DOCUMENTATION

A The SPRT Chart with Estimated Process Parameters

A.1 Programmes for Computing the Run Length Properties of the SPRT Chart with Known Process Parameters

The following subsections explain how to use the functions in the scripts to compute the theoretical properties of the SPRT chart with known process parameters derived using the Markov chain approach (see Section 3.2), as well as to verify the accuracy of their values via Monte Carlo simulations. Note that all scripts have been written in the C++ language, and users are required to install the `Rcpp` package in order to export the appropriate functions into the R language. The use of the RStudio integrated development environment (IDE) is highly encouraged. The facility OpenMP, which supports parallel computing, should be installed automatically with the `Rcpp` package on a computer.

A.1.1 Markov Chain Approach

The functions for computing the theoretical values of the ARL, ATS, SDRL, SDTS, ASN, ANOS, AEQL, and EANOS can be found in the script "A.1.1_SPRT_known_markov_chain.cpp". Some examples of implementing the functions are given below:

(i) To compute the ATS_0 , $SDTS_0$, and ASN_0 values of the SPRT chart with charting parameters $g = 0.317$, $h = 8.388$, $\gamma = 0.306$, and $d = 0.426$, run the following codes:

```
g = 0.317; h = 8.388; gamma = 0.306; d = 0.426; delta = 0
```

```
ATS(g, h, gamma, delta, d)
SDTS(g, h, gamma, delta, d)
ASN(g, h, gamma, delta)
```

(ii) To compute the ARL_1, $SDRL_1$, and $ANOS_1$ values of the SPRT chart with charting parameters $g = 0.317$, $h = 8.388$, and $\gamma = 0.306$ at $\delta = 0.4$, run the following codes:

```
g = 0.317; h = 8.388; gamma = 0.306; delta = 0.4

ARL(g, h, gamma, delta)
SDRL(g, h, gamma, delta)
ANOS(g, h, gamma, delta)
```

(iii) To compute the AEQL value of the SPRT chart with charting parameters $g = 0.317$, $h = 8.388$, $\gamma = 0.306$, and $d = 0.426$ in the range $[\delta_{\text{min}}, \delta_{\text{max}}] = [0.1, 2.0]$, run the following codes:

```
g = 0.317; h = 8.388; gamma = 0.306; d = 0.426; deltamin = 0.1;
deltamax = 2.0

AEQL(g, h, gamma, d, deltamin, deltamax)
```

(iv) To compute the EANOS value of the SPRT chart with charting parameters $g = 0.317$, $h = 8.388$, and $\gamma = 0.306$ in the range $[\delta_{\text{min}}, \delta_{\text{max}}] = [0.1, 2.0]$, run the following codes:

```
g = 0.317; h = 8.388; gamma = 0.306; deltamin = 0.1; deltamax = 2.0

EANOS(g, h, gamma, deltamin, deltamax)
```

A.1.2 Monte Carlo Simulation

The functions for approximating the values of the ARL, ATS, SDRL, SDTS, ASN, and ANOS through Monte Carlo simulation can be found in the script "A.1.2 _SPRT_known_monte_carlo.cpp". Some examples of implementing the functions are given below:

(i) To approximate the ATS_0, $SDTS_0$, and ASN_0 values of the SPRT chart with charting parameters $g = 0.317$, $h = 8.388$, $\gamma = 0.306$, and $d = 0.426$ using 100,000 iterations, run the following codes:

```
g = 0.317; h = 8.388; gamma = 0.306; d = 0.426; delta = 0; nsim =
100000

ATS_Simulation(g, h, gamma, delta, d, nsim)
SDTS_Simulation(g, h, gamma, delta, d, nsim)
ASN_Simulation(g, h, gamma, delta, nsim)
```

(ii) To approximate the ARL_1, $SDRL_1$, and $ANOS_1$ values of the SPRT chart with charting parameters $g = -0.122$, $h = 12.427$, and $\gamma = 0.164$ at $\delta = 0.8$ using 100,000 iterations, run the following codes:

```
g = -0.122; h = 12.427; gamma = 0.164; delta = 0.8; nsim = 100000

ARL_Simulation(g, h, gamma, delta, nsim)
SDRL_Simulation(g, h, gamma, delta, nsim)
ANOS_Simulation(g, h, gamma, delta, nsim)
```

To speed up the computation process, it is possible to run the program in parallel on multiple cores (assuming that the machine contains multi-core CPU's). To retrieve the current number of cores used by the machine, run the following code:

```
getOMPThreads()
```

To set the number of cores used as four, run the following code:

```
setOMPThreads(4)
```

A.2 Programmes for Computing the Run Length Properties of the SPRT Chart with Estimated Process Parameters

The following subsections explain how to use the functions in the scripts to compute the theoretical properties of the SPRT chart with estimated process parameters derived using the Markov chain approach (see Section 3.3), as well as to verify the accuracy of their values via Monte Carlo simulations.

