POLITECNICO
MILANO 1863

Scuola di Ingegneria Industriale e dell'Informazione
Corso di Laurea Magistrale in Mathematical Engineering - Quantitative Finance

Financial Engineering: Group 4
Assignment 3

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1 Asset Swap Spread

The objective of this analysis is to compute the Asset Swap Spread Over Euribor3m for a bond issued by issuer YY. The Asset Swap Spread, s_{aws}, is determined by the formula:

$$s_{asw} = \frac{C(0) - \bar{C}(0)}{BPV^{fl}(0)} \tag{1}$$

where:

- $C(0)$ represents the price of an interbank coupon bond.
- $\bar{C}(0)$ is the market observed clean price of the bond issued by YY.
- $BPV^{fl}(0)$ is the BPV of the floating leg part of an IRS, which is paid every 3 months.

The discount factors necessary for pricing were obtained via bootstrap procedure. In this computation, accrual calculations have been intentionally omitted, since both prices in the numerator are derived using identical accrual conventions, hence the accrual components naturally offset each other.

Asset Swap Spread
1.756595 %

2 Bootstrap CDS

Below, we derive a piecewise constant default intensity curve $\lambda(t)$ for ENI, using the provided CDS spreads. Initially we neglect the accrual term, then assess the accrual impact and compare the result with the Jarrow-Turnbull approximation.
Given the discount factors via bootstrap, we want to evaluate the survival probabilities $P(t_0, t_i)$ and, once computed, we derive the corresponding intensities $\lambda(t_i)$, just inverting the formula.

- **Negletting the accrual factor**: for each year i we solved the following equation with respect to $P(t_0, t_i)$:

$$\bar{s}_i \sum_{j=1}^{i} \delta(t_{j-1}, t_j)\, B(t_0, t_j)\, P(t_0, t_j) = (1 - \pi) \sum_{j=1}^{i} B(t_0, t_j)\, [P(t_0, t_{j-1}) - P(t_0, t_j)] \tag{2}$$

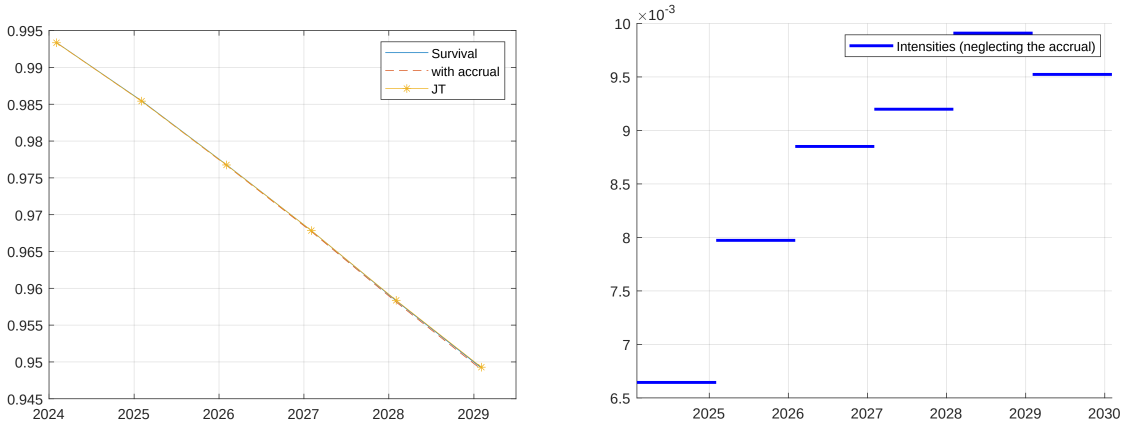
- **Considering the accrual factor**: we obtain $P(t_0, t_i)$ from the previous formula by adding the following term to the left-hand side:

$$\bar{s}_i \sum_{j=1}^{i} \frac{1}{2}\, \delta(t_{j-1}, t_j)\, B(t_0, t_j)\, [P(t_0, t_{j-1}) - P(t_0, t_j)] \tag{3}$$

- **Jarrow-Turnbull approximation**:

$$\lambda(t_i)^{JT} = \frac{\bar{s}_i}{1-\pi} \tag{4}$$

Notice that the differences in survival probabilities computed without and with accrual are on the order of 10^{-4}, rendering the accrual impact negligible. Moreover we can observe that also the Jarrow-Turnbull approximation, with far less computational effort, provide very precise results (up to an error of 10^{-4}), even if year by year the JT values are less precise.



(a) Survival probabilities in the 3 methods

(b) Intensities neglecting the accrual term

3 Credit Simulation

The purpose of this exercise is to simulate the default time, τ, for an obligor using a two-regime default intensity model and to estimate the model parameters λ_1 and λ_2. A sample of 10^5 simulations is used to generate default times, fit the survival probability, and provide estimators along with confidence intervals for λ_1 and λ_2.

