

# Natural Language Processing with Deep Learning

## CS224N/Ling284



Christopher Manning  
Lecture 16: Coreference Resolution

# 1. What is Coreference Resolution?

- Identify all **mentions** that refer to the same real world entity

Barack Obama nominated Hillary Rodham Clinton as his secretary of state on Monday. He chose her because she had foreign affairs experience as a former First Lady.

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A couple of years later, Vanaja met Akhila at the local park. Akhila's son Prajwal was just two months younger than her son Akash, and they went to the same school. For the pre-school play, Prajwal was chosen for the lead role of the naughty child Lord Krishna. Akash was to be a tree. She resigned herself to make Akash the best tree that anybody had ever seen. She bought him a brown T-shirt and brown trousers to represent the tree trunk. Then she made a large cardboard cutout of a tree's foliage, with a circular opening in the middle for Akash's face. She attached red balls to it to represent fruits. It truly was the nicest tree.

From The Star by Shruthi Rao, with some shortening.

# Applications

- Full text understanding
  - information extraction, question answering, summarization, ...
  - “He was born in 1961”        (Who?)

# Applications

- Full text understanding
- Machine translation
  - languages have different features for gender, number, dropped pronouns, etc.

The image displays two separate instances of a machine translation application interface. Both instances show a source text input field, a language selection bar at the top, and a translated output field with various interaction icons.

**Top Instance:**

- Source Text:** A Alicia le gusta Juan porque es inteligente
- Target Text:** Alicia likes Juan because he's smart
- Language Bar (Left):** Spanish, English, French, Detect language
- Language Bar (Right):** English, Spanish, Arabic
- Buttons:** Translate, Suggest an edit
- Input Tools:** Microphone, Keyboard
- Character Counter:** 44/5000

**Bottom Instance:**

- Source Text:** A Juan le gusta Alicia porque es inteligente
- Target Text:** Juan likes Alicia because he's smart
- Language Bar (Left):** Spanish, English, French, Detect language
- Language Bar (Right):** English, Spanish, Arabic
- Buttons:** Translate, Suggest an edit
- Input Tools:** Microphone, Keyboard
- Character Counter:** 44/5000

# Applications

- Full text understanding
- Machine translation
  - languages have different features for gender, number, dropped pronouns, etc.

o bir aşçı	she is a cook
o bir mühendis	he is an engineer
o bir doktor	he is a doctor
o bir hemşire	she is a nurse
o bir temizlikçi	he is a cleaner
o bir polis	He-she is a police
o bir asker	he is a soldier
o bir öğretmen	She's a teacher
o bir sekreter	he is a secretary

# Applications

- Full text understanding
- Machine translation
- Dialogue Systems

“Book tickets to see **James Bond**”

“**Spectre** is playing near you at 2:00 and 3:00 today. **How many tickets** would you like?”

“**Two** tickets for the showing at **three**”

# Coreference Resolution in Two Steps

## 1. Detect the mentions (easy)

“[I] voted for [Nader] because [he] was most aligned with [[my] values],” [she] said

- mentions can be nested!

## 2. Cluster the mentions (hard)

“[I] voted for [Nader] because [he] was most aligned with [[my] values],” [she] said

### 3. Mention Detection

- Mention: span of text referring to some entity
- Three kinds of mentions:

#### 1. Pronouns

- I, your, it, she, him, etc.

#### 2. Named entities

- People, places, etc.

#### 3. Noun phrases

- “a dog,” “the big fluffy cat stuck in the tree”

# Mention Detection

- Span of text referring to some entity
- For detection: use other NLP systems

## 1. Pronouns

- Use a part-of-speech tagger

## 2. Named entities

- Use a NER system (like hw3)

## 3. Noun phrases

- Use a parser (especially a constituency parser – next week!)

# Mention Detection: Not so Simple

- Marking all pronouns, named entities, and NPs as mentions over-generates mentions
- Are these mentions?
  - It is sunny
  - Every student
  - No student
  - The best donut in the world
  - 100 miles

# How to deal with these bad mentions?

- Could train a classifier to filter out spurious mentions
- Much more common: keep all mentions as “candidate mentions”
  - After your coreference system is done running discard all singleton mentions (i.e., ones that have not been marked as coreference with anything else)

# Can we avoid a pipelined system?

- We could instead train a classifier specifically for mention detection instead of using a POS tagger, NER system, and parser.
- Or even jointly do mention-detection and coreference resolution end-to-end instead of in two steps
  - Will cover later in this lecture!

## 4. On to Coreference! First, some linguistics

- **Coreference** is when two mentions refer to the same entity in the world
  - *Barack Obama traveled to ... Obama*
- A related linguistic concept is **anaphora**: when a term (anaphor) refers to another term (antecedent)
  - the interpretation of the anaphor is in some way determined by the interpretation of the antecedent
  - *Barack Obama said **he** would sign the bill.*  
antecedent                    anaphor

# Anaphora vs Coreference

- Coreference with named entities

text

Barack Obama

Obama

world



- Anaphora

text

Barack Obama

he

world



# Not all anaphoric relations are coreferential

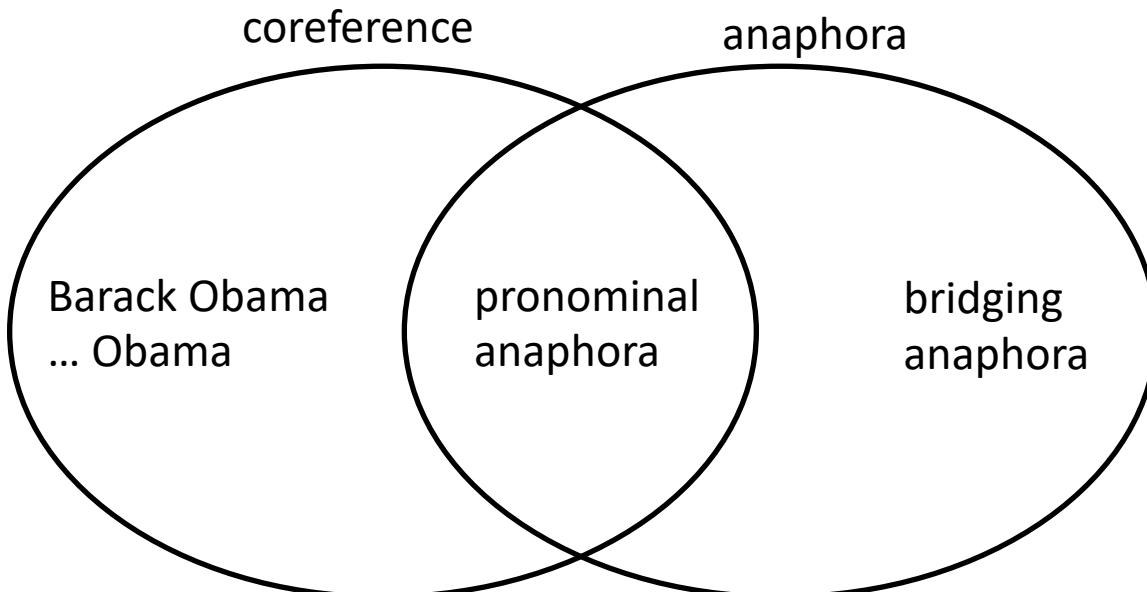
- Not all noun phrases have reference
- *Every dancer* twisted *her knee*.
- *No dancer* twisted *her knee*.
- There are three NPs in each of these sentences; because the first one is non-referential, the other two aren't either.

# Anaphora vs. Coreference

- Not all anaphoric relations are coreferential

*We went to see **a concert** last night. **The tickets** were really expensive.*

- This is referred to as **bridging anaphora**.



# Anaphora vs. Cataphora

- Usually the antecedent comes before the anaphor (e.g., a pronoun), but not always

# Cataphora

*“From the corner of the divan of Persian saddle-bags on which **he** was lying, smoking, as was his custom, innumerable cigarettes, Lord Henry Wotton could just catch the gleam of the honey-sweet and honey-coloured blossoms of a laburnum...”*

(Oscar Wilde – The Picture of



# Four Kinds of Coreference Models

- Rule-based (pronominal anaphora resolution)
- Mention Pair
- Mention Ranking
- Clustering

## 5. Traditional pronominal anaphora resolution: Hobbs' naive algorithm



1. Begin at the NP immediately dominating the pronoun
2. Go up tree to first NP or S. Call this X, and the path p.
3. Traverse all branches below X to the left of p, left-to-right, breadth-first. Propose as antecedent any NP that has a NP or S between it and X
4. If X is the highest S in the sentence, traverse the parse trees of the previous sentences in the order of recency. Traverse each tree left-to-right, breadth first. When an NP is encountered, propose as antecedent. If X not the highest node, go to step 5.

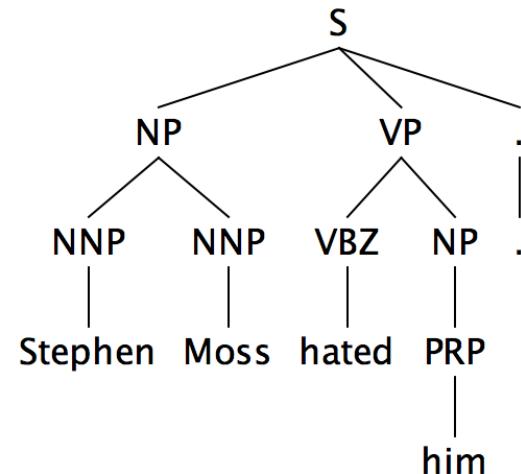
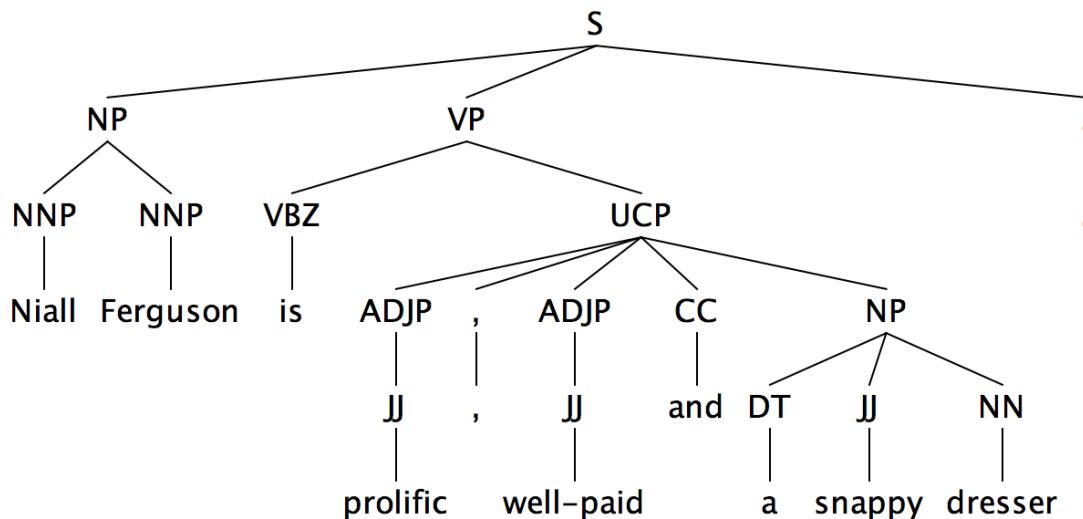
## Hobbs' naive algorithm (1976)

5. From node X, go up the tree to the first NP or S. Call it X, and the path p.
6. If X is an NP and the path p to X came from a non-head phrase of X (a specifier or adjunct, such as a possessive, PP, apposition, or relative clause), propose X as antecedent

(The original said “did not pass through the N’ that X immediately dominates”, but the Penn Treebank grammar lacks N’ nodes....)
7. Traverse all branches below X to the left of the path, in a left-to-right, breadth first manner. Propose any NP encountered as the antecedent
8. If X is an S node, traverse all branches of X to the right of the path but do not go below any NP or S encountered. Propose any NP as the antecedent.
9. Go to step 4

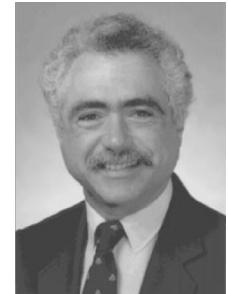
Until deep learning still often used as a feature in ML systems!

# Hobbs Algorithm Example



# Knowledge-based Pronominal Coreference

- She poured water from **the pitcher** into **the cup** until **it** was full
- She poured water from **the pitcher** into **the cup** until **it** was empty”
- **The city council** refused **the women** a permit because **they** feared violence.
- **The city council** refused **the women** a permit because **they** advocated violence.
  - Winograd (1972)
- These are called **Winograd Schema**
  - Recently proposed as an alternative to the Turing test
    - See: Hector J. Levesque “On our best behaviour” IJCAI 2013  
<http://www.cs.toronto.edu/~hector/Papers/ijcai-13-paper.pdf>
    - <http://commonsensereasoning.org/winograd.html>
  - If you’ve fully solved coreference, arguably you’ve solved AI



# Hobbs' algorithm: commentary

*"... the naïve approach is quite good. Computationally speaking, it will be a long time before a semantically based algorithm is sophisticated enough to perform as well, and these results set a very high standard for any other approach to aim for.*

*"Yet there is every reason to pursue a semantically based approach. The naïve algorithm does not work. Any one can think of examples where it fails. In these cases it not only fails; it gives no indication that it has failed and offers no help in finding the real antecedent."*

— Hobbs (1978), *Lingua*, p. 345

## 6. Coreference Models: Mention Pair

*“I voted for Nader because he was most aligned with my values,” she said.*

I

Nader

he

my

she

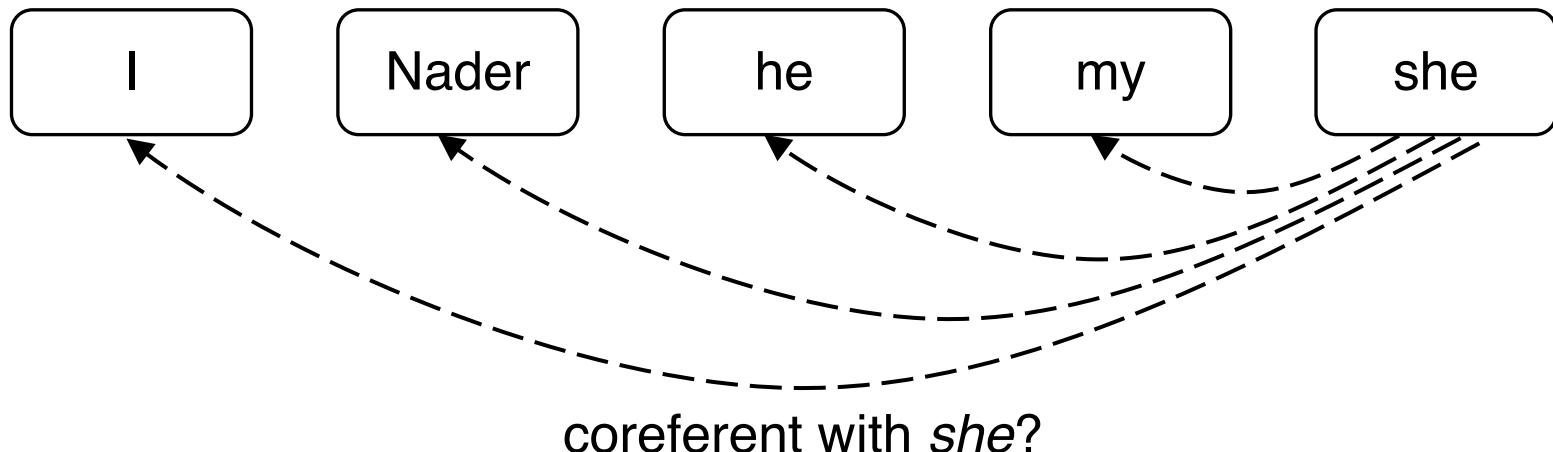
Coreference Cluster 1

Coreference Cluster 2

# Coreference Models: Mention Pair

- Train a binary classifier that assigns every pair of mentions a probability of being coreferent:  $p(m_i, m_j)$ 
  - e.g., for “she” look at all **candidate antecedents** (previously occurring mentions) and decide which are coreferent with it

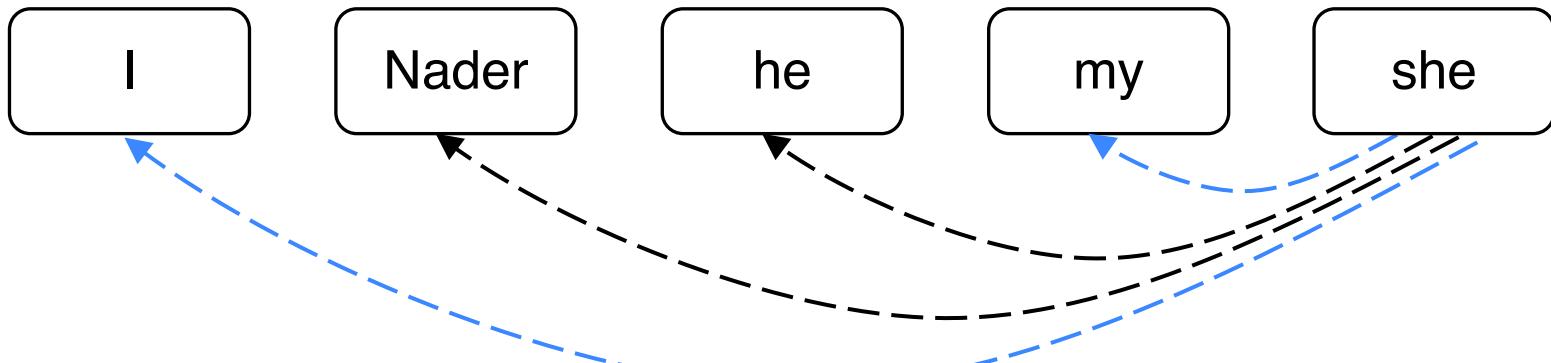
“I voted for **Nader** because **he** was most aligned with **my** values,” **she** said.



