Supplementary Information

1. Bayesian Data Analysis

This section outlines how the Bayesian Data Analysis was performed. We follow the recommendations and guidlines of [?, ?, 1] amd asses the Markov Chain Monte Carlo (MCMC) chains and Posterior Predictive Checks. i

1.1. **Assesing the MCMC chains.** All analysis were conducted using the R programming language and the cmdstarr library. Each model was computed using four parallel chains using 2000 iterations and an additional 200 warmup iterations. None of the iteration diverged as can be seen in the traceplots Figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 and therefore the MCMC simulation is considered valid.

In addition we investigate diagnostics of the posterior estimates, such as $Gelman-Rubin\ Potential\ Scale\ Reduction(\hat{R})$ [2] the number of efficient samples (n_{eff}) . Tables 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 contains three columns. The first column contains \hat{R} and the other two contains two different estiamtes of the number of effective samples. As a rule of thumb we should have $\hat{R} < 1.01$ which is satisfied in all scenarios. The number of efficient samples should be at least 200, which is satisfied in all cases.

Table 1.	Diagnostics	for the	posterior	ranks ((aggregated))
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Variable	Rhat	ess_bulk	ess_tail
a_{a} alg[1]	1.000	4363.596	4389.268
$a_a lg[2]$	1.002	4310.666	4704.940
$a_alg[3]$	1.002	4506.676	4641.121
$a_a lg[4]$	1.000	4511.913	5175.336
$a_a lg[5]$	1.001	4179.552	5407.158
$a_a g[6]$	1.000	4421.190	4853.736
$a_{a}[7]$	1.000	4431.419	4697.169
$a_a [8]$	1.000	4429.212	4462.071
$a_a g[9]$	1.001	4518.639	4809.640
$a_a lg[10]$	1.001	4581.093	4876.947
$a_a lg[11]$	1.001	4409.779	5259.614
$a_a lg[12]$	1.001	4903.602	5323.972
$a_a [13]$	1.001	4561.385	5199.520
$a_a lg[14]$	1.000	4635.099	4744.294
$a_a lg[15]$	1.001	4431.587	5018.196
a_alg[16]	1.000	4328.026	4618.103
$a_{-}alg[17]$	1.001	4493.896	4915.240

2. Posterior Predictive Cheks

For posterior predictive checks, we investigate how well the mean error rate is captured by the posterior distribution. Figures 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, and 24 shows the posterior distribution of the mean for each algorithm and the Figures show that the mean is well captured by the posterior distribution.

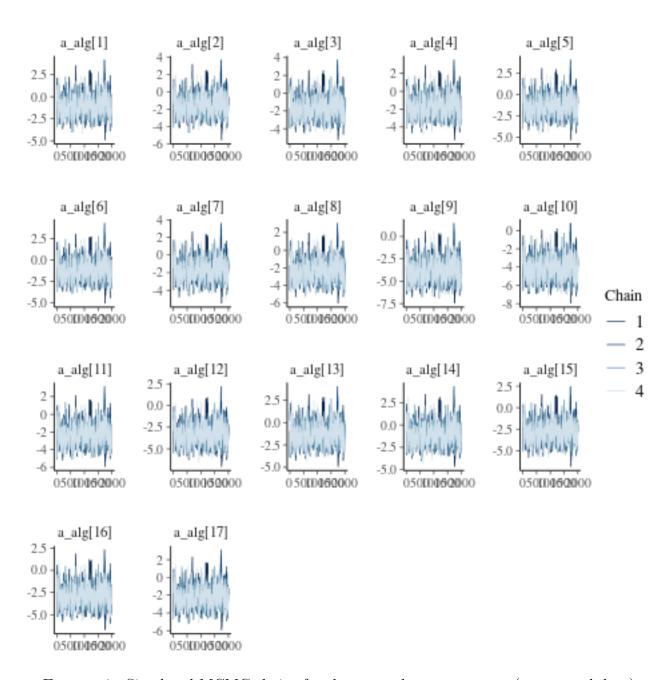


Figure 1. Simulated MCMC chains for the strength parameters a_i (aggregated data)

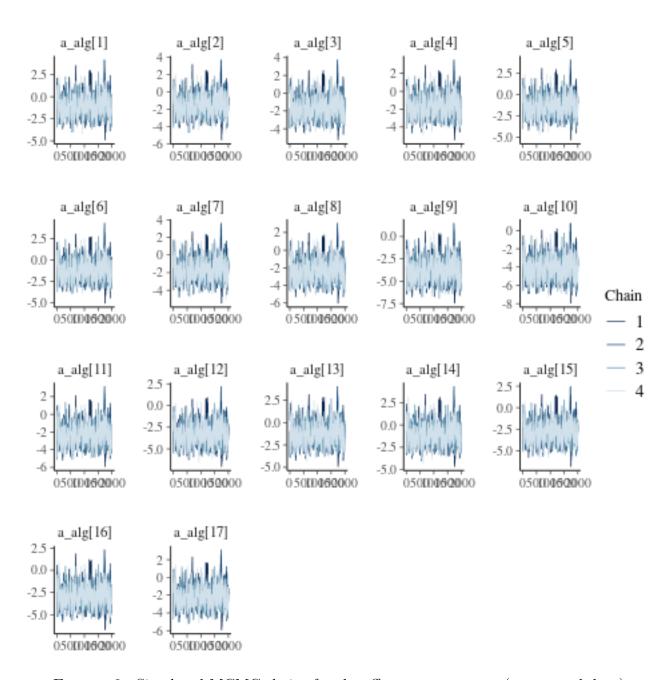


FIGURE 2. Simulated MCMC chains for the effect parameters a_i (aggregated data)

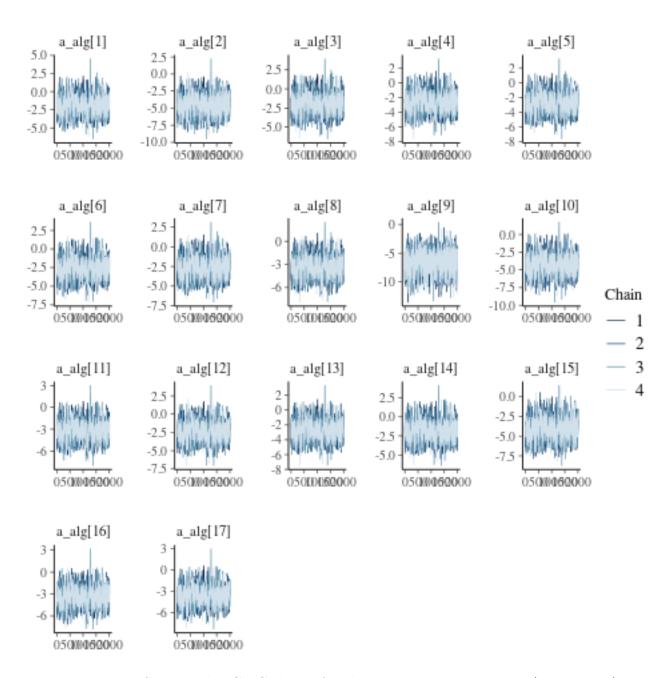


FIGURE 3. Simulated MCMC chains for the strength parameters a_i (image data)

