POLITECNICO
MILANO 1863

Scuola di Ingegneria Industriale e dell'Informazione
Corso di Laurea Magistrale in Mathematical Engineering - Quantitative Finance

Financial Engineering: Group 4 Assignment 3 Risk Management

Matteo Bovio: 272377
Alberto Busci: 278889
Matteo Campagnoli: 275975
Alice Sofia Casciani: 275720

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1 Case Study 1

1.1 Portfolio 1

The daily $VaR_{95\%}$ and $ES_{95\%}$ of the first portfolio have just been computed using the formula, assuming a gaussian parametric approach (returns distributed as normal random variables) over a 4y time horizon.

Daily VaR	**Daily ES**
€412˙698,59	€517˙873,58

1.2 Portfolio 2

In this case we firstly used an historical simulation approach over 3y, computing 767 simulations of losses, ordering them and then considering the useful ones.

Daily VaR (historical)	**Daily ES (historical)**
€317˙030.85	€469˙965.40

Then we used the statistical bootstrap method, randomly sampling (with replacement) 300 historical returns.

Daily VaR (bootstrap)	**Daily ES (bootstrap)**
€310˙549.79	€495˙486.10

As we can observe, the two methods provide similar results.

1.3 Portfolio 3

On this third portfolio we applied a weighted historical simulation approach, with λ equal to 0.95.

Daily VaR	**Daily ES**
€1˙211˙710.50	€2˙111˙448,45

To check the coherence between the weighted historical simulation and the non-weighted one, we took λ very close to 1 and confronted the obtained result with the one computed thanks to a non-weighted simulation on this third portfolio.

1.4 Portfolio 4

For this last portfolio, we performed a Principal Component Analysis. Since the first eigenvalues of a matrix are the most influential, we reduced the dimensionality of the covariance matrix.

We found that the results remained accurate even when considering only the first few eigenvalues and their corresponding eigenvectors, as they capture the majority of the total variance, as shown in the figure above.

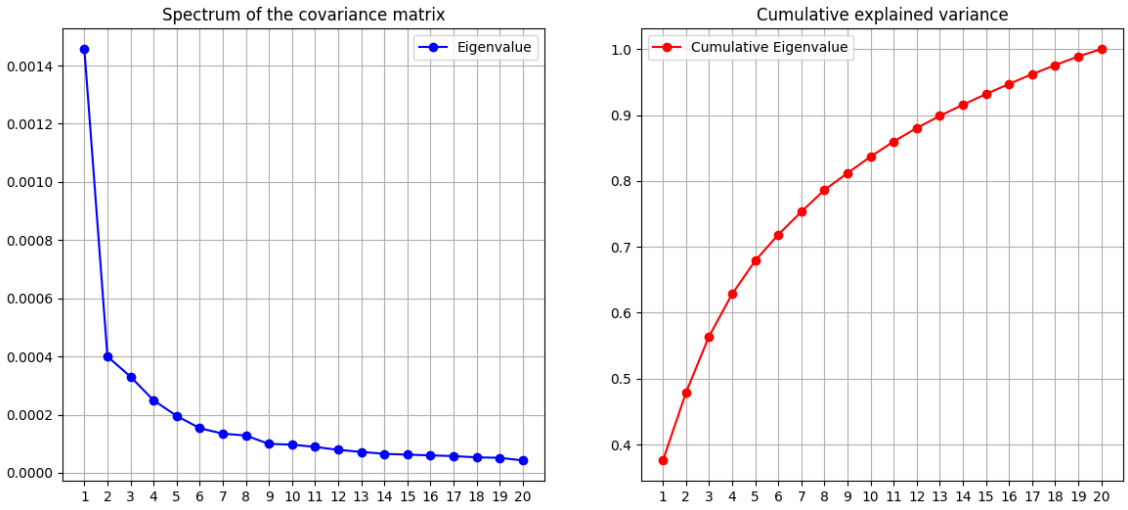

Figure 1: Left: Eigenvalues of the covariance matrix, after the first few they decay pretty quickly. Right: cumulative explained variance

10d VaR (2 eigenvalues)	**10d ES (2 eigenvalues)**
€1˙121˙445,33	€1˙390˙094,29

1.5 Plausibility check

We have finally performed a plausibility check of the VaR of the last three portfolios, obtaining the above results, which order of magnitude align with previous computations.