# Coreference Models: Mention Pair

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“I voted for **Nader** because **he** was most aligned with **my** values,” **she** said.

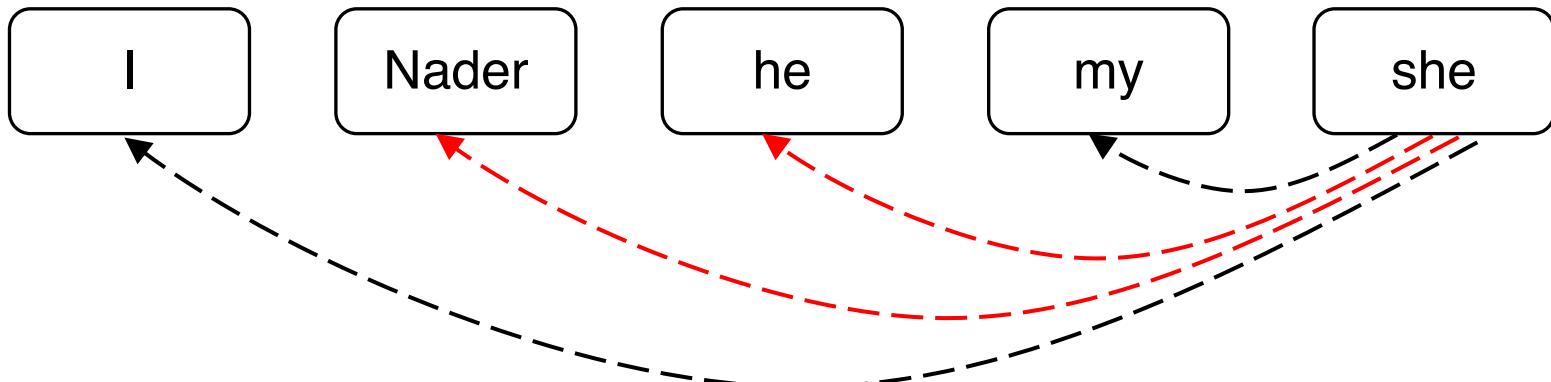


Positive examples: want  $p(m_i, m_j)$  to be near 1

# Coreference Models: Mention Pair

- Train a binary classifier that assigns every pair of mentions a probability of being coreferent:  $p(m_i, m_j)$ 
  - e.g., for “she” look at all **candidate antecedents** (previously occurring mentions) and decide which are coreferent with it

“I voted for **Nader** because **he** was most aligned with **my** values,” **she** said.



Negative examples: want  $p(m_i, m_j)$  to be near 0

# Mention Pair Training

- $N$  mentions in a document
- $y_{ij} = 1$  if mentions  $m_i$  and  $m_j$  are coreferent, -1 if otherwise
- Just train with regular cross-entropy loss (looks a bit different because it is binary classification)

$$J = - \sum_{i=2}^N \sum_{j=1}^i y_{ij} \log p(m_j, m_i)$$

Iterate through mentions      Iterate through candidate antecedents (previously occurring mentions)      Coreferent mentions pairs should get high probability, others should get low probability

The diagram illustrates the components of the cross-entropy loss function. The formula is  $J = - \sum_{i=2}^N \sum_{j=1}^i y_{ij} \log p(m_j, m_i)$ . Three arrows point to specific parts of the formula: one to the index  $i$  (labeled "Iterate through mentions"), one to the index  $j$  (labeled "Iterate through candidate antecedents (previously occurring mentions)"), and one to the probability term  $p(m_j, m_i)$  (labeled "Coreferent mentions pairs should get high probability, others should get low probability").

# Mention Pair Test Time

- Coreference resolution is a clustering task, but we are only scoring pairs of mentions... what to do?

I

Nader

he

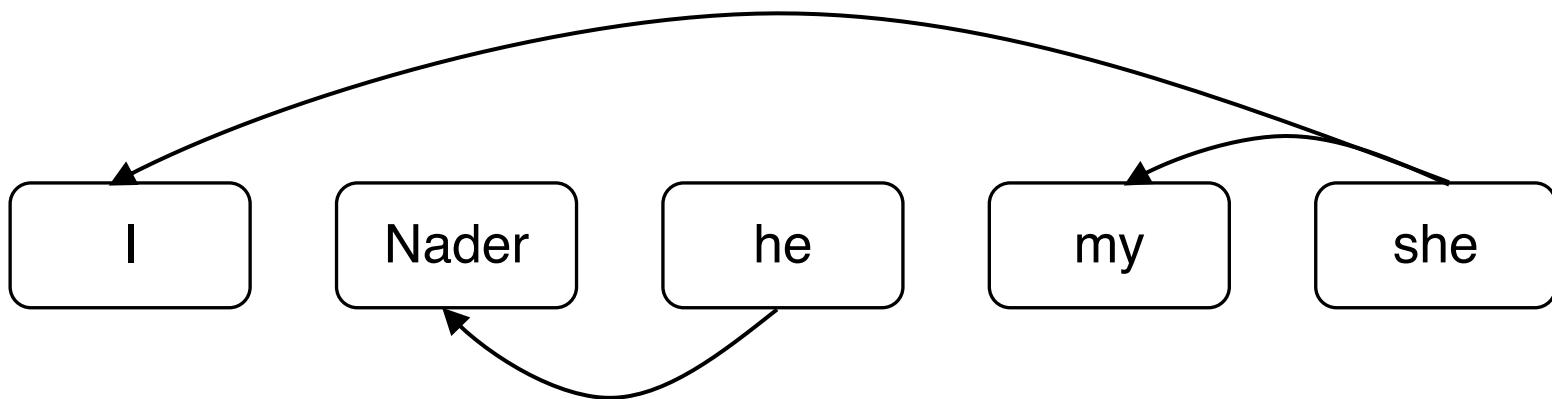
my

she

# Mention Pair Test Time

- Coreference resolution is a clustering task, but we are only scoring pairs of mentions... what to do?
- Pick some threshold (e.g., 0.5) and add **coreference links** between mention pairs where  $p(m_i, m_j)$  is above the threshold

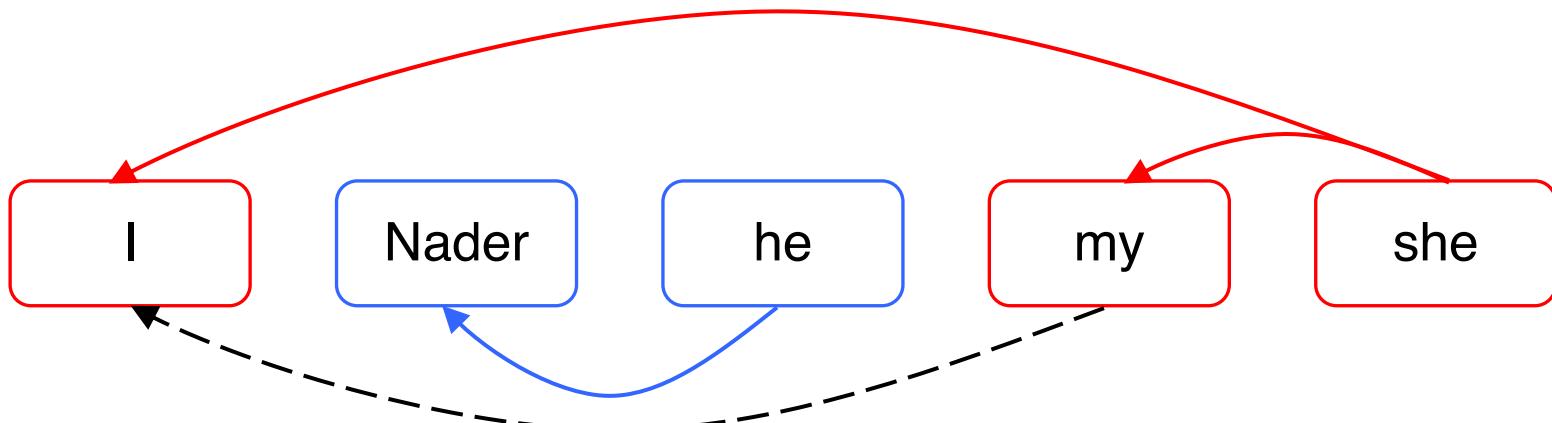
*"I voted for Nader because he was most aligned with my values," she said.*



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- Pick some threshold (e.g., 0.5) and add **coreference links** between mention pairs where  $p(m_i, m_j)$  is above the threshold
- Take the transitive closure to get the clustering

*"I voted for Nader because he was most aligned with my values," she said.*

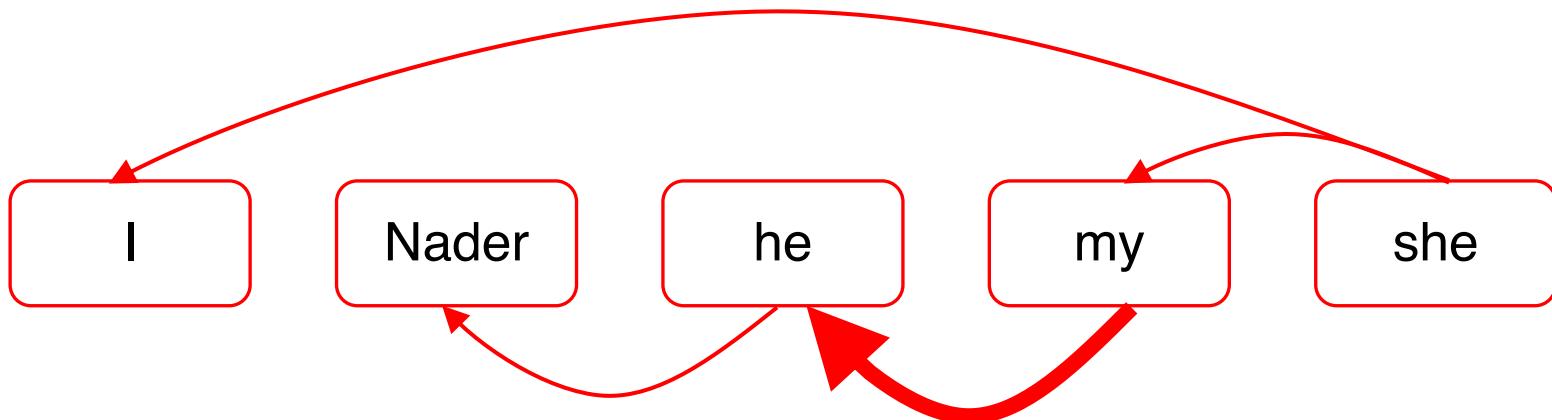


Even though the model did not predict this coreference link,  
/ and my are coreferent due to transitivity

# Mention Pair Test Time

- Coreference resolution is a clustering task, but we are only scoring pairs of mentions... what to do?
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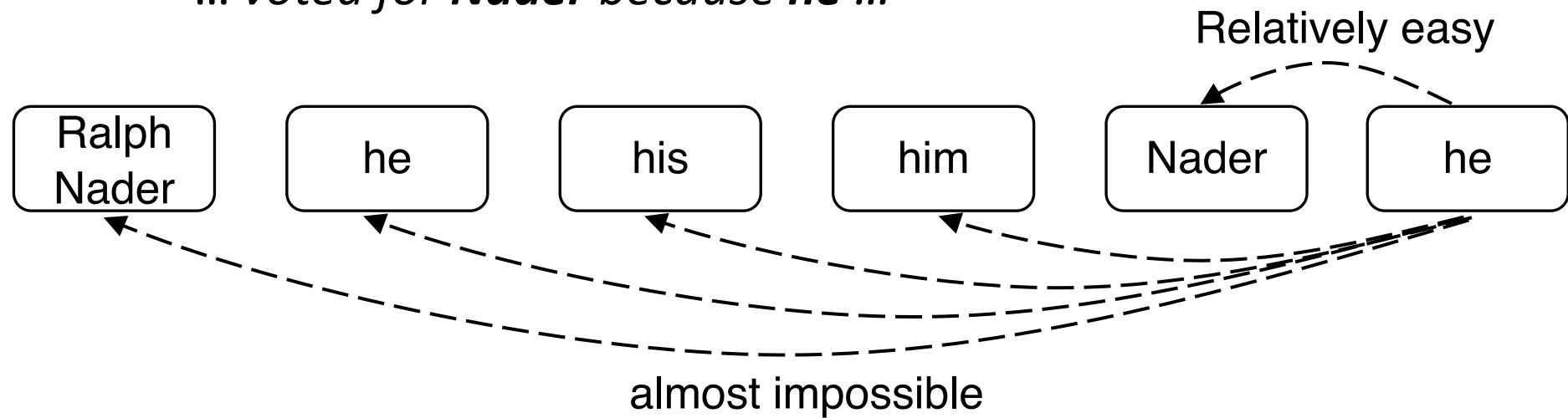
*"I voted for Nader because he was most aligned with my values," she said.*



Adding this extra link would merge everything  
into one big coreference cluster!