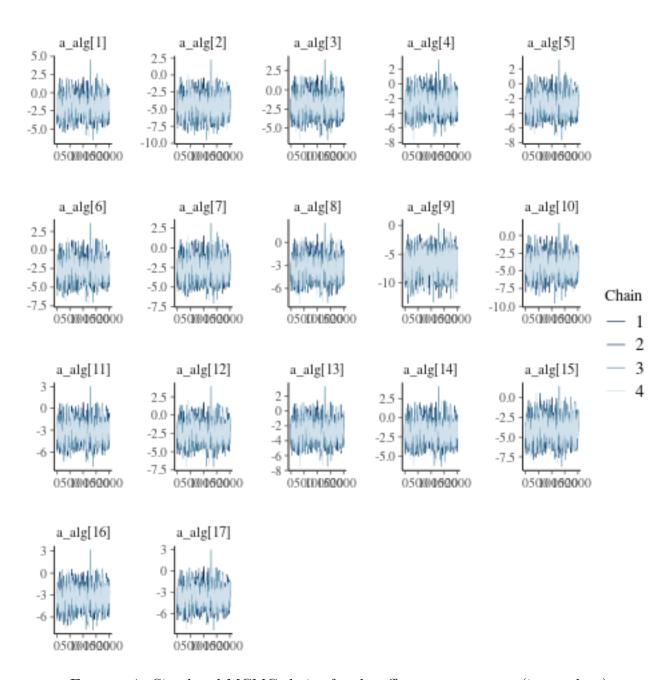


FIGURE 4. Simulated MCMC chains for the effect parameters a_i (image data)

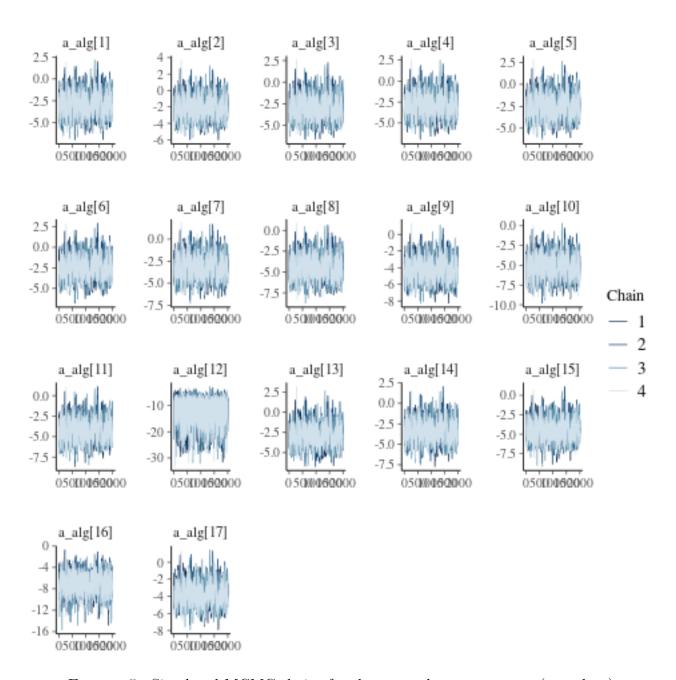


FIGURE 5. Simulated MCMC chains for the strength parameters a_i (text data)

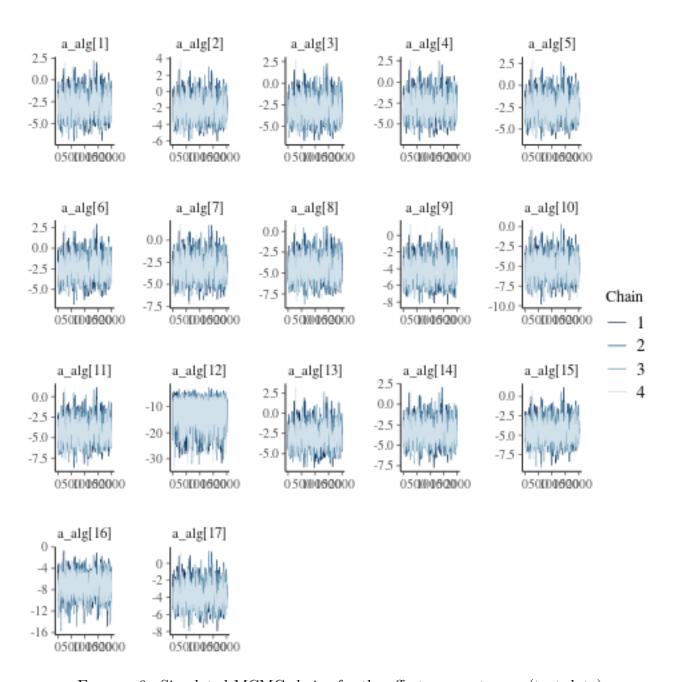


FIGURE 6. Simulated MCMC chains for the effect parameters a_i (text data)

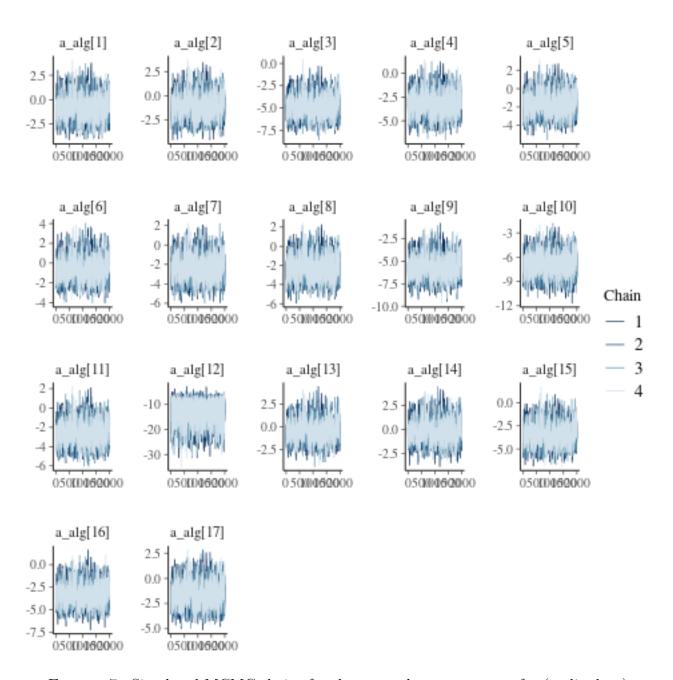


FIGURE 7. Simulated MCMC chains for the strength parameters a_i for (audio data)

