

EE 569 HOMEWORK 2

Issued: 9/18/2015 Due: 10/11/2015

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Problem 1: Texture Analysis and Classification

1.1 Motivation

Image texture is a set metrics to classify the different class image. In this problem, we will use Laws' filter to extract the feature of the image, and use Minimum Mean Distance Classifier and SVM (Support Vector Machine) Classifier to determine the image class. To reduce the dimension of feature and improve the performance, PCA (Principal Component Analysis) and LDA (Linear Discriminant Analysis) are used to the extracted feature vector.

1.2 Approach and procedure

1.2.1 Two Classes + Minimum Mean Distance Classifier

Feature extraction. Intuitively, using the whole image pixels as the information of feature can give the classifier the best data to determine the class. However, it will spend too much time, memory. [1] Here we use feature vector to represent the feature of one class. A n-dimension feature vector is a point in n-dimension space. And the classification is to find how to classify this n-dimension space. In this problem, we use Laws' Filter to extract the feature of each image.

The implementation of feature extraction is shown in Figure 1.

- 1) Subtract local mean from original image
- 2) Apply 2-D Laws' filter to subtracted image. The 1-D Law's Filter and 2-D 5 * 5 Laws' Filter are shown below in Table 1 and Table 2.
- 3) Get the average of energy of each pixel
- 4) Normalize the feature vector of the training set. This normalization applies for each feature separately. When extract the testing set, we use the training set result calculated before.

$$X' = \frac{X - \min}{\max - \min}$$

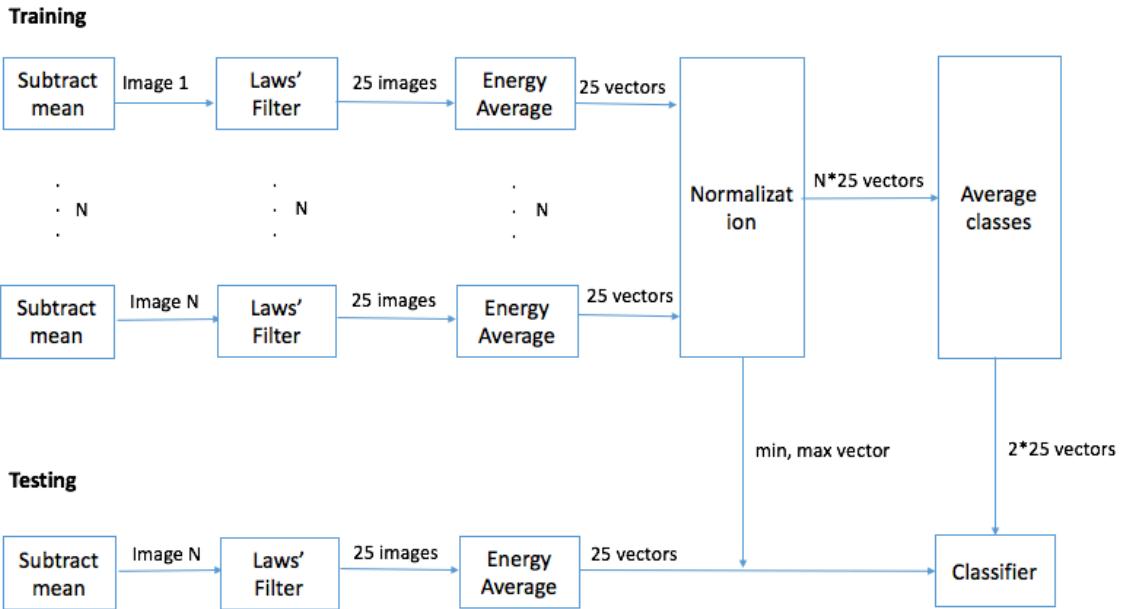


Figure 1. Flow chart of feature extraction

1 D Laws' Filter	Filter coefficients
L5	[1, 4, 6, 4, 1]
E5	[-1, -2, 0, 2, 1]
S5	[-1, 0, 2, 0, -1]
W5	[-1, 2, 0, -2, 1]
R5	[1, -4, 6, -4, 1]

Table 1. 1-D Laws' Filter

2-D Filter	0 (L5)	1 (E5)	2 (S5)	3 (W5)	4 (R5)
0 (L5)	$L5 * L5^T$	$L5 * E5^T$	$L5 * S5^T$	$L5 * W5^T$	$L5 * R5^T$
1 (E5)	$E5 * L5^T$	$E5 * E5^T$	$E5 * S5^T$	$E5 * W5^T$	$E5 * R5^T$
2 (S5)	$S5 * L5^T$	$S5 * E5^T$	$S5 * S5^T$	$S5 * W5^T$	$S5 * R5^T$
3 (W5)	$W5 * L5^T$	$W5 * E5^T$	$W5 * S5^T$	$W5 * W5^T$	$W5 * R5^T$

Table 2. 2-D Laws' Filter

Minimum mean distance classifier. To classify the class of each image, we can calculate the minimum distance between the feature vector of testing image and the feature vector of the mean of training image. The algorithm to get the minimum distance here is using Mahalanobis distance. [2]

$$d(x, y) = \sqrt{(x - y)^T S^{-1}(x - y)}$$

PCA (Principal Component Analysis). PCA is really a early algorithm which was invented by Karl Pearson in 1901[3]. The function of PCA here is to project the dimension of feature vector to lower dimension space using linear transformation [4]. Our goal is to find the eigenvectors in the n-dimension space and project other vectors to the space these eigenvectors formed, so that we preserve most of the information of the n-dimension feature vector. The implementation of PCA here is using OpenCV library cv::PCA. Flow of PCA is shown in Figure 2.

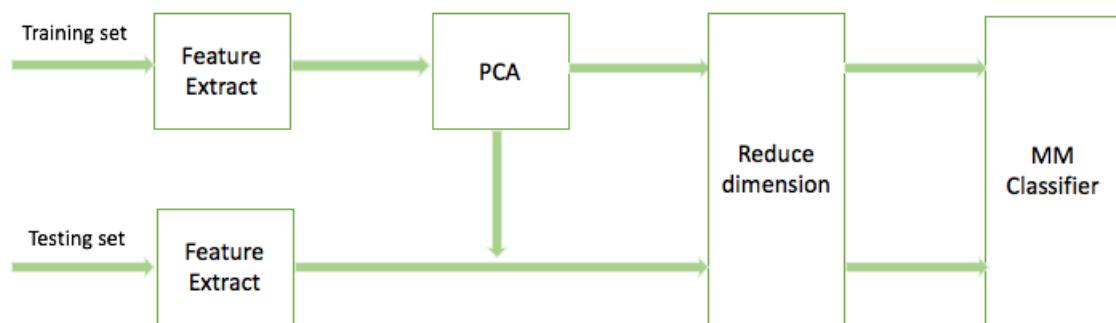


Figure 2. Flow chart of PCA

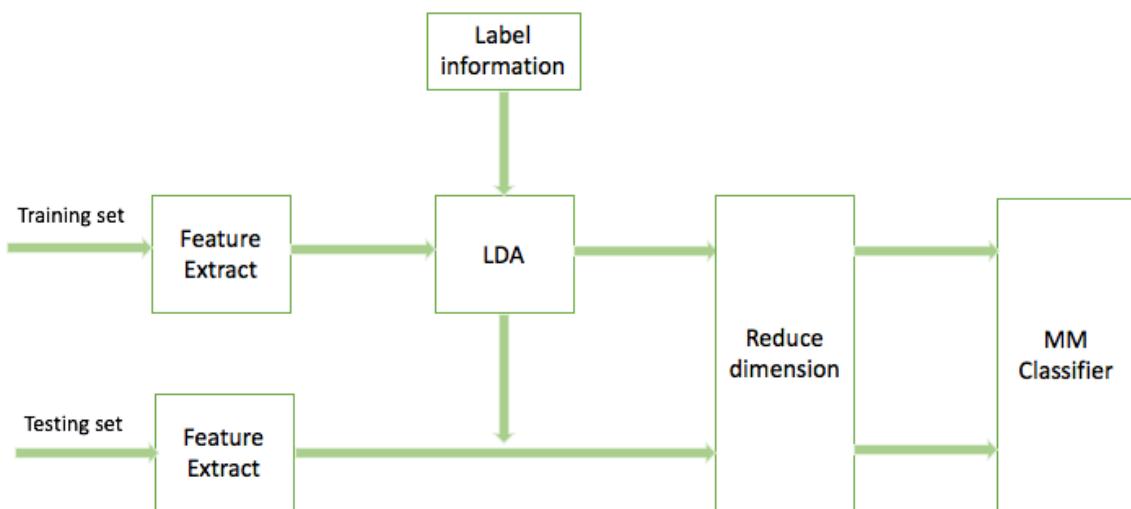


Figure 3. Flow chart of LDA

LDA (Linear Discriminant Analysis) is developed from Fisher's Linear Discriminant. This method is used to find the features that separated two or more classes points [5]. The implementation of LDA here is using OpenCV library cv::LDA. Flow of LDA is shown in Figure 3.

1.2.2 Multi-Classes + SVM

SVM (Support Vector Machine). SVM is a supervised learning models in machine learning [6]. The goal of SVM is not only finding a separating hyper-plane but also maximize the margin around the hyper-plane [7]. In the problem, we use OpenCV library to implement the SVM. The parameter and implementation are shown in Figure 4, 5.

```
// SVM train
Ptr<SVM> svm = SVM::create();
svm->setType(SVM::C_SVC);
svm->setKernel(SVM::LINEAR);
svm->setTermCriteria(TermCriteria(TermCriteria::MAX_ITER, 100, 1e-6));
svm->train(mat_train, ROW_SAMPLE, labelsMat);
```

Figure 4. Parameter used in SVM classifier

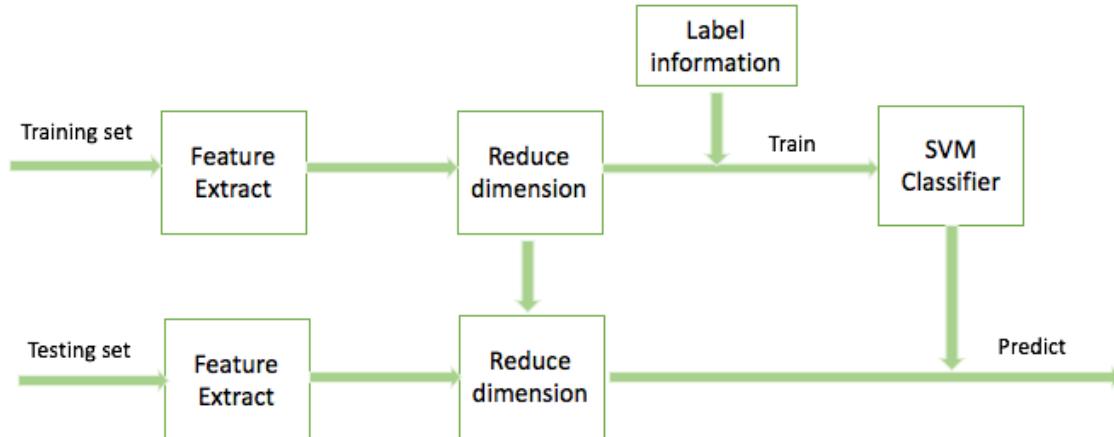


Figure 5. Flow chart of SVM classifier

1.3 Results

1.3.1 Two Classes + Minimum Mean Distance Classifier

1) Feature extraction: Here we use the training set of *grass 1-36* and *straw 1-36*. By subtracting the local mean of the image, we can get an extracted image. Figure 6 shows one the subtracted image. After apply Laws' Filter, we can get the mean of $72 * 25$ feature vector shown in Figure 7.

