Course 11

Push-Down Automata (PDA)

Intuitive Model

Definition

- A push-down automaton (APD) is a 7-tuple $M = (Q,\boldsymbol{\Sigma},\boldsymbol{\Gamma},\boldsymbol{\delta},q_0,Z_0,F)$ where:
  - Q – finite set of states
  - $\boldsymbol{\Sigma}$ - alphabet (finite set of input symbols)
  - $\boldsymbol{\Gamma}$ – stack alphabet (finite set of stack symbols)
  - $\boldsymbol{\delta} : Q \times (\boldsymbol{\Sigma} \cup \{\boldsymbol{\varepsilon}\}) \times \boldsymbol{\Gamma} \to \mathcal{P}(Q \times \boldsymbol{\Gamma}^*)$ –transition function
  - $q_0 \in Q$ – initial state
  - $Z_0 \in \boldsymbol{\Gamma}$ – initial stack symbol
  - $F \subseteq Q$ – set of final states

Push-down automaton

Transition is determined by:

- Current state
- Current input symbol
- Head of stack

Reading head -> input band:

- Read symbol
- No action

Stack:

- Zero symbols => pop
- One symbol => push
- Several symbols => repeated push

Configurations and transition / moves

- Configuration:

$$(\mathbf{q}, \mathbf{x}, \boldsymbol{\alpha}) \in Q \times \boldsymbol{\Sigma}^* \times \boldsymbol{\Gamma}^*$$

where:

  - PDA is in state **q**
  - Input band contains **x**
  - Head of stack is $\boldsymbol{\alpha}$

- Initial configuration (q_0, w, Z_0)

Configurations and moves(cont.)

- Moves between configurations:

p,q $\in$Q, a$\in\boldsymbol{\Sigma}$, Z $\in\boldsymbol{\Gamma}$, w $\in\boldsymbol{\Sigma^*}$,$\boldsymbol{\alpha},\boldsymbol{\gamma}$ $\in\boldsymbol{\Gamma^*}$

(q,aw,Z$\boldsymbol{\alpha}$) $\vdash$ (p,w,$\boldsymbol{\gamma}$Z$\boldsymbol{\alpha}$) iff $\boldsymbol{\delta}$(q,a,Z) $\ni$ (p,$\boldsymbol{\gamma}$Z)

(q,aw,Z$\boldsymbol{\alpha}$) $\vdash$ (p,w, $\boldsymbol{\alpha}$) iff $\boldsymbol{\delta}$(q,a,Z) $\ni$ (p, $\boldsymbol{\varepsilon}$)

(q,aw,Z$\boldsymbol{\alpha}$) $\vdash$ (p,aw,$\boldsymbol{\gamma}$Z$\boldsymbol{\alpha}$) iff $\boldsymbol{\delta}$(q,$\boldsymbol{\varepsilon}$,Z) $\ni$ (p,$\boldsymbol{\gamma}$Z)

($\boldsymbol{\varepsilon}$-move)

- $\overset{k}{\vdash}$, $\overset{+}{\vdash}$, $\overset{*}{\vdash}$

Language accepted by PDA

- Empty stack principle:

$L_{\boldsymbol{\varepsilon}}(M) = \{w|w\in\boldsymbol{\Sigma^*}, (q_0,w,Z_0) \vdash^* (q,\boldsymbol{\varepsilon},\boldsymbol{\varepsilon}), q\in Q\}$

- Final state principle:

$L_f(M) = \{w|w\in\boldsymbol{\Sigma^*}, (q_0,w,Z_0) \vdash^* (q_f,\boldsymbol{\varepsilon},\boldsymbol{\gamma}), q_f\in F\}$

Representations

- Enumerate
- Table
- Graphic

Construct PDA

- L = $\{0^n1^n \mid n \geq 1\}$
- States, stack, moves?

