

# Stochastic Block Model Prior with Ordering Constraints for Gaussian Graphical Models

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sim_id	n	p	data_gen	seed	part0	B_sigma	rho_true	accept	rho_est	VI	RI	KL	time
01	500	25	BD	22111996	25	0.2	8, 4, 8, 5	0.019	8, 4, 8, 5	1	1	0.1872	1.68
02	500	25	BD	31051999	25	0.2	8, 4, 8, 5	0.019	8, 4, 8, 5	1	1	0.1872	1.62
03	500	25	BD	27051999	25	0.2	8, 4, 8, 5	0.019	8, 4, 8, 5	1	1	0.1872	1.69
04	500	25	BD	29061999	25	0.2	8, 4, 8, 5	0.019	8, 4, 8, 5	1	1	0.1872	1.73
05	500	25	BD	12091997	25	0.2	8, 4, 8, 5	0.019	8, 4, 8, 5	1	1	0.1872	1.68
06	500	25	BD	27091999	25	0.2	8, 4, 8, 5	0.019	8, 4, 8, 5	1	1	0.1872	1.68
07	500	25	BD	27121996	25	0.2	8, 4, 8, 5	0.019	8, 4, 8, 5	1	1	0.1872	1.62
08	500	25	BD	22111996	25	0.2	8, 4, 8, 5	0.019	8, 4, 8, 5	1	1	0.1872	1.68
09	500	25	BD	27121996	25	0.062	8, 4, 8, 5	0.0677	8, 4, 8, 5	1	1	0.1904	1.82
10	500	25	BD	27121996	25	0.078	8, 4, 8, 5	0.0835	8, 4, 8, 5	1	1	0.1878	1.82
11	500	25	BD	27121996	25	0.093	8, 4, 8, 5	0.0751	8, 4, 8, 5	1	1	0.1883	1.81
12	500	25	BD	27121996	25	0.108	8, 4, 8, 5	0.08	8, 4, 8, 5	1	1	0.1895	1.81
13	500	25	BD	27121996	25	0.124	8, 4, 8, 5	0.079	8, 4, 8, 5	1	1	0.1862	1.81
14	500	25	BD	27121996	25	0.139	8, 4, 8, 5	0.06	8, 4, 8, 5	1	1	0.1881	1.8
15	500	25	BD	27121996	25	0.154	8, 4, 8, 5	0.0495	8, 4, 8, 5	1	1	0.1941	1.8
16	500	25	BD	27121996	25	0.169	8, 4, 8, 5	0.0467	8, 4, 8, 5	1	1	0.191	1.85
17	500	25	BD	27121996	25	0.185	8, 4, 8, 5	0.0421	8, 4, 8, 5	1	1	0.1894	1.56
18	500	25	BD	27121996	25	0.2	8, 4, 8, 5	0.019	8, 4, 8, 5	1	1	0.1872	1.56
19	500	25	BD	27121996	25	0.2	8, 4, 8, 5	0.019	8, 4, 8, 5	1	1	0.1872	1.56
20	500	25	BD	27121996	singletons	0.2	8, 4, 8, 5	0.019	8, 4, 8, 5	1	1	0.1872	1.55
21	500	25	BD	27121996	25	0.2	8, 4, 8, 5	0.019	8, 4, 8, 5	1	1	0.1872	1.56
22	500	25	BD	27121996	25	0.2	1, 10, 2, 9, 3	0.072	1, 10, 2, 9, 3	1	1	0.1813	1.65
23	500	25	BD	27121996	25	0.02	1, 3, 2, 4, 2, 3, 3, 4, 3	0.397	18, 7	1	0.1286	0.1263	1.5
24	500	25	BD	27121996	25	0.2	12, 13	0.1293	12, 13	1	1	0.2554	1.83
25	500	25	BD	27121996	25	0.1	8, 4, 8, 5	0.082	8, 4, 8, 5	1	1	0.1869	1.51
26	400	25	BD	27121996	25	0.1	8, 4, 8, 5	0.0722	8, 4, 8, 5	1	1	0.2214	1.5
27	300	25	BD	27121996	25	0.1	8, 4, 8, 5	0.0698	8, 4, 8, 5	1	1	0.336	1.49
28	200	25	BD	27121996	25	0.1	8, 4, 8, 5	0.0735	8, 4, 8, 5	1	1	0.5322	1.49
29	100	25	BD	27121996	25	0.1	8, 4, 8, 5	0.1111	8, 4, 8, 5	1	1	1.4868	1.46
30	50	25	BD	27121996	25	0.1	8, 4, 8, 5	0.2365	20, 5	1	0.2727	4.2121	1.43
31	20	25	BD	27121996	25	0.1	8, 4, 8, 5	0.2799	25	1	0	11.8537	1.38
32	500	5	BD	27121996	5	0.062	3, 2	0.543	3, 2	1	1	0.0064	1.16
33	500	10	BD	27121996	10	0.062	5, 5	0.1885	5, 5	1	1	0.0284	1.16
34	500	15	BD	27121996	15	0.062	5, 5, 5	0.1295	5, 5, 5	1	1	0.0454	1.24
35	500	20	BD	27121996	20	0.062	5, 5, 5, 5	0.1027	5, 5, 5, 5	1	1	0.1176	1.35
36	500	25	BD	27121996	25	0.062	5, 5, 5, 5, 5	0.1038	5, 5, 5, 5, 5	1	1	0.1699	1.49
37	500	30	BD	27121996	30	0.062	5, 5, 5, 5, 5, 5	0.0795	5, 5, 5, 5, 5, 5	1	1	0.1398	1.68
38	500	35	BD	27121996	35	0.062	5, 5, 5, 5, 5, 5, 5	0.0664	5, 5, 5, 5, 5, 5, 5	1	1	0.2045	1.98
39	500	40	BD	27121996	40	0.062	5, 5, 5, 5, 5, 5, 5, 5	0.0611	5, 5, 5, 5, 5, 5, 5, 5	1	1	0.2824	2.31
40	500	45	BD	27121996	45	0.062	5, 5, 5, 5, 5, 5, 5, 5, 5	0.0705	5, 5, 5, 5, 5, 5, 5, 5, 5	1	0.7556	0.396	2.84
41	500	50	BD	27121996	50	0.062	5, 5, 5, 5, 5, 5, 5, 5, 5, 5	0.0593	25, 5, 5, 5, 5, 5, 5	1	0.3636	0.3672	3.17
42	500	25	B	27121996	25	0.2	8, 4, 8, 5	0.0132	8, 4, 8, 5	1	1	0.1755	1.12

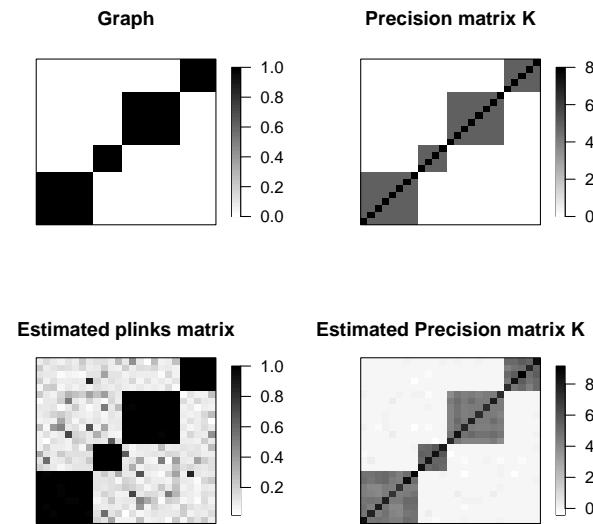
## 1 Recap table

## 2 Simulations

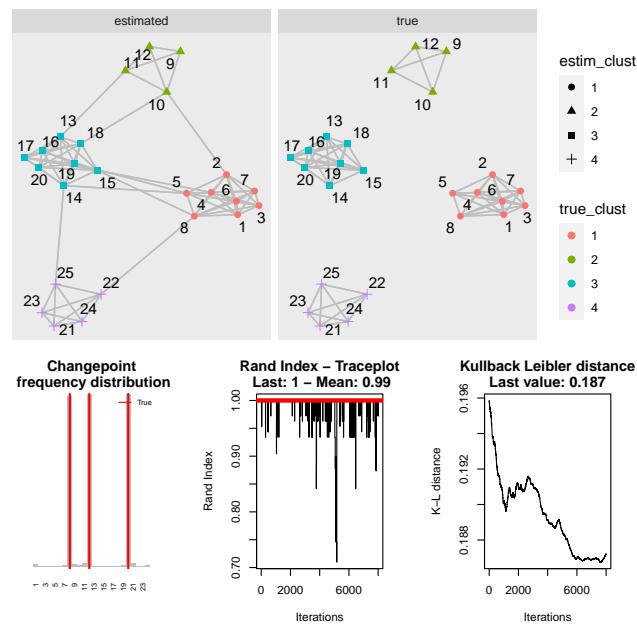
### 2.1 Group 1

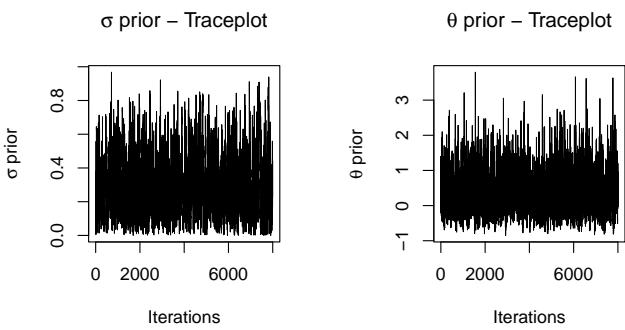
#### 2.1.1 Simulation number 1

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
01	500	25	BD	22111996	25	0.2	8,4,8,5



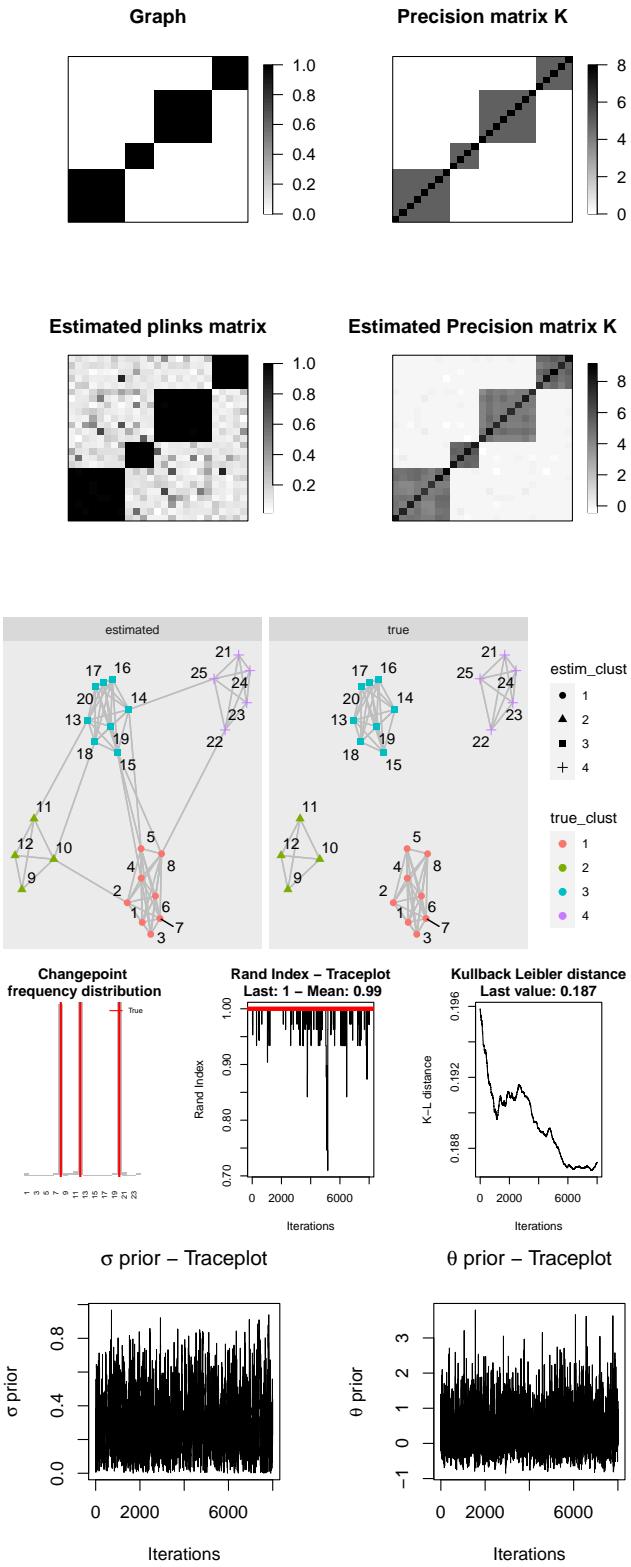
```
## Warning: Using the `size` aesthetic in this geom was deprecated in ggplot2 3.4.0.
## i Please use `linewidth` in the `default_aes` field and elsewhere instead.
```





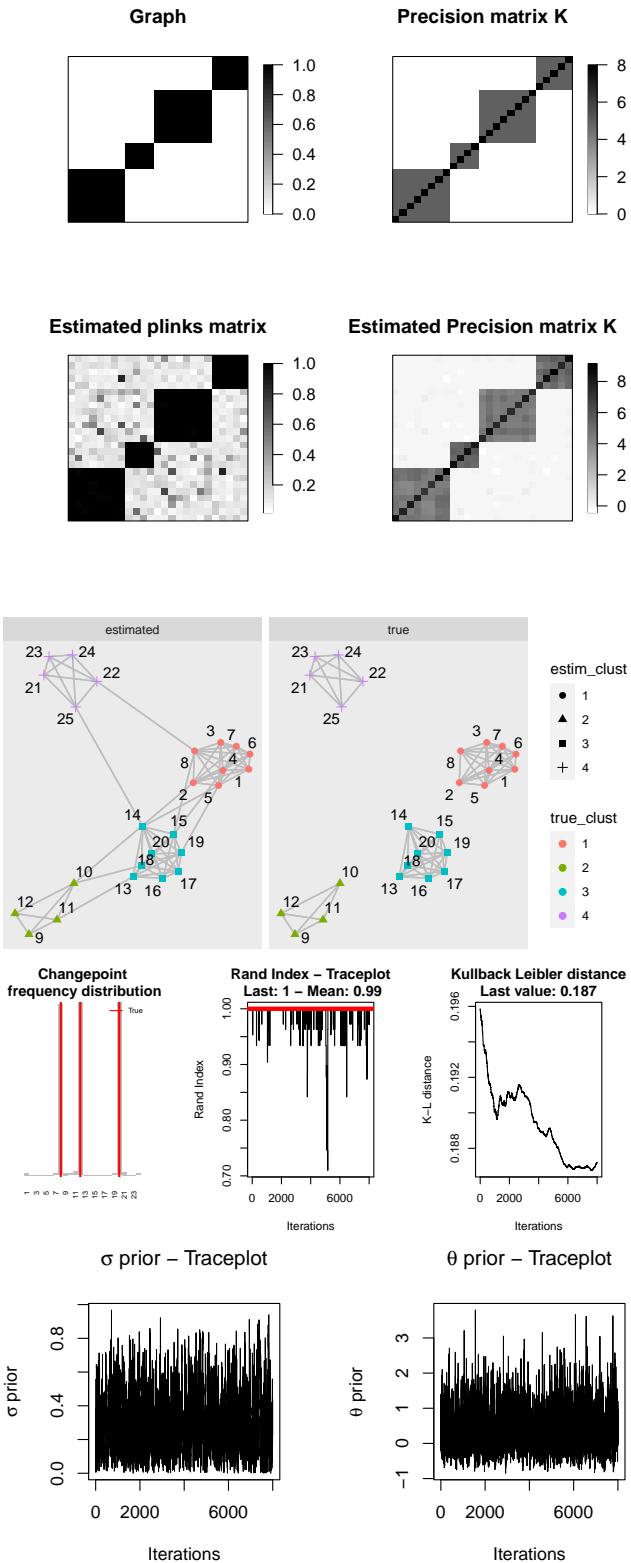
### 2.1.2 Simulation number 2

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
02	500	25	BD	31051999	25	0.2	8,4,8,5



