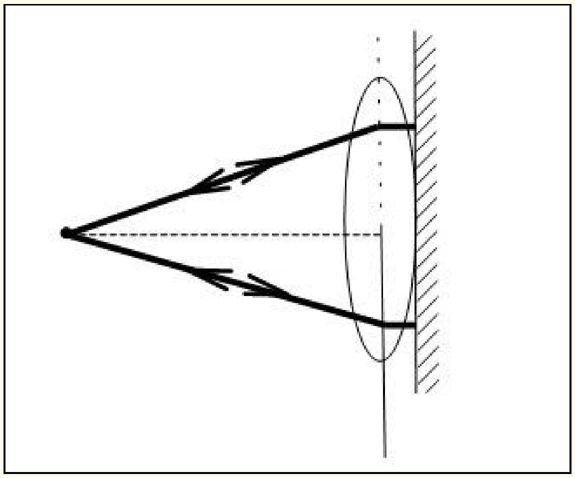
$$\frac{Q}{W} = \frac{C_p}{R} = \frac{\gamma}{\gamma - 1}$$

$$\frac{\gamma}{\gamma-1} = \frac{100}{40}$$

given, $\gamma = \frac{5}{3}$

8. (c)

$u = 40 \text{ cm}$



To form the image on the object, the rays must get parallel after refraction. For rays to be parallel, the object must be kept on the focus of the lens. Use the lens maker's formula to find the focus of the given convex lens.

$$\frac{1}{f} = \frac{\mu_L - \mu_S}{\mu_S}\left(\frac{1}{R_1} - \frac{1}{R2}\right)$$
$$= \frac{\frac{9}{8}-1}{1}\left(\frac{1}{10} - \left(\frac{1}{-10}\right)\right)$$
$$= \frac{9-8}{8} \times \frac{1}{5}$$
$$= \frac{1}{40}$$
$$\Rightarrow f = 40 \text{ cm}$$

Since $f = 40 \text{ cm}$, we get $u = 40 \text{ cm}$

9. (a)

$$H_1 = \frac{u^2 sin^2\theta}{2g}$$

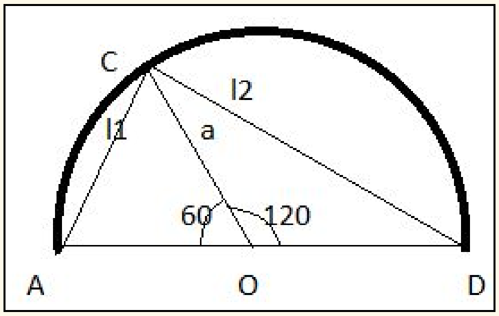
$$H_2 = \frac{u^2 sin^2(90-\theta)}{2g} = \frac{u^2 cos^2\theta}{2g}$$

$$H_1 H_2 = \frac{u^4 sin^2 2\theta}{16g^2} = \frac{R^2}{16}$$

$$\therefore R = 4\sqrt{H_1 H_2}$$

TENSORS

10 (c)



$$l_1 = 2a\sin\frac{60}{2} \quad = a$$
$$V_{C-A} = V_C - V_A \quad = \frac{Ba\omega^2}{2}$$
$$l_2 = 2a\sin\frac{120}{2} \quad = \sqrt{3}a$$
$$V_{C-D} = V_C - V_D \quad = \frac{3}{2}Ba\omega^2$$
$$V_A - V_D = Ba\omega^2$$

11. (c)

$$diameter = (4\cdot 0.5)mm + (23-3)\frac{0.5}{100}mm$$
$$= 2.10mm$$

12. (d)

$$E = \frac{1}{2}m\omega^2A^2$$
$$= \frac{1}{2}m(2\pi f)^2A^2$$
$$A = \frac{1}{2\pi f}\times\sqrt{\frac{2E}{m}}$$
$$E = K + U$$
$$= 0.6 + 0.2$$
$$= 0.8J$$
$$A = 0.1m$$

13. (a)

Let a_1 and a_2 be the accelerations of M and m respectively.

Then, $Mg - F = Ma_1$ ------------ (i)

And, $mg - F = ma_2$ ------------ (ii)

Now, $l + \frac{1}{2}a_1t^2 = \frac{1}{2}a_2t^2$ or $a_2 = \frac{2l}{t^2} + a_1$ --------- (iii)

Solving eqns (i), (ii) and (iii), we get

$F = \frac{2Mml}{(m-M)t^2}$

14. (c)

$$\frac{\rho_{air}}{\rho_{water}} = \frac{169}{169-121} \quad = \frac{169}{48}$$
$$\frac{\rho_{air}}{\rho_{liqiud}} = \frac{169}{169-144} \quad = \frac{169}{25}$$
$$\frac{\rho_{liquid}}{\rho_{water}} = \frac{25}{48}$$

15. (d)

$$X_l = l\omega \quad = 50\Omega$$
$$X_c = \frac{1}{c\omega} \quad = 10\Omega$$
$$Z = \sqrt{R^2 + (X_c - X_L)^2} \quad = 50\Omega$$
$$I_0 = \frac{V_0}{Z} \quad = 4A$$
$$\cos\alpha = \frac{R}{z} \quad = \frac{3}{5}$$
$$I = 4\sin(100 + 30^\circ - 53^\circ) = 4\sin(100t - 23^\circ)$$

TENSORS

16. (c) 1

The mass M is initially at rest. So from the conservation of momentum, the total momentum after decay is zero.