1. States:
   - Initial state:q_0 – beginning and process symbols ‘0’
   - When first symbol ‘1’ is found – move to new state => q_1
   - Final: final state q_2
2. Stack:
   - Z_0 – initial symbol
   - X – to count symbols:
     - When reading a symbol ’0’ – push X in stack
     - When reading a symbol ‘1’ – pop X from stack

Exemple 1 (enumerate)

$M = (\{q_0,q_1,q_2\}, \{0,1\}, \{Z_0,X\}, \boldsymbol{\delta}, q_0, Z_0, \{q_2\})$

$\boldsymbol{\delta}(q_0,0,Z_0) = (q_0,XZ_0)$

$\boldsymbol{\delta}(q_0,0,X) = (q_0,XX)$

$\boldsymbol{\delta}(q_0,1,X) = (q_1,\boldsymbol{\varepsilon})$

$\boldsymbol{\delta}(q_1,1,X) = (q_1,\boldsymbol{\varepsilon})$

$\cancel{\boldsymbol{\delta}(q_1,\boldsymbol{\varepsilon},Z_0) = (q_2,Z_0)}$

$\boxed{\boldsymbol{\delta}(q_1,\boldsymbol{\varepsilon},Z_0) = (q_1, \boldsymbol{\varepsilon})}$

Empty stack

$\vdash (q_1, \boldsymbol{\varepsilon}, \boldsymbol{\varepsilon})$

$(q_0,0011,Z_0) \vdash (q_0,011,XZ_0) \vdash (q_0,11,XXZ_0) \vdash (q_1,1,XZ_0) \vdash (q_1, \boldsymbol{\varepsilon}, Z_0) \vdash (q_2, \boldsymbol{\varepsilon}, Z_0)$

Final state

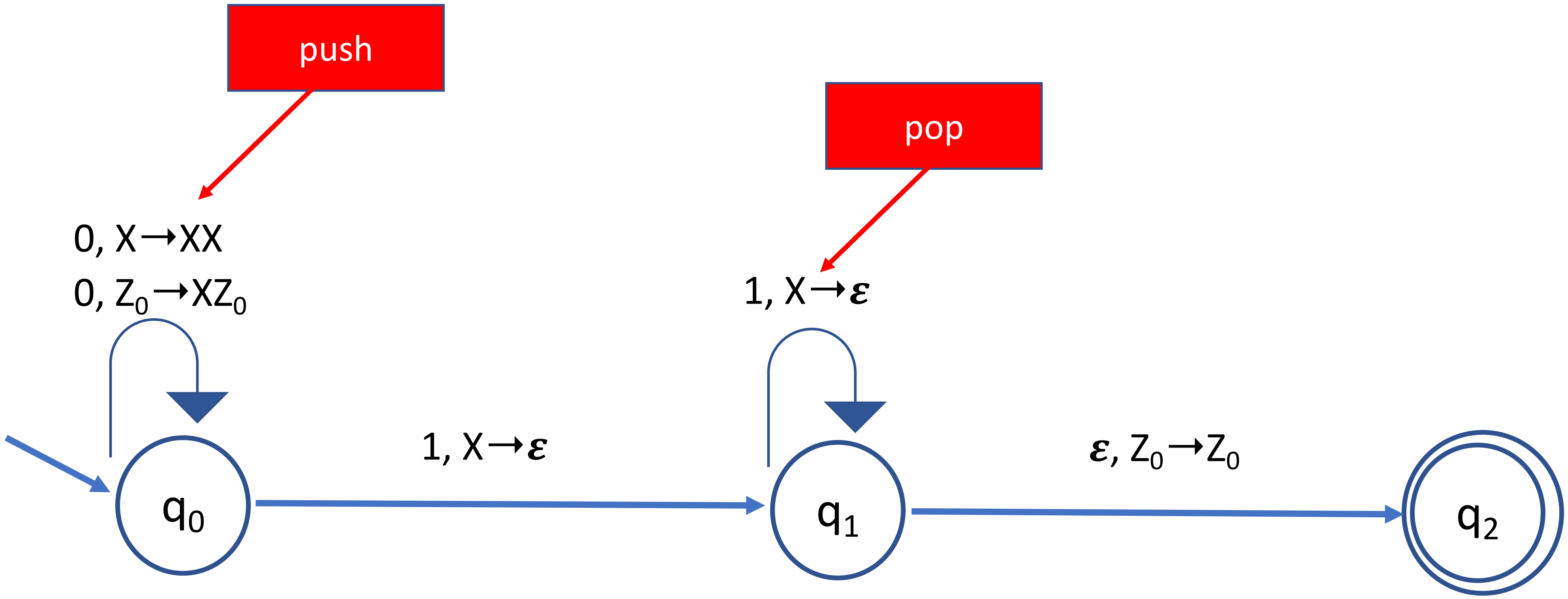
Exemple 1 (table)

