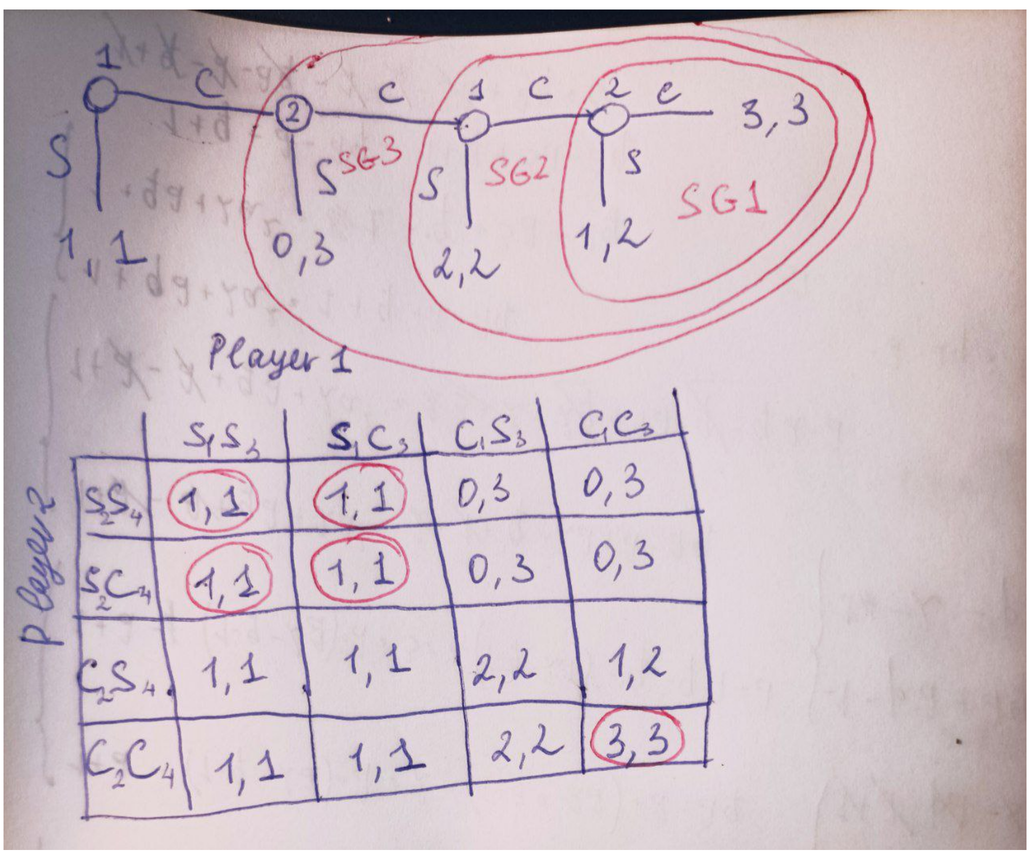
Multi Agents Systems
Group 48: Assignment 3

Mattera Francesco f.mattera@student.vu.nl,
Matteo De Rizzo m.de.rizzo@student.vu.n, and
Ekaterina Geraskina e.geraskina@student.vu.nl

