

The price of uncertainty during COVID-19

A sensitivity analysis on an extended infection model using UQpy

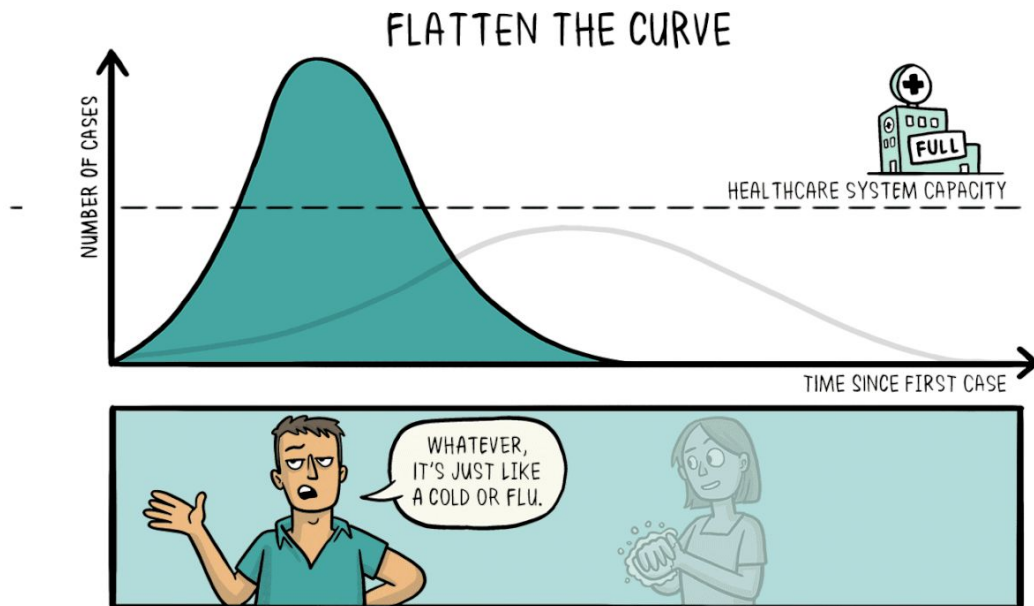
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March 28th, 2023



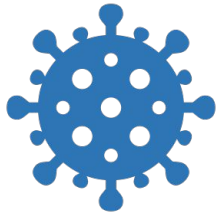
Summary

SIR models and
beyond

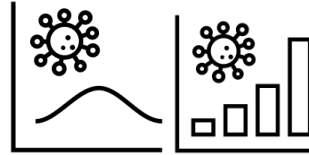
The UQpy
library

Results

Takeaways,
conclusions,
perspectives



UQpy



Project goal

Reproduce the results of a sensitivity analysis from a previously published work, using UQpy.






Focus: the dynamics of the first 180 days of a pandemic of COVID-19.

2458

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Understanding Dynamics of Pandemic Models to Support Predictions of COVID-19 Transmission: Parameter Sensitivity Analysis of SIR-Type Models

Chunfeng Ma , Member, IEEE, Xin Li , Senior Member, IEEE, Zebin Zhao , Feng Liu , Kun Zhang,
Adan Wu , and Xiaowei Nie

Crash course on compartmental models for epidemiology

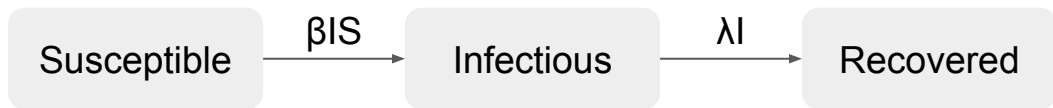
Compartmental models in epidemiology —

- Models in which people from a population of N individuals **move between compartments**
- The dynamics is essentially governed by the **initial conditions** and the **transition rates**

Compartmental models in epidemiology —●

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-

Simplest case: **SIR** (Susceptible, Infectious, Recovered)



