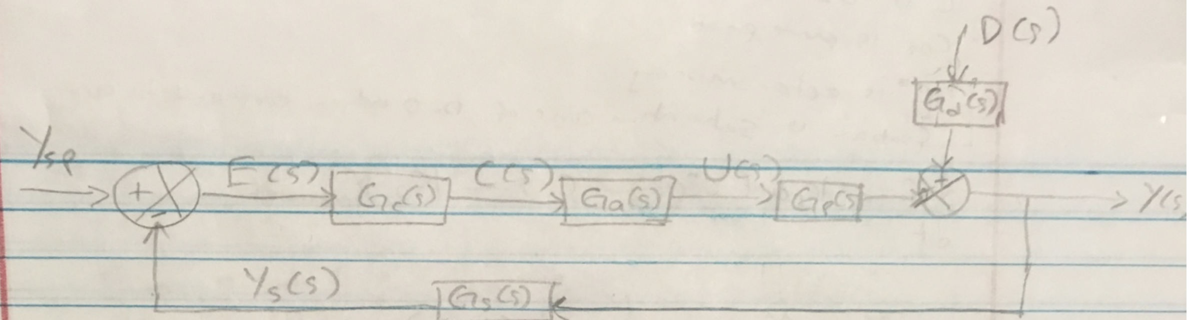
APPENDIX 2



D(s) is cell concentration X(s), $\dot V = 25$ SLPM

C(s) = Fspec = Flow rate of Oxygen

$K_a = \frac{\Delta U}{\Delta F_{spec}}$ i.e $\frac{\text{mole fraction}}{\text{SLPM}}$

$\dot V_1$ is flow rate of pure O_2 in SLPM, $\dot V_2$ is flow rate of air

Doing M.B around mixing point => $\dot V = \dot V_1 + \dot V_2$

Performing M.B on O_2 around mixing point => $U\dot V = \dot V_1 + U_{air}\dot V_2$

$U\dot V = \dot V_1 + U_{air}(\dot V - \dot V_1)$

$U\dot V = F_{spec} + U_{air}(\dot V - F_{spec})$

$U = \frac{U_{air}\dot V + (1 - U_{air})F_{spec}}{\dot V} \Rightarrow U = U_{air} + \frac{1 - U_{air}F_{spec}}{\dot V}$

Slope of the equation is actuator gain i.e Ka

When $F_{spec} = 0$ i.e no pure oxygen

$\bar U = 0.21$ i.e U_{air}

$F_{spec} = \hat F_{spec} + F_{spec} = \hat F_{spec}$

$U - U_{air} = U - \bar U = \frac{1 - U_{air}}{\dot V}\hat F_{spec} = \hat U$

Therefore $K_a = \frac{1 - U_{air}}{\dot V}$

$= \frac{0.79}{25 \text{ SLPM}}$

$U(s) = G_a F_{spec}(s)$

$G_a = K_a = \frac{1 - U_{air}}{\dot V}$

We ignore dynamics of actuator

C is OD

C_{O_2} is pure feed

C^* is after mixing

$C_{sat,air}$ is saturation conc of D.O when exposed to air

$$\frac{dC}{dt} = K_La\left(C^*(t) - C\right) - K_{O_2}\mu_{max}x$$

C^* !!

$$\frac{dx}{dt} = \mu_{max}x \quad , \quad x(t=0) = x_0$$

where C = actual dissolved oxygen content

C^* = dissolved oxygen at saturation

C_{O_2} = pure oxygen feed concentration

C^* is after mixing — $C_{sat,air}$ = saturation concentration of dissolved ox oxygen in liquid medium when exposed to air

H = Henry's constant

$C^* = uH$, U is mole fraction of O_2 in bubble

$C_{sat,air} = u_{air}H = 0.21H$

let $y = \frac{C}{C_{sat,air}} = \frac{C}{u_{air}H} = \frac{C}{0.21H}$

Substituting in dissolved oxygen equation

$$\frac{dy}{dt} = K_La\left(\frac{U}{u_{air}} - y\right) - K_{O_2}'\mu_{max}x$$

$$K_{O_2}' = K_{O_2}/u_{air}H$$

To find K_La using sterile run i.e $x = 0$ in O.D equation

$$\frac{dy}{dt} = K_La\left(\frac{U}{0.21} - y\right)$$

Putting in deviation format

$$\frac{d\bar{y}}{dt} = K_La\left(\frac{\bar{U}}{0.21} - \bar{y}\right)$$

Taking Laplace transform

$$\boxed{Y(s) = \frac{K_La}{0.21(s + K_La)}U(s)}$$

From sensor: $\frac{dC_s}{dt} = \frac{1}{\tau_s}(C - C_s)$ and $\tau_s = 10$ sec

$$\frac{d\hat{C}_s}{dt} = \frac{1}{10}(\hat{C} - \hat{C}_s)$$, divide by $C_{sat,air}$

