Multi Agents Systems
Group 48: Assignment 1

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1 Iterated Best response dynamics for a simple matrix game

1.1

No, this simple matrix game does not show signs of dominated strategies.

1.2

In that case, we would have an Iteration Equilibrium issue. This works by picking one action at the time in between players, watching your opponent play and react accordingly: not simultaneous moves but sequential ones.
Iterating like this, we end up reaching an equilibrium position that is stable for both players. In this exercise, such equilibrium is found in (M, C), with a payoff of (7, 6). Indeed, the players final's payoff of (7,6) is the Nash Equilibrium.

1.3

Not every simple game can yield such type of equilibrium given, from a rational thinking in steps. A perfect counterexample is trying to match the outcome of 2 coins. Doing that, the two players would keep on changing their best move in a infinite loop or cycle, being unable to find an equilibrium.

2 Traveler's Dillemma

2.1

The payoff matrix can be seen in table 1

2.2

In this example, we do have multiple pure Nash equilibria, specifically 3. These being 1,1 2,2 and 3,3.

Table 1: Payoff matrix

	1	2	3
1	1, 1	1+a, 1-a	1+a, 1-a
2	1-a, 1+a	2, 2	2+a, 2-a
3	1-a, 1+a	2-a, 2+a	3, 3

2.3

Person A
E1 = 1(q1) + (1+a)(q2) + (1+a)(1-q1-q2)
= 1 - aq1 +a
E2 = (1-a)(q1) + 2(q2) + (2+a)(1-q1-q2)
= 2 + a - q1 - 2aq1 - aq2
E3 = (1-a)(q1) + (2-a)(q2) + (3)(1-q1-q2)
= 3 - 2q1 - q2 - aq1 - aq2

Person B
E1 = 1(p1) + (1+a)(p2) + (1+a)(1-p1-p2)
= 1 - ap1 +a
E2 = (1-a)(p1) + 2(p2) + (2+a)(1-p1-p2)
= 2 + a - p1 - 2ap1 - ap2
E3 = (1-a)(p1) + (2-a)(p2) + 3(1-p1-p2)
= 3 - 2p1 - p2 - ap1 - ap2

Since the expected value formulas seem to be the same for both players, we're gonna derive the values of probabilities only once. To do so, we need to insist, that the players are indifferent between the outcomes:
1 - aq1 +a = 2 + a - q1 - 2aq1 - aq2 and
1 - aq1 +a = 3 - 2q1 - q2 - aq1 - aq2
bringing everything to the left side, gives:
- 1 + aq1 + q1 + aq2 = 0 and
- 2 + a + q2 + aq2 = 0
solving for this equations gives:
$q1 = \frac{a^2-a+1}{(a+1)^2}$
$q1 = \frac{2-a}{(a+1)}$

Which, therefore assumes that there is one MNE, if 0 < a < 0.5

2.4

This knowledge about the NE does not help the travelers in their decision.

2.5

The regret matrix can be seen in table 2

Table 2: Regret matrix

	1	2	3
1	a,a	0, 1+2a	0, 2+a
2	1+2a, 0	a,a	0, 1+a
3	2+a, 0	1+a,0	0,0

2.6

For both players, regret minimisation strategy is to play 1 all the time.

3 Cournot's Duopoly

3.1

In order to find best response, one needs to determine the profit formula:
profit1 = p*q1 - q1*c1 = q1*(p - c1) = q1*(a - b(q1+q2) - c1)
profit2 = p*q2 - q2*c2 = q2*(p - c2) = q2*(a - b(q1+q2) - c2)
and then take derivative of them in respect to quantity and solve them, setting them for zero.

$$\frac{\partial profit1}{\partial q2} = \alpha - 2\beta q1 - \beta q2 - c1$$
$$q1 = \frac{\alpha - c1}{2\beta} - \frac{q2}{2}$$
similairly, for the second company:

$$\frac{\partial profit1}{\partial q1} = \alpha - 2\beta q2 - \beta q1 - c1$$
$$q2 = \frac{\alpha - c2}{2\beta} - \frac{q1}{2}$$