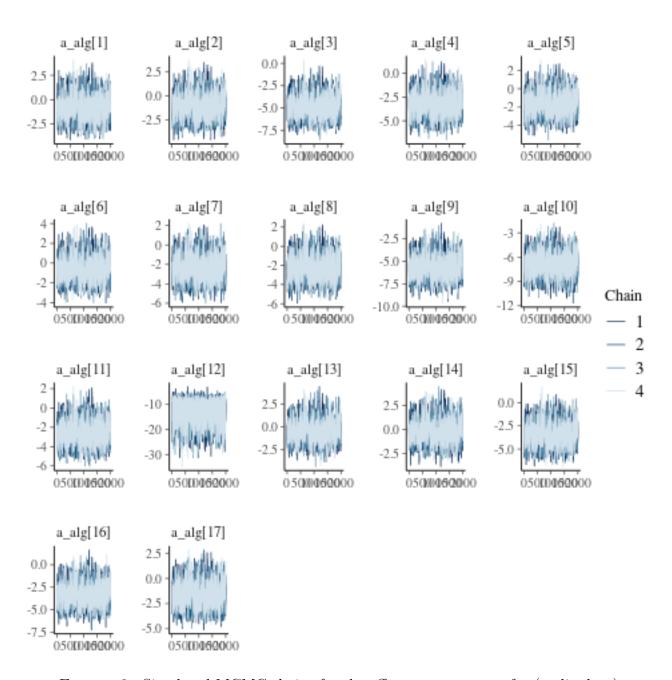


FIGURE 8. Simulated MCMC chains for the effect parameters a_i for (audio data)

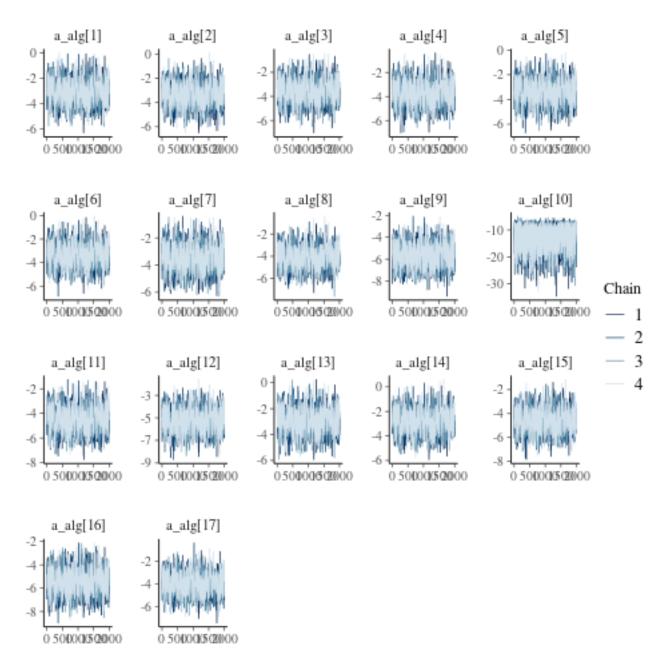


FIGURE 9. Simulated MCMC chains for the strength parameters a_i (small allocation of data)

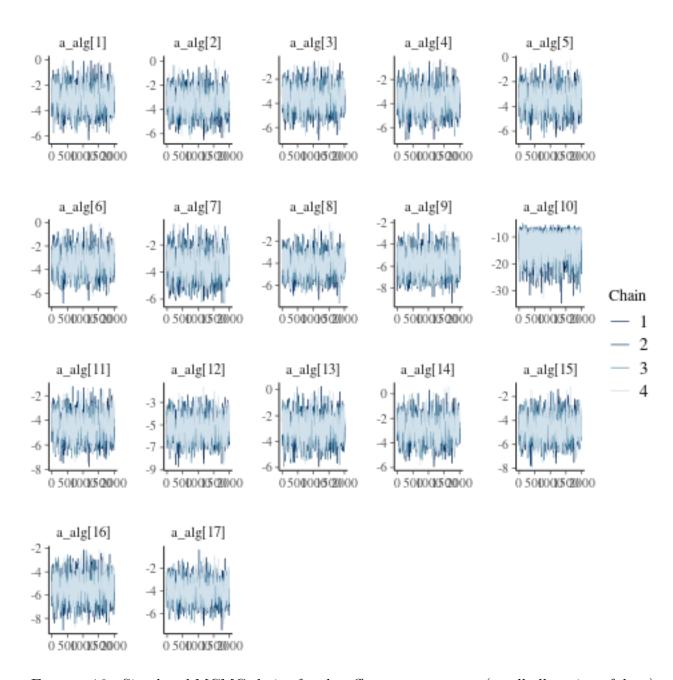


FIGURE 10. Simulated MCMC chains for the effect parameters a_i (small allocation of data)

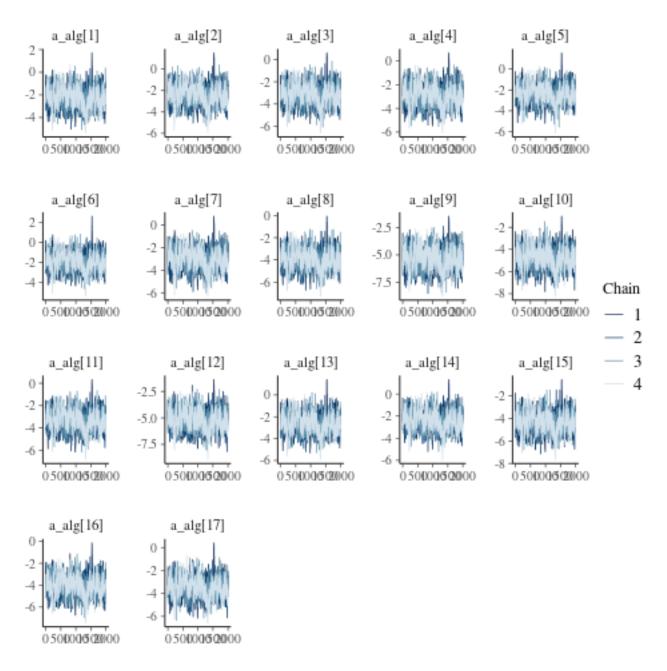


FIGURE 11. Simulated MCMC chains for the strength parameters a_i (large allocation of data)