$m_1v_1 + m_2v_2 = 0 \Rightarrow m_1v_1 = -m_2v_2.$

So the magnitude of momentum of both the particles is equal. Let it be P.

Now the De Broglie wavelength of the first particle $\lambda_1 = \frac{h}{m_1v_1} = \frac{h}{P}$ and the De Broglie wavelength of the second particle $\lambda_1 = \frac{h}{m_2v_2} = \frac{h}{P}$. From this , we obtain the result $\lambda_1 = \lambda_2$ which gives $\frac{\lambda_1}{\lambda_2} = 1$.

17. (a)

$$Initial\ KE = \tfrac{1}{2}mv^2 = \tfrac{1}{2} \times \tfrac{20}{1000} \times 600 \times 600 = 3600\ J$$

$$Change\ in\ KE = PE$$

$$\tfrac{1}{2}m\,(v^2 - u^2) = mg$$

$$3600 - \tfrac{1}{2}.\tfrac{20}{1000}.u^2 = 4 \times 10 \times 80$$

$$\therefore u = 200m/s$$

18. (d)

$$\mathrm{y} = \frac{4\mathrm{D}}{3}$$

For Maxima,

$$\mathrm{d}(\sin\theta) = n\lambda = 8\lambda$$

$$\sin\theta = \frac{4}{5}$$

$$\tan\theta = \frac{4}{3}$$

$$\mathrm{y} = \mathrm{D}\tan\theta = \frac{4\mathrm{D}}{3}$$

19. (d)

$$\begin{aligned} B &= 2|B|\cos\alpha \\ &= 2(\frac{x}{AP})(\frac{\mu_0}{2\pi})(\frac{I}{AP}) \end{aligned}$$

$$AP = \sqrt{a^2 + x^2}$$

$$B_{net} = (\frac{x}{\sqrt{a^2+x^2}})(\frac{\mu_0}{2\pi})(\frac{I}{\sqrt{a^2+x^2}})$$

$$\frac{\partial F}{\partial x} = (\frac{x}{\sqrt{a^2+x^2}})(\frac{\mu_0}{2\pi})(\frac{I}{\sqrt{a^2+x^2}})$$

$$\int_{x=0}^{x=L} dF = \frac{\mu_0 I^2}{2\pi}\ln\frac{L^2+a^2}{a^2}$$

20. (d)

$$\begin{aligned} a_{cm} &= \frac{F}{m} \\ &= \frac{6i+3j}{6} \\ &= i + \frac{1}{2}j \end{aligned}$$

$$\begin{aligned} |a_{cm}| &= \sqrt{1^2 + (\frac{1}{2})^2} \\ &= \frac{\sqrt{5}}{2}m/s^2 \end{aligned}$$

$$\tan\theta = \frac{1}{2}$$

$$\theta = \arctan\frac{1}{2}$$

21. (2)

Let the final charges be q1,q2 and q3 on A,B,C respectively

A and C are connected

q1+q3=Q-2Q=-Q..................(1)

Potential on A and C are equal

TENSORS

$$\frac{kq1}{a}+\frac{kq2}{2a}+\frac{kq3}{3a}=\frac{kq1}{3a}+\frac{kq2}{3a}+\frac{kq3}{3a}$$

On solving q1=-q2/4..............(2)

Potential of B =0

$$\frac{kq1}{2a}+\frac{kq2}{2a}+\frac{kq3}{3a}=0$$

Solving q3=-9q2/8.............(3)

Putting (2) and (3) in (1)

$$\frac{-q2}{4}+\frac{-9q2}{8}=-Q$$

q2=8Q/11

So n=8

n/4=2

22. (5)

Initial radius of wooden ring = 2.006m

Initial radius of iron ring = 2006 – 6mm = 2000mm

$$\Delta R=6mm$$

On increasing temperature by $\Delta\theta$, iron ring will fit on to the wooden ring.

Increment in the length(circumference) of the iron ring

$$\begin{aligned}\Delta l&=l\alpha\Delta\theta\\&=l\times\frac{\gamma}{3}\times\Delta\theta\end{aligned}$$

$$2\pi\Delta R=2\pi R\frac{\gamma}{3}\times\Delta\theta$$

$$\Delta\theta=250$$

$$\frac{250}{5}=5$$

23. (7)

$$\begin{aligned}\tau&=(10t-2t^2)\times 4\\&=40t-8t^2\end{aligned}$$

$$\alpha=\frac{\tau}{I}=\frac{40t-8t^2}{5}=8t-\frac{8t^2}{5}$$

$$\begin{aligned}\omega&=\int_0^t\alpha dt\\&=4t^2-\frac{8t^3}{15}\end{aligned}$$

Direction is reversed when ω is zero ,so

$$\begin{aligned}4t^2-\frac{8t^3}{15}&=0\\t&=\frac{15}{2}s\\a&=15\\b&=2\\a-4b&=15-8\\&=7\end{aligned}$$

24. (9)

Let P_{top} be the pressure above the liquid column, P_{bot} be the pressure below the liquid column and P_{liq} be the gauge pressure exerted by the liquid.

