MACM 316 HW #8: Why Planes Fly

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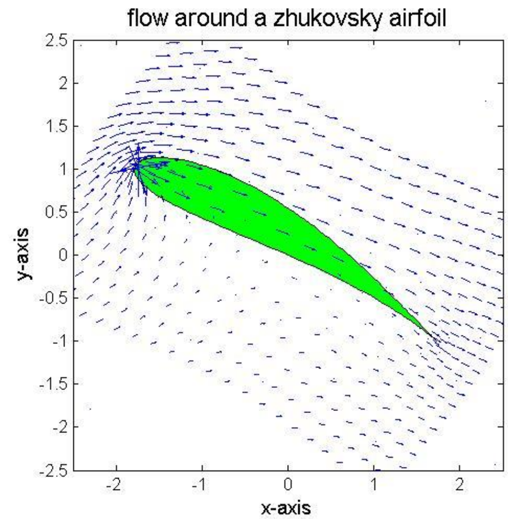
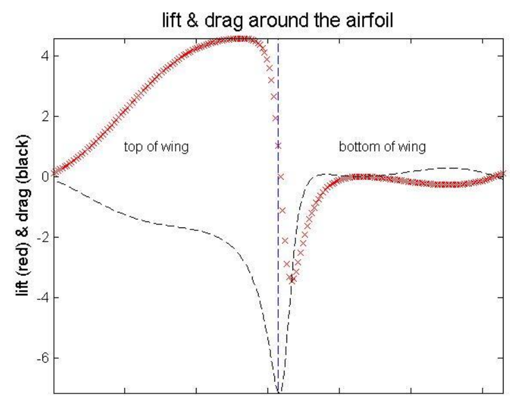
Method:

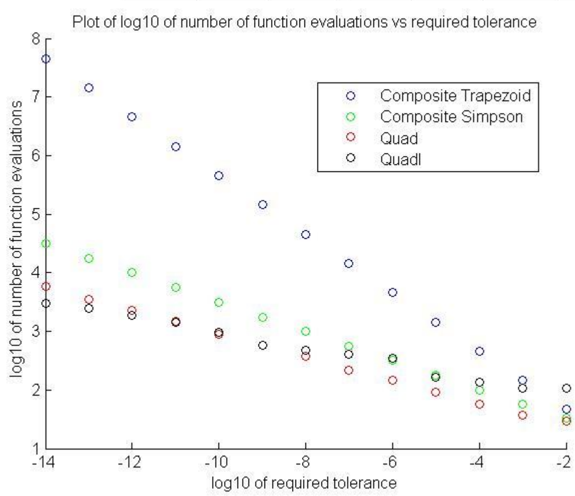
To compare the efficiencies between the composite trapezoidal (CT), composite simpsons (CS), quad, and quadl, I chose the following parameters for my airplane wing: $\mu = 0.1 + 0.1i$, and $\alpha = \pi/6$. This gives the zhukovsky airfoil shown on the right, and an integrand of $lift\left(\theta, 0.1 + 0.1i, \frac{\pi}{6}\right)$, where θ goes from 0 to 2π. Since the quad and the quadl routines are adaptive methods, points are automatically placed in the (a, b) interval according to the tolerance that is given. Thus, executing those two commands is straightforward. CT and CS are not as straightforward, since the points are uniformly spaced. Thus, for the uniformly spaced points, the decision comes down to how large of a subinterval h to use. I used theorems 4.4 and 4.5 to estimate the value of h. Since the scaling of the error depends on the exponent that h is being raised to (in each of the composite trapezoidal and composite simpsons rules), figuring out a value of h to use involves setting the respective error term to tolerance given, and solving for h. I implemented each of the CT and CS methods myself. To compare the numerical integration methods, I used tolerances values from the set $\{10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6}, 10^{-7}, 10^{-8}, 10^{-9}, 10^{-10}, 10^{-11}, 10^{-12}, 10^{-13}, 10^{-14}\}$, and recorded the number of function evaluations that each method performed for each tolerance value. The corresponding plot is shown on the bottom right of this page, where the common log of the number of function evaluations is on the vertical and the common log of the required tolerance is on the horizontal axis.





Observations and Analysis:

Without using any knowledge of fluid dynamics, of which I have none, the plot that shows the airflow past the wing seems to make sense intuitively; one would expect the direction of air flow to change direction with the presence of the wing. Based off the plot on the bottom right, the composite trapezoid method is the least capable of handling small tolerances, since the number of evaluations reaches the millions for the smallest tolerance tested. For the larger tolerance values, the composite simpson method seems to require the fewest, but is eventually outperformed by quad and quadl downwards of $tol = 10^{-11}$. Lastly, it makes sense that overall, for the smallest tolerance values, the quad and quadl values outperform the composite values, since the adaptive method specializes in reducing the number of function evaluations by adding points only in intervals that require it.



Conclusion:

While a reasonable first approach to reducing the error of a numerical method would be to add more points, one must realize that some intervals have a smaller error than others. This is how the quadl and the quad methods outperform the composite methods. Another conclusion from this investigation is that outputting the number of function evaluations as well as other diagnostics from a numerical method is more helpful than simply the answer, which is why MATLAB's higher level of abstraction with its new "integral" method is considered "backwards" by some.