POLITECNICO
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Scuola di Ingegneria Industriale e dell'Informazione
Corso di Laurea Magistrale in Mathematical Engineering -
Quantitative Finance

Financial Engineering: Group 4
Assignment 4

Matteo Bovio: 272377
Alberto Busci: 278889
Matteo Campagnoli: 275975
Alice Sofia Casciani: 275720

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Index

1 Certificate Pricing **2**

2 Pricing Digital Option **2**

3 Call Option Pricing - comparative study **3**

3.1 case $\alpha = 0$ - Variance Gamma 3

3.2 case $\alpha = \frac{1}{3}$ 5

4 Volatility Surface Calibration **5**

1 Certificate Pricing

The task involved determining the spread over Libor (in basis points) for a certificate issued by Bank XX, given an upfront payment of 3%. The certificate was linked to an equally weighted basket of two stocks (ENEL and AXA) and included a swap agreement where Bank XX payed Euribor 3m plus a spread, while Party B (an IB) payed a coupon based on the basket's performance. The spread was computed imposing the NPV of the swap contract equal to 0, obtaining the following formula:

$$\text{spread} = \frac{\text{upfront} + [\alpha \cdot \max(S(t) - P, 0) + P] \cdot DF(t_N) - 1}{\sum_{i=1}^{n} \Delta_i \cdot DF(t_i)}$$

Spread over Libor
-171 bps

The CI for the spread is [-173 ; -169].

As we can observe, the sign of the spread is negative, meaning that for Bank XX side it represents a positive cash flow. This likely stems from the fact that the contract's coupon structure isn't particularly advantageous for the commercial bank, which will receive it at maturity from the investment bank.

2 Pricing Digital Option

Firstly we computed the price of the digital option in a Black framework, hence discounting $N(d_2)$. This approach does not take into account the so called *digital risk*, given by the volatility dependency from the strike. To overcome this issue we added the extra term $-\nu \frac{\partial \sigma(K)}{\partial K}$, where the second quantity had been approximated as the slope of the straight line passing to the two closest points that we had in the dataset with respect to the target strike.

Black Price	**Corrected Price**
0.31 mln	0.41 mln

Coherently the second price is larger than the first one, as the volatility of our dataset decreases in the strikes.

3 Call Option Pricing - comparative study

We present a comparative study of European Call option pricing under a normal mean-variance mixture model. Three numerical techniques are employed: Fast Fourier Transform (FFT), numerical quadrature, and Monte Carlo simulation.

- **FFT**: we compute the Fourier transform in the Lewis formula using an FFT-based approach, leveraging MATLAB's built-in `fft` function scaled by an appropriate prefactor. We carefully choose the parameters M and dz to define the discretization of the Fourier space and the corresponding real-space grid. In our implementation, we set $M = 15$, which means that the total number of nodes is $N = 2^{15}$. The grid spacing in real space is defined as $dz = 0.1\%$. After computing the FFT, the results are interpolated over the desired log-moneyness grid to ensure that the output is aligned with the specific pricing points required for the option valuation.

- **Quadrature**: the quadrature method directly evaluates the integral in the Lewis formula using the MATLAB's built-in function `quadgk`.

- **Monte Carlo**: we generate two types of random variables: the first is a standard normal, representing the conventional stochastic shocks in financial returns; the second is a Gamma, which is used to modulate the volatility within the model. To ensure the accuracy of the simulation, the first four moments of the Gamma are numerically verified. We first simulate the log return of the asset's forward price, from which we derive the forward price. Finally, the option price is estimated by averaging the simulated payoffs and discounting them appropriately.

Theoretical	**Numerical**
1	0.999
2	1.993
6	5.946
24	23.538

Table 1: check of the four moments of the Gamma

3.1 case $\alpha = 0$ - Variance Gamma

FFT	**QUAD**	**MC**
230.45	230.45	229.90

Table 2: ATM Call Option Prices for $\alpha = 0$

Moreover, the option price is calculated using each of the three methods for all log-moneyness values, as shown in the figure below.

In the Monte Carlo approach, a 95% confidence interval is also computed. For the ATM Call, this interval is [209.49, 250.30].