A.2.1 Markov Chain Approach

The functions for computing the theoretical values of the unconditional metrics, i.e., AASN, AARL, AATS, AANOS, SDARL, SDATS, SDANOS, ASDRL, ASDTS, AAEQL, and EAANOS can be found in the script "A.2.1_SPRT_estimated_markov_chain.cpp". Some examples of implementing the functions are given below:

(i) To compute the AATS_1, ASDTS_1, and SDATS_1 values of the SPRT chart with charting parameters $g = -0.034$, $h = 9.283$, $\gamma = 0.420$, and $d = 0.450$ evaluated when $\delta = 0.6$ and $m = 200$, run the following codes:

```
g = -0.034; h = 9.283; gamma = 0.420; d = 0.450; delta = 0.6; m = 
200

AATS(g, h, gamma, delta, d, m)
ASDTS(g, h, gamma, delta, d, m)
SDATS(g, h, gamma, delta, d, m)
```

(ii) To compute the AASN_0, AARL_0, ASDRL_0, and SDARL_0 values of the SPRT chart with charting parameters $g = -0.034$, $h = 9.283$, and $\gamma = 0.420$ when $m = 1000$, run the following codes:

```
g = -0.034; h = 9.283; gamma = 0.420; delta = 0; m = 1000

AASN(g, h, gamma, delta, m)
AARL(g, h, gamma, delta, m)
ASDRL(g, h, gamma, delta, m)
SDARL(g, h, gamma, delta, m)
```

(iii) To compute the AANOS_1 and SDANOS_1 values of the SPRT chart with charting parameters $g = -0.122$, $h = 12.427$, and $\gamma = 0.164$ evaluated when $\delta = 0.8$ and $m = 2000$, run the following codes:

```
g = -0.122; h = 12.427; gamma = 0.164; delta = 0.8; m = 2000
```

```
AANOS(g, h, gamma, delta, m)
SDANOS(g, h, gamma, delta, m)
```

(iv) To compute the AAEQL value of the SPRT chart with charting parameters $g = 0.317$, $h = 8.388$, $\gamma = 0.306$, and $d = 0.426$ in the range $[\delta_{\text{min}}, \delta_{\text{max}}] = [0.1, 2.0]$ when $m = 2000$, run the following codes:

```
g = 0.317; h = 8.388; gamma = 0.306; d = 0.426; m = 2000; deltamin
= 0.1; deltamax = 2.0

AAEQL(g, h, gamma, d, m, deltamin, deltamax)
```

(v) To compute the EAANOS value of the SPRT chart with charting parameters $g = 0.317$, $h = 8.388$, and $\gamma = 0.306$ in the range $[\delta_{\text{min}}, \delta_{\text{max}}] = [0.1, 2.0]$ when $m = 2000$, run the following codes:

```
g = 0.317; h = 8.388; gamma = 0.306; m = 2000; deltamin = 0.1;
deltamax = 2.0

EAANOS(g, h, gamma, m, deltamin, deltamax)
```

A.2.2 Monte Carlo Simulation

The functions for approximating the values of the AASN, AARL, AATS, AANOS, ASDRL, ASDTS, $\Pr(\text{CATS}_0 \geq \tau_{\text{ATS}})$, and $\Pr(\text{CANOS}_0 \geq \tau_{\text{ANOS}})$ through Monte Carlo simulation can be found in the script "A.2.2_SPRT_estimated_monte_carlo.cpp". Some examples of implementing the functions are given below:

(i) To approximate the AASN_0 value, as well as the AATS_1 and ASDTS_1 values of the SPRT chart with charting parameters $g = -0.034$, $h = 9.283$, $\gamma = 0.420$, and $d = 0.450$ evaluated when $\delta = 0.6$ and $m = 200$ using 100,000 iterations, run the following codes:

```
g = -0.034; h = 9.283; gamma = 0.420; d = 0.450; delta = 0.6; m =
200; nsim = 100000

AASN_Simulation (g, h, gamma, 0, m, nsim)
AATS_Simulation (g, h, gamma, delta, d, m, nsim)
ASDTS_Simulation (g, h, gamma, delta, d, m, nsim)
```