		0	1	ε
q_0	Z_0	q_0,XZ_0		
	X	q_0,XX	q_1,ε	
q_1	Z_0			q_2,Z_0 (q_1, ε)
	X		q_1,ε	
q_2	Z_0			
	X			

(q0,0011,Z0) |- (q0,011,XZ0) |- (q0,11,XXZ0) |- (q1,1,XZ0)
|- (q1, ε,Z0) |- (q2, ε,Z0) q2 final seq. is acc based on final state

(q0,0011,Z0) |- (q0,011,XZ0) |- (q0,11,XXZ0) |- (q1,1,XZ0)
|- (q1, ε,Z0) |-(q1, ε, ε) seq is acc based on empty stack

Exemple 1 (graphic)



Properties

Theorem 1: For any PDA M, there exists a PDA M' such that

$$L_{\varepsilon}(M) = L_f(M')$$

Theorem 2: For any PDA M, there exists a context free grammar such that

$$L_{\varepsilon}(M) = L(G)$$

Theorem 3: For any context free grammar there exists a PDA M such that

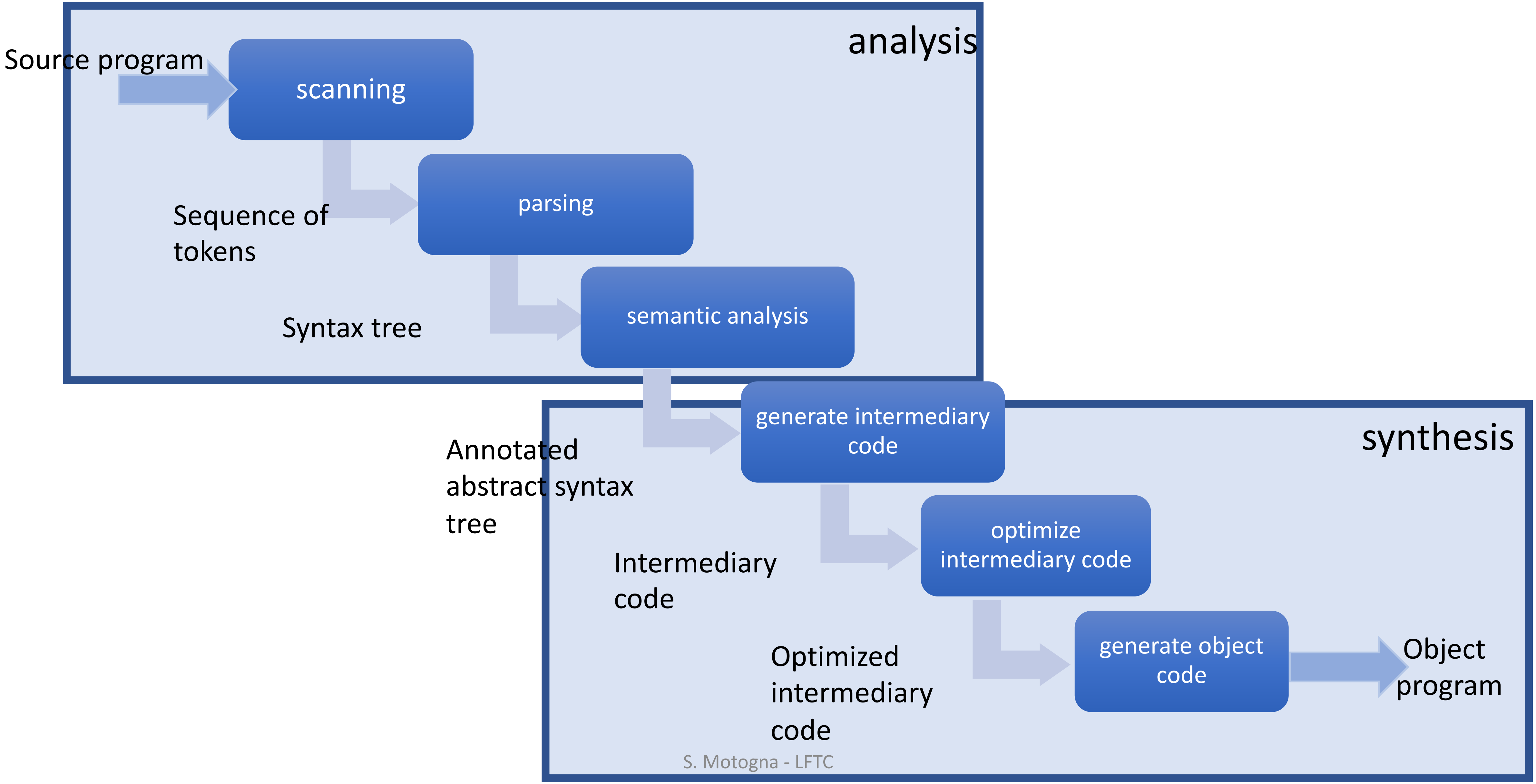
$$L(G) = L_{\varepsilon}(M)$$

HW

- Parser:
  - Descendent recursive
  - LL(1)
  - LR(0), SLR, LR(1)

Corresponding PDA

Structure of compiler



Semantic analysis

- Parsing – result: syntax tree (ST)
- Simplification: abstract syntax tree (AST)
- Annotated abstract syntax tree (AAST)
  - Attach semantic info in tree nodes

Example

Semantic analysis

- Attach meanings to syntactical constructions of a program
- What:
  - Identifiers -> values / how to be evaluated
  - Statements -> how to be executed
  - Declaration -> determine space to be allocated and location to be stored
- Examples:
  - Type checkings
  - Verify properties
- How:
  - **Attribute grammars**
  - Manual methods

Attribute grammar

- Syntactical constructions (nonterminals) – attributes

$$\forall X \in N \cup \Sigma : A(X)$$

- Productions – rules to compute/ evaluate attributes