β : infection rate

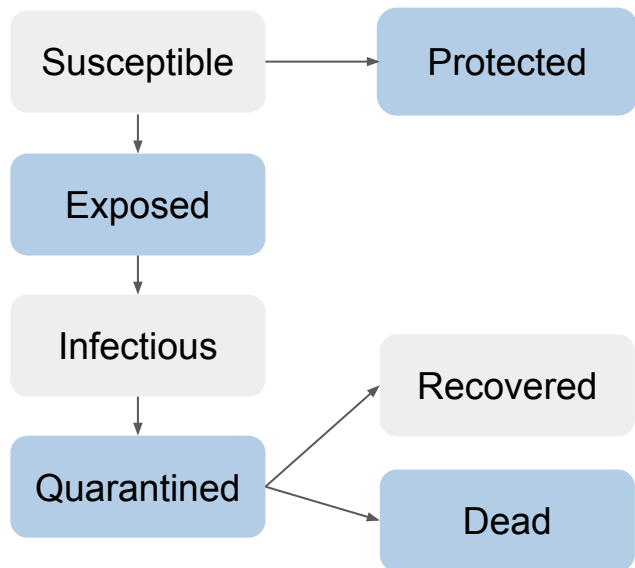
λ : recovery rate

$$\begin{aligned}\frac{dS(t)}{dt} &= -\beta \frac{S(t) I(t)}{N} \\ \frac{dI(t)}{dt} &= \beta \frac{S(t) I(t)}{N} - \lambda I(t) \\ \frac{dR(t)}{dt} &= \lambda I(t)\end{aligned}$$

Beyond SIR: SEIQRDP models

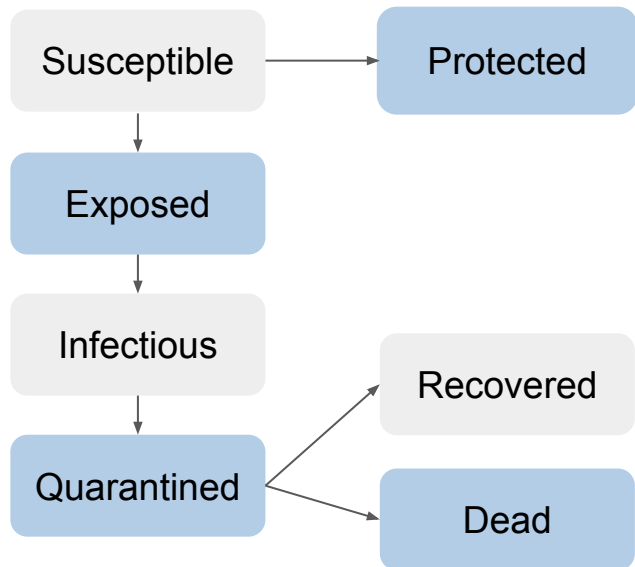


- **New compartments** to make the dynamics more realistic



Beyond SIR: SEIQRDP models

- **New compartments** to make the dynamics more realistic
- **New governing equations**



