Received 30 May 2022, accepted 10 June 2022, date of publication 27 June 2022, date of current version 8 July 2022.

Digital Object Identifier 10.1109/ACCESS.2022.3186485

Reordering and Partitioning of Distributed Quantum Circuits

DAVOOD DADKHAH[1], MARIAM ZOMORODI[2,3], SEYED EBRAHIM HOSSEINI[1], PAWEL PLAWIAK[3,4], AND XUJUAN ZHOU[5], (Member, IEEE)

[1]Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad 9177948974, Iran
[2]Department of Computer Engineering, Ferdowsi University of Mashhad, Mashhad 9177948974, Iran
[3]Department of Computer Science, Faculty of Computer Science and Telecommunications, Cracow University of Technology, 31-155 Krakow, Poland
[4]Institute of Theoretical and Applied Informatics, Polish Academy of Sciences, 44-100 Gliwice, Poland
[5]School of Business, University of Southern Queensland, Springfield Campus, Springfield, QLD 4300, Australia

Corresponding authors: Mariam Zomorodi (zomorodi@pk.edu.pl) and Xujuan Zhou (xujuan.zhou@usq.edu.au)

ABSTRACT A new approach to reduce the teleportation cost and execution time in Distributed Quantum Circuits (DQCs) was proposed in the present paper. DQCs, a well-known solution, have been applied to solve the problem of maintaining a large number of qubits next to each other. In the distributed quantum system, the qubits are transferred to another subsystem by a quantum protocol like teleportation. Hence, a novel method was proposed to optimize the number of teleportation and to reduce the execution time for generating DQC. To this end, first, the quantum circuit was reordered according to the qubits placement to improve the computational execution time, and then the quantum circuit was modeled as a graph. Finally, we combined the genetic algorithm (GA) and the modified tabu search algorithm (MTS) to partition the graph model in order to obtain a distributed quantum circuit aimed at reducing the number of teleportation costs. A significant reduction in teleportation cost (TC) and execution time (ET) was obtained in benchmark circuits. In particular, we performed a more accurate optimization than the previous approaches, and the proposed approach yielded the best results for several benchmark circuits.

INDEX TERMS Quantum computing, distributed quantum circuit, optimization, genetic algorithm, teleportation.

I. INTRODUCTION

In the field of computation, quantum computing is a new field with a high capacity to perform classical computing more efficiently [1]. The quantum computing was first introduced by researchers in the 1980s. It has presented new algorithms for computer science to accelerate problem solving compared to classical algorithms [2], [3]. Undoubtedly, the use of quantum computation is indispensable, since it employs fundamental techniques, such as encryption and optimization, causing classical problems, to be solved. Nevertheless, there are significant new trends and approaches into quantum computability system [4], quantum algorithms [5], quantum communication protocols [6], [7] and quantum mechanics [8], particularly with the advent of the quantum information theory [1].

But,there are some problems with quantum computing. For example, when the number of qubits grows, protecting them from destroyed interactions and decoherence rapidly becomes uncontrollable [9]. Therefore, for a distributed quantum circuit [10]–[12], development of a quantum circuit with limited quantum subsystems communicating with each other through quantum teleportation is a logical method to use. To build the distributed quantum circuit, it is necessary to make multi quantum circuits with a limited capacity connected together through a classical or quantum channel, and they implement the functionality of a distributed quantum circuit. DQCs are a model for quantum circuits consisting of subsystems, which are far from each other and have many qubits.

Briefly, DQCs are comprised of several subsystems having qubits and gates, so that qubits are exchanged between subsystems by teleportation (a technique for transferring qubits). Then, the qubits are returned to the first subsystem after completing the operation.

The associate editor coordinating the review of this manuscript and approving it for publication was Abdullah Iliyasu.

In the present paper, we intended to survey the performance of the distributed quantum circuit by reordering the qubits placement and implementing the combination genetic and modified tabu search algorithm to reduce the execution time and the number of teleportation cost.

In this work, we developed an optimization approach to improve the execution time for generating distributed quantum circuits in order to reduce the number of teleportation cost significantly. Since the general optimization of distributed quantum circuits is NP-hard, we presented the genetic algorithm and the modified tabu search algorithm to partition the graph model of quantum circuit in order to create a distributed quantum circuit with the minimum number of teleportation. Afterward, we implemented our optimization approach in several distributed quantum circuits. In this paper, as a benchmark, quantum circuits were selected from the RevLib website [13] and Quipper's library [14], [15] to test the proposed optimization. In all circuits, we focused on the number of teleportation and the execution time to optimize for the distributed quantum circuit.

Tabu search (TS) is one computational algorithm for optimization and is widely used in many sciences. Tabu search (TS)is a memory-based algorithm. However, TS cannot completely perform optimization problems. The modified version of TS (MTS) has been applied to improve its performance, for example, MTS has presented a significant performance when used to partition graph problems. Owing to motion mechanisms, MTS had rapid convergence to solution, and it performed a considerable search in the potential solution. Thus, we applied the modified tabu search, so that the problem of partitioning was best solved, and the optimal partitioning for the large distributed quantum circuits was obtained.

The long interaction distances between the qubits within the quantum circuit may increase the execution time for generating the distributed quantum circuit. Hence, the reordering leads to approach qubits for executing the quantum gates within a quantum circuit and ultimately reduces the execution time in DQC.

This work consists of four steps. In the first step, reordering operation is applied to the qubits placement in the initial quantum circuit; in the second step, reordered quantum circuit is modeled by a graph; in the third step, modified tabu search(MTS) partitions the graph model; and in the fourth step, genetic algorithm (GA) is presented to improve the graph partitioning.

The paper has been organized as follows: Section II explains some basic concepts of quantum computation. Section III presents the related work. Section IV offers the definitions required for the distributed quantum circuit. Sections V and VI discuss our proposed approach and results of simulation, respectively. Finally, Section VII concludes the paper.

II. BACKGROUND

Quantum computation is a novel computational field in which qubit is the basic unit of information. The state of a qubit is shown by a unit vector in a Hilbert space labeled as

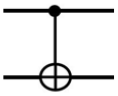


FIGURE 1. Circuit of the CNOT gate.

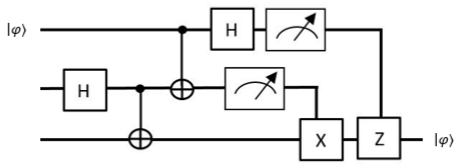


FIGURE 2. Quantum circuit of qubit teleportation.

$$|\Phi\rangle = \alpha|0\rangle + \beta|1\rangle \tag{1}$$

where $|0\rangle$ and $|1\rangle$ are the basis of space, and α as well as β are complex coefficients establishing $|\alpha|^2 + |\beta|^2 = 1$.We used some single- and two-qubit gates, such as single-qubit Hadamard, T and S, and two-qubit CNOT gate in our proposed method.

Fig. 1 shows the representation of the CNOT gate.

$$Hadamard = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \tag{2}$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \tag{3}$$

$$S = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix} \tag{4}$$

Let a quantum system be formed by two components that are placed in Hilbert space H_1 and H_2, respectively. Therefore, the whole system is placed in a Hilbert space $H = H_1 \otimes H_2$, so that the state vector is shown as

$$|\psi\rangle = \sum_{i,j} d_{i,j}|e_{1,i}\rangle|e_{2,j}\rangle \tag{5}$$

where $\{e_{a,i}\}$ is considered the basis in the Hilbert space and $\sum_{i,j} d_{i,j} = 1$.

In this paper, qubit transfer between subsystems was assumed to be accomplished through teleportation operation. In teleportation, the basic process is to transmit a quantum state of qubit, so that the destination receives the same state as the initial qubit state. Therefore, the initial state is removed, so that quantum teleportation matches the no-cloning theorem [16]. Moreover, the aim of quantum teleportation is to transfer a quantum state of qubit using two classical bits, so that the receiver generates the same state as the initial qubit state. Fig. 2 presents the quantum circuit of qubit teleportation [17].