$$\frac{d\hat{y}_s}{dt} = \frac{1}{10}(\hat{y} - \hat{y}_s)$$

$$\boxed{Y_s(s) = \frac{1}{10s+1} Y(s)}$$

Substituting value of $Y(s)$

$$Y_s(s) = \frac{1}{10s+1} \cdot \frac{K_La}{0.21(s+K_La)} U(s)$$, $U(s)$ is a constant

$$Y_s(s) = \frac{1}{10s+1} \frac{K_La}{0.21(s+K_La)} \frac{\hat{U}}{s} =$$

Let $A = \frac{K_La\hat{U}}{0.21}$

$$Y_s(s) = \frac{A}{s(10s+1)(s+K_La)} = \frac{C_0}{s} + \frac{C_1}{10s+1} + \frac{C_2}{s+K_La}$$

$$A = C_0(10s+1)(s+K_La) + C_1 s(s+K_La) + C_2 s(10s+1)$$

Let $s = 0$

$$A = C_0(K_La) \Rightarrow \boxed{C_0 = A/K_La}$$

Let $s = -1/10$

$$A = -\frac{C_1}{10}\left(\frac{-1}{10} + K_La\right) \Rightarrow C_1 = \left(\frac{100A}{1-10K_La}\right)$$

Let $s = -K_La$

$$A = -K_La C_2(-10K_La + 1) \Rightarrow C_2 = \frac{A}{10K_La^2 - K_La}$$

$$Y_s(s) = \frac{A}{K_Las} + \frac{100A}{(1-10K_La)(10s+1)} + \frac{A}{(10K_La^2 - K_La)(s+K_La)}$$

$$Y_s(s) = \frac{\hat{U}}{0.21s} + \frac{100K_La\hat{U}}{0.21(1-10K_La)(10s+1)} + \frac{\hat{U}}{0.21(10K_La-1)(s+K_La)}$$

Inverting back in time domain

$$y_s(t) = \frac{\hat{U}}{0.21} + \left[\frac{10K_La\hat{U}}{0.21(1-10K_La)}\right]e^{-t/10} + \left[\frac{\hat{U}}{0.21(10K_La-1)}\right]e^{-K_Lat}$$

$$\boxed{Y_s(t) = \frac{\hat{U}}{0.21} + \left[\frac{10K_La\hat{U}}{0.21[1-10K_La]}\right]e^{-t/10} + \left[\frac{\hat{U}}{0.21[10K_La-1]}\right]e^{-K_Lat} + \bar{Y}_s}$$

$\hat{U} = U - \bar{U}$

$\bar{U} = 0.88 \times 0.21 + 0.21 \times 1 = 0.3048$

$U = 0.76 \times 0.21 + 0.24 = 0.3996$

$\hat{U} = 0.0948$

Taking $Y_s(t=4.17)$ and $\bar{Y}_s(t=0) = 1.48$ and 1.45

$$\boxed{1.48 = \frac{0.0948}{0.21} + \left[\frac{0.948K_La}{0.21[1-10K_La}\right]e^{-\frac{4.17}{10}} + \left[\frac{0.0948}{0.21[10K_La-1]}\right]e^{-4.17K_La} + 1.45}$$

Solving with excel solver $\boxed{K_La = 0.5609421}$

Calculating μ_{max}

$$\frac{dx}{dt} = \mu_{max}\, x$$

Applying Laplace transform

$$s\,X(s) - X_0 = \mu_{max} X(s)$$

$$X(s)(s - \mu_{max}) = x_0$$

$$\boxed{X(s) = \frac{X_0}{s - \mu_{max}}}$$

Inverting back to the time domain

$$\boxed{X(t) = X_0\, e^{\mu_{max} t}}$$

Assuming H for oxygen at 37 °C =>

Calculating K_{O_2}':

Recall: $$\frac{dy}{dt} = K_L a \left(\frac{U}{U_{av}} - y\right) - K_{O_2}' \mu_{max} X$$

Putting in deviation variable format

$$\frac{d\hat{y}}{dt} = K_L a \left(\frac{\hat{U}}{U_{av}} - \hat{y}\right) - K_{O_2}' \mu_{max} \hat{X}$$

