Calcolo della risposta al gradino unitario per un sistema LTI-TC

```
In[•]:= ClearAll["Global`*"]
```

Inserisco la terna A, B, C

```
In[•]:= A = {{0, 1, 0}, {0, 0, 1}, {-21, -31, -11}}; B = {{0}, {0}, {1}}; C1 = {{1, 2, 0}};
```

Calcolo la FdT del sistema

```
In[•]:= G[s_] := Simplify[C1.Inverse[s IdentityMatrix[3] - A].B]
```

```
In[•]:= G[s]
```

Out[•]=

$$\left\{\left\{\frac{1+2\,s}{21+31\,s+11\,s^2+s^3}\right\}\right\}$$

```
In[•]:= Solve[Denominator[G[s][[1, 1]]] == 0, s]
```

Out[•]=

$$\{\{s\to -7\},\{s\to -3\},\{s\to -1\}\}$$

```
In[•]:= Factor[G[s]]
```

Out[•]=

$$\left\{\left\{\frac{1+2\,s}{(1+s)\,(3+s)\,(7+s)}\right\}\right\}$$

La risposta al Gradino unitario e' pari a $\frac{G(s)}{s}$

```
In[•]:= Factor[G[s][[1, 1]] × LaplaceTransform[UnitStep[t], t, s]]
```

Out[•]=

$$\frac{1+2\,s}{s\,(1+s)\,(3+s)\,(7+s)}$$

```
In[•]:= Y[s_] := G[s][[1, 1]] × LaplaceTransform[UnitStep[t], t, s]
```

In[•]:= $C_1 = \lim_{s\to 0} s\,Y[s]$

Out[•]=

$$\frac{1}{21}$$

In[•]:= $C_2 = \lim_{s\to -3} (s+3)\,Y[s]$

Out[•]=

$$-\frac{5}{24}$$

In[ ]:= $C_3 = \lim_{s \to -7} (s+7)\, Y[s]$

Out[ ]=

$$\frac{13}{168}$$

In[ ]:= $C_4 = \lim_{s \to -1} (s+1)\, Y[s]$

Out[ ]=

$$\frac{1}{12}$$

In[ ]:= `Apart[Y[s]]`

Out[ ]=

$$\frac{1}{21\,s} + \frac{1}{12\,(1+s)} - \frac{5}{24\,(3+s)} + \frac{13}{168\,(7+s)}$$

In[ ]:= $y_f[t\_] := \text{Expand}\left[\text{InverseLaplaceTransform}\left[\frac{G[s][[1,1]]}{s}, s, t\right]\right]$

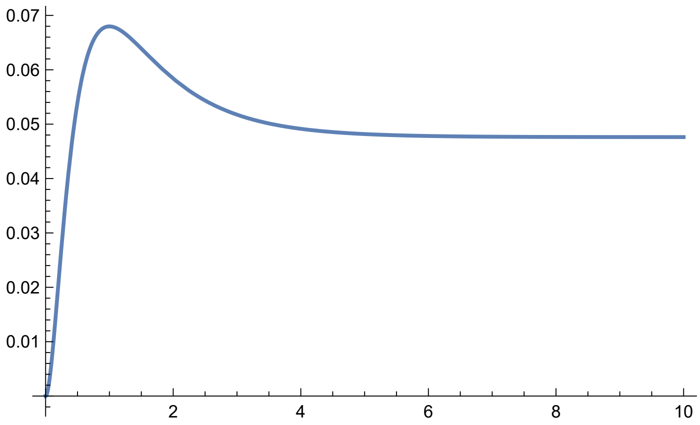
In[ ]:= $y_f[t]$

Out[ ]=

$$\frac{1}{21} + \frac{13\,e^{-7t}}{168} - \frac{5\,e^{-3t}}{24} + \frac{e^{-t}}{12}$$

In[ ]:= $\text{Plot}[y_f[t], \{t, 0, 10\}, \text{PlotRange} \to \text{All}]$

Out[ ]=



In[ ]:= $y_f[t]$

Out[ ]=

$$\frac{1}{21} + \frac{13\,e^{-7t}}{168} - \frac{5\,e^{-3t}}{24} + \frac{e^{-t}}{12}$$

In[ ]:= $y_f[t]\ /.\ \{t \to 0\}$

Out[ ]=

0

In[ ]:= `LaplaceTransform[Sin[t] × UnitStep[t], t, s]`

Out[ ]=

$$\frac{1}{1+s^2}$$

Calcolo la risposta forzata al segnale periodico elementare sin(t) 1(t)