$$\frac{dS(t)}{dt} = -\beta \frac{S(t)I(t)}{N} - \alpha S(t)$$

$$\frac{dE(t)}{dt} = \beta \frac{S(t)I(t)}{N} - \gamma E(t)$$

$$\frac{dI(t)}{dt} = \gamma E(t) - \delta I(t)$$

$$\frac{dQ(t)}{dt} = \delta I(t) - \lambda Q(t) - \kappa Q(t)$$

$$\frac{dR(t)}{dt} = \lambda Q(t)$$

$$\frac{dD(t)}{dt} = \kappa Q(t)$$

$$\frac{dP(t)}{dt} = \alpha S(t)$$

N: total number of people

α: protection rate

β: infectious rate

γ⁻¹: average incubation time

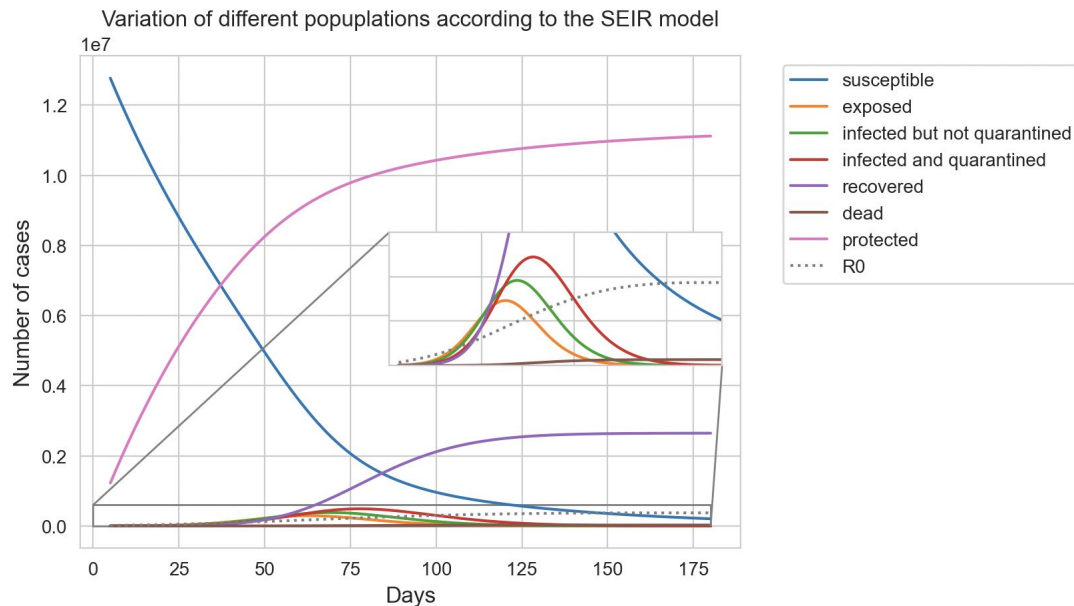
δ⁻¹: average quarantine time

κ: mortality rate

λ: cure rate

Ranges and values were set according to the paper.

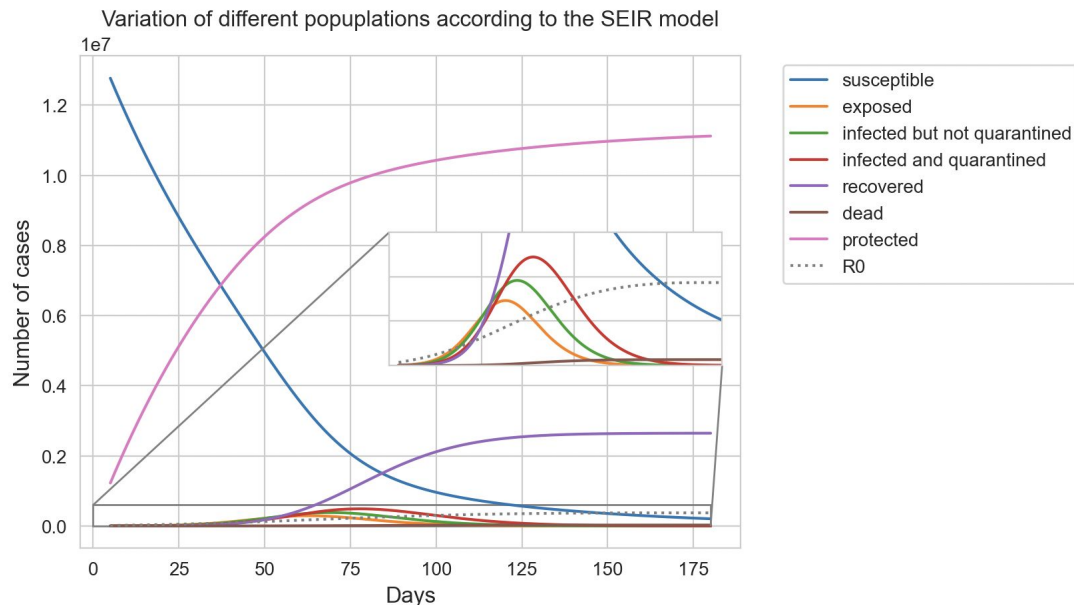
Reproducing the pandemic dynamics —●



The dynamics are close to the SIR model:

- the **initial** compartment S is slowly emptied
- the **intermediate** compartments E, I, Q present a bell shaped curve
- the **ending** compartments R, D, P are monotone non decreasing functions

