Analisi della risposta libera, caso TD, autovalori multipli

In[∘]:=
```
ClearAll["Global`*"]
```

In[∘]:=
```
A = {{-16 / 45, 38 / 45, 7 / 45, 7 / 45}, {8 / 45, -1 / 45, -8 / 45, -8 / 45},
   {1, -1, -1 / 5, 0}, {-68 / 45, 67 / 45, 23 / 45, 14 / 45}}
```

Out[∘]=

$$\left\{\left\{-\frac{16}{45}, \frac{38}{45}, \frac{7}{45}, \frac{7}{45}\right\}, \left\{\frac{8}{45}, -\frac{1}{45}, -\frac{8}{45}, -\frac{8}{45}\right\}, \left\{1, -1, -\frac{1}{5}, 0\right\}, \left\{-\frac{68}{45}, \frac{67}{45}, \frac{23}{45}, \frac{14}{45}\right\}\right\}$$

In[∘]:=
```
A // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{16}{45} & \frac{38}{45} & \frac{7}{45} & \frac{7}{45} \\ \frac{8}{45} & -\frac{1}{45} & -\frac{8}{45} & -\frac{8}{45} \\ 1 & -1 & -\frac{1}{5} & 0 \\ -\frac{68}{45} & \frac{67}{45} & \frac{23}{45} & \frac{14}{45} \end{pmatrix}$$

Mi calcolo lo spettro di A

In[∘]:=
```
Eigenvalues[A]
```

Out[∘]=

$$\left\{\frac{1}{3}, -\frac{1}{5}, -\frac{1}{5}, -\frac{1}{5}\right\}$$

In[∘]:=
```
MatrixMinimalPolynomial[a_List?MatrixQ, x_] :=
 Module[{i, n = 1, qu = {}, mnm = {Flatten[IdentityMatrix[Length[a]]]}},
  While[Length[qu] == 0, AppendTo[mnm, Flatten[MatrixPower[a, n]]];
   qu = NullSpace[Transpose[mnm]];
   n++];
  First[qu].Table[x^i, {i, 0, n - 1}]]
```

In[∘]:=
```
Factor[MatrixMinimalPolynomial[A, x]]
```

Out[∘]=

$$\frac{1}{375}\,(-1 + 3\,x)\,(1 + 5\,x)^3$$

In[∘]:=
```
Factor[CharacteristicPolynomial[A, x]]
```

Out[∘]=

$$\frac{1}{375}\,(-1 + 3\,x)\,(1 + 5\,x)^3$$

Mi calcolo intanto la forma di Jordan di A

In[ ]:= `{T, Λ} = JordanDecomposition[A]`

Out[ ]=

$$\left\{\{\{0, -1, -2, -1\}, \{0, 0, -1, -1\}, \{-1, -1, -3, 0\}, \{1, 0, 0, 1\}\}, \left\{\left\{-\frac{1}{5}, 1, 0, 0\right\}, \left\{0, -\frac{1}{5}, 1, 0\right\}, \left\{0, 0, -\frac{1}{5}, 0\right\}, \left\{0, 0, 0, \frac{1}{3}\right\}\right\}\right\}$$

In[ ]:= `Λ // MatrixForm`

Out[ ]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{5} & 1 & 0 & 0 \\ 0 & -\frac{1}{5} & 1 & 0 \\ 0 & 0 & -\frac{1}{5} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \end{pmatrix}$$

In[ ]:= `T // MatrixForm`

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & -1 & -2 & -1 \\ 0 & 0 & -1 & -1 \\ -1 & -1 & -3 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

In[ ]:= `MatrixPower[Λ, k] // MatrixForm`

Out[ ]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{1}{5}\right)^k & -(-1)^k 5^{1-k} k & \frac{1}{2} (-1)^k 5^{2-k} (-1+k) k & 0 \\ 0 & \left(-\frac{1}{5}\right)^k & -(-1)^k 5^{1-k} k & 0 \\ 0 & 0 & \left(-\frac{1}{5}\right)^k & 0 \\ 0 & 0 & 0 & 3^{-k} \end{pmatrix}$$

In[ ]:= x_0 `= {{1}, {0}, {0}, {1}}`

Out[ ]=

`{{1}, {0}, {0}, {1}}`

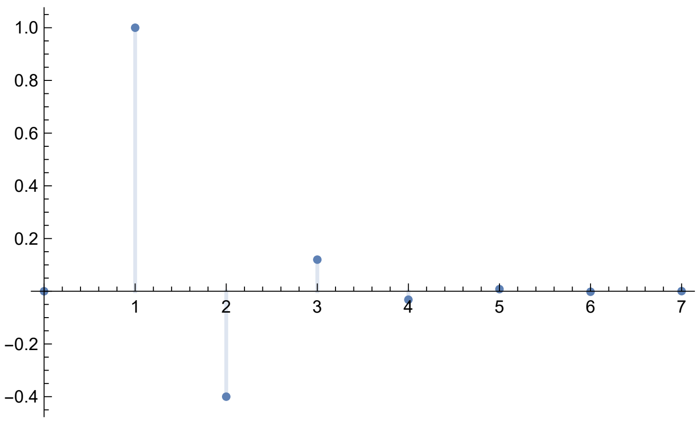
In[ ]:= z_0 `= Inverse[T].`x_0

Out[ ]=

`{{1}, {-1}, {0}, {0}}`

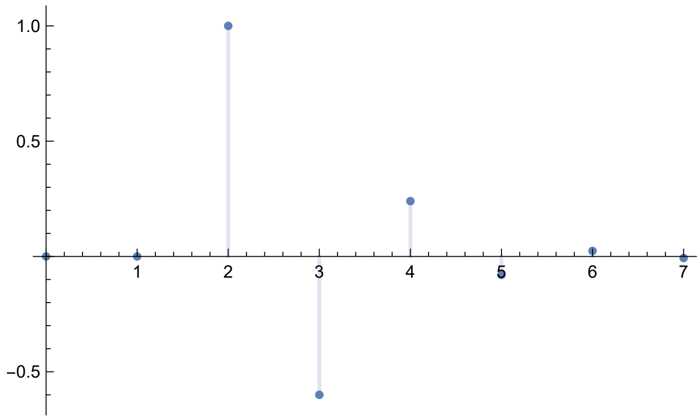
In[ ]:= `DiscretePlot[Binomial[k, 1]` $\left(-\frac{1}{5}\right)^{k-1}$ `, {k, 0, 7}, PlotRange → All]`

Out[ ]=



In[ ]:=
```
DiscretePlot[Binomial[k, 2] (-1/5)^(k-2), {k, 0, 7}, PlotRange → All]
```

Out[ ]=



Calcolo la risposta libera

In[ ]:=
```
C1 = {1, 0, 1, 0}
```

Out[ ]=
```
{1, 0, 1, 0}
```

In[ ]:= $x_1[k\_] := \text{Simplify}[T.\text{MatrixPower}[\Lambda, k].z_0]$

In[ ]:= $y_1[k\_] := \text{Simplify}[C1.x_1[k]]$

In[ ]:= $x_1[k]$ // MatrixForm

Out[ ]//MatrixForm=

$$\begin{pmatrix} \left(-\frac{1}{5}\right)^k \\ 0 \\ -(-1)^k\, 5^{1-k}\, k \\ \left(-\frac{1}{5}\right)^k (1+5k) \end{pmatrix}$$

In[ ]:= z_0

Out[ ]=
```
{{1}, {-1}, {0}, {0}}
```