Since the product PV remains constant,

$P_{atm}\times 45\text{ cm}\times A=P_{top}\times(45+\text{x})\text{ cm}\times A$

$P_{atm}\times 45\text{ cm}\times A=P_{bot}\times(45-\text{x})\text{ cm}\times A$

$\Rightarrow 45P_{atm}=(45+x)P_{top}$ and $45P_{atm}=(45-x)P_{bot}$

$\Rightarrow P_{top}=\dfrac{45P_{atm}}{45+x}$ and $P_{bot}=\dfrac{45P_{atm}}{45-x}$

Also,

$$\begin{aligned}P_{bot}&=P_{top}+P_{liq}\\&=P_{top}+\rho gh\end{aligned}$$

We are given $\rho g\times 20\text{ cm}=P_{atm}$.

So $P_{bot} = P_{top} + \rho g \times 10 \text{ cm}$ becomes

$$P_{bot} = P_{top} + \frac{P_{atm}}{2}$$

Substituting the values of P_{top} and P_{bot} in the expression $P_{bot} = P_{top} + \frac{P_{atm}}{2}$, we get the required quadratic equation.

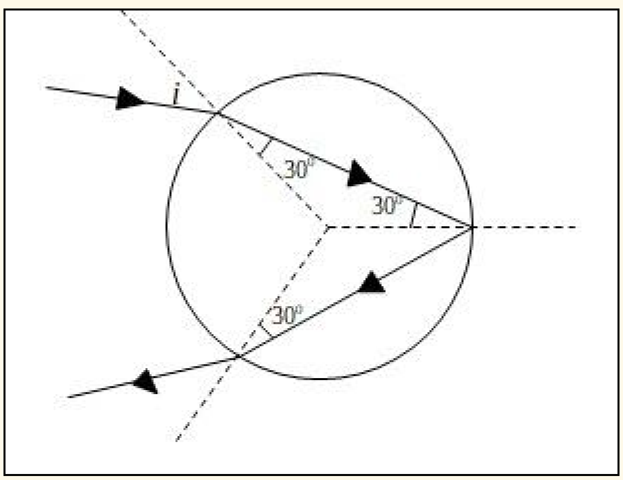
$$\frac{45P_{atm}}{45-x} = \frac{45P_{atm}}{45+x} + \frac{P_{atm}}{2}$$

$$90(45+x) = 90(45-x) + (45+x)(45-x)$$

$$x^2 + 180x - 45^2 = 0$$

Hence, $\alpha = 180$. Therefore the sum of the digits of α is 9.

25. (5)



$$\frac{\sin i}{\sin r} = \sqrt{2} \Rightarrow i = 45^0$$

Due to symmetry the emerging angle e will also be 45^0

$$\begin{aligned} \text{So the total deviation} &= (45^0 - 30^0) + 180^0 - 2 \times (30^0) + (45^0 - 30^0) \\ &= 30 + 120 \\ &= 150 \\ &= 30 \times 5 \end{aligned}$$

CHEMISTRY

1) (C)

SF_6 is sp^3d^2 or d^2sp^3 hybridised and has octahedral geometry. In octahedron, the bonds are formed parallel to the x, y, and z-axes, hence dx^2-dy^2 and dz^2 will be used to form the hybrid orbitals.

2) (C)

In π backbonding, bond angle, planarity and geometry are not changed.

3) (A)

With KOH and $Ca(OH)_2$ both HPh and MeOH requires x and 2y equivalents respectively.

In the case of Na_2CO_3

When methyl orange (MeOH) is used as indicator

$Na_2CO_3 + 2HCl \rightarrow 2NaCl + H_2O + CO_2$

This is the complete neutralization

When phenolphthalein (HPh) is used as indicator the reaction are given below.

$Na_2CO_3 + HCl \rightarrow NaHCO_3 + NaCl$

This is the half neutralization of Na_2CO_3

4) (D)

As nuclear charge is more, it is difficult to remove an electron from outermost shell of Ga, hence ionization energy is more than Al.

TENSORS

Because the electron is negatively charged you can see that there will be repulsion between the negative ion of oxygen and the electron making the process endothermic, hence the positive value for the second electron affinity of oxygen.

5) (B)

Common ion effect decreases solubility. Therefore, AgCl is most soluble in water due to the absence of common ions.