10^5 random numbers uniformly distributed between 0 and 1 are then generated, and each probability is converted into a default time.

Once the default times have been simulated, we computed the empirical survival probability for each year from 1 to 30 by counting, for each year, the number of simulations where the default time is greater than or equal to that year. The division of this count by the total number of simulations gives the probability of survival at that time.

For times up to 4 years, the survival probability is given by:

$$u(t) = e^{-\lambda_1 t}.$$

Taking the negative logarithm of both sides, we obtain:

$$-\ln(u(t)) = \lambda_1 t.$$

The estimation process involves finding the best-fitting line and the slope of this line is taken as the estimator for λ_1.

For times beyond 4 years, the survival probability is modeled as:

$$u(t) = e^{-\lambda_1 \cdot \theta - \lambda_2 (t-\theta)}.$$

This expression can be rearranged into:

$$\ln(u(t)) = -\lambda_2 t + (\lambda_2 \cdot \theta - \lambda_1 \cdot \theta).$$

In this linear relationship, the slope is $-\lambda_2$ and the intercept is $\lambda_2 \cdot \theta - \lambda_1 \cdot \theta$.

After estimating the parameters, it is essential to assess the reliability of these estimates. This is done by calculating 95% confidence intervals for the estimated values, by analyzing the residuals, which are the differences between the observed values (derived from the empirical survival probabilities) and the values predicted by the fitted regression line.

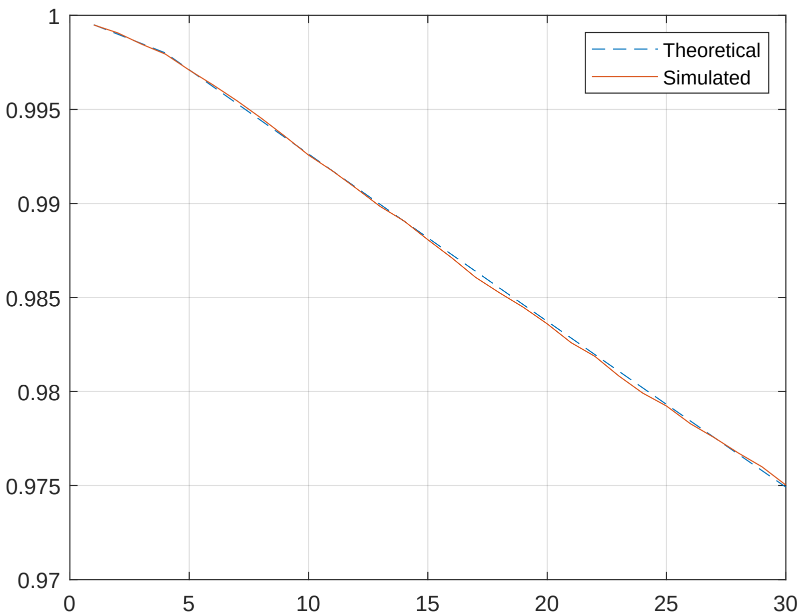


Figure 1: Accordance between theoretical and simulated survival probabilities. In the simulation, random seed was set to `rng('default')`

$\boldsymbol{\lambda_1}$	$\boldsymbol{\lambda_2}$
5.0710 bp	9.0354 bp
CI_{λ_1} [4.6674; 5.4156]	CI_{λ_2} [8.9568; 9.1140]

4 MBS Pricing

All the references to the formulas employed in this session are derived from *"A large deviation application to the securitized market"* by R. Baviera.
Firstly we want to find the price of the Mezzanine Tranche of a MBS (Mortgage Backed Security), using three different approaches:

- **HP approach**: using (1) we compute the expected loss for the tranche and then the price, discounting the difference between 1 and the expected loss. In this case we calculate the binomial coefficient in an exact way, thanks to *nchoosek* MatLab function, up to a number of obligors equal to 100.
- **Kullback-Leibler approach**: since computing the binomial coefficient in (1) becomes impractical for a large number of obligors, we approximate it using Stirling's formula, which leads us to the expression in (4).
- **LHP approach**: supposing a number of obligors I sufficiently large, we also analyze the impact on the results of this third method, described by (2) and not depending on I.