Applying Laplace transform

$$s\,Y(s) = K_L a \left(\frac{U(s)}{0.21s} - Y(s)\right) - K_{O_2}' \mu_{max} X(s)$$

$$Y(s) = \frac{K_L a\, U(s)}{0.21(s + K_L a)} - \frac{K_{O_2}' \mu_{max} X(s)}{(s + K_L a)}$$

$$X(s) = \frac{X_0}{s - \mu_{max}}$$

Substituting $X(s)$

$$Y(s) = \frac{K_L a\, U(s)}{0.21s(s + K_L a)} - \frac{K_{O_2}' \mu_{max} X_0}{(s - \mu_{max})(s + K_L a)}$$

let $$\frac{K_{O_2}' \mu_{max} x_0}{(s - \mu_{max})(s + K_L a)} = \frac{B}{(s - \mu_{max})(s + K_L a)} = \frac{C_0}{s - \mu_{max}} + \frac{C_1}{s + K_L a}$$

$$B = C_0 (s + K_L a) + C_1 (s - \mu_{max})$$

when $s = -K_L a$, $C_1 = -B / (K_L a + \mu_{max})$

when $s = \mu_{max}$, $C_0 = B / (K_L a + \mu_{max})$

$$\therefore \frac{K_{O_2}' \mu_{max} x_0}{(s - \mu_{max})(s + K_L a)} = \frac{B}{(K_L a + \mu_{max})(s - \mu_{max})} - \frac{B}{(K_L a + \mu_{max})(s + K_L a)}$$

$$Y(s) = \frac{K_L a\, U}{0.21s(s + K_L a)} - \frac{K_{O_2}' \mu_{max} x_0}{(K_L a + \mu_{max})(s - \mu_{max})} + \frac{K_{O_2}' \mu_{max} x_0}{(K_L a + \mu_{max})(s + K_L a)}$$

$$\frac{K_L a\, U}{0.21s(s + K_L a)} = \frac{C_2}{s} + \frac{C_3}{s + K_L a}$$

$$K_L a\, U / 0.21 = C_2 (s + K_L a) + C_3 s$$

when $s = 0$, $C_2 = U / 0.21$

when $s = -K_L a$, $C_3 = -U(s)/0.21$

$$\boxed{Y(s) = \frac{U}{0.21s} - \frac{U}{0.21(s + K_L a)} - \frac{K_{O_2}' \mu_{max} x_0}{(K_L a + \mu_{max})(s - \mu_{max})} + \frac{K_{O_2}' \mu_{max} x_0}{(K_L a + \mu_{max})(s + K_L a)}}$$

$H = \frac{S}{P}$

Inverting back to time domain

$$\hat{y}(t) = \frac{U(t)}{0.21}\left[1 - e^{-K_La t}\right] - \frac{K_{O_2}' \mu_{max} x_0}{(K_La + \mu_{max})}\left[e^{\mu_{max} t} - e^{-K_La t}\right] +$$

$$\boxed{y(t) = \frac{U(t)}{0.21}\left[1 - e^{-K_La t}\right] - \frac{K_{O_2}' \mu_{max} x_0}{K_La + \mu_{max}}\left[e^{\mu_{max} t} - e^{-K_La t}\right] + \bar{y}}$$

From excel sheets, using excel solver

$K_La = 0.126 s^{-1} = 7.56 \text{ min}^{-1}$

$x_0 = 0.19410D$

$\mu_{max} = 0.01875 \text{ min}^{-1}$

K_{O_2}' for 3SLPM $O_2 = 855.754 \text{ OD}^{-1}$

K_{O_2}' for 6SLPM $O_2 = 648.1895 \text{ OD}^{-1}$

} Average $K_{O_2}' = 751.9720D^{-1}$

To solve for K_{O_2}, we solve for Henry's constant of O_2 in H_2O at 37° C

$$H(T) = H_{25°C} \exp\left[\frac{d(\ln(H))}{d(1/T)}\left(\frac{1}{T} - \frac{1}{25°C}\right)\right]$$

From NIST, $H_{25°C} = 0.0012 \text{ mol kg}^{-1}\text{bar}^{-1}$

$d(\ln(H))/d(1/T) = 1800K$

$$H_{37°C} = 0.0012 \text{ mol kg}^{-1}\text{bar}^{-1} \exp\left[1800K\left(\frac{1}{310.15K} - \frac{1}{298.15K}\right)\right]$$

$H_{37°C} = 9.5003 \cdot 10^{-4} \text{ mol} \cdot L^{-1} \cdot \text{bar}^{-1}$, $1kg = 1L$

$K_H = 9.5003 \times 10^{-4} \text{ mol } L^{-1} \text{bar}^{-1} (1.013 \text{ bar})$