III. RELATED WORK

In this section, the pervious methods for distributed quantum circuit have been reviewed. Grover [18], Cleve and

Buhrman [19], and Cirac *et al.* [20] were the pioneers who started to survey the distributed quantum computing field. Grover divided a quantum circuit in which particles are far from each other, and each particle completed its computation. Thus, he presented a distributed quantum system. In this system, requiring information is transferred from one particle and received in another particle. He also showed the use of this distribution system and presented a quantum algorithm to calculate the total time required to compute the number of distributed particles. Beals *et al.* also indicated that a quantum circuit could be modeled as a distributed quantum circuit by introducing the hypercube graph, so that each subsystem was placed on each vertex of a hypercube graph in [21].

Yepez considered two communications structures for distributed quantum computing. Quantum subsystems are communicated through the quantum channel, while classical subsystems are communicated through classical channels. Quantum systems in which all qubits can be entangled to each other and subsystems are communicated to each other through classical channels in [22]. Streltsov *et al.* presented a way to distribute entanglement and indicated the minimum communication cost for sending entangled particles between subsystems. They also showed that the total entanglement between two particles sent should not be more than the total quantum communication required for moving each particle between two subsystems in [23].

Ying and Feng determined an algebraic language to introduce the distributed quantum system. They also defined some notations for distributed quantum computing in [24].

In addition, the hyper graph-partitioning approach was presented for the quantum circuit. Authors modeled the quantum circuit as a hypergraph model. Finally, they employed the proposed model to be applied on five quantum circuits in [25]. However, their approach did not consider the execution time for partitioning graph.

In [26], the authors presented an algorithm optimization, so that reduced the teleportation cost of the distributed quantum circuit (DQC) in terms of the number of teleportation. They showed that DQCs were the potential solution for a multi subsystem and proposed an algorithm to optimize the cost of communication consisting of two subsystems. In the end, the final configuration with the minimum number of teleportation was reported. However, their optimization algorithm divided quantum circuit into only two partitions, whereas they could consider their proposed approach to create multiple partitions. Moreover, in [27], a GA was introduced to perform the optimization process on distributed quantum circuits more efficiently. The authors found the proposed approach had executed the genetic algorithm with high speed. Shor's algorithm was introduced in [28]. In the proposed model, a distributed quantum circuit was implemented to execute the non-local gates of algorithm. Furthermore, the authors do not report the number of quantum teleportation for this distributed quantum circuit. Daei *et al.* presented a graph model to generate distributed quantum circuits from monolithic quantum circuits, so that communication between partitions of a distributed quantum circuit was minimized in [29]. However,their approach did not examine the execution time needed to generate graph-partitioning. Davarzani *et al.* proposed the algorithm consisting of two steps: The quantum circuit was converted into a bipartite graph model, and a dynamic programming approach was applied to generate distributed quantum circuits in [30].

IV. PROBLEM DEFINITION

A distributed quantum circuit (DQC) includes N number of quantum circuits that all of them construct a distributed quantum circuit. In a DQC, qubits are transferred between the subsystems by teleportation. In each subsystem, qubits are labeled from the top to the bottom line, where the ith line corresponds to the ith qubit. In the present work, we defined two kinds of quantum gates in a DQC as follows:

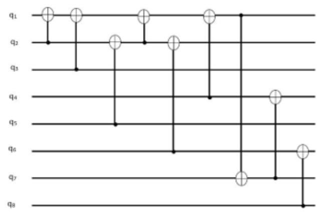
A local gate: Its control and target line are in the same partition applying on the local qubit. Each partition may have a finite number of local gates.

A global gate: Its control and target line are in different partitions. To run a global gate, a qubit in the current partition should be teleported to another partition to perform gate.

Moreover, in a distributed quantum circuit, when a qubit is sent into another subsystem, it does not execute in its initial subsystem anymore. To accomplish local gates, qubits can be performed in their initial subsystem. Furthermore, to execute global CNOT gates, qubits should be executed in the same subsystem. Quantum teleportation is a good solution to transfer qubits between partitions. Suppose that there is a two-partition system with Subsystem A and Subsystem B. In this system, there are two ways to perform any global gate. The qubit in Subsystem A is transferred to Subsystem B through teleportation. This is also true the other way around. Single-qubit and local CNOT gates should be executed in their local subsystems.

Although at first glance, DQCs are similar to quantum circuits (QCs), the problem with a DQC is essentially different from that of a QC. In a DQC, the optimization process is focused on reducing the computational execution time and teleportation costs. In the current study, to optimize the distributed quantum circuit, an approach was proposed based on optimization algorithms, so that we attempted to find a desirable DQC with reduced teleportation costs. Our proposed approach calculated the minimum number of teleportation for each configuration, where global gates had an individual position between partitions for each configuration. Ultimately, the minimum number of teleportation was reported for all configurations. We also strived to begin with an initial quantum circuit like basic gates (i.e., CNOT and single-qubit gates). In the current work, we considered a quantum circuit, including basic gates (i.e., CNOT and single-qubit), which were applied on qubits.

We consider a graph as G(V, E), where V is a set of vertices,and E is a set of edges formed by pairs of vertices, respectively. The graph-partitioning problem divides graph into K partitions, so that we try to minimize the connection



$$\begin{array}{c|cccccccc} & q_1 & q_2 & q_3 & q_4 & q_5 & q_6 & q_7 & q_8 \\ \hline q_1 & 0 & 2 & 1 & 1 & 0 & 0 & 1 & 0 \\ q_2 & 2 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ q_3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ q_4 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ q_5 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ q_6 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ q_7 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ q_8 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}$$

FIGURE 3. A quantum circuit without reordering the qubits placement.

among all different partitions.

$$\sum_{i=1,2,\cdots,n} \sum_{j=i,i+1,\cdots,n} W(v_i, v_j) \tag{6}$$

where $W(v_i, v_j)$ is the weight of between vertices v_i and v_j for all $v_i \in p_i$, $v_j \in p_j$.

A. REORDERING OF QUBITS PLACEMENT

We proposed reordering of the qubits placement in quantum circuits. This process can reduce the execution time to generate the distributed quantum circuit. In this section, our objective is to reorder the qubits placement based on the minimum nearest neighbor cost. Before partitioning of qubits into determined partitions as discussed in the next section, we apply the proposed approach in which a suitable qubit order is determined for each quantum circuit. For this goal, it is applied to each quantum circuit extensively. The reordering plays a crucial role in improving the execution time in DQC. So, we construct the matrix model for quantum circuit. In this matrix,the relationship between the qubits in the quantum circuit can be modeled as follows:

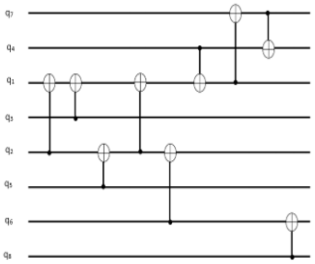
{1; if two qubits (q_i,q_j) are communicated.
{0; otherwise

Then, reordering of the qubits placement is carried out on the corresponding matrix according to the minimum nearest neighbor cost. The minimum nearest neighbor cost of the quantum circuit is determined as

$$NNCQC = \frac{1}{2}\left(\sum_{i=1,2,\cdots,n} \sum_{j=i,i+1,\cdots,n} A_{i,j}(|i-j|-1)\right) \tag{7}$$

The reordering operation was kept until the minimum NNCQC was attained. The optimization process depended on the number of qubits in the initial quantum circuit. Finally, we constructed a new quantum circuit whose qubits placement could be closed to each other and could improve the execution time when the distributed quantum circuit was generated.

For instance, consider the quantum circuit in Fig. 3. We reorder the qubits placement based on the minimum nearest neighbor cost from $\{q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8\}$ to $\{q_7, q_4, q_1, q_3, q_2, q_5, q_6, q_8\}$ as shown in Fig. 4.



$$\begin{array}{c|cccccccc} & q_7 & q_4 & q_1 & q_3 & q_2 & q_5 & q_6 & q_8 \\ \hline q_7 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ q_4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ q_1 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ q_3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ q_2 & 0 & 0 & 2 & 0 & 0 & 1 & 1 & 0 \\ q_5 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ q_6 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ q_8 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}$$

FIGURE 4. A quantum circuit with reordering the qubits placement.

B. GRAPH MODELING BASED ON LABELING OF QUANTUM CIRCUIT

The basic method presented in this section is to employ the graph structure to model the quantum circuit. In our quantum representation model, we consider a set of qubits (V), a set of input qubits (I), a set of output qubits (O) and a set of gates like U on the qubits.