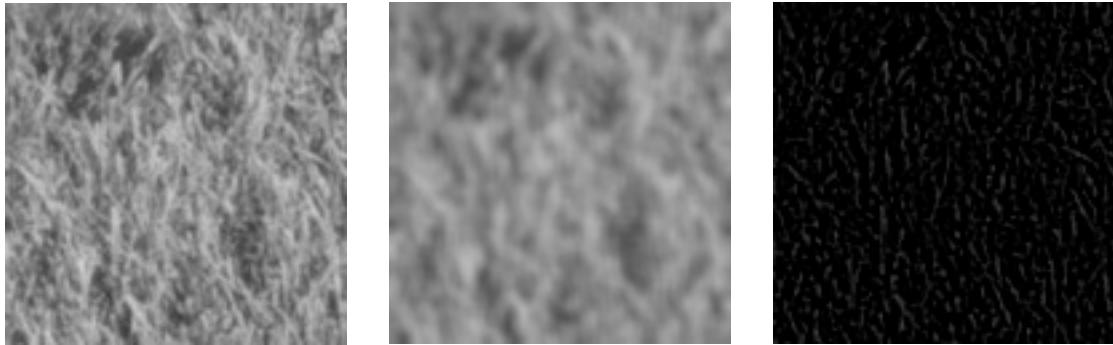


Figure 6. Left: original image. Middle: mean of original image. Right: subtracted image.

$[0.03848205192194842;$ $0.04220159784635971;$ $0.05365060428134978;$ $0.08098380690692716;$ $0.1366680918790449;$ $0.02657939646538538;$ $0.04559830283862562;$ $0.05511427522755147;$ $0.07505553212163041;$ $0.1326299996458987;$ $0.04536888506303546;$ $0.04084907050128248;$ $0.05229326011501036;$ $0.07233495032317311;$ $0.1374549630130937;$ $0.05452657015456856;$ $0.04262895297462142;$ $0.05909542746763417;$ $0.08035559758950366;$ $0.1813171788797291;$ $0.07803154076795031;$ $0.07561096858563127;$ $0.08912915247593965;$ $0.146675032064268;$ $0.2691393276794511]$	$[0.8353507428583575;$ $0.7909408823865719;$ $0.7414769124228408;$ $0.7007736196290062;$ $0.6793457131766513;$ $0.564962158815996;$ $0.6795425706824549;$ $0.6980653026402233;$ $0.7054668690587753;$ $0.6890481852753415;$ $0.6055963437208294;$ $0.5519789176907326;$ $0.5832473419619072;$ $0.5933804535224201;$ $0.6046466253885836;$ $0.6254063834685745;$ $0.4906626488599173;$ $0.5549258814403137;$ $0.5317057574221178;$ $0.5241357715492889;$ $0.5062951504993722;$ $0.5651469391143281;$ $0.615143106610675;$ $0.5790072091107065;$ $0.5233747768753533]$
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Figure 7: Average of feature for two classes

2) Minimum mean distance classification for the unlabeled data. Apply MM classifier to the extracted image. The distance of each image and the error rate is shown in Fig8. Training set: *grass 1-36, straw 1-36*. Testing set: *unknown 1-24*.

Distance: 0.130388 6.21013	label: grass	true_label: grass
Distance: 0.107998 6.60001	label: grass	true_label: grass
Distance: 0.0312302 7.24299	label: grass	true_label: grass
Distance: 0.151382 9.20841	label: grass	true_label: grass
Distance: 0.139576 9.25791	label: grass	true_label: grass
Distance: 0.0774479 7.03977	label: grass	true_label: grass
Distance: 0.318209 5.67047	label: grass	true_label: grass
Distance: 0.230096 5.70473	label: grass	true_label: grass
Distance: 0.0885114 6.33858	label: grass	true_label: grass
Distance: 0.044013 8.56792	label: grass	true_label: grass
Distance: 0.0167797 7.7865	label: grass	true_label: grass
Distance: 0.0744334 6.46774	label: grass	true_label: grass
Distance: 0.227594 5.63402	label: grass	true_label: grass
Distance: 0.284851 5.80412	label: grass	true_label: grass
Distance: 0.203659 6.05906	label: grass	true_label: grass
Distance: 0.0700409 8.70635	label: grass	true_label: grass
Distance: 0.0118362 7.96821	label: grass	true_label: grass
Distance: 0.131402 6.16442	label: grass	true_label: grass
Distance: 0.108633 6.75752	label: grass	true_label: grass
Distance: 0.0401573 8.30051	label: grass	true_label: grass
Distance: 0.105126 6.89533	label: grass	true_label: grass
Distance: 0.0406998 8.52588	label: grass	true_label: grass
Distance: 0.0486607 8.41392	label: grass	true_label: grass
Distance: 0.0487617 8.56948	label: grass	true_label: grass
Distance: 0.204333 9.50083	label: grass	true_label: grass
Distance: 0.205288 9.5037	label: grass	true_label: grass
Distance: 0.0409668 6.83774	label: grass	true_label: grass
Distance: 0.0230875 7.94107	label: grass	true_label: grass
Distance: 0.0250412 7.11859	label: grass	true_label: grass
Distance: 0.0670552 8.58065	label: grass	true_label: grass
Distance: 0.100608 8.895	label: grass	true_label: grass
Distance: 0.0199212 8.02845	label: grass	true_label: grass
Distance: 0.0888012 8.33889	label: grass	true_label: grass
Distance: 0.0760556 8.80717	label: grass	true_label: grass
Distance: 0.00071935 7.60649	label: grass	true_label: grass
Distance: 0.0507649 8.6373	label: grass	true_label: grass
Distance: 8.86619 0.202613	label: straw	true_label: straw
Distance: 8.59832 0.375951	label: straw	true_label: straw
Distance: 6.34181 0.151581	label: straw	true_label: straw
Distance: 7.85536 0.317811	label: straw	true_label: straw
Distance: 9.91513 0.22322	label: straw	true_label: straw
Distance: 4.71607 0.747597	label: straw	true_label: straw
Distance: 9.82847 0.324055	label: straw	true_label: straw
Distance: 9.93775 0.330621	label: straw	true_label: straw
Distance: 7.78543 0.15804	label: straw	true_label: straw
Distance: 9.35429 0.194933	label: straw	true_label: straw
Distance: 11.8611 0.661522	label: straw	true_label: straw
Distance: 6.66205 0.0731394	label: straw	true_label: straw
Distance: 15.8017 2.27982	label: straw	true_label: straw
Distance: 7.68964 0.188227	label: straw	true_label: straw
Distance: 9.19572 0.425407	label: straw	true_label: straw
Distance: 10.2246 0.389868	label: straw	true_label: straw
Distance: 6.97809 0.347755	label: straw	true_label: straw
Distance: 6.12844 0.391092	label: straw	true_label: straw
Distance: 13.3507 2.13434	label: straw	true_label: straw
Distance: 8.49627 0.114726	label: straw	true_label: straw
Distance: 8.31922 0.187929	label: straw	true_label: straw
Distance: 9.45523 0.15692	label: straw	true_label: straw
Distance: 8.66606 0.150311	label: straw	true_label: straw
Distance: 5.84709 0.181533	label: straw	true_label: straw
Distance: 6.75209 0.503913	label: straw	true_label: straw
Distance: 6.32117 0.128416	label: straw	true_label: straw
Distance: 8.11202 0.156886	label: straw	true_label: straw
Distance: 6.59206 0.346102	label: straw	true_label: straw
Distance: 6.90641 0.592398	label: straw	true_label: straw
Distance: 5.58116 0.783676	label: straw	true_label: straw
Distance: 5.48521 0.881249	label: straw	true_label: straw
Distance: 7.9613 0.341031	label: straw	true_label: straw
Distance: 7.45674 0.921797	label: straw	true_label: straw
Distance: 5.95413 0.332896	label: straw	true_label: straw
Distance: 6.02379 1.31029	label: straw	true_label: straw
Distance: 4.68133 0.847753	label: straw	true_label: straw

Distance: 9.32201 0.518406	label: straw	true_label: straw
Distance: 6.58798 0.0921584	label: straw	true_label: straw
Distance: 0.226423 9.63066	label: grass	true_label: grass
Distance: 0.0421551 8.24726	label: grass	true_label: grass
Distance: 0.145294 9.17196	label: grass	true_label: grass
Distance: 6.13949 0.274771	label: straw	true_label: straw
Distance: 0.0304415 8.17593	label: grass	true_label: grass
Distance: 9.47086 0.486005	label: straw	true_label: straw
Distance: 7.16212 0.514043	label: straw	true_label: straw
Distance: 0.13743 9.21217	label: grass	true_label: grass
Distance: 0.0473499 8.28605	label: grass	true_label: grass
Distance: 0.355197 10.3163	label: grass	true_label: grass
Distance: 10.1006 0.445806	label: straw	true_label: straw
Distance: 0.192446 6.15903	label: grass	true_label: grass
Distance: 6.00177 0.488666	label: straw	true_label: straw
Distance: 8.97213 0.444355	label: straw	true_label: straw
Distance: 9.22487 0.310115	label: straw	true_label: straw
Distance: 6.13087 0.471734	label: straw	true_label: straw
Distance: 0.0062801 7.6368	label: grass	true_label: grass
Distance: 0.368129 5.59009	label: grass	true_label: grass
Distance: 0.0626989 8.41421	label: grass	true_label: grass
Distance: 12.5748 0.847967	label: straw	true_label: straw
Distance: 6.18 0.491388	label: straw	true_label: straw
Distance: 0.177529 5.87818	label: grass	true_label: grass

error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %

Figure 8. MM classifier, no feature reduction. Left: training data for grass 1-36 and straw 1-36. Top-right: testing data for unknown 1-24. Button-right: Error rate of Minimum-mean distance classifier

3) Feature dimension reduction using PCA. Different features have different importance (1.4.1). Fig 9 is the result of PCA dimension reduction from 25 to 1.

```
-----mat_feature_average_pca-----
[-1.365780679872124]
[1.306259104714434]
```

Figure 9. PCA, dimension number = 1.

4) Feature dimension reduction using LDA (Figure 10)

```
-----mat_feature_average_lda-----
[-0.007088085703369104]
[-0.1276033155356752]
```

Figure 10. LDA, dimension number = 1

5) Classification with two reduced feature sets. PCA in Fig11, LDA in Fig12.

