Multi Agents Systems
Group 48: Assignment 2

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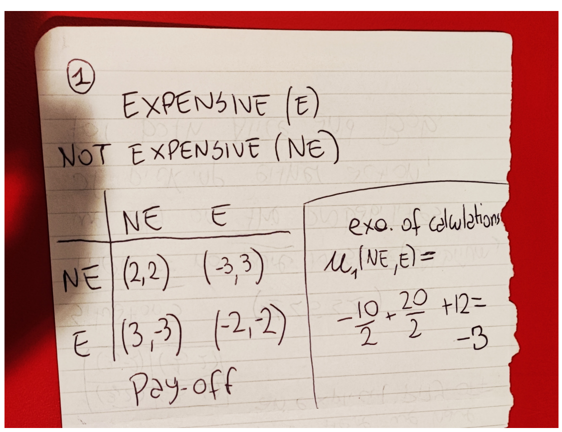
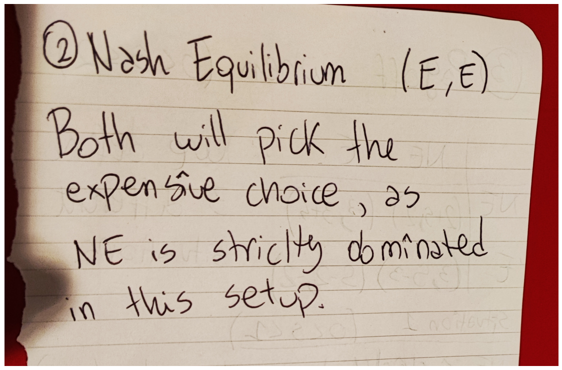
1 Dining out

For the sake of this exercise, the meal labels have been renamed to 'EXPENSIVE (E)' and 'NOT EXPENSIVE (NE), as it felt easier solving the issue with this terminology. We will keep this along the whole exercise.

1.1 Write down the pay-off matrix



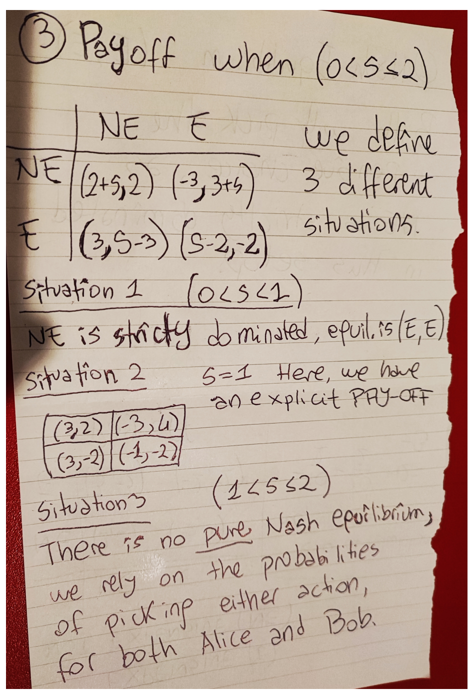
1.2 Assuming that they both order simultaneously and without coordinating, what will they order and why?



② Nash Equilibrium (E,E)

Both will pick the expensive choice, as NE is striclty dominated in this setup.

1.3 Alice is quite the romantic type and gets an additional s Euro's worth of pleasure if they happen to pick the same meal (either both cheap or both expensive). Bob, on the other hand, is a bit of a contrarian and gets an additional amount of pleasure (also equivalent to s Euro) when they happen to favour different meal choices. Assume that 0 < s 2. How does this change the pay-off matrix and the Nash equilibrium (or equilibria) of this game?



③ Payoff when $(0<s\leq 2)$

	NE	E
NE	$(2+s,2)$	$(-3,3+s)$
E	$(3,s-3)$	$(s-2,-2)$

we define 3 different situations.