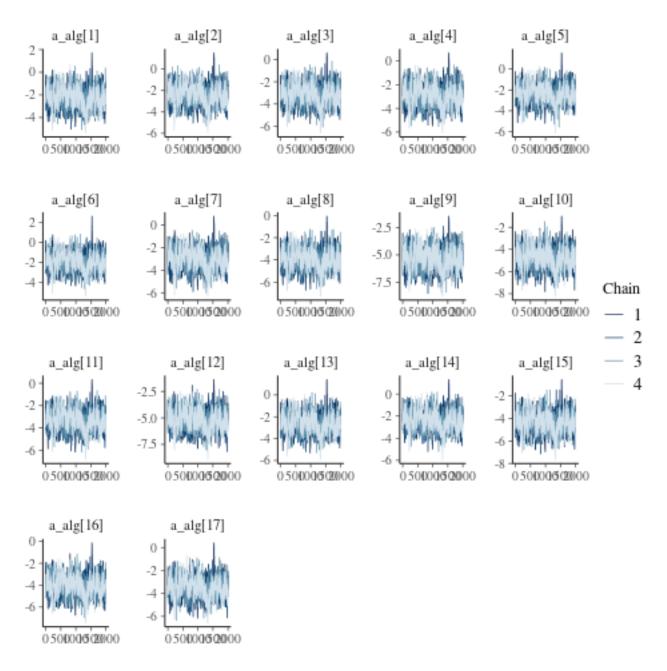


FIGURE 12. Simulated MCMC chains for the strength parameters a_i (large allocation of data)

Table 2. Diagnostics for the posterior ranks (image datat)

Variable	Rhat	ess_bulk	ess_tail
$a_{a}[1]$	1.001	5569.827	4926.560
$a_a g[2]$	1.000	5384.131	5833.872
$a_a g[3]$	1.000	5453.536	5380.084
$a_a lg[4]$	1.000	4758.031	5284.870
$a_a lg[5]$	1.000	4738.275	4432.089
$a_a g[6]$	1.000	5421.547	5321.455
$a_a [7]$	1.000	5538.582	5961.318
$a_a [8]$	1.000	5458.517	5340.038
$a_a g[9]$	1.000	5314.767	5418.474
$a_a lg[10]$	1.000	5526.175	5679.398
$a_a lg[11]$	1.000	5530.860	6031.030
$a_a lg[12]$	1.000	4838.689	5512.851
$a_a lg[13]$	1.000	5119.155	5921.655
$a_a lg[14]$	1.000	5301.094	5466.499
$a_a lg[15]$	1.000	5222.151	5542.210
$a_a lg[16]$	1.000	4757.067	5744.937
a_alg[17]	1.001	5019.732	5372.783

Table 3. Diagnostics for the posterior ranks (text datatype)

Variable	Rhat	ess_bulk	ess_tail
$a_a lg[1]$	1.001	3950.014	5012.816
$a_alg[2]$	1.001	3967.102	4947.424
$a_a lg[3]$	1.001	4034.655	5302.354
$a_{-}alg[4]$	1.001	4110.053	4768.079
$a_{-}alg[5]$	1.000	4124.764	5175.381
$a_a lg[6]$	1.001	4085.174	4895.489
$a_{-}alg[7]$	1.001	3927.361	4756.549
$a_a lg[8]$	1.001	4303.844	4836.293
$a_a lg[9]$	1.001	4407.529	5267.432
$a_a lg[10]$	1.000	4110.240	4805.624
$a_a lg[11]$	1.001	4178.923	5332.661
$a_a lg[12]$	1.001	4659.343	5859.878
$a_a lg[13]$	1.000	4142.509	5047.237
$a_a lg[14]$	1.001	3822.927	4777.553
$a_a lg[15]$	1.001	3859.451	4303.521
$a_a lg[16]$	1.000	4199.281	5302.039
a_alg[17]	1.001	4283.987	5291.580

Table 4. Diagnostics for the posterior ranks (audio datatype)

Variable	Rhat	ess_bulk	ess_tail
$a_{a}[1]$	1.002	8472.718	5296.022
$a_{a}[2]$	1.000	9903.935	5834.777
$a_a lg[3]$	1.000	9027.025	5998.672
$a_a lg[4]$	1.000	9146.187	5669.856
$a_alg[5]$	1.001	8125.924	5011.553
$a_a lg[6]$	1.001	8687.339	5981.227
$a_a [7]$	1.000	8628.567	5128.435
$a_a [8]$	1.000	8184.104	5895.332
$a_a [9]$	1.001	8197.165	6069.792
$a_a lg[10]$	1.001	7761.948	5446.681
$a_a lg[11]$	1.001	8807.185	5967.099
$a_a lg[12]$	1.001	8913.463	6830.820
$a_a lg[13]$	1.001	8633.998	5469.254
$a_a lg[14]$	1.000	8748.116	5787.659
$a_a lg[15]$	1.001	8118.402	5409.835
$a_a lg[16]$	1.001	8752.863	6160.065
a_alg[17]	1.000	9061.207	5681.185

Table 5. Diagnostics for the posterior ranks (small allocation)

Variable	Rhat	ess_bulk	ess_tail
$a_alg[1]$	1.000	4735.897	4112.013
$a_{a}[2]$	1.000	4793.097	4387.702
$a_a lg[3]$	1.001	4694.239	4751.959
$a_{-}alg[4]$	1.001	4675.820	4336.875
$a_alg[5]$	1.001	4835.159	4383.011
$a_alg[6]$	1.000	4435.438	4177.937
$a_alg[7]$	1.000	4555.595	4329.844
$a_{a}[8]$	1.000	4474.490	4002.949
$a_a lg[9]$	1.000	4748.329	4499.903
$a_a lg[10]$	1.000	4955.164	4542.496
$a_a lg[11]$	1.001	5020.975	4230.675
$a_a lg[12]$	1.001	5478.078	4698.688
$a_a lg[13]$	1.001	4471.626	3814.573
$a_{a} = alg[14]$	1.001	4459.145	4289.903
$a_a lg[15]$	1.000	4488.908	4743.348
$a_a lg[16]$	1.001	4503.813	4560.218
a_alg[17]	1.001	4666.474	3592.517

Table 6. Diagnostics for the posterior ranks (large allocation)