The set of U gates consists of gates like $U_1 U_2 \ldots$. Each of these gates may be dependent on the gates before them. It should be noted that these gates are applied from left to right, and to show the labeling model, we use the graph structure displayed as $G = (V, E)$ based on $\{I, O\}$ these components:

V: Represents the set of nodes of graph G that each node of the graph represents a qubit.

I: Represents the set of the input qubit of the U gate.

O: Represents the set of the output qubit of the U gate.

E: Represents the edges of graph G, which indicates the relationship between two nodes.

To determine the CNOT gate in the graph model, we use the following method to label the input and output in a quantum circuit and we model a quantum circuit into the graph based on differences labeling the target of the CNOT gate.

This labeling is such that if the two-part input and output of CNOT gates have the same label, these gates realize the same function. Hence, all CNOT gates having a different functionality are implemented in this model.

1. First, we label all quantum circuit input lines as 0 and consider an empty list L_i for each quantum circuit. Each list L_i keeps the input and output labeling of CNOT gates.
2. For each CNOT gate from the inputs toward the outputs of the quantum circuit, we apply the following step for CNOT gates:

If there are k CNOT gates $C_1, \cdots, C_k$ that realize the quantum circuit, i.e. $C_1 . \cdots . C_k = I$, then the label output side of the target for the latest CNOT gate equals the label on the input side of the target for the first CNOT gate. Therefore, we omit these CNOT gates from the graph model.
We propose creating an edge in graph model from the labeling by applying the following procedure:

$$comp\left((c_i, t_j) labeling, (c_i', t_j') labeling\right)$$

where comp represents the comparison operation for the labeling of the inputs and outputs of each CNOT gate. Using the labeling method, we can find all changes in the input and output qubits of a CNOT gate.

In other words, by labeling the input qubits of a CNOT gate as (c_i, t_j) and its output qubits as (c'_i, t'_j), changes to the labeled inputs and outputs of CNOT gates can be saved in the L_i list,and with the help of this list, we can recognize the existence of the CNOT gate. After we find changes in labeling according to list L_i, we must regard these changes for CNOT gates and begin to model a quantum circuit into a graph based on the order of qubits. The reason for this is that the qubits of CNOT gates depend on other possible qubits.

To determine CNOT gates, we should consider the labeling method to represent the graph model of a quantum circuit. Because no edge is constructed,we do not consider the one-qubit in the graph model like $\{H, T, T^t\}$.

However, for the CNOT gate, we assume two qubits as control and target as $CNOT(q_1, q_2)$. Thus, a connection is needed for showing it with an edge between q_1 and q_2. The graph models all quantum qubits and gates in a quantum circuit to all interconnections needed to implement it.

Algorithm: Creating a Graph Modeling From a Quantum Circuit

```
Input:  The CNOT gates of a quantum circuit QC
          sorted by the order they are applied.
Result: A graph $G = (V, E)$ based on $\{I, O\}$
        apply the labeling method to the quantum
          circuit for each CNOT gate
        if qubit labeling is changed then
        $q_i \leftarrow$ the first qubit acts on;
        $q_j \leftarrow$ the second qubit acts on;
        $G \leftarrow$ Add CNOT gate $(q_i, q_j, G)$
        else
        remove the CNOT gate
        return G;
```

We consider a sample quantum circuit in Fig. 5. In this figure, the sets of qubits are $Q = \{q_1, q_2, q_3, q_4\}$ and CNOT gates are $G = \{g_1, g_2, g_3, g_4, g_5, g_6, g_7\}$, respectively. To create a graph model as shown in Fig. 6, first, we consider the labeling $(q_1, q_2, q_3, q_4) = (0, 0, 0, 0)$.

For the first CNOT gate, the target (q_1, q_2) is assigned labeling 1. The line that is not empty $(\bigoplus)$ should be labeled by 1 or 0. The qubits of the first CNOT gate are recorded in the list L_i, so that the obtained labeling is (0,0,0,0) and (0,1,0,0) for the input and output, respectively.

The implementation of the proposed model for a quantum circuit will continue until all CNOT gates of a quantum circuit are modeled as shown in Table 1. Afterward, we find labeling changes to model the CNOT gate in a quantum circuit into the graph model based on the obtained list L_i.

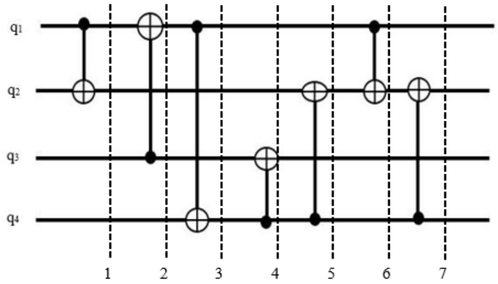


FIGURE 5. Sample quantum circuit.

TABLE 1. Labeling of Figure 5 for creating the list L_i.

Step	Input labeling	Output labeling	CNOT Gates (Edges)
1	$L_i = (0, 0, 0, 0)$	$L_i = (C_0, 1, 0, 0)$	E_{12}
2	$L_i = (0, 1, 0, 0)$	$L_i = (1, 1, C0, 0)$	E_{31}
3	$L_i = (1, 1, 0, 0)$	$L_i = (C1, 1, 0, 1)$	E_{14}
4	$L_i = (1, 1, 0, 1)$	$L_i = (1, 1, 1, C1)$	E_{43}
5	$L_i = (1, 1, 1, 1)$	$L_i = (1, 0, 1, C1)$	E_{42}
6	$L_i = (1, 0, 1, 1)$	$L_i = (C1, 1, 1, 1)$	E_{12}
7	$L_i = (1, 1, 1, 1)$	$L_i = (1, 0, 1, C1)$	E_{42}

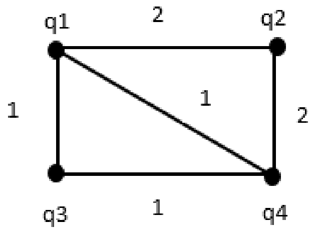


FIGURE 6. Graph model for Figure 5.

Therefore, edges $E_{12}(q_1, q_2)$ are added to the graph model. Other edges are modeled as

$$E = \{E_{31}(q_3, q_1), E_{14}(q_1, q_4), E_{43}(q_4, q_3), E_{42}(q_4, q_2), E_{12}(q_1, q_2), E_{42}(q_4, q_2)\}$$

C. GENETIC ALGORITHM

Genetic algorithm has been introduced as an optimization model for evolutionary computation. It is used to generate optimal solutions to optimize difficult problems. The partitioning of graph is an NP-hard problem, which can be solved by the genetic algorithm. Such an algorithm is commonly applied to generate the converged solutions by recombination, mutation operation, and production of new generations. Through a genetic model, a distributed quantum circuit was modeled to a graph, and then it was attempted to be partitioned with a minimum number of cuts (number of teleportation cost) by using the genetic algorithm. First, we presented a chromosome to indicate the way in which the proposed method was used in the genetic algorithm model. In this algorithm, a chromosome structure, whose elements represented partition of the vertices, was used. It assigns each gene a number 1 to k depending on which the gene belongs

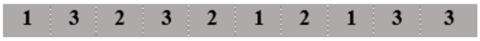


FIGURE 7. The representation of chromosome.

to the specified partition of vertices. For example, in the following chromosome, Vertex 1 is in Partition 1, and Vertex 2 is in Partition 3. The number of cuts (the teleportation cost of the distributed quantum circuit) is calculated when vertex (v_i) in the partition (p_i) is connected to the vertex (v_j) in the partition (p_j) by edges (global gates) as the considered chromosome. The points of cut are changed by using the genetic algorithm until the lowest quantum teleportation cost is achieved. In a genetic algorithm, a population of candidate solutions always evolves to obtain an optimized solution. Each chromosome represents a set of properties, which can be mutated and replaced. GA search is also an iterative process with a generation (a population in each iteration). To implement genetic algorithms, first we start to define the structure of the genetic algorithm used for finding best solutions. The following procedure scheme was used in this paper.

