2024 Exemplos questões freq 2

1 $x(t) = 1 + (\sin(90\pi t))^2 + 6\sin(60\pi t)\sin(180\pi t)$

$6\sin(60\pi t)\sin(180\pi t) = 3\cos(120\pi t) - \cos(240\pi t)$

$\sin^2(90\pi t)$ → $\sin^2\theta$ → $\frac{1-\cos(180\pi t)}{2}$ → $\frac{180\pi}{2\pi} = 90$ Hz

↳ tem 90 Hz frequencia

↳ tem frequs 60 Hz e 120 Hz

A $f_{max} = 120$ Hz

$f_s > 2f_{max} \Leftrightarrow f_s > 240$

↳ 241 Hz

2 $f_s = 600$ Hz

$1 + \frac{1-\cos(180\pi t)}{2} + 3\cos(120\pi t) - \cos(240\pi t)$

$T_s = \frac{1}{f_s} = \frac{1}{600}$ s

$\omega_0 = \{0, 180\pi, 120\pi, 240\pi\}$

$= 60\pi$

$\Omega_0 = 60\pi \times \frac{1}{600} = \frac{\pi}{10}$

$N = \frac{f_s}{f_0} = \frac{600}{30} = 20$

$\omega_0 = 2\pi f_0 \Leftrightarrow 30 = f_0$

180	2
90	2
45	3
15	3
5	5
1	

120	2
60	2
30	2
15	3
5	5
1	

240	2
120	2
60	2
30	2
15	3
5	5
1	

$2\times2\times3\times5 = 60$

3 $X_{FT}(\omega) = \begin{cases} 0 & , \omega < -40\pi \vee \omega > 40\pi \\ \frac{(40\pi-\omega)(40\pi+\omega)}{200\pi^2} & , -40\pi < \omega < 40\pi \end{cases}$

a) $X_{DFT}[0] = 400$ $\quad \omega_s = 2\pi f_s$

$X_{DFT}[0] = f_s X_{FT}[\underset{0}{\omega}\ \Delta\omega] \Leftrightarrow 400 = f_s \times 8 \Leftrightarrow f_s = 50$ Hz

O valor da frequência considerada foi 50 Hz.

b) $T_0 = 8$ s $\quad c_0$

$c_0 = \frac{X_{FT}(0)}{T_0} = \frac{8}{8} = 1$

c) eliminar as superiores a 5 Hz $\quad f_s = 50$ Hz

$\Omega_c = \frac{\omega_c}{f_s} = \frac{2\pi f_c}{f_s} = \frac{2\pi \times 5}{50} = \frac{\pi}{5}$ rad/s

Usaríamos um filtro passa-baixo para reter as ≤ 5 Hz

④ 2 componentes – 100 Hz , 202,5 Hz $f_s = 1000$ Hz $\Delta t = 1$ s

Poderíamos usar 1 de 1000 amostras, $N = f_s \times \Delta t = 1000$ amostras
com zero padding para

$\Delta f = \frac{f_s}{N} = \frac{1000}{1000} = 1$ → logo não conseguimos identificar a 202,5Hz

$\Delta f_\omega = \frac{1000}{2000} = 0{,}5$ Hz → aumentamos a resolução para conseguirmos ver

⑤ $N = 32$ $T_s = 0{,}25$ s $f_s = \frac{1}{0{,}25} = 4$ Hz

↓

$X_{DFT}[k]$

$X_{DFT}[k] = 4\, X_{DTFT}(k\frac{\pi}{4})$ – falso $X_{DFT}[k] = \frac{1}{4} X_{DTFT}(k\frac{1}{4})$ – falso.

$X_{DFT}[k] = X_{DTFT}(k\frac{\pi}{4})$ – verdadeiro

⑥ $x[n] = 1 - 2\sin[0{,}03\pi n + \frac{\pi}{2}] + \cos[0{,}07\pi n]$

$\Omega_0 = mdc(0{,}03\pi, 0{,}07\pi)$ $\left(n - 2\cos\left(0{,}03\pi n + \frac{\pi}{2} - \frac{\pi}{2}\right)\right) = -2\cos(0{,}03\pi n)$

↓

$mdc(3\pi, 7\pi)/100 = 1/100$ $\begin{array}{r|l} 7 & 4 \\ 1 & 1 \\ 0 & \end{array}$ $\begin{array}{r|l} 3 & 3 \\ 1 & 1 \\ 0 & \end{array}$