(ii) To approximate the AARL_1, ASDRL_1, and AANOS_1 values of the SPRT chart with charting parameters $g = -0.122$, $h = 12.427$, and $\gamma = 0.164$ evaluated when $\delta = 0.8$ and $m = 600$ using 100,000 iterations, run the following codes:

```
g = -0.122; h = 12.427; gamma = 0.164; delta = 0.8; m = 600; nsim
= 100000

AARL_Simulation(g, h, gamma, delta, m, nsim)
ASDRL_Simulation(g, h, gamma, delta, m, nsim)
AANOS_Simulation(g, h, gamma, delta, m, nsim)
```

(iii) To approximate the exceedance probability $\Pr(\text{CATS}_0 \geq \tau_{\text{ATS}})$ of the SPRT chart with charting parameters $g = -0.034$, $h = 9.283$, $\gamma = 0.420$, and $d = 0.450$, when $m = 200$ and $\tau_{\text{ATS}} = 370.4$, using 100,000 iterations, run the following codes:

```
g = -0.034; h = 9.283; gamma = 0.420; d = 0.450; m = 200; tau =
370.4; epsilon = 0; nsim = 100000

CATSEP(g, h, gamma, d, m, tau, epsilon, nsim)
```

(iv) To approximate the exceedance probability $\Pr(\text{CANOS}_0 \geq \tau_{\text{ANOS}})$ of the SPRT chart with charting parameters $g = -0.122$, $h = 12.427$, and $\gamma = 0.164$, when $m = 600$, $\tau_{\text{ANOS}} = 1500$, using 100,000 iterations, run the following codes:

```
g = -0.122; h = 12.427; gamma = 0.164; m = 600; tau = 1500; epsilon
= 0; nsim = 100000

CANOSEP(g, h, gamma, m, tau, epsilon, nsim)
```

A.3 Optimisation Programmes for the SPRT Chart with Known Process Parameters

This section explains how to use the functions in the script to compute the optimal charting parameters of the SPRT chart with known process parameters. The functions for computing the optimal charting parameters of the AEQL-optimal and EANOS-optimal SPRT chart with known process parameters can be found in the script "A.3_SPRT_known_optimisation.cpp".

(i) To compute the charting parameters of the AEQL-optimal SPRT chart detailed in Section 3.4, run the following codes:

```
gammamin = 0.05; gammamax = 1;
asnmin = 1.25; asnmax = 5;
tol = 0.001;
RATE = 5;
tauATS = 370.4;
deltamin = 0.1;
deltamax = 2;
dmin = 0.2;

optim_search_AEQL(gammamin, gammamax, asnmin, asnmax, tol, tol,
RATE, tauATS, deltamin, deltamax, dmin)
```

The code will return a vector of length 6, with values correspond to the optimum (AEQL, ASN_0, γ, d, g, h).

(ii) To compute the charting parameters of the EANOS-optimal SPRT chart detailed in Section 3.4, run the following codes:

```
gammamin = 0.05; gammamax = 1;
tol = 0.001;
asn = 5;
tauANOS = 1500;
deltamin = 0.1;
deltamax = 2;

optim_search_EANOS(gammamin, gammamax, tol, asn, tauANOS, deltamin,
deltamax)
```

The code will return a vector of length 4, with values correspond to the optimum (EANOS, γ, g, h).

A.4 Optimisation Programmes for the SPRT Chart with Estimated Process Parameters

The following subsections explain how to use the functions in the scripts to compute the optimal charting parameters of the SPRT chart with estimated process parameters designed under the AATS-matching approach (Section 3.5) and GICP approach (Section 3.6). In Appendix A.4.1, the optimisation programmes for the AAEQL-optimal design under the AATS-matching approach, as well as the AATS-optimal and AAEQL-optimal SPRT chart under the GICP approach, are discussed. On the other hand, in Appendix A.4.2, the optimisation programmes for the EAANOS-optimal design under the AANOS-matching approach, as well as the AANOS-optimal and EAANOS-optimal SPRT chart under the GICP approach, are discussed.

A.4.1 Optimisation Programmes for the AAEQL-optimal SPRT Chart Based on the AATS-matching Approach, as well as the AATS-optimal and AAEQL-optimal SPRT Chart Based on the GICP Approach

The functions for computing the optimal charting parameters of the AAEQL-optimal SPRT chart with estimated process parameters based on the AATS-matching approach, as well as the AATS-optimal and AAEQL-optimal SPRT chart with estimated process parameters based on the GICP approach, can be found in the script "A.4.1_SPRT_estimated_optimisation.cpp". Some examples of implementing the functions are given below:

(i) To compute the optimal charting parameters of the AAEQL-optimal SPRT chart with specifications m = 1000, IR = 5, τ_{ATS} = 370.4, $\delta_{\min}$ = 0.1, $\delta_{\max}$ = 2.0, and $d_{\min}$ = 0.2 under the AATS-matching approach, run the following codes:

```
gammamin = 0.1; gammamax = 0.5;
asnmin = 1.25; asnmax = 5;
tol = 0.001;
m = 1000;
RATE = 5;
```

```
tauATS = 370.4;
deltamin = 0.1;
deltamax = 2;
dmin = 0.2;

optim_search_ats(list(type = "AAEQL", deltamin = deltamin, deltamax
= deltamax), list(approach = "AATS-matching"), gammamin, gammamax,
asnmin, asnmax, tol, tol, m, RATE, tauATS, dmin)
```

The code will return a vector of length 6, with values correspond to the optimum (AAEQL, AASN_0, γ, d, g, h).

(ii) To compute the optimal charting parameters of the AATS-optimal SPRT chart with specifications $m = 1000$, $p_e = 0.05$, $\varepsilon = 0.2$, IR $= 5$, $\tau_{\text{ATS}} = 370.4$, $\delta_{\text{opt}} = 1.0$, and $d_{\text{min}} = 0.2$ under the GICP approach, run the following codes:

```
gammamin = 0.45; gammamax = 0.55;
asnmin = 1.5; asnmax = 15;
tol = 0.001;
m = 1000;
p = 0.05;
epsilon = 0.2;
RATE = 5;
tauATS = 370.4;
deltaopt = 1.0;
dmin = 0.2;

optim_search_ats(list(type = "AATS", delta = deltaopt),
list(approach = "GICP", p = p, epsilon = epsilon), gammamin,
gammamax, asnmin, asnmax, tol, tol, m, RATE, tauATS, dmin)
```

The code will return a vector of length 6, with values correspond to the optimum (AATS_1, AASN_0, γ, d, g, h).

(iii) To compute the optimal charting parameters of the AAEQL-optimal SPRT chart with specifications $m = 1000$, $p_e = 0.05$, $\varepsilon = 0.2$, IR $= 5$, $\tau_{\text{ATS}} = 370.4$, $\delta_{\text{min}} = 0.1$, $\delta_{\text{max}} = 2.0$, and $d_{\text{min}} = 0.2$ under the GICP approach, run the following codes:

```
gammamin = 0.1; gammamax = 0.5;
asnmin = 1.25; asnmax = 5;
tol = 0.001;
m = 1000;
p = 0.05;
epsilon = 0.2;
RATE = 5;
tauATS = 370.4;
deltamin = 0.1;
deltamax = 2.0;
dmin = 0.2;

optim_search_ats(list(type = "AAEQL", deltamin = deltamin, deltamax
= deltamax), list(approach = "GICP", p = p, epsilon = epsilon),
gammamin, gammamax, asnmin, asnmax, tol, tol, m, RATE, tauATS, dmin)
```

The code will return a vector of length 6, with values correspond to the optimum (AAEQL, AASN_0, γ, d, g, h).

NOTE: To speed up the computation process, it is possible to further restrict the search ranges of γ and $\bar{n}$ based on the results obtained from the previous sections and/or the optimality theorem stated in Sections 3.6.1 and 3.6.2. It is also possible to increase the tolerance of the golden section search algorithm `tol`.

A.4.2 Optimisation Programmes for the EAANOS-optimal SPRT Chart Based on the AANOS-matching Approach, as well as the AANOS-optimal and EAANOS-optimal SPRT Chart Based on the GICP Approach

The functions for computing the optimal charting parameters of the EAANOS-optimal SPRT chart with estimated process parameters based on the AANOS-matching approach, as well as the AANOS-optimal and EAANOS-optimal SPRT chart with estimated process parameters based on the GICP approach, can be found in the script "A.4.2_SPRT_estimated_optimisation.cpp".

(i) To compute the optimal charting parameters of the EAANOS-optimal SPRT chart with specifications $m = 1000$, $\text{AASN}_0 = 5$, $\tau_{\text{ANOS}} = 1500$, $\delta_{\min} = 0.1$, and $\delta_{\max} = 2.0$ under the AANOS-matching approach, run the following codes:

```
gammamin = 0.01; gammamax = 0.3;
tol = 0.001;
m = 1000;
aasn = 5;
tauANOS = 1500;
deltamin = 0.1;
deltamax = 2;

optim_search_anos(list(type = "EAANOS", deltamin = deltamin,
deltamax = deltamax), list(approach = "AANOS-matching"), gammamin,
gammamax, tol, aasn, m, tauANOS)
```

The code will return a vector of length 4, with values correspond to the optimum (EAANOS, γ, g, h).