Fitness function (f), which is the number of communication costs between the partitions, is evaluated by using an optimization function $F(v)$. This function tries to attain the minimum number of teleportation,which is the main objective, and they often include multiple limitations. An optimization function is defined as

$$\begin{aligned} O = F(v) \ &= \text{minimize}\Big(f_1(v), f_2(v), \cdots, f_n(v)\Big) \\ &\quad \textit{with } v = (v_1, v_2, \cdots, v_n) \in V \\ &\quad o = \Big(o_1 = f_1(v), o_2 = f_2(v), \\ &\quad \cdots, o_n = f_n(v)\Big) \in O \qquad (8) \end{aligned}$$

$$\text{Subject to } \ W = \sum_{i \neq j} w(v_i, v_j) \leq \lambda \qquad (9)$$

where v is a vector of vertices variables, n is the number of vertices, V is the input space of the problem, o is an output vector of vertices, and O is the output space of the problem. Each fitness function $f(v)$ is an objective function, W is the sum of weights of edges between partitions that is considered the feasible limitation for possible solutions, and λ is a real number implying that v_i is connected to v_j. The optimal solutions are the set of all possible solutions obtained by the optimization function within feasible limitations. The total cost of edges (n_c) for each vertex (v_i) connected to other vertices (v_j) for different partitions is obtained by

$$n_c = \min\left(\sum_{i=1}^{n} \sum_{j=1, i \neq j}^{n} w(v_i, v_j)\right) \qquad (10)$$

The best individuals are selected from the current population by using the roulette wheel selection strategy, and genes of each chromosome are modified to form a new generation so that they are recombined and randomly mutated. GA applies a two-point crossover operation to randomly generate two new offsprings by swapping genes of the chromosomes. As Fig. 8 depicts, Crossover operation starts by randomly selecting and moving parts of the parents with each other. It is applied to cut each of the selected parents from the determined points and to generate a new chromosome by recombining them with probability-pc. We implement the mutation operation to flip the chosen genes of a chromosome to generate new solutions with probability-pm. It randomly selects one or more partitions of the vertex and replaces them with other ones depending on the probability of the mutation operation. Fig. 9 presents the mutation procedure. As Fig. 10 shows, using the crossover, mutation, and recombination operation, we attain high searching performance while maintaining the population's diversity. Genetic operators are applied to the selected chromosome to generate a new population. Ultimately, the genetic algorithm terminated is obtained for the population when the number of generations exceeded from acceptable number or minimum fitness function. We apply the modified tabu search algorithm [31], which is used in our optimization step to improve the quality of the obtained partitions, is based on the tabu search algorithm, in which the disruption mechanism is used to diversify the search. According to the results, the combination of these two mechanisms provides a highly effective improvement method to generate high quality partitions.

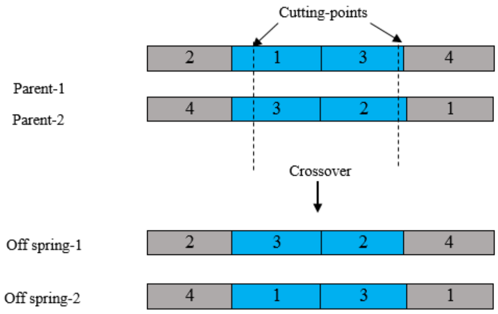


FIGURE 8. Representation of the crossover.

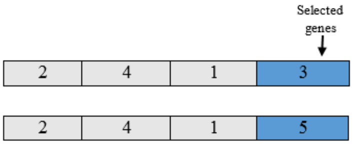


FIGURE 9. Representation of the mutation.

We explain the modified tabu search algorithm, and examine how the modified version of this algorithm is presented on the graph partitioning problem.

D. MODIFIED TABU SEARCH ALGORITHM

We define two motion operators T_1 and T_2. The application of these two motion operators causes to transfer the vertices to other subsets in order to reduce the connections. These operators play a crucial role in minimizing the sum of the connections of the edges between the partitions in the graph

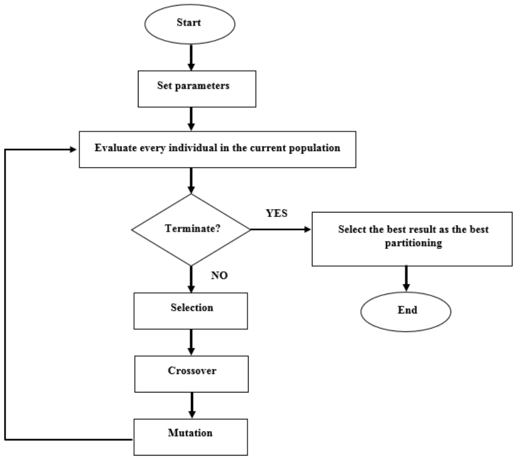

Start
Set parameters
Evaluate every individual in the current population
Terminate?
YES
Select the best result as the best partitioning
End
NO
Selection
Crossover
Mutation

FIGURE 10. The flowchart of the genetic algorithm in the proposed approach.

model. These operators move one vertex or two vertices between the subsets. Assuming that vertex v_m in partition S_m, the gain move $g(v, n)$ can be calculated when vertex v_m in partition S_m goes to any arbitrary $S_n(n \neq m)$. The concept of gain move shows how motion operators cause to minimize the connection edges between partitions. Therefore, the high gain move decreases the connections in the graph model.

Suppose $P = \{s_1, s_2, \cdots, s_k\}$ is a K-partition. For example, in Fig. 7, $\{v_1, v_6, v_8\}$ are considered in partition s_1, $\{v_3, v_5, v_7\}$ are in partition s_2, and $\{v_2, v_4, v_9, v_{10}\}$ are in partition s_3.

Additionally, $S_{\max}$ is the subset with the most edges so that:

$$S_{\max} = \max_{i \in 1,2,\cdots,k} W(s_i) \tag{11}$$

In this case, we define two motion operators as follows:

Single-move operator (T_1): We transfer a vertex like v_m, which is the maximum gain move. We randomly select a partition like $S_m(m \neq \max)$, then the vertex like v_m, which is in partition $S_c \in \{S_i \in P | W(S_i) > W(S_m)\}$ is transferred to S_m.

Two-move operator (T_2): We select the two vertices v_x and v_y,which have the highest gain move to transfer. We move vertex v_x to partition S_m based on the single-move operator. Then, we randomly select a subset like $S_n \in P(n \neq \max, n \neq m)$. Thus, we select the vertex v_y that has the highest gain move from the subset $S_c \in \{S_i \in P | S_i \neq S_m, S_i \neq S_n\}$. Finally, we transfer v_x to S_m and v_y to S_n. It is important to note that vertex v is chosen to move to S_i only v is adjacent to at least one vertex of S_i.

To improve the assessment of the motion, we use the bucket structure, which arranges the vertices according to the amount of gain move. This sorting is used to avoid unnecessary searching and reduces the time needed for finding vertices with a high gain move. Accordingly, we select the vertex that has the highest gain move from the bucket. After moving the vertex to the desired partition, we update the bucket structure. The complexity is removing a vertex and inserting it into bucket $O(1)$. Thus, the complexity of moving vertex v from partition S_c to S_m is equal to the number of adjacent vertices.

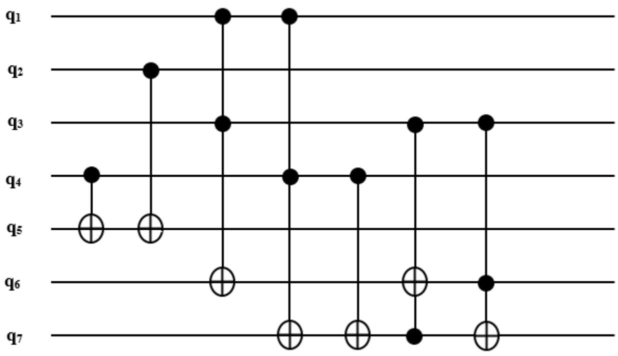


FIGURE 11. 4*mod*5 quantum circuit.

Suppose V_{sel} is a set of candidate vertices with the highest gain to be transferred to subset S_m. We select a vertex like $v \in V_{sel}$, which is the current subset of S_c, to transfer to S_m whenever it is not in the tabu list or the move from v to S_m causes the new partition to be better than the partition achieved thus far. If there is more than one vertex to move, we select the vertex to move to S_m, which has the least connection with S_c.

