

## Multi Agents Systems

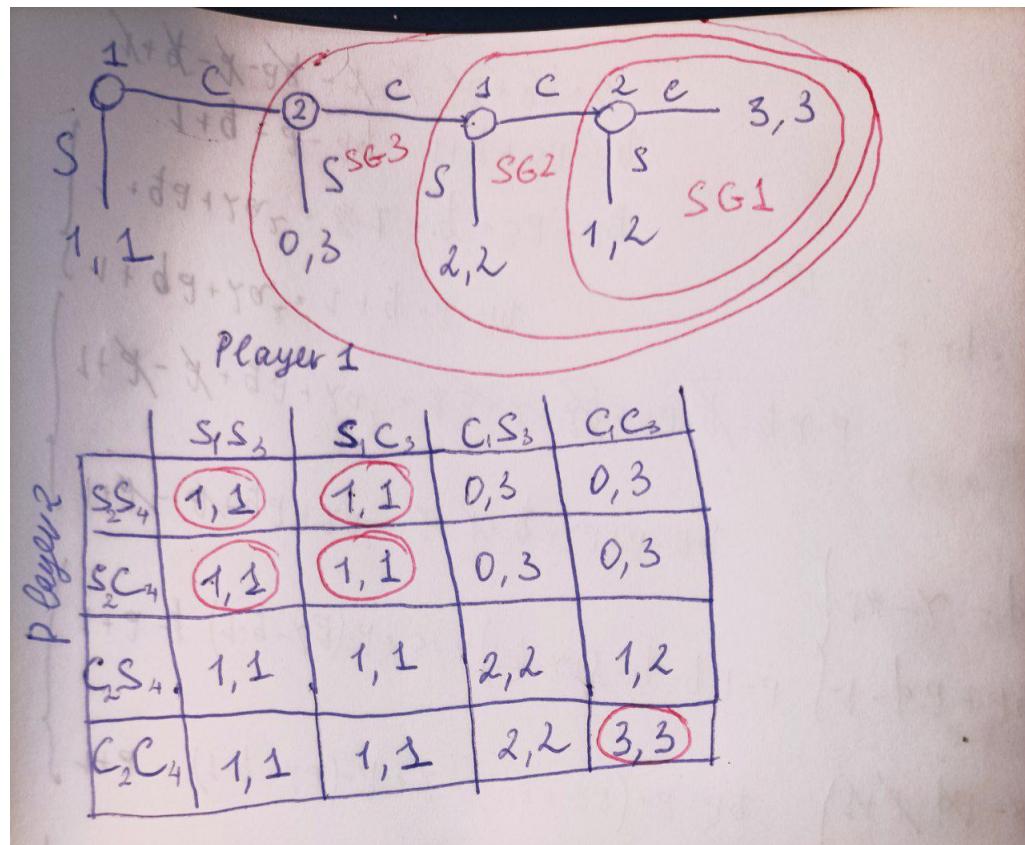
### Group 48: Assignment 3

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### 1 Reduced centipede game

- 1.1 Write the normal form for this game and find all Nash equilibria in pure strategies (PNEs).



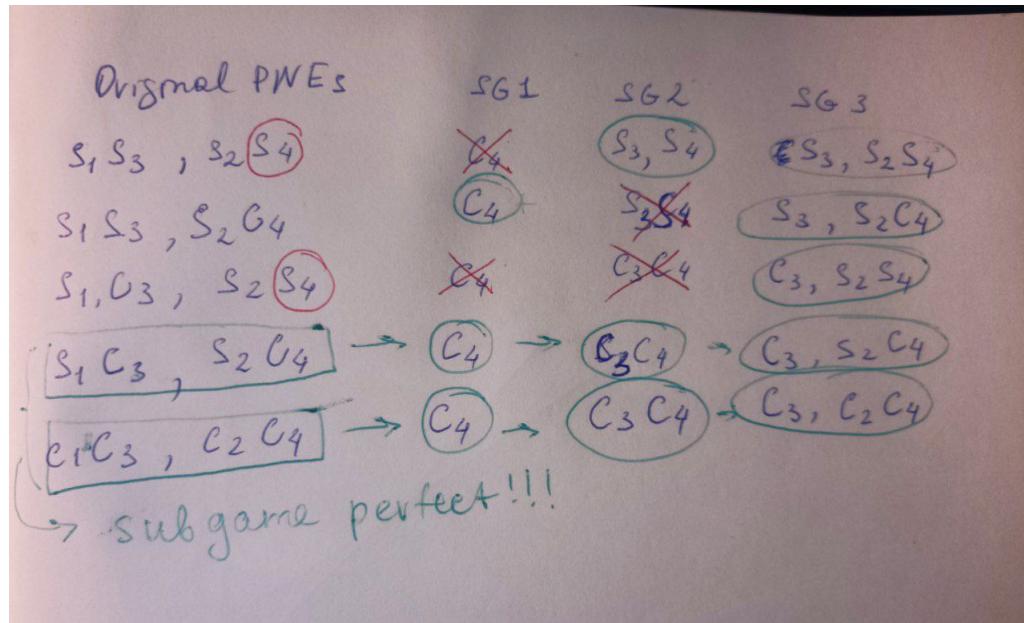
**1.2 List all subgames and determine which of these PNEs are also subgame-perfect?**