(ii) To compute the optimal charting parameters of the AANOS-optimal SPRT chart with specifications m = 1000, p_e = 0.05, ε = 0.2, $AASN_0$ = 5, τ_{ANOS} = 1500, and δ_{opt} = 1.0 under the GICP approach, run the following codes:

```
gammamin = 0.45; gammamax = 0.55;
tol = 0.001;
m = 1000;
p = 0.05;
epsilon = 0.2;
aasn = 5;
tauANOS = 1500;
deltaopt = 1.0;

optim_search_anos(list(type = "AANOS", delta = deltaopt),
list(approach = "GICP", p = p, epsilon = epsilon), gammamin,
gammamax, tol, aasn, m, tauANOS)
```

The code will return a vector of length 4, with values correspond to the optimum ($AANOS_1$, γ, g, h).

(iii) To compute the optimal charting parameters of the EAANOS-optimal SPRT chart with specifications m = 1000, p_e = 0.05, ε = 0.2, $AASN_0$ = 5, τ_{ANOS} = 1500, δ_{min} = 0.1, and δ_{max} = 2.0 under the GICP approach, run the following codes:

```
gammamin = 0.05; gammamax = 0.3;
tol = 0.001;
m = 1000;
p = 0.05;
epsilon = 0.2;
aasn = 5;
tauANOS = 1500;
deltamin = 0.1;
deltamax = 2.0;

optim_search_anos(list(type = "EAANOS", deltamin = deltamin,
deltamax = deltamax), list(approach = "GICP", p = p, epsilon =
epsilon), gammamin, gammamax, tol, aasn, m, tauANOS)
```

The code will return a vector of length 4, with values correspond to the optimum (EAANOS, γ, g, h).

NOTE: To speed up the computation process, it is possible to further restrict the search ranges of γ based on the results obtained from the previous sections and/or the optimality theorem stated in Sections 3.6.3 and 3.6.4. It is also possible to increase the tolerance of the golden section search algorithm `tol`.

B The SPRT Chart for Joint Monitoring of the Mean and Dispersion

B.1 Programmes for Computing the Run Length Properties of the OSPRT Chart

The following subsections explain how to use the functions in the scripts to compute the theoretical properties of the OSPRT chart using the Markov chain approach (see Section 4.2.1), as well as to verify the accuracy of their values via Monte Carlo simulations.

B.1.1 Markov Chain Approach

The functions for computing the theoretical values of the ARL, ATS, SDRL, SDTS, ASN, ANOS, AEQL, and EANOS can be found in the script "B.1.1_ OSPRT_markov_chain.cpp".. To calculate the run-length properties using the "chi-squared distribution" method, choose 1 for the argument method; to calculate the run-length properties using the "normal distribution" method, choose 2 for the argument method. Some examples of implementing the functions are given below:

(i) To compute the ATS_0 , $SDTS_0$, and ASN_0 values of the OSPRT chart with charting parameters g = –0.662, h = 15.617, k = 0.128, γ = 1.585, and d = 0.462 using the "normal distribution" method, run the following codes:

```
g = -0.662; h = 15.617; k = 0.128; gamma = 1.585; d = 0.462; delta
= 0; eta = 1

ATS(2, g, h, k, gamma, delta, eta, d)
SDTS(2, g, h, k, gamma, delta, eta, d)
ASN(2, g, h, k, gamma, delta, eta)
```

(ii) To compute the ARL_1, $SDRL_1$, and $ANOS_1$ values of the OSPRT chart with charting parameters g = –1.416, h = 14.852, k = 0.256, and γ = 1.711 at (δ, η) = (0.4, 1.6) using the “chi-squared distribution” method, run the following codes:

```
g = -1.416; h = 14.852; k = 0.256; gamma = 1.711; delta = 0.4; eta
= 1.6

ARL(1, g, h, k, gamma, delta, eta)
SDRL(1, g, h, k, gamma, delta, eta)
ANOS(1, g, h, k, gamma, delta, eta)
```

(iii) To compute the AEQL value of the OSPRT chart with charting parameters g = –0.662, h = 15.617, k = 0.128, γ = 1.585, and d = 0.462 in the domain $[\delta_{\min}, \delta_{\max}] \times [\eta_{\min}, \eta_{\max}]$ = [0.05, 2.0] × [1.05, 2.0] using the “normal distribution” method, run the following codes:

```
g = -0.662; h = 15.617; k = 0.128; gamma = 1.585; d = 0.462; deltamin
= 0.05; deltamax = 2.0; etamin = 1.05; etamax = 2.0

AEQL(2, g, h, k, gamma, d, deltamin, deltamax, etamin, etamax)
```

