POLITECNICO
MILANO 1863

Scuola di Ingegneria Industriale e dell'Informazione
Corso di Laurea Magistrale in Mathematical Engineering - Quantitative Finance

Financial Engineering: Group 4 Assignment 4 Risk Management

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Index

1 Portfolio Mark-to-Market

The portfolio's current value is determined by discounting the future cash flow of 100 zero-coupon bonds, each paying €1 million at maturity. The valuation method discounts the cash flow (single payment at maturity) by applying the appropriate discount factor. In addition, credit risk is integrated by adjusting the cash flow based on the probability of default derived from the historical transition matrix. Finally, the Mark-to-Market value of a single bond is multiplied by the total number of bonds, yielding the portfolio's overall market value.

Portfolio MtM
91,3 mln

2 1y Forward Price - Investment Grade Scenario

We focus on estimating the 1-year forward price of a zero-coupon bond, under the assumption that the issuer maintains its Investment Grade rating. Essentially, the forward price is computed as a combination of two outcomes: the issuer survives as Investment Grade (thus receiving the full forward factor for the period) or it defaults (in which case the cash flow is reduced by the recovery rate).

1y Forward Price
958 K

3 1y Forward Price - High Yield Scenario

A similar approach is then used to derive the forward price assuming the issuer downgrades to High Yield. The procedure uses the default probability specific to a High Yield issuer, extracted from the transition matrix, to weight the forward cash flow. The resulting forward price is lower than that of the Investment Grade scenario, thereby quantifying the additional risk premium associated with a High Yield rating.

1y Forward Price
941 K

4 Monte Carlo Simulation

A Monte Carlo simulation is performed with one million scenarios; for the firms value we consider a Vasicek model with correlation equal to 15%. In each simulated scenario, random variables v_i, generated as a combination of a common factor and idiosyncratic risk, are used to determine whether an issuer defaults or is downgraded.

→ **Losses** are calculated by multiplying the number of default events by the loss given default, with additional adjustments when migration (downgrades) is considered.

→ **Threshold values for default** are computed using the inverse cumulative distribution function based on the default probability from the transition matrix. For each issuer in every scenario, if the variable v_i falls below the default threshold, that issuer is marked as having defaulted.

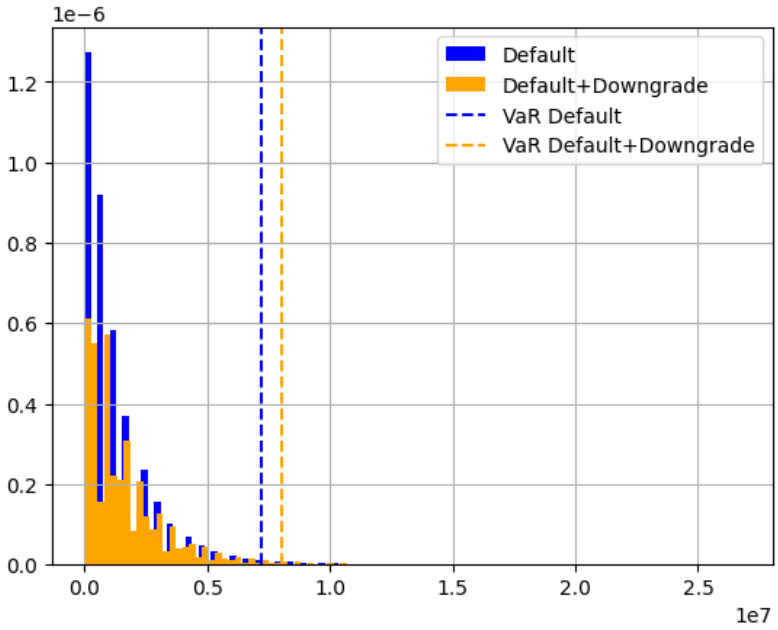
→ Similarly, a **downgrade threshold** is established by considering the cumulative probability of downgrade events. Issuers whose v_i falls below this second threshold, but not below the default threshold, are flagged as downgraded.

→ Finally, the **99% VaR** is determined by taking the 99^{th} percentile of the aggregated loss distribution across all scenarios.

Average defaults	**Average downgrades**
2%	25%

$\text{VaR}_{0.99}$ (Defaults Only)	**$\text{VaR}_{0.99}$ (Defaults & Migrations)**
7,2 mln	8 mln

In the figure below we observe the loss distributions in case of only defaults and defaults & migrations, notice that in the latter the density is more uniform, leading to a right shift of the quantile.