$$\forall p \in P : R(p)$$

Definition

AG = (G,A,R) is called ***attribute grammar*** where:

- G = (N,$\boldsymbol{\Sigma}$,P,S) is a context free grammar
- A = {A(X) | X ∈N U $\boldsymbol{\Sigma}$} – is a finite set of attributes
- R = {R(p) | p ∈P} – is a finite set of rules to compute/evaluate attributes

Example 1

- $G = (\{N,B\},\{0,1\}, P, N\}$

P: $N_1 \rightarrow N_2B$

$N \rightarrow B$

$B \rightarrow 0$

$B \rightarrow 1$

$N_1.v = 2 * N_2.v + B.v$

$N.v = B.v$

$B.v = 0$

$B.v = 1$

Attribute – value of number = **v**

- **Synthetized attribute: A(lhp) depends on rhp**
- **Inherited attribute: A(rhp) depends on lhp**

Evaluate attributes

- Traverse the tree: can be an infinite cycle

- Special classes of AG:
  - L-attribute grammars: for any node the depending attributes are on the "*left*";
    - can be evaluated in one left-to-right traversal of syntax tree
    - Incorporated in top-down parser (LL(1))
  - S-attribute grammars: synthetized attributes
    - Incorporated in bottom-up parser (LR)

Steps

- What? - decide what you want to compute (type, value, etc.)
- Decide attributes:
  - How many
  - Which attribute is defined for which symbol
- Attach evaluation rules:
  - For each production – which rule/rules

Example 2 (L-attribute grammar)

Decl -> DeclTip ListId

ListId -> Id

ListId -> ListId, Id

ListId.type = DeclTip.type
Id.type = ListId.type
$\text{ListId}_2\text{.type} = \text{ListId}_1\text{.type}$
$\text{Id.type} = \text{ListId}_1\text{.type}$

Attribute – type

int i,j

Example 3 (S-attribute grammar)