Reproducing the pandemic dynamics —●



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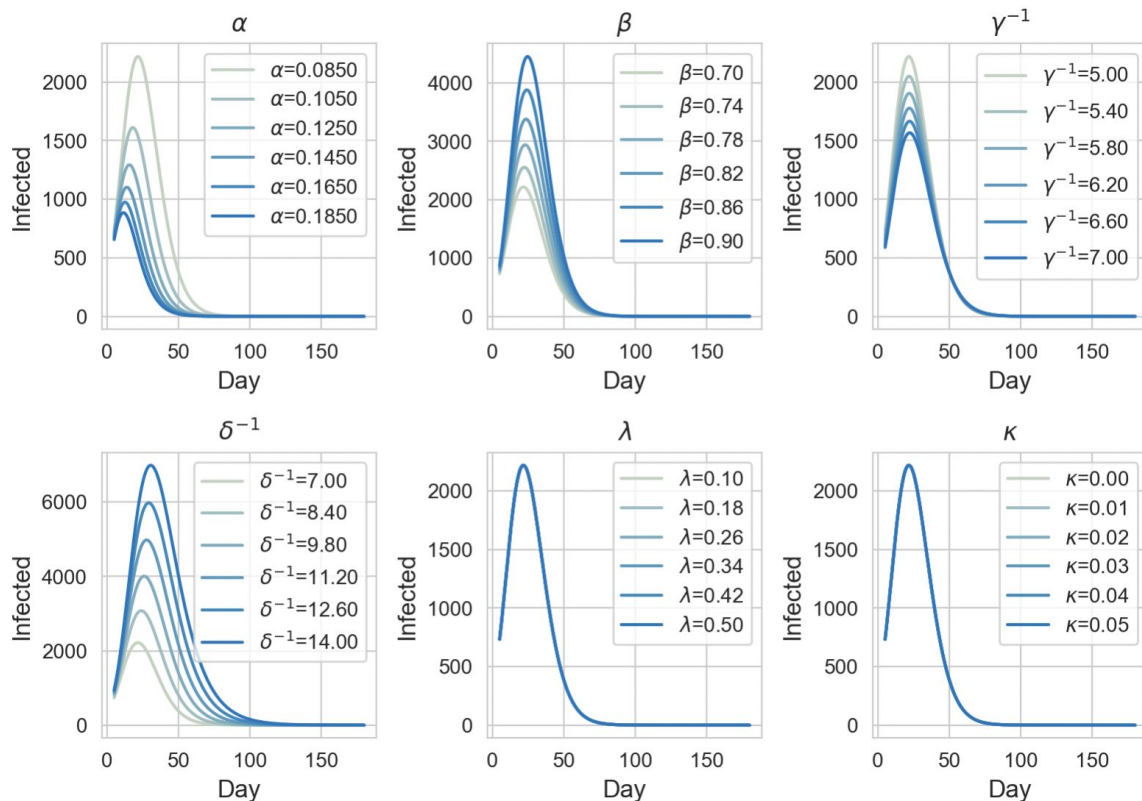
- the **initial** compartment S is slowly emptied
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Following the approach of the paper, from now on our output of interest will be the number of the **infectious cases I**

Varying parameters one-at-a-time



One-at-a-time effect of single parameters on the output



- With the light-blue parameter combination, the **peak of infectious cases** is around 2220, and is reached around day 16
- The **mortality rate κ** and **cure rate λ** do not seem to be influential on the infected cases
- The other parameters exert a higher influence, especially the **protection rate α**

On with the Global Sensitivity Analysis!

Quick recap of Sobol' indices

Exploiting the decomposition of variance, they quantify how much variability in the output each input accounts for.

First order

$$S_i = \frac{\text{Var}_{X_i} [E_{\mathbf{X}_{\sim i}} [Y | X_i]]}{\text{Var}[Y]}$$

Quantifies the **individual** effect of X_i on Y , **without interactions**

Total order

$$S_{T_i} = 1 - \frac{\text{Var}_{\mathbf{X}_{\sim i}} [E_{X_i} [Y | \mathbf{X}_{\sim i}]]}{\text{Var}[Y]}$$

Quantifies the **overall** effect of X_i on Y , **including interactions**

Quick recap of Sobol' indices

Exploiting the decomposition of variance, they quantify how much variability in the output each input accounts for.