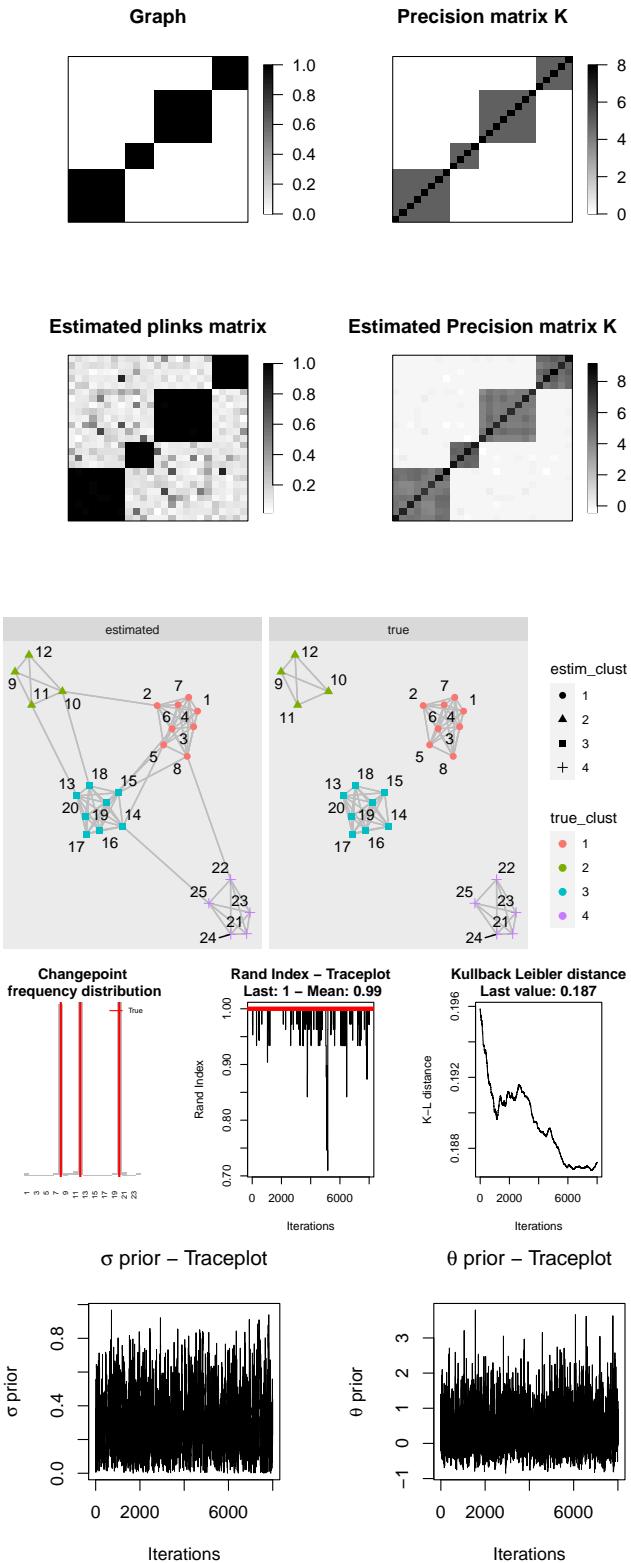
### 2.1.3 Simulation number 3

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
03	500	25	BD	27051999	25	0.2	8,4,8,5



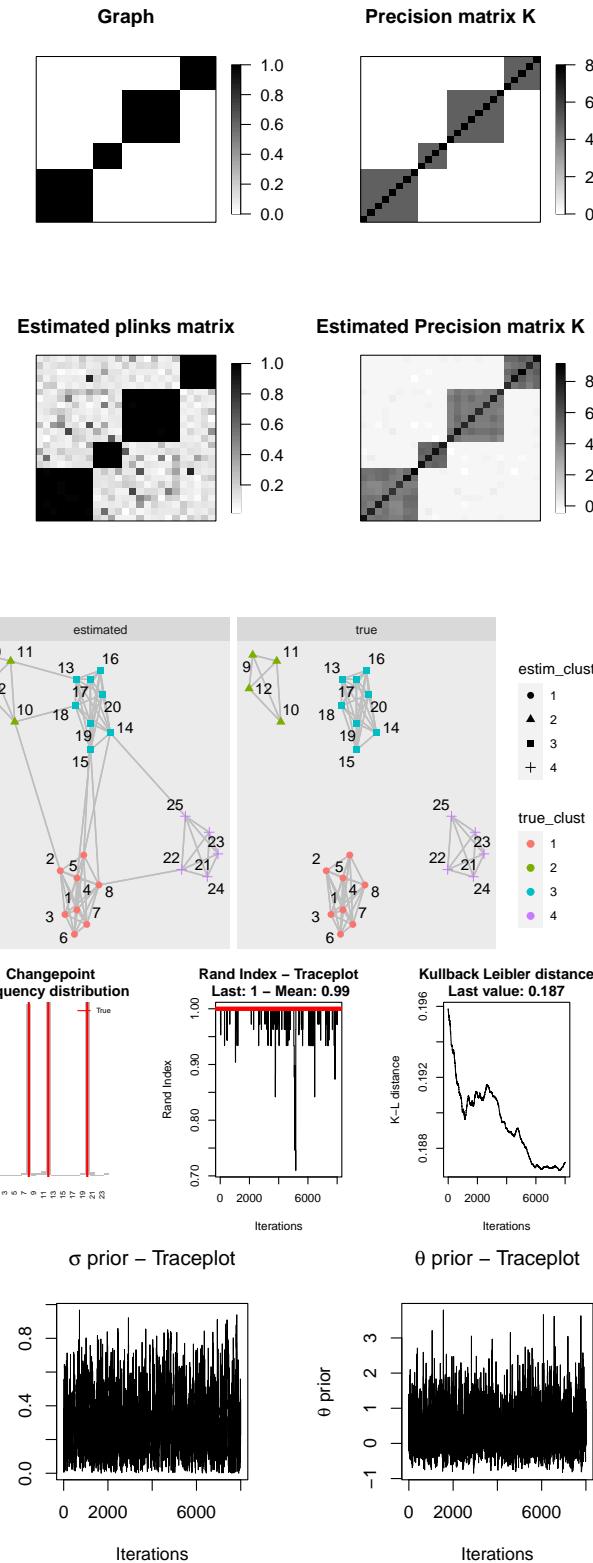
#### 2.1.4 Simulation number 4

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
04	500	25	BD	29061999	25	0.2	8,4,8,5



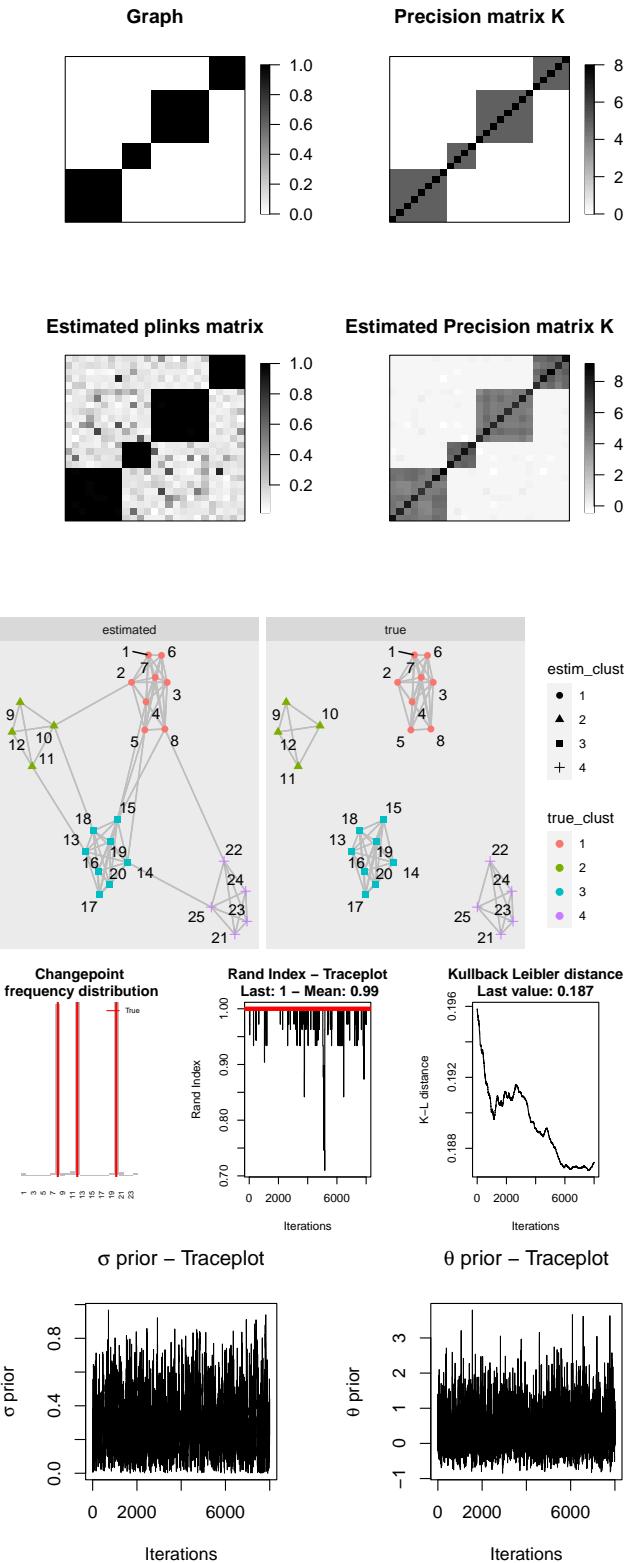
### 2.1.5 Simulation number 5

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
05	500	25	BD	12091997	25	0.2	8,4,8,5



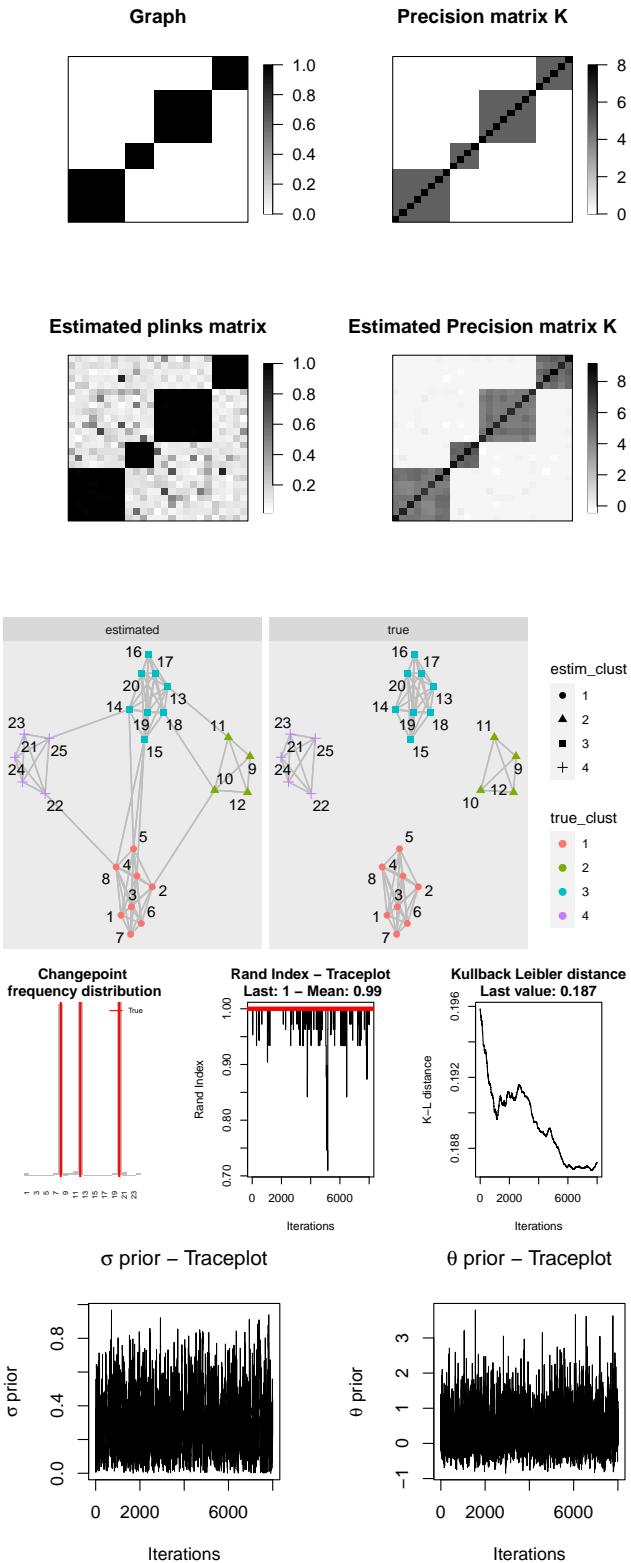
### 2.1.6 Simulation number 6

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
06	500	25	BD	27091999	25	0.2	8,4,8,5



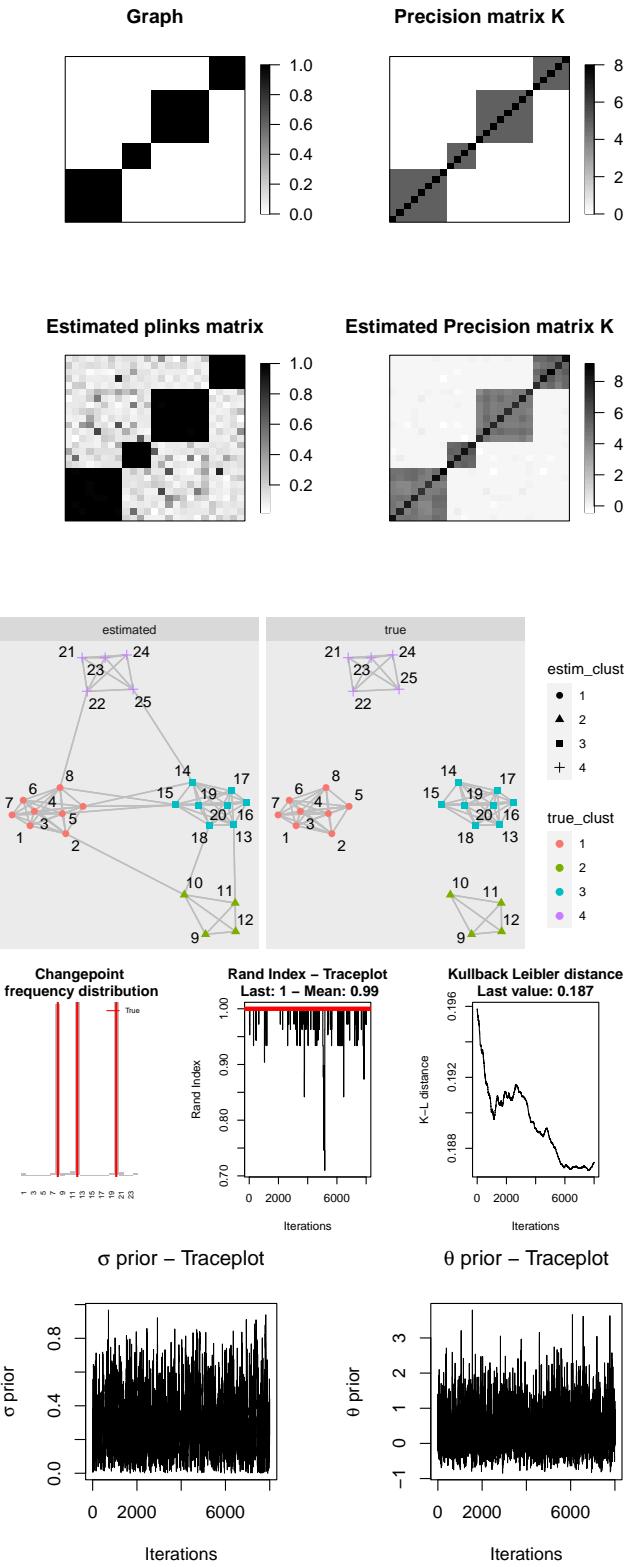
### 2.1.7 Simulation number 7

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
07	500	25	BD	27121996	25	0.2	8,4,8,5



### 2.1.8 Simulation number 8

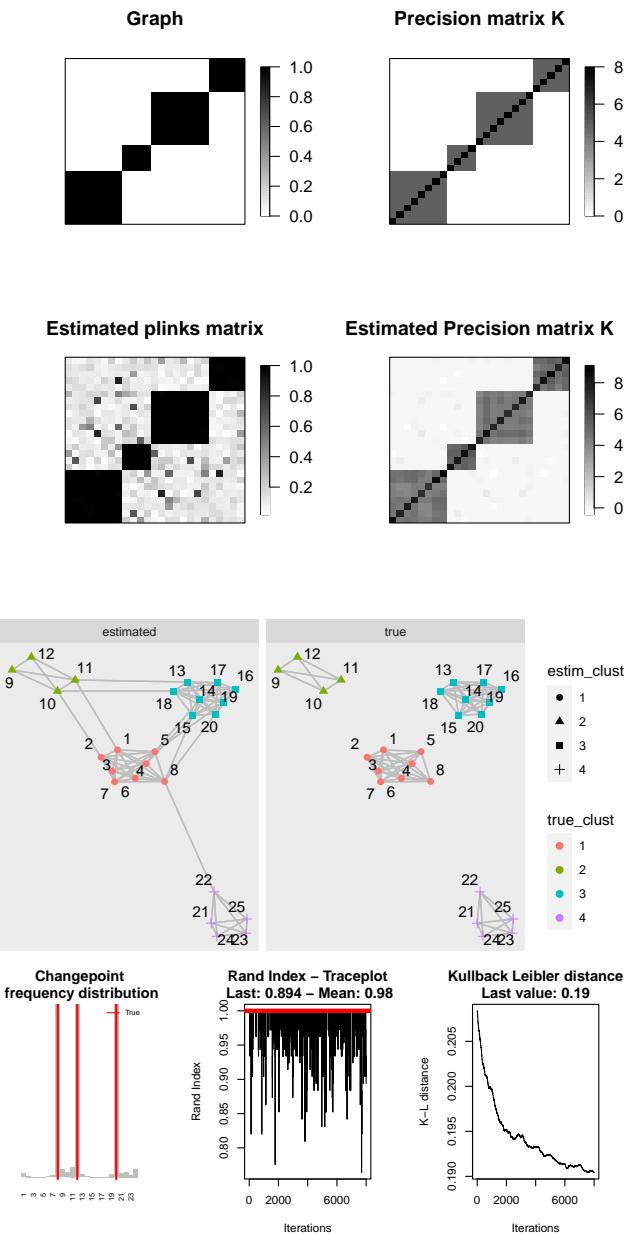
simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
08	500	25	BD	22111996	25	0.2	8,4,8,5

