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The electric field at a point near a uniformly charged spherical shell can be calculated by integrating over infinitesimal rings to make up the sphere. The electric field due to each ring is calculated starting from Coulomb's law $E = \frac{kQ}{r^2}$. Using the variables from the ring diagram in Figure 1, integrating over the circumference of the ring, given a uniform line charge density ρ gives $E = \frac{k2\pi\lambda R'}{r^2}$. However, due to the radial symmetry of the ring, only the z component of the electric field will remain for a point along the ring's axis so

$$E_{ring} = \frac{k2\pi\rho R'}{r^2}\cos\theta = \frac{k2\pi\rho R'}{r^2}\frac{z}{r} = \frac{k2\pi\rho R'z}{r^3}$$
.

Finally, r can be expressed in terms of z and R':

$$E_{ring} = \frac{k2\pi\rho R'z}{(z^2 + R'^2)^{3/2}} \ .$$

Figure 1: Charged ring diagram