To improve and complete the modified tabu search algorithm, we use the disruption mechanism for diversity in the search space. In this case, if the best partition P does not change after T repetitions, the disruption mechanism selects a vertex randomly. To be more precise, suppose that the current partition is $\{s_1, s_2, \cdots, s_k\}$. We use an independent step as

Algorithm: Disruption Mechanism

```
1. Select randomly $S_m \in \{S_1, S_2, \cdots, S_k\}$
2. Select randomly v from $S_c \in \{S | W(S) > W(S_m)\}$
3. Transfer v to $S_m$
4. Do steps 1 to 3 for T repetition
```

This mechanism is similar to (T_1). However, there is a significant difference, and this vertex, which is considered for partition S_m, is not necessarily adjacent to S_m. This causes us to go to a different and more diverse space in the process of finding the solution.

As Fig.11 depicts, to illustrate the functionality of MTS, for example, we use $4mod5$ quantum circuit, which has 7-inputs and outputs. Fig.12 presents the graph model of $4mod5$. The graph is partitioned into two parts by the modified tabu search partitioning algorithm, as follows: $P_1\{q_1,q_4\}, P_2\{q_2,q_3,q_5,q_6,q_7\}$. Fig. 13 shows the $4mod5$ distributed quantum circuit,and Table 2 presents the steps of executing quantum gates for the $4mod5$ circuit. Therefore, $4mod5$ is divided into two partitions with specified qubits.

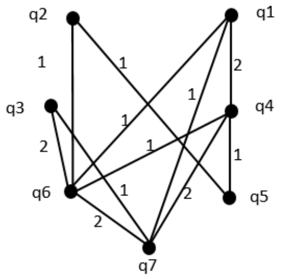


FIGURE 12. Graph model of the 4*mod*5 circuit.

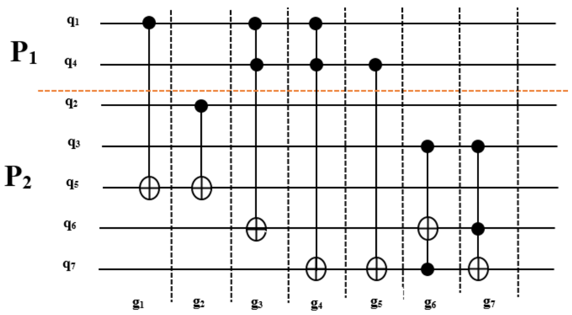


FIGURE 13. 4*mod*5 distributed quantum circuit.

TABLE 2. The steps of executing quantum gates in the MTS algorithm for the 4*mod*5 circuit.

# of gates	type of gate
g_1	global
g_2	local
$g3$	global
g_4	global
g_5	global
g_6	local
g_7	local

V. THE PROPOSED APPROACH

We attempted to reorder the qubit placement in the initial quantum circuit to construct a new quantum circuit to improve the execution time and modeled the new quantum circuit into a graph, and finally partitioned the graph model by the modified tabu search algorithm as local optimization,and then applied the genetic algorithm as global optimization to obtain a distributed quantum circuit with at least the number of teleportation cost. We implement the proposed approach in benchmark circuits by applying steps (reordering- graph modeling- modified tabu search algorithm- genetic algorithm).

Teleportation in DQCs is a costly operation; therefore, we attempt to reduce it as far as possible. Our approach is based on reducing the number of teleportation cost and execution time in DQC. Hence, we model the quantum circuit with a graph and assume that the quantum circuit is composed of the quantum gates $\{H, T, T^{t}\} + CNOT$. It is also noteworthy that quantum gates with multiple qubits can be decomposed to the basic gates [32], [33]. To construct the graph model, we begin from the first quantum gate in the quantum circuit until the set of edges is completed. The weight of each edge is related to the number of connection between vertices. We model the quantum circuit as an undirected graph in which the weight of each edge represents the connection of two qubits through quantum gates.

Since connections between partitions in a distributed quantum circuit indicate the number of teleportation cost, one of the main goals is to reduce the connections between partitions by applying the modified tabu search algorithm and genetic algorithm. We can divide the graph model into K-partitions using the modified tabu search algorithm for graph partitioning,and then apply the genetic algorithm to improve the graph partitioning in order to obtain the minimum number of teleportation cost. It is important to note that the best partitioning of vertices attained in the modified tabu search algorithm is as an initial partitioning in the genetic algorithm. We implement these partitioning algorithms for benchmark circuits and finally present the best number of teleportation cost. After applying the modified tabu search to the graph model, we attain the partition of vertices so that we have K-partitions $\{p_1, p_2, \cdots, p_k\}$ in which each partition has a number of vertices, which may be connected to several other partitions through some edges. Then, the results of the modified tabu search algorithm transfer to the genetic algorithm to run the global optimization, Since our proposed approach attempts to partition a graph to obtain the least number of teleportation cost after applying the partitioning algorithms. All of the connected edges in different partitions determine the number of teleportation cost in DQC. Once the proposed approach is executed, DQC is achieved by reducing the number of teleportation cost. If the vertices are properly placed in partitions, the execution time will be reduced,and the least teleportation cost will be obtained for the distributed quantum circuit. The algorithm for the proposed approach is run as

- First, the quantum circuit is reordered based on the proposed method in Section IV-A.
- The graph model is constructed according to the proposed approach in Section IV-B.
- The graph is partitioned by the modified tabu search and genetic algorithm into K-partitions and a partitioned graph model is created.
- If the partitioning result is obtained by the modified tabu search algorithm is better than the genetic algorithm, this result is regarded as the best partitioning. Otherwise, the partitioning obtained by the genetic algorithm will be the best result.
- In this paper, genetic algorithm terminates when the number of generations reaches the maximum value,and MTS terminates when the number of iterations reaches the maximum size.

As Fig. 14 shows, we reordered the qubits placement in the initial quantum circuit,and then constructed a graph model from the new quantum circuit. Finally, partitioning

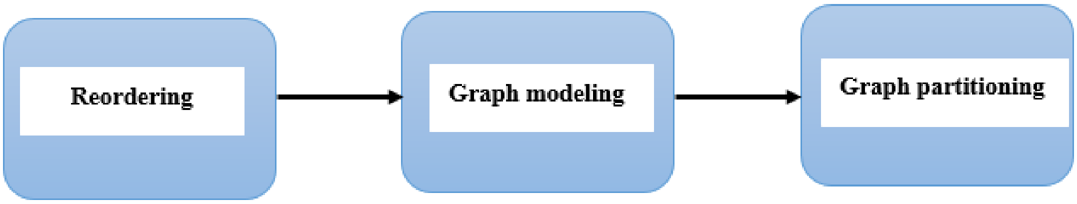


FIGURE 14. The flowchart of the proposed approach.

TABLE 3. The parameters in the proposed approach.

The size of population	400
The number of generations	200
Replacement probability, pr	0.01
Crossover probability, pc	0.7
Mutation probability, pm	0.2
The maximum iteration of MTS	1000

algorithms were implemented by the modified tabu search and the genetic algorithm to create a DQC.