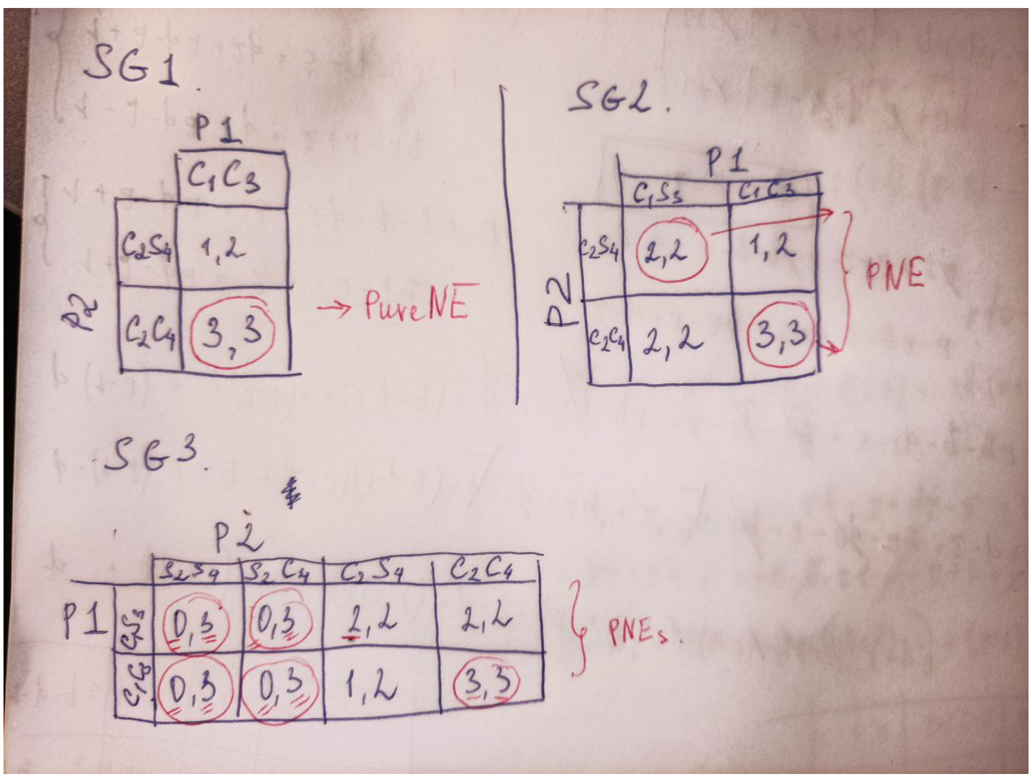
Vrije Universiteit Amsterdam

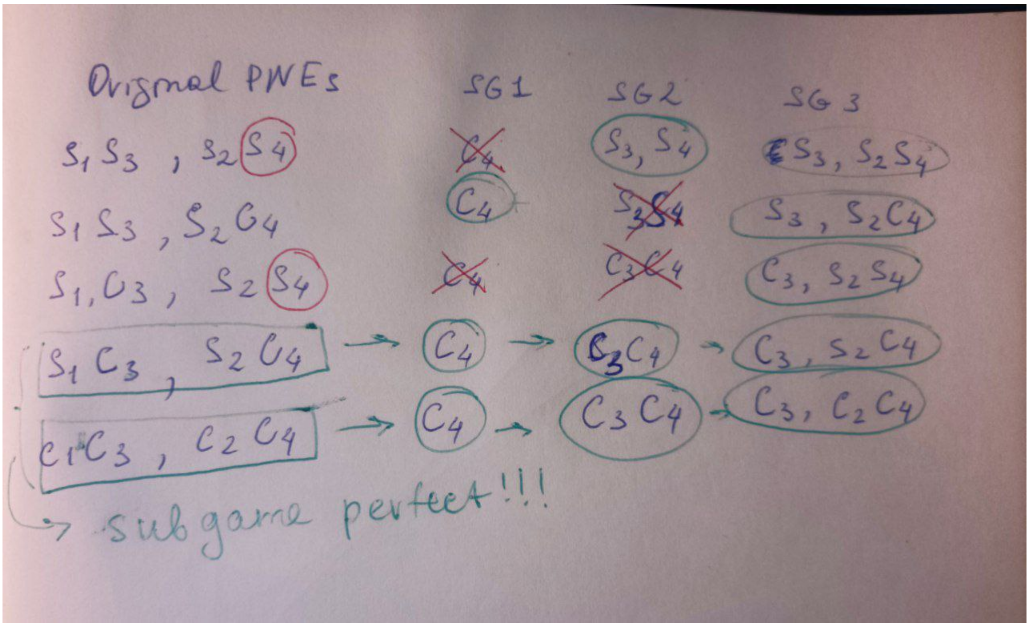
1 Reduced centipede game

1.1 Write the normal form for this game and find all Nash equilibria in pure strategies (PNEs).



1.2 List all subgames and determine which of these PNEs are also subgame-perfect?



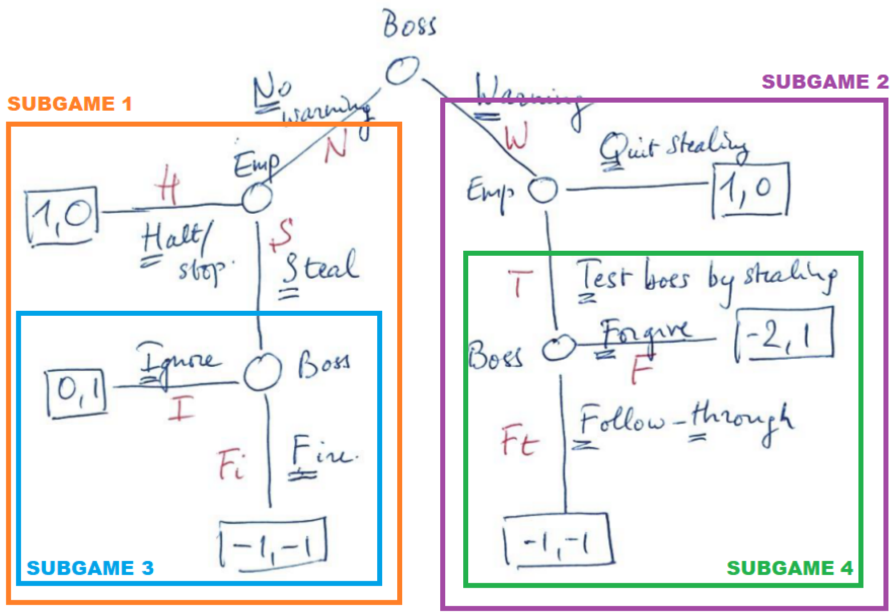


