Inserisco la matrice del sistema tempo discreto

In[ ]:= $A = \left\{\left\{\frac{1}{8}, \frac{1}{8}\right\}, \left\{-\frac{13}{8}, \frac{3}{8}\right\}\right\}$

Out[ ]=

$$\left\{\left\{\frac{1}{8}, \frac{1}{8}\right\}, \left\{-\frac{13}{8}, \frac{3}{8}\right\}\right\}$$

calcolo il polinomio caratteristico di A e i suoi autovalori

In[ ]:= `CharacteristicPolynomial[A, λ]`

Out[ ]=

$$\frac{1}{4} - \frac{\lambda}{2} + \lambda^2$$

In[ ]:= `λ = Eigenvalues[A]`

Out[ ]=

$$\left\{\frac{1}{4}\left(1 + i\sqrt{3}\right), \frac{1}{4}\left(1 - i\sqrt{3}\right)\right\}$$

Mi calcolo modulo e argomento del primo autovalore (l'ho scelto io)

In[ ]:= `ρ = Abs[λ[[1]]]`

Out[ ]=

$$\frac{1}{2}$$

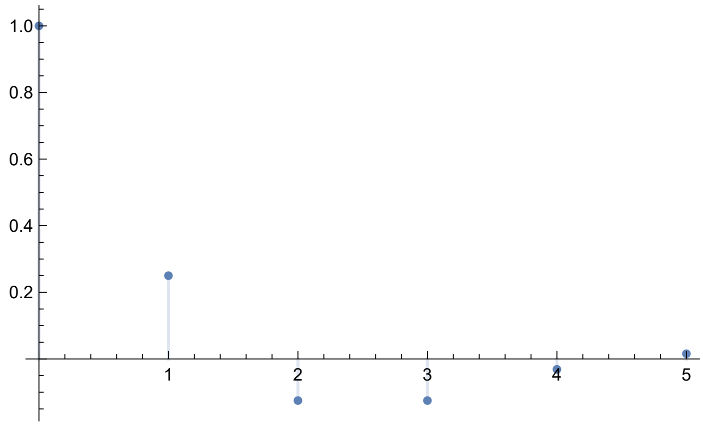
In[ ]:= `θ = Arg[λ[[1]]]`

Out[ ]=

$$\frac{\pi}{3}$$

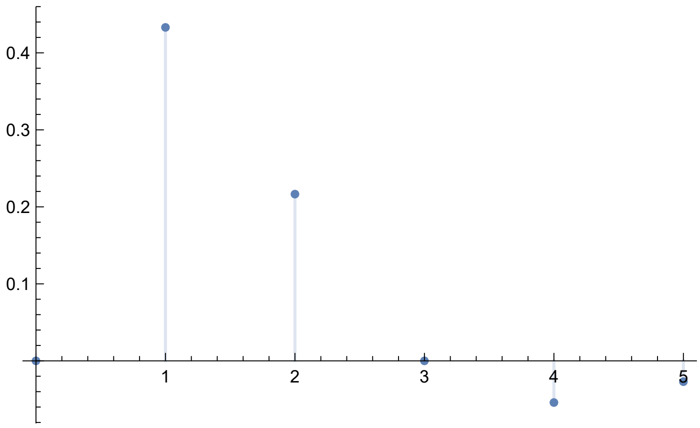
In[ ]:= `DiscretePlot[`ρ^k` Cos[θ k], {k, 0, 5}]`

Out[ ]=



In[∘]:=
```
DiscretePlot[ρ^k Sin[θ k], {k, 0, 5}]
```

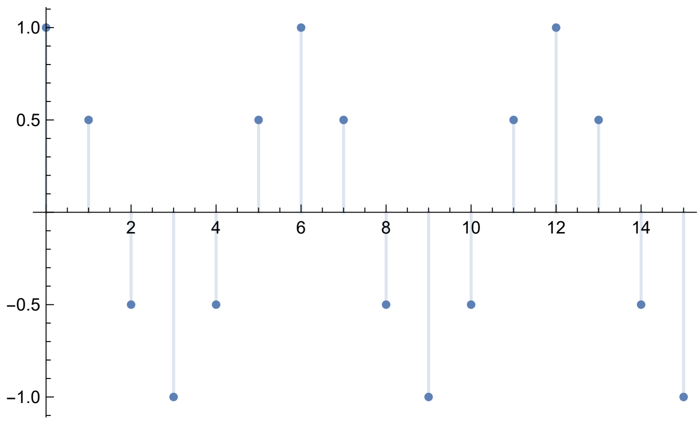
Out[∘]=



In[∘]:=

In[∘]:=
```
DiscretePlot[Cos[θ k], {k, 0, 15}]
```

Out[∘]=



Calcolo della matrice di cambiamento di base “reale” a partire dagli autovettori complessi e coniugati di A

In[∘]:=
```
T = Simplify[Transpose[Eigenvectors[A]]]
```

Out[∘]=

$$\left\{\left\{\frac{1}{13}\left(1-2\,i\,\sqrt{3}\right),\ \frac{1}{13}\left(1+2\,i\,\sqrt{3}\right)\right\},\ \{1,\ 1\}\right\}$$