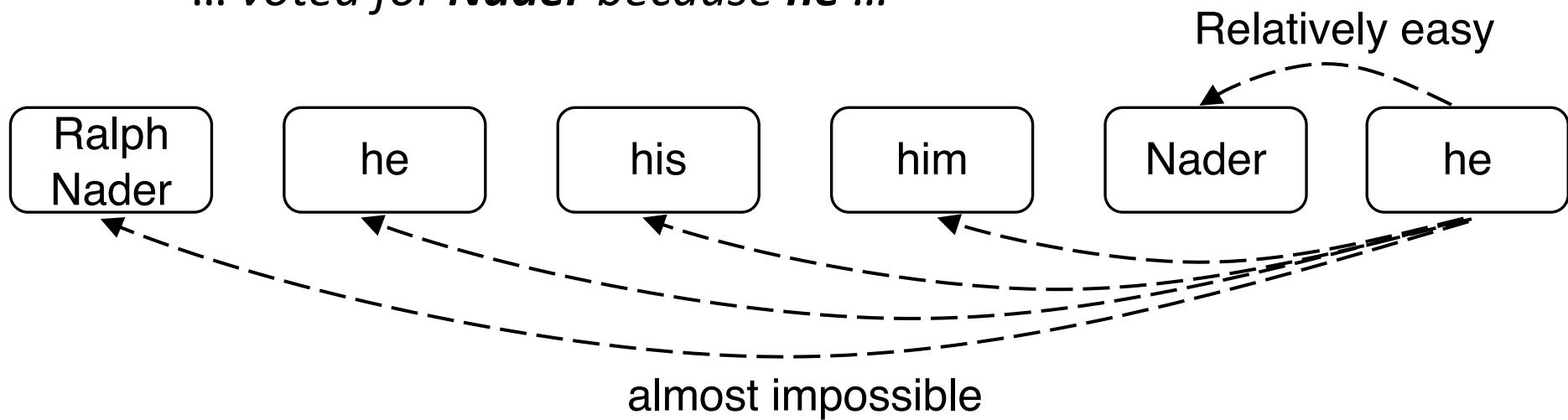
# Mention Pair Models: Disadvantage

- Suppose we have a long document with the following mentions
  - **Ralph Nader** ... **he** ... **his** ... **him** ... <several paragraphs>  
... *voted for Nader because he* ...



# Mention Pair Models: Disadvantage

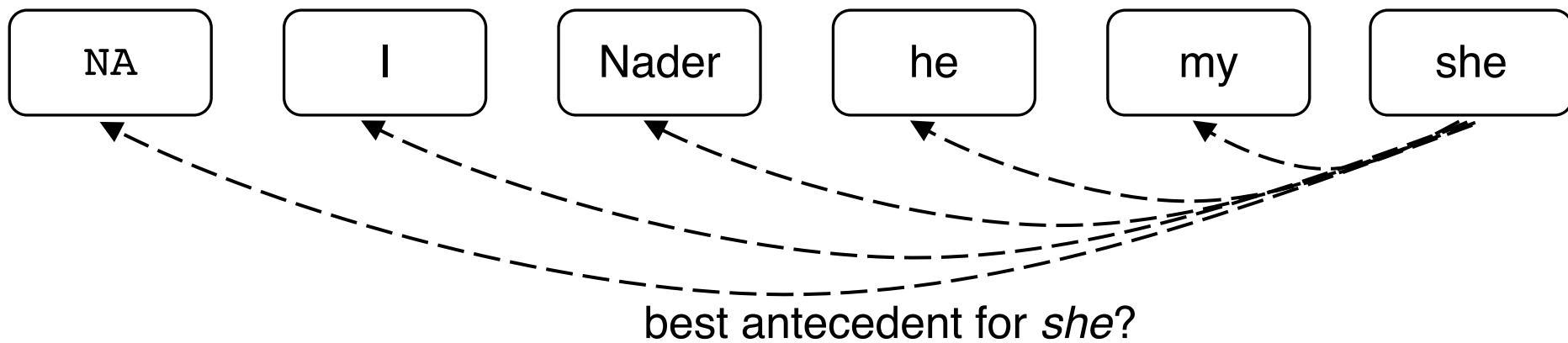
- Suppose we have a long document with the following mentions
  - **Ralph Nader** ... **he** ... **his** ... **him** ... <several paragraphs>  
... voted for **Nader** because **he** ...



- Many mentions only have one clear antecedent
  - But we are asking the model to predict all of them
- Solution: instead train the model to predict only one antecedent for each mention
  - More linguistically plausible

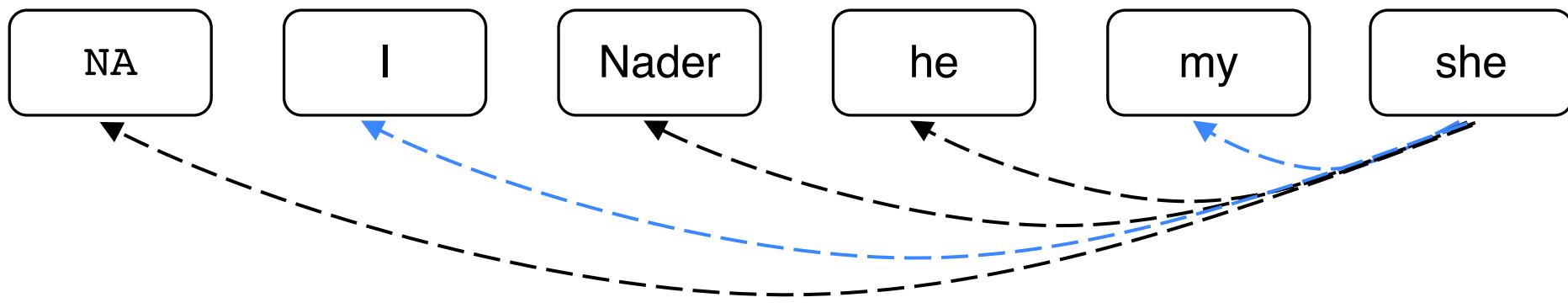
## 7. Coreference Models: Mention Ranking

- Assign each mention its highest scoring candidate antecedent according to the model
- Dummy NA mention allows model to decline linking the current mention to anything (“singleton” or “first” mention)



# Coreference Models: Mention Ranking

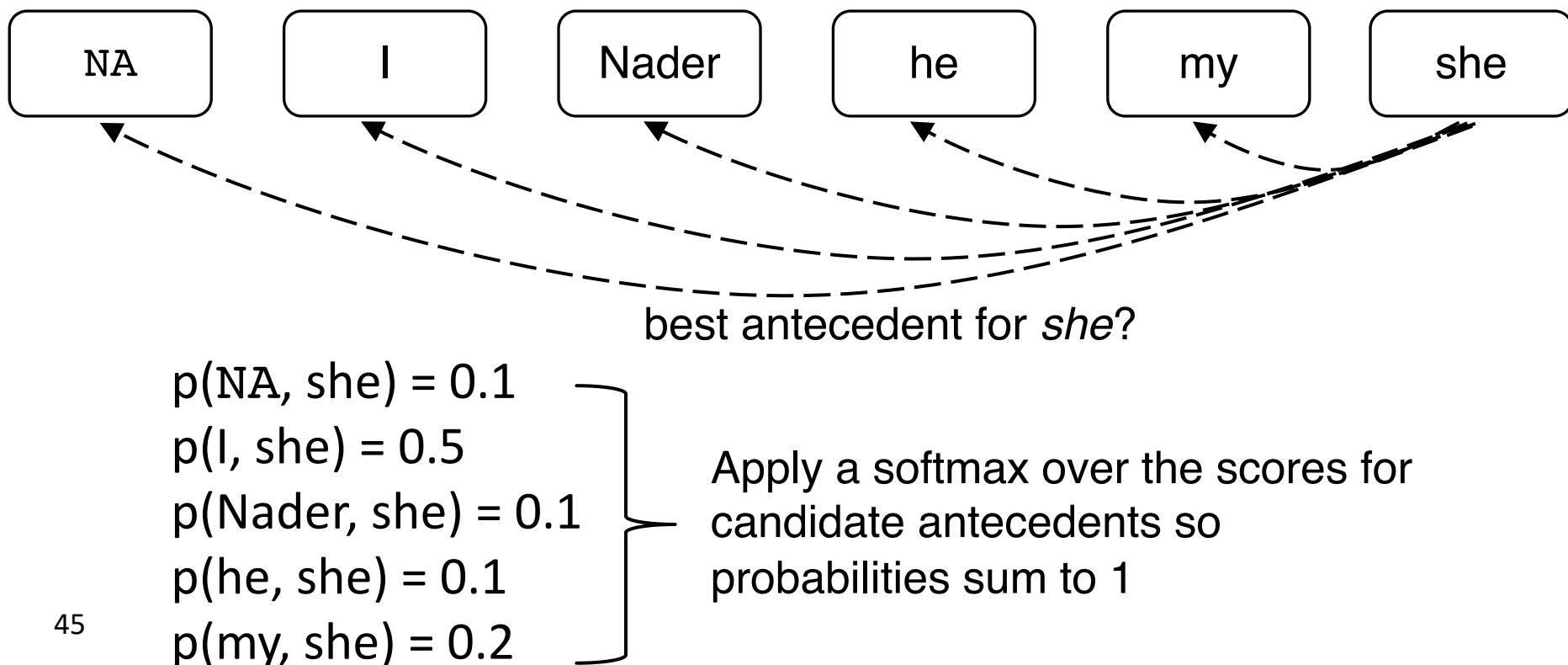
- Assign each mention its highest scoring candidate antecedent according to the model
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**Positive examples:** model has to assign a high probability to either one (but not necessarily both)

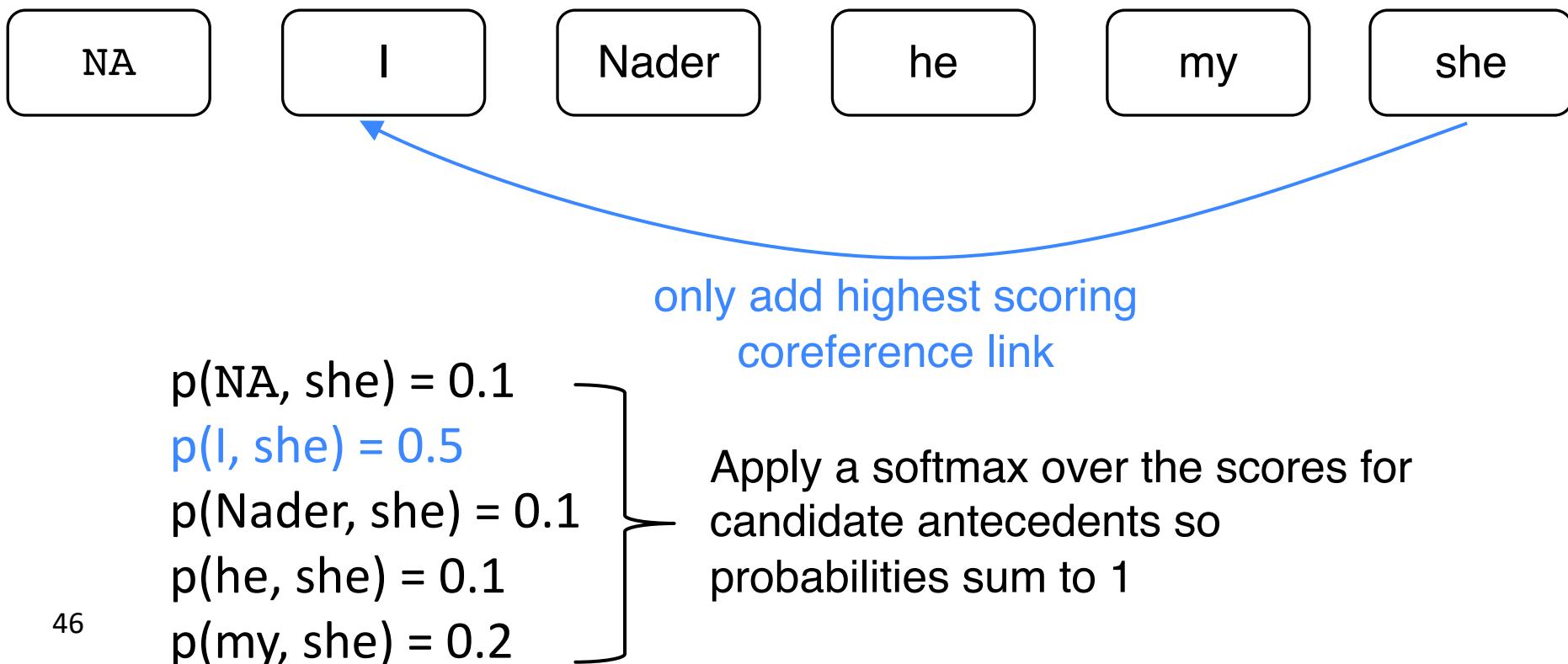
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# Coreference Models: Training

- We want the current mention  $m_j$  to be linked to *any one* of the candidate antecedents it's coreferent with.
- Mathematically, we might want to maximize this probability:

$$\sum_{j=1}^{i-1} \mathbb{1}(y_{ij} = 1) p(m_j, m_i)$$

Iterate through candidate antecedents (previously occurring mentions)

For ones that are coreferent to  $m_j$ ...

...we want the model to assign a high probability

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...we want the model to assign a high probability

- The model could produce 0.9 probability for one of the correct antecedents and low probability for everything else, and the sum will still be large

# Coreference Models: Training

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- Mathematically, we want to maximize this probability:

$$\sum_{j=1}^{i-1} \mathbb{1}(y_{ij} = 1) p(m_j, m_i)$$

- Turning this into a loss function:

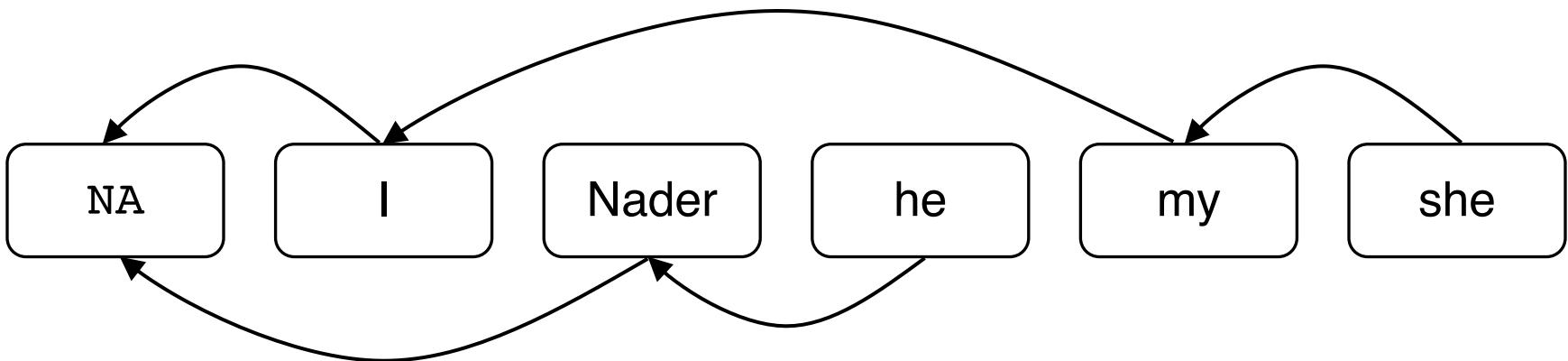
$$J = \sum_{i=2}^N -\log \left( \sum_{j=1}^{i-1} \mathbb{1}(y_{ij} = 1) p(m_j, m_i) \right)$$

Iterate over all the mentions  
in the document

Usual trick of taking negative  
log to go from likelihood to loss

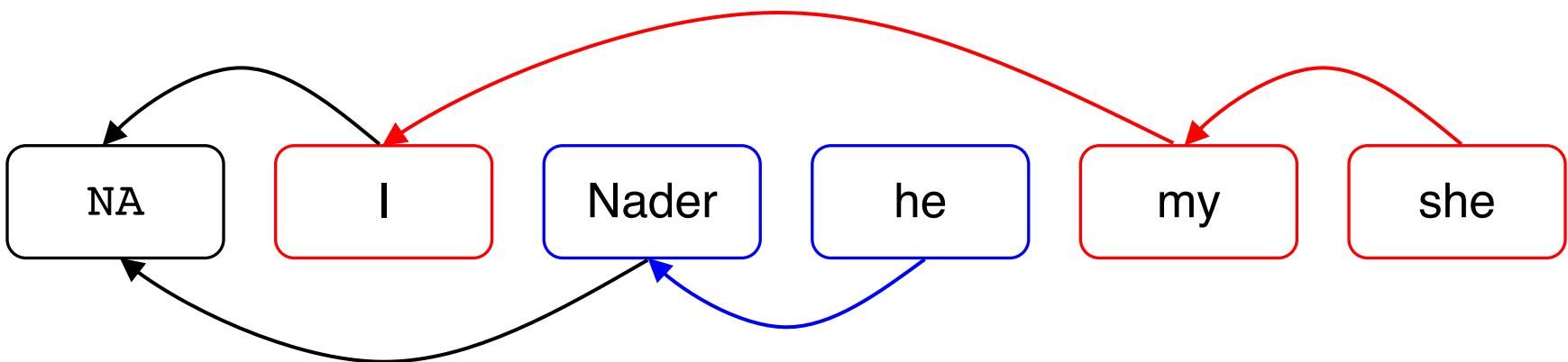
# Mention Ranking Models: Test Time

- Pretty much the same as mention-pair model except each mention is assigned only one antecedent



# Mention Ranking Models: Test Time

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# How do we compute the probabilities?