KCl = 0.1 * 1

$AgNO_3$ = 0.5*1

$CaCl_2$ = 0.2*2

$BaCl_2$ =0.05*2

6) (C)

$$H_3A \rightleftharpoons H^+ + H_2A^- \quad ka_1 - (i)$$
$$H_2A^- \rightleftharpoons H^+ + HA^{2-} \quad ka_2 - (ii)$$
$$HA^{2-} \rightleftharpoons H^+ + A^{3-} \quad ka_3 - (iii)$$
$$\left[HA^{2-}\right] \text{ is max at}$$
$$P^H = \left[\frac{Pka_2 + Pka_3}{2}\right]$$

Average of the Pka's of the reactions in which HA^{2-} is formed (ii) and reaction in which HA^{2-} disproportionate (iii)

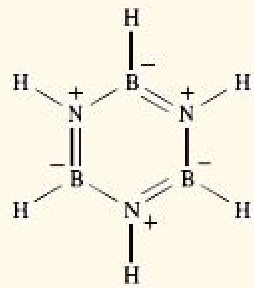
7) (D)

$Fe + 2HCl \longrightarrow FeCl_2 + H_2$

A catalyst is a substance that decreases the activation energy of a chemical reaction without itself being changed at the end of the chemical reaction. A promoter is an accelerator of catalysis, but not a catalyst by itself.

8) (D)

Structure :



9) (A)

$$CH_2(COOH)_2 \xrightarrow[140^o-150^o]{P_4O_{10}} C_3O_2 + 2H_2O$$

10) (A)

As the negative charge on metal carbonyl complex increases, back pi bonding increases and hence the bond length of C-O bond increases.

11) (B)

X atoms at corners and face centres - 4

Y atoms at body centre and edge centres - 4

Along a face diagonal - 2 corner atoms of X + 1 face centre of X

Atoms of X removed = (2/8) + (1/2) = 3/4

Therefore, atoms of X left = 13/4

Formula : X 13/4Y 4

X 13Y 16

12) (C)

13) (B)

This medication is used to treat symptoms of anxiety and nervousness.

14) (A)

15) (A)

In thiosulphuric acid there is a S=S linkage present. One S atom is in -2 and other sulphur atom is in +6 oxidation state.

16) (A)

A(Solution) ----> B(Solution) + C(Solution)

At time $t=0$ (a) 0 0

At time $t=t$ (a-x) x x

At time $t=\infty$ (0) a a

$$C_A a = r_0 - (1)$$
$$C_a(a-x) + C_b x + C_c x = r_t - (2)$$
$$r_0 + x(C_b + C_c - C_a) = r_t$$

$$x = \frac{r_t - r_0}{C_b + C_c - C_a}$$

$$a(C_b + C_c) = r_\infty - (3)$$

(3)-(1)

$$a(C_b + C_c - C_a) = r_\infty - r_0$$
$$a = \frac{r_\infty - r_0}{C_b + C_c - C_a}$$
$$a - x = \frac{r_\infty - r_t}{C_b + C_c - C_a}$$

$$k = \frac{1}{t}\ln\frac{a}{a-x}$$
$$k = \frac{1}{t}\ln\frac{r_\infty - r_0}{r_\infty - r_t}$$
$$k = \frac{2.303}{t}\log\frac{r_\infty - r_0}{r_\infty - r_t}$$

17) (D)

In Compound 3, the compound is stabilised by resonance with the lone pair of Cl atom. Therefore, the Cl atom will not combine with Ag+ to form white precipitate (AgCl).

The two compounds which form white ppt will become aromatic once they lose the Cl^- ion

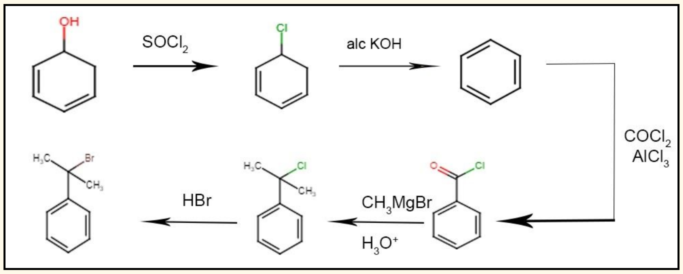
18) (D)

As there are 2 cumulated double bonds the terminal carbons are mutually perpendicular to each other & they have the same groups attached to them i.e CH3& H. So the molecule becomes chiral (form enantiomers) as plane passing through it does not cut it in a symmetrical manner.

19) (B)

Aldehydes and ketones having at least one α-hydrogen undergo aldol condensation. Due to +I effect of 2-methyl propanal, it gives Cannizzaro's reaction although it has alpha hydrogen.

20) (A)



21) (7)

The equation for packing fraction is:
Packing fraction = (N atoms) x (V atom) / V unit cell
N atoms is the number of atoms in a unit cell. V atom is the volume of the atom, and V unit cell is the volume of a unit cell.
Diamond has eight atoms per unit cell, so the diamond packing fraction equation now becomes:
Packing fraction = 8 x (V atom) / V unit cell
Substitute the volume of the atom into the equation. Assuming atoms are spherical, the volume is: $V = 4/3 \times \pi \times r3$
The equation for packing fraction now becomes:
Packing fraction = $8 \times 4/3 \times \pi \times r3$/ V unit cell
Substitute the value for the unit cell volume. Since the unit cell is cubic, the volume is V unit cell = a3
The formula for packing fraction then becomes:
Packing fraction = $8 \times 4/3 \times \pi \times r3/ a3$
The radius of an atom r is equal to $\sqrt{3} \times a/8$
The equation is then simplified to : $\sqrt{3} \times \pi /16 = 0.3401$

Therefore, packing fraction = 34.01%

22) (5)

Galena - PbS

Malachite - $CuCO_3 . Cu(OH)_2$

Fluorspar - CaF_2

Pyrolusite -MnO_2

Cinnabar - HgS

Zincite - ZnO

Horn Silver - AgCl

Rutile - TiO_2

23) (6)

If the solution is diluted to 100 times, molar conc of Cu^{2+} is reduced to 1/100 of its original value.