SG 1		SG 2	
P1	C1C3	P1	C1C3
C2S4	1, 2	C2S4	2, 2
C2C4	(3, 3)	1, 2	(3, 3)
P2	Pure NE	P2	PNE

SG 3		P2	
P1	C1C3	C2S4	C2C4
S2S4	0, 3	0, 3	1, 2
S2C4	0, 3	0, 3	1, 2
C1S4	0, 3	0, 3	(3, 3)
C1C3	0, 3	0, 3	(3, 3)

} PNEs.

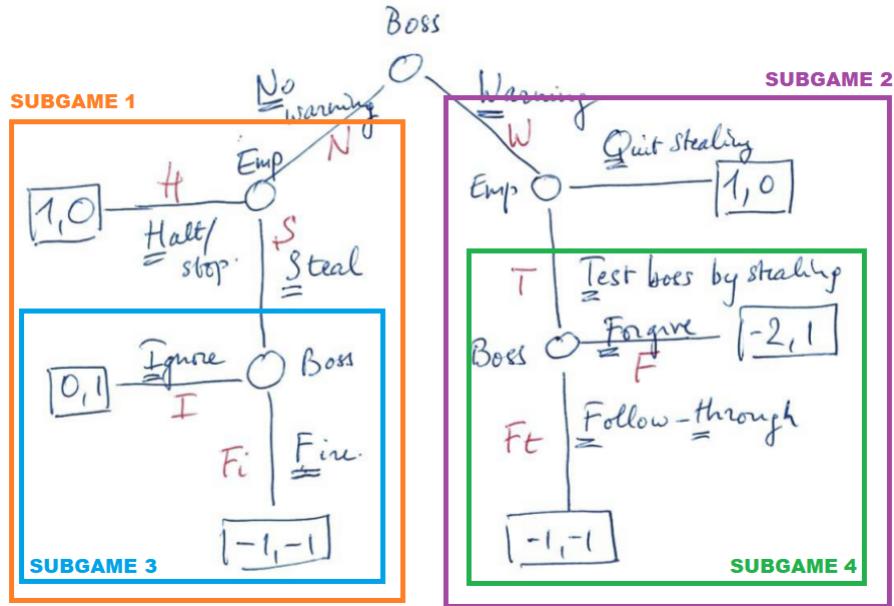


### 1.3 Can you solve this game using backward induction? Discuss.

It is not possible to solve this game with backward induction, since applying this method shows that player 2 is indifferent between S and C at first decision point, since both will yield 3 for him, therefore it is impossible to identify action profile for player 1, hence the game cannot be solved.