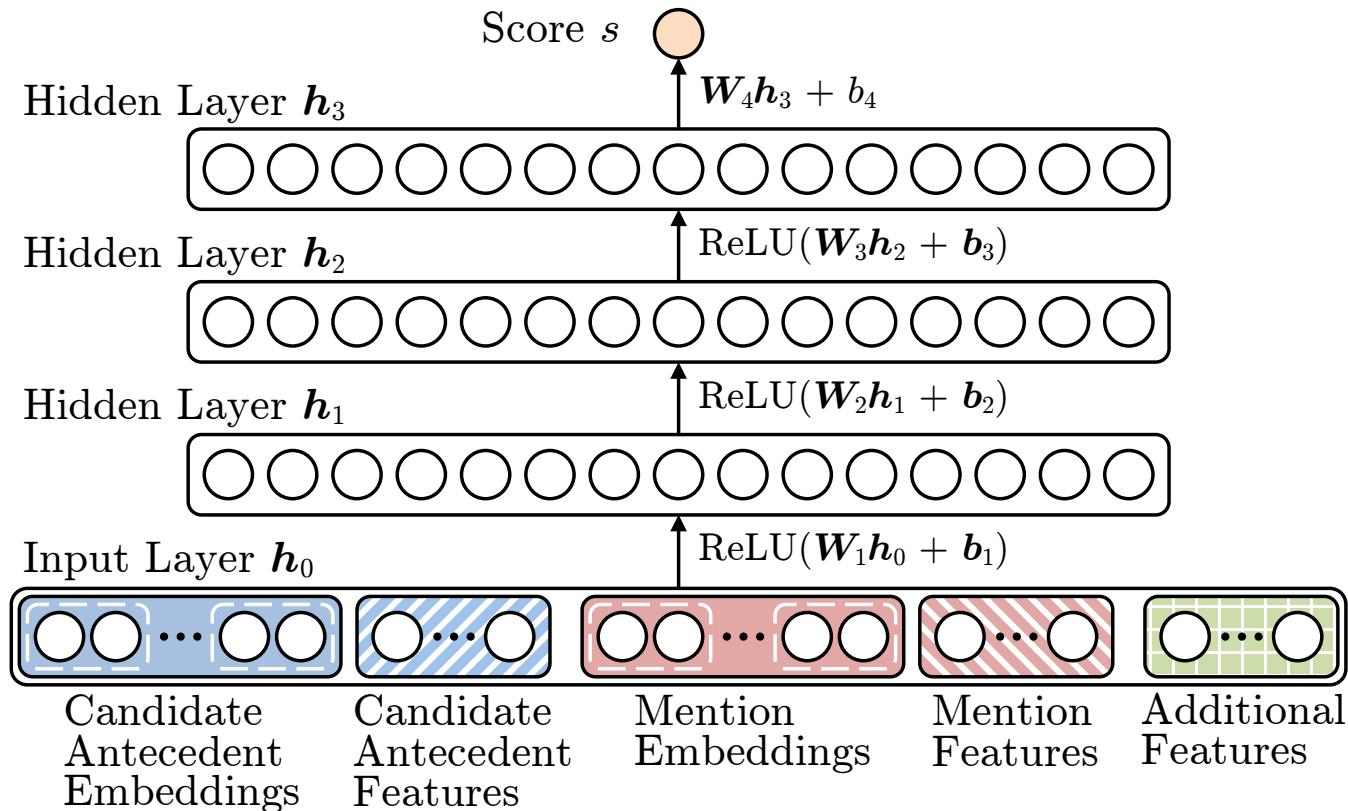
- A. Non-neural statistical classifier
- B. Simple neural network
- C. More advanced model using LSTMs, attention

## A. Non-Neural Coref Model: Features

- Person/Number/Gender agreement
  - Jack gave Mary a gift. She was excited.
- Semantic compatibility
  - ... the mining conglomerate ... the company ...
- Certain syntactic constraints
  - John bought him a new car. [him can not be John]
- More recently mentioned entities preferred for referenced
  - John went to a movie. Jack went as well. He was not busy.
- Grammatical Role: Prefer entities in the subject position
  - John went to a movie with Jack. He was not busy.
- Parallelism:
  - John went with Jack to a movie. Joe went with him to a bar.
- ...

## B. Neural Coref Model

- Standard feed-forward neural network
  - Input layer: word embeddings and a few categorical features



# Neural Coref Model: Inputs

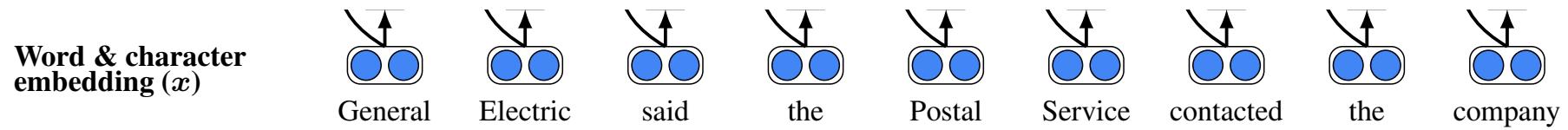
- Embeddings
  - Previous two words, first word, last word, head word, ... of each mention
    - The **head** word is the “most important” word in the mention – you can find it using a parser. e.g., *The fluffy **cat** stuck in the tree*
- Still need some other features:
  - Distance
  - Document genre
  - Speaker information

## C. End-to-end Model

- Current state-of-the-art model for coreference resolution (Kenton Lee et al. from UW, EMNLP 2017)
- Mention ranking model
- Improvements over simple feed-forward NN
  - Use an LSTM
  - Use attention
  - Do mention detection and coreference end-to-end
    - No mention detection step!
    - Instead consider every **span** of text (up to a certain length) as a candidate mention
      - a **span** is just a contiguous sequence of words

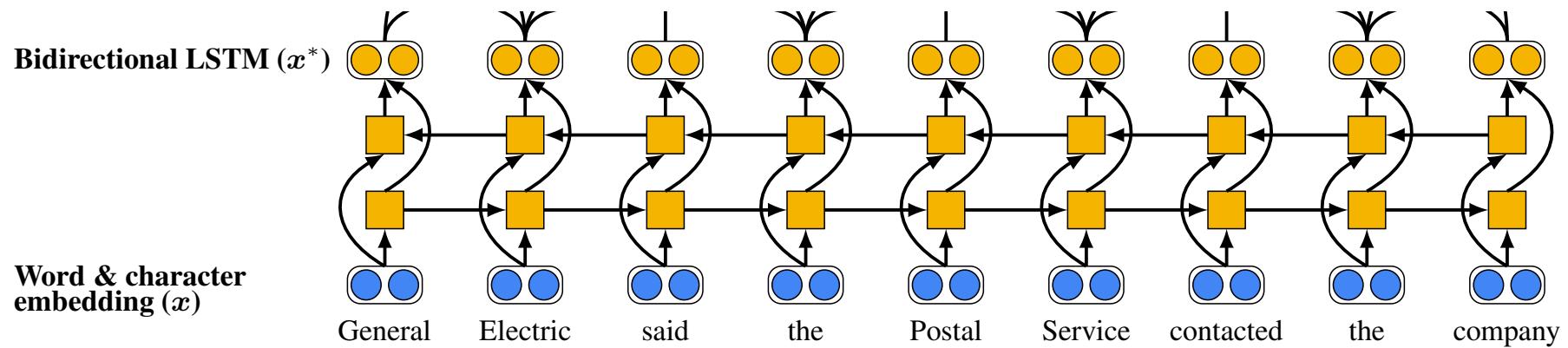
# End-to-end Model

- First embed the words in the document using a word embedding matrix and a character-level CNN



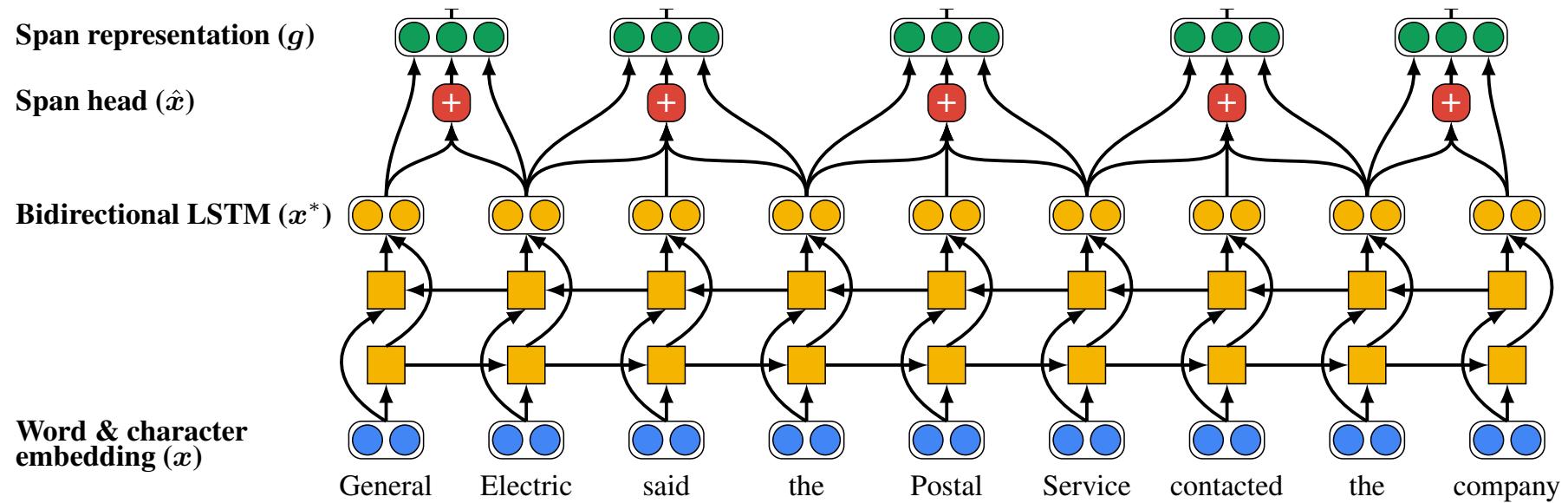
# End-to-end Model

- Then run a bidirectional LSTM over the document



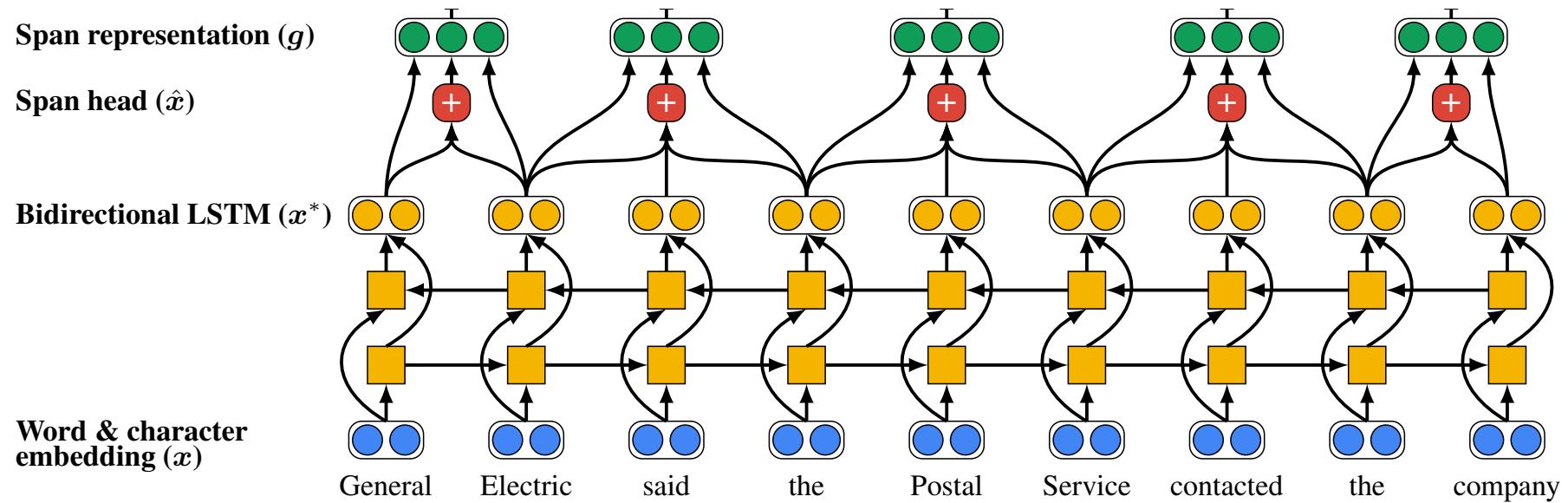
# End-to-end Model

- Next, represent each span of text  $i$  going from  $\text{START}(i)$  to  $\text{END}(i)$  as a vector



# End-to-end Model

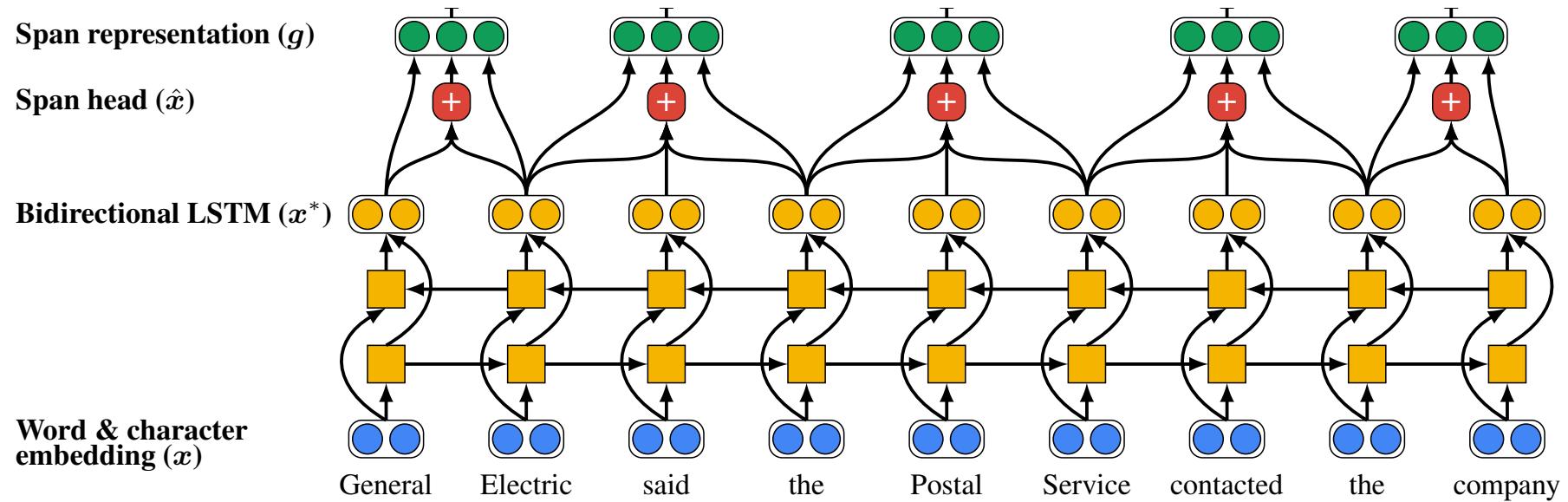
- Next, represent each span of text  $i$  going from  $\text{START}(i)$  to  $\text{END}(i)$  as a vector