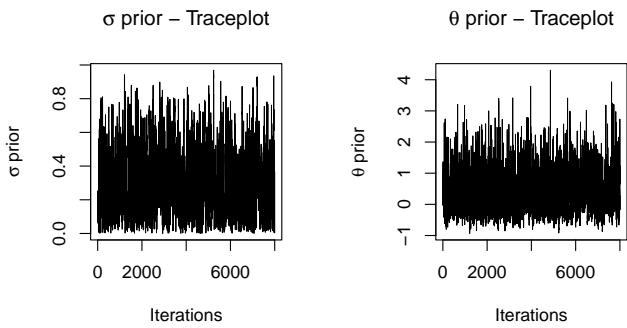


## 2.2 Group 2

### 2.2.1 Simulation number 9

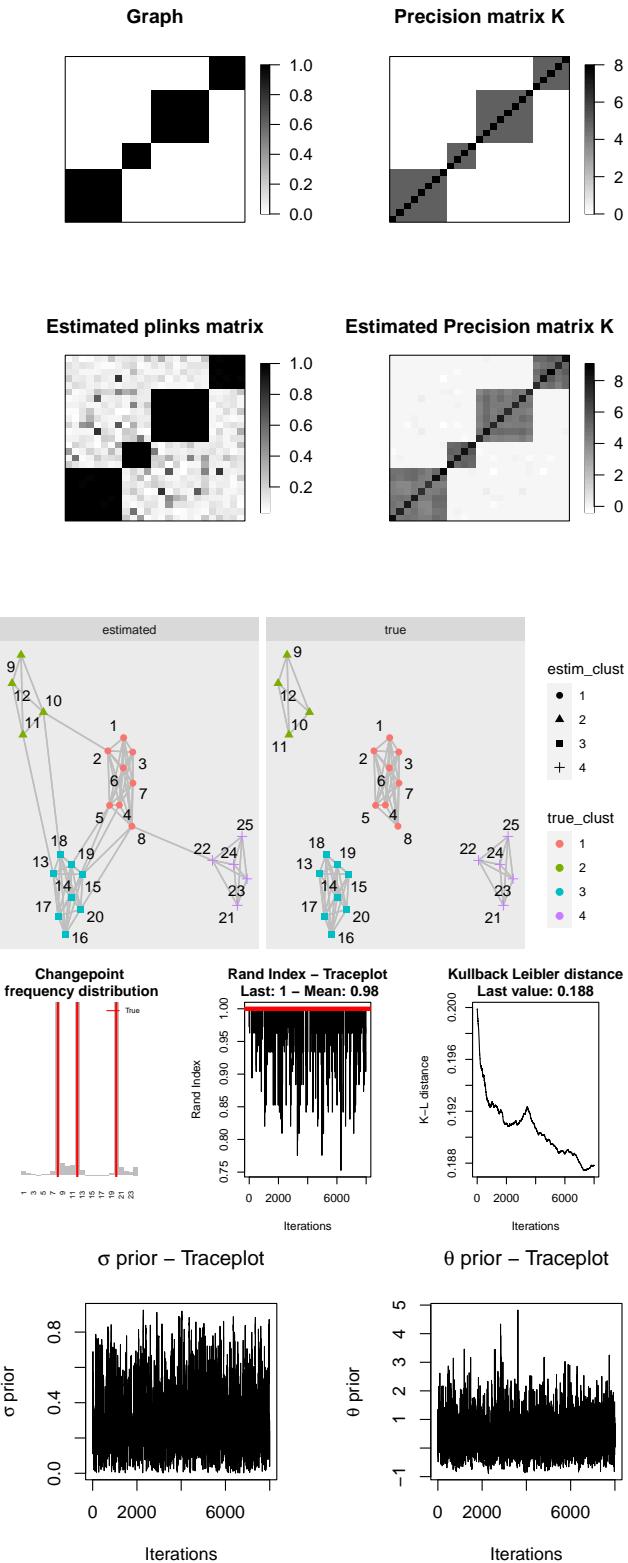
simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
09	500	25	BD	27121996	25	0.0625	8,4,8,5





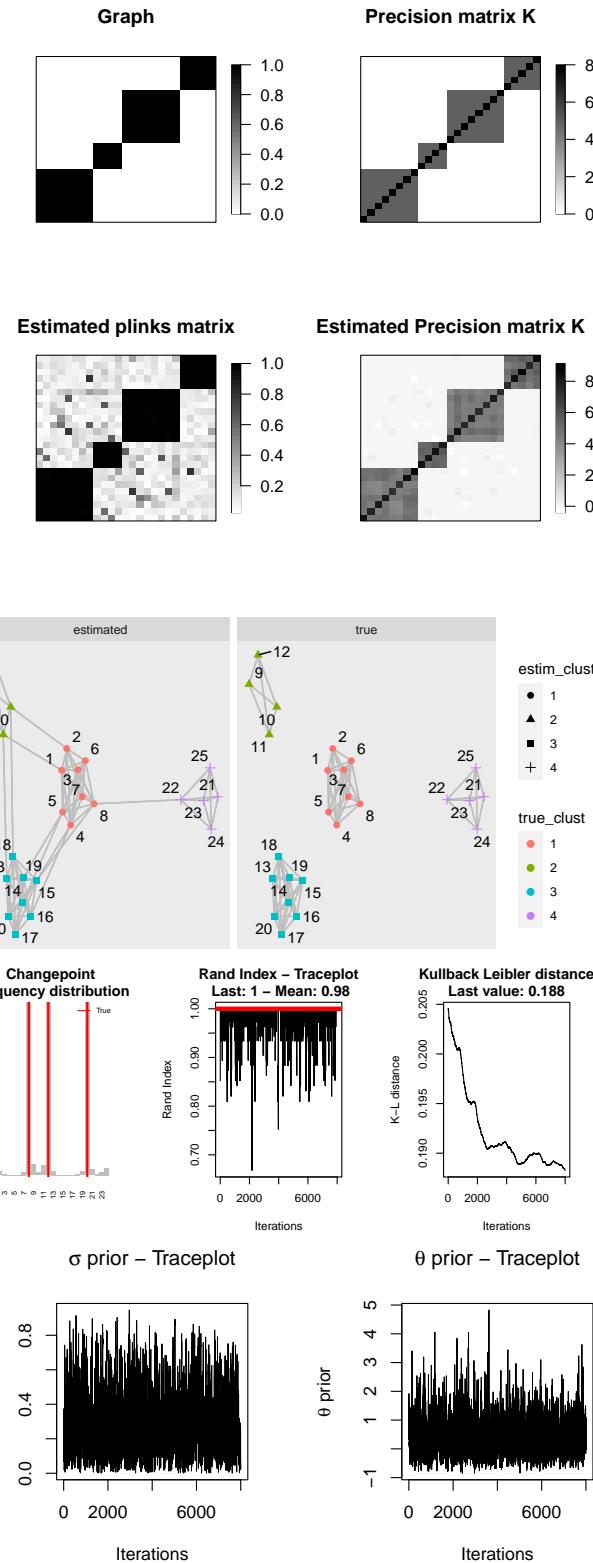
## 2.2.2 Simulation number 10

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
10	500	25	BD	27121996	25	0.0777778	8,4,8,5



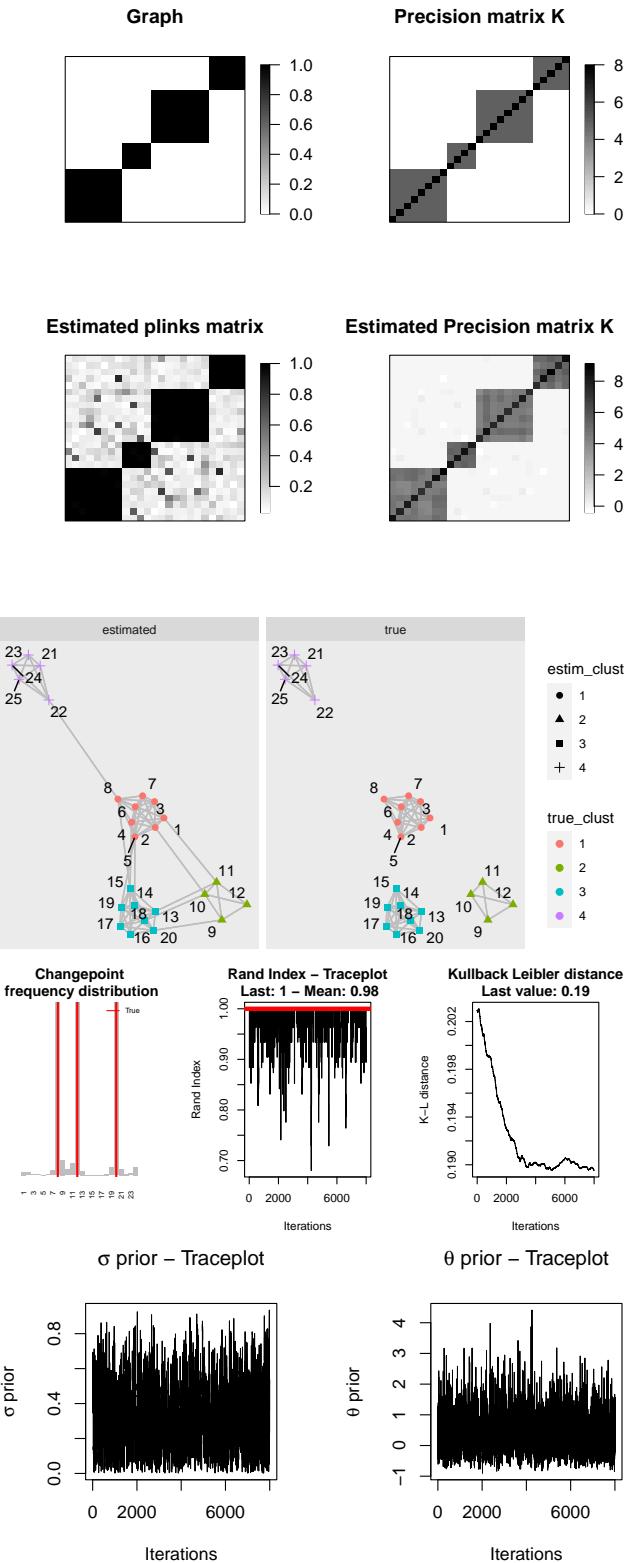
### 2.2.3 Simulation number 11

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
11	500	25	BD	27121996	25	0.0930556	8,4,8,5



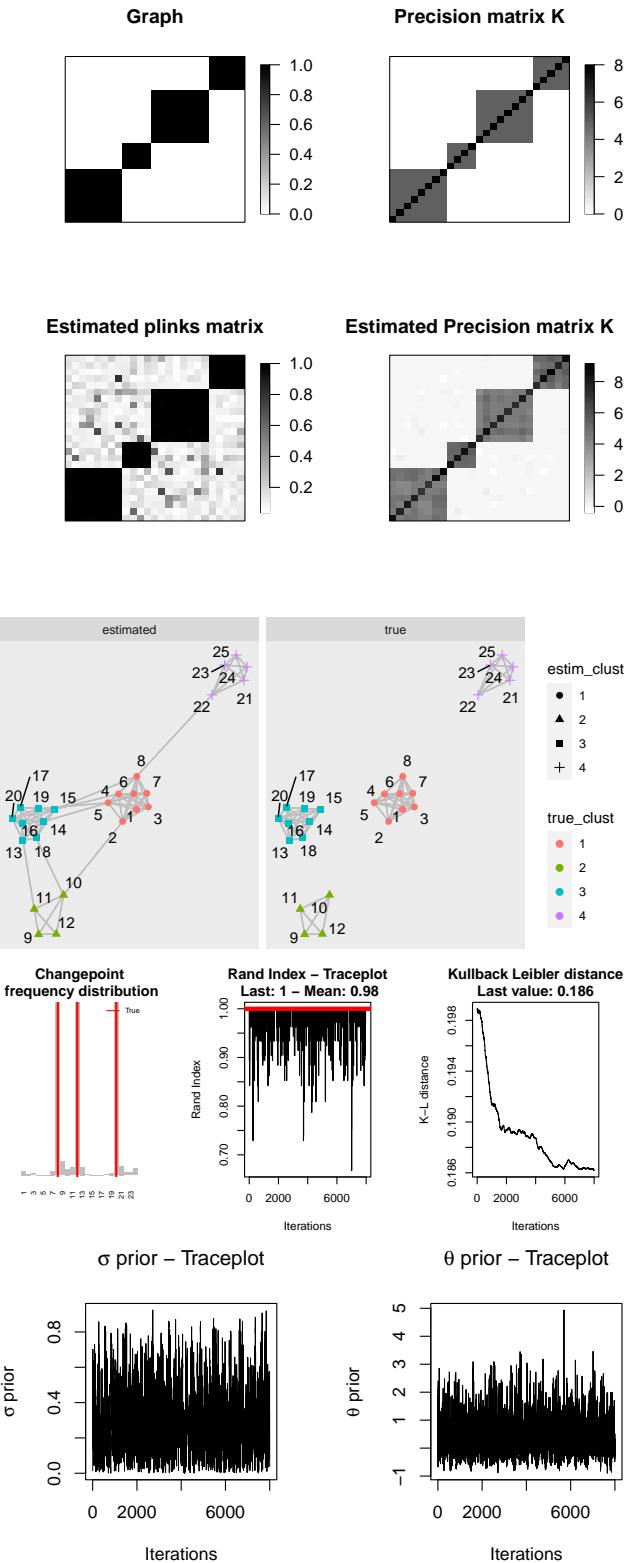
#### 2.2.4 Simulation number 12

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
12	500	25	BD	27121996	25	0.1083333	8,4,8,5



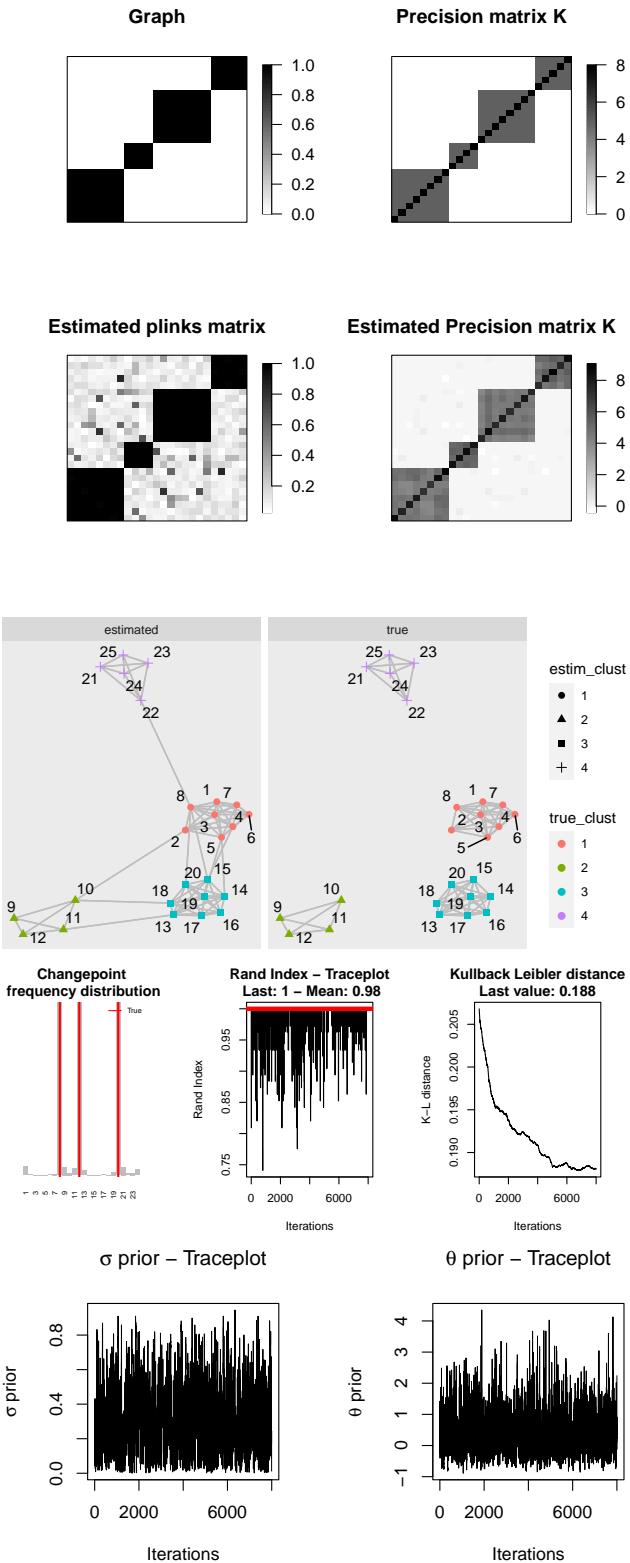
### 2.2.5 Simulation number 13

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
13	500	25	BD	27121996	25	0.1236111	8,4,8,5



