

$$x_t = rx_{t-1}(1-x_{t-1})$$

1. Find the eq points:

$$x_{eq} = rx_{eq}(1-x_{eq})$$

$$rx_{eq} - rx_{eq}^2 - x_{eq} = 0$$

$$x_{eq}(r - rx_{eq} - 1) = 0$$

$$\begin{cases} x_{eq} = 0 \\ r - rx_{eq} - 1 = 0 \end{cases} \Rightarrow \begin{cases} x_{eq} = 0 \\ x_{eq} = \frac{r-1}{r} \end{cases}$$

We calculate $\frac{dF}{dt} [rx(1-x)] = r - 2rx$

$$\left| \frac{dF}{dt} \right|_{x=0} = r - 2r \cdot 0 = r$$

$$\left| \frac{dF}{dt} \right|_{x=\frac{r-1}{r}} = r - 2r \left(\frac{r-1}{r} \right) = r - 2(r-1) = r - 2r + 2 = 2 - r$$

Since this is a discrete-time model, the critical condition

$$\text{for } \left| \frac{dF}{dt} \right|_{x=x_{eq}} = 1 \Rightarrow \begin{cases} r = 1 \\ 2 - r = 1 \end{cases} \Rightarrow \boxed{r = 1}$$

We can determine the stability at the eq point by checking the value of $\frac{dF}{dt}$ at each point

Eq point	$r < 1$	$r > 1$
$x_{eq} = 0$	$ F'(x_{eq}) < 1 \rightarrow \text{stable}$	$ F'(x_{eq}) > 1 \rightarrow \text{unstable}$
$x_{eq} = \frac{r-1}{r}$	$ F'(x_{eq}) < 1 \rightarrow \text{stable}$	$ F'(x_{eq}) > 1 \rightarrow \text{unstable}$