↳ $0{,}01\pi$

$N = \frac{2\pi}{\Omega_0} = \frac{2\pi}{0{,}01\pi} = 200$

↳ período

⑦ $N = 50$ $f_s = 1$ Hz $\Omega_0 = \frac{2\pi}{50} = \frac{\pi}{25}$

$X_{DFT}[1] = -50j$ $C_1 = 2\left|\frac{-50j}{50}\right| = 2$, $\theta_1 = -\frac{\pi}{2}$

$X_{DFT}[-1] = 50j$

$X_{DFT}[50] = -100$ $C_{50} = 2\left|\frac{-100}{50}\right| = 4$, $\theta_{50} = \pi$

$X_{DFT}[-50] = -100$

$x[n] = 2\cos\left(\frac{2\pi}{25}n - \frac{\pi}{2}\right) + 4\cos\left(\frac{4\pi}{25}n + \pi\right)$ (sin – ímpar; par)

$x[n]$ não é par nem é ímpar

$k = 1$

$k = 50$ → maior $f = k\Delta f = 50 \times \frac{1}{50} = 1$ Hz ($\Delta f = \frac{1}{50}$ Hz)

↳ maior freq.

(8) $N=50$ $\quad C_1 = 2\left|\frac{-50j}{50}\right| = 2 \quad \theta_2 = -\frac{\pi}{2}$

$\Omega_0 = \frac{2\pi}{50} = \frac{\pi}{25} \quad C_5 = 2\left|\frac{-100}{50}\right| = 4 \quad \theta_5 = \pi$

$x[n] = 2\cos_2\left(\frac{2\pi}{25}n + \left(-\frac{\pi}{2}\right)\right) + 4\cos_2\left(\frac{4\pi}{25}n + \pi\right)$

(9) $f_2 = 1000\,Hz$ $\quad 2^a$ janela $\quad K=50 \quad \Delta t = 500ms \quad N = 1000\times 0{,}5 = 500$

$\Delta f = \frac{1}{0{,}5} = 2\,Hz$

$49 = \frac{f_2}{\Delta f} \Leftrightarrow 49\times 2 = f_s \Leftrightarrow f_s = 98\,Hz$

$50°$, ou seja, $k=49$

(10) $f_2 = 4000\,Hz \quad$ Ré $- 294\,Hz \quad 2\bar{a} - 440\,Hz \quad \Delta t = 100ms = 0{,}1$

a) $K = \frac{f}{\Delta f} = \frac{440}{10} = 44 \qquad \Delta f = \frac{1}{\Delta t} = \frac{1}{0{,}1} = 10\,Hz$

b) $|Erro| = |294 - (29\times 10)| = 4\,Hz$

$K_{Ré} = \frac{294}{10} \approx 29$

c) $C_5 = 2\left|\frac{-40j}{400}\right| = 0{,}2 \qquad \theta_5 = -\frac{\pi}{2} \qquad N = 4000\times 0{,}1 = 400$

$C_2 = 2\left|\frac{80}{400}\right| = 0{,}4 \qquad \theta_2 = 0 \qquad \Omega = \frac{2\pi}{400} = \frac{\pi}{200} \qquad 3N \le n < 4N$

$x_4[n] = \left(0{,}4\cos_2\left(\frac{0{,}4\,\pi}{200}n - \frac{\pi}{2}\right) + 0{,}2\cos_2\left(\frac{0{,}2\pi}{200}n\right)\right)\left(u[n-1200]\,u[n-1600]\right)$

(11) - já fiz antes não me apetece repetir (exame 2021 EN)

(12)

a) extrapolação ordem 0: $T(16) = T(12) = 14$

extrapolação linear: $m = \frac{T(12)-T(8)}{12-8} = \frac{14-10}{4} = \frac{4}{4} = 1$

$T(16) = T(12) + m\times(16-12)$
$= 14 + 1\times 4 =$
$= 18$

Interpolação linear:

$m = \frac{T(20)-T(12)}{20\times 12} = -\frac{1}{4}$

$T(16) = T(12) + m\,(16-12)$
$= 14 - \frac{1}{4}\times 4$
$= 13$

b) $a_1 = 2,0 \quad a_2 = -1,2 \quad a_3 = 0,1$

$$T(16) = 2\,T(12) - 1,2\,T(8) + 0,1\,T(4) =$$
$$= 16,8$$