(iv) To compute the EANOS value of the OSPRT chart with charting parameters g = –1.416, h = 14.852, k = 0.256, and γ = 1.711 in the domain in the domain $[\delta_{\min}, \delta_{\max}] \times [\eta_{\min}, \eta_{\max}]$ = [0.05, 3.0] × [1.05, 3.0] using the “chi-squared distribution” method, run the following codes:

```
g = -1.416; h = 14.852; k = 0.256; gamma = 1.711; deltamin = 0.05;
deltamax = 3.0; etamin = 1.05; etamax = 3.0

EANOS(1, g, h, k, gamma, deltamin, deltamax, etamin, etamax)
```

B.1.2 Monte Carlo Simulation

The functions for approximating the values of the ARL, ATS, SDRL, SDTS, ASN, and ANOS of the OSPRT chart through Monte Carlo simulation can be found in the script "B.1.2_OSPRT_markov_chain.cpp". Some examples of implementing the functions are given below:

(i) To approximate the ATS_0, $SDTS_0$, and ASN_0 values of the OSPRT chart with charting parameters $g = -0.662$, $h = 15.617$, $k = 0.128$, $\gamma = 1.585$, and $d = 0.462$ using 100,000 iterations, run the following codes:

```
g = -0.662; h = 15.617; k = 0.128; gamma = 1.585; d = 0.462; delta
= 0; eta = 1; nsim = 100000

ATS_Simulation(g, h, k, gamma, delta, eta, d, nsim)
SDTS_Simulation(g, h, k, gamma, delta, eta, d, nsim)
ASN_Simulation(g, h, k, gamma, delta, eta, nsim)
```

(ii) To approximate the ARL_1, $SDRL_1$, and $ANOS_1$ values of the OSPRT chart with charting parameters $g = -1.416$, $h = 14.852$, $k = 0.256$, and $\gamma = 1.711$ at $(\delta, \eta) = (0.4, 1.6)$ using 100,000 iterations, run the following codes:

```
g = -1.416; h = 14.852; k = 0.256; gamma = 1.711; delta = 0.4; eta
= 1.6; nsim = 100000

ARL_Simulation(g, h, k, gamma, delta, eta, nsim)
SDRL_Simulation(g, h, k, gamma, delta, eta, nsim)
ANOS_Simulation(g, h, k, gamma, delta, eta, nsim)
```

B.2 Optimisation Programmes for the OSPRT Chart

The following subsections explain how to use the functions in the scripts to compute the charting parameters of the optimal OSPRT chart. Appendix B.2.1 presents the optimisation programmes for the ATS-optimal and AEQL-optimal designs, whereas Appendix B.2.2 presents the optimisation programmes for the ANOS-optimal and EANOS-optimal designs.

B.2.1 Optimisation Programmes for the ATS-optimal and AEQL-optimal OSPRT Chart

The functions for computing the optimal charting parameters of the ATS-optimal and AEQL-optimal OSPRT chart can be found in the script "B.2.1_ OSPRT_optimisation.cpp". Some examples of implementing the functions are given below:

(i) To compute the optimal charting parameters of the ATS-optimal OSPRT chart with specifications IR = 5, $d_{\min} = 0.2$, $\tau_{\text{ATS}} = 370.4$, $\delta_{\text{opt}} = 0.2$, and $\eta_{\text{opt}} = 1.2$ using the "normal distribution" method, run the following codes:

```
asnmin = 1.3; asnmax = 5;
asntol = 0.001;
RATE = 5;
tauATS = 370.4;
delta = 0.2;
eta = 1.2;
dmin = 0.2;

optim_search_ats(list(type = "ATS", delta = delta, eta = eta),
                 list(asnmin = asnmin, asnmax = asnmax),
                 list(asntol = asntol), 2, RATE, tauATS, dmin)
```

The code will return a vector of length 7, with values correspond to the optimum (ATS_1, ASN_0, k, γ, d, g, h).

