POLITECNICO
MILANO 1863

Scuola di Ingegneria Industriale e dell'Informazione
Corso di Laurea Magistrale in Mathematical Engineering - Quantitative Finance

Financial Engineering: Group 4 Assignment 2 Risk Management

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1 Constant Intensity Analysis

The model estimates the average default intensities for two bonds with different maturities by matching the theoretical dirty prices to the observed market prices. The resulting intensities are nearly identical, this similarity is expected because both bonds are issued by the same entity, *Beta*. Since we model the intensity as constant and it reflects the issuer's intrinsic credit risk, assuming that Beta's credit quality remains constant over time, the intensity should not depend on the bond's maturity.

Intensity over 1y	**Intensity over 2y**
2.43896%	2.43377%

2 Default Probability Estimation

Using the calibrated intensities, survival probabilities are calculated based on the exponential decay relationship. The analysis confirms that, as maturity increases, the overall default probability also increases, reflecting the natural accumulation of risk over time. For the computation, we take the average of the intensities above, since in this section the intensity is supposed constant over time ans so independent from the maturity of the two issued bonds.

1y default probability	**2y default probability**
2.40693%	4.76863%

3 Z-Spread Computation

In this section, we calibrate the Z-spread for each bond by solving the equation that sets the theoretical price equal to the observed dirty price. The Z-spreads are nearly identical for both maturities and are lower than the default intensities, this outcome is consistent with theoretical expectations, as the Z-spread reflects not only the credit risk but also adjustments for recovery. Indeed, from a mathematical point of view, we observe:

$$\begin{gathered} \hat{B}(t_0, t_n) > \bar{B}(t_0, t_n) \\ \exp\{-z(t_n - t_0)\} > \exp\{-\lambda(t_n - t_0)\} \\ z < \lambda \end{gathered}$$

With a recovery rate of 30%, we expect the Z-spread to be around 70% of λ (i.e., about $0.70 \times 2.43\% \approx 1.70\%$), which aligns very well with the obtained values.

Z-spread over 1y	**Z-spread over 2y**
1.7214%	1.7273%

In practice it is preferred the use of Z-spread instead of intensity because we do not assume anything about the distribution of the survival probabilities.

4 Piecewise Intensity Estimation

We modeled the intensity as a piecewise constant function over time. We recalibrated these values to match the observed prices of the bonds, then computed survival probabilities by integrating over the respective time intervals. This method typically provides a more refined approximation of the issuer's creditworthiness, as it captures potential changes in risk over time. The model returns nearly identical values for the intensities; and this suggests that the credit risk of the issuer remains stable over the two-year horizon.

Intensity over 1y	**Intensity over 2y**
2.43896%	2.42823%

1y default probability	**2y default probability**
2.40946%	4.76332%

5 Historical Default Analysis

In this section, we derive the historical default probabilities for Beta using a rating transition matrix. To estimate the two-year default probability, we compute the two-year transition matrix by squaring the one-year matrix (i.e. P^2). These historical probabilities serve as a benchmark. While the market-implied probabilities (derived from intensities and Z-spreads) capture forward-looking risk, the transition matrix reflects observed migration frequencies. Any discrepancies between the two approaches may indicate shifts in market sentiment compared to historical trends; in our case study we observe similar values with respect to the ones above.

1y real world default probability	**2y real world default probability**
2.00%	4.71%

6 Shock Scenario on Mid-Term Survival

In this scenario, the dirty price of the 2y bond drops significantly and the 1y bond price remains unchanged. We recalibrated the default intensities using the new market price for the 2y bond and then recomputed the survival and default probabilities accordingly.

The recalibration process shows that the 1y default probability remains the same, since the 1y bond's price has not been affected by the shock. However, for the 2y bond, the default probability increases substantially. This marked increase is consistent with expectations, as a lower bond price indicates a higher risk premium for longer maturities. Consequently, the heightened medium-term credit risk is reflected in the significant rise in the 2y default probability.

1y default probability	**2y default probability**
2.4095%	11.9259%

7 Shock Scenario on Overall Creditworthiness

For scenario 2, we assume that an acquisition by a high-quality company improves Beta's creditworthiness, reflected in higher dirty prices. New intensity values were computed using these revised market prices, and the corresponding default probabilities were computed. These results align with theoretical expectations: improved credit outlook should lead to lower intensities and hence lower default probabilities.

1y default probability	**2y default probability**
1.00%	3.43%

8 Comparison of Conditional Default Probabilities

We compute the analytical probability for Beta to default between the first and the second year, conditional to the fact that we do not have default in the first year. To this purpose we use the intensity of Section 6 in the 1y-2y time horizon, obtaining a default probability equal to 9.75%. For the historical one, to replicate scenario 1, we assume that after one year the rating class of Beta drop to the HY one (with probability 1). From the one step matrix we then infer that the desired probability is 5%, noticing that the historical data do not correctly replicate the downgrade of Beta creditworthness.

Appendix: Errors in the code

- `ex2_notebook`: the maturity of the bond (with 6% coupon and market dirty price 102 €) is 2 years

  ```
  maturity2 = 3
  ```

- `ex2_utilities`: in the function `bond_cash_flows`:
  - we replaced

    ```
    cash_flows_dates = cash_flows_dates.sort()
    ```

    with

    ```
    cash_flows_dates.sort()
    ```

  - we generated the cash flow dates as

    ```
    cash_flows_dates = date_series(ref_date, expiry, coupon_freq)[1:]
    ```

  - we replaced the coupon payments with

    ```
     cash_flows = pd.Series(
    data=notional * coupon_rate * yfs,
    index=cash_flows_dates,
     )
    ```