$\boxed{K_H = 9.6238 \times 10^{-4} \text{ mol}/L}$

$$K_{O_2}' = \frac{K_{O_2}}{U \times H}$$

$K_{O_2} = 751.972 \text{ OD}^{-1} \times 0.21 \times 9.6238 \times 10^{-4} \text{ mol}/L$

$= 0.15197 \text{ mol } L^{-1} \text{ OD}^{-1}$

Proportional Band for 1st run = 30%, $\tau_I = 100s = 1.6667 min$

Proportional Band for 2nd run = 15%, $\tau_I = 100s = 1.6667 mm$

$$PB = \frac{100\%}{K_c^D} \qquad K_c^D = \frac{\Delta y_s}{\Delta C}$$

$$K_c = \frac{100\% \Delta C}{PB \Delta y_s}$$

$$\frac{dy}{dt} = K_L a \left(\frac{U}{U_{air}} - y \right) \Rightarrow \frac{dC}{dt} = K_L a \left(C^*(t) - C \right)$$

To get range of controller = 25 SLPM − 0 = 25 SLPM

To get range of $y_{s,max} \Rightarrow 0 = \frac{1}{0.21} - y_{max} = 4.7619$

$$y_{s,min} \Rightarrow 0 = \frac{1}{1} - y_{min}$$

$$y_{min} = 1$$

$$\Delta y_s = 4.7619 - 1 = 3.7619$$

For 1st run: $K_c = \frac{100 \times 25}{30 \times 3.7619}$ SLPM

$$\boxed{K_c = 22.1519 \text{ SLPM}}$$

For 2nd run: $K_c = \frac{100 \times 25}{15 \times 3.7619}$

$$\boxed{= 44.3038 \text{ SLPM}}$$

$$Y(s) = \frac{K_L a}{0.21(s + K_L a)} U(s) - \frac{K_{O_2}' M_{max} X(s)}{(s + K_L a)}$$

Dividing all through by $K_L a$

$$Y(s) = \frac{1/0.21}{1/K_L a \, s + 1} U(s) - \frac{K_{O_2}' M_{max} / K_L a \, X(s)}{1/K_L a \, s + 1}$$

$K_p = 1/0.21 = 4.761905$ $\qquad K_d = -1.865010 D^{-1}$

$\tau_p = \frac{1}{K_L a} = 0.132275 min$ $\qquad \tau_d = 4.761905$

$$G_a(s) = \frac{U(s)}{F_{spec}(s)} = \frac{1 - U_{aw}}{\forall} = K_a$$

$$G_s = \frac{1}{\tau_s S + 1} \qquad G_c = K_c\left(1 + \frac{1}{\tau_I S}\right)$$

Characteristic Equation With No Deadtime

$$G_a G_c G_p G_s + 1 = 0$$

$$K_a K_c\left(1 + \frac{1}{\tau_I S}\right)\left(\frac{K_p}{\tau_p S + 1}\right)\left(\frac{1}{\tau_s S + 1}\right) + 1 = 0$$

$$K_a K_c\left(\frac{\tau_I S + 1}{\tau_I S}\right)\left(\frac{K_p}{\tau_p S + 1}\right)\left(\frac{1}{\tau_s S + 1}\right) + 1 = 0$$

$$K_a K_p K_c(\tau_I S + 1) + (\tau_I S(\tau_p S + 1)(\tau_s S + 1)) = 0$$

$$K_a K_p K_c \tau_I S + K_a K_p K_c + \tau_I \tau_p \tau_s S^3 + \tau_p \tau_I S^2 + \tau_s \tau_I S^2 + \tau_I S = 0$$

$$\boxed{\tau_s \tau_I \tau_p S^3 + (\tau_p \tau_I + \tau_s \tau_I) S^2 + (1 + K_a K_p K_c)\tau_I S + K_a K_p K_c = 0}$$

$\tau_s = 0.16667$ min

$\tau_p = 0.132275$ min^{-1}

$\tau_I = 1.6667$ min

$K_a = \frac{1 - U_{aw}}{\forall} = 0.03165$ SLPM^{-1}

$K_p = \frac{1}{0.21} = 4.7619$

For the 1st run (30% PB, $K_c = 22.15195$ SLPM

$S_1 = -0.4767$

S_2 and $S_3 = -6.5416 \pm 12.1461i$ {Using Matlab}

You have one real root and two complex roots which results in instabilities.