Variable	Rhat	ess_bulk	ess_tail
$a_{alg}[1]$	1.000	9164.379	5867.408
$a_{a}[2]$	1.001	9215.643	5816.067
$a_{-}alg[3]$	1.000	9621.344	5915.346
$a_{a}[4]$	1.000	9415.307	5461.207
$a_alg[5]$	1.000	9607.589	5958.987
$a_{-}alg[6]$	1.001	8800.023	5865.614
$a_{-}alg[7]$	1.000	9822.973	6074.739
$a_{a}[8]$	1.001	8850.728	5606.480
$a_{a}[9]$	1.000	8739.513	5854.369
$a_{a} = alg[10]$	1.003	9746.317	5910.605
$a_{a}[11]$	1.001	8805.959	5627.071
$a_{a} = alg[12]$	1.001	9520.813	6643.876
$a_{a}[13]$	1.001	8688.475	6033.553
$a_{a} = alg[14]$	1.001	8779.775	6017.763
$a_{a} = alg[15]$	1.000	9485.947	5613.972
$a_{a} = alg[16]$	1.001	9502.301	6621.634
a_alg[17]	1.001	9140.726	5586.761

Table 7. Diagnostics for the posterior effects (aggregated)

variable	rhat	ess_bulk	ess_tail
$a_{a}[1]$	1.006	440.182	541.622
$a_{a}[2]$	1.006	439.851	507.177
$a_{a}[3]$	1.006	437.147	522.022
$a_{-}alg[4]$	1.006	442.510	523.726
$a_alg[5]$	1.006	440.479	535.330
$a_alg[6]$	1.005	439.403	531.731
$a_alg[7]$	1.006	441.416	530.607
$a_{a}[8]$	1.006	443.915	552.155
$a_a [9]$	1.006	455.324	549.392
$a_a lg[10]$	1.005	454.539	548.766
$a_alg[11]$	1.005	436.243	532.145
$a_a lg[12]$	1.005	446.804	552.072
a_{a} alg [13]	1.006	443.509	522.109
$a_{a} = alg[14]$	1.006	436.285	518.366
$a_alg[15]$	1.005	447.754	514.499
$a_a lg[16]$	1.005	446.589	518.547
$a_alg[17]$	1.006	438.401	525.700

Table 8. Diagnostics for the posterior ranks (image datatype)

Variable	Rhat	ess_bulk	ess_tail
$a_a lg[1]$	1.002	694.306	1165.395
$a_{a}[2]$	1.002	848.293	1373.391
$a_a [3]$	1.002	711.569	1215.024
$a_{-}alg[4]$	1.002	705.012	1163.763
$a_{-}alg[5]$	1.002	714.125	1100.587
$a_alg[6]$	1.001	696.947	1189.039
$a_alg[7]$	1.002	696.537	1208.653
$a_alg[8]$	1.002	746.903	1307.071
$a_{a} = alg[9]$	1.001	1298.246	2323.478
$a_a lg[10]$	1.001	753.645	1406.142
$a_alg[11]$	1.002	703.356	1172.664
$a_alg[12]$	1.002	684.666	1234.366
$a_alg[13]$	1.002	713.963	1315.901
$a_a lg[14]$	1.002	716.643	1163.742
$a_a lg[15]$	1.002	766.597	1409.595
$a_a lg[16]$	1.002	728.488	1288.008
a_alg[17]	1.001	723.604	1322.541

Table 9. Diagnostics for the posterior effects (text datatype)

variable	rhat	ess_bulk	ess_tail
a_alg[1]	1.011	600.486	901.568
$a_{a}[2]$	1.011	619.436	935.044
$a_a lg[3]$	1.012	613.667	910.521
$a_{-}alg[4]$	1.011	628.431	891.711
$a_alg[5]$	1.012	605.999	902.718
$a_alg[6]$	1.012	612.309	950.841
$a_a lg[7]$	1.012	612.771	850.828
$a_{a}[8]$	1.011	636.753	905.439
$a_a lg[9]$	1.011	649.210	961.074
$a_a lg[10]$	1.010	693.142	1043.443
$a_a lg[11]$	1.010	629.357	961.024
$a_a lg[12]$	1.000	4197.822	4444.723
$a_a lg[13]$	1.011	599.856	811.524
$a_a lg[14]$	1.011	626.391	906.396
$a_a lg[15]$	1.009	646.797	888.692
a_alg[16]	1.005	1014.138	1357.790
$a_{a} = alg[17]$	1.012	620.882	895.637

Table 10. Diagnostics for the posterior effects (audio datatype)

variable	rhat	ess_bulk	ess_tail
$a_{a} = alg[1]$	1.003	870.761	1374.035
$a_{-}alg[2]$	1.004	849.701	1351.964
$a_alg[3]$	1.003	770.054	1325.369
$a_a lg[4]$	1.003	833.502	1442.465
$a_alg[5]$	1.003	842.617	1358.068
$a_{-}alg[6]$	1.003	770.569	1398.594
$a_{-}alg[7]$	1.002	862.501	1389.156
$a_{a}[8]$	1.003	845.793	1565.068
$a_a [9]$	1.003	841.420	1339.853
$a_a lg[10]$	1.003	1028.792	1773.255
$a_alg[11]$	1.003	839.701	1495.961
$a_{a}[12]$	1.001	4585.525	4785.559
$a_alg[13]$	1.003	764.878	1392.853
$a_a lg[14]$	1.003	799.794	1362.592
$a_a lg[15]$	1.003	832.628	1402.115
$a_a lg[16]$	1.003	830.624	1467.762
a_alg[17]	1.003	879.117	1481.837

Table 11. Diagnostics for the posterior effects (small allocation)

variable	rhat	ess_bulk	ess_tail
a_alg[1]	1.003	614.066	1004.753
$a_{a}[2]$	1.004	618.395	1045.639
$a_a [3]$	1.003	627.256	1019.701
$a_a lg[4]$	1.004	616.507	1038.961
$a_alg[5]$	1.004	617.869	955.493
$a_alg[6]$	1.004	615.979	1151.694
$a_alg[7]$	1.003	630.325	1006.059
$a_a lg[8]$	1.004	632.262	1038.843
$a_a g[9]$	1.003	717.147	1206.127
$a_a lg[10]$	1.001	3588.698	3392.161
$a_a lg[11]$	1.003	623.126	1198.584
$a_alg[12]$	1.003	684.711	1284.887
$a_a lg[13]$	1.003	623.283	1096.985
$a_a lg[14]$	1.003	611.306	1008.557
$a_alg[15]$	1.003	648.600	1073.667
$a_alg[16]$	1.002	694.782	1184.188
$a_alg[17]$	1.004	629.064	1135.959