HP price	**KL price**	**LHP price**
2.995227e+07 (74.88%)	2.996298e+07 (74.88%)	3.010181e+07 (74.88%)

Table 1: The percentage is with respect to the Notional of the Mezzanine Tranche and I is equal to 100

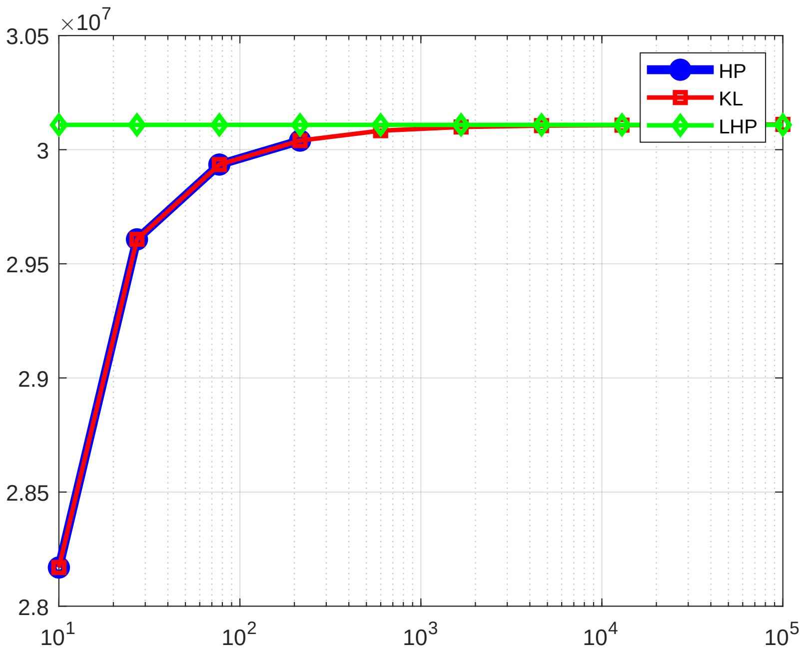


Figure 2: Prices vs number of obligors I (log-linear scale)

Lastly we price the Equity Tranche. The HP and LHP approaches still work fine in this situation; instead the problem arise with KL, since the Stirling approximation does not hold. This follows by the fact that a small number of defaulting mortgages already have a significant impact on the loss of the Equity Tranche. To overcome this problem we find the price as: $price_{equity} = price_{ref\ ptf} - price_{mezzanine}$

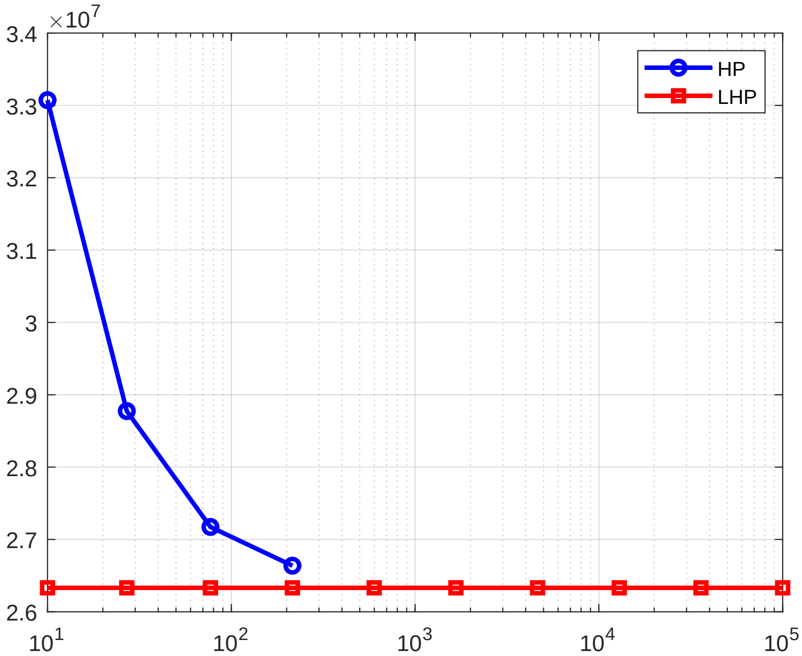


Figure 3: Prices vs number of obligors I (log-linear scale)

KL Equity Price
2.644707e+07

Table 2: As in the table above I is equal to 100

Appendix

Pricing the *Mortgage Backed Security* for a large number of bond holders I has proven to be quite computationally intensive. Computing binomial coefficients and factorials for large numbers is challenging and can lead to numerical instabilities and underflow. Running the `nchoosek` function with $I > 70$ already raises some warnings. On the other hand, numerical integration with adaptive grids, like the ones used in the `quadgk` and `integral2` functions, requires more time and memory than usual. Our function to plot the price of MBS with respect to the number of holders takes almost 10 seconds to run. According to the `MATLAB profiler`, more than 60% of the computational time is devoted to integration.

Of the two criticalities mentioned above, the first one might be partially avoided by use of the following *trick*: `MATLAB` provides a *Symbolic Math Toolbox*, which enables us to compute symbolic expression. If instead of computing the factorial of standard numerical variables, we instead pass *symbolic variables* `factorial(sym(x))`, the result will have a much higher precision. However, this approach is far from optimal, as evaluation of symbolic expression takes up substantially more computational time and memory.

References

[1] P. J. Schonbucher, *Credit Derivatives Pricing Models*, John Wiley and Sons, New York, 2003.

[2] R. Jarrow and S. Turnbull, *Pricing derivatives on financial securities subject to credit risk*, Journal of Finance 50, 53-85.