<pre>MM Mode: 1 num: 1 Dimension_PCA Distance: 0.0700299 5.79561 label: grass true_label: grass Distance: 0.0363682 6.15702 label: grass true_label: grass Distance: 0.00311724 6.84454 label: grass true_label: grass Distance: 0.0836169 8.76874 label: grass true_label: grass Distance: 0.0894645 8.82771 label: grass true_label: grass Distance: 0.0102907 6.60797 label: grass true_label: grass Distance: 0.16254 5.1478 label: grass true_label: grass Distance: 0.141264 5.27248 label: grass true_label: grass Distance: 0.0541518 8.95035 label: grass true_label: grass Distance: 0.0347913 8.17139 label: grass true_label: grass Distance: 0.00227116 7.39675 label: grass true_label: grass Distance: 0.0430924 6.07353 label: grass true_label: grass Distance: 0.150515 5.21701 label: grass true_label: grass Distance: 0.139626 5.28252 label: grass true_label: grass Distance: 0.0960755 5.57942 label: grass true_label: grass Distance: 0.0436893 8.3005 label: grass true_label: grass Distance: 0.00664452 7.58206 label: grass true_label: grass Distance: 0.0757138 5.74503 label: grass true_label: grass Distance: 0.0272271 6.28522 label: grass true_label: grass Distance: 0.0196734 7.90904 label: grass true_label: grass Distance: 0.0408684 8.26102 label: grass true_label: grass Distance: 0.0314248 8.11857 label: grass true_label: grass Distance: 0.0253381 8.0158 label: grass true_label: grass Distance: 0.0347524 8.17079 label: grass true_label: grass Distance: 0.11028 9.02476 label: grass true_label: grass Distance: 0.110398 9.02583 label: grass true_label: grass Distance: 0.0170951 6.45816 label: grass true_label: grass Distance: 0.00576237 7.55123 label: grass true_label: grass Distance: 0.00618675 6.72564 label: grass true_label: grass Distance: 0.0351445 8.17679 label: grass true_label: grass Distance: 0.0571656 8.4747 label: grass true_label: grass Distance: 0.00819563 7.63179 label: grass true_label: grass Distance: 0.0197194 7.90996 label: grass true_label: grass Distance: 0.0515724 8.40499 label: grass true_label: grass Distance: 0.00025463 7.22533 label: grass true_label: grass Distance: 0.0394444 8.24061 label: grass true_label: grass Distance: 8.6701 0.0742362 label: straw true_label: straw Distance: 8.29235 0.0430996 label: straw true_label: straw Distance: 6.21222 0.0322588 label: straw true_label: straw Distance: 7.47044 0.00374194 label: straw true_label: straw Distance: 9.79674 0.209704 label: straw true_label: straw Distance: 4.28525 0.36235 label: straw true_label: straw Distance: 9.60817 0.182897 label: straw true_label: straw Distance: 9.72543 0.199382 label: straw true_label: straw Distance: 7.64735 0.00871335 label: straw true_label: straw Distance: 9.23469 0.13456 label: straw true_label: straw Distance: 11.5779 0.533774 label: straw true_label: straw Distance: 6.55487 0.0124975 label: straw true_label: straw Distance: 14.8952 1.40989 label: straw true_label: straw Distance: 7.41128 0.0025327 label: straw true_label: straw Distance: 8.84541 0.0912543 label: straw true_label: straw Distance: 10.0425 0.246953 label: straw true_label: straw Distance: 6.53486 0.0133872 label: straw true_label: straw Distance: 5.77949 0.0718143 label: straw true_label: straw Distance: 11.7296 0.566725 label: straw true_label: straw Distance: 8.32219 0.0452754 label: straw true_label: straw Distance: 8.15939 0.0340123 label: straw true_label: straw Distance: 9.3672 0.150969 label: straw true_label: straw Distance: 8.50976 0.060079 label: straw true_label: straw Distance: 5.72542 0.0779836 label: straw true_label: straw Distance: 6.2778 0.0277175 label: straw true_label: straw Distance: 6.18679 0.0341193 label: straw true_label: straw Distance: 7.88371 0.0184291 label: straw true_label: straw Distance: 6.22832 0.0311097 label: straw true_label: straw Distance: 6.25285 0.0294022 label: straw true_label: straw Distance: 4.91314 0.207461 label: straw true_label: straw Distance: 4.85826 0.218925 label: straw true_label: straw Distance: 7.5676 0.00622325 label: straw true_label: straw Distance: 6.50626 0.0147143 label: straw true_label: straw Distance: 5.64968 0.0871046 label: straw true_label: straw Distance: 4.82058 0.227012 label: straw true_label: straw Distance: 4.21019 0.384605 label: straw true_label: straw</pre>	<pre>Distance: 8.79315 0.0860161 label: straw true_label: straw Distance: 6.45537 0.0172392 label: straw true_label: straw Distance: 0.124907 9.15342 label: grass true_label: grass Distance: 0.0167862 7.84897 label: grass true_label: grass Distance: 0.0798061 8.7293 label: grass true_label: grass Distance: 5.85679 0.0634832 label: straw true_label: straw Distance: 0.0141468 7.78957 label: grass true_label: grass Distance: 8.94935 0.102085 label: straw true_label: straw Distance: 6.57241 0.0117436 label: straw true_label: straw Distance: 0.0844064 8.77681 label: grass true_label: grass Distance: 0.0189124 7.89364 label: grass true_label: grass Distance: 0.207831 9.78391 label: grass true_label: grass Distance: 9.8126 0.212029 label: straw true_label: straw Distance: 0.0826398 5.68617 label: grass true_label: grass Distance: 5.54772 0.100286 label: straw true_label: straw Distance: 8.45257 0.0553616 label: straw true_label: straw Distance: 8.93474 0.100529 label: straw true_label: straw Distance: 5.67668 0.0837884 label: straw true_label: straw Distance: 0.000555044 7.26625 label: grass true_label: grass Distance: 0.183395 5.03461 label: grass true_label: grass Distance: 0.0250884 8.01135 label: grass true_label: grass Distance: 12.2893 0.694841 label: straw true_label: straw Distance: 5.69992 0.0800911 label: straw true_label: straw Distance: 0.112593 5.45919 label: grass true_label: grass</pre> <p>error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 %</p>
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Figure 11. MM classifier, PCA = 1. Left: training data for grass 1-36 and straw 1-36. Top-right: testing data for unknown 1-24. Button-right: Error rate of Minimum-mean distance classifier

MM Mode: 2 num: 1	label: straw true_label: straw	label: straw true_label: straw
Dimension_LDA		
Distance: 9.82261e-06 0.0137783	label: grass true_label: grass	Distance: 0.0126609 6.39149e-05 label: straw true_label: straw
Distance: 2.81366e-06 0.014931	label: grass true_label: grass	Distance: 0.0149617 3.25066e-06 label: straw true_label: straw
Distance: 2.27352e-06 0.0141628	label: grass true_label: grass	Distance: 4.11776e-07 0.0143697 label: grass true_label: grass
Distance: 1.46528e-07 0.0144318	label: grass true_label: grass	Distance: 7.92135e-07 0.0143102 label: grass true_label: grass
Distance: 1.087e-06 0.0142737	label: grass true_label: grass	Distance: 2.85455e-05 0.0132647 label: grass true_label: grass
Distance: 5.51542e-07 0.0143455	label: grass true_label: grass	Distance: 0.0165157 6.39721e-05 label: straw true_label: straw
Distance: 2.91205e-06 0.0149381	label: grass true_label: grass	Distance: 6.94388e-06 0.0138957 label: grass true_label: grass
Distance: 1.12858e-06 0.014269	label: grass true_label: grass	Distance: 0.0215347 0.000688103 label: straw true_label: straw
Distance: 2.25166e-07 0.0146385	label: grass true_label: grass	Distance: 0.0157976 2.67647e-05 label: straw true_label: straw
Distance: 1.07669e-05 0.0153256	label: grass true_label: grass	Distance: 3.11831e-07 0.0146588 label: grass true_label: grass
Distance: 2.62781e-06 0.0141358	label: grass true_label: grass	Distance: 1.73691e-05 0.0135368 label: grass true_label: grass
Distance: 1.16898e-07 0.0146064	label: grass true_label: grass	Distance: 1.79492e-06 0.0148486 label: grass true_label: grass
Distance: 1.87555e-05 0.0134988	label: grass true_label: grass	Distance: 0.0139351 0.09191e-06 label: straw true_label: straw
Distance: 9.46457e-06 0.0152749	label: grass true_label: grass	Distance: 1.91683e-06 0.0141921 label: grass true_label: grass
Distance: 5.44618e-07 0.0147023	label: grass true_label: grass	Distance: 0.0117545 0.000146338 label: straw true_label: straw
Distance: 1.6928e-06 0.014212	label: grass true_label: grass	Distance: 0.0195381 0.000371077 label: straw true_label: straw
Distance: 3.42463e-06 0.0140813	label: grass true_label: grass	Distance: 0.0150587 4.83368e-06 label: straw true_label: straw
Distance: 2.88448e-05 0.0132583	label: grass true_label: grass	Distance: 0.0159522 3.34882e-05 label: straw true_label: straw
Distance: 3.49091e-06 0.0149778	label: grass true_label: grass	Distance: 3.3201e-06 0.0140881 label: grass true_label: grass
Distance: 5.54904e-07 0.0143449	label: grass true_label: grass	Distance: 8.46416e-06 0.0152336 label: grass true_label: grass
Distance: 9.26194e-06 0.0152667	label: grass true_label: grass	Distance: 0.020683 0.000542916 label: straw true_label: straw
Distance: 1.57263e-05 0.0154955	label: grass true_label: grass	Distance: 0.0157139 2.34252e-05 label: straw true_label: straw
Distance: 1.24461e-06 0.0142563	label: grass true_label: grass	Distance: 9.20848e-06 0.0138017 label: grass true_label: grass
Distance: 2.13335e-06 0.0148781	label: grass true_label: grass	
Distance: 1.18696e-06 0.0147877	label: grass true_label: grass	
Distance: 3.03074e-06 0.0149466	label: grass true_label: grass	
Distance: 2.06743e-07 0.0146337	label: grass true_label: grass	
Distance: 1.85311e-06 0.0141977	label: grass true_label: grass	
Distance: 3.8491e-06 0.0149549	label: grass true_label: grass	
Distance: 1.09465e-07 0.0144443	label: grass true_label: grass	
Distance: 3.21049e-06 0.0140953	label: grass true_label: grass	
Distance: 3.94604e-06 0.0150067	label: grass true_label: grass	
Distance: 4.46543e-06 0.0140191	label: grass true_label: grass	
Distance: 2.31014e-07 0.01464	label: grass true_label: grass	
Distance: 8.899e-05 0.0168867	label: grass true_label: grass	
Distance: 3.68438e-06 0.0140465	label: grass true_label: grass	
Distance: 0.0148478 1.78599e-06	label: straw true_label: straw	
Distance: 0.0139855 5.08537e-06	label: straw true_label: straw	
Distance: 0.0136908 1.23025e-05	label: straw true_label: straw	
Distance: 0.015671 2.17943e-05	label: straw true_label: straw	
Distance: 0.0136665 1.30431e-05	label: straw true_label: straw	
Distance: 0.0152691 9.32143e-06	label: straw true_label: straw	
Distance: 0.0160112 3.62442e-05	label: straw true_label: straw	
Distance: 0.0152722 9.39684e-06	label: straw true_label: straw	
Distance: 0.0126824 6.23994e-05	label: straw true_label: straw	
Distance: 0.014399 2.69853e-07	label: straw true_label: straw	
Distance: 0.0191151 0.000314782	label: straw true_label: straw	
Distance: 0.0159827 3.48995e-05	label: straw true_label: straw	
Distance: 0.0172521 0.000117333	label: straw true_label: straw	
Distance: 0.0148943 2.33106e-06	label: straw true_label: straw	
Distance: 0.0148995 2.39705e-06	label: straw true_label: straw	
Distance: 0.0149957 3.76944e-06	label: straw true_label: straw	
Distance: 0.0170575 0.000101792	label: straw true_label: straw	
Distance: 0.0155678 1.81119e-05	label: straw true_label: straw	
Distance: 0.0158554 2.191927e-05	label: straw true_label: straw	
Distance: 0.0161939 4.54301e-05	label: straw true_label: straw	
Distance: 0.0154662 1.48865e-05	label: straw true_label: straw	
Distance: 0.0167803 8.1425e-05	label: straw true_label: straw	
Distance: 0.0147486 8.62305e-07	label: straw true_label: straw	
Distance: 0.014858 1.89878e-06	label: straw true_label: straw	
Distance: 0.0161842 4.49164e-05	label: straw true_label: straw	
Distance: 0.0153312 1.09177e-05	label: straw true_label: straw	
Distance: 0.0171702 0.000110671	label: straw true_label: straw	
Distance: 0.0156907 2.25372e-05	label: straw true_label: straw	
Distance: 0.0154901 1.55555e-05	label: straw true_label: straw	
Distance: 0.0144728 4.50568e-08	label: straw true_label: straw	
Distance: 0.0148031 1.32927e-06	label: straw true_label: straw	
Distance: 0.0143173 7.39808e-07	label: straw true_label: straw	
Distance: 0.0150219 4.197e-06	label: straw true_label: straw	
Distance: 0.0138608 7.74733e-06	label: straw true_label: straw	
Distance: 0.015075 5.13083e-06	label: straw true_label: straw	
Distance: 0.0157634 2.53729e-05	label: straw true_label: straw	

error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %

Figure 12. MM classifier, LDA = 1. Left: training data for grass 1-36 and straw 1-36. Top-right: testing data for unknown 1-24. Button-right: Error rate of Minimum-mean distance classifier

6) Performance comparison and discussion. Please see discussion 1.4.1

1.3.2 Multi-Classes + SVM

1) Repeat the feature extraction and feature dimension reduction to 3-dimension. Figure 13 is the results. The training set and testing set of four cases are in Table 3.

2) Use four minimum-distance-to-class-means classifiers. The error rate is also shown in Figure 13. The training set and testing set of four cases are in Table 3.

<pre> -----grass----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 1 num: 3 Decrease_Dimension_PCA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----straw----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 1 num: 3 Decrease_Dimension_PCA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----sand----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 1 num: 3 Decrease_Dimension_PCA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----leather----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 1 num: 3 Decrease_Dimension_PCA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % </pre>	<pre> -----grass----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 2 num: 3 Decrease_Dimension_LDA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----straw----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 2 num: 3 Decrease_Dimension_LDA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----sand----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 2 num: 3 Decrease_Dimension_LDA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----leather----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 2 num: 3 Decrease_Dimension_LDA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % </pre>
--	--

Figure 13. MM classifier. Left: PCA feature reduction, number from 1 to 3.
Right: LDA feature reduction, number from 1 to 3.