1.3 Can you solve this game using backward induction? Discuss.

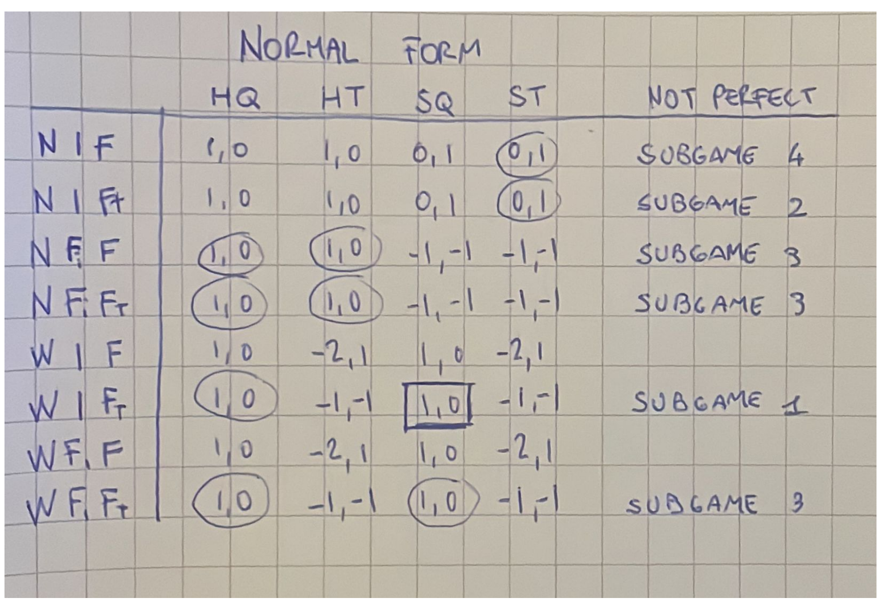
It is not possible to solve this game with backward induction, since applying this method shows that player 2 is indifferent between S and C at first decision point, since both will yield 3 for hime, therefore it is impossible to identify action profile for player 1, hence the game cannot be solved.