In[∘]:= $Y[s\_] := G[s][[1,1]] \left(\frac{1}{s^2+1}\right)$

In[∘]:= Y[s]

Out[∘]=

$$\frac{1+2\,s}{\left(1+s^2\right)\,\left(21+31\,s+11\,s^2+s^3\right)}$$

In[∘]:= Factor[Y[s]]

Out[∘]=

$$\frac{1+2\,s}{(1+s)\,(3+s)\,(7+s)\,\left(1+s^2\right)}$$

In[∘]:= Apart[Y[s]]

Out[∘]=

$$-\frac{1}{24\,(1+s)}+\frac{1}{16\,(3+s)}-\frac{13}{1200\,(7+s)}+\frac{7-s}{100\,\left(1+s^2\right)}$$

Calcolo i coefficienti dell'espansione in fratti semplici

In[∘]:= $\frac{D_1}{s+i}+\frac{D_2}{s-i}+\frac{D_3}{s+1}+\frac{D_4}{s+3}+\frac{D_5}{s+7}$

Out[∘]=

$$-\frac{\frac{1}{200}+\frac{7\,i}{200}}{-i+s}-\frac{\frac{1}{200}-\frac{7\,i}{200}}{i+s}-\frac{1}{24\,(1+s)}+\frac{1}{16\,(3+s)}-\frac{13}{1200\,(7+s)}$$

In[∘]:= $D_1 = \lim_{s\to -i}\,(s+i)\,Y[s]$

Out[∘]=

$$-\frac{1}{200}+\frac{7\,i}{200}$$

In[∘]:= $D_2 = \lim_{s\to i}\,(s-i)\,Y[s]$

Out[∘]=

$$-\frac{1}{200}-\frac{7\,i}{200}$$

In[∘]:= $D_3 = \lim_{s\to -1}\,(s+1)\,Y[s]$

Out[∘]=

$$-\frac{1}{24}$$

In[∘]:= $D_4 = \lim_{s\to -3}\,(s+3)\,Y[s]$

Out[∘]=

$$\frac{1}{16}$$

In[∘]:=
```
D5 = Limit[(s + 7) Y[s], s -> -7]
```
$$D_5 = \lim_{s\to -7} (s+7)\, Y[s]$$

Out[∘]=
$$-\frac{13}{1200}$$

In[∘]:=
$$\frac{D_1}{s+i}+\frac{D_2}{s-i}+\frac{D_3}{s+1}+\frac{D_4}{s+3}+\frac{D_5}{s+7}$$

Out[∘]=
$$-\frac{\frac{1}{200}+\frac{7 i}{200}}{-i+s}-\frac{\frac{1}{200}-\frac{7 i}{200}}{i+s}-\frac{1}{24\,(1+s)}+\frac{1}{16\,(3+s)}-\frac{13}{1200\,(7+s)}$$

Voglio evidenziare la risposta a regime, la formula e' questa 2 Re $(D \exp(j\,t))$

In[∘]:=
```
ComplexExpand[2 Re[D2 Exp[I t]]]
```
$$\text{ComplexExpand}[2\,\text{Re}[D_2\,\text{Exp}[I\,t]]]$$

Out[∘]=
$$-\frac{\text{Cos}[t]}{100}+\frac{7\,\text{Sin}[t]}{100}$$

Come trasformare la combinazione lineare di due segnali periodici elementari nella forma amplitude-phase $X \sin(\omega\, t + \theta)$

In[∘]:=
$$\text{Solve}\left[\left\{\left(X\,\text{Sin}[t+\theta] == -\frac{\text{Cos}[t]}{100}+\frac{7\,\text{Sin}[t]}{100}\right)\text{ /. }\{t\to 0\},\right.\right.$$
$$\left.\left.\left(D[X\,\text{Sin}[t+\theta], t] == D\left[-\frac{\text{Cos}[t]}{100}+\frac{7\,\text{Sin}[t]}{100}, t\right]\right)\text{ /. }\{t\to 0\},\ X>0\right\},\ \{X,\theta\}\right]$$

Out[∘]=
$$\left\{\left\{X\to \boxed{\frac{1}{10\sqrt{2}}\ \text{if}\ c_1\in\mathbb{Z}},\ \theta\to \boxed{2\,\text{ArcTan}\left[\frac{-10+7\sqrt{2}}{\sqrt{2}}\right]+2\,\pi\,c_1\ \text{if}\ c_1\in\mathbb{Z}}\right\}\right\}$$