For 2nd run (15% PB, $K_c = 44.3038$ SLPM)

$S_1 = -0.5323$

S_2 and $S_3 = -6.5138 \pm 17.2739i$ {Using Matlab}

You have one real root and two complex roots which results in instabilities

Characteristic Equation With 10sec Deadtime $= 0.16667\,\text{min}$

$$\frac{dY_s}{dt} = \frac{1}{\tau_s}\left(y(t-\theta) - Y_s\right)$$

$$\tau_s s Y_s(s) = Y(s)\, e^{-\theta s} - Y_s(s)$$

$$Y_s(s) = \frac{e^{-\theta s}}{\tau_s s + 1} Y(s)$$

$$G_s(s) = \frac{e^{-\theta_s}}{\tau_s s + 1}$$

Using padé approximation $e^{-\theta_s} \approx \dfrac{1 - \frac{1}{2}\theta_s s}{1 + \frac{1}{2}\theta_s s}$

$$G_s(s) = \frac{1 - \frac{1}{2}\theta_s s}{(1 + \frac{1}{2}\theta_s s)(\tau_s s + 1)}$$

$$K_a K_c \left(\frac{\tau_I s + 1}{\tau_I s}\right)\left(\frac{K_p}{\tau_p s + 1}\right)\left(\frac{1 - \frac{1}{2}\theta_s s}{\tau_s s + \frac{1}{2}\theta_s \tau_s s^2 + \frac{1}{2}\theta_s s + 1}\right) + 1 = 0$$

$$\underbrace{K_p K_a K_c (\tau_I s + 1)(1 - \tfrac{1}{2}\theta_s s)}_{A} + \underbrace{(\tau_I \tau_p s^2 + \tau_I s)(\tau_s s + \tfrac{1}{2}\theta_s \tau_s s^2 + \tfrac{1}{2}\theta_s s + 1)}_{B} = 0$$

$$A = K_a K_p K_c \tau_I s - \frac{K_a K_p K_c \theta_s \tau_I s^2}{2} + K_a K_p K_c - \frac{K_a K_p K_c \theta_s s}{2}$$

$$B = \tau_I \tau_p \tau_s s^3 + \frac{\tau_I \tau_p \theta_s \tau_s s^4}{2} + \frac{\tau_I \tau_p \theta_s s^3}{2} + \tau_I \tau_p s^2 + \tau_I \tau_s s^2 + \frac{\tau_I \tau_s \theta_s s^3}{2} + \frac{\tau_I \theta_s s^2}{2} + \tau_I s$$

① — $2A = 2K_a K_p K_c \tau_I s - K_a K_p K_c \theta_s \tau_I s^2 + 2K_a K_p K_c - K_a K_p K_c \theta_s s$

② — $2B = 2\tau_I \tau_p \tau_s s^3 + \tau_I \tau_p \theta_s \tau_s s^4 + \tau_I \tau_p \theta_s s^3 + 2\tau_I \tau_p s^2 + 2\tau_I \tau_s s^2 + \tau_I \tau_s \theta_s s^3 + \tau_I \theta_s s^2 + 2\tau_I s$

Adding both equations:

$$\boxed{\tau_I \tau_p \tau_s \theta_s s^4 + (2\tau_I \tau_p \tau_s + \tau_I \tau_p \theta_s + \tau_I \tau_s \theta_s) s^3 + (2\tau_I \tau_p + 2\tau_I \tau_s + \tau_I \theta_s - K_a K_p K_c \theta_s \tau_I) s^2 + (2K_a K_p K_c \tau_I - K_a K_p K_c \theta_s + 2\tau_I) s + (2K_a K_p K_c)}$$

For 1st run (30% PB, $K_c = 22.1519\,\text{SLPM}$)

$S_1 = -26.5716$
$S_2 = -0.4846$ } Matlab

S_3 and $S_4 = 0.7493 \pm 9.1629i$

You get positive complex roots that results in instabilities

For 2nd run (15% PB, K_c = 44.3038 SLPM)

S_1 = -32.2539

S_2 = -0.5376

S_3 and S_4 = 3.6170 ± 10.6040i

Matlab

This results in instabilities as you have positive complex poles in characteristics equation.

Looking at the controlled run data, the lag can be calculated to tell if instabilities is from τ_I or K_c

Controlled Run 1:

Lag = $C_{trough} - Y_{s,peak}$

= 208.15 - 206.63 = 1.52 min

1/4 Period = 1/4 (210.65 min - 208.65 min)

= 0.5 min

Lag >> 1/4 Period so τ_I is too small.

Controlled Run 2:

Lag = 173.14 - 172.07

= 1.07 min

1/4 Period = 1/4 (174.92 - 173.40)

= 0.38 min

Lag >> 1/4 Period