

Lab 10, Numerical Calculus

Linear Systems; Direct Methods

1. Implement back substitution for solving an upper triangular linear system and forward substitution for solving a lower triangular linear system.
2. Implement Gaussian elimination with partial pivoting.
3. Implement a routine that solves the system $Ax = b$ using
 - LUP factorization;
 - Cholesky factorization;
 - QR factorization.

Applications

Use both methods (Gaussian elimination and factorizations) to solve the following general linear systems (that can be generated of any order $n \geq 3$, the first one, and $n \geq 7$, the second):

1.

$$\begin{cases} 5x_1 - x_2 & = 4 \\ -x_{j-1} + 5x_j - x_{j+1} & = 3, \quad j = \overline{2, n-1} \\ -x_{n-1} + 5x_n & = 4 \end{cases}$$

2.

$$\begin{bmatrix} 5 & -1 & 0 & -1 & \dots & \dots & 0 \\ -1 & 5 & -1 & 0 & -1 & \dots & \dots \\ 0 & -1 & 5 & -1 & \ddots & \ddots & \vdots \\ -1 & & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 & 5 & -1 & 0 \\ 0 & \dots & \ddots & 0 & -1 & 5 & -1 \\ 0 & \dots & & -1 & 0 & -1 & 5 \\ 0 & \dots & & & -1 & 0 & -1 \end{bmatrix} x = \begin{bmatrix} 3 \\ 2 \\ 2 \\ 1 \\ \vdots \\ 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}$$