POLITECNICO
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Scuola di Ingegneria Industriale e dell'Informazione
Corso di Laurea Magistrale in Mathematical Engineering - Quantitative Finance

Financial Engineering: Group 4 Assignment 1 Risk Management

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1 MtM the portfolio

Knowing that the NPV of an IRS at par is equal to zero, we just computed the price of the swaption.

Forward Swap Rate	**Swaption Price**
2.97979%	€57˙062˙717,42 (8.15%)

Table 1: The percentage is with respect to the Swaption Notional

2 DV01 up parallel shift

Since we considered an up parallel shift of the rates of 1 bp, as we expected, we obtained:

- **Swaption Receiver**: negative DV01, due to the fact that in this contract the fixed leg is received and the floating one is paid and an increase in the rates will lead to higher floating leg payments for the holder.

- **Swap Payer**: positive DV01, because in this case the fixed leg is paid and the floating is received.

DV01-Swaption	**DV01-Swap**	**DV01-Portfolio**
-97˙265,47	514˙969,19	417˙703,71

3 DV01 via analytical approximation

- **Swaption Receiver**: we used the Swaption Delta (**-0.3548**), but this is not correct in practice, since if we move the underlying swap we will get different discount factors, so it does not make any sense to consider all constant except for the forward swap rate. Indeed, our numerical results do not match those presented above.

- **Swap Payer**: in this situation we considered the Macalauy Duration (**8.84**), that is a good approximation of the $DV01^{(z)}$, but provides a slightly different result with respect to the DV01.

DV01-Swaption	**DV01-Swap**	**DV01-Portfolio**
-24˙924,50	530˙630,66	505˙706,16

4 Delta Hedge 10y IRS

In order to make our position Delta-neutral with a 10y IRS we considered the DV01 of the Swaption and the DV01 of the IRS weighted for their Notionals and we imposed the total DV01 equal to zero. It is important to underline the fact that the result is not exactly zero since, by assumption, the IRS is negotiated between institutional investors in multiples of €$1Mln$. The same computations with the approximated DV01 lead to an overestimation of the sensitivity of the Interest Rate Swap.

New IRS Notional	DV01-Portfolio
€113˙000˙000,00	€-280
€28˙000˙000	€-73˙234

Table 2: The first line refer to the exact DV01, while the second to the approximated one

5 Coarse Grained Bucket DV01 (10y, 15y)

- **Bucket 10y**: we summed 1 bp to all the rates up to the tenth year, then we used linear decreasing weights up to the fifteenth year and finally we did not change the remaining ones.
- **Bucket 15y**: all the rates up to the tenth year were not changed, while we applied linear increasing weights up to the fifteenth year and finally we summed 1 bp to the last swap rates.

Bucket 10y	Bucket 15y	DV01-Bucket
559˙114,34	-140˙977,46	418˙136,88

In accordance with the theory, the DV01-Bucket, obtained as the sum of the two Buckets, is similar to DV01 parallel shift (section 2).

6 Delta Hedge 10y IRS and 15y IRS

We opted to hedge our portfolio using a 10y IRS Payer and a 15y IRS Payer. By short selling 600˙000˙000 of the 10y IRS Payer, we neutralized our long position in that instrument. This left us with a Swaption receiver exposure, which we hedged using the 15y IRS Payer, resulting in the outcome shown below.

Notional IRS 15y
€81˙000˙000

Then we evaluated the portfolio DV01-Bucket as in the previous section.

Bucket 10y	**Bucket 15y**	**DV01-Bucket**
44˙145,15	-43˙457,90	687,26

Since we built an hedged portfolio, the DV01-Bucket is approximately zero.

7 Profit and Loss

We considered a steepening scenario, in which the IRS 10y goes down of 1 bp, while the IRS 15y goes up of 1 bp.

- **Portfolio section 4**: we can observe that the Swaption Price depends only on the IRS 15y, indeed we notice a decrease in its value, while the NPV of a IRS 10y Payer becomes negative. In conclusion our portfolio is interested in a Loss of money.

Profit and Loss
€-694˙660,77

- **Portfolio section 6**: for the Swaption price, the same considerations as mentioned above still apply. The IRS 15y Payer is affected in the opposite direction by the steepening scenario; however, because the impact of a 1 bp decrease on the IRS 10y is more significant than the corresponding increase on the IRS 15y, its NPV remains negative. In summary, our portfolio continues to register a loss.

Profit and Loss
€-81˙841,63

As we expected the second Loss is lower than the first one, so our second hedging strategy is better.

More accurate hedging strategies using Bucket-DV01s

The use of Bucket-DV01s enables a more precise hedge by covering risk across multiple maturities. This can be achieved by using multiple IRS contracts to match exposures in different buckets or by optimizing hedge composition through regression techniques to minimize residual risk.

Pros: This approach improves risk coverage across different buckets and reduces exposure to yield curve shifts.

Cons: However, it increases management complexity and monitoring efforts while also leading to higher transaction costs.

The choice of hedging strategy depends on portfolio structure. If exposure spans multiple maturity buckets, a Bucket-DV01-based approach provides a more tailored and effective hedge. Conversely, for simpler risk profiles, a single IRS may be sufficient, offering a cost-effective and easier-to-manage solution.

Appendix: Errors in the code

Throughout the provided code, in particular in the `utilities_ex1.py` file, we spotted several mistakes, which we now briefly recall.

First, in the `swap_par_rate()` function, which computes either the spot or the forward swap rate, the original code, reported below, divided the difference between the discounts by the `discount_factor_tN` instead of the basis point value.

```
return (discount_factor_t0 - discount_factor_tN) / discount_factor_tN
```

Later on, we found an error in the `swap_mtm`, which *marks to market* the value of an Interest Rates Swap. If the swap is a payer, like the ones we are dealing with, the Net Present Value (NPV) should be the difference between the floating leg, which the holder of the contract receives, and the fixed leg, which the holder pays. The erroneous code was indeed

```
if swap_type == SwapType.RECEIVER:
    multiplier = 1
elif swap_type == SwapType.PAYER:
    multiplier = -1
else:
    raise ValueError("Unknown swap type.")
```

The last error we encountered was instead inside of the `ex1_notebook.py`, related to the exact computation of the $DV01$. To compute the portfolio sensitivity to a $1bp$ parallel shift of the interest rate curve, we had to re-price our instruments: the Swaption and the Interest Rate Swap. However, the Swaption has to be priced according to the new forward swap rate, affected by the $1bp$ bump, not the original one as wrongly suggest by the code.

```
    swaption_price_up = swaption_price_calculator(
    fwd_swap_rate, strike, today, swaption_expiry,
    underlying_expiry, sigma_black, swaption_fixed_leg_freq,
    discount_factors_up, swaption_type,
)
```

References

[1] P. J. Schonbucher, *Credit Derivatives Pricing Models*, John Wiley and Sons, New York, 2003.

[2] John C. Hull, *Options, Futures, and Other Derivatives*, Prentice Hall, 7th edition, 2009

[3] Uri Ron, *A Practical Guide to Swap Curve Construction*, Bank of Canada, 2000