(ii) To compute the optimal charting parameters of the AEQL-optimal OSPRT chart with specifications IR = 5, $d_{\min} = 0.2$, $\tau_{\text{ATS}} = 370.4$, $(\delta_{\min}, \delta_{\max}) = (0.05, 2.0)$, and $(\eta_{\min}, \eta_{\max}) = (1.05, 2.0)$ using the "normal distribution" method, run the following codes:

```
deltamin = 0.05; deltamax = 2;
etamin = 1.05; etamax = 2;
kmin = 0.3; kmax = 0.7; ktol = 0.001;
gammamin = 1.5; gammamax = 4; gammatol = 0.001;
asnmin = 1.3; asnmax = 5; asntol = 0.001;
RATE = 5;
tauATS = 370.4;
dmin = 0.2;

optim_search_ats(list(type = "AEQL", deltamin = deltamin, deltamax
= deltamax, etamin = etamin, etamax = etamax), list(kmin = kmin,
kmax = kmax, gammamin = gammamin, gammamax = gammamax, asnmin =
asnmin, asnmax = asnmax), list(ktol = ktol, gammatol = gammatol,
asntol = asntol), 2, RATE, tauATS, dmin)
```

The code will return a vector of length 7, with values correspond to the optimum (AEQL, ASN_0, k, γ, d, g, h).

NOTE: To speed up the computation process of the AEQL-optimisation design, it is possible to restrict the search ranges of k, γ, and $\bar{n}$ based on the results obtained from the ATS-optimisation design. It is also possible to increase the tolerance of the golden section search algorithm in order to the reduce the computation time.

B.2.2 Optimisation Programmes for the ANOS-optimal and EANOS-optimal OSPRT Chart

The functions for computing the optimal charting parameters of the ANOS-optimal and EANOS-optimal OSPRT chart can be found in the script "B.2.2_ OSPRT_optimisation.cpp". Some examples of implementing the functions are given below:

(i) To compute the optimal charting parameters of the ANOS-optimal OSPRT chart with specifications $ASN_0 = 3$, $\tau_{ANOS} = 1500$, $\delta_{opt} = 0.8$, and $\eta_{opt} = 1.4$ using the "normal distribution" method, run the following codes:

```
asn = 3;
tauANOS = 1500;
delta = 0.8;
eta = 1.4

optim_search_anos(list(type = "ANOS", delta = delta, eta = eta),
                  list(), list(), 2, asn, tauANOS)
```

The code will return a vector of length 5, with values correspond to the optimum ($ANOS_1$, k, γ, g, h).

(ii) To compute the optimal charting parameters of the EANOS-optimal OSPRT chart with specifications $ASN_0 = 3$, $\tau_{ANOS} = 1500$, $(\delta_{min}, \delta_{max}) = (0.05, 3.0)$, and $(\eta_{\min}, \eta_{\max}) = (1.05, 3.0)$ using the "normal distribution" method, run the following codes:

```
deltamin = 0.05; deltamax = 3;
etamin = 1.05; etamax = 3;
kmin = 0.3; kmax = 0.7; ktol = 0.001;
gammamin = 1.5; gammamax = 4; gammatol = 0.001;
asn = 3;
tauANOS = 1500;

optim_search_anos(list(type = "EANOS", deltamin = deltamin,
deltamax = deltamax, etamin = etamin, etamax = etamax), list(kmin
= kmin, kmax = kmax, gammamin = gammamin, gammamax = gammamax),
list(ktol = ktol, gammatol = gammatol), 2, asn, tauANOS)
```

The code will return a vector of length 5, with values correspond to the optimum (EANOS, k, γ, g, h).

NOTE: To speed up the computation process of the EANOS-optimisation design, it is possible to restrict the search ranges of k and γ based on the results obtained from the ANOS-optimisation design. It is also possible to increase the tolerance of the golden section search algorithm in order to the reduce the computation time.

C The Impact of Non-normality on the Performances of the SPRT Charts

C.1 Monte Carlo Simulation Programmes for Computing the Run Length Properties of the SPRT Chart with Estimated Process Parameters under Non-normal Distributions

The functions for approximating the values of the exceedance probability $\Pr(\text{CATS}_0 \geq \tau_{\text{ATS}})$, AATS, and ASDTS values of the SPRT chart with estimated process parameters through Monte Carlo simulation can be found in the script "C.1_SPRT_estimated_skewed.cpp".

(i) To approximate the $\Pr(\text{CATS}_0 \geq 370.4)$, AATS_0, and ASDTS_0 values of the GICP-adjusted SPRT chart with charting parameters $g = 0.553$, $h = 7.297$, $\gamma = 0.363$, $d = 0.544$, and $m = 1000$ under the gamma distribution with $\alpha' = 160000$ using 100,000 iterations, run the following codes:

```
g = 0.553; h = 7.297; gamma = 0.363; d = 0.544; delta = 0; m =
1000; tau = 370.4; nsim = 100000

CATSEP_Simulation(list(distribution = "gamma", shape = 160000,
scale = 1), g, h, gamma, d, m, tau, nsim)
AATS_Simulation(list(distribution = "gamma", shape = 160000, scale
= 1), g, h, gamma, delta, d, m, nsim)
ASDTS_Simulation(list(distribution = "gamma", shape = 160000, scale
= 1), g, h, gamma, delta, d, m, nsim)
```

