Multi Agents Systems
Group 48: Assignment 4

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1 Monte Carlo Simulation

1.1 MC Sampling

1) When we test our function on a sample of size n = 100000, we find a mean E(X)= 0.727 and a standard deviation sd = 0.002.

2) For the purpose of this assignment, we assume A and S to be independent and normally distributed. then, with a random sample of 10 numbers taken from each, we calculate the correlation. This was done for a total of 1000 times. We then compared the correlation coefficient between the two samples to the observed correlation of 0.3. Then, a one-sample t-test was performed, to compare the resulting sample of correlation coefficient with observed r = 0.3. The results yielded t = -28.52 with p-value < 0.001, meaning that the sample mean differs significantly from the observed population, yet, taking into consideration that 21% of samples had a correlation bigger than 0.3, one can say, that correlation is not, in fact significant.

1.2 Importance Sampling

1) First of all, we pulled 100k samples from the uniform distribution having parameters [-5, 5]. After that, we divided two elements. On the numerator, we had the density coming from the normal distribution; on the denominator, we had the density coming from the uniform distribution. Finally, we took the result of that and multiplied it by the square of the samples, being the goal function. Results show that the following values (sd is standard deviation):

$$\bar{\mu} = 1.005$$

$$sd = 0.003$$

2) This point uses a very similar procedure of importance sampling w.r.t. the previous exercise. Again, out focus is in finding an estimation for the second moment $E[X^2]$ through importance sampling. The only difference here is that, instead of pulling samples from a [-5, 5] uniform, our new parameters for the uniform are [-1, 1].

1.3 Kullback-Leibler divergence

1)

Given:

$$KL(f\|g) = \int_{-\infty}^{\infty} f(x) \log\left(\frac{f(x)}{g(x)}\right) dx$$

where $f(x)$ & $g(x)$ P distributions are: $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$$\Rightarrow \log\left(\frac{f(x)}{g(x)}\right) = \log\left(\frac{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\frac{1}{r\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-v}{r}\right)^2}}\right)$$

$$= \log\left(\frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}} \cdot \frac{r\sqrt{2\pi}}{e^{-\frac{1}{2}\left(\frac{x-v}{r}\right)^2}}\right)$$

$$= \log\left(\frac{r\sqrt{2\pi}}{\sigma\sqrt{2\pi}}\right) - \log\left(e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2 + \frac{1}{2}\left(\frac{x-v}{r}\right)^2}\right)$$

$$= \log\left(\frac{r}{\sigma}\right) - \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2 + \frac{1}{2}\left(\frac{x-v}{r}\right)^2$$

THUS:

$$KL(f\|g) = \int f(x) \log\left(\frac{r}{\sigma}\right) + \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2 + \frac{1}{2}\left(\frac{x-v}{r}\right)^2 \, dx$$

SEMPLIFIED:

$$-\frac{1}{2} + \frac{\sigma + (\mu - v)^2}{2r^2} \Longrightarrow KL(f\|g) = \log\left(\frac{r}{\sigma}\right) - \frac{1}{2} + \frac{\sigma + (\mu - v)^2}{2r^2}$$

The densities cannot be interchanged, as they are not symmetric. It can be seen by the very divergent position of the densities in the final expression 2)Theoretical KL-divergence estimate was calculated based on the formula above, resulting in KL = 0.35. Simulated KL = 0.31 with se = 0.007.