DEPARTMENT OF CHEMICAL & BIOMOLECULAR ENGINEERING
NORTH CAROLINA STATE UNIVERSITY

CHE 225 **Summer I 2017**
Homework Set 4 **Due in class on June 7, 2017**

Use Matlab to solve numerically the following problems. On each problem, compare the Matlab results with the corresponding numerical results of the analytical solutions by plotting both sets of results on the same figure. Submit your Matlab files to CHE225ncsu@gmail.com.

Problem 1 (18%) A cylindrical tank with an inside radius R_o (= 1.0 m) is initially filled with water to a height H_o (= 1.0 m). At time t = 0, a drain opening at the bottom of the pool is opened and water is drained from the pool at a volumetric rate v_{out} given by (a, 10%) $v_{out} = \alpha H$ [α = 157.08 L/(min.m)], and (b, 10%) $v_{out} = -\alpha' H^{1/2}$ [α' = 100 L/(min.m$^{1/2}$)], where H is the height of the water level in pool at time t, and α and α' are constants with the appropriate units. For each of the two cases, solve the differential equation governing $H(t)$ over the 0−50 minute interval.

Problem 2 (18%) A reservoir which is used to collect water runoff from a hill side is a cylindrical structure with a radius R (= 10 m) that is large compared to the axial dimension z. One day, when the water level in the reservoir is at h_o (= 0.25 m), it starts to rain heavily and water is collected by the reservoir at a steady volumetric rate of $v_{in} = v_o$ (= 10 L/s). The water level in the reservoir rises steadily until it reaches h_1 (= 0.50 m), whereupon a gate at the bottom of the reservoir is opened and water is drained from the reservoir to a treatment station at a volumetric rate given by $v_{out} = -\alpha h$, where α is a known constant and h is the height of the water level in the reservoir at time t. Solve the following differential equations that govern h,

$$\frac{dh}{dt} = \frac{v_o}{\pi R^2} \quad \text{for} \quad 0 \le t \le t_1 \qquad\qquad \frac{dh}{dt} = \frac{v_o - \alpha h}{\pi R^2} \quad \text{for} \quad t \ge t_1$$

over the 0-250 minute interval for (i) α = 1,000 L/(m.min) and (ii) α = 1,500 L/(m.min).

Problem 3 (15%) At time zero, an aqueous stream containing a substance B at a molar concentration of C_{Bo} (= 1.0 M = 1.0 mol/L) is fed at a constant volumetric rate of v_o (= 15 L/min) to a large mixing tank. Simultaneously, an outlet stream is withdrawn from the tank at a constant volumetric rate of v' (= 9 L/min). The tank initially contains a volume V_o (= 50 L) of an aqueous solution with an initial concentration C_B^* (= 0.5 M) of B. As a result of a vigorous mixing provided by a stirrer, the outlet stream leaving the tank has the same concentration of B as the solution within the tank at any time t.

Solve the following differential equations that apply to V and C_B over the 0-10 minute interval:

$$\frac{dV}{dt} = v_o - v' \quad \text{and} \quad \frac{dN_B}{dt} = \frac{d(VC_B)}{dt} = v_o C_{B_o} - v' C_B$$

Problem 4 (18%) A liquid-phase reactor is operated isothermally on a semi-batch mode with an influent feed stream and **no** effluent stream. The feed stream has a steady volumetric flow rate v_o (= 10 L/min) and contains a reactant B at a constant molar concentration C_{Bo} (= 2.0 M) Upon entering the reactor, it is mixed instantaneously with the reactor solution by a mechanical stirrer, and reactant B undergoes a first-order reaction with a rate constant k (= 0.5/min). Given that the reactor solution initially has a volume V^* (= 20 l) and contains B at the molar concentration C_B^*

(= 0.10 m), solve the differential equations governing the reactor solution volume (V) and mole number of reactant B (N_B) and determine the molar concentration of B in the reactor solution (C_B) over the time interval 0−50 min . Plot, on a single figure, V, N_B, and C_B, along with their analytical counterparts (V_a, N_{Ba}, and C_{Ba}) as a function of reaction time t.

Problem 5 (16%) The hydrolytic decomposition of aqueous urea (denoted by A) to ammonia and carbon dioxide, $H_2NCONH_{2\,(aq)} + H_2O_{(aq)} \rightarrow 2NH_{3(aq)} + CO_{2(g)}$, is catalyzed by the enzyme urease (denoted by E). Given that the rate of the reaction in a batch reactor at 25°C and 1 atm is given by the rate expression, $-r_A \equiv -\dfrac{dC_A}{dt} = \dfrac{kKC_E^{\,o}C_A}{1+KC_A}$, where k, the reaction rate constant, is 0.0133 mol/(g.min), K, the binding constant of urea to urease, is 5.081 $M^{-1}$and $C_E{}^{o}$, the total urease concentration (i.e., the sum of the concentration of free urease (C_E) and the concentration of urease bound to the substrate ($C_{A.E}$)) is 5.0 g/L. Solve the differential equation and plot C_A versus t over a 30-minute time period for C_{Ao} = 1.0 M.

Problem 6 (15%) An insulated water tank receives and discharges water at a mass flow rate $\dot{m}$ (= 5 kg/min). Initially, it contains a mass M^* (= 25 kg) of water at temperature T^* (= 30°C). The temperatures of the inlet and outlet streams are T_o (= 20°C) and T, respectively, where T is a function of time. The water in the tank is well mixed by stirrer that has a power rating $\dot{W}_S$ (= 2.0 kcal/min). The tank has a heating coil that has a heat-transfer surface area A [A is specified in conjunction with U as UA = 2.0 kcal/(min.°C)]. At time 0, condensing steam is passed through the coil to provide heating. The heat transfer rate between the coil and the water in the tank is given by $Q = UA(T_s - T)$, where U is the overall heat transfer coefficient, A and T are defined previously, and T_s (= 200°C) is the temperature of the pressurized condensing steam. Solve the differential equation governing $T(t)$ over the 0-20 minute period.