$E = E^0 + 0.059/2 \log (Cu^{2+})$

If Cu^{2+} concentration becomes 1/100,

E changes by (0.059/2)*2 = 0.059

24) (4)

$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$
x   2x
$2CO + O_2 \rightarrow 2 CO_2$

57.5-x.   (57.5-x)/2

2x + (57.5-x)/2 = 85

Solving for x,

x = 37.5 ml

25) (8)

The enthalpy change for dissolution is the sum of the lattice enthalpy and the enthalpy change for hydration :
ΔH sol = ΔH lattice + ΔH hyd
3 = 150 + ΔH hyd
ΔH hyd = -147

The hydration energy of XY is the sum of enthalpies of hydration of X ions and Y ions
ΔH hyd(X ions) + ΔH hyd(Y ions) = -147
The hydration energies are in the ratio 6:5
Therefore, ΔH hyd (X ions) = $\frac{6*(-147)}{11}$
= -80.18 Kcal/mol

TENSORS

MATHEMATICS

1. (C)

$$\begin{aligned} n(X \cap Y) &= 25 \\ n(X \times Y) \cap (Y \times X) &= n(X \cap Y) \times (Y \cap X) \\ &= 25 \times 25 \\ &= 625 \end{aligned}$$

2. (A)

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^3y^3 - xy$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} + xy = x^3y^3$$

$$\frac{1}{y^3}\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{x}{y^2} = x^3$$

Put $\frac{1}{y^2} = z$

$$-\frac{2}{y^3}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}z}{\mathrm{d}x}$$

$$-\frac{1}{2}\frac{\mathrm{d}z}{\mathrm{d}x} + zx = x^3$$

$$\frac{\mathrm{d}z}{\mathrm{d}x} - 2zx = -2x^3$$

$$\mathrm{IF} = e^{\int -2x dx} = e^{-x^2}$$

$$e^{-x^2}\frac{\mathrm{d}z}{\mathrm{d}x} - 2e^{-x^2}xz = -2x^3e^{-x^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(ze^{-x^2}) = -2x^3e^{-x^2}$$

$$\mathrm{d}(ze^{-x^2}) = -2x^3e^{-x^2}\mathrm{d}x$$

Substitute, $x^2 = t$

$$2xdx = dt$$

$$ze^{-x^2} = -\int te^{-t}\mathrm{d}t$$

$$ze^{-x^2} = te^{-t} + \int e^{-t}\mathrm{d}t$$

$$ze^{-x^2} = te^{-t} + e^{-t} + c$$

$$\frac{1}{y^2}e^{-x^2} = x^2e^{-x^2} + e^{-x^2} + c$$

$$(x^2 + 1 + ce^{x^2})y^2 = 1$$

3. (B)

Let S = $(1 + x)^{1000} + 2x(1+x)^{999} + \ldots + 1000x^{999}(1+x) + 1001\, x^{1000}$

This is an Arithmetic Geometric Series with r = x/(1+x) and d = 1.

Now

$(x/(1+x))\, S = x(1 + x)^{999} + 2x^2 (1 + x)^{998} + \ldots + 1000x^{1000} + 1001x^{1001}/(1+x)$

Subtracting we get,

$(1 - (x/(x+1)))\, S = (1+x)^{1000} + x(1+x)^{999} + \ldots + x^{1000} - 1001x^{1000}/(1+x)$

$S = (1+x)^{1001} + x(1+x)^{1000} + x^2(1+x)^{999} + \ldots + x^{1000} (1+x) - 1001x^{1001}$

This is G.P. and sum is

$S = (1+x)^{1002} - x^{1002} - 1002x^{1001}$

So the coeff. of x^{50} is = ${}^{1002}C_{50}$

4. (D)

$answer : -|a-b| = 2$

$(x+a)^2 = (x+b)^2 = 1$

$(x+a)^2 - (x+b)^2 = 0 \Rightarrow (a-b)(2x+a+b) = 0$

$\Rightarrow x = -1/2(a+b)$

so put this in the first equation which gives

$(a-b)^2 = 1 \Rightarrow (a-b)^2 = 4$

$so, |a-b| = 2$

5. (C)

$\frac{1+z+z^2}{1-z+z^2} \in R$

$\therefore \frac{1+z+z^2}{1-z+z^2} = \overline{\frac{1+z+z^2}{1-z+z^2}}$

$\Rightarrow |z| = 1$

Since z is non real $S = \{z : |z| = 1 \text{ and } z \neq 1, -1\}$

Also $\frac{w - \overline{w}z}{1-z} \in R$

$\Rightarrow \frac{w-\overline{w}z}{1-z} = \overline{\frac{w-\overline{w}z}{1-z}} \quad \Rightarrow |z| = 1$

So, $P = \{z : |z| = 1 \text{ and } z \neq 1$, So $S \subset P$

Centroid of the polygon = $\frac{z_1+z_2+\ldots z_n+\frac{1}{z_1}+\ldots\ldots\frac{1}{z_n}}{2n}$

=

$\frac{z_1+z_2+\ldots z_n+\overline{z}_1+\overline{z}_2+\ldots\overline{z}_n}{2n} \left(\because |z| = 1, \frac{1}{z} = \overline{z}\right)$

$\in R \left(\because Z + \overline{Z} \varepsilon R\right)$

ω, ω^2 are the roots of the given equation. So they belong to P.