5 Monte Carlo Simulation under alternative correlation scenarios

The Monte Carlo simulation is repeated under alternative correlation settings, 0% and 30%. When the correlation is reduced to 0%, indicating that issuer outcomes are independent, the overall risk is substantially lower. On the other hand, increasing the correlation to 30% concentrates the risk, as adverse outcomes are more likely to occur simultaneously, leading to a higher VaR.

VaR with $\rho = 0\%$	**VaR with $\rho = 30\%$**
4,01 mln	12,3 mln

Table 1: VaRs take into account defualts and migrations

6 Impact of Migration versus Default Risk and the role of Correlation

The differences in the resulting VaR illustrate the significant impact that correlation has on the overall risk level of the portfolio. When the correlation among issuers is low, individual outcomes are largely independent, so adverse events tend not to occur at the same time. This diversification reduces the overall risk, resulting in a lower VaR. In contrast, a higher correlation means that if one issuer experiences

a loss, others are more likely to do so simultaneously. This clustering of negative outcomes amplifies the total loss, leading to a higher VaR. Notably, the baseline case (15% correlation) lies between these two extremes.

$\rho = 0\%$	$\rho = 15\%$	$\rho = 30\%$
4,01 mln	8 mln	12,3 mln

Table 2: VaRs take into account defaults and migrations

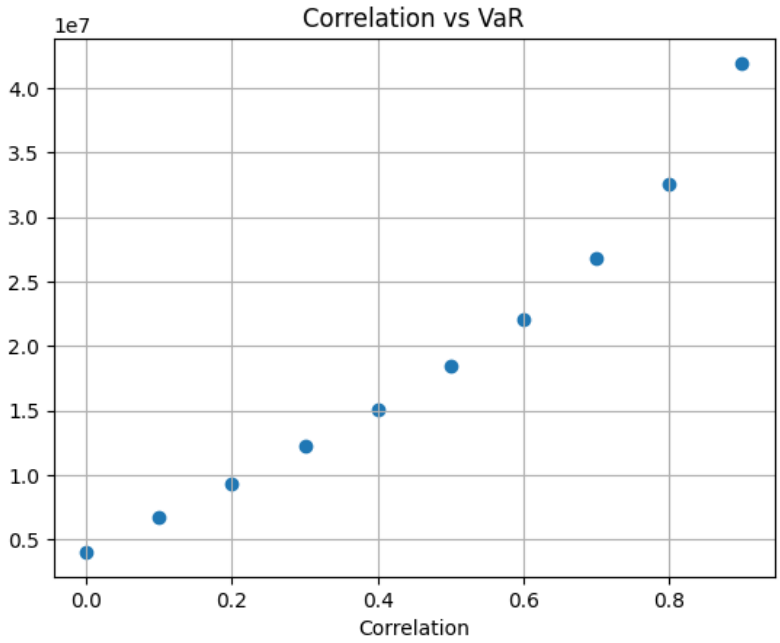


Figure 1: VaR vs correlation with ρ in [0%, 100%]

Additionally, the inclusion of migration (downgrade) risk markedly impacts the VaR. The numerical results show that incorporating migration risk increases the VaR. This significant rise underlines that migration risk also plays an essential role, as downgrades affect the bond's value even if a full default does not occur.

Defaults only	**Defaults and migrations**
7,2 mln	8 mln

Table 3: VaR calculations are based on a 15% correlation

7 Modification for Reduced Recovery in Downgrade Events

We analyze the impact of a change in recovery assumptions, specifically a reduction in recovery rate from 40% to 10% in downgrade events. The framework is modified

by adjusting the loss given default accordingly, so that the expected loss in the case of a downgrade becomes significantly larger. Moreover, we computed the new 1y forward price in the HY scenario, re-evaluating the loss given downgrade. The numerical results indicate that a lower recovery rate in downgrade scenarios leads to substantially higher losses.

$\mathbf{VaR}_{0.99}$ **(Defaults & Migrations)**
8,7 mln

Table 4: VaR calculations are based on a 15% correlation

Appendix: Errors in the code

- `ex4_notebook`: We replaced:

  ```
  recovery_rate = 0.3
  ```

  with:

  ```
  recovery_rate = 0.4
  ```

- `ex4_notebook`: We replaced:

  ```
  issuers_num = 200
  ```

  with:

  ```
  issuers_num = 100
  ```