ListDecl -> ListDecl; Decl

ListDecl -> Decl

Decl -> Type ListId

Type -> int

Type -> long

ListId -> Id

ListId -> ListId, Id

$ListDecl_1.dim = ListDecl_2.dim + Decl.dim$

$ListDecl.dim = Decl.dim$

$Decl.dim = Type.dim * ListId.no$

$Type.dim = 4$

$Type.dim = 8$

$ListId.no = 1$

$ListId_1.no = ListId_2.no + 1$

Attributes – dim + no – **for which symbols**

int i,j; long k

Proposed problems (HW):

1) Define an attribute grammar for arithmetic expressions
2) Define an attribute grammar for logical expressions
3) Define an attribute grammar for if statement

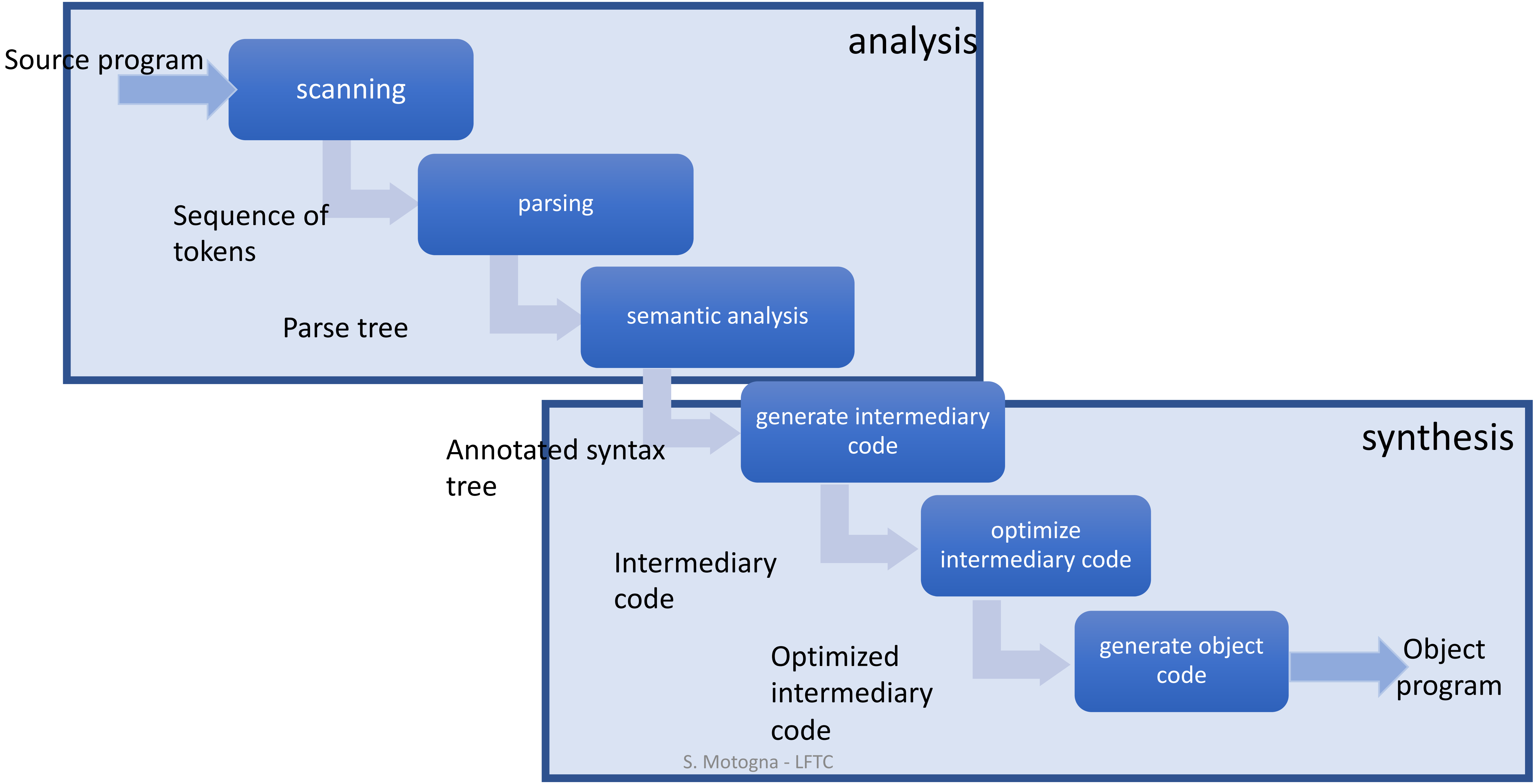
Manual methods

- Symbolic execution
  - Using control flow graph, simulate on stack how the program will behave
  - [Grune – Modern Compiler Design]