6. (D)

$$I_n = \int \csc^n x dx$$
$$= \int \csc^2 x \csc^{n-2} x dx$$
$$= -\cot x \csc^{n-2} x + \int (\cot x)(n-2)(-\csc^{n-3} x \cot x \csc x) dx$$
$$= -\cot x \csc^{n-2} x - (n-2) \int (\cot^2 x \csc^{n-2} x) dx$$
$$= -\cot x \csc^{n-2} x - (n-2) \int (\csc^2 x - 1) \csc^{n-2} x dx$$
$$I_n = -\cot x \csc^{n-2} x + (n-2) \int \csc^{n-2} x dx - (n-2) I_n$$
$$(n-1) I_n = -\cot x \csc^{n-2} x + (n-2) I_{n-2}$$
$$I_n = \frac{-\cot x \csc^{n-2} x}{n-1} + \frac{n-2}{n-1} I_{n-2}$$
$$a = -\frac{\mathrm{d}}{\mathrm{d}x} cosecx$$
$$c = \frac{n-2}{n-1}$$
$$b = 1 - n$$

7. (A)

Let $P(x, y)$ be a point on the curve.

Equation of tangent at P

$$Y - y = m(X - x)$$
$$\frac{y - mx}{\sqrt{1+m^2}} = x$$
$$y^2 + m^2x^2 - 2mxy = x^2(1+m^2)$$
$$\frac{y^2 - x^2}{2xy} = \frac{\mathrm{d}y}{\mathrm{d}x}$$

Put $y = vx$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = v + x\frac{\mathrm{d}v}{\mathrm{d}x}$$
$$v + x\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{v^2 - 1}{2v}$$
$$x\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{v^2 - 1 - 2v^2}{2v}$$
$$\frac{2v}{v^2+1}\mathrm{d}v = -\frac{\mathrm{d}x}{x}$$
$$\ln(v^2+1) = -\ln x + \ln C$$
$$x(v^2+1) = C$$
$$x\left(\frac{y^2}{x^2} + 1\right) = C$$
$$x^2 + y^2 = Cx$$

8. (C)

$$\sum_{r=0}^{n} \frac{n-2r}{{}^n\mathrm{C}_r} = \sum_{r=0}^{n} \frac{n-r-r}{{}^n\mathrm{C}_r}$$
$$= \sum_{r=0}^{n} \frac{n-r}{{}^n\mathrm{C}_r} - \sum_{r=0}^{n} \frac{r}{{}^n\mathrm{C}_r}$$
$$= \sum_{r=0}^{n} \frac{n-r}{{}^n\mathrm{C}_{n-r}} - \sum_{r=0}^{n} \frac{r}{{}^n\mathrm{C}_r}$$
$$= \sum_{r=0}^{n} \frac{r}{{}^n\mathrm{C}_r} - \sum_{r=0}^{n} \frac{r}{{}^n\mathrm{C}_r}$$
$$= 0$$

9. (C)

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$
$$y=\frac{3}{4}\sqrt{16-x^2}$$
$$\text{Area}=2\int_2^4 y dx$$
$$\text{Area}=2\int_2^4 \frac{3}{4}\sqrt{16-x^2}dx$$
$$=\frac{3}{2}\int_2^4 \sqrt{16-x^2}dx$$
$$=\frac{3}{2}\left[\frac{1}{2}x\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_2^4$$
$$==\frac{3}{2}\left[0+8\sin^{-1}1\right]-\frac{3}{2}\left[\sqrt{12}+8\sin^{-1}\left(\frac{1}{2}\right)\right]$$
$$=4\pi-3\sqrt{3}$$

10. (B)

Let,
$P(n):(2n+7)<(n+3)^2$
$P(1):(2+7)<(4)^2$
$P(1):9<16$

$P(k):(2k+7)<(k+3)^2$

Now $n=k+1$
$P(k+1):(2(k+1)+7)<((k+1)+3)^2$

$2(k+1)+7=(2k+7)+2$
$2(k+1)+7<(k+3)^2+2$
$2(k+1)+7<k^2+6k+9+2$
$2(k+1)+7<k^2+8k+16-2k-5$
$2(k+1)+7<(k+4)^2-(2k+5)$
$2(k+1)+7<(k+4)^2$
$2(k+1)+7<((k+1)+3)^2$

Hence by mathematical Induction
$(2n+7)<(n+3)^2$

11. (D)

Rolle's Theorem cannot be applied here;f(x) can be any function.