Label	Training 72	Testing 24
Grass	Grass 36, Straw 12, Sand 12, Leather 12	Grass 12, Straw 4, Sand 4, Leather 4
Straw	Straw 36, Grass 12, Sand 12, Leather 12	Straw 12, Grass 4, Sand 4, Leather 4
Sand	Sand 36, Straw 12, Grass 12, Leather 12	Sand 12, Straw 4, Grass 4, Leather 4
Leather	Leather 36, Straw 12, Sand 12, Grass 12	Leather 12, Straw 4, Sand 4, Grass 4

Table 3. Training and testing data

```

-----grass-----
Set_Feature_Train: 72
Set_Feature_Train: 24

Classify_SVM Mode: 1 num: 3
Decrease_Dimension_PCA
error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %

-----straw-----
Set_Feature_Train: 72
Set_Feature_Train: 24

Classify_SVM Mode: 1 num: 3
Decrease_Dimension_PCA
error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %

-----sand-----
Set_Feature_Train: 72
Set_Feature_Train: 24

Classify_SVM Mode: 1 num: 3
Decrease_Dimension_PCA
error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %

-----leather-----
Set_Feature_Train: 72
Set_Feature_Train: 24

Classify_SVM Mode: 1 num: 3
Decrease_Dimension_PCA
error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %

```

Figure 14. SVM Classifier. PCA number = 3

1.4 Discussion

1.4.1 Two Classes + Minimum Mean Distance Classifier

Different feature vectors have different importance for the classification. What we want is to enhance the feature that have a large difference between two classes, and suppress the feature similar. Too many dimension of the feature vector is a disaster that increases the time and memory complexity of the computation. When we apply a dimension reduction to the high-dimension feature vector, in this case, from 25 to 1 or 2 or 3, we enhance the major feature and suppress the feature less useful, which compensates the information we lose during the dimension reduction.

Here is one test case that using 1, 2, 3 three-dimension reduction parameter to PCA, LDA. We can see there is no large difference between three dimension.

<pre> Classify_MM Mode: 1 num: 1 Decrease_Dimension_PCA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----mat_feature_average_pca----- [-1.365780679872124] [1.306259104714434] </pre>	<pre> Classify_MM Mode: 2 num: 1 Decrease_Dimension_LDA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----mat_feature_average_lda----- [-0.007088085703369104] [-0.1276033155356752] </pre>
<pre> Classify_MM Mode: 1 num: 2 Decrease_Dimension_PCA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----mat_feature_average_pca----- [-1.365780679872124] [1.306259104714434] [-1.365780679872124] [1.306259104714434] </pre>	<pre> Classify_MM Mode: 2 num: 2 Decrease_Dimension_LDA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----mat_feature_average_lda----- [-0.007088085703369104] [-0.1276033155356752] [-0.007088085703369104] [-0.1276033155356752] </pre>
<pre> Classify_MM Mode: 1 num: 3 Decrease_Dimension_PCA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----mat_feature_average_pca----- [-1.365780679872124] [1.306259104714434] [-1.365780679872124] [1.306259104714434] [-1.365780679872124] [1.306259104714434] </pre>	<pre> Classify_MM Mode: 2 num: 3 Decrease_Dimension_LDA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----mat_feature_average_lda----- [-0.007088085703369104] [-0.1276033155356752] [-0.007088085703369104] [-0.1276033155356752] [-0.007088085703369104] [-0.1276033155356752] </pre>

Figure 15 Special case for discussion. Classifier MM. Left: PCA, number 1, 2, 3. Right: LDA, number 1, 2, 3 (Mode 1 is PCA, Mode 2 is LDA)

Compared PCA and LDA dimension reduction algorithm, LDA is better than PCA in this case. By taking label information into account, LDA get a more accurate result. Though no feature reduction and and feature reduction get the same result, apparently feature reduction is better. Because PCA and LDA reduce the time and memory of computation. Also, Figure 16 is a result that I use classifier MM and PCA, LDA WITHOUT feature normalization. We can see in this case, LDA apparently performs better than PCA and no-feature reduction. No-feature reduction classification performance slightly better than classification with PCA. Also there is a more clearly result in Figure 19 and Figure 20. LDA in the four-class MM classifiers performs better than PCA.

```
Problem1a
Set_Feature_Train: 72
Set_Feature_Test: 24

Classify_MM Mode: 0 num: 0
error_rate_total: 2.08333 %
error_rate_train: 2.77778 %
error_rate_test: 0 %

Classify_MM Mode: 1 num: 1
Decrease_Dimension_PCA
error_rate_total: 3.125 %
error_rate_train: 2.77778 %
error_rate_test: 4.16667 %

Classify_MM Mode: 1 num: 2
Decrease_Dimension_PCA
error_rate_total: 3.125 %
error_rate_train: 2.77778 %
error_rate_test: 4.16667 %

Classify_MM Mode: 2 num: 1
Decrease_Dimension_LDA
error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %
```

Figure 16. Special case for discussion to compare PCA and LDA. (Mode 0 is no-feature reduction, Mode 1 is PCA, Mode 2 is LDA)

1.4.2 Multi-Classes + SVM

Because the error rates of SVM are all 0. I change PCA dimension number to 1 , 2, and 3 to see if there is any change (Figure 17). We can see when number of dimension in PCA decrease from 3 to 1, the performance of classification become

inaccurate. And the performance of each texture classification is texture-dependent. Leather class, in this case, is the hardest predicting class.

```

-----grass-----
Set_Feature_Train: 72
Set_Feature_Train: 24

Classify_SVM Mode: 1 num: 1
Decrease_Dimension_PCA
error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %

-----straw-----
Set_Feature_Train: 72
Set_Feature_Train: 24

Classify_SVM Mode: 1 num: 1
Decrease_Dimension_PCA
error_rate_total: 18.75 %
error_rate_train: 18.0556 %
error_rate_test: 20.8333 %

-----sand-----
Set_Feature_Train: 72
Set_Feature_Train: 24

Classify_SVM Mode: 1 num: 1
Decrease_Dimension_PCA
error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %

-----leather-----
Set_Feature_Train: 72
Set_Feature_Train: 24

Classify_SVM Mode: 1 num: 1
Decrease_Dimension_PCA
error_rate_total: 22.9167 %
error_rate_train: 23.6111 %
error_rate_test: 20.8333 %

-----grass-----
Set_Feature_Train: 72
Set_Feature_Train: 24

Classify_SVM Mode: 1 num: 2
Decrease_Dimension_PCA
error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %

-----straw-----
Set_Feature_Train: 72
Set_Feature_Train: 24

Classify_SVM Mode: 1 num: 2
Decrease_Dimension_PCA
error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %

-----sand-----
Set_Feature_Train: 72
Set_Feature_Train: 24

Classify_SVM Mode: 1 num: 3
Decrease_Dimension_PCA
error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %

-----leather-----
Set_Feature_Train: 72
Set_Feature_Train: 24

Classify_SVM Mode: 1 num: 3
Decrease_Dimension_PCA
error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %

-----grass-----
Set_Feature_Train: 72
Set_Feature_Train: 24

Classify_SVM Mode: 1 num: 3
Decrease_Dimension_PCA
error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %

-----straw-----
Set_Feature_Train: 72
Set_Feature_Train: 24

Classify_SVM Mode: 1 num: 3
Decrease_Dimension_PCA
error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %

-----sand-----
Set_Feature_Train: 72
Set_Feature_Train: 24

Classify_SVM Mode: 1 num: 3
Decrease_Dimension_PCA
error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %

-----leather-----
Set_Feature_Train: 72
Set_Feature_Train: 24

Classify_SVM Mode: 1 num: 3
Decrease_Dimension_PCA
error_rate_total: 0 %
error_rate_train: 0 %
error_rate_test: 0 %

```

Figure 17. Classifier SVM, Dimension reduction PCA. Left: number of dimension is 1. Middle number of dimension is 2. Right: number of dimension is 3.

To compare with the Minimum-Mean distance classifier, I also set dimension reduction number to 1, 2, and 3 to run the four MM classifiers again and get the results in Figure 19, Figure 20. We will see that these texture classification performances a little better than SVM. The results are not as much texture-dependent as SVM results. Another interesting thing is, in MM classifier case, the error rates of training set and testing set (PCA number = 1) are not similar. In my opinion, if we want to get a "looks good" result in this case, we should use MM classifier. However, if we want to get a more reasonable result and have more training data, we can try to use SVM classifier.

Here is a deep explanation using an example in 2-D in Figure 18. Assume we only train "blue" points, to optimize the minimum-mean distance, MM classifier may get a line which has a "good" result but is not as useful as SVM classification. In other words, minimum-mean distance may lead to "over fit" result.

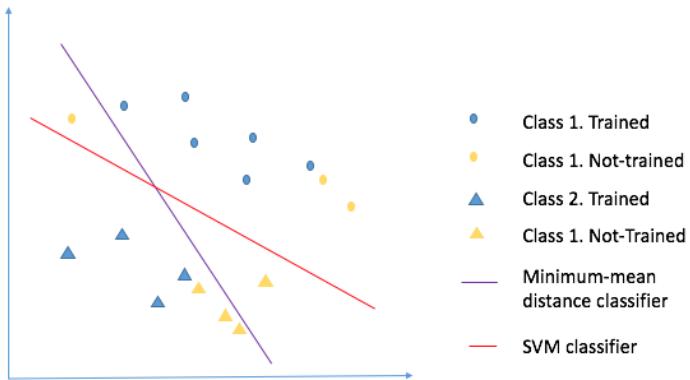


Figure 18. Explanation for Minimum-mean distance classifier and SVM classifier

<pre>-----grass----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 1 num: 1 Decrease_Dimension_PCA error_rate_total: 12.5 % error_rate_train: 11.1111 % error_rate_test: 16.6667 % -----straw----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 1 num: 1 Decrease_Dimension_PCA error_rate_total: 7.29167 % error_rate_train: 8.33333 % error_rate_test: 4.166667 % -----sand----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 1 num: 1 Decrease_Dimension_PCA error_rate_total: 3.125 % error_rate_train: 1.38889 % error_rate_test: 8.33333 % -----leather----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 1 num: 1 Decrease_Dimension_PCA error_rate_total: 20.8333 % error_rate_train: 26.3889 % error_rate_test: 4.16667 %</pre>	<pre>-----grass----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 1 num: 2 Decrease_Dimension_PCA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----straw----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 1 num: 2 Decrease_Dimension_PCA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----sand----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 1 num: 2 Decrease_Dimension_PCA error_rate_total: 2.08333 % error_rate_train: 1.38889 % error_rate_test: 4.16667 % -----leather----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 1 num: 2 Decrease_Dimension_PCA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 %</pre>	<pre>-----grass----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 1 num: 3 Decrease_Dimension_PCA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----straw----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 1 num: 3 Decrease_Dimension_PCA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----sand----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 1 num: 3 Decrease_Dimension_PCA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 % -----leather----- Set_Feature_Train: 72 Set_Feature_Train: 24 Classify_MM Mode: 1 num: 3 Decrease_Dimension_PCA error_rate_total: 0 % error_rate_train: 0 % error_rate_test: 0 %</pre>
--	--	--

Figure 19. Classifier MM, Dimension reduction PCA. Left to right: numbers of dimension are 1, 2, 3.

	grass	straw	sand	leather
Set_Feature_Train:	72	72	72	72
Set_Feature_Train:	24	24	24	24
Classify_MM Mode:	2 num: 1	2 num: 2	2 num: 3	2 num: 3
Decrease_Dimension_LDA				
error_rate_total:	0 %	0 %	0 %	0 %
error_rate_train:	0 %	0 %	0 %	0 %
error_rate_test:	0 %	0 %	0 %	0 %
-----	straw	sand	leather	leather
Set_Feature_Train:	72	72	72	72
Set_Feature_Train:	24	24	24	24
Classify_MM Mode:	2 num: 1	2 num: 2	2 num: 3	2 num: 3
Decrease_Dimension_LDA				
error_rate_total:	2.08333 %	0 %	0 %	0 %
error_rate_train:	1.38889 %	0 %	0 %	0 %
error_rate_test:	4.16667 %	0 %	0 %	0 %
-----	sand			
Set_Feature_Train:	72	72	72	72
Set_Feature_Train:	24	24	24	24
Classify_MM Mode:	2 num: 1	2 num: 2	2 num: 3	2 num: 3
Decrease_Dimension_LDA				
error_rate_total:	5.20833 %	0 %	0 %	0 %
error_rate_train:	5.55556 %	0 %	0 %	0 %
error_rate_test:	4.16667 %	0 %	0 %	0 %
-----	leather			
Set_Feature_Train:	72	72	72	72
Set_Feature_Train:	24	24	24	24
Classify_MM Mode:	2 num: 1	2 num: 2	2 num: 3	2 num: 3
Decrease_Dimension_LDA				
error_rate_total:	3.125 %	0 %	0 %	0 %
error_rate_train:	0 %	0 %	0 %	0 %
error_rate_test:	12.5 %	0 %	0 %	0 %

Figure 20. Classifier MM, Dimension reduction LDA. Left to right: numbers of dimension are 1, 2, 3.