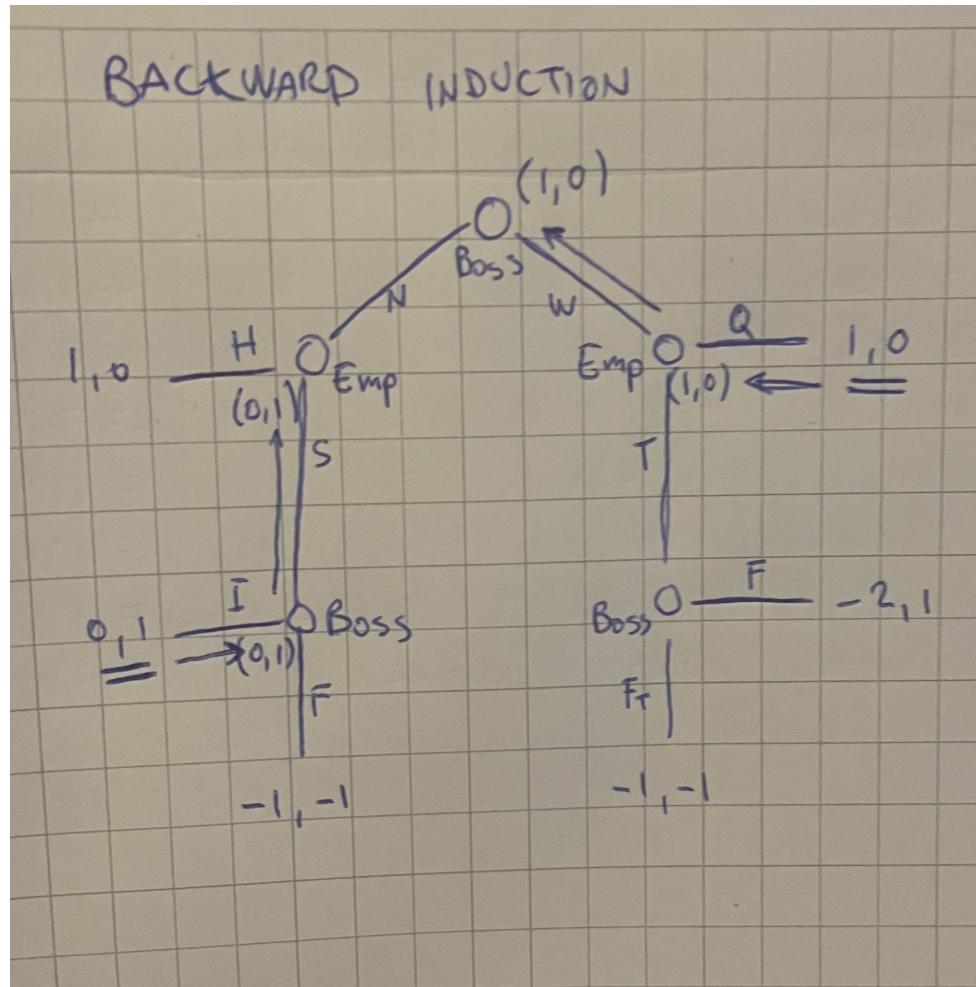
## 2 Boss and stealing employee

We can divide the problem in 4 subgames:



Here below, the normal form matrix is represented, showing the subgame-perfect equilibrium in square. The circled NF are all nash equilibria. Lastly, if NE does not induce a subgame NE, said subgame is reported on the right.

	HQ	HT	SQ	ST	NOT PERFECT
N I F	(1, 0)	(1, 0)	(0, 1)	(0, 1)	SUBGAME 4
N I Ft	(1, 0)	(1, 0)	(0, 1)	(0, 1)	SUBGAME 2
N Ef	(1, 0)	(1, 0)	-1, -1	-1, -1	SUBGAME 3
N FFt	(1, 0)	(1, 0)	-1, -1	-1, -1	SUBGAME 3
W I F	(1, 0)	-2, 1	1, 0	-2, 1	
W I Ft	(1, 0)	-1, -1	(1, 0)	-1, -1	SUBGAME 1
WF, F	(1, 0)	-2, 1	1, 0	-2, 1	
WF, Ft	(1, 0)	-1, -1	(1, 0)	-1, -1	SUBGAME 3



### 3 Stackelberg's Duopoly

The setup makes it a SEQUENTIAL GAME. We need BACKWARD INDUCTION.

Quantities are  $q_i$ .

$$\begin{aligned} \textcircled{1} \quad \frac{dU_2}{dq_2} &= \frac{d[\alpha - c - \beta(q_1 + q_2)]}{dq_2} = \\ &= (\alpha - c) - \beta(q_1 + 2q_2) \stackrel{\substack{\text{set it} \\ \text{to zero}}}{=} 0 \end{aligned}$$

$$\hat{q}_2 = \left[ \frac{\alpha - c}{\beta} - q_1 \right] \cdot \frac{1}{2}$$

②  $\left\{ q_1 + \hat{q}_2 = \left[ \frac{\alpha - c}{B} + q_1 \right] \cdot \frac{1}{2} \right\}$  is the total

$$u_1(q_1, \hat{q}_2) = \left[ (\alpha - c) - Bq_1 \right] \frac{q_1}{2}$$

Again, we set its derivative w.r.t.  $q_1$  to zero. (we want the best response).

$$\frac{du_1}{dq_1} = 0 \Rightarrow \hat{q}_1 = \frac{\alpha - c}{2B}$$

$$\hat{q}_2 = \frac{\alpha - c}{4B}$$

③  $P(\hat{q}_1, \hat{q}_2) - c = \alpha - c - B \hat{q}_{\text{TOTAL}}$

$$\hat{u}_1 = \left[ \frac{\alpha - c}{B} \right]^2 \cdot \frac{B}{8}$$

$$\hat{u}_2 = \left[ \frac{\alpha - c}{B} \right]^2 \cdot \frac{B}{16}$$

(5) Cournot results from  
before

$$\hat{q}_1 = \left[ \left( \frac{\alpha - c}{\beta} \right) - q_2 \right] \cdot \frac{1}{2}$$

$$\hat{q}_2 = \left[ \left( \frac{\alpha - c}{\beta} \right) - q_1 \right] \cdot \frac{1}{2}$$