Table 12. Diagnostics for the posterior effects (large allocation)

variable	rhat	ess_bulk	ess_tail
		00010 4111	
$a_alg[1]$	1.023	258.317	463.035
$a_alg[2]$	1.021	268.310	538.120
$a_alg[3]$	1.021	253.876	458.608
$a_alg[4]$	1.023	248.612	503.228
$a_alg[5]$	1.022	258.077	465.426
$a_{-}alg[6]$	1.023	255.613	533.282
$a_{a}[7]$	1.022	252.546	457.869
$a_{a}[8]$	1.021	263.317	584.556
$a_a [9]$	1.019	301.839	709.768
$a_a lg[10]$	1.018	289.796	539.473
$a_a lg[11]$	1.021	264.149	504.409
$a_{a}[12]$	1.021	281.814	665.613
$a_{a}[13]$	1.022	257.686	480.975
$a_{a} = alg[14]$	1.022	252.151	576.368
$a_{a} = alg[15]$	1.020	262.272	499.345
a_alg[16]	1.020	266.885	570.303
a_alg[17]	1.023	259.107	516.268

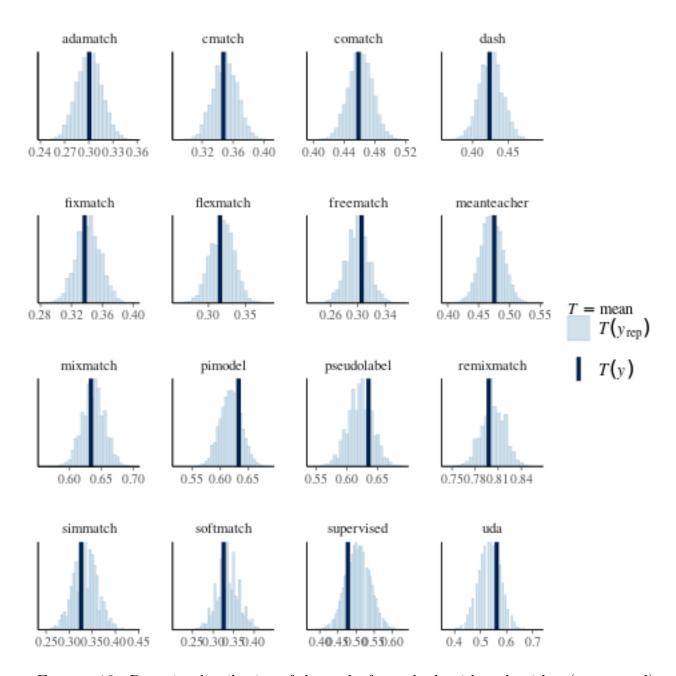


FIGURE 13. Posterior distribution of the ranks for each algorithm algorithm (aggregated)

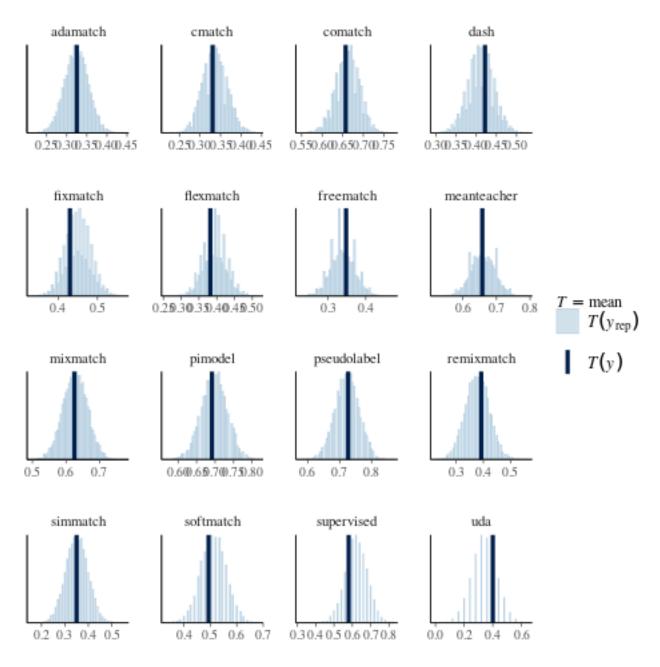


FIGURE 14. Posterior distribution of the ranks for each algorithm algorithm (image datatype)

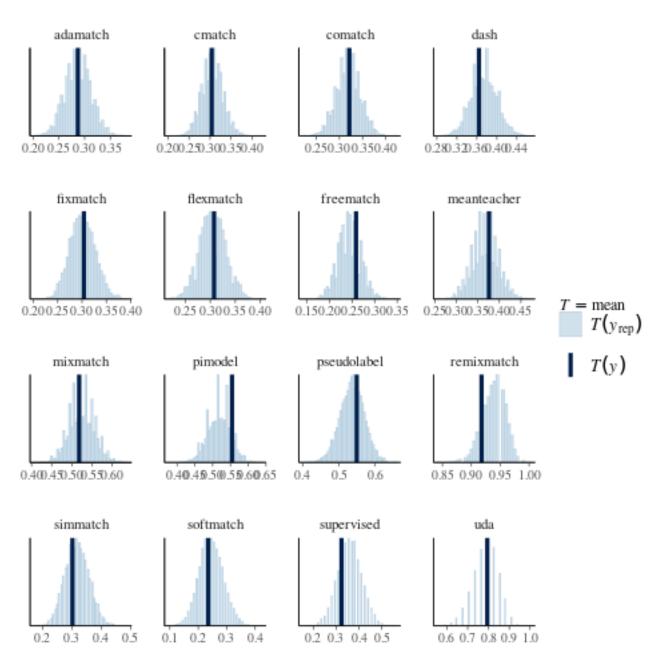


FIGURE 15. Posterior distribution of the ranks for each algorithm algorithm (text datatype)

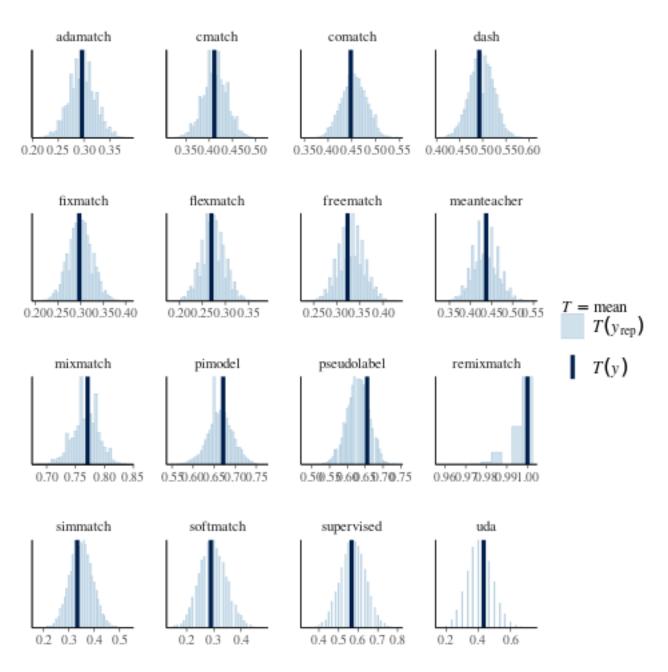


FIGURE 16. Posterior distribution of the ranks for each algorithm algorithm (audio datatype)