- *General, General Electric, General Electric said, ... Electric, Electric said, ...* will all get its own vector representation

# End-to-end Model

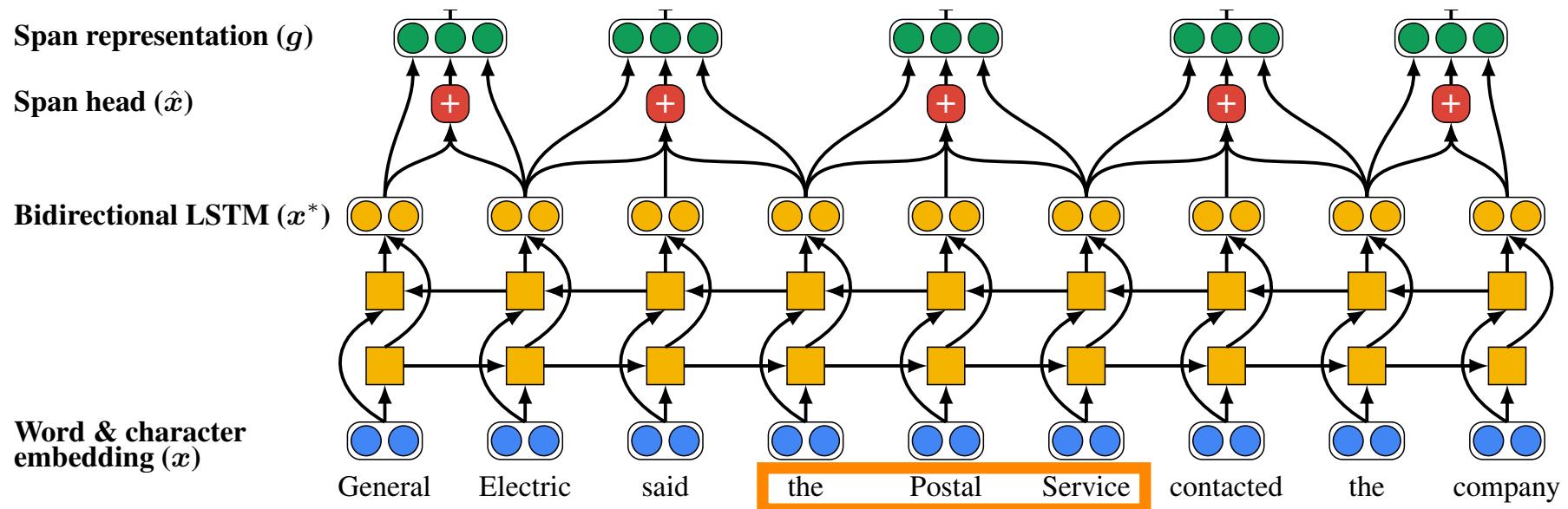
- Next, represent each span of text  $i$  going from  $\text{START}(i)$  to  $\text{END}(i)$  as a vector.



Span representation:  $g_i = [x_{\text{START}(i)}^*, x_{\text{END}(i)}^*, \hat{x}_i, \phi(i)]$

# End-to-end Model

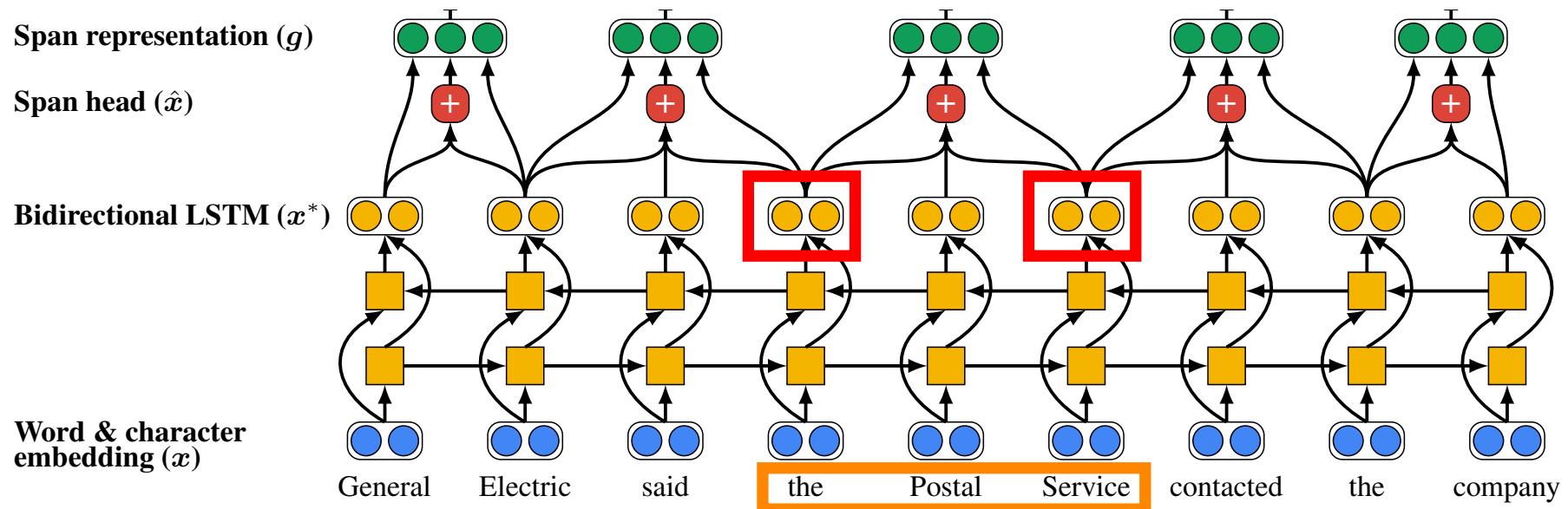
- Next, represent each span of text  $i$  going from  $\text{START}(i)$  to  $\text{END}(i)$  as a vector. For example, for “**the postal service**”



Span representation:  $g_i = [x_{\text{START}(i)}^*, x_{\text{END}(i)}^*, \hat{x}_i, \phi(i)]$

# End-to-end Model

- Next, represent each span of text  $i$  going from  $\text{START}(i)$  to  $\text{END}(i)$  as a vector. For example, for “the postal service”

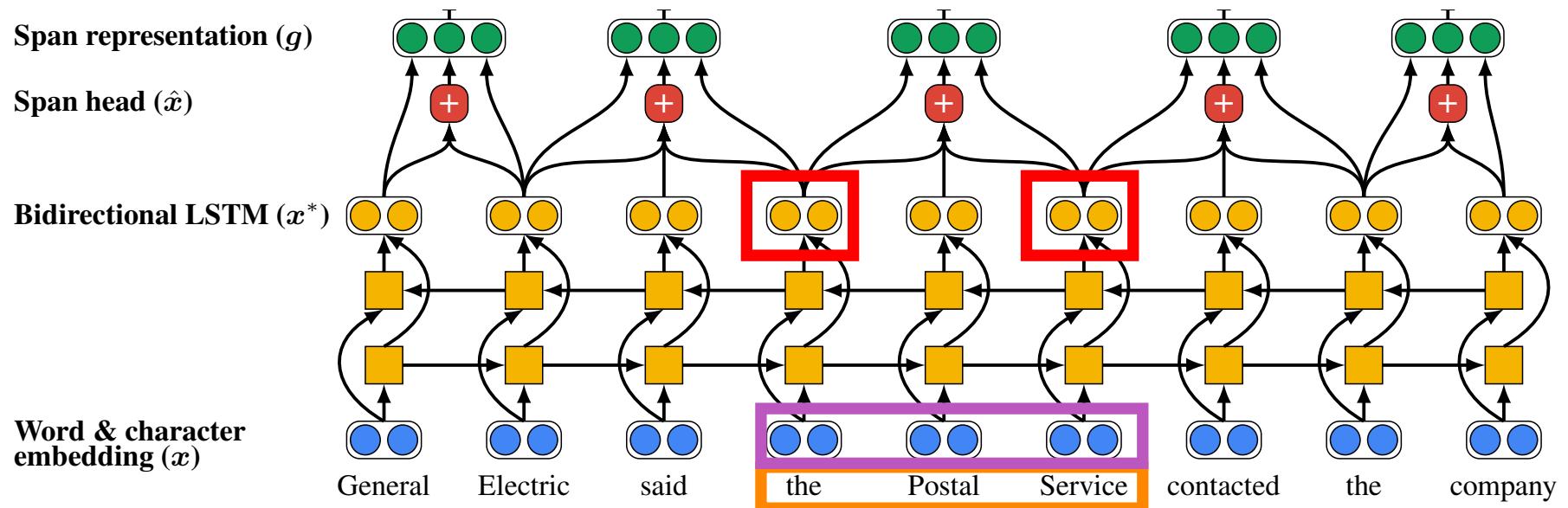


Span representation:  $g_i = [x_{\text{START}(i)}^*, x_{\text{END}(i)}^*, \hat{x}_i, \phi(i)]$

BILSTM hidden states for  
span's start and end

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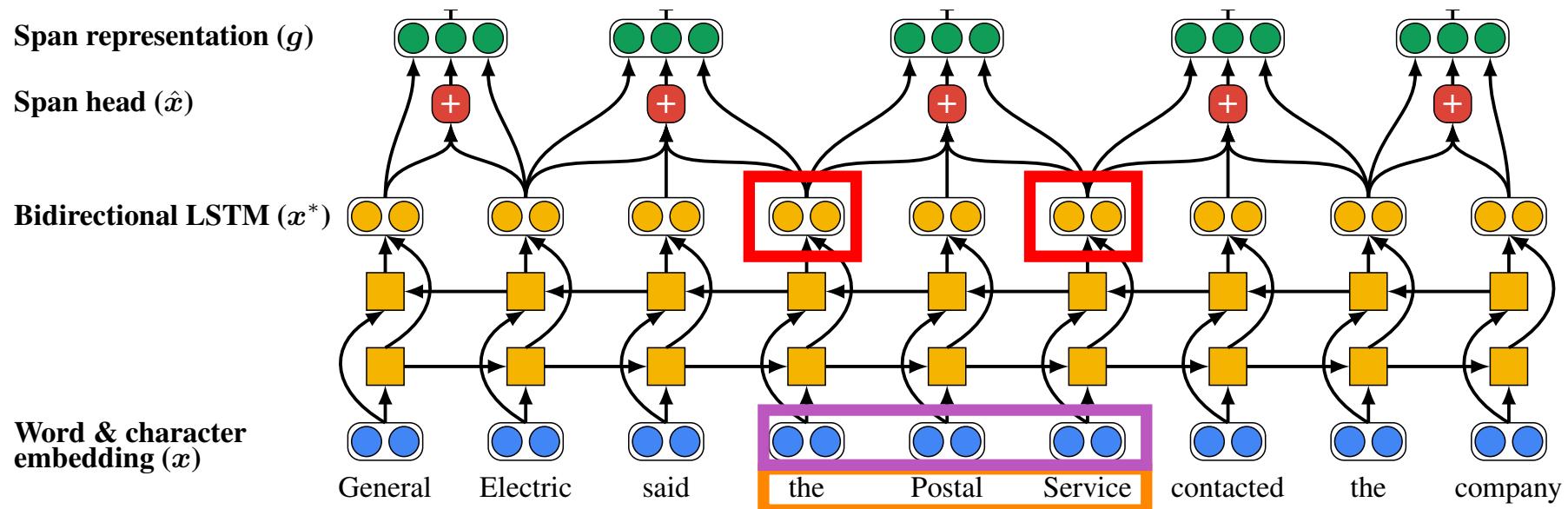
Span representation:  $g_i = [x_{\text{START}(i)}^*, x_{\text{END}(i)}^*, \hat{x}_i, \phi(i)]$

BILSTM hidden states for span's start and end

Attention-based representation (details next slide) of the words in the span

# End-to-end Model

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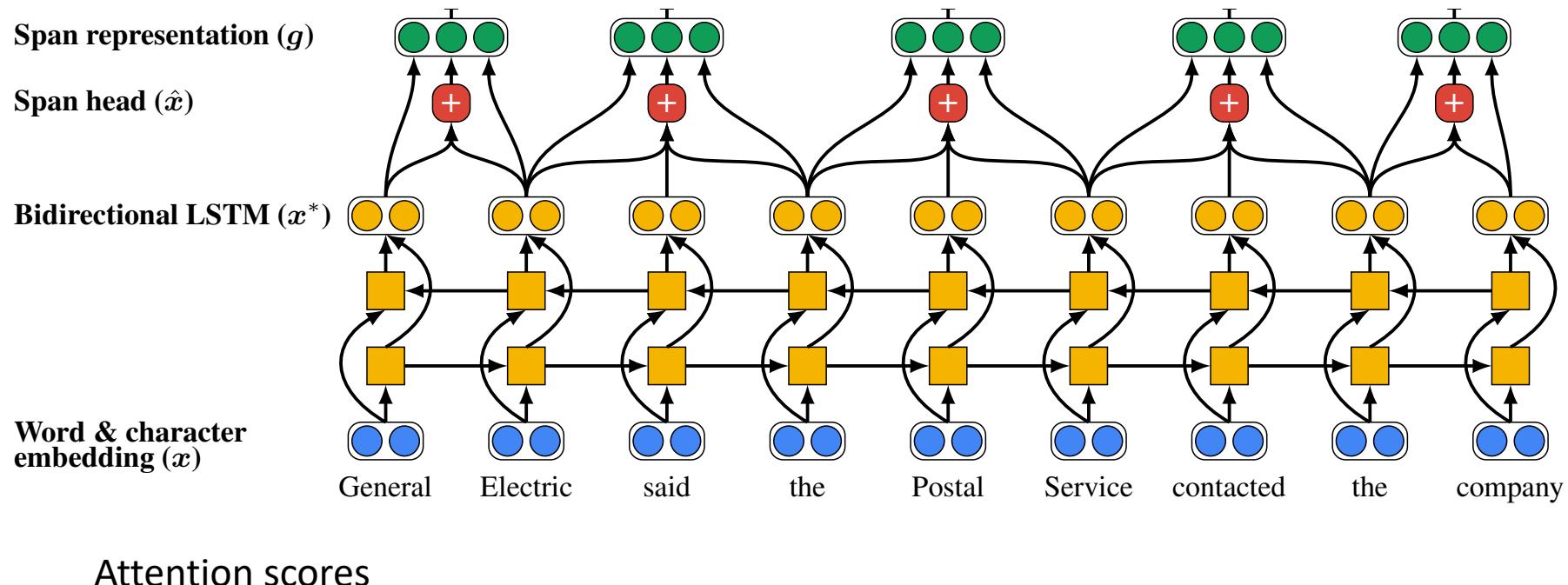
BILSTM hidden states for  
span's start and end