First order

$$S_i = \frac{V_i}{V} = \frac{\text{Var}_{\mathbf{x} \sim_i}(\mathbb{E}(f(\mathbf{x})|\mathbf{x}_i))}{\text{Var}(f(\mathbf{x}))}$$

Total order

$$S_{T_i} = 1 - \frac{\text{Var}_{\mathbf{x} \sim_i}(\mathbb{E}(f(\mathbf{x})|\mathbf{x}_i))}{\text{Var}(f(\mathbf{x}))}$$

Main goals:

- factor prioritization
- factor fixing

The heavy artillery: UQpy

A general purpose Python **toolbox** for modeling uncertainty in physical and mathematical systems, exploiting a black-box approach.

Several tools for sensitivity analysis, including computation of **Sobol' indices**.

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 **Journal of Computational Science**

journal homepage: www.elsevier.com/locate/jocs



UQpy: A general purpose Python package and development environment for uncertainty quantification

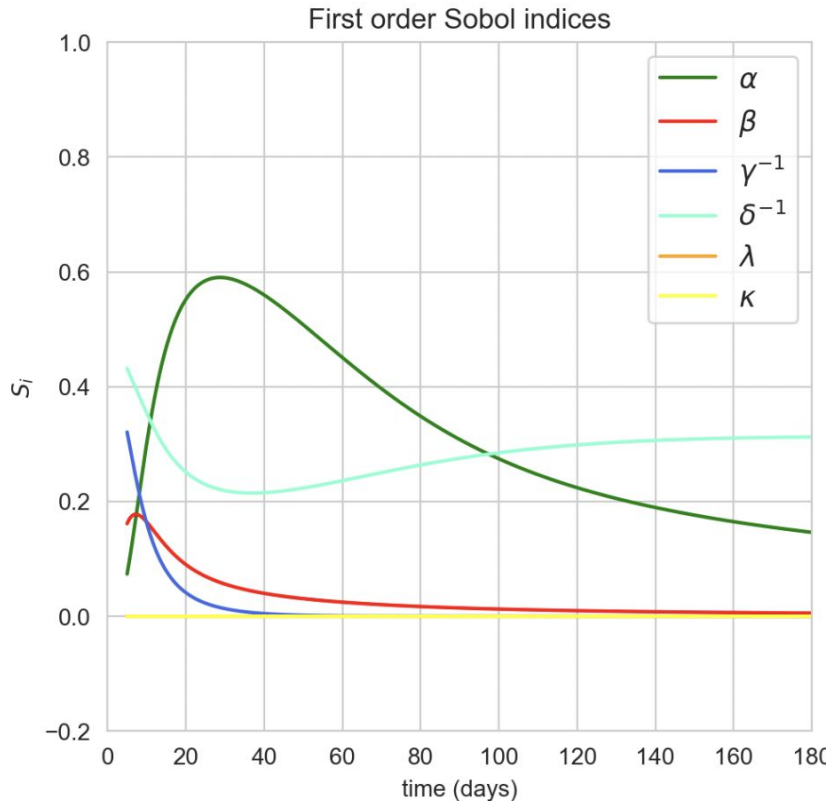
Audrey Olivier, Dimitris G. Giovanis, B.S. Aakash, Mohit Chauhan, Lohit Vandanapu, Michael D. Shields*

Department of Civil and Systems Engineering, Johns Hopkins University, Baltimore, MD, United States