12. (B)

$5x-x^2>0\to x(5-x)\to x\in(0,5)$
$so, \log_{10}(5x-x^2/6)\geqslant 0\Rightarrow(5x-x^2/6)\geq 1$
$x^2-5x+6\leq 0$
$x^2-3x-2x+6\leq 0$
$\Rightarrow x\leq 3\ and\ x\geq 2$
$so, x\in[3,2]$

13. (B)

We have, $\cos^2(\theta)-6\sin(\theta)\cos(\theta)+3\sin^2(\theta)+2$

$=(1-\sin^2(\theta))-3\sin(2\theta)+3\sin^2(\theta)+2$

$=2\sin^2(\theta)-3\sin(2\theta)+3$

$=(1-\cos(2\theta))-3\sin(2\theta)+3$

$=4-(\cos(2\theta)+3\sin(2\theta))$ .....(i)

As we have , $-\sqrt{10}\leq\cos(2\theta)+3\sin(2\theta)\leq\sqrt{10}$

$\therefore\quad -\sqrt{10}\leq-(\cos(2\theta)+3\sin(2\theta))\leq\sqrt{10}$

Or, $4-\sqrt{10}\leq 4-(\cos(2\theta)+3\sin(2\theta))\leq 4+\sqrt{10}$ ......(ii)

From (i) and (ii), we get

$4-\sqrt{10}\leq\cos^2(\theta)-6\sin(\theta)\cos(\theta)+3\sin^2(\theta)+2\leq 4+\sqrt{10}$

Hence, $4+\sqrt{10}$ and $4-\sqrt{10}$ are the maximum and minimum values

14. (B)

$(1/20)\Sigma^{20}_{i=1}(x_i-\bar{x})^2=5$

$\Sigma^{20}_{i=1}(x_i-\bar{x})^2=100$

New observations are $2x_1,2x_2,\ldots,2x_{20}$

Their mean = $\bar{x}1=2(x_1+x_2+\ldots+x_{20})/20=2\bar{x}$

Now, variance = $(1/20)\ \Sigma^{20}_{i=1}(2x_i-2\bar{x})^2$

$=(1/20)\times 4\Sigma^{20}_{i=1}(x_i-\bar{x})^2$

$=(1/20)\times 4\times 100$

$=20$

15. (B)

$$|a \times b| = 3$$
$$|c - a|^2 = |c|^2 - 2a \cdot c + |a|^2$$
$$|c|^2 - 2|c| + 9 = 8$$
$$|c| = 1$$
$$|(a \times b) \times c| = |a \times b| \times |c| \times \sin(\pi/6)$$
$$= 3 \times 1 \times \frac{1}{2}$$
$$= \frac{3}{2}$$

16. (A)

The normal passes through the centre.

The slope of the tangent m is –x/y

$$m = \frac{-1}{\sqrt{3}}$$
$$y - y_1 = m(x - x_1)$$
$$y - \sqrt{3} = \frac{-1}{\sqrt{3}}(x - 1)$$

The tangent meets the x axis at (a,0)

$$0 - \sqrt{3} = \frac{-1}{\sqrt{3}}(a - 1)$$
$$a = 4$$

The vertices of the triangle are (0,0),(4,0), $(1,3^{1/2})$.

$$area = \frac{1}{2} \times 4 \times \sqrt{3}$$
$$= 2\sqrt{3}$$

17. (A)

Since the system of equations has a non trivial solution, det(coeff.)=0

$$\Rightarrow \begin{vmatrix} -1 & c & b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$
$$\Rightarrow -1(1 - a^2) - c(-c - ab) + b(ca + b) = 0$$
$$\Rightarrow a^2 + b^2 + c^2 = 1 - 2abc$$

18. (C)

$$(p \to q) \leftrightarrow (q \cap p)$$
$$= (\sim p \cup q) \leftrightarrow (q \cap p)$$
$$= [(\sim p \cup q) \to (q \cap p)] \cap [(q \cap p) \to (\sim p \cup q)]$$
$$= [\sim (\sim p \cup q) \cup (q \cap p)] \cap [\sim (q \cap p) \cup (\sim p \cup q)]$$
$$= [(p \cap \sim q) \cup (q \cap p)] \cap [(\sim q \cup \sim p) \cup (\sim p \cup q)]$$
$$= [\, p \,] \cap [\mathrm{T}]$$
$$= p$$

19. (A)

Let m_0, m_1 and m_2 be the slopes of AC , BC and AB respectively.

Since AB = AC, the angles ABC = ACB = a

Tan a = | (m_2 -m_1)/(1+m_1m_2) | = | (m_1 -m_0)/(1+m_1m_0) | —Equation (1)

Here, m_1 = 3/4 and m_2 = -4

Solving equation 1 we get, m_0 = -52/89

Therefore equation of line AC is y+7 = -52/89(x-2)

52x+89y+519 = 0

a+b+c = 660

20. (A)

C1: t_1 and t_2 are in the same group

C2: t_1 and t_2 are in a different group

E: exactly one of the two players t_1 and t_2 is among the eight winners.