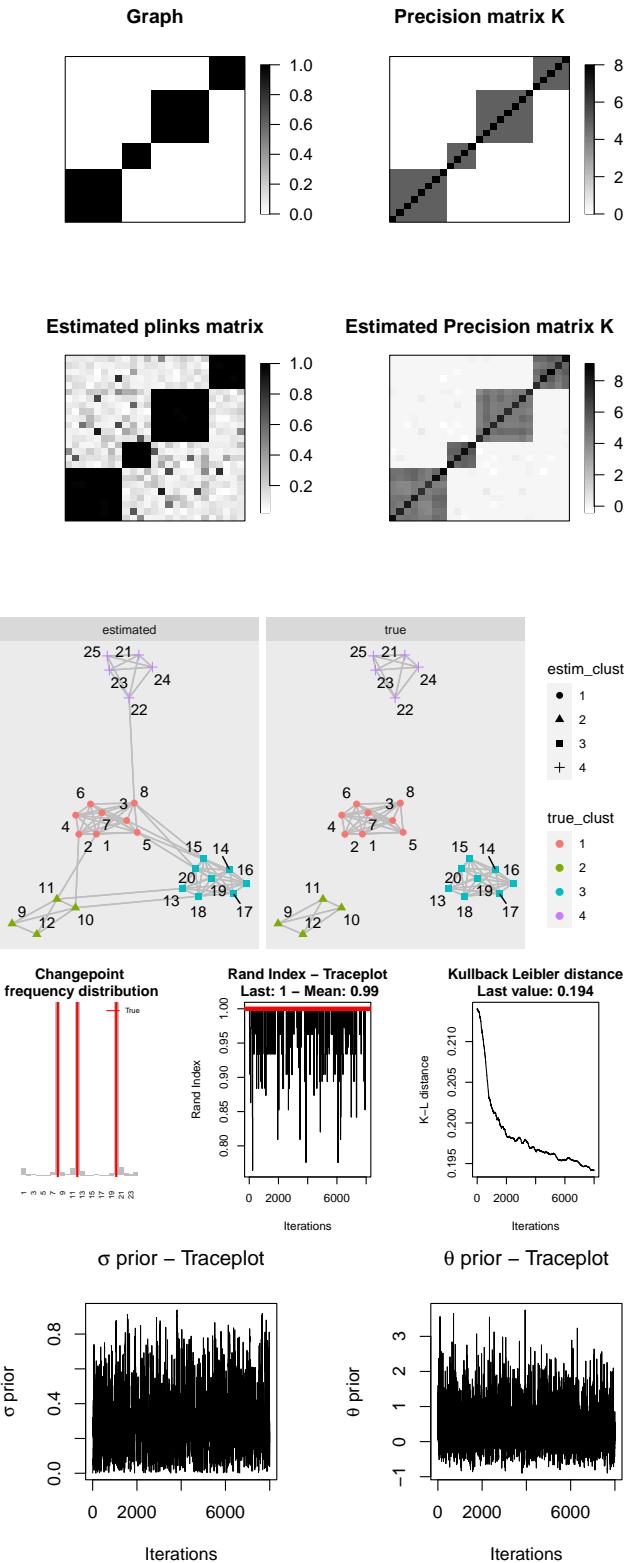
### 2.2.6 Simulation number 14

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
14	500	25	BD	27121996	25	0.1388889	8,4,8,5



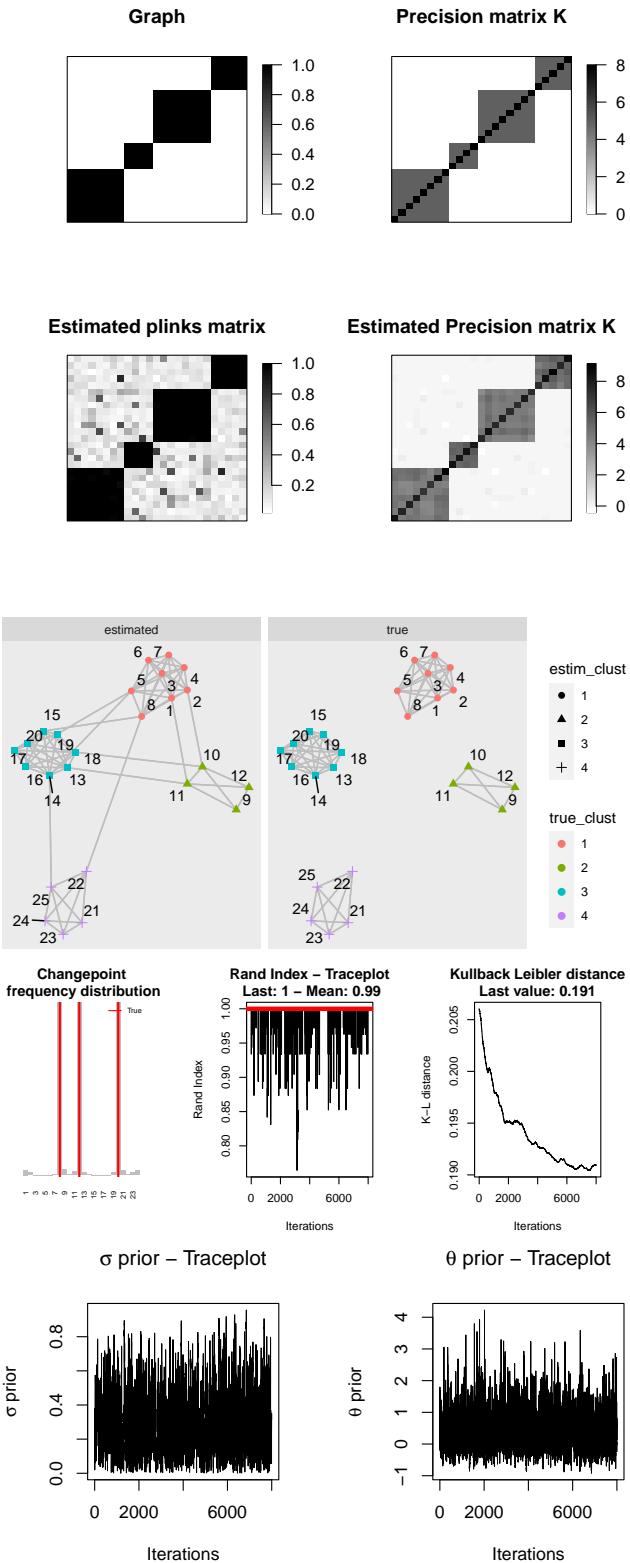
### 2.2.7 Simulation number 15

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
15	500	25	BD	27121996	25	0.1541667	8,4,8,5



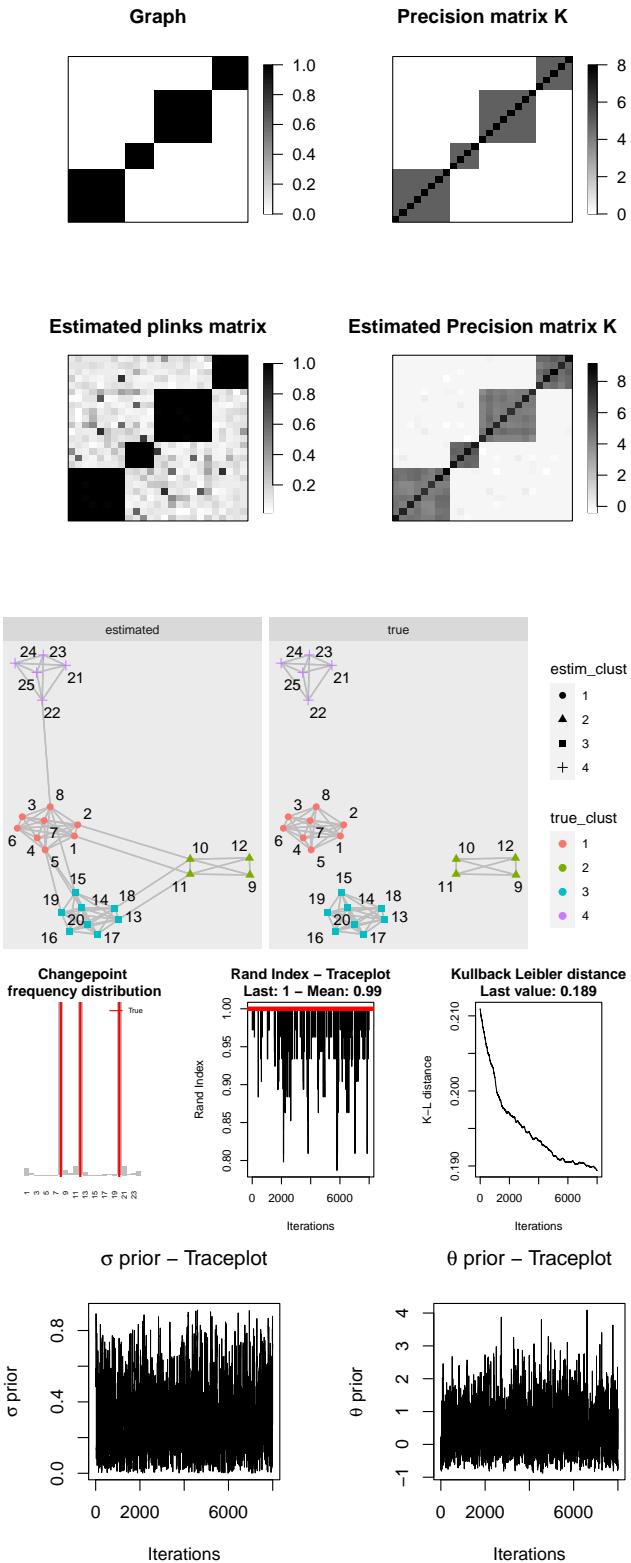
## 2.2.8 Simulation number 16

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
16	500	25	BD	27121996	25	0.1694444	8,4,8,5



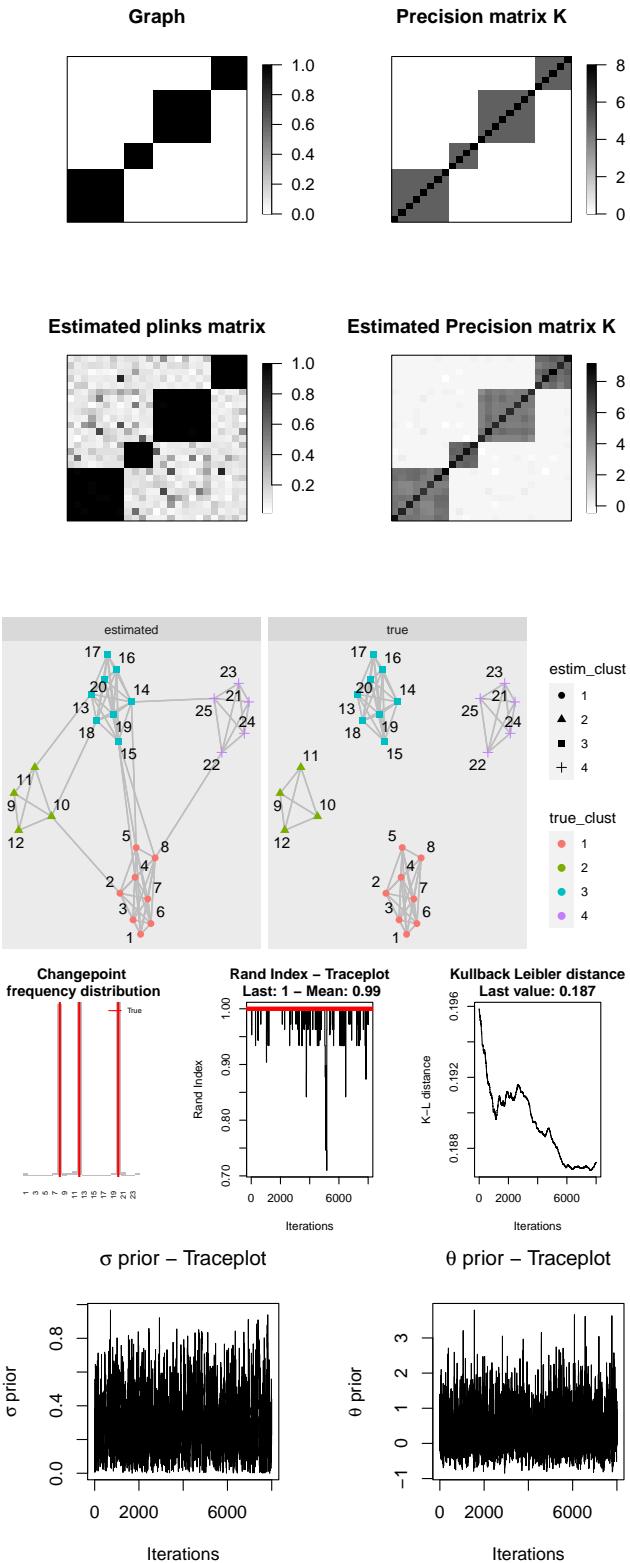
### 2.2.9 Simulation number 17

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
17	500	25	BD	27121996	25	0.1847222	8,4,8,5



### 2.2.10 Simulation number 18

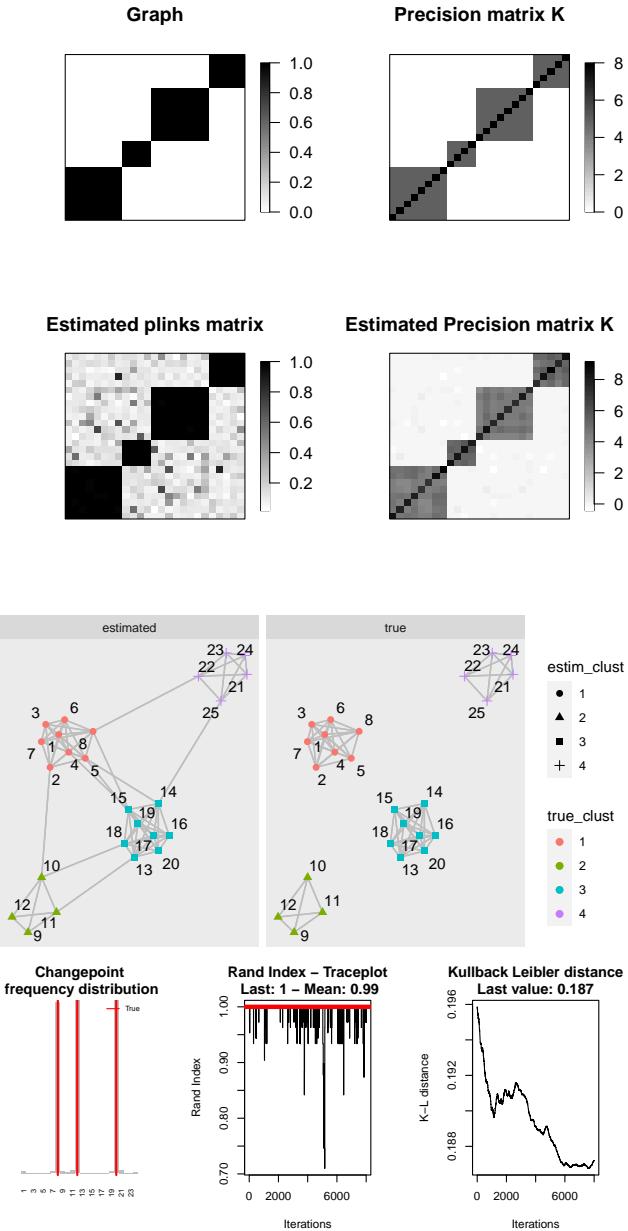
simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
18	500	25	BD	27121996	25	0.2	8,4,8,5

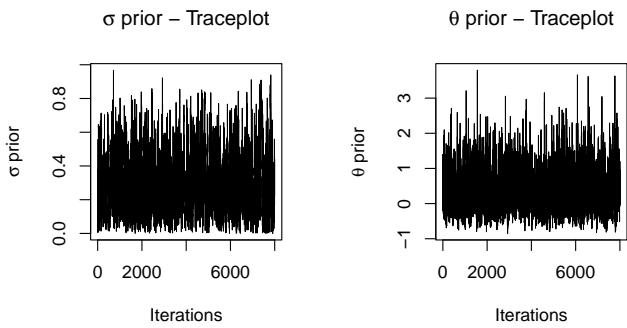


## 2.3 Group 3

### 2.3.1 Simulation number 19

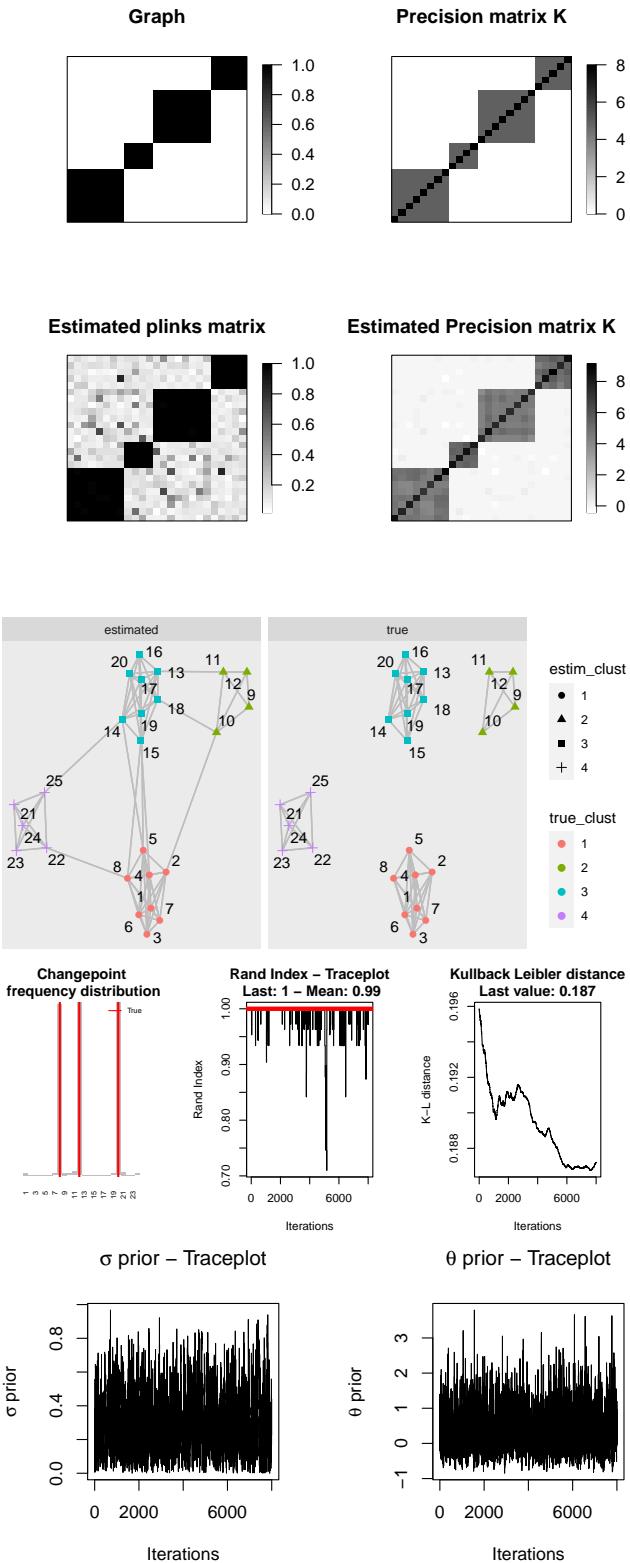
simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
19	500	25	BD	27121996	25	0.2	8,4,8,5