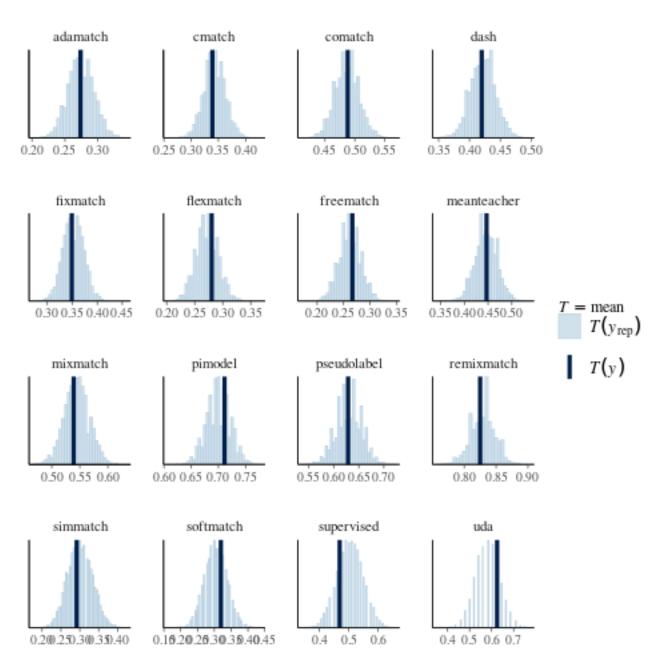


FIGURE 17. Posterior distribution of the ranks for each algorithm algorithm (small allocation)

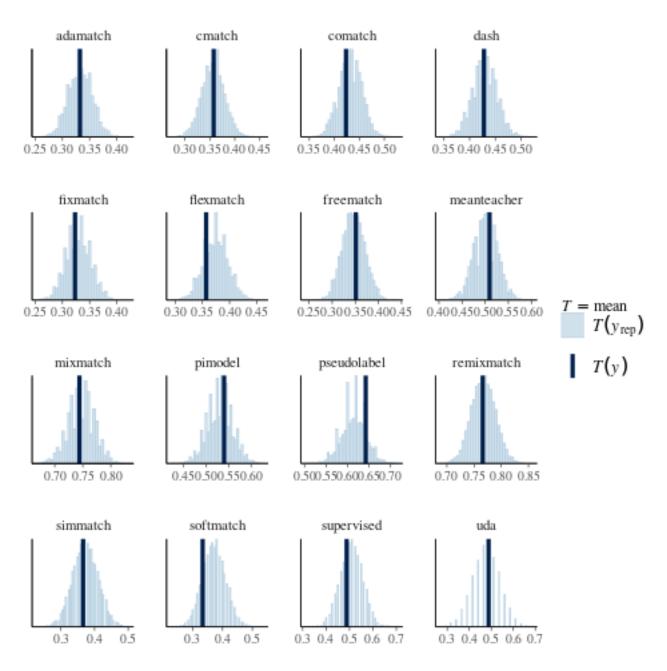


FIGURE 18. Posterior distribution of the ranks for each algorithm algorithm (large allocation)

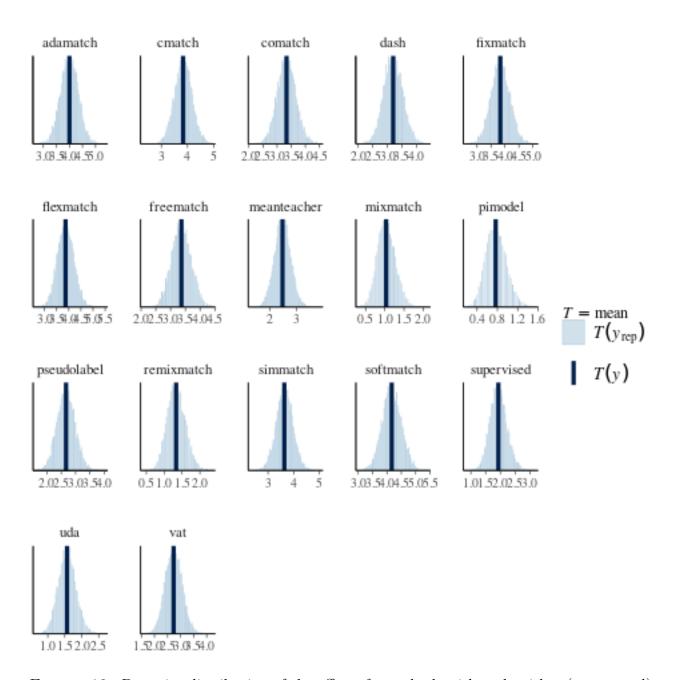


FIGURE 19. Posterior distribution of the effects for each algorithm algorithm (aggregated)

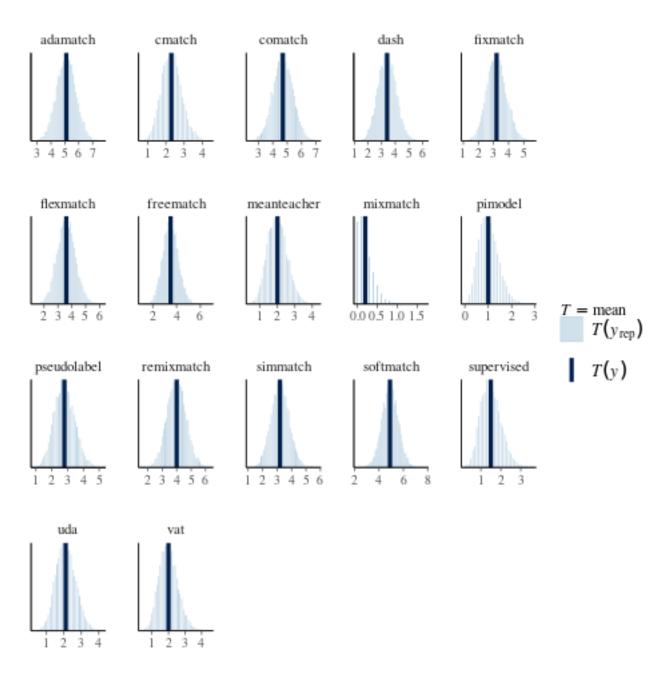


FIGURE 20. Posterior distribution of the effects for each algorithm algorithm (image datatype)