Pseudo Code: Improvement of Graph Partitioning With Modified Tabu Search

```
Input: Graph $G_i(V_i, E_i)$
Result: Best partitioning ($P_i$) from Graph $G_i$
    1. initialize tabu list T=0
    2. initialize ($P_i$) as the best partitioning
    3. while (not termination-condition) do
    4. select vertex v from $S_c \in P_i$
    5. apply motion Set $T_1$ and $T_2$
    6. update $P_i$ after transferring v to $S_m \in P_i$
    7. update tabu list
    8. calculate gain move $g(v, m)$
    9. if $P_i$ is better than the best partitioning
    10. update ($P_i$) as the best partitioning
    11. if no improvement ($P_i$), apply the
    disruption mechanism
    12. end while
```

VI. RESULTS

The proposed approach leads to attain the best results for the most distributed quantum circuits according to the results of implementation. For many distributed quantum circuits, the algorithms of the proposed approach are better than the results of other algorithms. Therefore, the results illustrate that the proposed approach can obtain the minimum number of teleportation and reduce the execution time in the distributed quantum circuit effectively. Some of the results are new for benchmark circuits, which were not found previously. As the number of connections between the partitions of the DQC increases, the execution time increases exponentially. For this reason, we reorder the qubits placement to improve the execution time in DQC. The obtained teleportation cost of the proposed approach was also compared to that of other approaches, and it appeared that teleportation cost and execution time were lower than those of other approaches. As Fig. 15 depicts, Our proposed method causes to execute distributed quantum circuits with less execution time compared to [27]. Therefore, the results prove that our proposed approach has superior performance and high efficiency for DQCs. The reasons are as follows: The proposed approach is the combination algorithm of MTS and GA. It combines the global search by using GA and local search by employing MTS to perform the proposed algorithms. The genetic algorithm has the ability for global searching, and the modified tabu search has the ability for local searching. Furthermore, genetic operators in GA and the motion structure in MTS are applied and implemented in the proposed approach. To indicate the performance and effectiveness of the proposed approach, we used RevLib circuits, Binary welded tree (BWT),and Ground state estimation (GSE) as benchmark circuits. Afterward, we applied our proposed method to benchmark circuits in order to determine teleportation cost. The partitioning of graph is applied, so that the least number of communications between partitions is required, and this causes to reduce the teleportation cost in distributed quantum circuits as shown in Fig. 16. We implemented our proposed method to benchmark circuits for $K = 2$ and 3 partitions, respectively. The number of teleportation is related to the placement of gates in the partitions. Compared to the proposed approach in [27], our proposed method causes to execute distributed quantum circuits with less execution time. Our proposed method changes the order of the qubits and causes quantum qubits to move closer to each other to run on quantum gates, thereby reducing the execution time of the partitioning algorithm. For each quantum circuit, we used a genetic algorithm and the modified tabu search to partition the graph model, so that the minimum number of teleportation costs was obtained. We performed the proposed approach over different quantum circuits to show the execution time of our proposed approach. The results indicate that the proposed approach can reduce the execution time compared to [27] by approximately 80% that is a considerable improvement. The execution time of the method presented in [27] for quantum circuit alu_primitive and sym9_147 is 1182.51 and 10930.47 seconds, respectively. However, the proposed algorithm can considerably reduce the execution time needed for these distributed quantum circuits. By determining the various number of partitions for the benchmark quantum circuits, the proposed approach attained an average of 50%

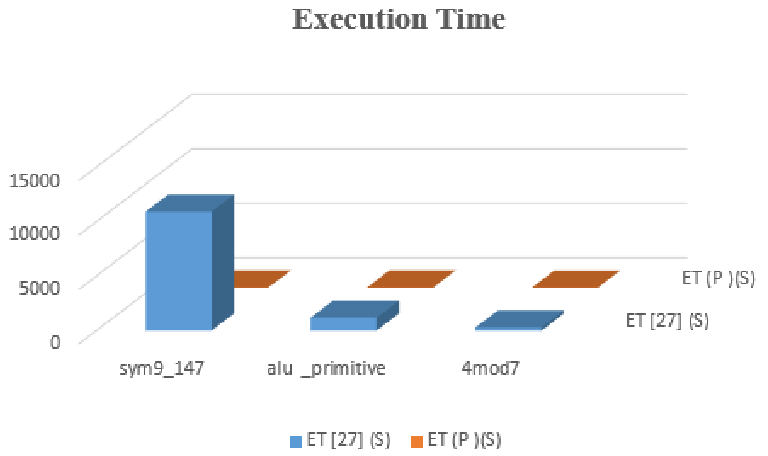


FIGURE 15. Improvement in the execution time of the proposed approach (P) compared to [27].

TABLE 4. Execution time comparison of the proposed approach (P) with the results in [26] and [27].

circuits	# of qubits	# of partitions	ET [26] (S)	ET [27] (S)	ET (P) (S)
sym9_147	12	2	N.A	10930.47	24.55
flip-flop	8	3	N.A	N.A	18.21
alu_primitive	6	2	154020	1182.51	24.48
4gt5_75	5	2	N.A	N.A	12.20
4gt5_76	5	2	N.A	275	37.74
4mod7	5	2	N.A	292.26	15.97
cm42a_207	14	2	N.A	N.A	40.54
4gt4-v0_79	6	2	N.A	N.A	38.50
ham15_107	15	2	N.A	N.A	29.86
4_49_17	15	2	N.A	N.A	12.78
mod5d1_63	15	2	N.A	N.A	15.40

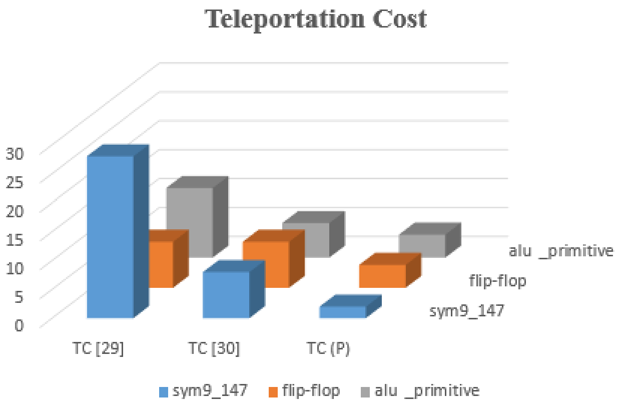


FIGURE 16. Improvement in the teleportation cost of the proposed approach (P) compared to [29] and [30].

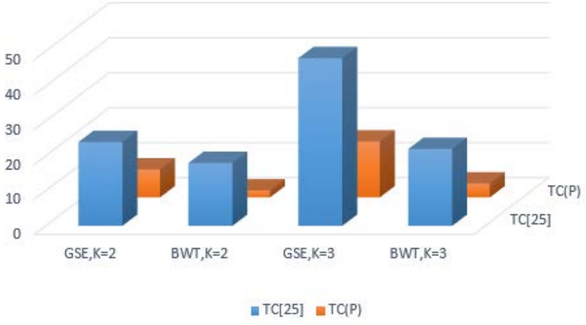


FIGURE 17. Number of the teleportation cost for GSE and BWT circuits in comparison to [25].

improvement in the number of teleportation compared to [29] and [30].

Table 4 displays the execution time of proposed approach compared to [26], [27] and other quantum circuits. Table 5 compares the results of our proposed approach to those in [29], [30] and the RevLib website in terms of teleportation cost (TC) for 21 different quantum circuits. Thus, we proposed the novel approach to find the number of teleportation required for the communication between subsystems. Table 6 presents the results of the comparison of our proposed approach to [25] in terms of teleportation cost. Fig. 17 depicts TC for various numbers of partitions (K) in comparison to [25] for two quantum circuits: BWT,and GSE circuits.

Fig.18 also shows the improved teleportation cost of the proposed method in comparison to the proposed methods

TABLE 5. Teleportation cost comparison of the proposed approach (P) with the results in [29] and [30].

circuits	# of qubits	# of gates	# of partitions	TC [29]	TC [30]	TC (P)
sym9_147	12	28	2	28	8	2
flip-flop	8	17	3	8	8	4
alu_primitive	6	13	2	12	6	4
4gt5_75	5	5	2	N.A	N.A	6
4gt5_76	5	16	2	N.A	N.A	8
4mod7	5	11	2	10	N.A	4
cm42a_207	14	20	2	N.A	N.A	2
4gt4-v0_79	6	9	2	N.A	N.A	6
4_49_16	4	16	2	N.A	N.A	8
4_49_17	4	12	2	N.A	N.A	8
alu_200	18	94	2	N.A	N.A	12
clip_206	14	174	2	N.A	N.A	4
ham 15_107	15	132	3	N.A	N.A	44
ham 15_108	15	70	3	N.A	N.A	42
mod5d1_63	5	7	2	N.A	N.A	6
z4ml_269	11	48	2	N.A	N.A	10
root_255	13	99	2	N.A	N.A	6
radd_250	13	48	2	N.A	N.A	10
pm1_249	14	35	2	N.A	N.A	4
cyc1e10_22	12	19	2	N.A	N.A	4
z4_268	11	48	2	N.A	N.A	10

N.A means the author did not provide the results.