### 2.3.2 Simulation number 20

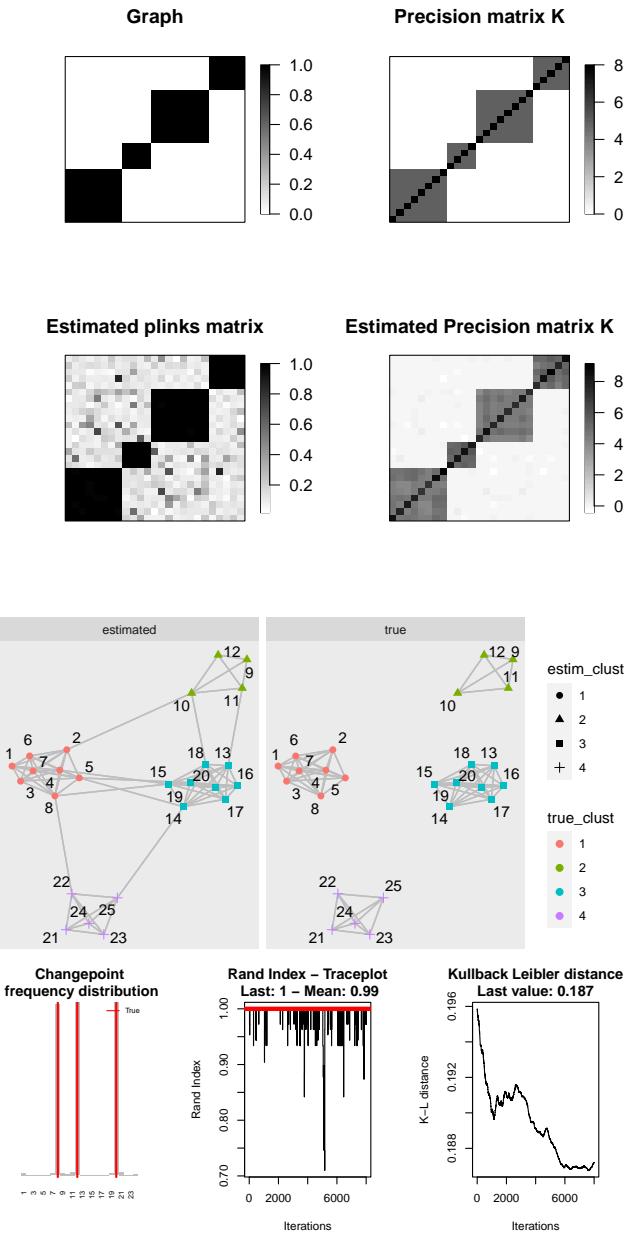
simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2
20	500	25	BD	27121996	1,1	0.2

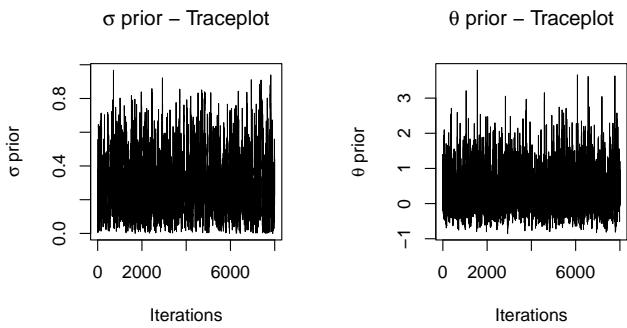


## 2.4 Group 4

### 2.4.1 Simulation number 21

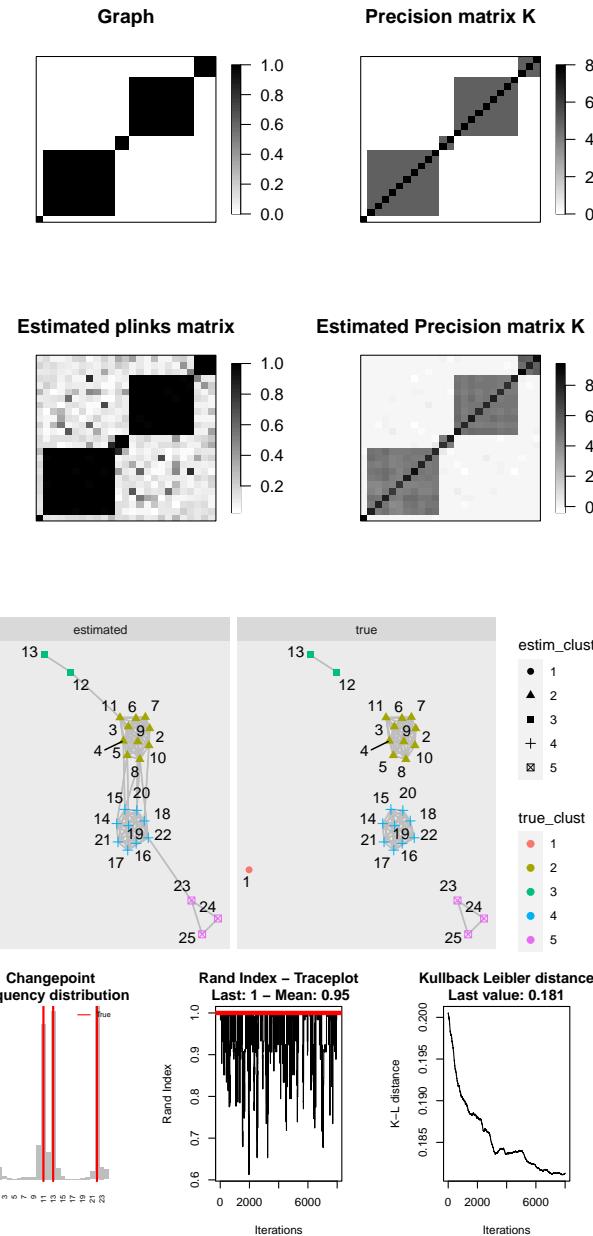
simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
21	500	25	BD	27121996	25	0.2	8,4,8,5





#### 2.4.2 Simulation number 22

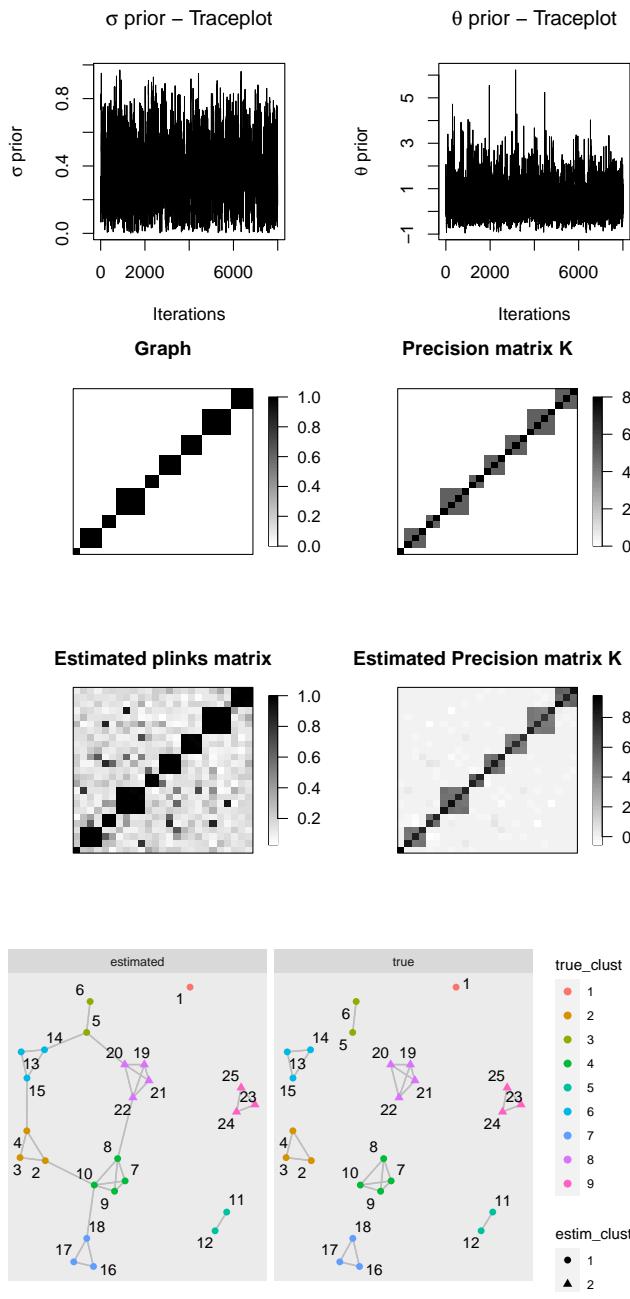
simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
22	500	25	BD	27121996	25	0.2	1,10,2,9,3

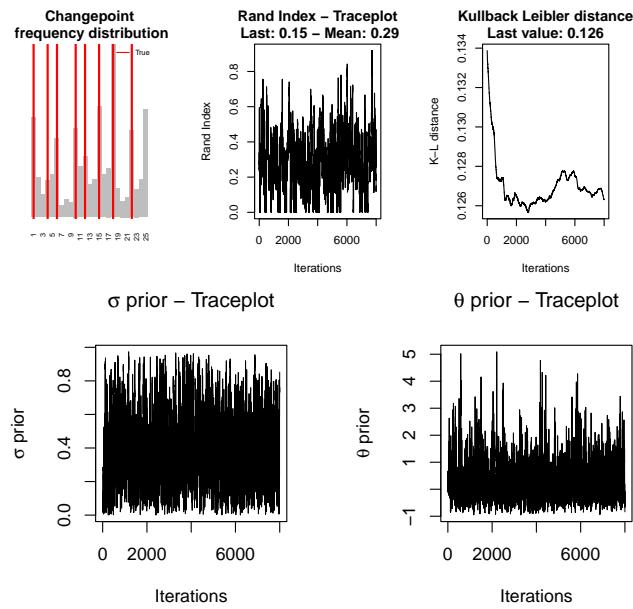


### 2.4.3 Simulation number 23

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
23	500	25	BD	27121996	25	0.02	1,3,2,4,2,3,3,4,3

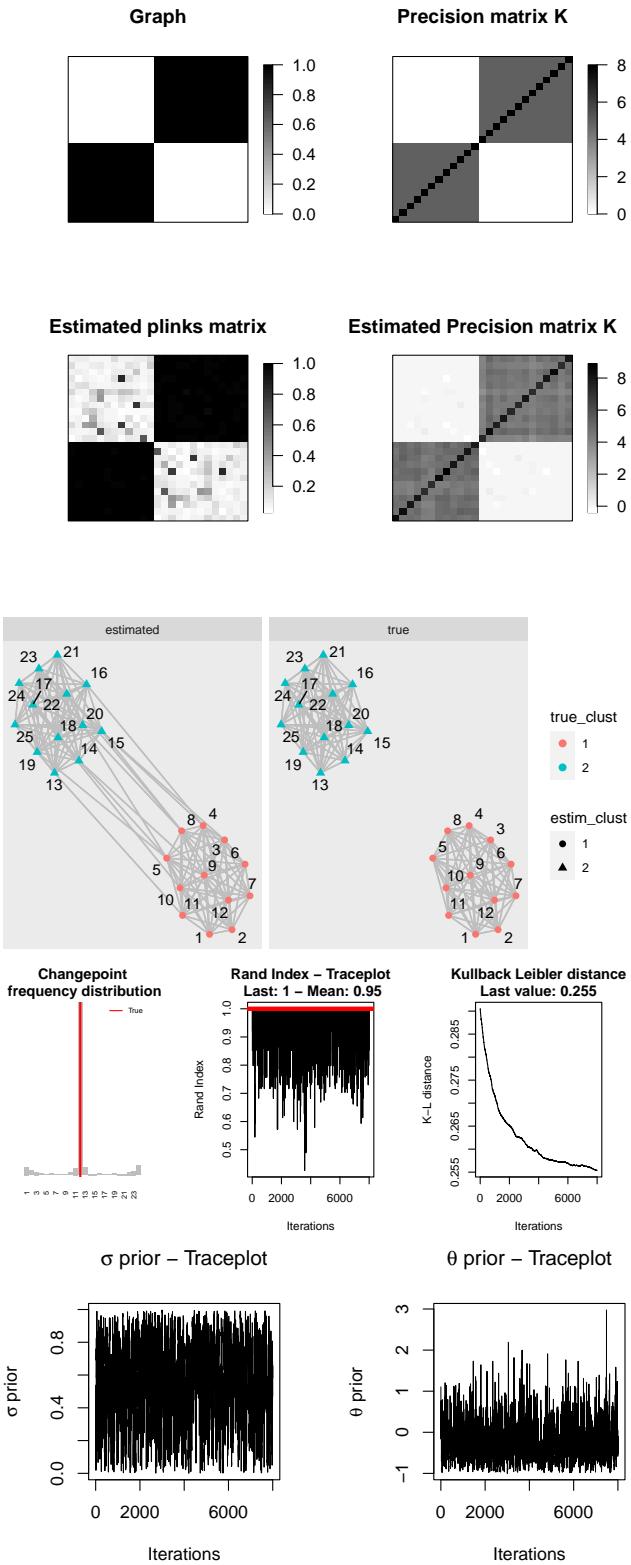
```
## Warning in (function (... , deparse.level = 1) : number of columns of result is
## not a multiple of vector length (arg 1)
```





#### 2.4.4 Simulation number 24

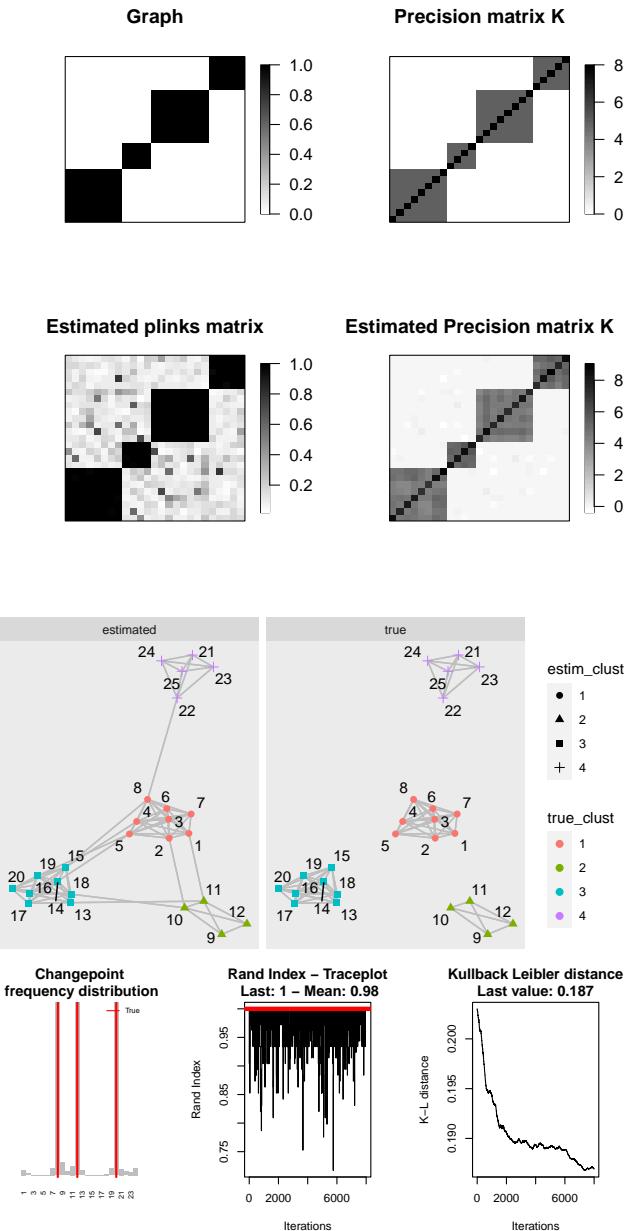
simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
24	500	25	BD	27121996	25	0.2	12,13

