POLITECNICO
MILANO 1863

Scuola di Ingegneria Industriale e dell'Informazione
Corso di Laurea Magistrale in Mathematical Engineering - Quantitative Finance

Financial Engineering: Group 4
Assignment 2

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1 Bootstrap

Considering the data provided, we boostrapped the *Discount Factor Curve*. First of all, we looked at deposits rates, up to the fourth expiry $03 - Apr - 2023$; we did not consider the last two deposits since futures are more liquid instruments and the first settlement date of the futures was $15 - Mar - 2023$, to obtain which we interpolated thanks to the fourth depos. In this first part we simply computed the discount factor B(t0, ti) inverting the Euribor rate.

Afterwards, we had to take into consideration the Eurodollar futures. We limited our analysis to the first seven contracts, as, *by market practice*, are considered to be the most liquid. When needed, such as for the first future contract, we interpolated the zero rates on suitable dates, and later on we computed the actual discount factor at expiry. Moreover to find the discount at the settlement date of the second future $21 - Jun - 2023$ we used a 6 day linear extrapolation.

Lastly, we looked at Interest Rates Swaps. As prescribed by the Assignment, we generated a *complete set* of swap rates, one for each year from 2024 to 2073, using a spline interpolation. To start our computation, we had to infer the discount factor at 1 year through interpolation, while the latter were calculated with a the closed recursive formula.

The results of our bootstrap is found in the figure below.

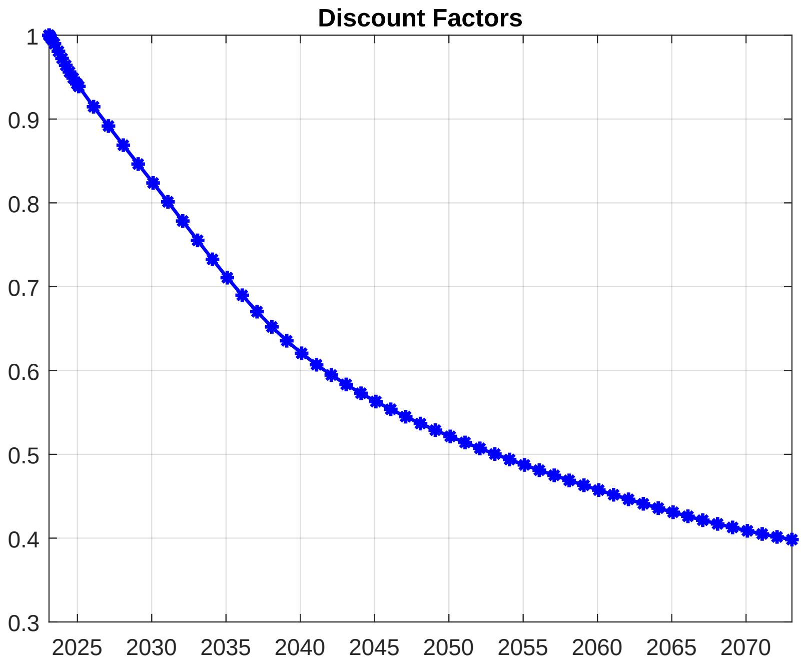

Figure 1: The discount factors decreases in time, according to the theory. The first few expiries of the curve are bootstrapped from a higher number of instruments.

Q: Compared to other approaches, Bootstrap, ensuring arbitrage free discount factors, is widely used in finance as it helps build a complete discount curve from available market rates, even when maturities are limited. This method maintains consistency across financial instruments like swaps, bonds, and FRAs, making it essential for accurate pricing. Additionally, it adapts to changing market conditions, supporting effective risk management and hedging. Its flexibility and reliability make it a standard approach in yield curve construction.

Moreover, we report the zero rate curve, obtained directly from the discount factors thanks to their well-known relation.