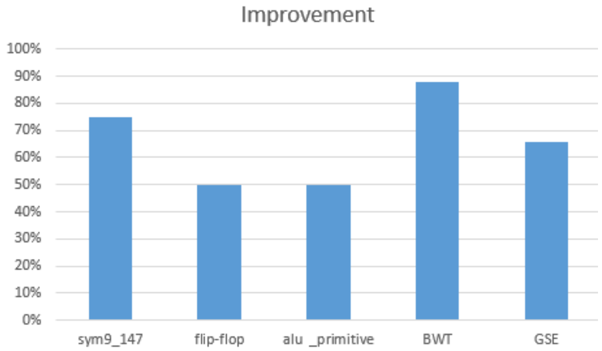


FIGURE 18. Percentage of the improved teleportation cost of the proposed approach compared to [25] and [30].

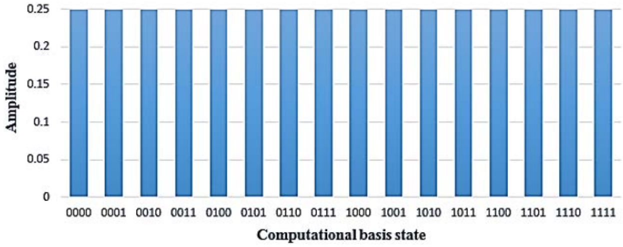


FIGURE 19. State vector for the quantum circuit in [26].

TABLE 6. The number of teleportation cost for $K = 2$ and 3 for GSE and BWT circuits compared to [25].

circuits	# of partitions	TC [25]	TC (P)
GSE	2	24	8
GSE	3	48	16
BWT	2	18	2
BWT	3	22	4

in [25] and [30]. Table 3 presents the parameters for the proposed approach.

International Business Machines (IBM) has made the quantum computers which is available for researchers [34]. IBM Quantum (IBM Q) provided the quantum computing services based on the IBM Quantum Composer and the IBM Quantum Lab which builds a platform to create the quantum circuits and quantum models. Also, the configurations with several quantum gates are available for running the test circuits on an IBM Q computer. However, each configuration can be implemented in a quantum computer including the single-qubit and CNOT gates. For the quantum circuit in [26], by running IBM quantum computer on the distributed quantum circuit constructed by proposed approach in [26] and the distributed quantum circuit generated by our proposed approach, we verified that our proposed approach significantly minimized the number of teleportations based on an equal state vector obtained for this quantum circuit as shown in Fig. 19. To demonstrate the crucial aspects of each quantum circuit, we developed certain quantum circuits in the quantum composer and run them on the IBM Q platform. In the IBM Q platform, transpiling time (TPT) and transpiled quantum circuit help us to assess the findings. TPT is the time which it takes for quantum circuit to be translated into a circuit that can be run on backend. This process includes converting quantum gates into standard basis gates and optimizes the quantum circuit in own method as condensing gates in the platform IBM Q. The time it takes to convert gates into basis gates is mostly independent of the backend system, and it relies on the number and complexity of quantum gates in the original quantum circuit. The IBM Q takes longer to make quantum circuits compatible and better efficient for the backend because of the lengthier transpiling time.

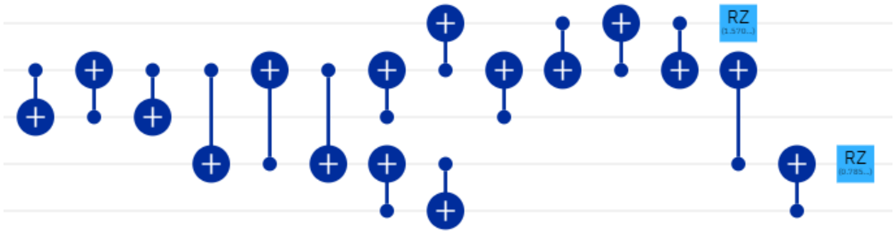


FIGURE 20. The representation of the transpiled quantum circuit for the quantum circuit in [26].

TABLE 7. The status timeline and the number of teleportation cost for the transpiled quantum circuits.

circuits	TPT(mS)	EXT(S)	TPC
mod5d1	745	20.42	6
4_49_16	920	10.65	4
Figure of [23]	790	10.04	12

Execution time (EXT) is the time which it takes to generate the distributed quantum circuit in our proposed approach based on transpiled quantum circuit. Regarding the transpiled quantum circuit, teleportation cost (TPC) is the number of teleportations that is obtained by our proposed approach. Fig. 20 shows the transpiled quantum circuit for the quantum circuit in [26]. Table 7 shows the obtained status timeline and the number of teleportation cost to generate the distributed quantum circuits due to the quantum circuits, which are transpiled on the IBM Q.

VII. CONCLUSION

We introduced a new method to optimize the number of teleportation cost and to improve the execution time in a distributed quantum circuit. As we proved in the results, the proposed approach reduced the number of teleportation and execution time. We also implemented our optimization approach in different quantum circuits. We used the RevLib website as benchmark circuits [10]. Furthermore, all implementations were performed on an Intel Celeron Dual Core 3 GHz with 2 GB of main memory. We employed the genetic algorithm and modified tabu search for a graph model in which reduction of teleportation cost was possible. Moreover, we perform four steps in our proposed approach to construct a DQC. Finally, we demonstrated that the proposed approach could be easily extended to upgrade the performance of DQCs.

The contributions of this paper include:

- Reordering of the qubits placement can cause to change qubits placement in QC, leading to reduction of the execution time when partitioning the algorithm is implemented.
- A combination of MTS and GA has been proposed to generate a distributed quantum circuit with the least number of teleportation cost.

The results indicate that the proposed approach has been successfully used to generate DQCs.

REFERENCES

[1] M. A. Nielsen and I. Chuang, *Quantum Computation and Quantum Information*. Annapolis, MD, USA: American Association of Physics Teachers, 2002.

[2] L. K. Grover, "A fast quantum mechanical algorithm for database search," in *Proc. 28th Annu. ACM Symp. Theory Comput.*, 1996, pp. 212–219.

[3] P. W. Shor, "Polynomial-time algorithms for prime factorization and discrete logarithms on a quantum computer," *SIAM Rev.*, vol. 41, no. 2, pp. 303–332, 1999.

[4] D. Deutsch and R. Jozsa, "Rapid solution of problems by quantum computation," *Proc. Roy. Soc. London A, Math. Phys. Sci.*, vol. 439, no. 1907, pp. 553–558, Dec. 1992.

[5] L. K. Grover, "A fast quantum mechanical algorithm for database search," 1996, *arXiv:quant-ph/9605043*.

[6] C. H. Bennett and S. J. Wiesner, "Communication via one-and two-particle operators on Einstein-Podolsky-Rosen states," *Phys. Rev. Lett.*, vol. 69, no. 20, p. 2881, Nov. 1992.

[7] C. H. Bennett, G. Brassard, R. Jozsa, D. Mayers, A. Peres, B. Schumacher, and W. K. Wootters, "Reduction of quantum entropy by reversible extraction of classical information," *J. Modern Opt.*, vol. 41, no. 12, pp. 2307–2314, Dec. 1994.

[8] A. Peres, "Quantum cryptography with orthogonal states?" 1995, *arXiv:quant-ph/9509003*.

[9] J. Stolze and D. Suter, *Quantum Computing*, vol. 29. Hoboken, NJ, USA: Wiley, 2004, pp. 30–31.

[10] R. Wille, D. Gro, L. Teuber, G. W. Dueck, and R. Drechsler, "RevLib: An online resource for reversible functions and reversible circuits," in *Proc. 38th Int. Symp. Multiple Valued Log. (ISMVL)*, May 2008, pp. 220–225.

[11] O. Daei, K. Navi, and M. Zomorodi, "Improving the teleportation cost in distributed quantum circuits based on commuting of gates," *Int. J. Theor. Phys.*, vol. 60, no. 9, pp. 3494–3513, Sep. 2021.

[12] D. Dadkhah, M. Zomorodi, and S. E. Hosseini, "A new approach for optimization of distributed quantum circuits," *Int. J. Theor. Phys.*, vol. 60, no. 9, pp. 3271–3285, Sep. 2021.

