# Computer Science 685 Midterm

Paul McKerley (G00616949)

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1. If I understand the question correctly, it is to define the forward kinematics of the point at the end of $L2$. Since this is the position of the gripper, I will refer to it as $G$. I interpret the diagram to mean that $L1$ is a fixed bar, which slides up and down $y1$, causing $d1$ to vary. Also, $\theta_2$ can change, causing $L2$, which is also fixed, to swing around.

   So the position of $G$ in the second coordinate frame is simply how far it is translated from the end of $L1$, which is

   $$G_2 = \begin{bmatrix} L2 \\ 0 \end{bmatrix}$$

   Now in the first coordinate frame, $G$ is rotated by $\theta_2$ and translated by $L1$ on the $x$ axis and $d1$ on the $y$ axis. So

   $$G_1 = \begin{bmatrix} L1 \\ d1 \end{bmatrix} + R(\theta_2) \cdot \begin{bmatrix} L2 \\ 0 \end{bmatrix} = \begin{bmatrix} L1 \\ d1 \end{bmatrix} + \begin{bmatrix} L2cos(\theta_2) \\ L2sin(\theta_2) \end{bmatrix} = \begin{bmatrix} L1 + L2cos(\theta_2) \\ d1 + L2sin(\theta_2) \end{bmatrix}$$

   As mentioned above, the kinematic parameters are $L1$, $L2$, and implicitly the angles of $d1$ with respect to $x1$, and $L1$ with respect to to $d1$. The joint variables are $d1$ and $\theta_2$.

2. a) The function needs $U_r$ to be expanded to include repulsion from robot $R2$ in position $x_2$. So it would look something like
   $U(x_1) = k_1||x_1 - x_g||^2 + k_2\frac{1}{||x_1 - x_o||^2} + k_3\frac{1}{||x_2 - x_1||^2}$
   with the repulsion of the $R2$ possibly having a different constant $k_3$, since a moving obstacle might require speedier avoidance.

   b) The control function will be to figure out the forces applied to $R1$ at that position. We need to define distances of influence for the obstacle and the other robot, so, we will define $q_o$ and $q_2$ to be the minimum distances from the obstacle and other robot, respectively, before they have a repulsive effect.

   $$F(x_1) = -k_1 \cdot (x_1 - x_g) - \begin{cases} (k_2(\frac{1}{x_1} - \frac{1}{x_o})), & \text{if} x = 1 \\ 0, & \text{otherwise} \end{cases}$$

   c) The repulsive and attractive forces at a certain location can cancel each other out perfectly, so that the net force applied at that location is $0$; ie, the robot encounters a local mininum. When this happens the robot could adopt a number of strategies. When if finds itself stuck, it could go on a random walk and hope to leave the local minimum. It could use a strategy like one of the Bug algorithms. For example, the Bug2 algorithm tells the robot that when it can't move, it should remember its bearing to the goal, and circle around the obstacle until it hits that bearing line again, at which point it should continue toward the goal.

3. I'm assuming that the $d_x^2$ and $d_z^2$ variables are constant for a given run.

4.