

# Math 124 - Programming for Mathematical Applications

UC Berkeley, Spring 2023

## Homework 12

Due Wednesday April 26

### Problem 1

It is well known that  $\sum_{i=1}^n i = \frac{1}{2} n (1 + n)$ . Make a table with similar formulas for  $\sum_{i=1}^n i^k$ , with  $k$  ranging from 1 to 8.

### Problem 2

Use the Factor function to prove that the product of four consecutive numbers plus one is always a squared number.

### Problem 3

Show that the formula  $n^2 + n + 41$  produces prime numbers for  $n$  from 0 to 39.

### Problem 4

11 is the first prime number with all digits equal to 1. Find the next one (using a loop).

### Problem 5

Define the function  $f(x)$  as follows:

$$f(xy) = f(x) + f(y)$$

$$f(x^n) = nf(x)$$

$$f(n) = 0$$

where  $n$  is an integer. Show that

$$f\left(\prod_{k=1}^{20} k! (x_k)^k\right) = \sum_{k=1}^{20} k f(x_k)$$

### Problem 6

- a)** Plot the function  $f(x) = e^{-x}/(2 + \sin(x^2))$  and its tangent line  $g(x)$  at  $x = 1$  for  $x \in [0, 3]$ .
- b)** Calculate the integral of  $f(x) - g(x)$  between  $x = 0$  and  $x = 1$  numerically with 100 digits.

**Problem 7**

Define the following piecewise function:

$$f(x) = \begin{cases} -x & \text{if } |x| < 1 \\ \sin(x) & \text{if } 1 \leq |x| < 2 \\ \cos(x) & \text{otherwise.} \end{cases}$$

- a)** Plot  $f(x)$  between  $x = -3$  and  $x = 3$ .
- b)** Calculate the integral of  $1/(1 + f(x)^2)$  between  $x = -3$  and  $x = 3$  (symbolically).