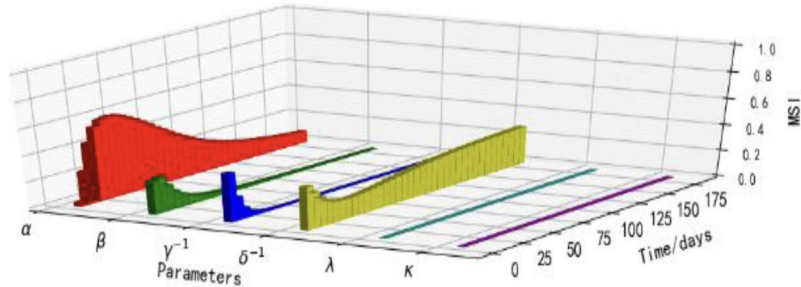
UQpy

First-order Sobol' indices on Infectious —●

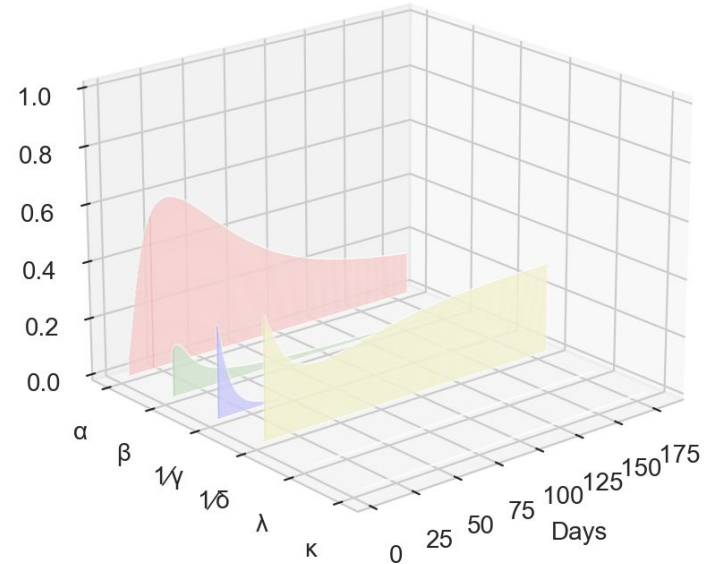


- As expected, the **protection rate α** and the **average quarantine time δ^{-1}** are highly influential on the infectious cases
- The influence of the **infectious rate β** and **average incubation time γ^{-1}** quickly decrease to 0
- The **mortality rate κ** and the **cure rate λ** always remain close to 0

First-order Sobol' indices on Infectious —●



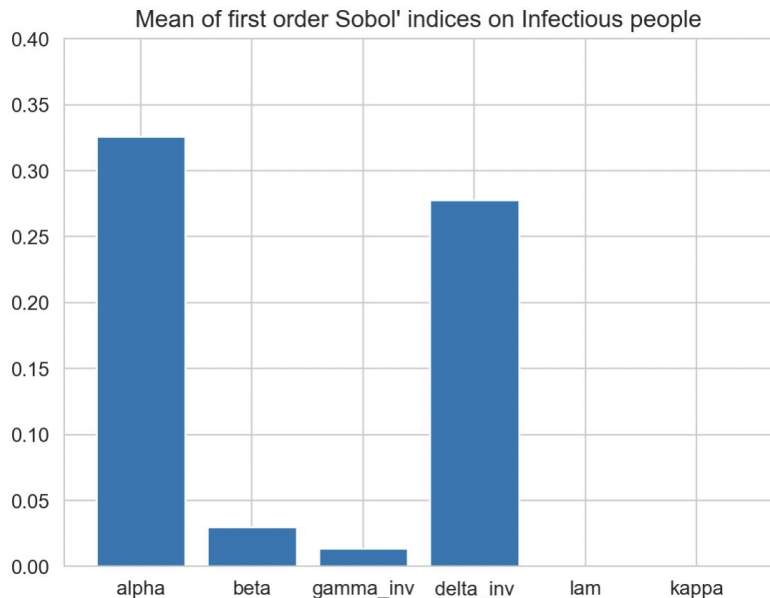
Original results



Reproduced results

Factor prioritization using Sobol' indices —●

We computed the **mean over time** of the first-order Sobol' indices.