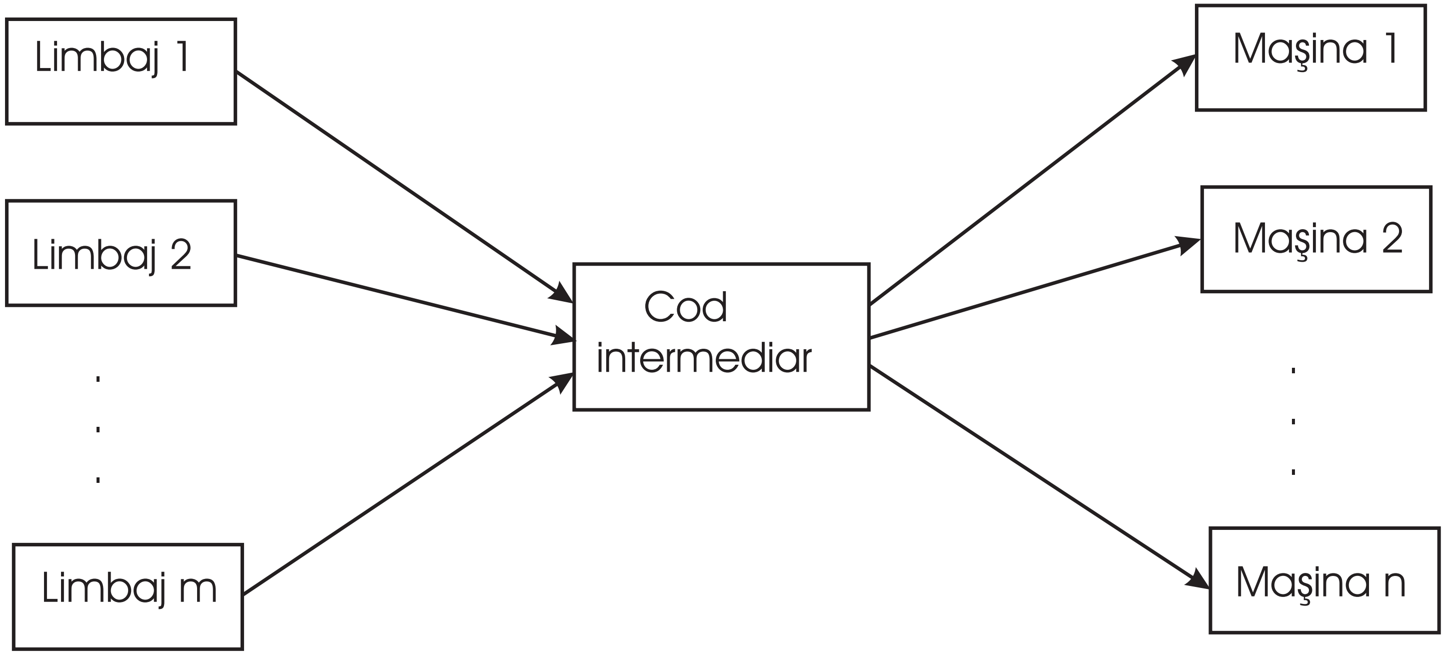
- Data flow equations
  - Data flow – associate equations based on data consumed in each node (statement) of the control flow graph: In, Out, Generated, Killed
  - [Grune – Modern Compiler Design], [Kildall], [course]

