Exame Normal 2022 (en materia 2ª frequencia)

$$f_{max} = m_{max} \times f_0$$

$$105 = 7 \times f_0 \Leftrightarrow$$

$$\Leftrightarrow f_0 = 15 Hz$$

(1)

c) $f_a = 300$ Hz

$$\Omega_0 = \frac{2\pi}{N} = \omega_0 T_a \qquad \omega_0 = 2\pi f_0 = 2\pi \times 15 = 30\pi \text{ rad/s}$$

$$T_a = \frac{1}{f_a} = \frac{1}{300}$$

$$\Omega_0 = 30\pi \times \frac{1}{300} = \frac{\pi}{10} \text{ rad/s}$$

d) $N=14$

$$f_{max} = 105 Hz$$

$$\Omega_0 = \frac{2\pi}{N} \Leftrightarrow \Omega_0 = \frac{\pi}{7}$$

$$\omega_{max} = 2\pi f_{max} = 210\pi$$

$$f_s > 2 f_{max} \Leftrightarrow$$

$$\Leftrightarrow f_s > 210 \text{ Hz}$$

mínimo 211

$\omega_s > 420$

$\hookrightarrow 421$

$$\frac{\pi}{7} = \omega_0 \frac{1}{f_s} \Leftrightarrow$$

$$\Leftrightarrow f_s \frac{\pi}{7} = \omega_0$$

$$211 \times \frac{\pi}{7} < 420 \longrightarrow$$

$$\left\{ f_s \frac{\pi}{7} = \omega_0 \right.$$

Apenas se a f_s for maior que 210 e ω_s maior que 420, mas substituindo pelos valores mínimos, isso não é cumprido com aquele $N=14$.

(6)

$$X_{FT}(\omega) = \begin{cases} 0 & , \omega < -20\pi \vee \omega > 20\pi \\ 2\left|\frac{\omega}{\pi}\right| & , -20\pi \leq \omega \leq 20\pi \end{cases}$$

$N = 240$ $\qquad X_{DFT}[6] = 360$

a)

$$X_{DFT}[n] = f_s \, X_{FT}\left[n \frac{\omega_s}{N}\right] \longrightarrow \Delta\omega = \frac{\omega_s}{N} = \frac{2\pi f_s}{N}$$

$$360 = f_s \times 2 \left| \frac{12\pi f_s}{240\pi} \right| \Leftrightarrow 180 = f_s \times \frac{f_s}{20} \Leftrightarrow$$

$$\Leftrightarrow 3600 = f_s^2 \Leftrightarrow$$

$$f_s = -60 \vee f_s = 60$$

Segundo o teorema da amostragem $\omega_s > 2\omega_{max} \Leftrightarrow f_s > 2 f_{max}$

$$f_{max} = \frac{20\pi}{2\pi} = 10 \text{ Hz}$$

$60 > 2 \times 10 \Leftrightarrow 60 > 20$, logo como o teorema é cumprido é possível garantir a reconstrução sem aliasing.

b) $T_0 = 2s$ $\quad C_1$ e θ_1? $\quad N = \frac{T_0}{T_s} = \frac{2}{\frac{1}{60}} = 120 \quad T_s = \frac{1}{60}$

$X_{DFT}[k] = f_s \, X_{FT}\left[n \frac{2\pi}{T_0}\right] \Leftrightarrow X_{DFT}[1] = 60 \, X_{FT}[\pi] \Leftrightarrow X_{DFT}[1] = 60 \times 2\left|\frac{\pi}{\pi}\right| = 120$

$C_m = 2\left|\frac{X_{DFT}[m]}{N}\right| \Leftrightarrow C_1 = 2\left|\frac{120}{120}\right| = 2 \qquad \theta_1 = 2\pi$

c) filtro passa-baixo $\quad w_c = 1.6\pi$ rad/s $\quad \Omega \quad f_s = 60$ Hz

$\Omega_c = \frac{w_c}{f_s} = \frac{1,6\pi}{60} = \frac{2\pi}{75} \longrightarrow 0 \leq \Omega < \frac{2\pi}{75} \qquad \Omega_0 = \frac{2\pi}{N} \Leftrightarrow$

$\Leftrightarrow \Omega_0 = \frac{2\pi}{240} = \frac{\pi}{120}$

Dá-nos o intervalo

$\Omega \in$ multiplos de $\frac{\pi}{120}$ no intervalo $\left[0, \frac{1,6\pi}{60}\right[\Rightarrow \Omega \in \left\{0, \frac{0,8\pi}{60}, \frac{\pi}{60}, \frac{1.8\pi}{60}\right\}$.