Situation 1 $(0<s<1)$

NE is stricty dominated, equil. is (E,E)

Situation 2 $s=1$ Here, we have an explicit PAY-OFF

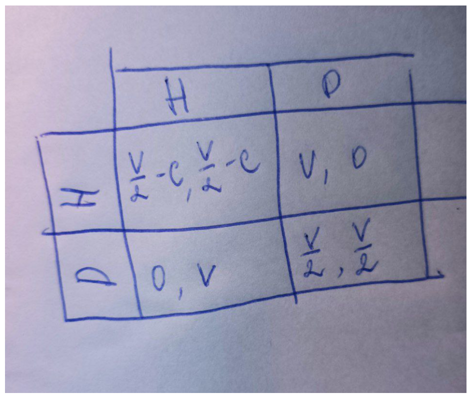
(3,2)	(-3,4)
(3,-2)	(-1,-2)

Situation3 $(1<s\leq 2)$

There is no pure Nash equilibrium, we rely on the probabilities of picking either action, for both Alice and Bob.

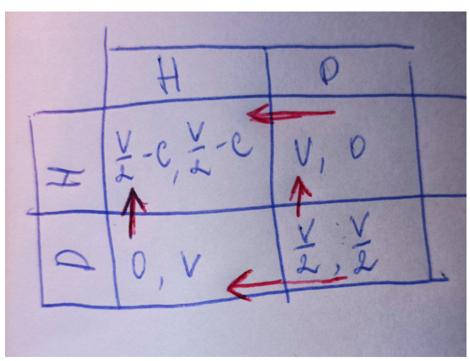
2 Hawk versus Dove

2.1 Write down the pay-off matrix for this game

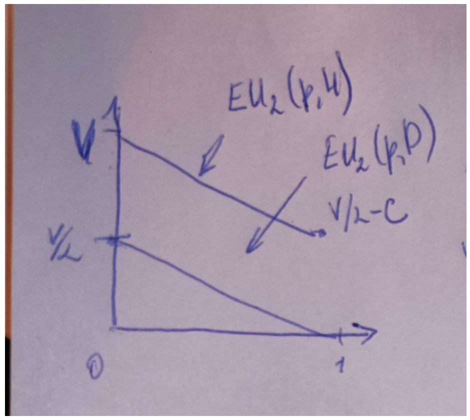


2.2 Determine the Nash equilibria for this game and discuss how they change as the cost of aggression (c) increases. Do your results make sense?

Case 1. v/2 > c => v/2 - c >0

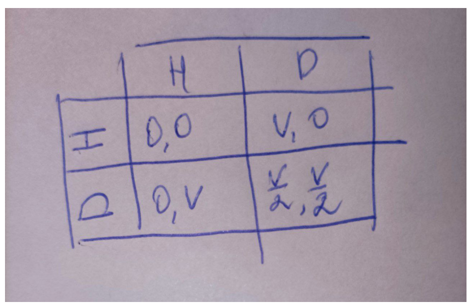


As can be seen from the graph, if v/2>c, there is a Pure Strategy nash equilibrium (H,H)



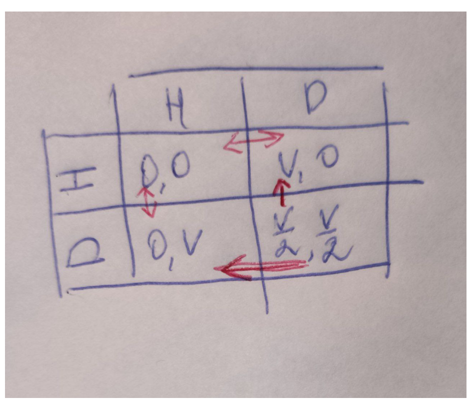
Case 2. V/2 = c => v/2 - c = 0

First of all, the payoff matrix needs to be updated:

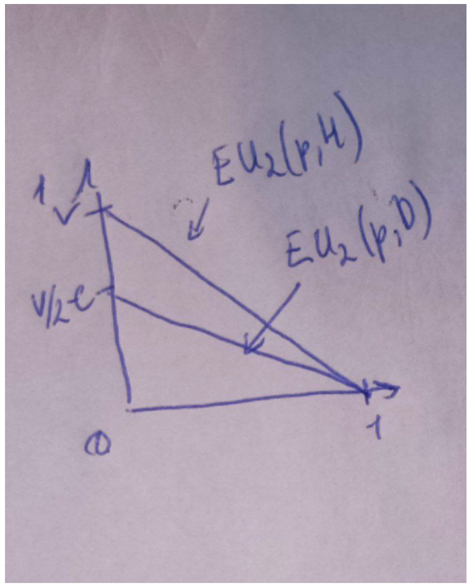


	H	D
H	0,0	V,0
D	0,V	V/2,V/2

If we add arrows, we see that H weakly dominates D for both animals



	H	D
H	0,0	V,0
D	0,V	V/2,V/2



Case 3. v/2 < c => v/2 - c < 0

First, we need to update payof matrice, with - a = c-v/2

		q	1-q
		H	D
p	H	$-a,-a$	$v, 0$
1-p	D	$0, v$	$\frac{v}{2}, \frac{v}{2}$

=> Mixed Nash Equilibrium
2 ~~PH~~PNE (H,D) & (D,H)

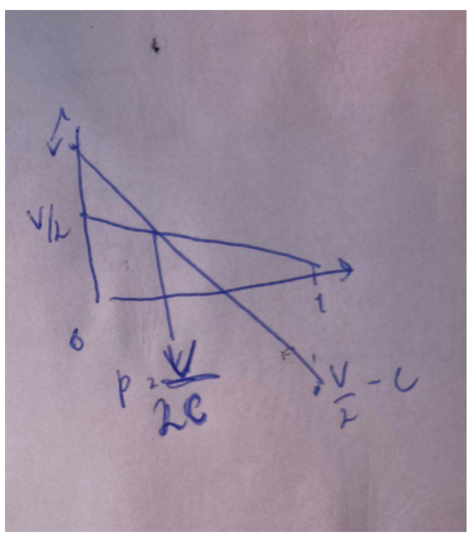
Then, we calculate the p and q

$$EU_2(p,H) = -a \cdot p + v(1-p)$$
$$EU_2(p,D) = 0 \cdot p + \frac{v}{2}(1-p)$$
$$-ap + v - vp = \frac{v}{2} - \frac{vp}{2} \quad | \cdot 2$$
$$-2ap + 2v - 2vp = v - vp$$
$$-2ap + 2v - vp = v$$
$$-p(2a+v) + 2v = v$$
$$-p(2a+v) = -v$$
$$p(2a+v) = v$$

$$EU_1(q,H) = -aq + 0(1-q)$$
$$EU_1(q,D) = vq + \frac{v}{2}(1-q)$$

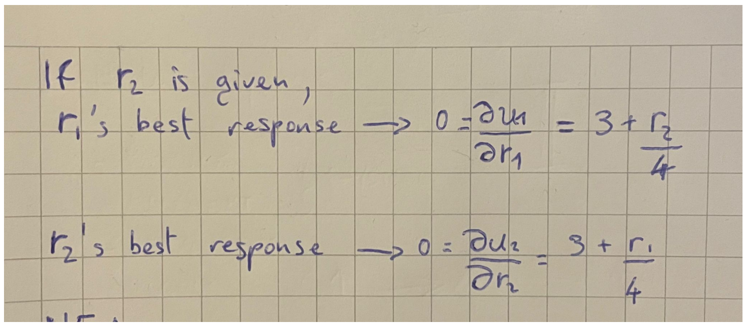
$$-aq = vq + \frac{v}{2}(1-q)$$
$$-aq - vq = \frac{v}{2} - \frac{vq}{2} \quad | \cdot 2$$
$$2aq - 2vq = v - vq$$
$$2aq - 2vq + vq = v$$
$$-2aq - vq = v$$
$$-q(2a+v) = v$$

$$\Rightarrow p = \frac{v}{2a+v} = \frac{v}{2(c-\frac{v}{2})+v} = \frac{v}{2c-v+v} = \frac{v}{2c}$$
$$q = -p.$$