Course 12

Structure of compiler



Generate intermediary code



Forms of intermediary code

- Java bytecode – source language: Java
  – machine language (dif. platforms) JVM
- MSIL (Microsoft Intermediate Language)
  – source language: C#, VB, etc.
  – machine language (dif. platforms) Windows
- GNU RTL (Register Transfer Language)
  – source language: C, C++, Pascal, Fortran etc.
  – machine language (dif. platforms)

Representations of intermediary code

- Annotated tree: intermediary code is generated in semantic analysis
- Polish postfix form:
  - No parenthesis
  - Operators appear in the order of execution
  - Ex.: MSIL

Exp = a + b * c	ppf = abc*+
Exp = a * b + c	ppf = ab*c+
Exp = a * (b + c)	ppf = abc+*

- 3 address code

3 address code

= sequence of simple format statements, close to object code, with the following general form:

< result >=< arg1 >< op >< arg2 >

Represented as:

- Quadruples
- Triples
- Indirected Triples

- Quadruples:

  < op > < arg1 > < arg2 > < result >

- Triples:

  < op > < arg1 > < arg2 >

(considered that the triple is storing the result)

Special cases:

1. Expressions with unary operator**: < result >=< op >< arg2 >**
2. Assignment of the form **a := b** => the 3 addresss code is **a = b** (no operatorand no 2nd argument)
3. Unconditional jump: statement is **goto L**, where L is the label of a 3 address code
4. Conditional jump: **if c goto L**: if **c** is evaluated to **true** then unconditional jump to statement labeled with L, else (if c is evaluated to false), execute the next statement
5. Function call p(x1, x2, ..., xn) – sequence of statements: **param x1, param x2 , param xn, call p, n**
6. Indexed variables: < arg1 >,< arg2 >,< result > can be array elements of the form **a[i]**
7. Pointer, references: **&x, ∗x**

Example: $b * b - 4 * a * c$

op	arg1	arg2	rez
*	b	b	t1
*	4	a	t2
*	t2	c	t3
-	t1	t3	t4

nr	op	arg1	arg2
(1)	*	b	b
(2)	*	4	a
(3)	*	(2)	c
(4)	-	(1)	(3)

Example 2

If (a<2) then a=b else a=b*b

Optimize intermediary code

- Local optimizations:
  - Perform computation at compile time – constant values
  - Eliminate redundant computations
  - Eliminate inaccessible code – if…then...else...

- Loop optimizations:
  - Factorization of loop invariants
  - Reduce the power of operations

Eliminate redundant computations

Example:

D:=D+C*B
A:=D+C*B
C:=D+C*B

(1)	*	C	B
(2)	+	D	(1)
(3)	:=	(2)	D
~~(4)~~	~~*~~	~~C~~	~~B~~
(5)	+	D	(4)
(6)	:=	(5)	A
~~(7)~~	~~*~~	~~C~~	~~B~~
~~(8)~~	~~+~~	~~D~~	~~(7)~~
(9)	:=	(8)	C

Determine redundant operations

- Operation (j) is redudant to operation (i) with i<j if the 2 operations are identical and if the operands in (j) did not change in any operation between (i+1) and (j-1)
- Algorithm [Aho]

Factorization of loop invariants

What is a loop invariant?

```
for(i=0, i<=n,i++)
      { x=y+z;
      a[i]=i*x}
```

```
x=y+z;
for(i=0, i<=n,i++)
      { a[i]=i*x}
```

Challenge

```
V1:
P = a[0]
For i=1 to n
   P = P + a[i]*v^i
```

```
V2:
P = a[0]
Q=v
For i=1 to n
   P = P + a[i]*Q
   Q = Q*v
```

Consider n, and a[i] i=0,n the coefficients of a polynomial P.

Given v, write an algorithm that computes the value of P(v)

```
V3
P=a[n]
For i=1 to n
  P = P*v + a[n-i]
```

3 solutions

P(x) = a[n]*x^n+ … + a[1]*x + a[0] = (a[n]*x^(n-1)+ … + a[1])*x + a[0]

Reduce the power of operations

```
for(i=k, i<=n,i++)
    { t=i*v;
    . . .}
```

```
t1=k*v;
for(i=k, i<=n,i++)
    { t=t1;
    t1=t1+v;...}
```