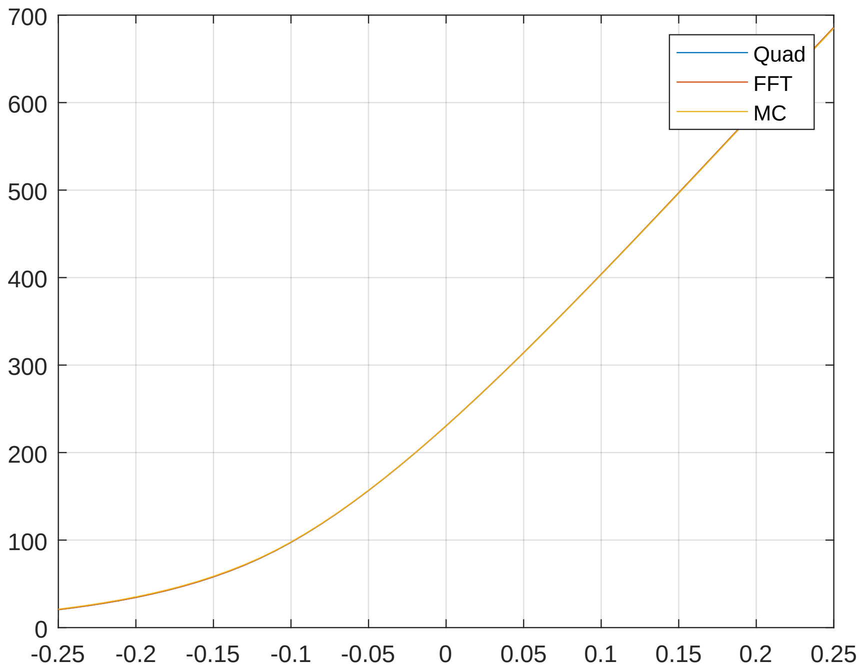

Figure 1: Call Option Prices vs. Log-Moneyness

Moreover, a confidence interval in the MC simulation is calculated for each log-moneyness value, as illustrated in the figure below.

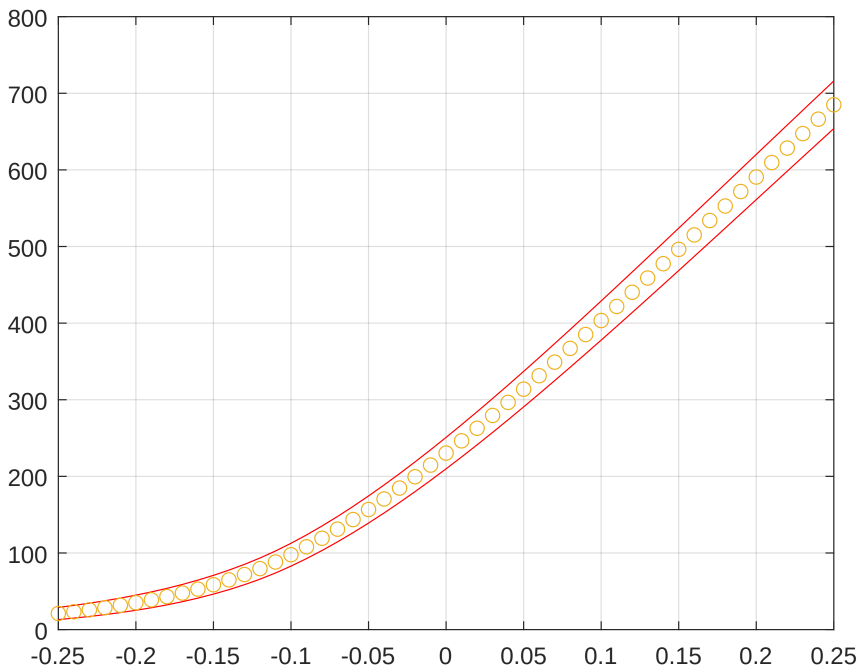

Figure 2: Confidence intervals for Call option prices across all log-moneyness values

3.2 case $\alpha = \frac{1}{3}$

FFT	QUAD
228.26	228.26

Table 3: ATM Call Option Prices for $\alpha = \frac{1}{3}$

Moreover, the option price is calculated using the above two methods for all log-moneyness values, as shown in the figure below.