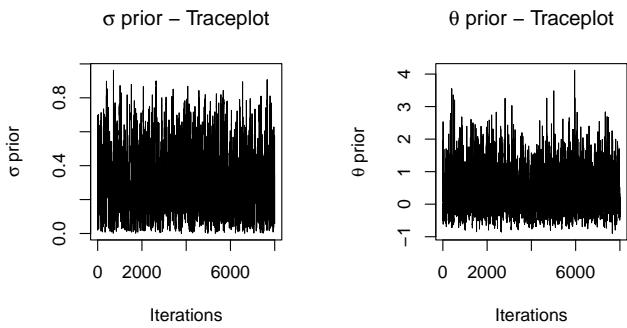


## 2.5 Group 5

### 2.5.1 Simulation number 25

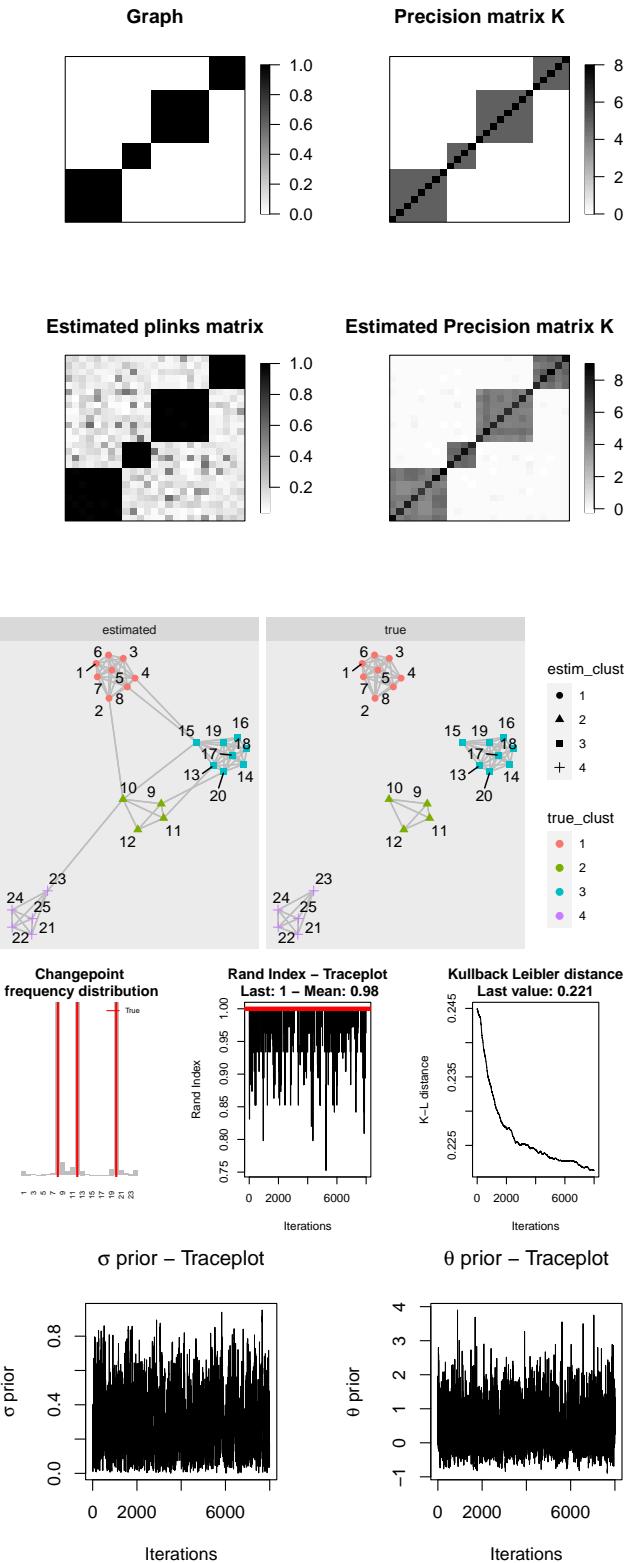
simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
25	500	25	BD	27121996	25	0.1	8,4,8,5





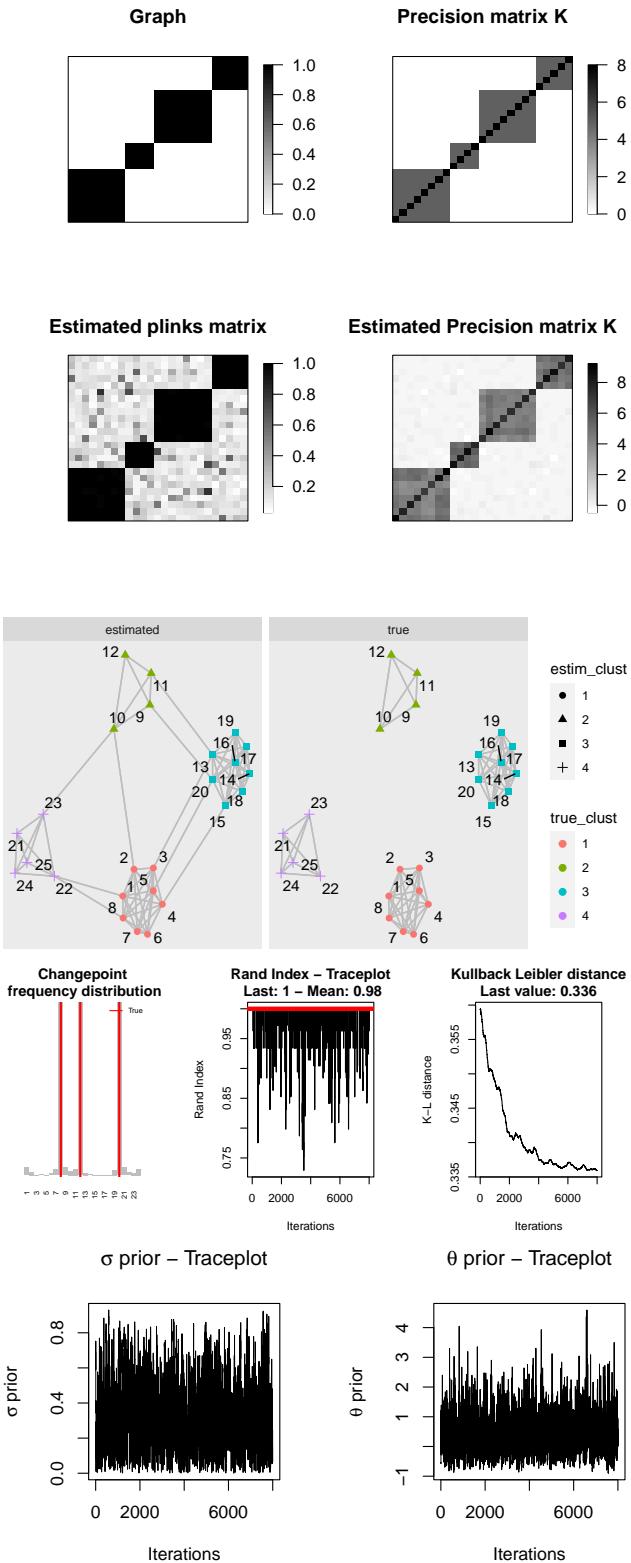
### 2.5.2 Simulation number 26

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
26	400	25	BD	27121996	25	0.1	8,4,8,5



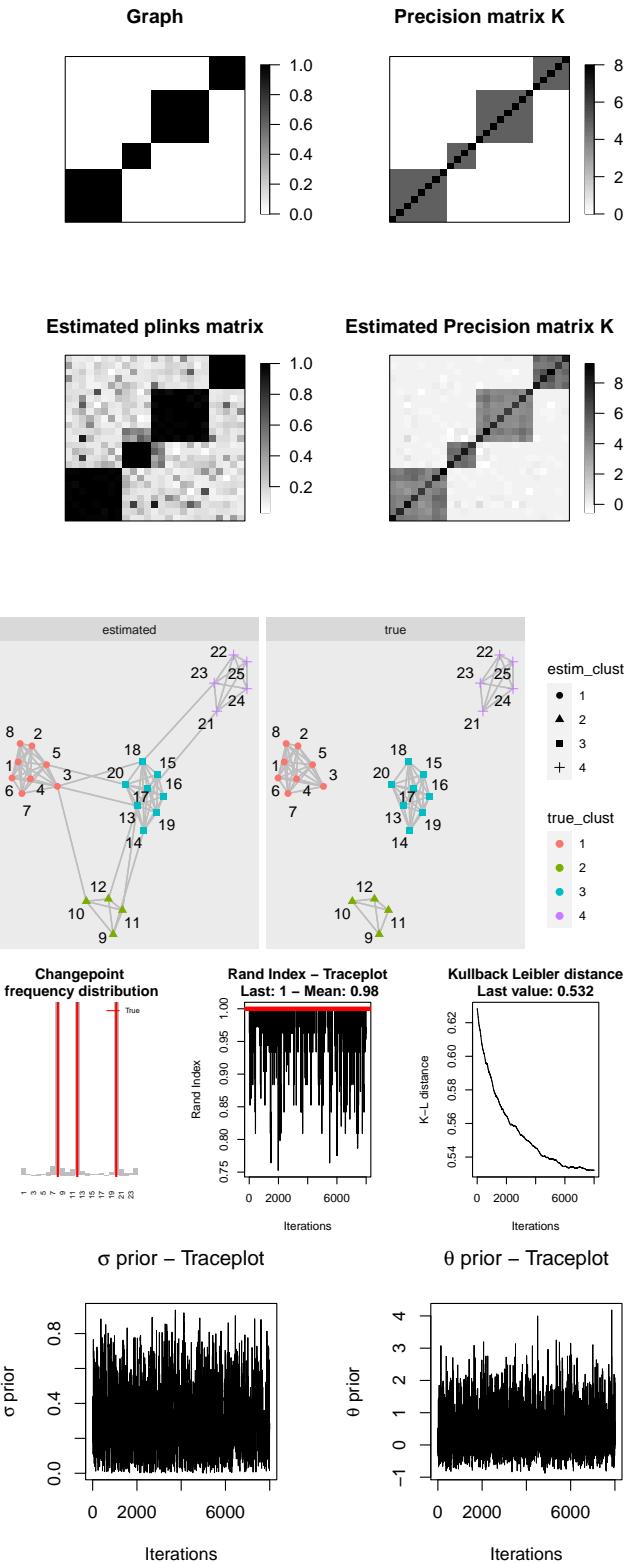
### 2.5.3 Simulation number 27

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
27	300	25	BD	27121996	25	0.1	8,4,8,5



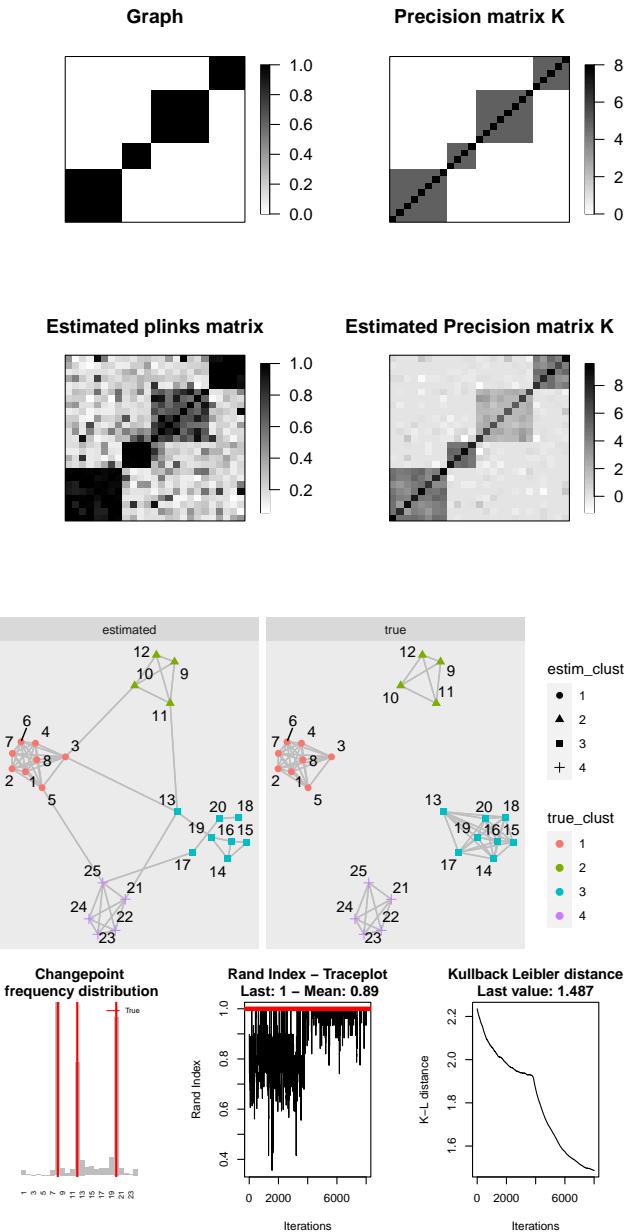
#### 2.5.4 Simulation number 28

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
28	200	25	BD	27121996	25	0.1	8,4,8,5



### 2.5.5 Simulation number 29

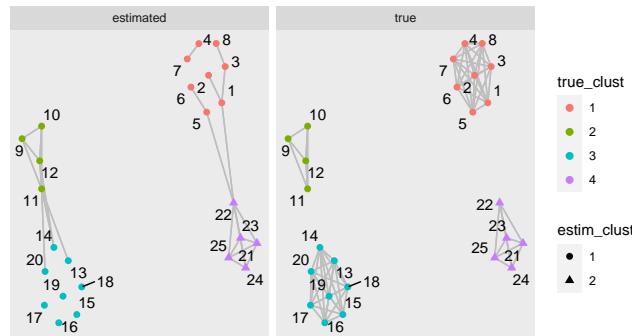
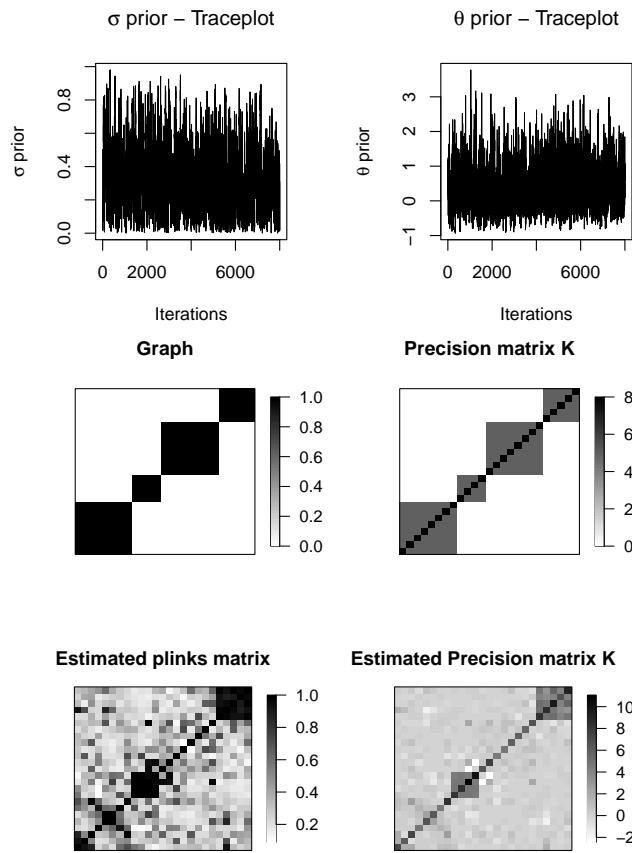
simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
29	100	25	BD	27121996	25	0.1	8,4,8,5

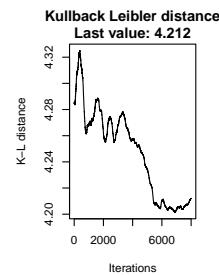
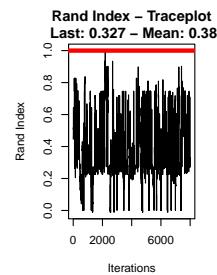
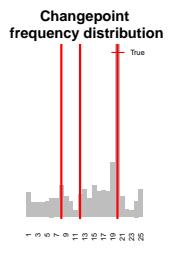


### 2.5.6 Simulation number 30

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
30	50	25	BD	27121996	25	0.1	8,4,8,5

```
## Warning in (function (... , deparse.level = 1) : number of columns of result is
## not a multiple of vector length (arg 1)
```

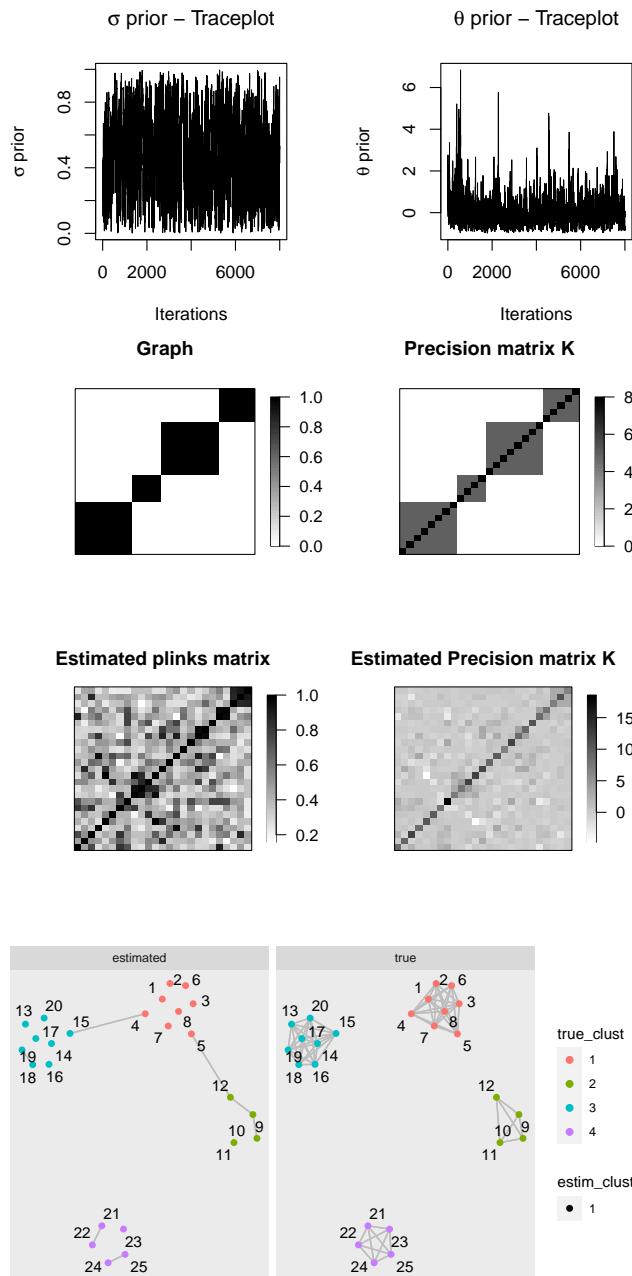


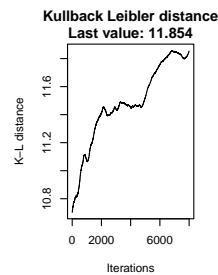
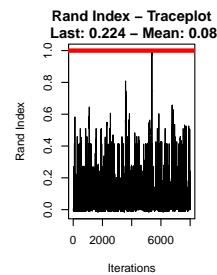
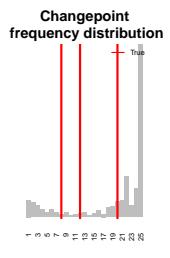