Problem 2: Edge Detection

2.1 Motivation

The edge of image is one of the basic feature of the digital image. The edge points here mean that the intensity scale of certain field in the image has a sharp change or discontinuities [8]. Edge detection is one of the basic problem in the computer vision and image processing. The aim of edge detection is to find the edge points which has a large brightness changing in the digital image.

In this problem, I will implement three methods of edge detection using: Sobel edge detector and non-maximal suppression, Canny edge detector, and Structured edge detector. F-measure is used to measure the performance of these three methods.

2.2 Approach and procedure

2.2.1 Sobel edge detector and non-maximal suppression

The Sobel edge detector algorithm is mentioned by Sobel I. and Feldman G., in a talk at Stanford Artificial Intelligence Project (SAIL) in 1968 [9]. This is a discrete differentiation operator, which searches the edge by computing the gradient of each x direction and y direction.

Non-maximum suppression algorithm is always used in thin the edge in edge detection in image processing field. The idea of NMS is to suppress the intensity value of image to 0 except the local maximal point [10].

The implementation for above algorithm is following by these steps:

(1) Convert the color image to gray image.

$$Gray\ intensity = R \times 0.21 + G \times 0.72 + B \times 0.07$$

(2) Apply Sobel Filter to gradient x, gradient y. Generate Sobel mask:

$$\frac{1}{4} \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \frac{1}{4} \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

A Sobel mask to x direction of the image, and get gradient x image, and apply Sobel mask to y direction of the image, and get gradient y image. Get magnitude image by

$$Magnitude = \sqrt{\text{gradient } x^2 + \text{gradient } y^2}$$

(3) Normalize the magnitude matrix to 0 ~ 255.

$$X' = \frac{(X - \min) \times 255}{\max - \min}$$

(4) Use threshold to binary the edge and non-edge. First sort the normalized matrix and get the intensity value by threshold. Then set black to edge, white to non-edge in the normalized image.

(5) Apply non-maximum suppression to get thin edge. Find the major direction by comparing gradient x and gradient y calculated before. Then set the edge point which is larger than both its neighbor in major direction.

2.2.2 Canny edge detector

Canny algorithm is first raised by Canny in 1986. He mentioned that there have three performance criteria to edge detection: 1) Good detection. The probability of error in the edge detection should be low. 2) Good localization. The edge detected should be as close as the true edge. 3) Only one response to a single edge. [11]

To find the best solution to match the above three criteria, the basic idea of Canny edge detector is first using Gaussian filter to smooth the image, and then use non-maximum suppression to get the final edge detection image. The details of Canny edge detector are following:

- 1) Use Gaussian filter to smooth the noise image.
- 2) Use first-derivative differentiation to get the magnitude and orientation gradient of the image.
- 3) Apply non-maximum suppression to the magnitude image.
- 4) Use double threshold algorithm to get rid of the false response.
- 5) Edge tracking by hysteresis to connect the edge [10].

I implement the Canny edge detector by OpenCV library. The Gaussian filter and gradient filters are [12]:

$$K = \frac{1}{159} \begin{bmatrix} 2 & 4 & 5 & 4 & 2 \\ 4 & 9 & 12 & 9 & 4 \\ 5 & 12 & 15 & 12 & 5 \\ 4 & 9 & 12 & 9 & 4 \\ 2 & 4 & 5 & 4 & 2 \end{bmatrix} \quad G_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix} \\ G_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix}$$

For performance evaluation, I convert all the result images to black edge and white non-edge images.

2.2.3 Structured edge

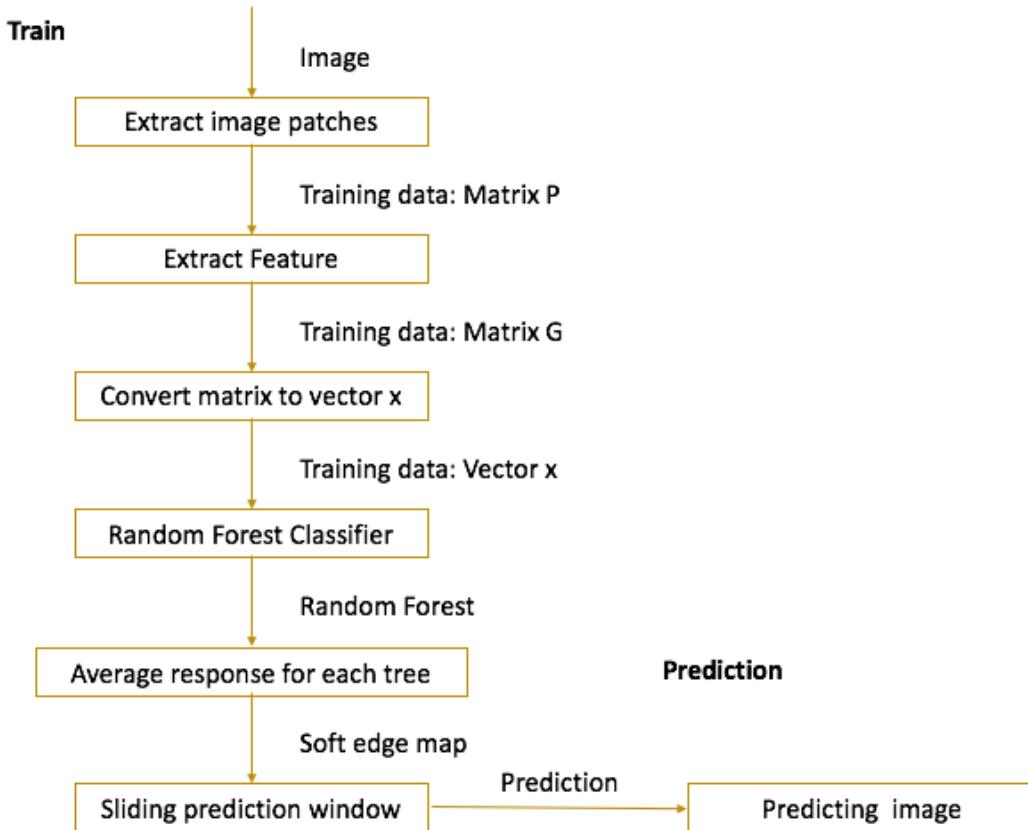


Figure 21. Flow chart of Structured Edge detector

Structured edge detector. Structured edge detection is first mentioned by Dollár, Piotr, and C. Lawrence Zitnick 2013. This is a algorithm developed from Structured Token Detection. This algorithm can be divided to two parts: Train and Predict.

Train: 1) Extract all image patches to get Matrix P; 2) Extract image feature from Matrix P to get Matrix G; 3) Convert each Matrix G to a vector x; 4) Use Random Forest Classifier to classify these vector x [13][15].

Predict: after apply RF classifier, we get a training tree result to predict the image. To predict the image, we need average response for each tree to get a soft edge map, and then sliding it to $16 * 16$ prediction window [14].

Random Forest classifier. Compared to Structure Token using predefined labels, Structured Edge use edge structure directly, which preserves more information, but larger dimensions (2^{256}). To reduce the dimension, we should

apply the following strategies: 1) Reduced by space Y. Y is not random because patch is segmented by close contours. We can use Z to encode the information that measure the similarity of every pair patch in Y belong to the same segment and split them into two classes. Then the dimension reduced to 32640. 2) Reduced by PCA. Now we sample 256 dimensions of 32640 dimensions in Z and reduce the data to 5 dimensions using PCA. 3). K-mean to binary classify into two classes. The dimensions are 2 now [14].

The implementation of SE is provided by Structured Edge Detection Toolbox V3.0, Piotr Dollar [17].

2.2.4 Performance evaluation

Ground Truth. To evaluate the performance of the detection above (Sobel, Canny, SE), we need a "correct" answer for each image. Because we goal to develop an edge detection algorithm is making machine predict the edge like a human being. So for edge detection, we use the edge maps provided by human to evaluate the performance of edge detection. The data here we use is from the Berkeley Segmentation Dataset 500 (BSD500) [16].

F-measure	Edge map	Ground Truth
True Positive (TP)	+	+
True Negative (TN)	-	-
False Positive (FP)	+	-
False Negative (FN)	-	+

Table 4. F-measure coefficients

F-measure. We use F-measure here to evaluation the edge detection performance. First we can divide the edge map result to 4 classes: TP, TN, FP, FN, shown in Table 4. "+" means edge pixels are shown in the map, and "-" means edge pixels are not in the map. After counting the number of pixels in four classes, we calculate P and R and then get F-measure:

$$\text{Precision: } P = \frac{\text{number of True Positive}}{\text{number of True Positive} + \text{number of False Positive}}$$

$$\text{Recall: } R = \frac{\text{number of True Positive}}{\text{number of True Positive} + \text{number of False Negetive}}$$

$$F - measure = 2 \times \frac{P \times R}{P + R}$$

The implementation of F-measure is also provided by Structured Edge Detection Toolbox V3.0, Piotr Dollar [17].

2.3 Results

2.3.1 Sobel edge detector and non-maximal suppression

First convert RGB image of "Farm" and "Cougar" to gray image. Then apply Sobel Filter to both images. The figure of the gradient image of two directions and magnitude map of "Farm" and "Cougar" images are shown below in Figure 22, 23.

Apply 10% and 15% threshold to "Farm" and "Cougar" magnitude images. For displaying, set black color to edge, and white color to non-edge. Then apply non-maximal-suppression to the above images. The results are shown in Figure 24, 25.

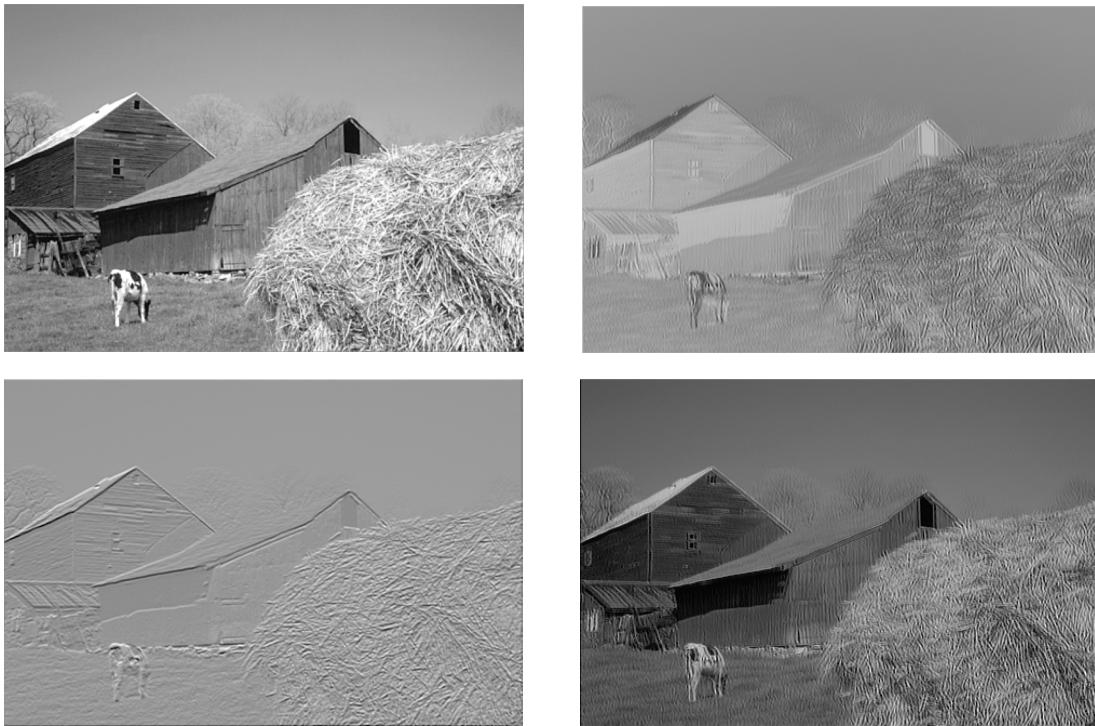


Figure 22. Top-left: The gray scale of *Farm*. Top-right: the gradient x image of *Farm*. Button-left: the gradient y image of *Farm*. Button-right: the magnitude image of *Farm*.