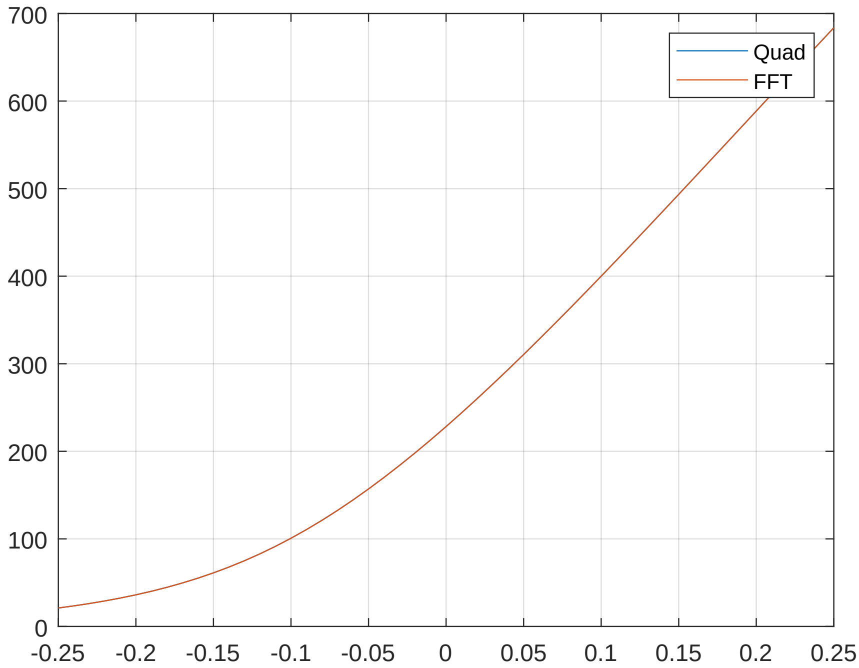

Figure 3: Call Option Prices vs. Log-Moneyness

We compare the FFT-based prices under $\alpha = \frac{1}{3}$ against those obtained with $\alpha = 0$ by computing the average difference and expressing it as a fraction of the ATM price from the $\alpha = 0$ case. This provides a relative measure of how sensitive the results are to changes in the α parameter.
The numeric result is: 0.0035.

4 Volatility Surface Calibration

In the fourth case study, we calibrated a normal mean-variance mixture model with $\alpha = \frac{2}{3}$, using the S&P 500 volatility surface via a global calibration with constant weights (equal to 1). Initially, we evaluated the real prices with the Matlab function `blkprice` and discounting back the results with the one year discount factor. Subsequently, we calculated the prices of the call options with the function implemented in this laboratory as functions of our set of parameters $\mathbf{p}$=$\{\sigma, \kappa, \eta\}$. Then we evaluated the quantity to be minimized, which is:

$$D = \sum_{i=1}^{N} (C(x_i, t_i, \mathbf{p}) - C_i)^2 \tag{1}$$

The following constraints have also been imposed (the first is always satisfied in the equity world):

$$\eta \geq -\underline{\omega} \tag{2}$$

$$\underline{\omega} := \frac{1-\alpha}{k\sigma^2} \geq 0 \tag{3}$$

Finally we calibrated the model using `fmincon` with an initial guess for the parameters and additional constraints to ensure that the volatility and the κ were positive.

$\hat{\sigma}$	$\hat{\kappa}$	$\hat{\eta}$
0.0646	0.6134	58.6311

The second part of the exercise focused on computing implied volatilities. To this purpose, we first evaluated the call option prices using the calibrated parameters. Then we computed the same option prices as functions of volatility using MATLAB's built-in `blkprice` function. The differences between the two sets of prices were calculated for each strike, and the function `fmincon` was employed to minimize these differences (with an initial guess) and obtain the implied volatilities.

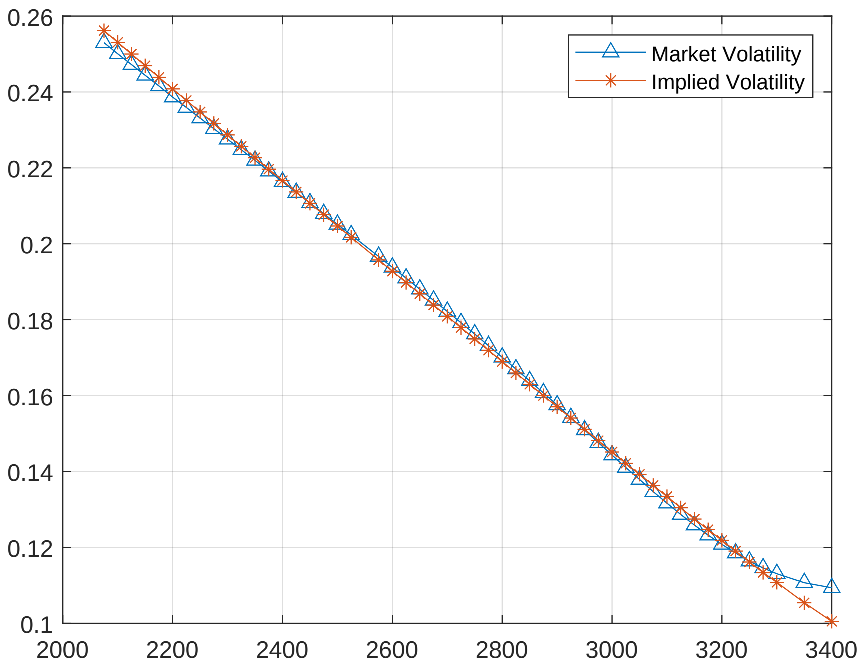

Figure 4: Volatilities vs Strikes. The graph refers to $\alpha = \frac{2}{3}$.