Attention-based representation  
(details next slide) of the words  
in the span

Additional features

# End-to-end Model

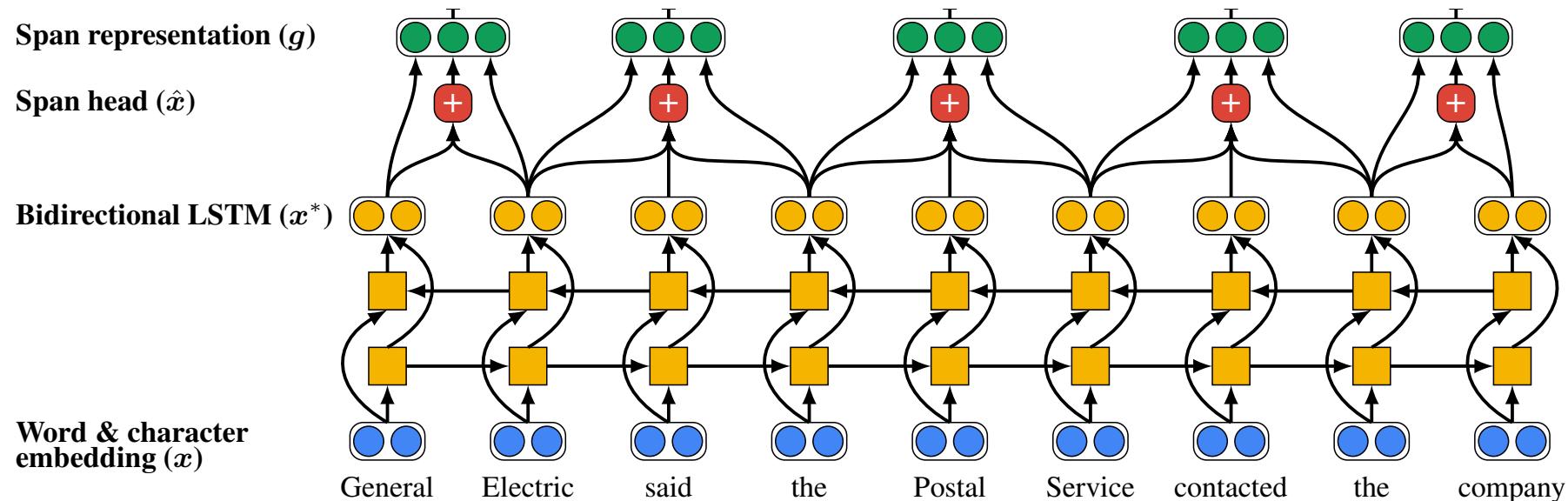
- $\hat{x}_i$  is an attention-weighted average of the word embeddings in the span



dot product of weight  
vector and transformed  
hidden state

# End-to-end Model

- $\hat{x}_i$  is an attention-weighted average of the word embeddings in the span



Attention scores

$$\alpha_t = \mathbf{w}_\alpha \cdot \text{FFNN}_\alpha(\mathbf{x}_t^*)$$

dot product of weight vector and transformed hidden state

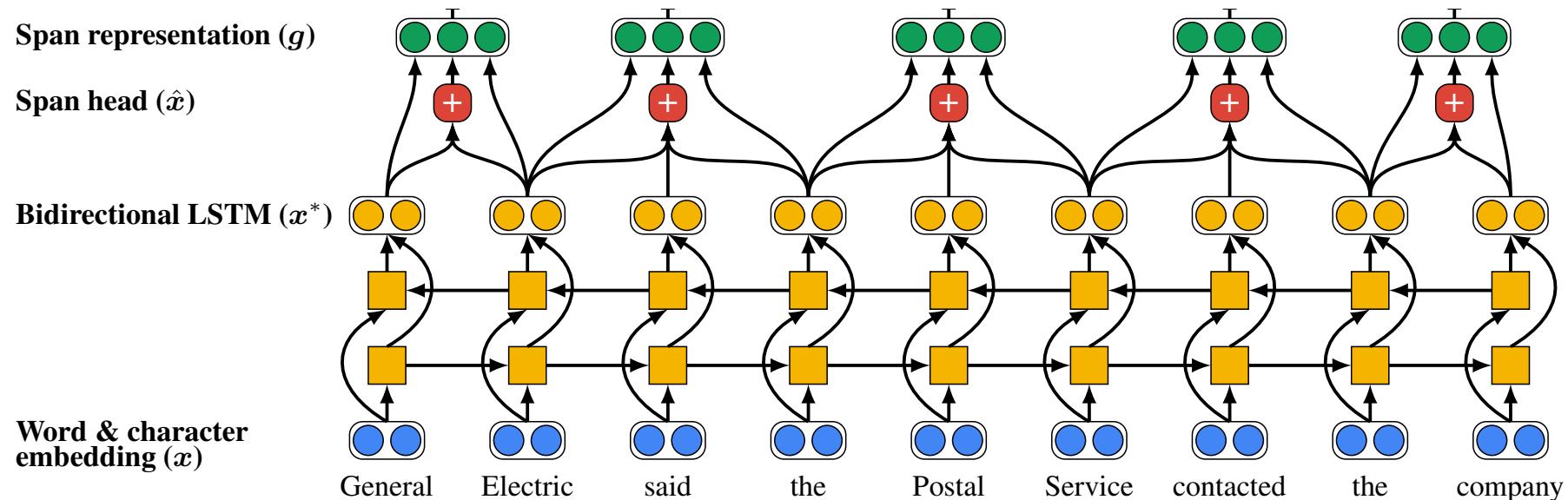
Attention distribution

$$a_{i,t} = \frac{\exp(\alpha_t)}{\sum_{k=\text{START}(i)}^{\text{END}(i)} \exp(\alpha_k)}$$

just a softmax over attention scores for the span

# End-to-end Model

- $\hat{x}_i$  is an attention-weighted average of the word embeddings in the span



Attention scores

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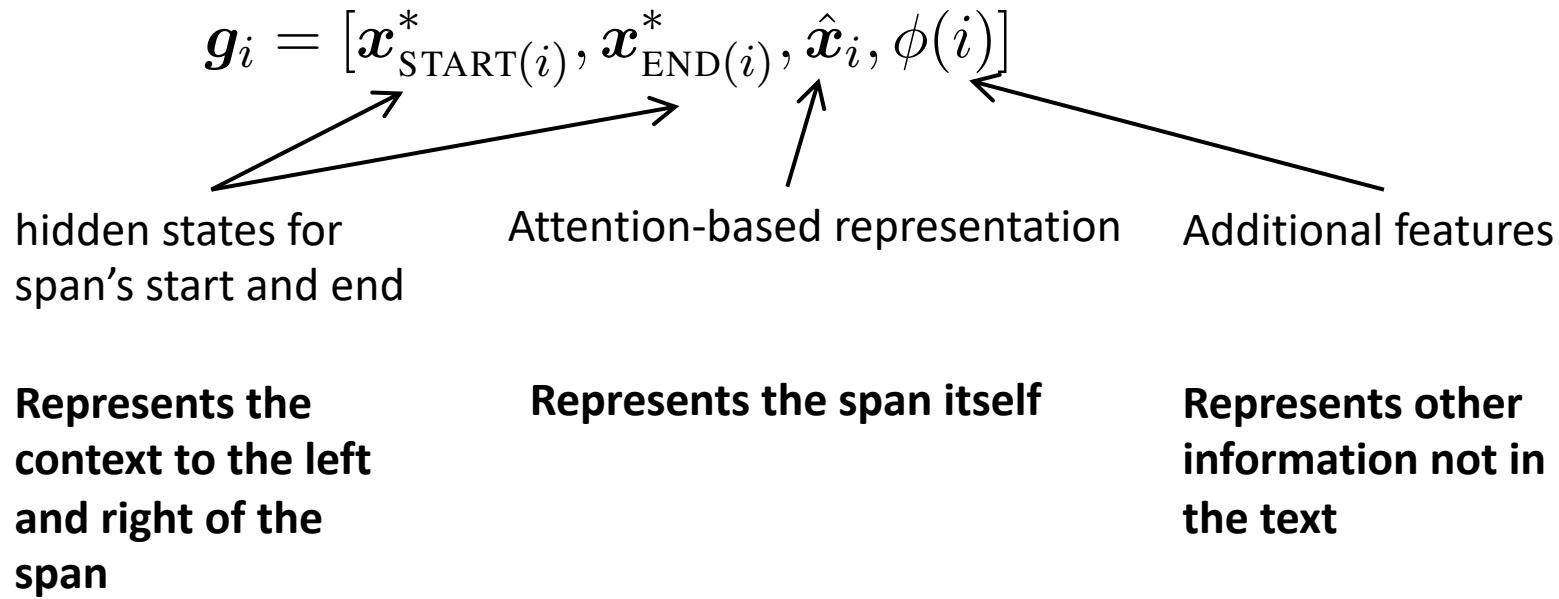
Final representation

$$\hat{x}_i = \sum_{t=\text{START}(i)}^{\text{END}(i)} a_{i,t} \cdot \mathbf{x}_t$$

Attention-weighted sum of word embeddings

# End-to-end Model

- Why include all these different terms in the span?



# End-to-end Model

- Lastly, score every pair of spans to decide if they are coreferent mentions

$$s(i, j) = s_m(i) + s_m(j) + s_a(i, j)$$

Are spans  $i$  and  $j$  coreferent mentions?      Is  $i$  a mention?      Is  $j$  a mention?      Do they look coreferent?

The diagram illustrates the formula for calculating the coreference score  $s(i, j)$ . The formula is 
$$s(i, j) = s_m(i) + s_m(j) + s_a(i, j)$$
. Below the formula, four arrows point from text labels to the corresponding terms in the equation:

- An arrow points from "Are spans  $i$  and  $j$  coreferent mentions?" to  $s_a(i, j)$ .
- An arrow points from "Is  $i$  a mention?" to  $s_m(i)$ .
- An arrow points from "Is  $j$  a mention?" to  $s_m(j)$ .
- An arrow points from "Do they look coreferent?" to the plus sign between  $s_m(j)$  and  $s_a(i, j)$ .

# End-to-end Model

- Lastly, score every pair of spans to decide if they are coreferent mentions

$$s(i, j) = s_m(i) + s_m(j) + s_a(i, j)$$

Are spans  $i$  and  $j$  coreferent mentions?      Is  $i$  a mention?      Is  $j$  a mention?      Do they look coreferent?

```
graph TD; Eq[s(i, j) = s_m(i) + s_m(j) + s_a(i, j)] --> Q1[Are spans i and j coreferent mentions?]; Eq --> Q2[Is i a mention?]; Eq --> Q3[Is j a mention?]; Eq --> Q4[Do they look coreferent?]
```

- Scoring functions take the span representations as input

$$s_m(i) = \mathbf{w}_m \cdot \text{FFNN}_m(\mathbf{g}_i)$$

$$s_a(i, j) = \mathbf{w}_a \cdot \text{FFNN}_a([\mathbf{g}_i, \mathbf{g}_j, \mathbf{g}_i \circ \mathbf{g}_j, \phi(i, j)])$$

# End-to-end Model

- Lastly, score every pair of spans to decide if they are coreferent mentions

$$s(i, j) = s_m(i) + s_m(j) + s_a(i, j)$$

Are spans  $i$  and  $j$  coreferent mentions?      Is  $i$  a mention?      Is  $j$  a mention?      Do they look coreferent?

```
graph LR; Eq[s(i, j) = s_m(i) + s_m(j) + s_a(i, j)] --> A[Are spans i and j coreferent mentions?]; Eq --> B[Is i a mention?]; Eq --> C[Is j a mention?]; Eq --> D[Do they look coreferent?]
```

- Scoring functions take the span representations as input

$$s_m(i) = \mathbf{w}_m \cdot \text{FFNN}_m(\mathbf{g}_i)$$

$$s_a(i, j) = \mathbf{w}_a \cdot \text{FFNN}_a([\mathbf{g}_i, \mathbf{g}_j, \mathbf{g}_i \circ \mathbf{g}_j, \phi(i, j)])$$

include multiplicative interactions between the representations

again, we have some extra features

# End-to-end Model

- Intractable to score every pair of spans
  - $O(T^2)$  spans of text in a document ( $T$  is the number of words)
  - $O(T^4)$  runtime!
  - So have to do lots of pruning to make work (only consider a few of the spans that are likely to be mentions)
- Attention learns which words are important in a mention (a bit like head words)

(A **fire** in a Bangladeshi garment factory) has left at least 37 people dead and 100 hospitalized. Most of the deceased were killed in the crush as workers tried to flee (the **blaze**) in the four-story building.

## 8. Last Coreference Approach: Clustering-Based

- Coreference is a clustering task, let's use a clustering algorithm!
  - In particular we will use agglomerative clustering
- Start with each mention in its own singleton cluster
- Merge a pair of clusters at each step
  - Use a model to score which cluster merges are good

# Coreference Models: Clustering-Based

Google recently ... **the company** announced **Google Plus** ... **the product** features ...

Cluster 1

Cluster 2

Cluster 3

Cluster 4

Google

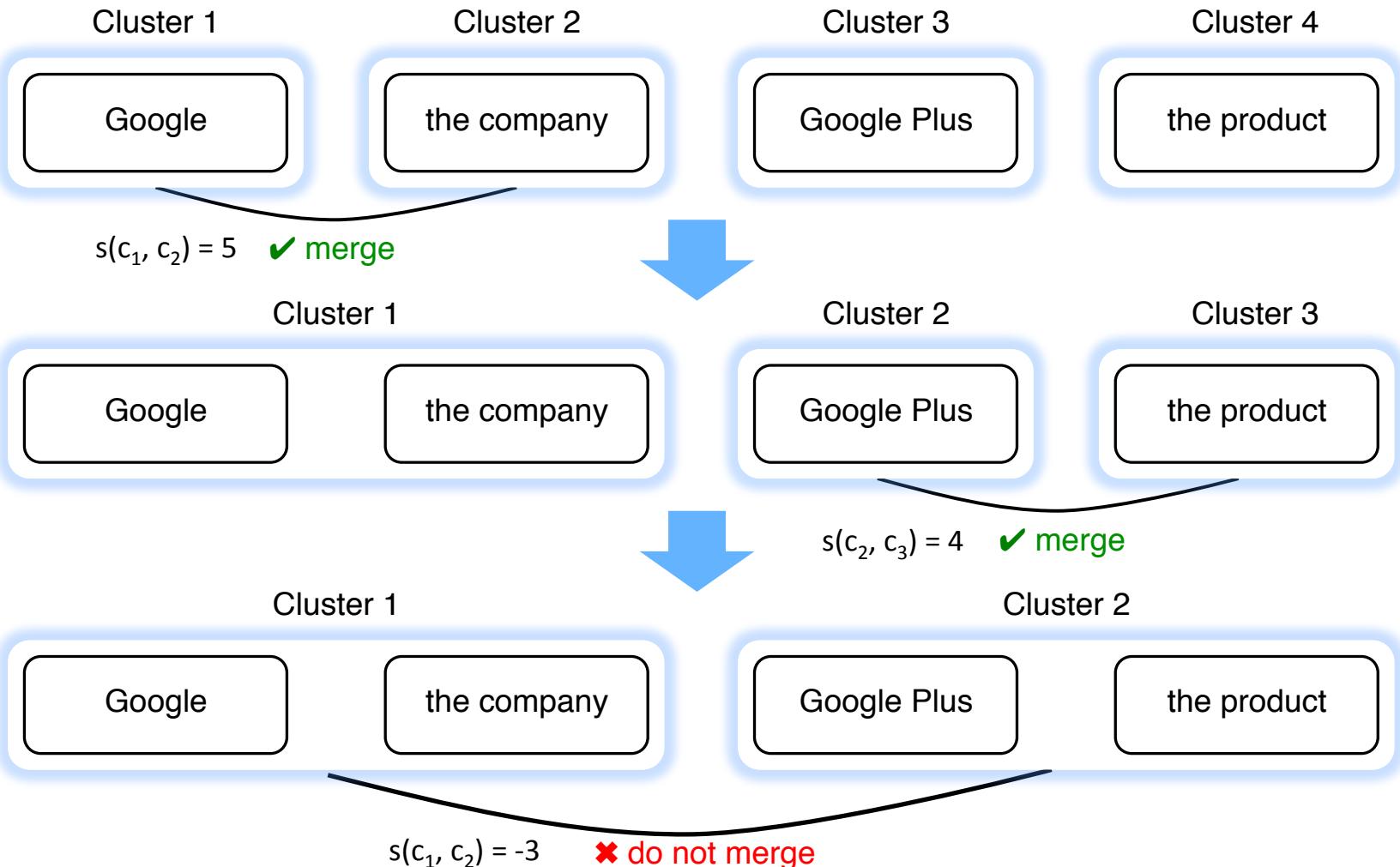
the company

Google Plus

the product

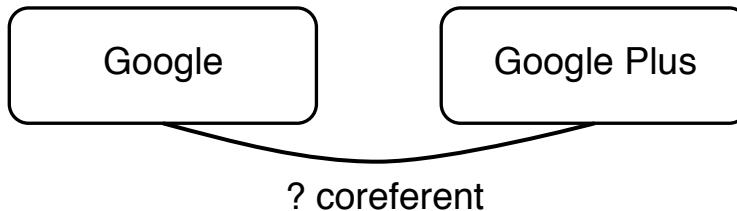
# Coreference Models: Clustering-Based

Google recently ... the company announced Google Plus ... the product features ...