2 Boss and stealing employee

We can divide the problem in 4 subgames:

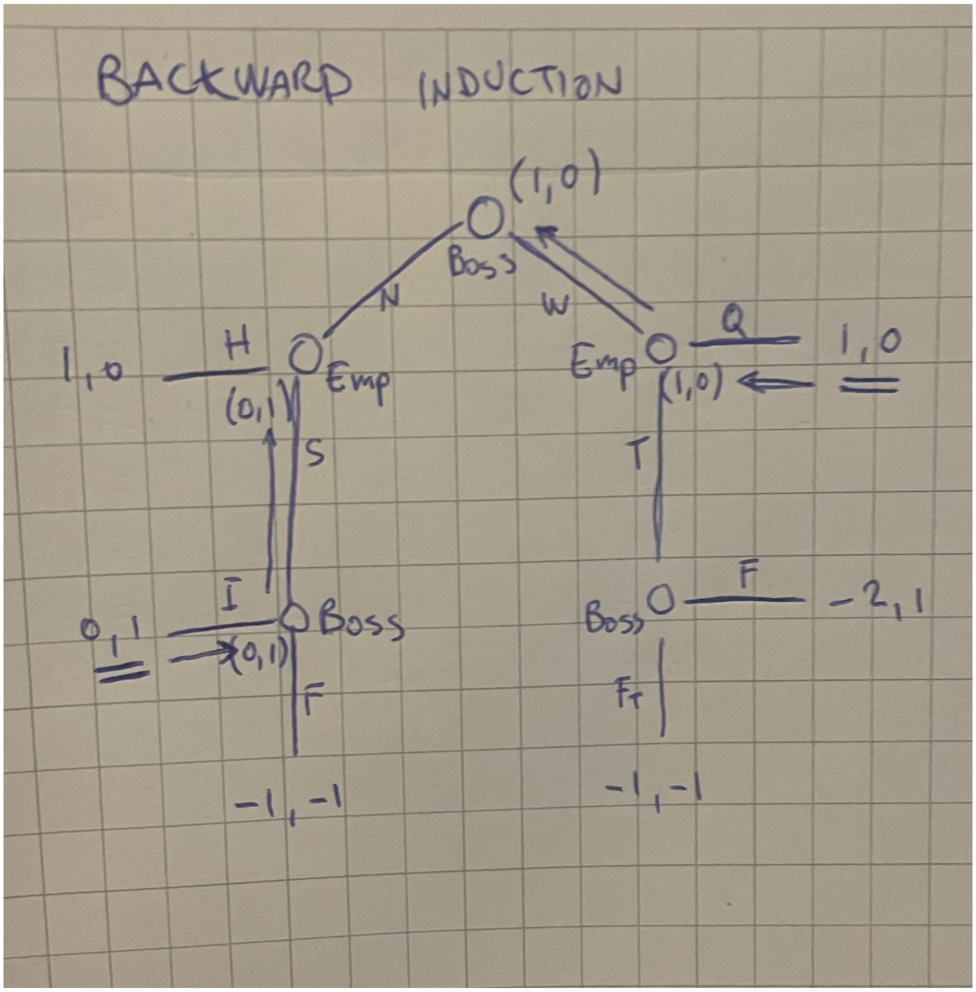


Here below, the normal form matrix is represented, showing the subgame-perfect equilibrium in square. The circled NF are all nash equilibria. Lastly, if NE does not induce a subgame NE, said subgame is reported on the right.