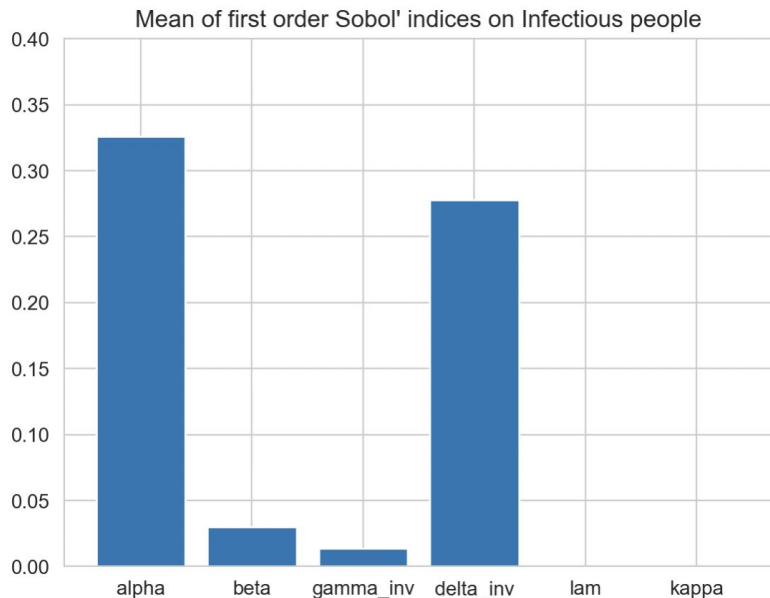


Factor ordered by influence:

1. α : protection rate
2. δ^{-1} : average quarantine time
3. β : infectious rate
4. γ^{-1} : average incubation time
5. κ, λ : mortality and cure rate

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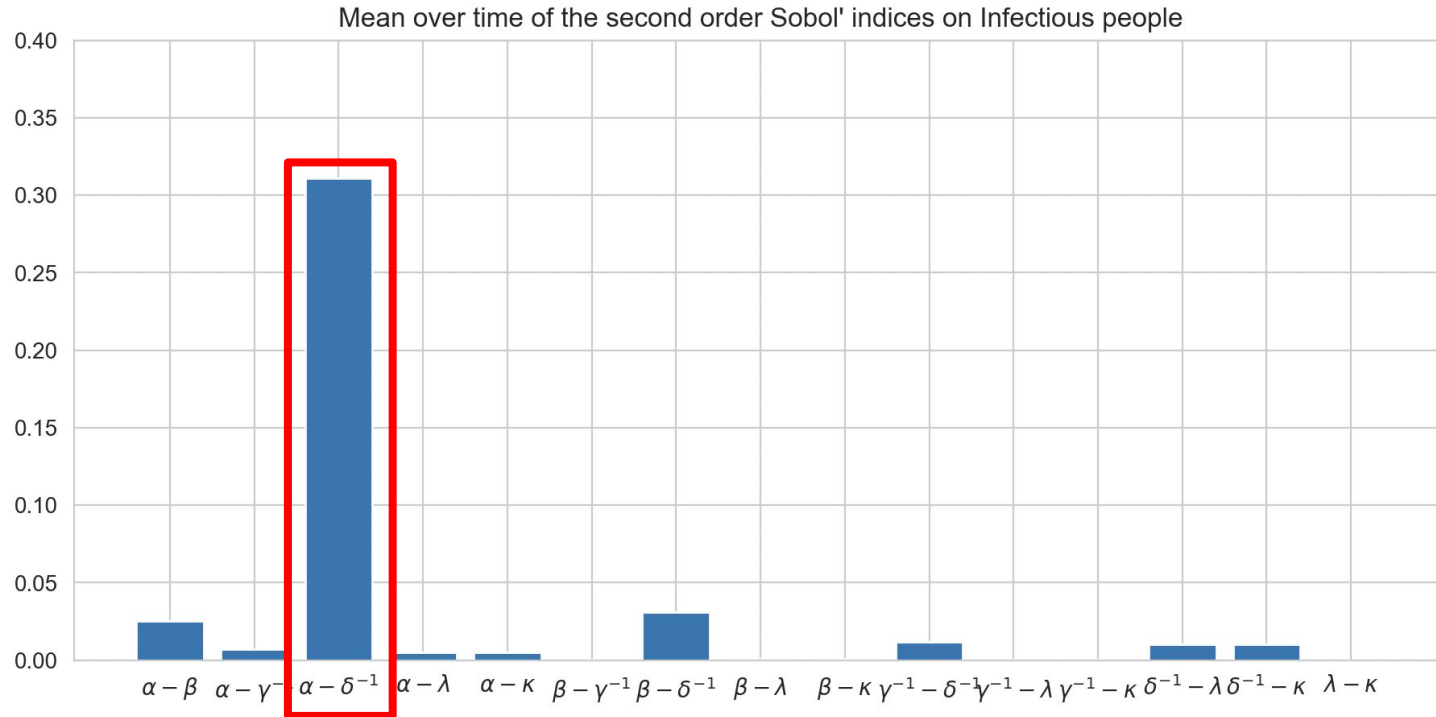
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Same conclusions obtained in the paper

Extending the results of the paper:
second order interactions

Mean of second-order Sobol' indices



The most influential interaction is between the **protection rate α** and the **average quarantine time δ^{-1}** , while the other interactions are less relevant.