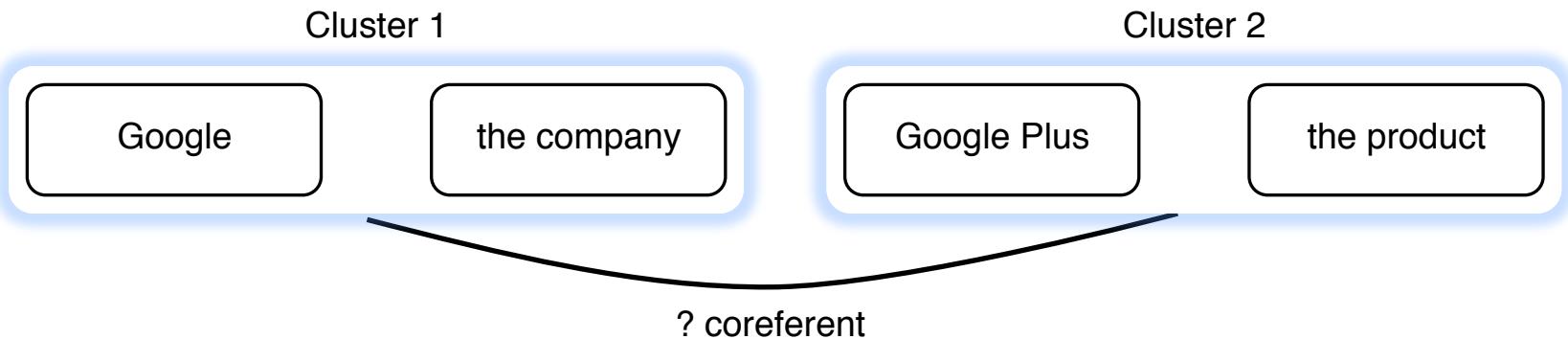


# Coreference Models: Clustering-Based

Mention-pair decision is difficult



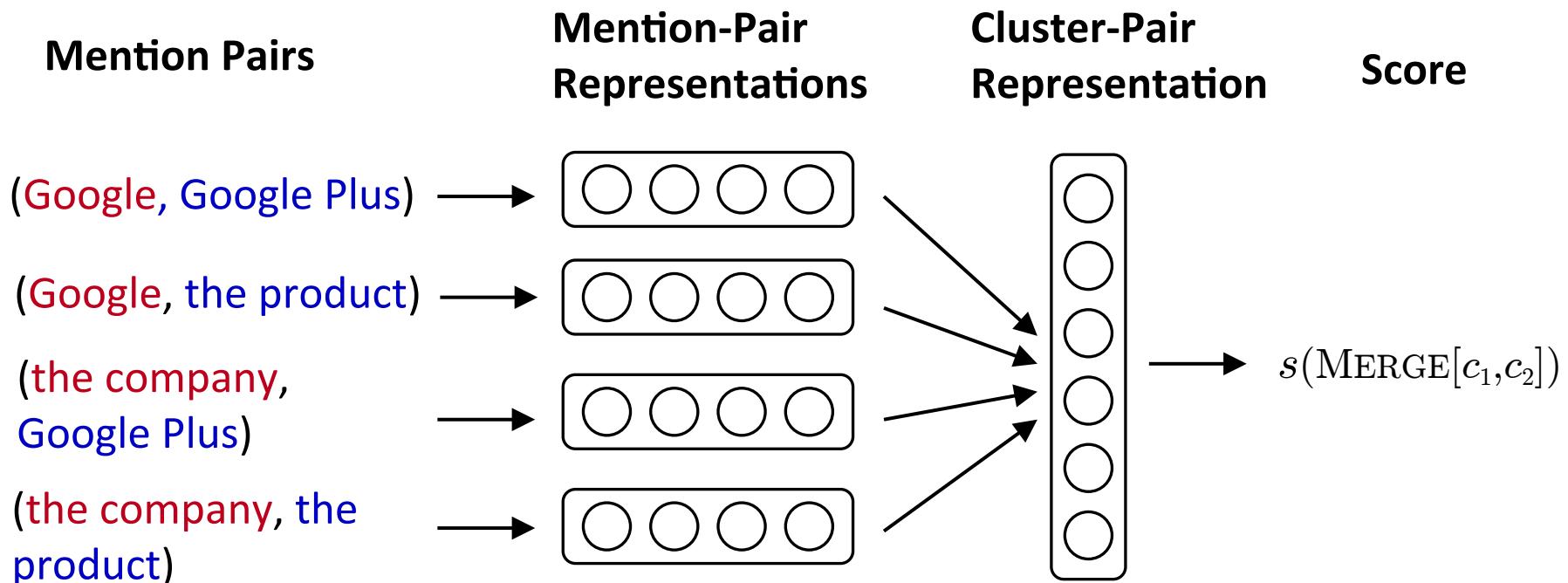
Cluster-pair decision is easier



# Clustering Model Architecture

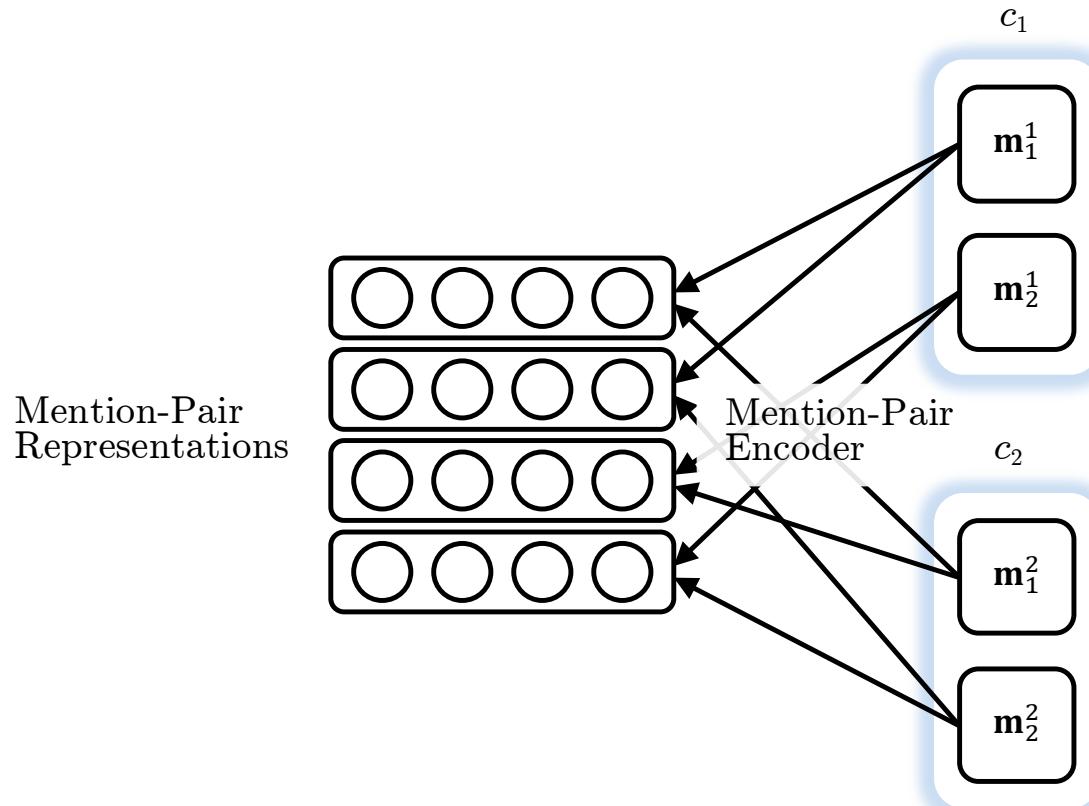
From Clark & Manning, 2016

Merge clusters  $c_1 = \{\text{Google, the company}\}$  and  
 $c_2 = \{\text{Google Plus, the product}\}$  ?



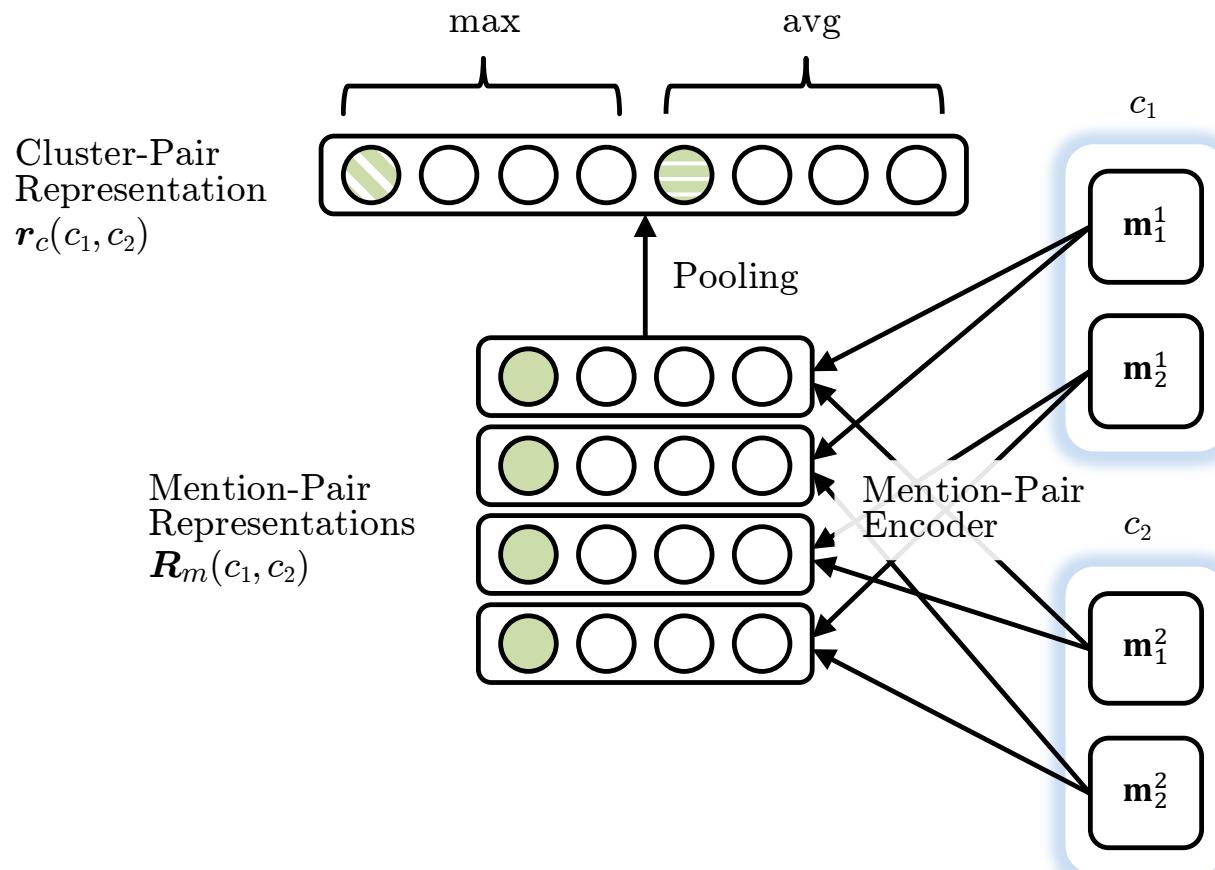
# Clustering Model Architecture

- First produce a vector for each pair of mentions
  - e.g., the output of the hidden layer in the feedforward neural network model



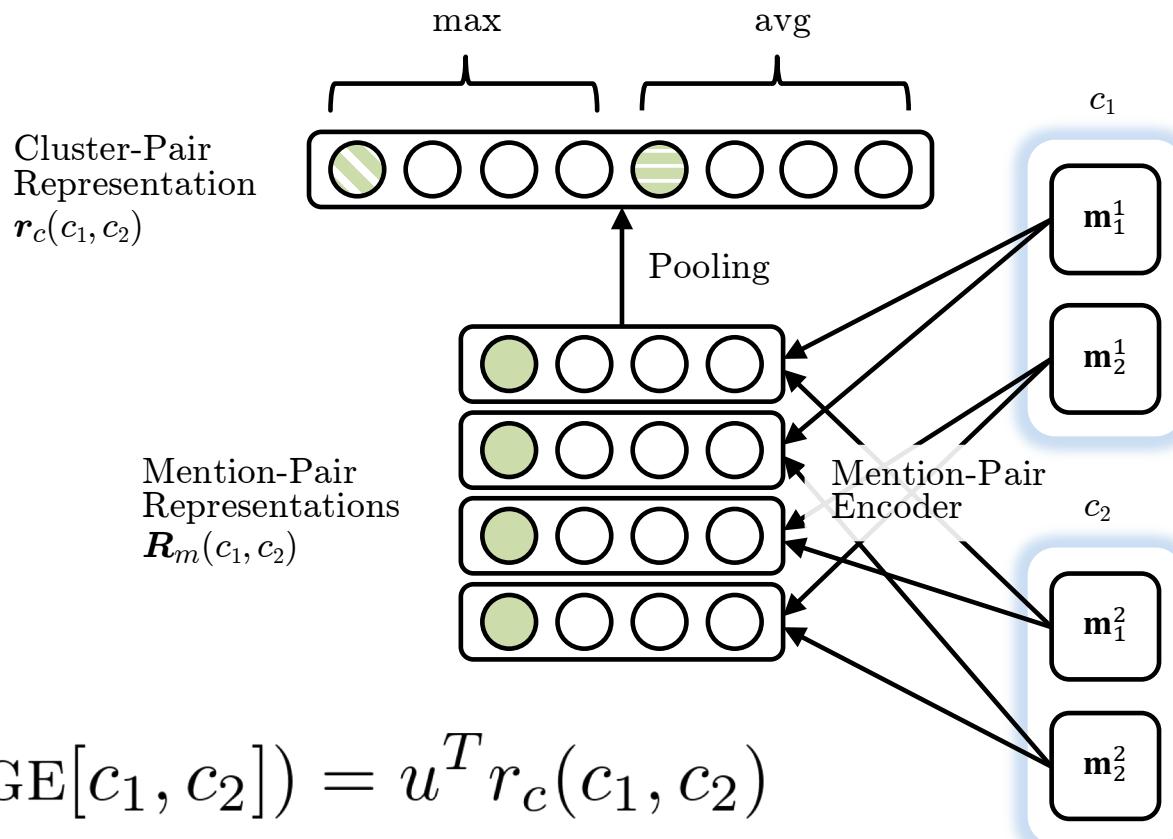
# Clustering Model Architecture

- Then apply a pooling operation over the matrix of mention-pair representations to get a cluster-pair representation



# Clustering Model Architecture

- Score the candidate cluster merge by taking the dot product of the representation with a weight vector