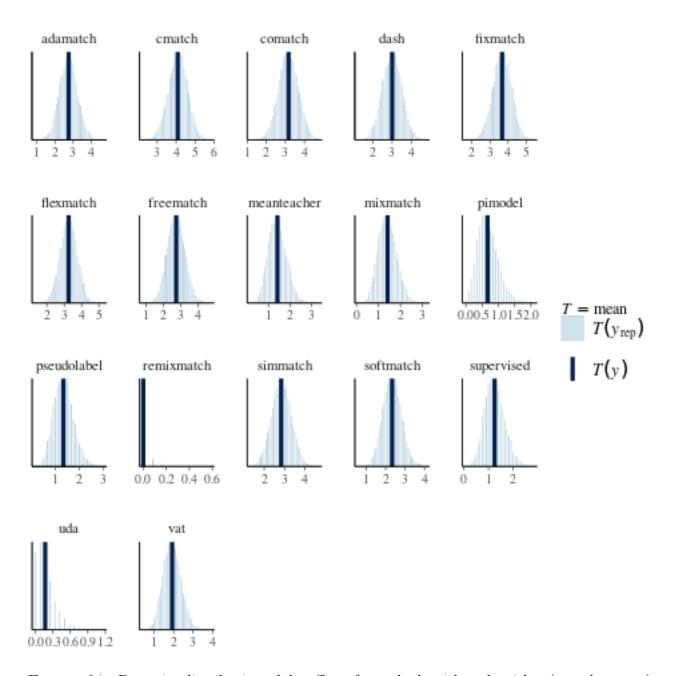


FIGURE 21. Posterior distribution of the effects for each algorithm algorithm (text datatype)

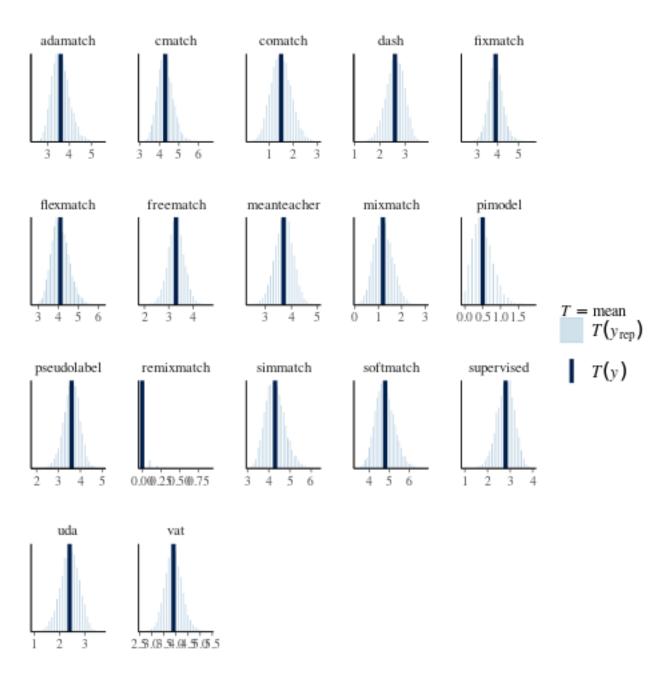


FIGURE 22. Posterior distribution of the effects for each algorithm algorithm (audio datatype)

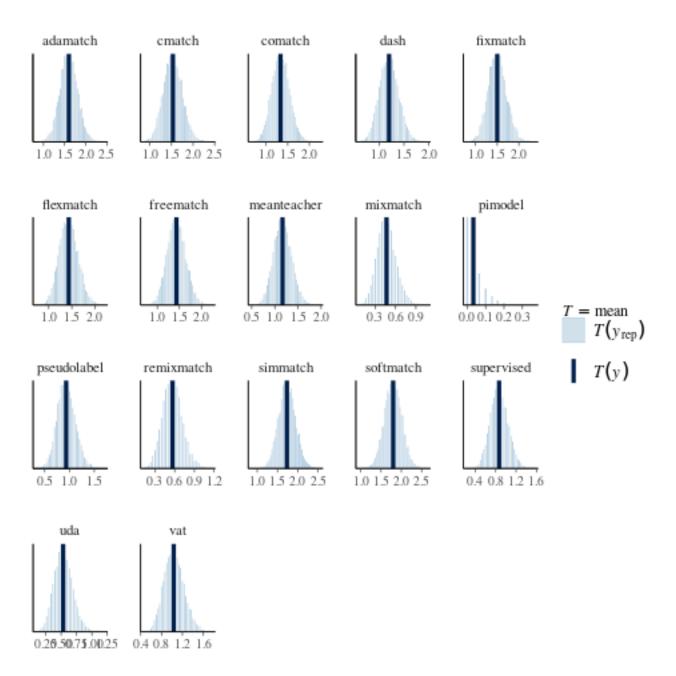


FIGURE 23. Posterior distribution of the effects for each algorithm algorithm (small allocation)

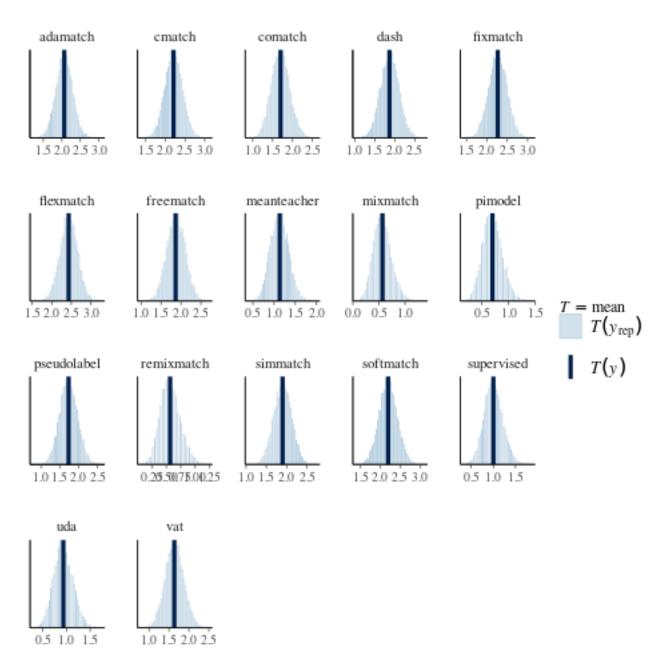


FIGURE 24. Posterior distribution of the effects for each algorithm algorithm (large allocation)

References

- [1] David Issa Mattos, Jan Bosch, and Helena Holmström Olsson. Statistical models for the analysis of optimization algorithms with benchmark functions. $IEEE\ Transactions\ on\ Evolutionary\ Computation,\ 25(6):1163-1177,\ 2021.$
- [2] Richard McElreath. Statistical rethinking: A Bayesian course with examples in R and Stan. CRC press, 2020.