In[∘]:=
```
T // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{1}{13}\left(1-2\,i\,\sqrt{3}\right) & \frac{1}{13}\left(1+2\,i\,\sqrt{3}\right) \\ 1 & 1 \end{pmatrix}$$

In[∘]:=
```
λ
```

Out[∘]=

$$\left\{\frac{1}{4}\left(1+i\,\sqrt{3}\right),\ \frac{1}{4}\left(1-i\,\sqrt{3}\right)\right\}$$

In[∘]:= `Re[T[[All, 1]]]`

Out[∘]=

$$\left\{\frac{1}{13}, 1\right\}$$

In[∘]:= `Im[T[[All, 1]]]`

Out[∘]=

$$\left\{-\frac{2\sqrt{3}}{13}, 0\right\}$$

In[∘]:= `T̂ = Transpose[{Re[T[[All, 1]]], Im[T[[All, 1]]]}]`

Out[∘]=

$$\left\{\left\{\frac{1}{13}, -\frac{2\sqrt{3}}{13}\right\}, \{1, 0\}\right\}$$

In[∘]:= `T̂ // MatrixForm`

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{1}{13} & -\frac{2\sqrt{3}}{13} \\ 1 & 0 \end{pmatrix}$$

In[∘]:= `T // MatrixForm`

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{1}{13}\left(1 - 2\,i\sqrt{3}\right) & \frac{1}{13}\left(1 + 2\,i\sqrt{3}\right) \\ 1 & 1 \end{pmatrix}$$

Una volta individuata la matrice di cambiamento di base mi calcolo la forma canonica Rotation-Scaling di A

In[∘]:= `Λ̂ = Simplify[Inverse[T̂].A.T̂]`

Out[∘]=

$$\left\{\left\{\frac{1}{4}, \frac{\sqrt{3}}{4}\right\}, \left\{-\frac{\sqrt{3}}{4}, \frac{1}{4}\right\}\right\}$$

In[∘]:= `Λ̂ // MatrixForm`

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{1}{4} & \frac{\sqrt{3}}{4} \\ -\frac{\sqrt{3}}{4} & \frac{1}{4} \end{pmatrix}$$

Inserisco lo stato iniziale e lo proietto lungo le colonne della matrice di cambiamento di base

In[∘]:= `x0 = {{-1}, {1}}`

Out[∘]=

$$\{\{-1\}, \{1\}\}$$

In[∘]:= `z0 = Simplify[Inverse[T̂].x0]`

Out[∘]=

$$\left\{\{1\}, \left\{\frac{7}{\sqrt{3}}\right\}\right\}$$

Scrivo ora la risposta libera sfruttando la forma canonica Rotation-Scaling

In[∘]:=
```
x[k_] := Simplify[T̂.( ρ^k Cos[θ k]    ρ^k Sin[θ k]
                     -ρ^k Sin[θ k]   ρ^k Cos[θ k] ).z0]
```

In[∘]:=
```
x[k]
```

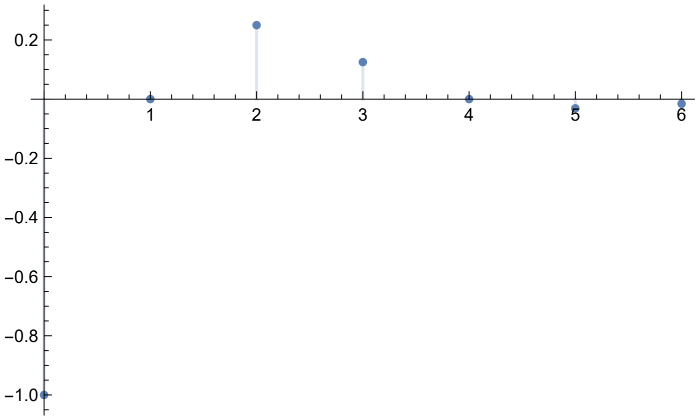
Out[∘]=

$$\left\{\left\{\frac{1}{3}\times 2^{-k}\left(-3\,\mathrm{Cos}\left[\frac{k\,\pi}{3}\right]+\sqrt{3}\,\mathrm{Sin}\left[\frac{k\,\pi}{3}\right]\right)\right\},\left\{\frac{1}{3}\times 2^{-k}\left(3\,\mathrm{Cos}\left[\frac{k\,\pi}{3}\right]+7\,\sqrt{3}\,\mathrm{Sin}\left[\frac{k\,\pi}{3}\right]\right)\right\}\right\}$$

Voglio rappresentare graficamente, ad esempio, la prima componente della risposta libera

In[∘]:=
```
DiscretePlot[x[k]⟦1⟧, {k, 0, 6}, PlotRange → All]
```

Out[∘]=



Inserisco la matrice di uscita C e mi calcolo la risposta libera (nell'uscita)

In[∘]:=
```
C1 = {1, -3}
```

Out[∘]=

```
{1, -3}
```

In[∘]:=
```
y[k_] := C1.x[k]
```

In[∘]:=
```
y[k]
```

Out[∘]=

$$\left\{\frac{1}{3}\times 2^{-k}\left(-3\,\mathrm{Cos}\left[\frac{k\,\pi}{3}\right]+\sqrt{3}\,\mathrm{Sin}\left[\frac{k\,\pi}{3}\right]\right)-2^{-k}\left(3\,\mathrm{Cos}\left[\frac{k\,\pi}{3}\right]+7\,\sqrt{3}\,\mathrm{Sin}\left[\frac{k\,\pi}{3}\right]\right)\right\}$$

Rappresento graficamente la risposta libera

In[ ]:= `DiscretePlot[y[k], {k, 0, 8}, PlotRange → All]`

Out[ ]=