### 2.5.7 Simulation number 31

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
31	20	25	BD	27121996	25	0.1	8,4,8,5

```
## Warning in (function (... , deparse.level = 1) : number of columns of result is
## not a multiple of vector length (arg 13)
```



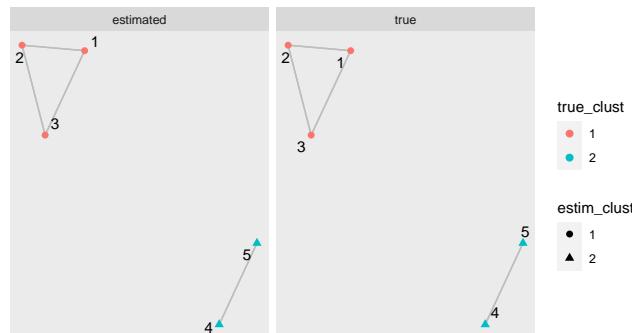
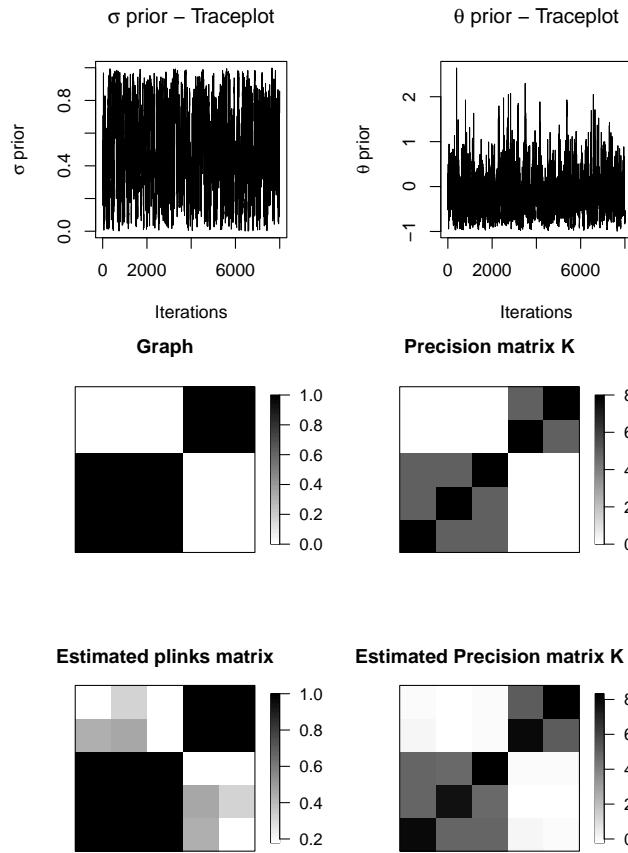


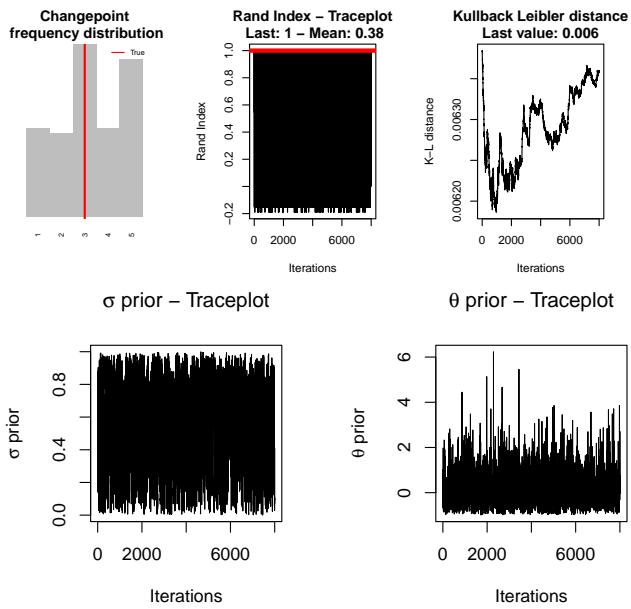
## 2.6 Group 6

### 2.6.1 Simulation number 32

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
32	500	5	BD	27121996	5	0.0625	3,2

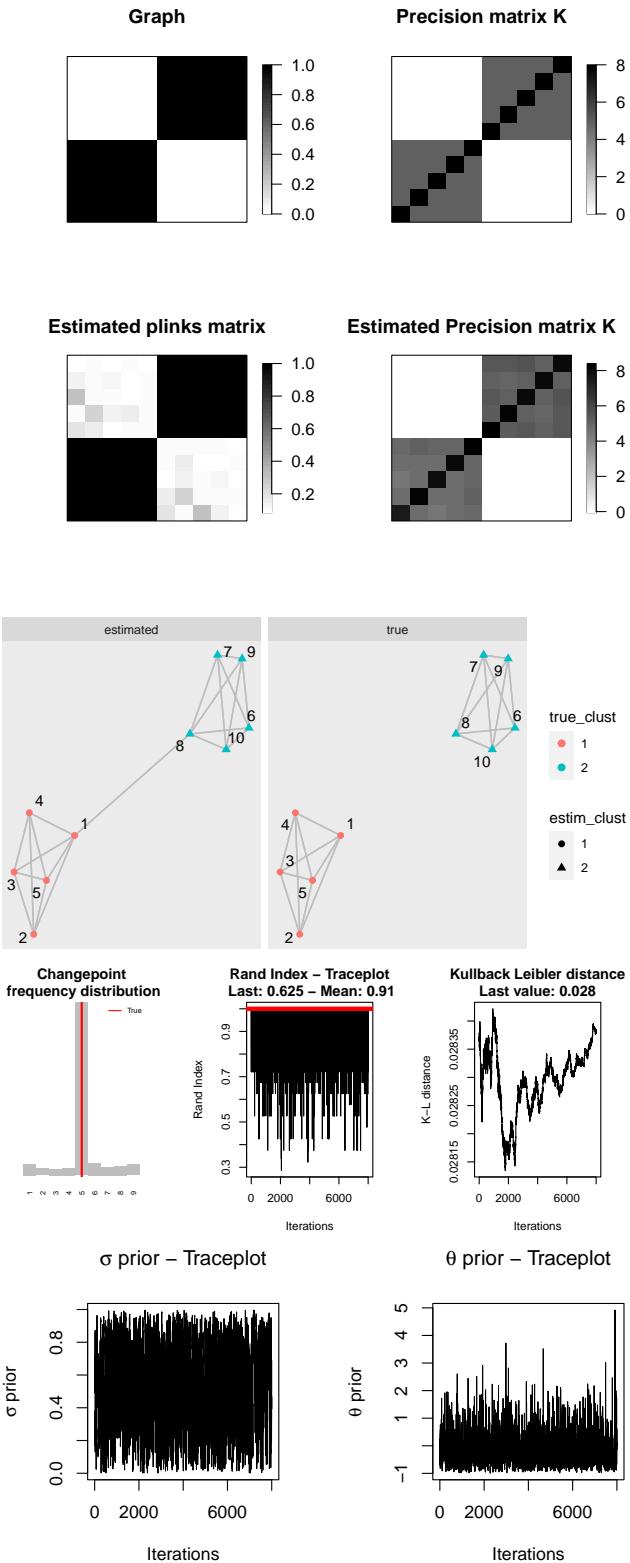
```
## Warning in (function (... , deparse.level = 1) : number of columns of result is
## not a multiple of vector length (arg 1)
```





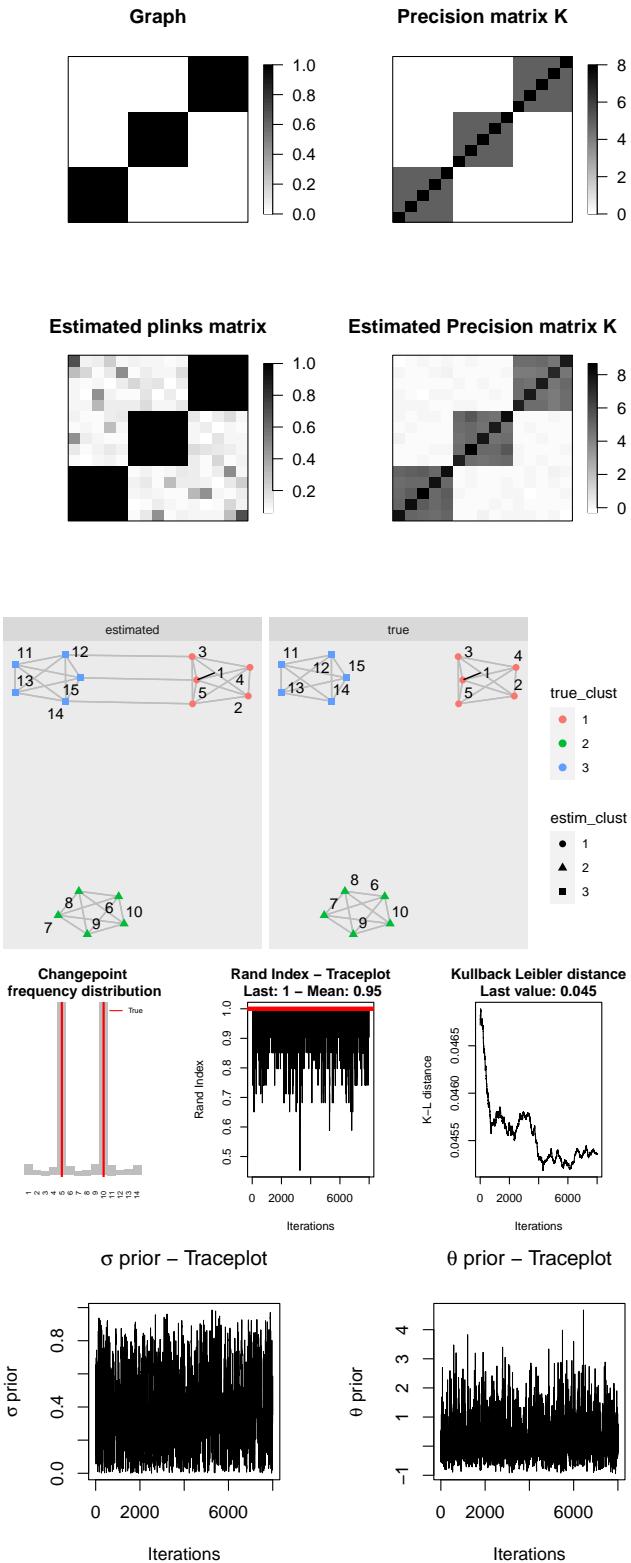
## 2.6.2 Simulation number 33

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
33	500	10	BD	27121996	10	0.0625	5,5



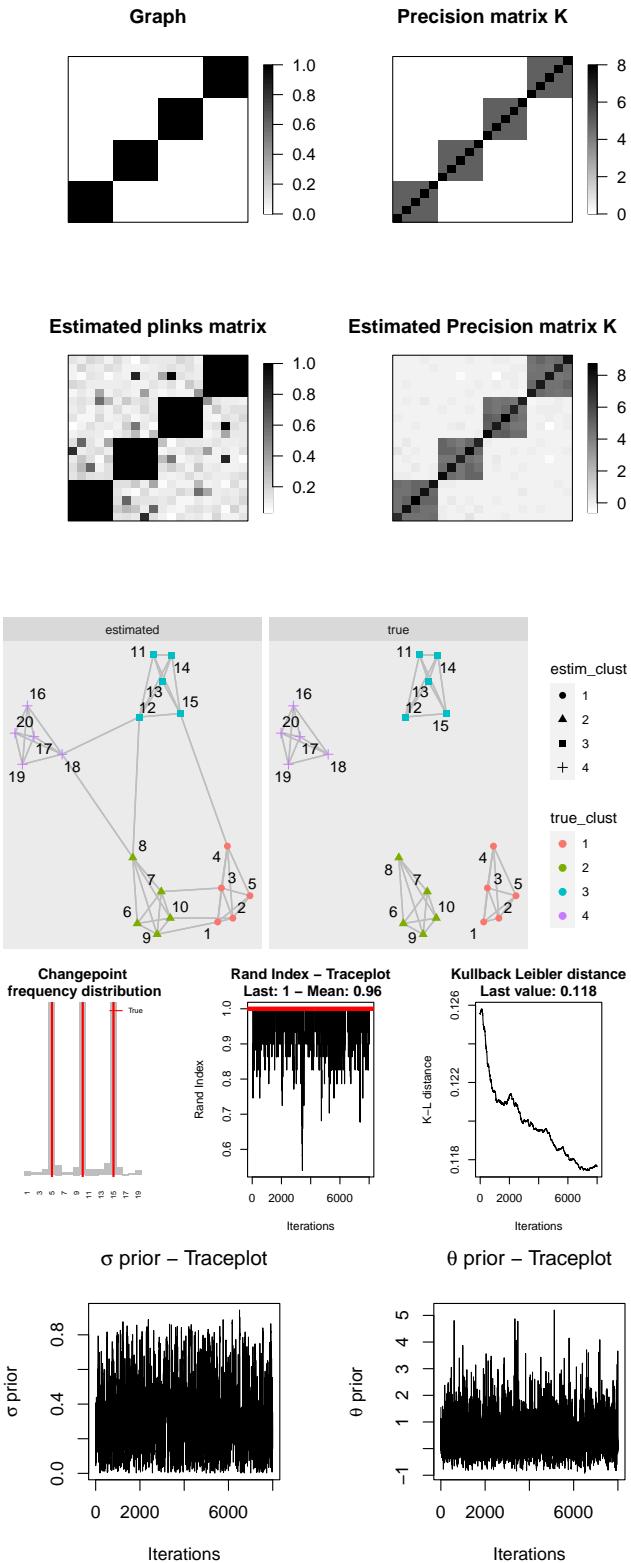
### 2.6.3 Simulation number 34

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
34	500	15	BD	27121996	15	0.0625	5,5,5



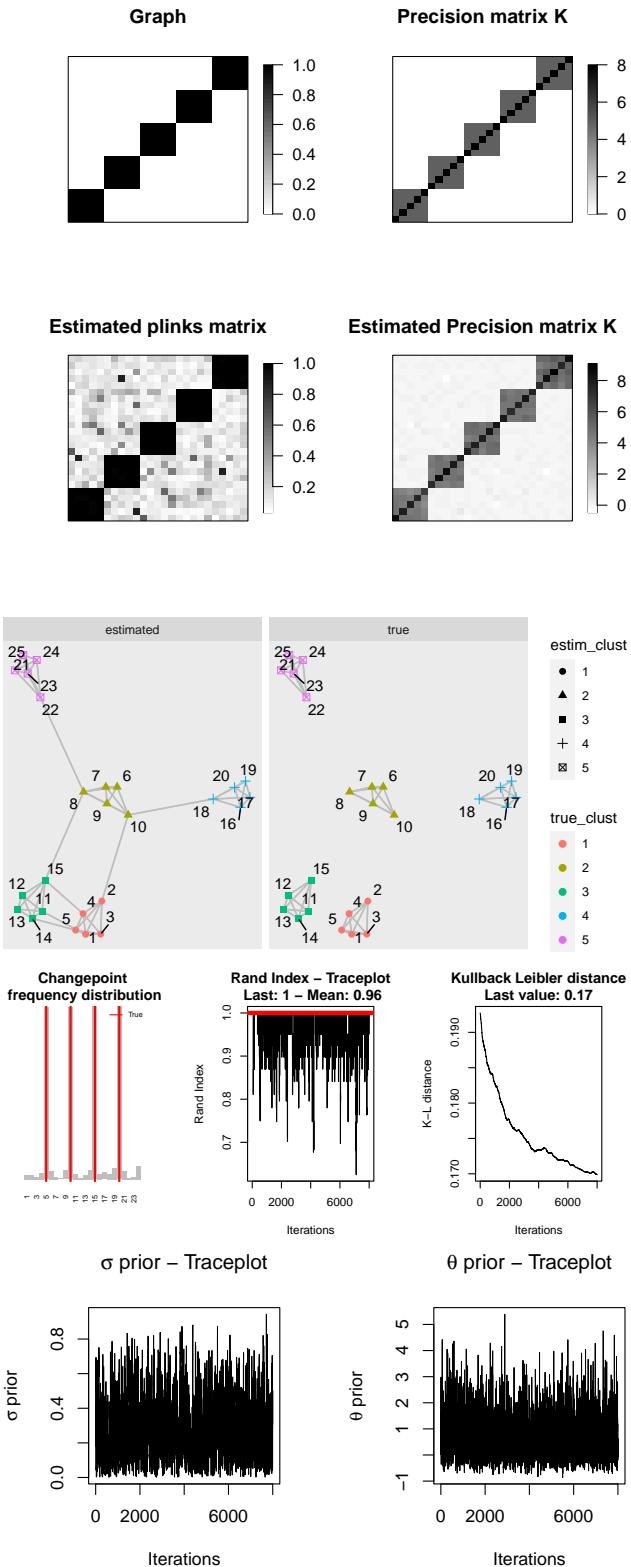
#### 2.6.4 Simulation number 35

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
35	500	20	BD	27121996	20	0.0625	5,5,5,5