(ii) To approximate the AATS_1 and ASDTS_1 values of the GICP-adjusted SPRT chart with charting parameters $g = 0.553$, $h = 7.297$, $\gamma = 0.363$, $d = 0.544$, and $m = 1000$ evaluated at $\delta = 0.5$ under the lognormal distribution with $\sigma_{\text{LN}} = 0.16405$ using 100,000 iterations, run the following codes:

```
g = 0.553; h = 7.297; gamma = 0.363; d = 0.544; delta = 0; m =
1000; nsim = 100000

AATS_Simulation(list(distribution = "lognormal", mu = 0, sigma =
0.16405), g, h, gamma, delta, d, m, nsim)
```

```
ASDTS_Simulation(list(distribution = "lognormal", mu = 0, sigma =
0.16405), g, h, gamma, delta, d, m, nsim)
```

(iii) To approximate the $AATS_1$ and $ASDTS_1$ values of the GICP-adjusted SPRT chart with charting parameters $g = 0.553$, $h = 7.297$, $\gamma = 0.363$, $d = 0.544$, and $m = 1000$ evaluated at $\delta = 2.0$ under the Weibull distribution with $k' = 2.21560$ using 100,000 iterations, run the following codes:

```
g = 0.553; h = 7.297; gamma = 0.363; d = 0.544; delta = 2; m =
1000; nsim = 100000

AATS_Simulation(list(distribution = "weibull", shape = 2.21560,
scale = 1), g, h, gamma, delta, d, m, nsim)
ASDTS_Simulation(list(distribution = "weibull", shape = 2.21560,
scale = 1), g, h, gamma, delta, d, m, nsim)
```

C.2 Monte Carlo Simulation Programmes for Computing the Run Length Properties of the OSPRT Chart under Non-normal Distributions

The functions for approximating the values of the ATS and SDTS values of the OSPRT chart through Monte Carlo simulation can be found in the script "C.2_OSPRT_skewed.cpp". Some examples of implementing the functions are given below:

(i) To approximate the ATS_0 and $SDTS_0$ values of the OSPRT chart with charting parameters $g = -0.061$, $h = 12.596$, $k = 0.389$, $\gamma = 2.288$, and $d = 0.469$ under the gamma distribution with $\alpha' = 160000$ using 100,000 iterations, run the following codes:

```
g = -0.061; h = 12.596; k = 0.389; gamma = 2.288; d = 0.469; delta
= 0; eta = 1; nsim = 100000

ATS_Simulation(list(distribution = "gamma", shape = 160000, scale
= 1), g, h, k, gamma, delta, eta, d, nsim)
SDTS_Simulation(list(distribution = "gamma", shape = 160000, scale
= 1), g, h, k, gamma, delta, eta, d, nsim)
```

(ii) To approximate the ATS_1 and $SDTS_1$ values of the OSPRT chart with charting parameters $g = -0.061$, $h = 12.596$, $k = 0.389$, $\gamma = 2.288$, and $d = 0.469$ evaluated at $(\delta, \eta) = (0.5, 1.5)$ under the lognormal distribution with $\sigma_{LN} = 0.16405$ using 100,000 iterations, run the following codes:

```
g = -0.061; h = 12.596; k = 0.389; gamma = 2.288; d = 0.469; delta
= 0.5; eta = 1.5; nsim = 100000

ATS_Simulation(list(distribution = "lognormal", mu = 0, sigma =
0.16405), g, h, k, gamma, delta, eta, d, nsim)
SDTS_Simulation(list(distribution = "lognormal", mu = 0, sigma =
0.16405), g, h, k, gamma, delta, eta, d, nsim)
```

(iii) To approximate the ATS_1 and $SDTS_1$ values of the OSPRT chart with charting parameters $g = -0.061$, $h = 12.596$, $k = 0.389$, $\gamma = 2.288$, and $d = 0.469$ evaluated at $(\delta, \eta) = (2.0, 2.0)$ under the Weibull distribution with $k' = 2.21560$ using 100,000 iterations, run the following codes:

```
g = -0.061; h = 12.596; k = 0.389; gamma = 2.288; d = 0.469; delta
= 2; eta = 2; nsim = 100000

ATS_Simulation(list(distribution = "weibull", shape = 2.21560,
scale = 1), g, h, k, gamma, delta, eta, d, nsim)
SDTS_Simulation(list(distribution = "weibull", shape = 2.21560,
scale = 1), g, h, k, gamma, delta, eta, d, nsim)
```