3 Investment in recycling

3.1 Determine each country's best-response function

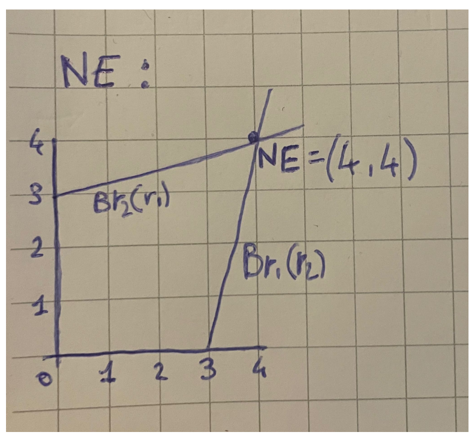


If r_2 is given,

r_1's best response $\longrightarrow 0 = \frac{\partial u_1}{\partial r_1} = 3 + \frac{r_2}{4}$

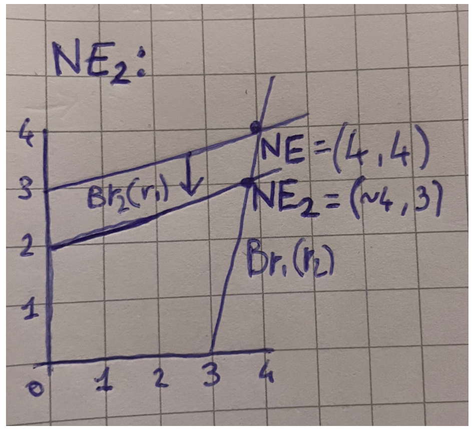
r_2's best response $\longrightarrow 0 = \frac{\partial u_2}{\partial r_2} = 3 + \frac{r_1}{4}$

3.2 Indicate the pure strategy Nash Equilibrium (r1, r2) on the graph



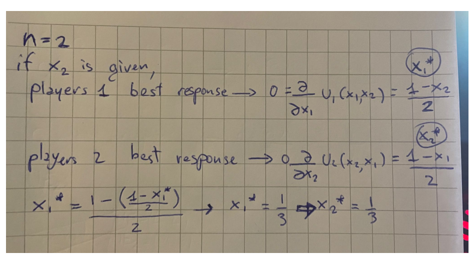
3.3 On your previous figure, show how the equilibrium would change if the intercept of one of the countries' average benefit functions bi fell from 10 to some smaller number. What would this mean for the recycling efforts of both countries?

In this case, where the intercept is reduced but the slope remained constant, country 2's effort has been reduced to a very high extend, while effort of country one did only reduce to a very small extend.



4 Tragedy of the commons

4.1 Consider the special case where there are only two players (i.e. n = 2). Determine the individual shares x1 and x2 in the Nash equilibrium for this game.



4.2 Does this Nash equilibrium optimise social welfare which is the aggregated utility of all players (i.e. $\sum_{i=1}^{n} u_i(x)$)?

We get optimal social welfare when x_i claims $\frac{1}{2n}$. Though, NE shows much larger amounts: $x_i^* = \frac{1}{n+1}$. This is the reason that social welfare is reduced.

4.3 Can you generalise this result to arbitrary n?

$n =$ arbitrary

player i best response $\rightarrow 0 = \frac{\partial}{\partial x_i} u_i(x_i, x_{-i}) = 1 - 2x_i - S_{-i}$

so : $x_i^* = \frac{1 - S_{-i}}{2}$

$x_1^* = x_2^* = x_3^* = \dots = x_n^* \longrightarrow S_{-i}^* = (n-1)x_i^*$

$$x_i^* = \frac{1 - S_{-i}^*}{2}$$

$$2x_i^* = 1 - (n-1)x_i^*$$

$$x_i^* = \frac{1}{n+1}$$