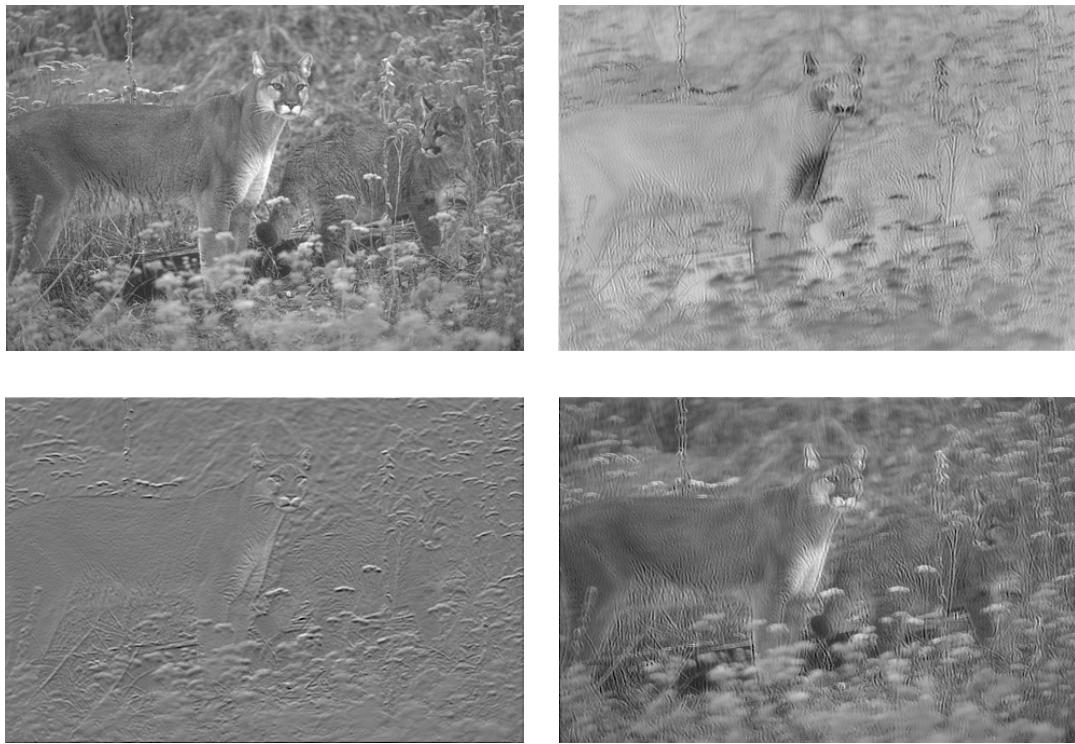


Figure 23. Top-left: The gray scale of *Cougar*. Top-right: the gradient x image of *Cougar*. Button-left: the gradient y image of *Cougar*. Button-right: the magnitude image of *Cougar*.

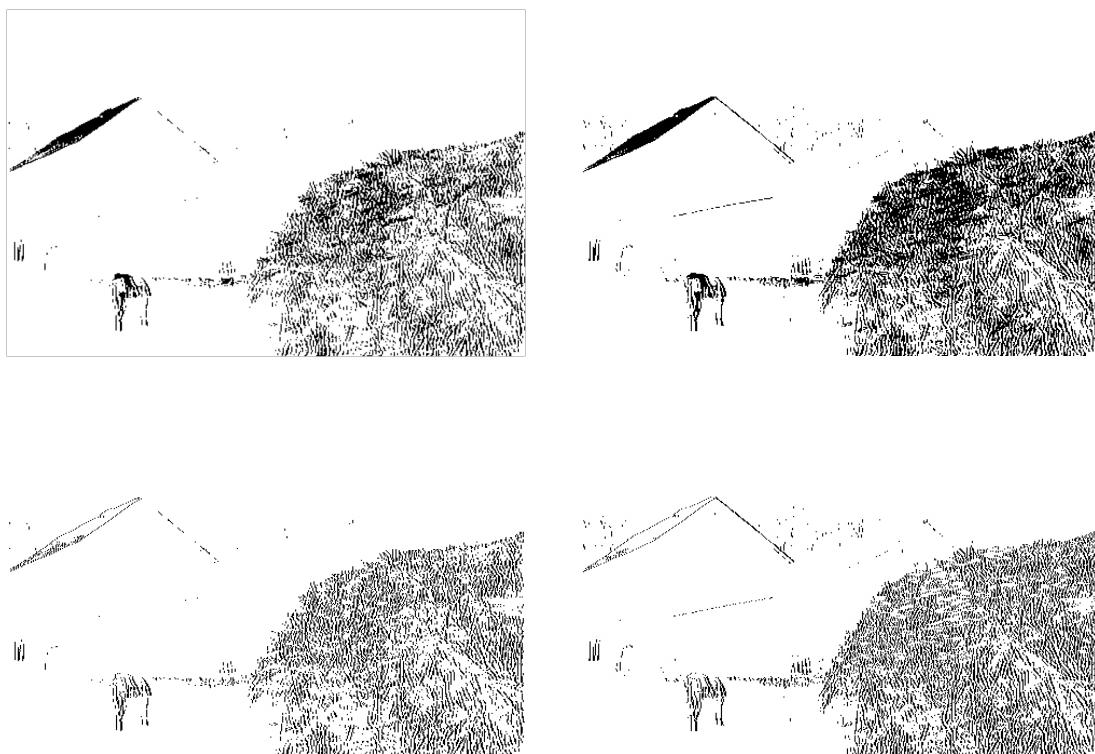


Figure 24. Top-left: the binary-threshold 10% image of *Farm*. Top-right: the binary-threshold 15% image of *Farm*. Button-left: the non-maximum suppression result of threshold 10% image of *Farm*. Button-right: the non-maximum suppression result of threshold 10% image of *Farm*.

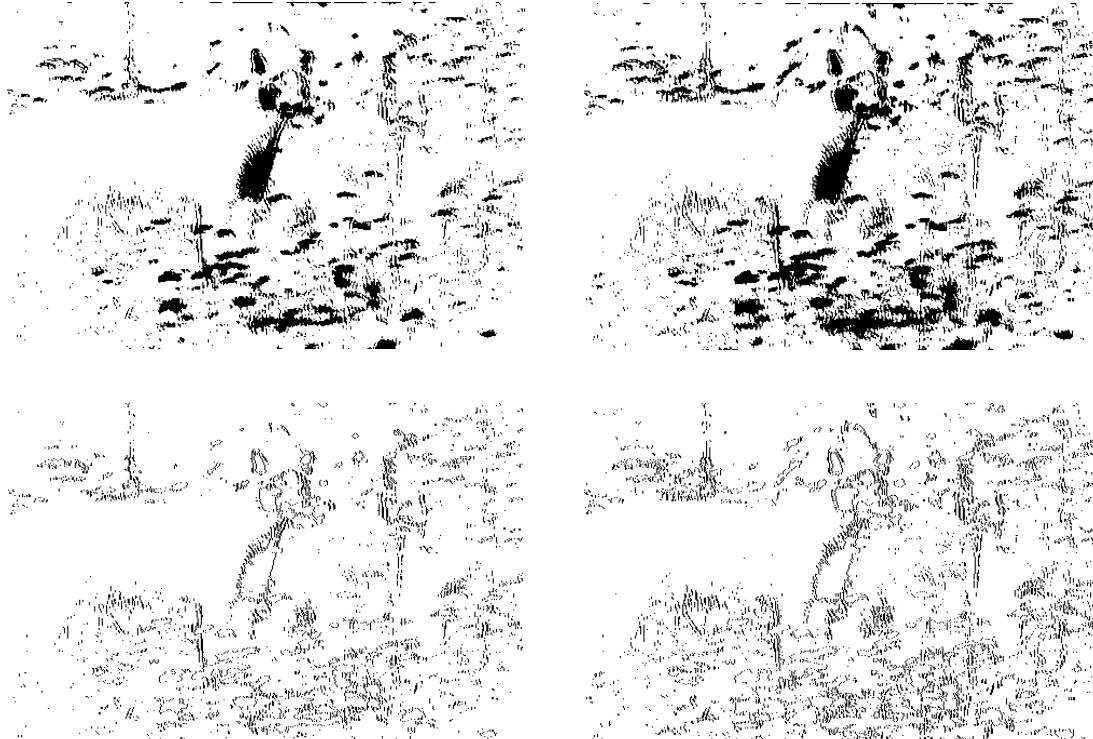


Figure 25. Top-left: the binary-threshold 10% image of *Cougar*. Top-right: the binary-threshold 15% image of *Cougar*. Button-left: the non-maximum suppression result of threshold 10% image of *Cougar*. Button-right: the non-maximum suppression result of threshold 10% image of *Cougar*.

2.3.2 Canny edge detector

Generate edge images by Canny edge detector using five different thresholds: Parameters table is below(Table 5). The results are Figure 26 - 30.

1	A: 0.3	B: 0.6	Default	Default
2	A: 0.2	B: 0.7	Default A - Δ	Default B + Δ
3	A: 0.2	B: 0.5	Default A - Δ	Default B - Δ
4	A: 0.4	B: 0.7	Default A + Δ	Default A + Δ
5	A: 0.4	B: 0.5	Default A + Δ	Default A - Δ

Table 5. Two threshold for Canny Edge detector

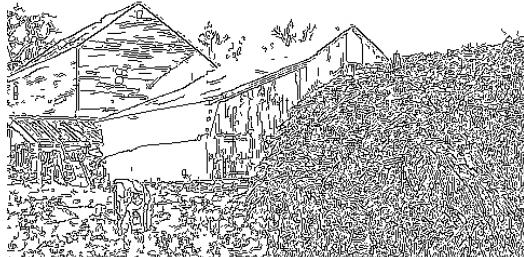


Figure 26. Canny edge detector, Threshold = (0.3, 0.6)

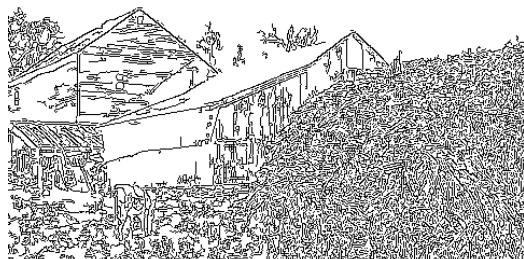
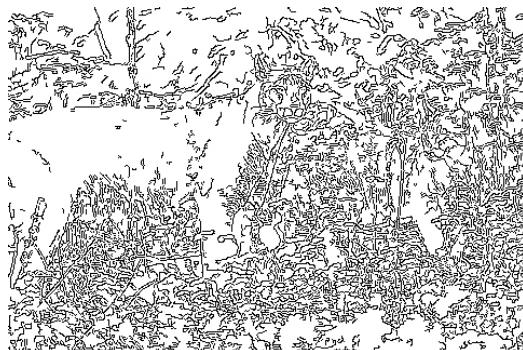