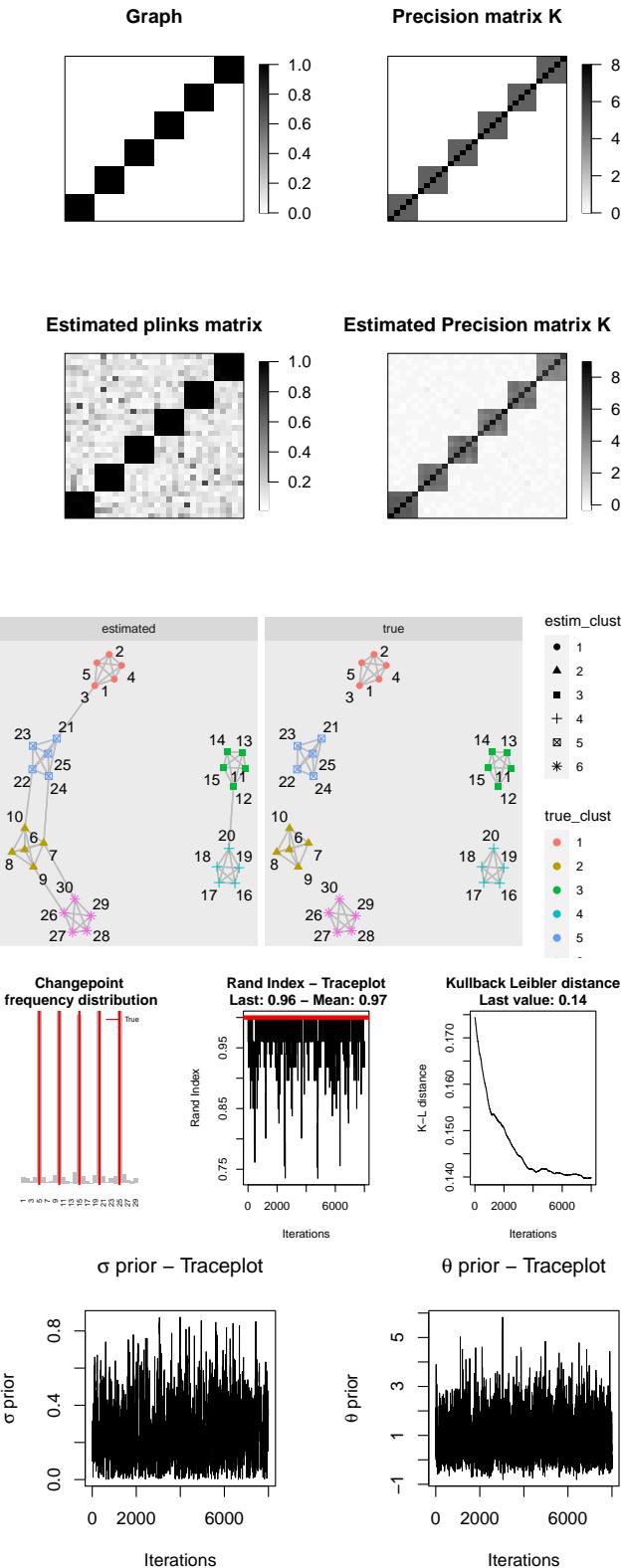
### 2.6.5 Simulation number 36

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
36	500	25	BD	27121996	25	0.0625	5,5,5,5,5



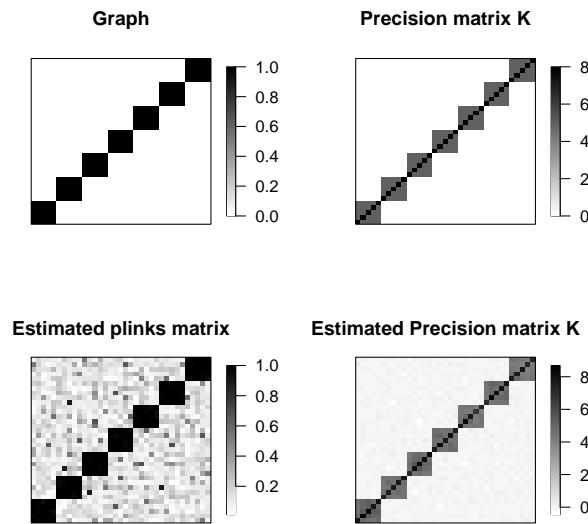
### 2.6.6 Simulation number 37

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
37	500	30	BD	27121996	30	0.0625	5,5,5,5,5,5



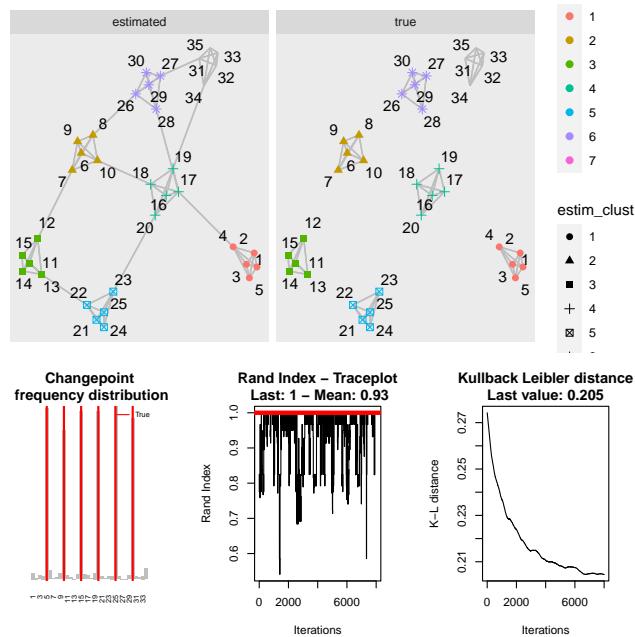
### 2.6.7 Simulation number 38

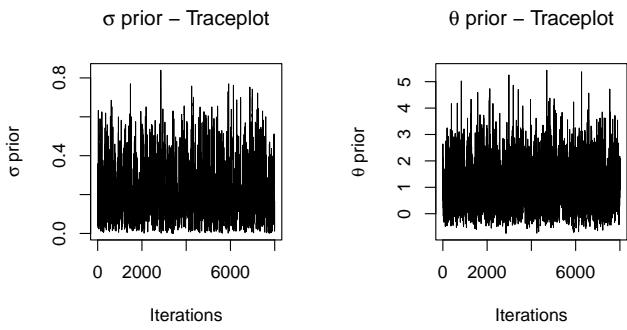
simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
38	500	35	BD	27121996	35	0.0625	5,5,5,5,5,5,5



```
## Warning: The shape palette can deal with a maximum of 6 discrete values because
## more than 6 becomes difficult to discriminate; you have 7. Consider
## specifying shapes manually if you must have them.

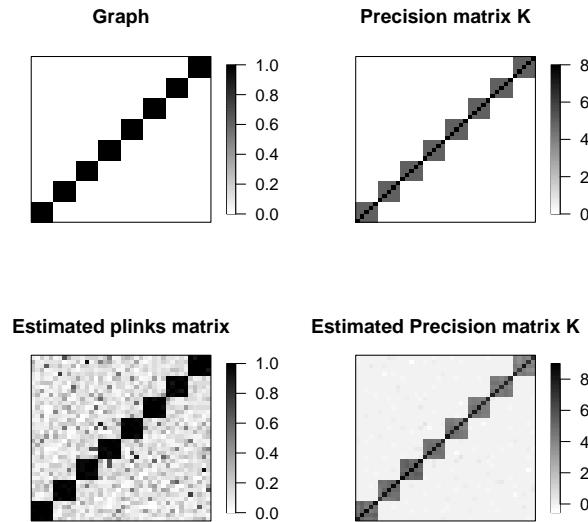
## Warning: Removed 10 rows containing missing values (`geom_point()`).
```





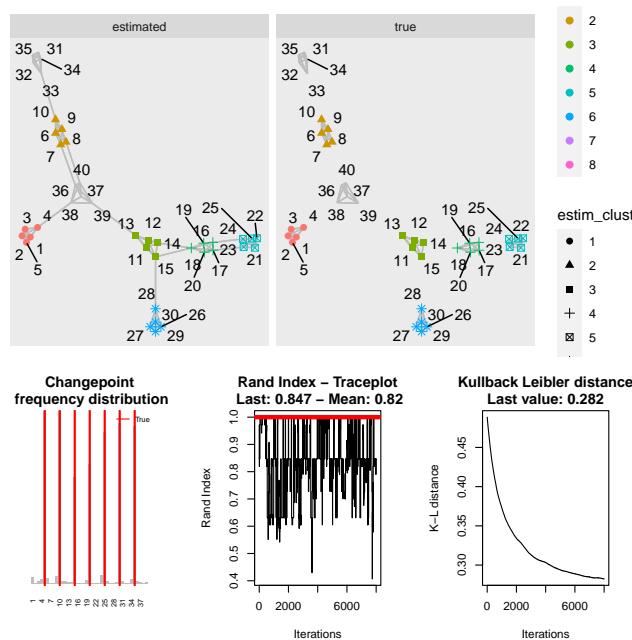
## 2.6.8 Simulation number 39

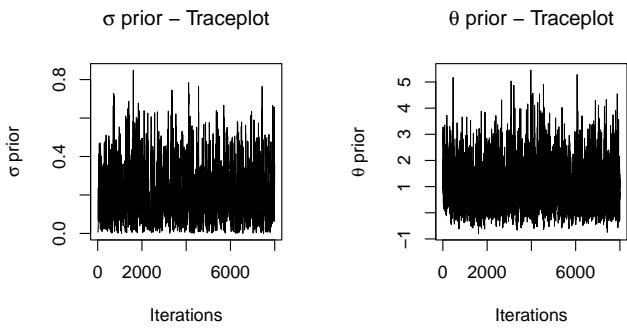
simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
39	500	40	BD	27121996	40	0.0625	5,5,5,5,5,5,5



```
## Warning: The shape palette can deal with a maximum of 6 discrete values because
## more than 6 becomes difficult to discriminate; you have 8. Consider
## specifying shapes manually if you must have them.

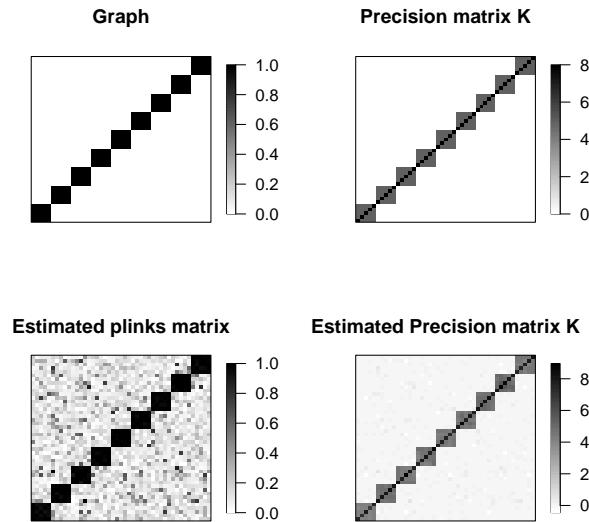
## Warning: Removed 20 rows containing missing values (`geom_point()`).
```





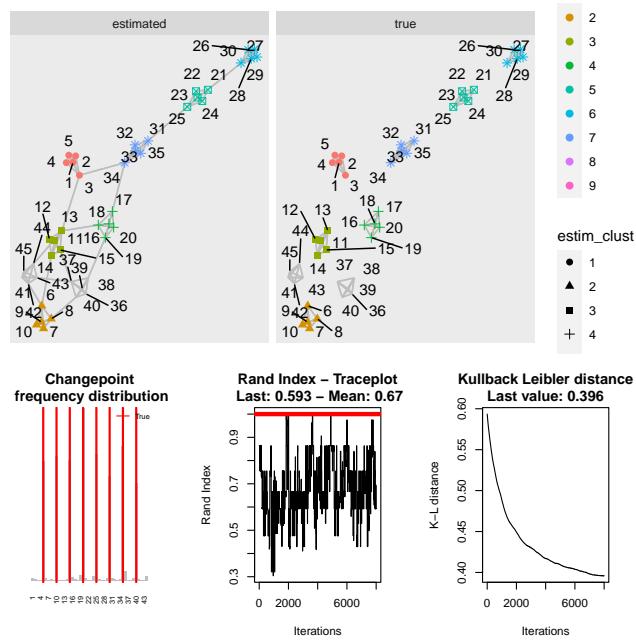
### 2.6.9 Simulation number 40

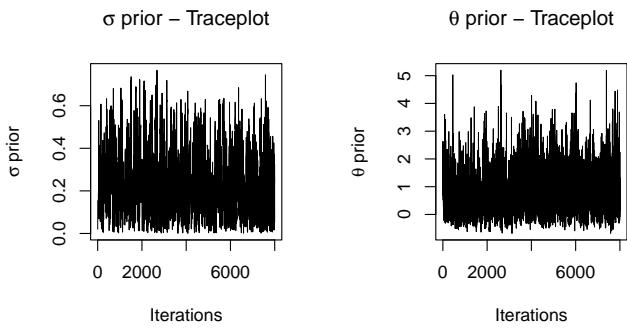
simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
40	500	45	BD	27121996	45	0.0625	5,5,5,5,5,5,5,5



```
## Warning: The shape palette can deal with a maximum of 6 discrete values because
## more than 6 becomes difficult to discriminate; you have 7. Consider
## specifying shapes manually if you must have them.
```

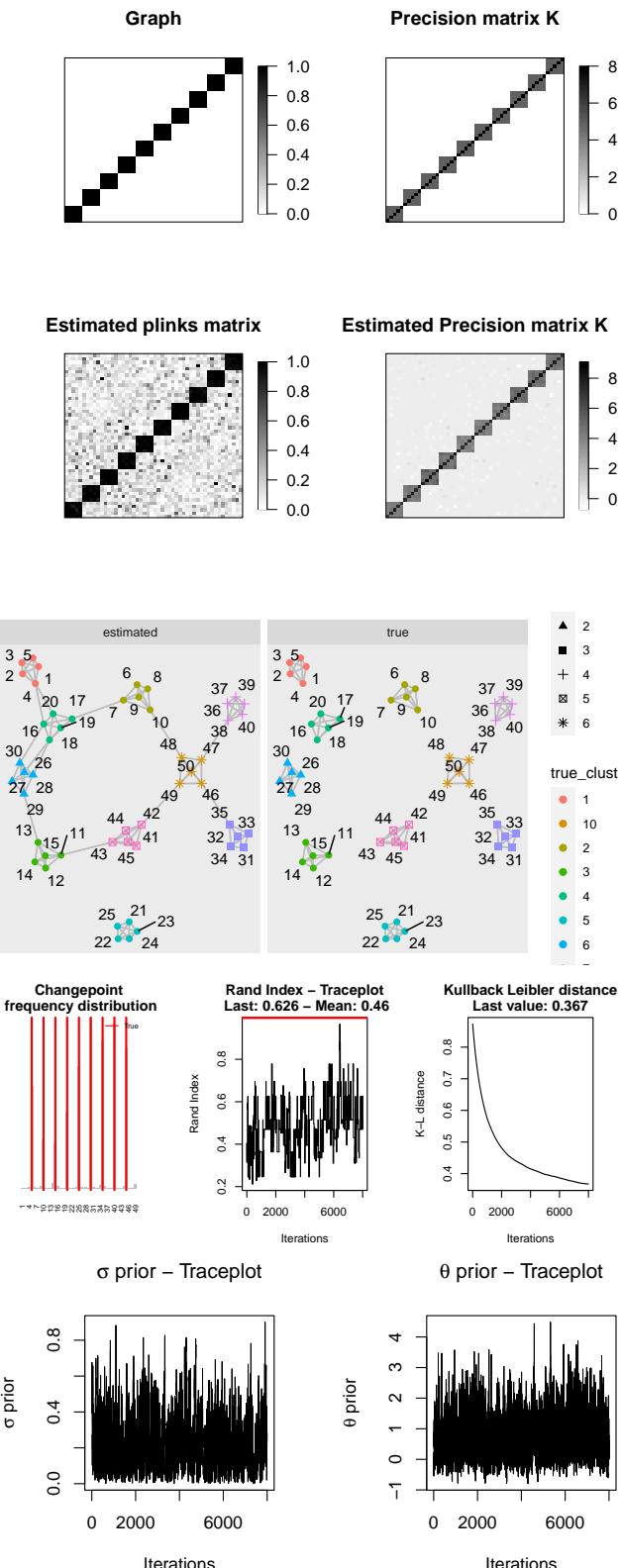
```
## Warning: Removed 20 rows containing missing values (`geom_point()`).
```





### 2.6.10 Simulation number 41

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
41	500	50	BD	27121996	50	0.0625	5,5,5,5,5,5,5,5,5,5



## 2.7 Group 7

### 2.7.1 Simulation number 42

simulation_id	n	p	type_data_gen	seed_data_gen	initial_partition	beta_sig2	rho_true
42	500	25	B	27121996	25	0.2	8,4,8,5

