Report: Homework 3 Math/CS 471

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Abstract

This report will explore two methods of approximating the following integral

$$I = \int_{-1}^{1} e^{cos(kx)} dx,$$

for $k = pi$ or pi^2. The first method is known as the trapezoidal rule and the second as Gauss quadrature.

1 Trapezoidal Rule

The trapezoidal rule is given by the following expression

$$\int_{X_L}^{X_R} f(x)dx \approx h\left(\frac{f(x_0) + f(x_n)}{2} + \sum_{i=1}^{n-1} f(x_i)\right)$$

where the grid is given by $x_i = X_L + ih$, $i = 0, ..., n$, $h = \frac{X_R - X_L}{n}$.

2 Gauss Quadrature

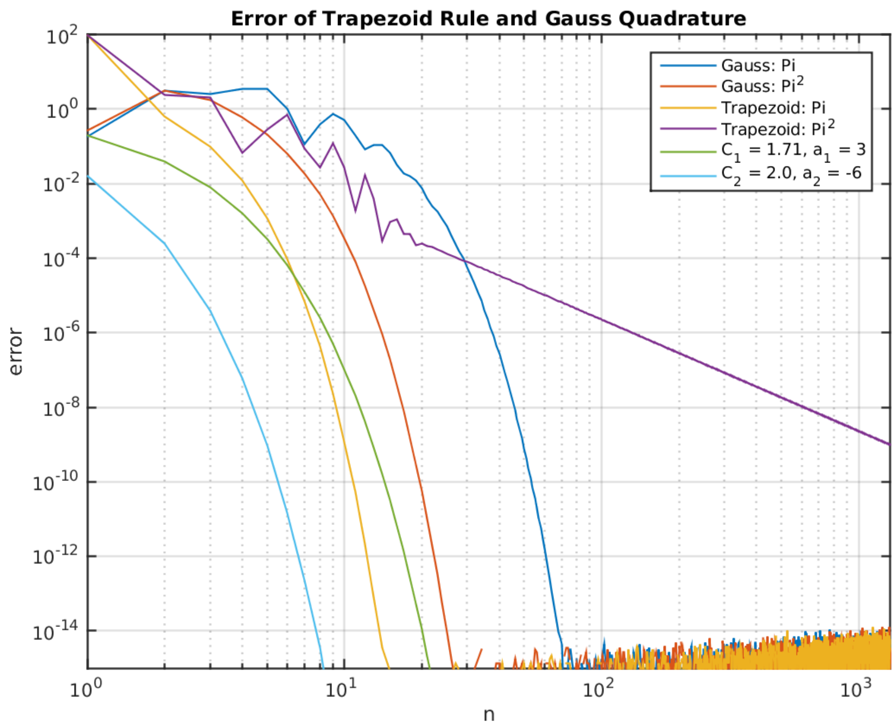
From the definition offered in [1] Gauss quadrature involves the location of the grid-points and weights, ω_i, are chosen so that the order of the approximation to the weighted integral

$$\int_{-1}^{1} f(z)w(z)dz \approx \sum_{i=0}^{n} \omega_i f(z_i),$$

is maximized. (The function $w(z)$ is positive and integrable. In this report we will only consider the case when $w(z) = 1$ in order to simplify things.

3 Results

Figure 1: Error plot for trapezoidal and Gauss estimations



In the figure shown above, different rates of convergence are observed for each of the methods and for each of the values of k. The trapezoidal method where $k = \pi^2$ is the only case in which the order of the method may be read from the slope of its plot. This slope is ≈ -3 which is consistent with the theory as shown in [2]

$$error = -\frac{(b-a)^2}{12N^2}\left[f\prime(b) - f\prime(a)\right] + O(N^{-3})$$

The error plot above (Fig.1) shows that the fastest rate of convergence occurred with the trapezoid approximation when $k = \pi$, followed by the Gauss quadrature approximations ($k = \pi^2$ converging faster than $k = \pi$), and then the slowest rate of convergence was demonstrated by the trapezoid approximation with $k = \pi^2$. This observation is interesting because the trapezoid

approximation with $k = \pi$ converges faster than expected.
For Gauss quadrature the error was expected to decrease as $e(n) \approx C^{-an}$. We tried $C_1 = 1.71$ and $C_2 = 2$ with respective $a_1 = 3$ and $a_2 = -6$. The second plot appeared to resemble the Gauss approximation.

4 Appendix

To run the program, simply navigate to Homework3/Code and enter the command *"perl approx_error.p"*. This may take a minute to complete, but when it concludes a png file named *"ErrorPlot.png"* will be created with the plots of the errors for both the Trapezoidal Rule and Gauss Quadrature for both values of k (π and π^2) and the file is saved in the Report directory.

References

[1] Daniel Appelo *Homework 3*. referenced Sep. 26, 2015

[2] Trapezoidal rule. September 17, 2015. Retrieved from
https://en.wikipedia.org/wiki/Trapezoidal_rule#Error_analysis