[13] I. Ghodsollahee, Z. Davarzani, M. Zomorodi, P. Pławiak, M. Houshmand, and M. Houshmand, "Connectivity matrix model of quantum circuits and its application to distributed quantum circuit optimization," *Quantum Inf. Process.*, vol. 20, no. 7, pp. 1–21, Jul. 2021.

[14] J. D. Whitfield, J. Biamonte, and A. Aspuru-Guzik, "Simulation of electronic structure Hamiltonians using quantum computers," *Mol. Phys.*, vol. 109, no. 5, pp. 735–750, Mar. 2011.

[15] M. Ghosh, A. Chakrabarti, and N. K. Jha, "Automated quantum circuit synthesis and cost estimation for the binary welded tree Oracle," *ACM J. Emerg. Technol. Comput. Syst.*, vol. 13, no. 4, pp. 1–14, Aug. 2017.

[16] M. A. Nielsen and I. L. Chuang, "Quantum computation and quantum information," *Phys. Today*, vol. 54, pp. 2–60, Nov. 2001.

[17] M. Nakahara and T. Ohmi, *Quantum Computing: From Linear Algebra to Physical Realizations*. Boca Raton, FL, USA: CRC Press, 2008.

[18] L. K. Grover, "Quantum telecomputation," 1997, *arXiv:quant-ph/9704012*.

[19] R. Cleve and H. Buhrman, "Substituting quantum entanglement for communication," *Phys. Rev. A, Gen. Phys.*, vol. 56, no. 2, p. 1201, 1997.

[20] J. I. Cirac, A. K. Ekert, S. F. Huelga, and C. Macchiavello, "Distributed quantum computation over noisy channels," *Phys. Rev. A, Gen. Phys.*, vol. 59, no. 6, p. 4249, Jun. 1999.

[21] R. Beals, S. Brierley, O. Gray, A. W. Harrow, S. Kutin, N. Linden, D. Shepherd, and M. Stather, "Efficient distributed quantum computing," *Proc. Roy. Soc. A, Math., Phys. Eng. Sci.*, vol. 469, no. 2153, 2013, Art. no. 20120686.

[22] J. Yepez, "Type-II quantum computers," *Int. J. Modern Phys. C*, vol. 12, no. 9, pp. 1273–1284, Nov. 2001.

[23] A. Streltsov, H. Kampermann, and D. Bruß, "Quantum cost for sending entanglement," *Phys. Rev. Lett.*, vol. 108, no. 25, Jun. 2012, Art. no. 250501.

[24] M. Ying and Y. Feng, "An algebraic language for distributed quantum computing," *IEEE Trans. Comput.*, vol. 58, no. 6, pp. 728–743, Jun. 2009.

[25] P. Andrés-Martínez and C. Heunen, "Automated distribution of quantum circuits via hypergraph partitioning," *Phys. Rev. A, Gen. Phys.*, vol. 100, no. 3, Sep. 2019, Art. no. 032308.

[26] M. Zomorodi-Moghadam, M. Houshmand, and M. Houshmand, "Optimizing teleportation cost in distributed quantum circuits," *Int. J. Theor. Phys.*, vol. 57, no. 3, pp. 848–861, Mar. 2018.

[27] M. Houshmand, Z. Mohammadi, M. Zomorodi, and M. Houshmand, "An evolutionary approach to optimizing teleportation cost in distributed quantum computation," *Int. J. Theor. Phys.*, vol. 59, no. 4, pp. 1315–1329, 2020.

[28] A. Yimsiriwattana and S. J. Lomonaco, Jr., "Distributed quantum computing: A distributed Shor algorithm," *Proc. SPIE*, vol. 5436, pp. 360–372, Aug. 2004.

[29] O. Daei, K. Navi, and M. Zomorodi-Moghadam, "Optimized quantum circuit partitioning," *Int. J. Theor. Phys.*, vol. 59, no. 12, pp. 3804–3820, Dec. 2020.

[30] Z. Davarzani, M. Zomorodi-Moghadam, M. Houshmand, and M. Nouri-Baygi, "A dynamic programming approach for distributing quantum circuits by bipartite graphs," *Quantum Inf. Process.*, vol. 19, no. 10, pp. 1–18, Oct. 2020.

[31] U. Benlic and J.-K. Hao, "An effective multilevel tabu search approach for balanced graph partitioning," *Comput. Oper. Res.*, vol. 38, no. 7, pp. 1066–1075, Jul. 2011.

[32] D. Maslov, C. Young, D. M. Miller, and G. W. Dueck, "Quantum circuit simplification using templates," in *Proc. Design, Autom. Test Eur.*, 2005, pp. 1208–1213.

[33] D. Maslov, G. W. Dueck, and D. M. Miller, "Toffoli network synthesis with templates," *IEEE Trans. Comput.-Aided Design Integr.*, vol. 24, no. 6, pp. 807–817, Jun. 2005.

[34] A. Cross, "The IBM Q experience and QISKit open-source quantum computing software," in *Proc. Bull. Amer. Phys. Soc.*, vol. 63, 2018, p. L58-003.

DAVOOD DADKHAH received the B.S. degree in electrical engineering from Imam Reza University, Mashhad, Iran, in 2012, and the M.S. degree in electrical engineering from the Ferdowsi University of Mashhad, Mashhad, in 2014, where he is currently pursuing the Ph.D. degree in electrical engineering.

MARIAM ZOMORODI received the B.Sc. degree in computer engineering from the Iran University of Science and Technology, in 2002, the M.Sc. degree in computer engineering from the Amirkabir University of Technology (Tehran Polytechnic), in 2007, and the Ph.D. degree in computer engineering from Shahid Beheshti University, in 2014. She is currently with the Department of Information and Communications Technology, Cracow University of Technology, and the Ferdowsi University of Mashhad. Her current research interests include quantum computing, evolutionary computation, and machine learning.

SEYED EBRAHIM HOSSEINI is currently a Full Professor of electrical engineering at the Ferdowsi University of Mashhad, Iran. His research interests include quantum information and semiconductor science and technology.

PAWEL PLAWIAK was born in Ostrowiec, Poland, in 1984. He received the B.Eng. and M.Sc. degrees in electronics and telecommunications and the Ph.D. degree (Hons.) in biocybernetics and biomedical engineering from the AGH University of Science and Technology, Kraków, Poland, in 2012 and 2016, and the D.Sc. degree in technical computer science and telecommunications from the Silesian University of Technology, Gliwice, Poland, in 2020. He is currently the Dean of the Faculty of Computer Science and Telecommunications and an Associate Professor with the Cracow University of Technology, Kraków, and the Deputy Director for research with the National Institute of Telecommunications, Warsaw, and an Associate Professor with the Institute of Theoretical and Applied Informatics, Polish Academy of Sciences, Gliwice. He has published more than 50 articles in refereed international SCI-IF journals. His research interests include machine learning and computational intelligence (e.g., artificial neural networks, genetic algorithms, fuzzy systems, support vector machines, k-nearest neighbors, and hybrid systems), ensemble learning, deep learning, evolutionary computation, classification, pattern recognition, signal processing and analysis, data analysis and data mining, sensor techniques, medicine, biocybernetics, biomedical engineering, and telecommunications. He is an academic editor and a reviewer of many prestigious and reputed journals.

XUJUAN ZHOU (Member, IEEE) received the Ph.D. degree from the Queensland University of Technology, Australia, in 2009. She is currently an Associate Professor with the University of Southern Queensland (USQ), Australia. In 2021, she received a three-year (2022–2024) Advance Queensland Industry Research Fellowship grant for the project titled "Early detection of chronic health conditions using AI prediction model." She has completed the Commonwealth Innovation Connections Grant research project titled "Prediction of cancer recurrence using innovative machine learning approaches" (2017–2019). She has published articles in prestige journals (*Artificial Intelligence Review*, *Decision Support Systems*, *Pattern Recognition Letter*, *Artificial Intelligence in Medicine*, *Journal of Medical Intent Research*, *Knowledge-Based Systems*, and IEEE Access). Her research interests include machine learning and deep learning, health informatics, information retrieval/filtering, recommender systems, sentiment analysis, and natural language processing. She is an Editorial Board Member of *Web Intelligence*–an international journal and a reviewer for many journals and conferences related to her research fields.

• • •