Lab 9, Numerical Calculus
Gaussian Quadratures

Implement a Matlab routine (or separate routines for each case) that, for the 5 cases in the table (Laguerre, Chebyshev of the 1st kind, Chebyshev of the 2nd kind, Legendre and Hermite), computes the nodes, the coefficients and the approximate value of the integral, for a Gaussian quadrature, in the form

$$[I, gn, gc] = Name(f, n, case) \ \ \text{(or for each case } [I, gn, gc] = Name(f, n)\text{)},$$

where
f is the integrand,
n is the number of nodes,
I is the approximate value of $\displaystyle\int_a^b w(x)f(x)\,dx$,
gn and gc are vectors containing the nodes and the coefficients of the Gaussian quadrature formula.

Applications

Use appropriate Gaussian quadratures with $n = 2$ and 8 nodes, to approximate the following integrals:

1. $\displaystyle\int_{-\infty}^{\infty} e^{-x^2} dx \ \ (= \sqrt{\pi})$

2. $\displaystyle\int_{-1}^{1} \sqrt{1-x^2}\,dx \ \ \left(= \frac{\pi}{2}\right)$

3. $\displaystyle\int_{-1}^{1} \sin x^2\,dx$

4. $\displaystyle\int_{0}^{\infty} xe^{-x} \sin x\,dx \ \ \left(= \frac{1}{2}\right)$

5. $\displaystyle\int_{-1}^{1} \frac{x^2}{\sqrt{1-x^2}}\,dx \ \ \left(= \frac{\pi}{2}\right)$

Orthogonal Polynomials and Recurrence Coefficients

Name	Notation	Polynomial	Weight Fn.	Interval	$\boldsymbol{\alpha_k}$	$\boldsymbol{\beta_k}$
Legendre	l_m	$\left[(x^2-1)^m\right]^{(m)}$	1	$[-1,1]$	0	$\beta_0 = 2$, $\beta_k = (4-k^{-2})^{-1}, k \geq 1$
Chebyshev 1^{st}	T_m	$\cos(m \arccos x)$	$(1-x^2)^{-\frac{1}{2}}$	$[-1,1]$	0	$\beta_0 = \pi$, $\beta_1 = \frac{1}{2}$, $\beta_k = \frac{1}{4}, k \geq 2$
Chebyshev 2^{nd}	Q_m	$\dfrac{\sin[(m+1)\arccos x]}{\sqrt{1-x^2}}$	$(1-x^2)^{\frac{1}{2}}$	$[-1,1]$	0	$\beta_0 = \frac{\pi}{2}$, $\beta_k = \frac{1}{4}, k \geq 1$
Laguerre	L_m^a	$x^{-a}e^x\left(x^{m+a}e^{-x}\right)^{(m)}$	$x^a e^{-x},\ a > -1$	$[0,\infty)$	$2k+a+1$	$\beta_0 = \Gamma(1+a)$, $\beta_k = k(k+a), k \geq 1$
Hermite	H_m	$(-1)^m e^{x^2}\left(e^{-x^2}\right)^{(m)}$	e^{-x^2}	$\mathbb{R}$	0	$\beta_0 = \sqrt{\pi}$, $\beta_k = \frac{k}{2}, k \geq 1$