Figure 27. Canny edge detector, Threshold = (0.2, 0.7)

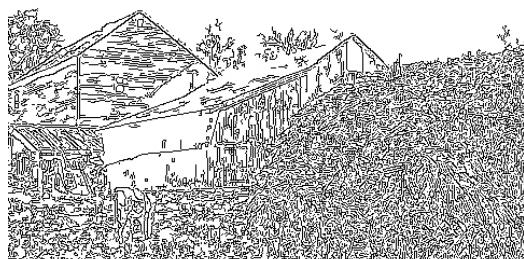
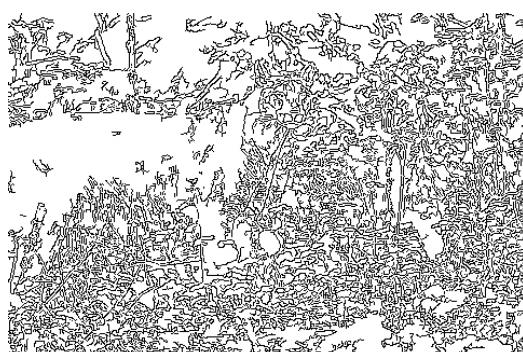


Figure 28. Canny edge detector, Threshold = (0.2, 0.5)

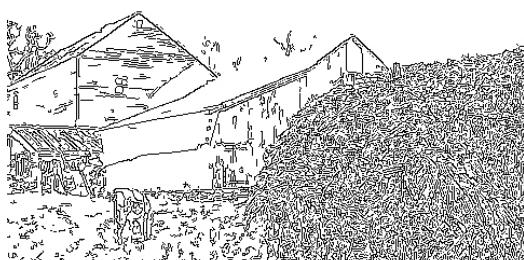
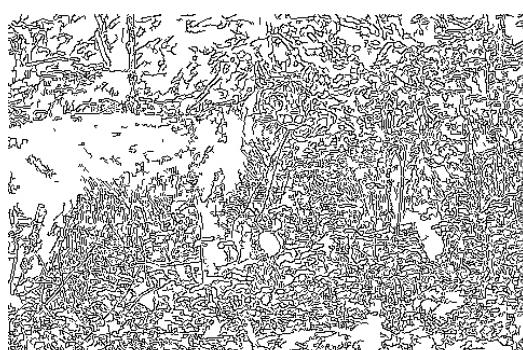


Figure 29. Canny edge detector, Threshold = (0.4, 0.7)



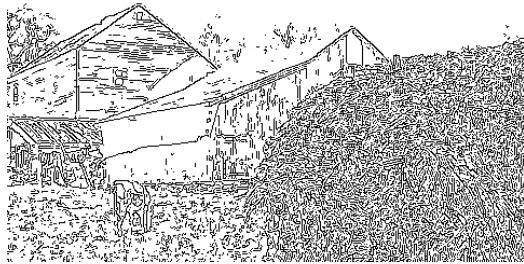


Figure 30. Canny edge detector, Threshold = (0.4, 0.5)

2.3.3 Structured edge

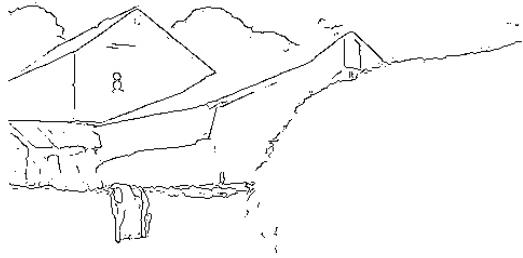


Figure 31. Structured edge detector results, Threshold = 0.02

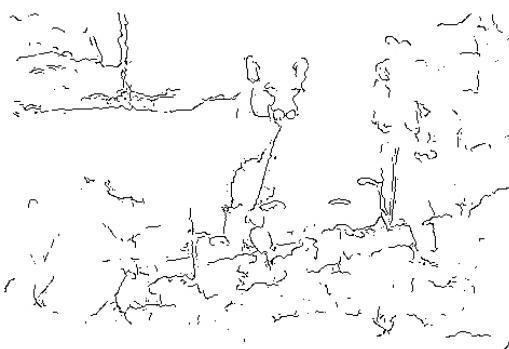
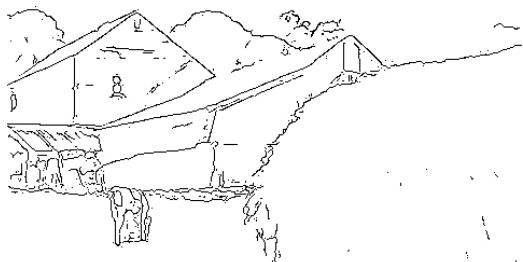


Figure 32. Structured edge detector results, Threshold = 0.03

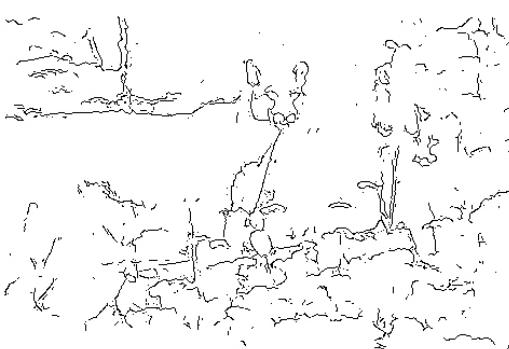
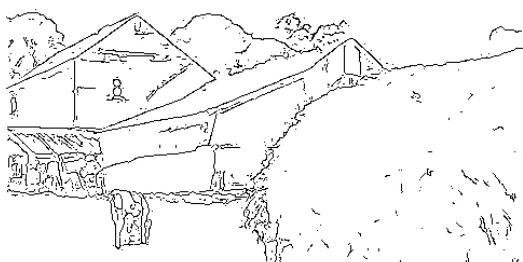


Figure 33. Structured edge detector results, Threshold = 0.04

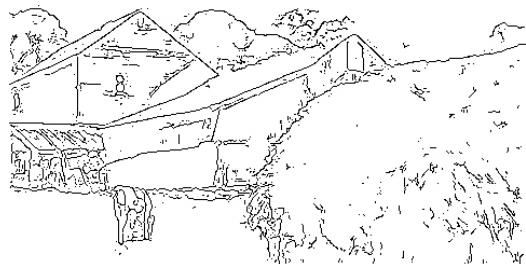


Figure 34. Structured edge detector results, Threshold = 0.05

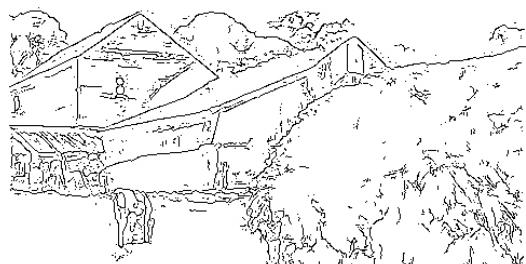
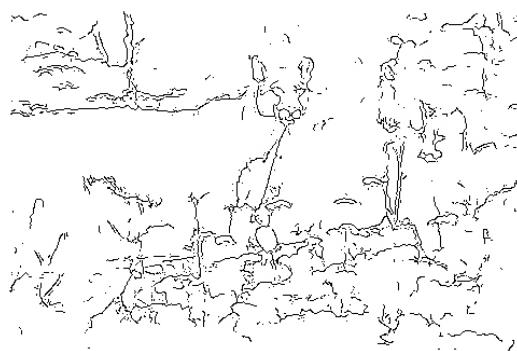


Figure 35. Structured edge detector results, Threshold = 0.06

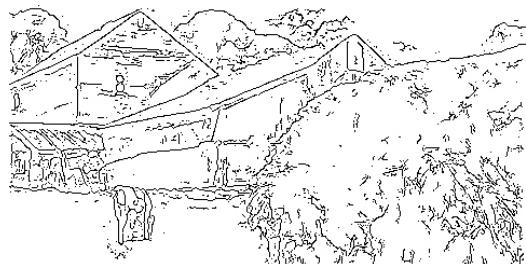
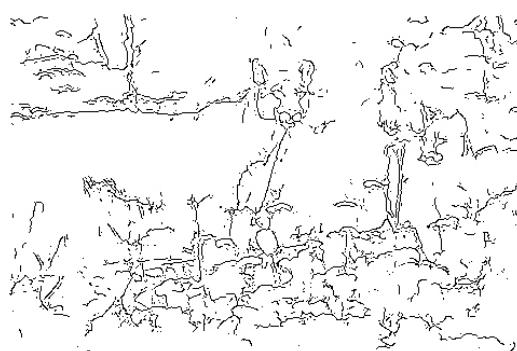


Figure 36. Structured edge detector results, Threshold = 0.07

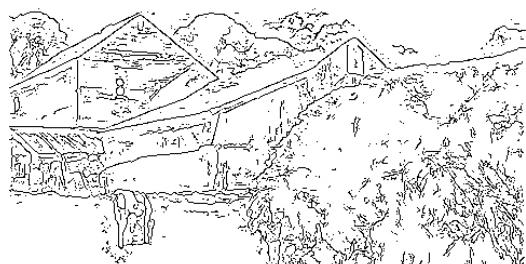
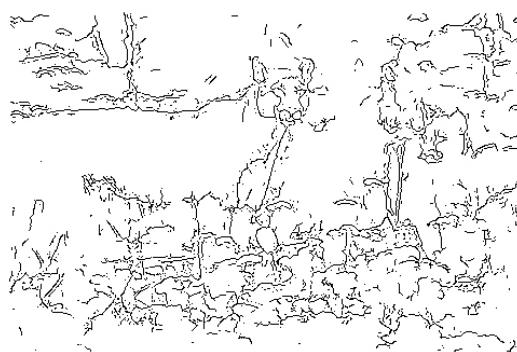
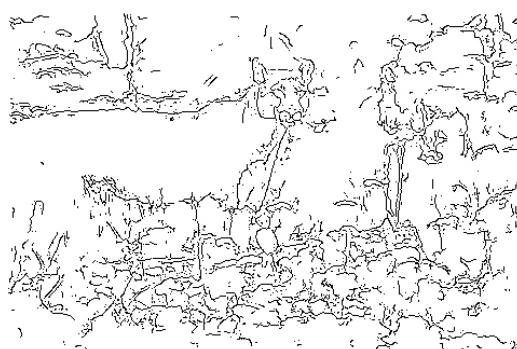


Figure 37. Structured edge detector results, Threshold = 0.08



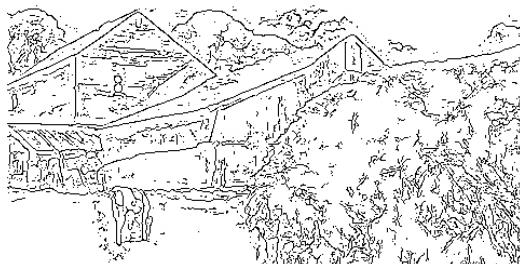


Figure 38. Structured edge detector results, Threshold = 0.09

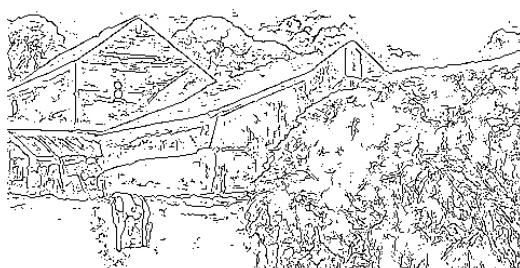
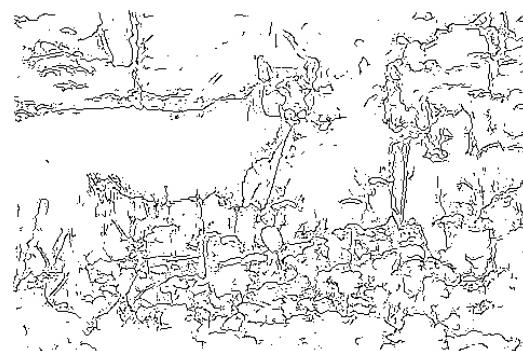
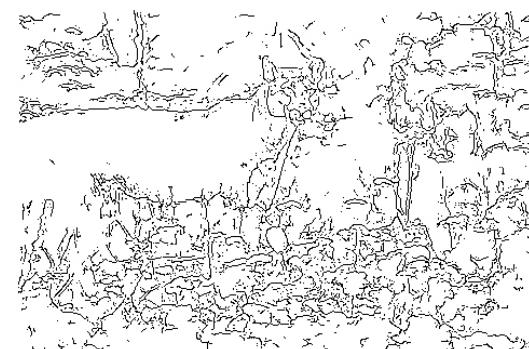


Figure 39. Structured edge detector results, Threshold = 0.10



To analysis the influence of threshold in Structured edge detector, here I choose 9 thresholds for Structure edge detector to see. These 9 thresholds are (0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09, 0.10). The results are from Figure 31 to Figure 39.