We're talking about pandemics,
so what about R_0 ?

The basic reproduction number R_0



The average number of secondary infected cases generated by a primary case, crucial for transmissibility:

- $R_0 < 1 \rightarrow$ infection-free steady state is reached asymptotically
- $R_0 > 1 \rightarrow$ **the pandemic will outbreak**

Here computed as:

$$R_0 = \left(1 + \frac{\ln(I(t)/t)}{\gamma}\right) \left(1 + \frac{\ln(I(t)/t)}{\lambda}\right)$$

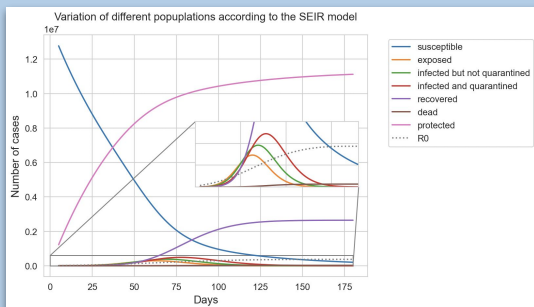
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Remember this image?

The values for R_0 do not really make sense (**R_0 plateauing around 10^6**)

R0: an unsolved mystery



After many attempts and doubts on the formula we **contacted the author.**

He confirmed that everything was correct.

Dear Teo and Flavia,

Sorry for the delayed reply—I was doing fieldwork in a remote mountainous area with no communication signal.

(1) According to Equation 11 and reference 11, $Y(t)$ is the total number of infected people given by $I(t)$, without $Q(t)$.

(2) inside Log.

(3) No, γ and λ are variables to be analyzed for their sensitivity.

Hope this reply is clear.

Please hesitate to contact us if you have further questions.

Best regards,

Chunfeng Ma

More investigating on this topic
is needed.

Conclusions

- **Main takeaways**

- In the extended SIR model, the **protection rate α** is the most influential on the infectious cases, together with the **quarantine time δ^{-1}** , and their **interaction**
- Other factors such as the **mortality rate κ** and the **cure rate λ** can be considered as **fixed** for modeling purposes

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- Other factors such as the **mortality rate κ** and the **cure rate λ** can be considered as **fixed** for modeling purposes

- **Future improvements**

- **Solving the mystery of R_0**
- Extend the analysis to other **quantities of interest** such as the number of quarantined cases (Q)
- Apply different methods (e.g., **moment-independent sensitivity measures**)

Thank you for the attention!

References

- **Main reference paper**

Ma, C., Li, X., Zhao, Z., Liu, F., Zhang, K., Wu, A., & Nie, X. (2022). Understanding Dynamics of Pandemic Models to Support Predictions of COVID-19 Transmission: Parameter Sensitivity Analysis of SIR-Type Models. *IEEE Journal of Biomedical and Health Informatics*, 26(6), 2458-2468. <https://doi.org/10.1109/JBHI.2022.3168825>

- **UQpy paper**

Olivier, A., Giovanis, D. G., Aakash, B. S., Chauhan, M., Vandanapu, L., & Shields, M. D. (2020). UQpy: A general purpose Python package and development environment for uncertainty quantification. *Journal of Computational Science*, 47, 101204. <https://doi.org/10.1016/j.jocs.2020.101204>

Any questions?

UQpy methods



Several methods to compute the Sobol indexes

- **First-order:**
 - Jensen (2014)
 - **Saltelli (2002)**
 - Sobol (1983)
- **Second-order:**
 - **Saltelli (2002)**
- **Total order:**
 - Homma (1996)
 - **Saltelli (2002)**