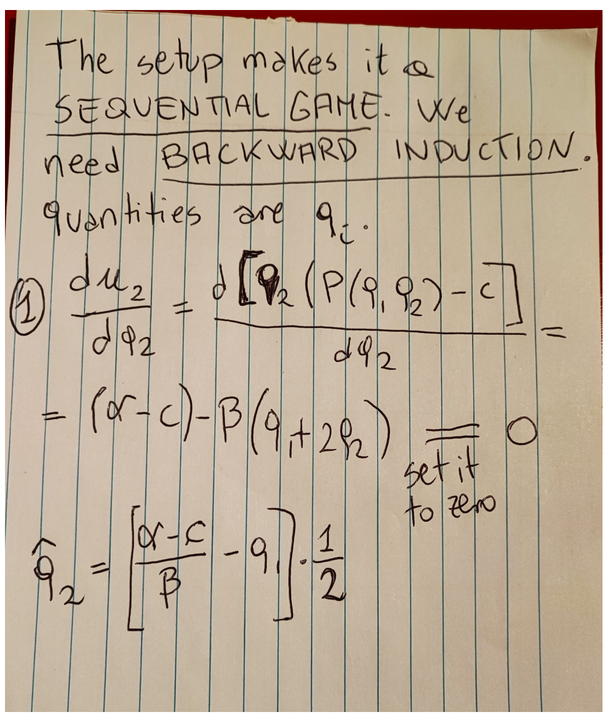


NORMAL FORM

	HQ	HT	SQ	ST	NOT PERFECT
N I F	1,0	1,0	0,1	0,1	SUBGAME 4
N I Ft	1,0	1,0	0,1	0,1	SUBGAME 2
N Fi F	1,0	1,0	-1,-1	-1,-1	SUBGAME 3
N Fi Ft	1,0	1,0	-1,-1	-1,-1	SUBGAME 3
W I F	1,0	-2,1	1,0	-2,1	
W I Ft	1,0	-1,-1	$\boxed{1,0}$	-1,-1	SUBGAME 1
W Fi F	1,0	-2,1	1,0	-2,1	
W Fi Ft	1,0	-1,-1	1,0	-1,-1	SUBGAME 3


BACKWARD INDUCTION
(1,0)
Boss
N
W
H
1,0
Emp
(0,1)
S
Q
1,0
Emp
(1,0)
T
I
0,1
Boss
(0,1)
F
-1,-1
F
-2,1
Boss
F_T
-1,-1

3 Stackelberg's Duopoly



The setup makes it a SEQUENTIAL GAME. We need BACKWARD INDUCTION.

quantities are q_i.

(1) $$\frac{du_2}{dq_2} = \frac{d[q_2(P(q_1, q_2) - c]}{dq_2} =$$

$$= (\alpha - c) - \beta(q_1 + 2q_2) = 0$$

set it to zero

$$\hat{q}_2 = \left[\frac{\alpha - c}{\beta} - q_1\right] \cdot \frac{1}{2}$$

(2) $$\left\{ q_1 + \hat{q}_2 = \left[\frac{\alpha - c}{\beta} + q_1\right] \cdot \frac{1}{2} \right\}$$ is the total

$$u_1(q_1, \hat{q}_2) = \left[(\alpha - c) - \beta q_1\right] \frac{q_1}{2}$$

Again, we set its derivative w.r.t. q_1 to zero. (we want the best response).

$$\frac{du_1}{dq_1} = 0 \Rightarrow \hat{q}_1 = \frac{\alpha - c}{2\beta}$$

$$\hat{q}_2 = \frac{\alpha - c}{4\beta}$$

(3) $$P(\hat{q}_1, \hat{q}_2) - c = \alpha - c - \beta \hat{q}_{TOTAL}$$

$$\hat{u}_1 = \left[\frac{\alpha - c}{\beta}\right]^2 \cdot \frac{\beta}{8} \qquad \hat{u}_2 = \left[\frac{\alpha - c}{\beta}\right]^2 \cdot \frac{\beta}{16}$$

(4) Cournot results from before

$$\hat{q}_1 = \left[\left(\frac{\alpha - c}{\beta}\right) - q_2\right] \cdot \frac{1}{2}$$

$$\hat{q}_2 = \left[\left(\frac{\alpha - c}{\beta}\right) - q_1\right] \cdot \frac{1}{2}$$