3.2

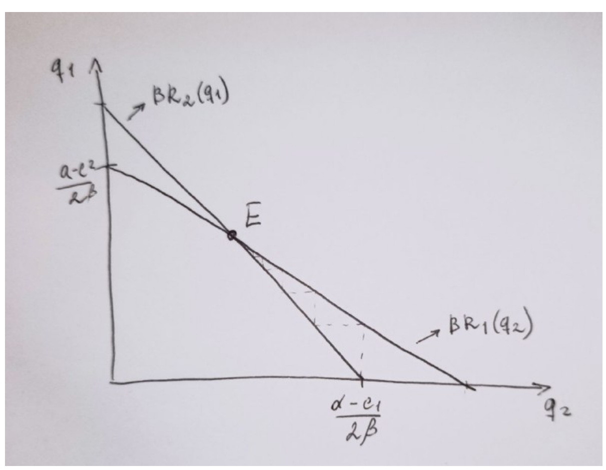

Fig. 1: Expected Utility at different positions x

4 Ice cream time!

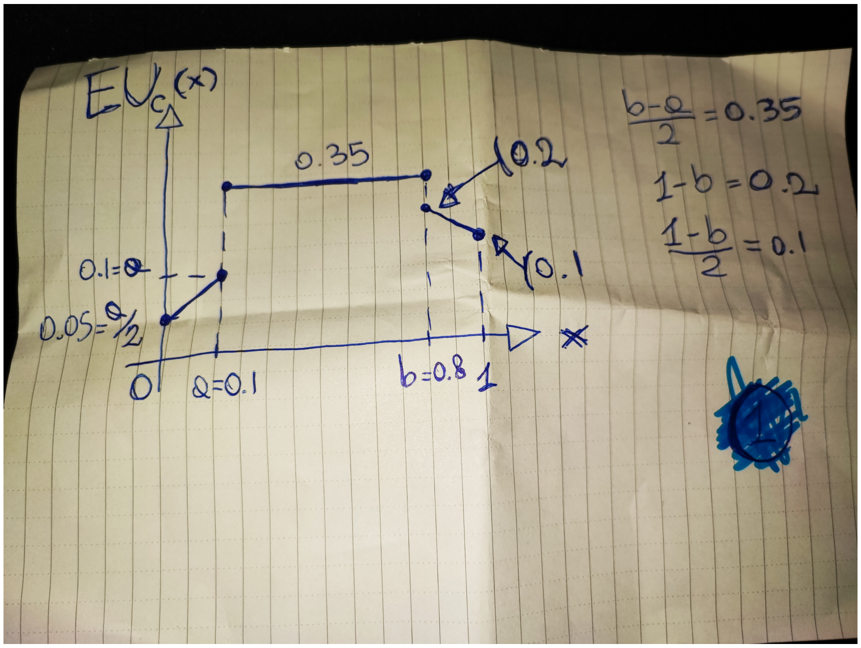

Fig. 2: Expected Utility at different positions x

4.1

We found a function for the Expected Utility (EU), given a position x, seen in Figure 2. Before position 0.1, the Expected Utility is rising from 0.05 to 0.1. Between 0.1 and 0.8, it keeps stable at 0.35. After Bob's positioning at 0.8, it lowers from 0.2 to 0.1.

4.2

We now have a parameter for Bob's positioning: we again split the position domain into 3 parts. For x ranging from 0 to a (where a is the position of Alice), we have an EU following the formula:

$$EU_c(x, b) = \frac{x+a}{2}$$

If the position is in between Alice and Bob, we have an EU of (b-a)/2.
Finally, if our position is to Bob's right b<x<1, the yielded EU is the following:

$EU_c(x,b) = \frac{b-a}{2}$

Finally, when b<x<1, we have an EU of:

$EU_c(x,b) = 1 - \frac{x-b}{2}$

4.3

We split Bob's decision, based on the context we are in. For instance, Bob's decision does not change based on Charlize if:

$\frac{b-a}{2} = 1 - b \qquad b = \frac{a+2}{3}$

Having an utility value set to (1-b). Applying the equation for b to this formula, we get $\frac{1-a}{3}$

In the case where $\frac{b-a}{2} > 1 - b$, Bob will end up sitting to the right of Charlize: his utility would then be $(1-b) < 0.3$.
In the case where $\frac{b-a}{2} < 1 - b$, Bob will end up sitting to the left of Charlize: his utility would then be $\frac{b-a}{2} < 0.3$.

Finally, Bob should choose the $\frac{a+2}{3}$ position, yielding an utility of $\frac{1-a}{3}$.

4.4

Bob will always come in after her, meaning that he will always put himself in a position where his utility is at its maximux point. This can be done because he knows everybody's previous positions before hand, and therefore can come in right next to Alice, increasing his utility.