7) $f_s = 5000$ Hz

a) $\Delta t \in \left]\frac{1}{20}, 1\right[s \qquad \Delta f = \frac{1}{\Delta t} = 10$ Hz $\rightarrow$ para $\Delta t = \frac{1}{10}s \qquad K = \frac{valor}{\Delta f}$

$|erro| = |valor - (K\Delta f)|$

$k_1 = 33 \qquad k_2 = 37 \qquad k_3 = 44$ ↗ dão todos inteiros, erro nulo

b) $N = 200 \qquad 370$ Hz

$K = \frac{x}{\Delta f} \qquad \Delta f = \frac{f_s}{N} \Leftrightarrow \Delta f = \frac{5000}{200} = 25$

$K = \frac{370}{25} = \frac{74}{5} \simeq 15 \qquad$ Erro absoluto $= |370 - (15 \times 25)| =$

$= 5$ Hz

c) $\Delta f = \frac{f_s}{N} = 10$ Hz $\qquad 5^a$ janela $\qquad N = \frac{f_s}{\Delta f} = \frac{5000}{10} = 500 \qquad 4K \leq n < 5N$

$X_{DFT}[3] = 500j \qquad C_3 = 2\left|\frac{500j}{500}\right| = 2 \times 1 = 2 \quad , \theta_3 = \frac{\pi}{2} \qquad \Omega_0 = \frac{2\pi}{N} = \frac{\pi}{250}$

$X_{DFT}[-3] = 500j$

$X_{DFT}[7] = 250 \qquad C_7 = 2\left|\frac{250}{500}\right| = 2 \times \frac{1}{2} = 1 \quad , \theta_7 = 0$

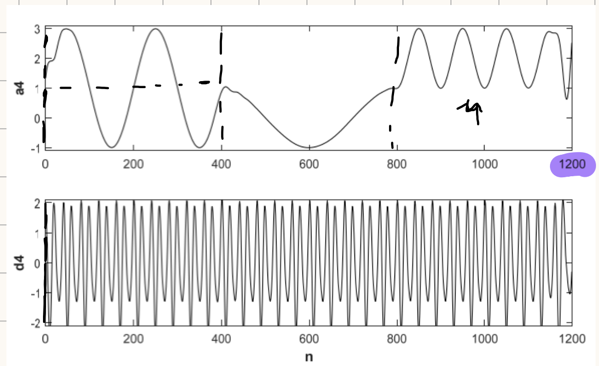
$X_{DFT}[-7] = 250$

$x_5[n] = \left(2\cos\left(\frac{2\pi}{250} n + \frac{\pi}{2}\right) + \cos\left(\frac{\pi}{250} n\right)\right)\left(u[n-2000]\, u[n-2500]\right)$

8 $f_a = 800$ Hz decomposição nível 4

n	0 – 399	400 – 799	800 -1199
A partir de **d4**:	$f \in [$ 25 , 50 $[$ Hz, $C =$ 2		
A partir de **a4**:	$f = 0$ Hz, C = 1 $f =$ 9 Hz, C = 2	$f =$ 4,5 Hz, C= 1	$f = 0$ Hz, C = 2 $f =$ 18 Hz, C = 1

$2 \times \frac{9}{2}$ $4 \times \frac{9}{2}$



$t = \frac{800}{1200} \times \frac{1}{3} = \frac{2}{9}$ tempo por intervalo

0-400

1. 0-200 200-400

2. 0-100 100-200

3. 0-50 50-100

4. 0-25 25-50

9 ?