Numerical Calculus - Recap

1. Give the first two iterations of bisection, secant and Newton's methods to solve the equation $x^3+x-1=0$. How many steps are required to obtain an accuracy of 10^{-3} for the bisection method?

2. Approximate $\sqrt{3}$ using the first two iterations of Newton's method.

3. Find a fixed point iteration for the equation $3x^2 - e^x = 0$ on the interval $[0,1]$ that satisfies the Banach Theorem conditions. Write the first two iterations and estimate the number of iterations needed for an accuracy of 10^{-5} for the fixed point iteration.

4. Knowing that $\lg 2 = 0.301,\ \lg 3 = 0.477,\ \lg 5 = 0.699$, approximate $\lg 76$ using a suitable interpolation polynomial. Estimate the error of approximation.

5. Approximate $f\left(\frac{1}{2}\right)$ knowing that $f(0) = 1,\ f'(0) = 2,\ f'(1) = -1$.

6. Complete the following forward difference table:

x_i	f_i	$\Delta_1 f_i$	$\Delta_2 f_i$	$\Delta_3 f_i$
2	6			
4		3		
6	10	5		
8				

7. Write the polynomial that interpolates the data

x_i	f_i	f_i'
0	1	1
1	3	4

8. (a) Write the constant least squares approximant for N points $P_i(x_i, y_i),\ i = \overline{1,N}$.
   (b) Using the result from (a), write the approximant in the case $P_1(1,0),\ P_2(2,1),\ P_3(0,3)$.

9. Determine n such that the approximation error for the integral $I = \int_0^\pi \sin x\, dx$ is less that $2 \cdot 10^{-3}$ for the composite Simpson's rule. With the n obtained, approximate the integral.

10. Find a quadrature formula of the form

$$\int_{-1}^{1} f(x)\, dx = A_1 f(x_1) + A_2 f(x_2)$$

that has the degree of precision $d = 3$ using the relations for the remainder and then using orthogonal polynomials.

11. Solve the following system using Gauss elimination with partial pivoting:

$$\begin{cases} x_1 + x_2 - x_3 = 1 \\ x_1 + x_2 + 4x_3 = 2 \\ 2x_1 - x_2 + 2x_3 = 3 \end{cases}$$

12. Solve the previous linear system using LUP decomposition.