Assignment 4
Structured Products

1. Case study: Certificate Pricing.

On the 31st of January 2023 at 10:45 C.E.T., the Bank XX issues a certificate whose hedging termsheet is described in the annex.
Determine the spread over Libor (in bps) and given an upfront of 3% at the Certificate issue, in a single-curve interest rate modeling setting, knowing that the two counterparties have signed an ISDA with CSA that allows neglecting the counterparty risk. Neglect also the dynamics of interest rates.
Consider the dynamics for the underlyings.

Market parameters:
Basket underlyings: ENEL, AXA. Stock prices at the 31st of January 2023 at 10:45 C.E.T. are 100 and 200 euros respectively.
Correlation = 45%, $\sigma_1 = 16.2\%, \sigma_2 = 20.0\%$,
Dividend Yields: $d_E = 2.5\%, d_{CS} = 2.9\%$,
Discounts: consider values of the 31st of January 2023 at 10:45 C.E.T.

2. Exercise: Pricing Digital option

Verify the difference between the price of a digital option computed according to the Black model and in the case where one takes into account the smile in the curve of the implied volatility.
Consider the provided dataset. Moreover:

- Reference in dataset is ATM Spot
- Notional 10 Mln €
- Digital Payoff 7% Notional
- Expiry: 1y (Act/365)
- Pricing date: the 31st of January 2023 at 10:45 C.E.T.

3. Exercise: Pricing

Compute call prices C on the 31st of January 2023 at 10:45 C.E.T., according to a normal mean-variance mixture with $\alpha = 0$ (VG) model with parameters $\sigma = 20\%; \kappa = 1; \eta = 3;$ t=1; for moneyness x between -25% and 25% (in a grid with 1% steps). F_0, discount and ttm are the values in the Exercise 2. Compute prices according to the following methods:

a. FFT. Choose an adequate value for M and, and either ξ_1 or dz. Discuss the results for different choices of the FFT parameters;
b. Quadrature;
c. MonteCarlo.
d. Facultative: Consider a α=1/3 model with same parameters (same values for a different model). Compute C with FFT and quadrature. Comment the results: Do you observe significant differences with respect to the above case in terms of precision of results?

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Remark: structure the code with i) a script: runPricingFourier, ii) some functions that compute I (with the two methodologies: FFT & Quadrature) that call iii) an integrand function. It is suggested not to use the parameters in the excel spreadsheet but to deduct them within the code (in order to avoid rounding errors).
Hint: The parameters of matlab Gamma simulation should be reconciled with the ones in the assignment e.g. using the Laplace transform of the corresponding mixture r.v. Compute numerically the first four moments and verify with the analytical ones.

4. Case study: Volatility surface calibration

Calibrate a normal mean-variance mixture with α=2/3 model parameters considering SP 500 volatility surface via a global calibration with constant weights. F_0, discount and ttm are the values in the Exercise 2.
Plot implied volatility obtained with the model and compare it with market's one.

Exercise 1, Annex: Indicative Terms and Conditions as of 31st of January 2023

Swap Termsheet

Principal Amount (N):	100 MIO EUR
Party A:	Bank XX
Party B:	I.B.
Trade date:	today
Start Date:	2th of February 2023
Maturity Date (t):	4 years after the Start Date, subject to the Following Business Day Convention.

Party A pays:	Euribor 3m + Spol
Party A payment dates:	Quarterly, subject to Following Business Convention
Day-count:	Act/360
Party A pays @ Maturity Date:	$(1-P)$ of the Principal Amount
Protection (P):	90%

Party B pays @ Start Date:	3% of the Principal Amount
Party B pays @ Maturity Date:	Coupon
Participation coefficient:	α=110%
Coupon:	Pays at expiry date the participation coefficient of the performance, if positive, of an equally weighted basket of ENEL S.p.A. and AXA S.A. $\alpha(S(t)-P)^+$
Basket:	$S(t)=\sum_{s=1}^{4}\sum_{n=1}^{2} W_n \frac{E_s^n}{E_{s-1}^n}$
Monitoring dates:	yearly, subject to the Following Business Day Convention from the Start Date.
Initial monitoring date (s=0)	2th of February 2023

and:

E_s^n: Value of the n^{th} element of the basket (the stock price) at monitoring date s;

W_n : Weight of the n^{th} element of the basket, each one equal to one half.

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