Portfolio 2	**Portfolio 3**	**Portfolio 4**
€325˙154.23	€490˙536,90	€1˙061˙700,77

Table 1: The rule of thumb for portfolio 3 is in accordance with the non-weighted simulation

2 Case Study 2: Monte Carlo vs Δ-Normal & Δ-Γ Approaches

For the second case study we had to consider as reference day the 31^{st} of January 2023 and a portfolio composed by stocks of Anheuser-Busch for an amount of €5.509.000 and the same number of put options with expiry 5^{th} April 2023, strike € 53 and implied volatility equal to 18.5%. First of all we computed the log-returns from the provided Excel file, the discount factor with the function written in the previous laboratories, the zero rate and the price of the put option with its sensitivities via Black & Scholes formula with the provided python function, obtaining **0.84024** (single put price). Subsequently we followed a Monte Carlo approach to simulate the stock prices and the put ones, sampling 10 log returns for 10^5 times (we then considered the sum of each sample to get the 10d risk factor). Finally,

we computed the losses and sorted them to obtain the $\text{VaR}_{99\%}^{10 \text{ days}}$ with a 2-years historical simulation for the underlying. We also computed the requested VaR with a Delta-Normal approach. The results are shown below:

Monte Carlo VaR	**Delta-Normal VaR**
€ 271˙465,82	€462˙820,46

From the table it is possible to observe that the second methodology does not provide a VaR in line with the one observed with the Monte Carlo simulation, in particular we are overestimating it; to improve the result we tried with the Delta-Gamma approach by adding the well known extra term in the computation of the loss. The result can be found below and it is quite similar to the first one.

Delta-Gamma VaR
€ 286˙883,13

3 Case Study 3: Pricing in presence of counterparty risk

In this section, we analyze a pricing methodology and a seller strategy for a 5-year Cliquet option transacted between Polimi Bank and ISP. The payoff structure of the Cliquet is made up of 5 contingent cash flows $(S_{t_{i-1}} - S_{t_i})^+$, to be paid every year. In this sense, the payoff is very similar to a forward starting put, with strike $S_{t_{i-1}}$. Let's derive an expression for the Net Present Value of such instrument:

$$\text{Cl} = \mathbb{E}\left[\sum_{i=1}^{5}(S_{t_{i-1}} - S_{t_i})^+ D(t_0, t_i)\right] = \sum_{i=1}^{5}\mathbb{E}\Big[(S_{t_{i-1}} - S_{t_i})^+\Big]\bar{B}(t_0, t_i)$$

As will be further discussed below, the discount factor in use *should* account for the default risk of our trading counterparty, effectively multiplying the original discount factor, $B(t_0, t_i)$, by the corresponding survival probability, $P(t_0, t_i)$. In the end, this means the contract should be worth less. It is in the ISP's interest to be perceived as secure as possible, often leveraging the *asymmetric information* available to other market players.

We priced the contract through a Monte Carlo method, where the stock price $S(t)$ is assumed to follow a Geometric Brownian Motion with a constant volatility. We generated 10^5 simulated paths for the underlying asset over the 5-year period. For each simulated path, we computed the annual payoff and the option's coupon was determined as the mean of these payoffs. Note that the coupon is not fixed, instead, there are five different coupons, one for each year, because the Cliquet option resets annually. Each coupon reflects the underlying asset's performance during that specific period.

Our simulations give a theoretical fair price of Cl = 1.0364 euros; which fits in the following $\alpha = 95\%$ confidence interval

$$CI_\alpha = \mu \pm z_{1-\frac{\alpha}{2}}\sigma = \left[1.0330, 1.0397\right]$$

What should be the correct price?
The theoretical fair price of the option is estimated by summing, for each year, the present value of the annual coupon payment, discounted by the appropriate discount factor, and adjusted for the probability that the counterparty, ISP, survives to the next period.

At what price ISP would try to sell it?
ISP assumes a survival probability of 1, effectively eliminating any credit risk from the pricing model. This means that future payoffs are not reduced to account for the chance of default, leading to a higher computed price for the option. This approach makes sense for several reasons: if ISP is confident in its own financial stability, assuming a survival probability of 1 highlights that confidence and also make ISP may appear more attractive to potential buyers.
The numerical results are presented in the table below:

Theoretical Fair Price	**ISP Selling Price**
€51˙818˙274,35	€53˙084˙751,31

Table 2: The prices refer to a notional amount of € 50mln

As expected, ISP selling price is higher than the other: on the total value of the trade, $50Mln$, this would increase ISP revenue by more than a million; almost a 2.5% increase.

Appendix: Errors in the code

- `ex3_notebook`: the simulation number is 300 and not 200

  ```
  simulations_num = 200
  ```

- `ex3_utilities`: in the function `black_scholes_option_pricer`:
  - We replaced:

    ```
    -np.exp(d * ttm) * norm.pdf(-d1) * (-1 / (S * sigma
    ↪  * ttm ** (0.5)))
    ```

    with:

    ```
    np.exp(-d * ttm) * norm.pdf(d1) / (S * sigma * ttm
    ↪  ** (0.5))
    ```

  - We replaced the given d_1:

    ```
    d1 = (np.log(S / K) + (r + d + sigma**2 / 2) * ttm)
    ↪  / (sigma * np.sqrt(ttm))
    ```

    with:

```
d1 = (np.log(S / K) + (r - d + sigma**2 / 2) * ttm)
↪  / (sigma * np.sqrt(ttm))
```

– Finally we replaced:

```
d2 = d1 + sigma * np.sqrt(ttm)
```

with:

```
d2 = d1 - sigma * np.sqrt(ttm)
```