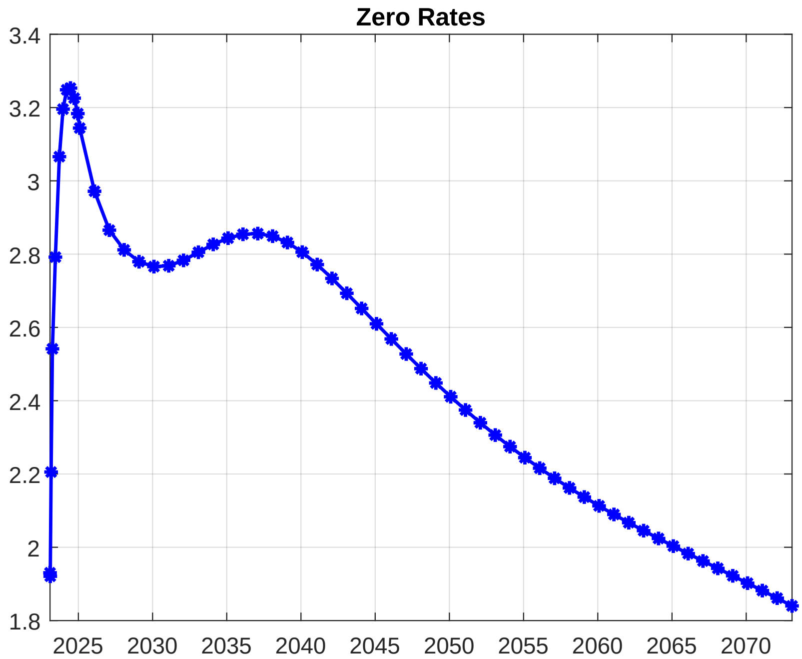

Figure 2: The values of the Zero Rates are expressed in percentage

2 Pricing a IB Coupon Bond *(at par)*

We were asked to price an InterBank Coupon Bond, issued on the $31st$ of January 2023, with coupon equal to the *mid-market* 7 years swap rate. Before diving into the numerical implementation, we observed that the coupon structure is equivalent to the fixed leg of the corresponding 7 years Interest Rate Swap. Let $P(t, T)$ be the price of the coupon bond and S be the swap rate, so that

$$P(t_0, t) = B(t_0, t_N) + \sum_{i=1}^{N} \delta(t_{i-1}, t_i) S B(t_0, t_i)$$

However, the Interest Rate Swap has null *Net Present Value (NPV)*. Hence, the NPV of the fixed leg is equal to the floating one. The latter can be expressed as $NPV_{float} = 1 - B(t_0, t_N)$. Putting everything together

$$P(t_0, t) = B(t_0, t_N) + 1 - B(t_0, t_N) = 1$$

If the Face Value of the bond is $100Mln$, then the bond will also be priced at this amount. This is in total accordance with the numerical results.

3 Sensitivities

We studied the main risk factors in the context of interest rates derivatives, for a portfolio composed by a single 7 year plain vanilla swap, vs 3 months Euribor. The swap rate was fixed at $S = 2.8175\%$, with a Notional of $100Mln$.

Computing the $DV01$ involves shifting the interest rate curve by 1bp. We calculated the $DV01$ as the difference between the NPV of the shifted curve and the original one; which is zero, since the swap is traded at par.

Arguing as before, we shifted the zero coupon rate curve of 1bp to find the $DV01^{(z)}$.

The last sensitivity of the swap we considered was the Basis Point Value (BPV). Results of our numerical calculations (considering a Notional of $100Mln$) are shown below.

DV01	**DV01$^{(z)}$**	**BPV**
6.2546e4	6.4604e4	6.2569e4

Being traded at par, the $DV01$ and the BPV for the plain vanilla swap are quite similar. However, it's important to recall that the former is the actual sensitivity, while the latter is a simpler quantity to compute.

Considering a portfolio containing the InterBank coupon bond described in the previous section, we analyzed one of its main sensitivities indicator, the *Macaulay Duration* $MacD$. Leveraging on the previously computed quantities, we found the duration to be equal to $MacD = 6.457$ years. Notice that, up to a scaling factor (due to the basis point), this is equivalent to the $DV01^{(z)}$ of the corresponding swap.

4 Monthly Cash Flows NPV

We were given two different initial monthly cash flows (1500 and 6000 euros), with and AAGR equal to 5%, starting from 2026 to 2046. We discounted (thanks to our bootstrap) the 240 $(12 \cdot 20)$ cash flows, increasing the monthly value by 5% each year, with respect to the previous year. We obtained the two NPVs below.

1.5K	**6K**
397˙158	1˙588˙631

References

[1] John C. Hull, *Options, Futures, and Other Derivatives*, Prentice Hall, 7th edition, 2009

[2] Uri Ron, *A Practical Guide to Swap Curve Construction*, Bank of Canada, 2000