P(E) = P(E∩C1) + P(E∩C2)

P(E)=P(C1).P(E/C1)+P(C2).P(E/C2)

Now P(C1)=(14!/((2^7).7!))÷(16!/((2^8).8!))= 1/15

P(C2)=1 - (1/15) =14/15

Hence P(E)= (1/15).1 + (14/15).p (p:exactly one of either t_1 or t_2 wins)

=1/15 + (14/15).((½).(½)+(½).(½))

= 1/15 + (14/15).(½)

= 1/15 + 7/15

= 8/15

21. (1)

$$p=\frac{a(r^n-1)}{r-1}$$
$$q=a^n r^{\frac{n(n-1)}{2}}$$
$$s=\frac{\frac{1}{a}\left(\frac{1}{r^n}-1\right)}{\frac{1}{r}-1}$$
$$=\frac{1-r^n}{ar^{n-1}(1-r)}$$

Now,

$$q^2\left(\frac{s}{p}\right)^n=a^{2n}r^{n(n-1)}\left(\frac{\frac{1-r^n}{ar^{n-1}(1-r)}}{\frac{a(r^n-1)}{r-1}}\right)^n$$
$$=a^{2n}r^{n(n-1)}\left(\frac{1-r^n}{ar^{n-1}(1-r)}\cdot\frac{r-1}{a(r^n-1)}\right)^n$$
$$=a^{2n}r^{n(n-1)}\left(\frac{1}{a^2r^{n-1}}\right)^n$$
$$=\frac{a^{2n}r^{n(n-1)}}{a^{2n}r^{n(n-1)}}$$
$$=1$$

22. (6)

f_1 & f_2 are 1 & -1 respectively.

Since $c^2=a^2-b^2$

T1 : $y=m_1x+\frac{1}{m_1}$

T1 passes through (-3,0)

$$0=-3m_1+\frac{1}{m_1}$$
$$m_1{}^2=\frac{1}{3}$$

T2 : $y=m_2x-\frac{3}{m_2}$

T2 passes through (1,0)

$$0=m_2-\frac{3}{m_2}$$
$$m_2{}^2=3$$
$$m_1^2+\frac{1}{m_2^2}=\frac{1}{3}+\frac{1}{3}=\frac{2}{3}=0.66$$

k/0.11 = 0.66/0.11

= 6

23. (5)

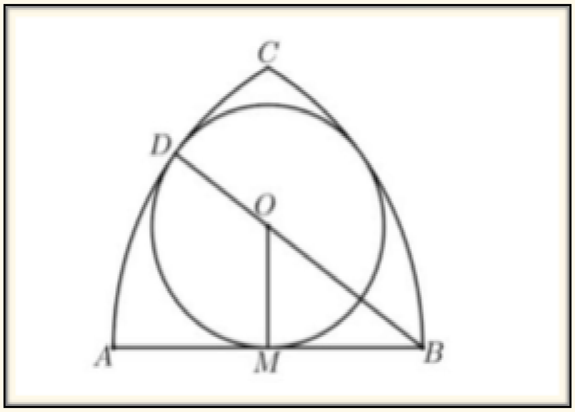
Consider points M and D as shown in figure.

Let the radius of the circle be r.

Since M bisects AB, MA = MB = 6

BD = BA = 12 (radii of arc AC)

BO = BD - OD = 12 - r



In triangle OMB,

$BO^2 = BM^2 + OM^2$

$(12-r)^2 = 6^2 + r^2$

24r = 108

r=4.5

k= r+0.5

= 5

24. (8)

$$g(1)=\lim_{x\to 0}\frac{\int_0^{x^2}\cos(t^2)\,dt}{x\sin x}$$

By, Leibniz Integral Rule,

$$g(1)=\lim_{x\to 0}\frac{2x\cos x^4}{\sin x+x\cos x}$$
$$=\lim_{x\to 0}\frac{2\cos x^4-2x\sin x^4 4x^3}{\cos x+\cos x-x\sin x}$$
$$=1$$

Since g(x) is continuous at 1; $\lim_{x\to 1^+} g(1)=1$

Note: g(x) is not defined to the left of 1

$$\lim_{x\to 1^+}\left(\lim_{m\to\infty}\frac{x^m f(1)+8}{2x^m+3x+3}\right)=1$$

Dividing by x^m

$$\lim_{x\to 1^+}\left(\lim_{m\to\infty}\frac{f(1)+\frac{8}{x^m}}{2+\frac{3}{x^{m-1}}+\frac{3}{x^m}}\right)=1$$
$$\frac{f(1)}{2}=1$$
$$f(1)=2$$

So $2g(1)+3f(1)$ = 2 x 1 + 3 x 2 = 8

25. (1)

$$adj\,(adj\ldots\ldots ntimes\ldots(adj\,(A)))=|A|^{(n-1)^n}=n$$
$$\therefore n=1\ or\ 2$$

But according to the conditions given $, n=2$

Second part:

Degree of the determinant=degree of the polynomial

$$\Rightarrow 3\alpha+5=2$$
$$\therefore \alpha=-1$$

Then $\alpha+n=1$