2.3.4 Performance evaluation

To evaluate the performance of above results, I use F-measure to measure each image. Figure 40 is the F-measure value for *Farm* image. First two suppressed images are using Sobel, threshold is 10% and 15%. Then in "p2b: Canny", Farm_3_6 means Canny Edge Map for threshold (0.3, 0.6). In "p2c: SE", Farm_SE_02 means Structure Edge Map for threshold (0.02) of *Farm* image. Figure 41 represent the result of *Cougar* image.

Farm			
p2a: Sobel			
p2a_result/Farm_suppression_010.raw	P: 0.870841%	R: 4.97846%	F: 1.48238%
p2a_result/Farm_suppression_015.raw	P: 1.23254%	R: 8.89844%	F: 2.16518%
p2b: Canny			
p2b_result/Farm_3_6.raw	P: 2.15417%	R: 27.4086%	F: 3.9944%
p2b_result/Farm_2_7.raw	P: 2.09968%	R: 28.0373%	F: 3.98678%
p2b_result/Farm_2_5.raw	P: 1.9994%	R: 28.7185%	F: 3.73852%
p2b_result/Farm_4_7.raw	P: 2.32816%	R: 26.0039%	F: 4.27369%
p2b_result/Farm_4_5.raw	P: 2.18582%	R: 26.4114%	F: 4.03749%
p2c: SE			
p2c_result/Farm_SE_02.raw	P: 14.8139%	R: 19.5327%	F: 16.8491%
p2c_result/Farm_SE_03.raw	P: 11.1116%	R: 22.0384%	F: 14.7742%
p2c_result/Farm_SE_04.raw	P: 8.69354%	R: 23.0587%	F: 12.6266%
p2c_result/Farm_SE_05.raw	P: 7.01723%	R: 23.2712%	F: 10.7829%
p2c_result/Farm_SE_06.raw	P: 5.87156%	R: 23.3769%	F: 9.38571%
p2c_result/Farm_SE_07.raw	P: 5.0458%	R: 23.4419%	F: 8.30415%
p2c_result/Farm_SE_08.raw	P: 4.41836%	R: 23.4625%	F: 7.43634%
p2c_result/Farm_SE_09.raw	P: 3.93178%	R: 23.4913%	F: 6.73613%
p2c_result/Farm_SE_10.raw	P: 3.54511%	R: 23.5343%	F: 6.162%

Figure 40. F-measure for Farm edge map

Cougar			
p2a: Sobel			
p2a_result/Cougar_suppression_010.raw	P: 3.85445%	R: 8.9006%	F: 5.37934%
p2a_result/Cougar_suppression_015.raw	P: 3.71168%	R: 12.5372%	F: 5.72767%
p2b: Canny			
p2b_result/Cougar_3_6.raw	P: 3.06524%	R: 29.8389%	F: 5.55938%
p2b_result/Cougar_2_7.raw	P: 2.85325%	R: 31.0242%	F: 5.22588%
p2b_result/Cougar_2_5.raw	P: 2.6609%	R: 32.4649%	F: 4.91866%
p2b_result/Cougar_4_7.raw	P: 3.34247%	R: 25.3973%	F: 5.90747%
p2b_result/Cougar_4_5.raw	P: 3.12919%	R: 27.5605%	F: 5.62026%
p2c: SE			
p2c_result/Cougar_SE_02.raw	P: 8.83134%	R: 7.79869%	F: 8.28295%
p2c_result/Cougar_SE_03.raw	P: 8.08547%	R: 10.6099%	F: 9.17724%
p2c_result/Cougar_SE_04.raw	P: 7.26566%	R: 12.6428%	F: 9.22807%
p2c_result/Cougar_SE_05.raw	P: 6.60795%	R: 14.2821%	F: 9.03545%
p2c_result/Cougar_SE_06.raw	P: 6.1047%	R: 15.7496%	F: 8.79886%
p2c_result/Cougar_SE_07.raw	P: 5.60274%	R: 16.8391%	F: 8.40796%
p2c_result/Cougar_SE_08.raw	P: 5.21655%	R: 17.8503%	F: 8.07366%
p2c_result/Cougar_SE_09.raw	P: 4.87299%	R: 18.7357%	F: 7.73435%
p2c_result/Cougar_SE_10.raw	P: 4.57095%	R: 19.519%	F: 7.40726%

Figure 41. F-measure for Cougar edge map

2.4 Discussion

2.4.1 Sobel edge detector and non-maximal suppression

Sobel edge detector is a first-derivative algorithm. It takes account to the neighbor information and applies a weighted smooth effect to each x and y direction. This smooth effect reduces the noise but we can see there are a lots of true edge missing (Figure 42). Also Sobel edge detector is too sensitive to texture like barn in the *Farm* image (Figure 43).



Figure 42. Roof in Farm image. Left: Ground truth image. Right: Edge map image.



Figure 43. Barn in Farm image. Left: Ground truth image. Right: Edge map image.

2.4.2 Canny edge detector

Compared to Sobel edge detector, Canny detector has two thresholds to detect the edges. By using two thresholds T_1 and T_2 , $T_1 < T_2$, we can get two edge images Image1 and Image2. Because Image2 is calculated by a higher threshold, it has less false edges, but more discontinuities than Image1. So we can use Image2 to find the true edge and use Image1 to get the edge points which can connect the edge points in Image2.

Compared Figure 26 to Figure 30 fives image. The most clearly image is Figure 29, Threshold $A = 0.4$, $B = 0.7$. Compared Figure 29, Figure 27, which both B threshold is 0.7 (Details in Figure 44), we can see, though the correct edge in two figures are similar, Figure 27 (threshold $A = 0.2$, $B = 0.7$) have too many false positive points which have a connection to the true positive points. This is because A threshold is used to binary a map for link discontinuities in B threshold map.

Also Figure 45 is a comparison of Figure 29 ($A = 0.4$, $B = 0.7$) and Figure 30 ($A = 0.4$, $B = 0.5$). Both two images have the same A threshold, but the results here are really different. Figure 30 ($A = 0.4$, $B = 0.5$) have too many noise that we even cannot recognize what it is. I think it is because B threshold is used for reduced the false edges.

For all the five results, we can see that Canny Edge detector is also sensitive to the texture and has too many noise.

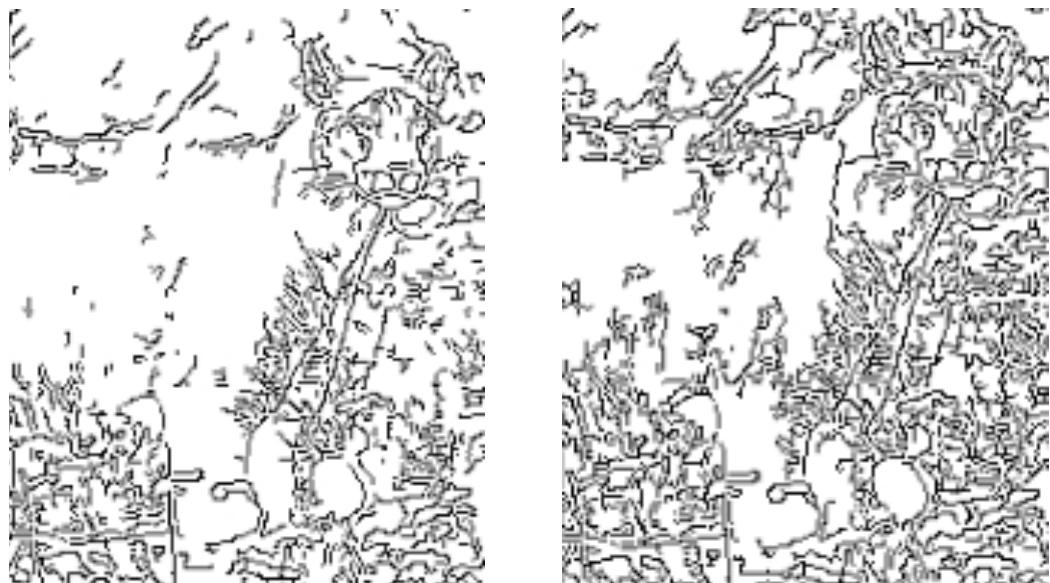


Figure 44. Cougar image. Left: threshold (0.4, 0.7). Right: threshold (0.2, 0.6)



Figure 45. Cougar image. Left: threshold (0.4, 0.7). Right: threshold (0.2, 0.7)

2.4.3 Structured edge

Compared to Sobel and Canny edge detector, I find Structured edge detector perform much better than I thought. In all my 9 results, the best performance of detector is threshold 0.02 in *Farm* and threshold 0.03 in *Cougar*.

Texture. First, it reduces the false detection of texture. We can clearly see this in Figure 46. The texture reduction effect of SE is close to human being.



Figure 46. Barn in Farm image. Left: SE (threshold = 0.02). Right: Ground truth

Continuity for above three structure edge. Compared Figure 24, 29, and 31, Sobel detector and SE detector are all have a efficient effect to increase the continuity of the image. And Sobel detector did a bad job here in detect the edge line of Roof in Farm image.

Noise. SE reduce most of false edges and perform the best in these three detector. Sobel detector has a less noise.

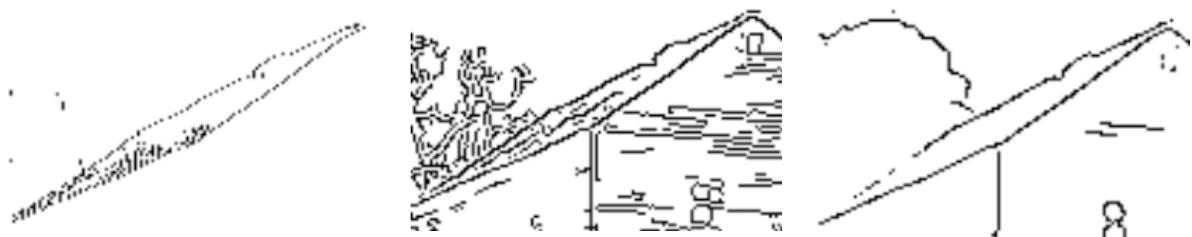


Figure 47. Roof in Farm image. Left: Sobel detector. Middle: Canny detector. Right: SE detector

2.4.4 Performance evaluation

Let us go back to the formula of P, R, F in the 2.2.4.

$$\text{Precision: } P = \frac{\text{number of True Positive}}{\text{number of True Positive} + \text{number of False Positive}}$$

$$\text{Recall: } R = \frac{\text{number of True Positive}}{\text{number of True Positive} + \text{number of False Negative}}$$

$$F - \text{measure} = 2 \times \frac{P \times R}{P + R}$$

P value means the probability of correct pixels in the whole edge map. And R value means the probability of the correct pixels compared with the ground truth. Sometimes, we make a mistake that we think Precision P or Recall R is enough to measure the detection performance. However, here is a special case, if we only have one pixel in the edge map, and it is correct, then the precision P for the edge map is 100%, though Recall R in this case is really small. In contrast, if we fill all the pixels in the edge map, then the recall R here is 100%, but Precision P is low. To make a effective measure of edge detection, F-measure combines Precision P and Recall R to one parameter and get a optimal of P and R.

Performance of three edge detector. As the discussion above in 2.4.3, I think the best edge detector is SE detector, the worst edge detector is Sobel detector, which is shown in F-measure result in Figure 40, Figure 41. We can see that F-measure is image-dependent. *Farm* image detection always get a higher value of F-measure than *Cougar* image detection.

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