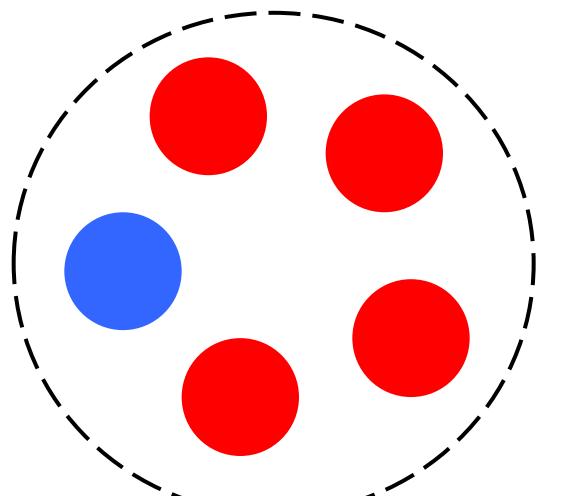


# Clustering Model: Training

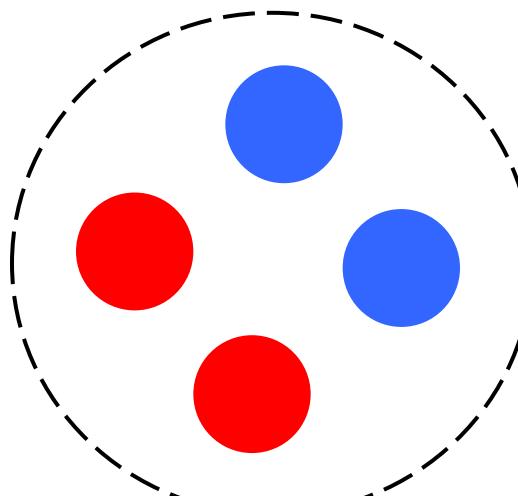
- Current candidate cluster merges depend on previous ones it already made
  - So can't use regular supervised learning
  - Instead use something like Reinforcement Learning to train the model
    - Reward for each merge: the change in a coreference evaluation metric

## 9. Coreference Evaluation

- Many different metrics: MUC, CEAFF, LEA, B-CUBED, BLANC
  - Often report the average over a few different metrics



System Cluster 1



System Cluster 2

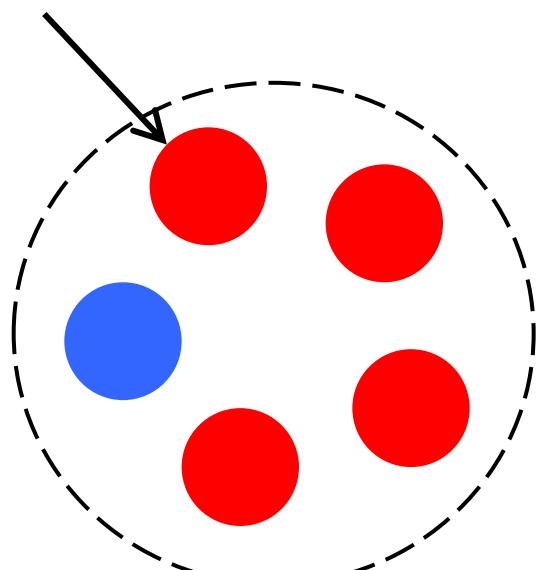
Gold Cluster 1  
Gold Cluster 2

# Coreference Evaluation

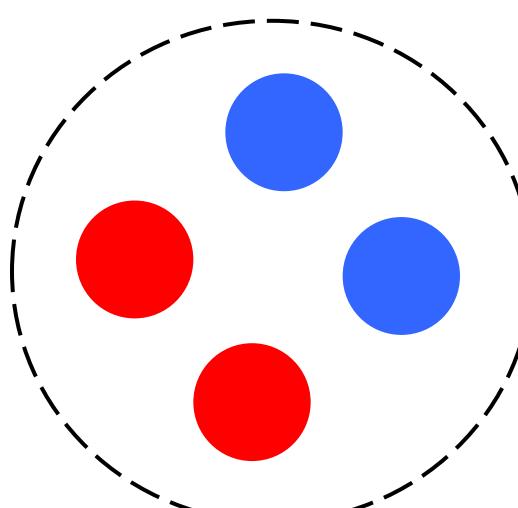
- An example: B-cubed
  - For each mention, compute a precision and a recall

$$P = 4/5$$

$$R = 4/6$$



System Cluster 1



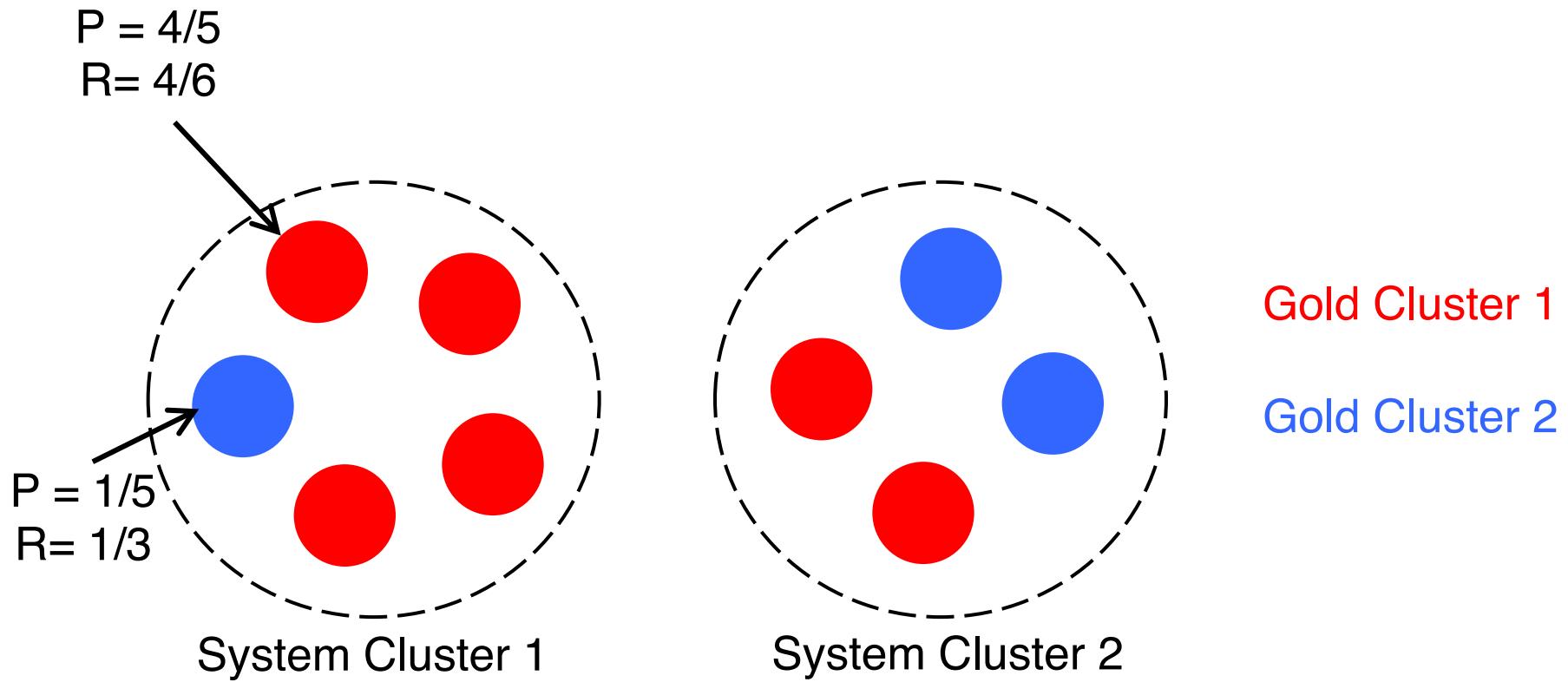
System Cluster 2

Gold Cluster 1

Gold Cluster 2

# Coreference Evaluation

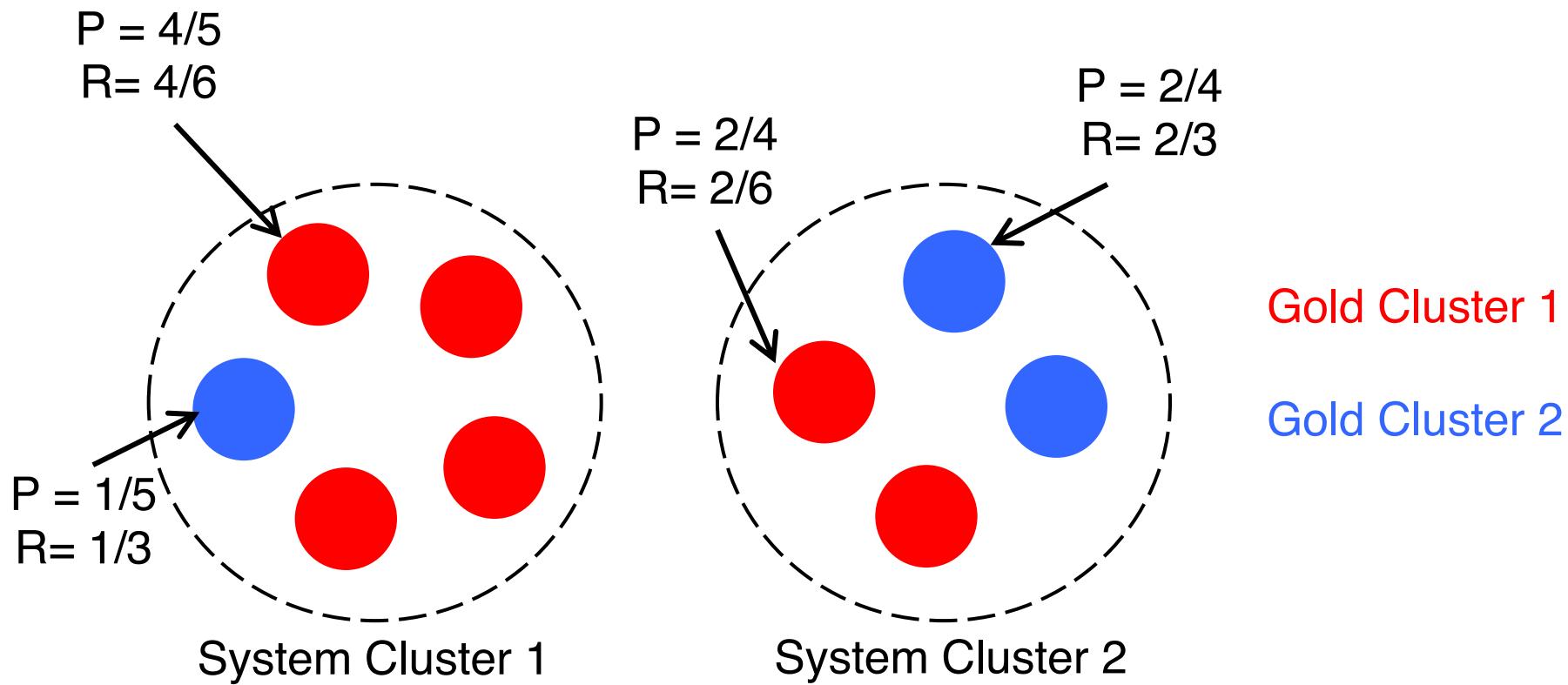
- An example: B-cubed
  - For each mention, compute a precision and a recall



# Coreference Evaluation

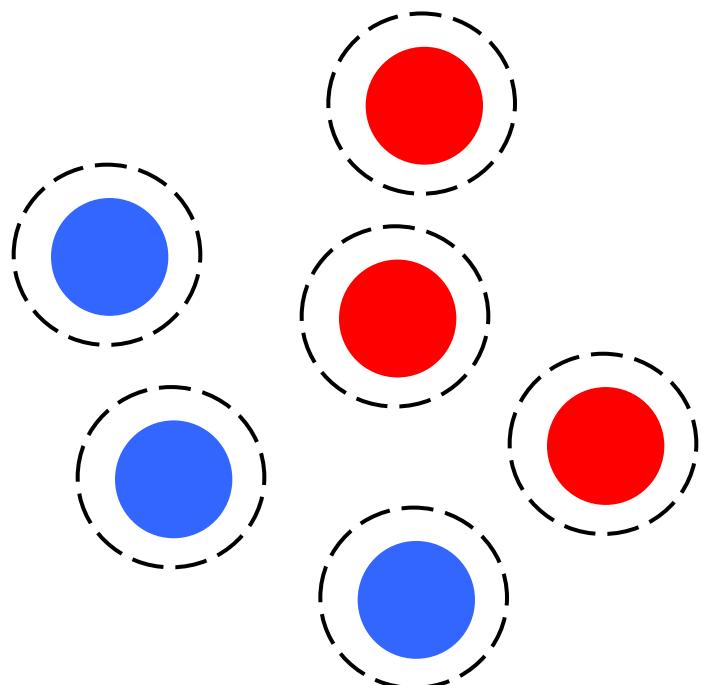
- An example: B-cubed
  - For each mention, compute a precision and a recall
  - Then average the individual Ps and Rs

$$P = [4(4/5) + 1(1/5) + 2(2/4) + 2(2/4)] / 9 = 0.6$$

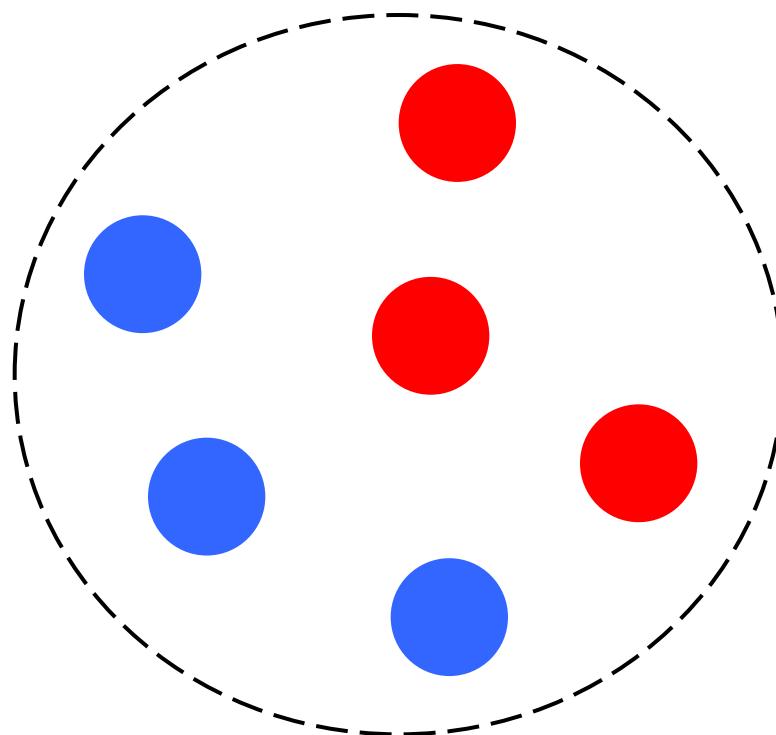


# Coreference Evaluation

100% Precision, 33% Recall



50% Precision, 100% Recall,



# System Performance

- OntoNotes dataset: ~3000 documents labeled by humans
  - English and Chinese data
- Report an F1 score averaged over 3 coreference metrics

# System Performance

Model	English	Chinese	
Lee et al. (2010)	~55	~50	Rule-based system, used to be state-of-the-art!
Chen & Ng (2012) [CoNLL 2012 Chinese winner]	54.5	57.6	
Fernandes (2012) [CoNLL 2012 English winner]	60.7	51.6	Non-neural machine learning models
Wiseman et al. (2015)	63.3	—	Neural mention ranker
Clark & Manning (2016)	65.4	63.7	Neural clustering model
Lee et al. (2017)	67.2	--	End-to-end neural mention ranker

# Where do neural scoring models help?

- Especially with NPs and named entities with no string matching.  
Neural vs non-neural scores:

18.9  $F_1$  vs 10.7  $F_1$  on this type compared to 68.7 vs 66.1  $F_1$

These kinds of coreference are hard and the scores are still low!

## Example Wins

Anaphor	Antecedent
the country's leftist rebels	the guerillas
the company	the New York firm
216 sailors from the ``USS cole''	the crew
the gun	the rifle

# Conclusion

- Coreference is a useful, challenging, and linguistically interesting task
  - Many different kinds of coreference resolution systems
- Systems are getting better rapidly, largely due to better neural models
  - But overall, results are still not amazing
- Try out a coreference system yourself!
  - <http://corenlp.run/> (ask for coref in Annotations)
  - <https://huggingface.co/coref/>