PHYS 512 Problem Set 5

Teophile Lemay, 281081252

Data columns: The columns are 1) multipole (starting with the l=2 quadrupole), 2) the variance of the sky at that multipole, 3) the 1σ lower uncertainty, and 4) the 1σ upper uncertainty.

Assume errors in data are Gaussian and uncorrelated.

1

Running the provided code with the initial hard-coded parameters gives $\chi^2 = 15267.937150261654$. Given 2501 degrees of freedom, the fit for the initial values has a reduced $\chi_r^2 = \frac{\chi^2}{n_{dof}} = 6.1047329669178945$. A good fit within uncertainty should produce $\chi_r^2 \approx 1$, or $\chi^2 \approx n_{dof}$. The initial guesses produce $\chi^2 \approx 6n_{dof}$ which is more than 2 times the variance $\sigma_{\chi^2}^2 = 2n_{dof}$ away from the mean so I do not believe that the initial guesses make a good fit to the data.

Adjusting the values of the parameters to

$$H_0 = 69$$
$$\Omega_b h^2 = 0.022$$
$$\Omega_c h^2 = 0.12$$
$$\tau = 0.06$$
$$A_s = 2.1e9$$
$$n_s = 0.95$$

produces a fit with $\chi^2 = 3272.2053559202204$. This gives a value of $\chi_r^2 = 1.3083587988485488$. With the new parameters, χ^2 for the fit is approximately equal to n_{dof} so the new parameters are an acceptable fit.

2

I used Newton's method with numerical derivatives to fit the parameters to the data. The gradients in parameter space were computed using central differences for each parameter. Optimal step sizes for the central difference derivative is given by

$$dp = \left(\frac{\epsilon f}{f'''}\right)^{1/3} p$$

estimated as $p \cdot \epsilon^{1/3}$. Parameter errors were calculated according to

$$\langle ()\Delta m)^2 \rangle = \left(\boldsymbol{\nabla} A^T N^{-1} \boldsymbol{\nabla} A\right)^- 1$$

where $\boldsymbol{\nabla} A$ is the gradient of the model with respect to the parameters, and N is a diagonal matrix containing the noise at each point in the data. Best fit parameters and error are saved to "planck_ fit_params.txt" with the data in the first column (ordered as $H_0, \Omega_b h^2, \Omega_c h^2, \tau, A_s, n_s$), and estimated error in the second column.

The best fit parameters from Newton's method are
6.893937484734925647e+01 1.203114364497524374e-01 2.237485308855360205e-02 2.455875613831389617e-05 1.173978076698
01 2.530294034097717803e-04 5.229301155576019516e-02 4.123714427583602751e-03 2.077948885767722189e-09 1.63708594946210
11 9.683974911565972343e-01 7.189904611656261664e-04

3

MCMC best fit parameters

4

Run new MCMC with constrained parameters