RISPOSTA = RISPOSTA LIBERA + RISPOSTA FORZATA

= RISPOSTA LIBERA + RISPOSTA TRANSITORIA +

RISPOSTA A REGIME

$$A = \begin{bmatrix} 0 & 1/2 & 0 & 1/2 \\ -10 & -11 & 14 & 9 \\ -10 & -11 & 14 & 10 \\ 8 & 9 & -12 & -9 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

DETERMINARE x_0 (O C.I.) T.C.

LA RISPOSTA AL GRADINO COINCIDA

ESATTAMENTE CON IL SUO VALORE

DI REGIME_

$$Y(s) = C(sI-A)^{-1}x_0 + G(s)U(s)$$

$$U(s) = \frac{1}{s}$$

$$Y(s) = G(s)\cdot\frac{1}{s} = \frac{n_G(s)}{s(s-p_1)(s-p_2)\dots(s-p_n)} =$$

$$= \frac{C_1}{s} + \sum_{i=1}^{n}\frac{D_i}{s-p_i}$$

$$C_1 = \lim_{s\to 0} s\cdot Y(s) = \lim_{s\to 0} \cancel{s}\cdot G(s)\cdot\frac{1}{\cancel{s}} =$$

$$= G(0)$$

$$Y_{ss}(s) = \frac{G(0)}{s}$$

$$\underbrace{Y(s) - Y_{ss}(s)}$$

CASO ID

$$A = \begin{bmatrix} \frac{11}{12} & -1 & \frac{31}{12} \\ 1 & -1 & 1 \\ \frac{1}{12} & 0 & -\frac{7}{12} \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 3/2 & -1 \end{bmatrix}$$

$$Y(z) = C\, z\, (zI - A)^{-1} x_0 + G(z)\, U(z)$$

$$Y_F(z) = G(z) \cdot \frac{z}{z-1}$$

$$\frac{Y_F(z)}{z} = G(z) \frac{1}{z-1} =$$

$$\frac{Y_F(z)}{z} = \frac{C_1}{z-1} + \sum_{i=1}^{n} \frac{D_i}{z - p_i}$$

$$C_1 = \lim_{z \to 1} (z-1) \frac{Y_f(z)}{z} =$$

$$= \lim_{z \to 1} (\cancel{z-1}) \frac{1}{\cancel{z}} G(z) \frac{\cancel{z}}{\cancel{z-1}} = G(1)$$

$$G(1) \frac{z}{z-1}$$

ASINTOTICA STABILITÀ $\Rightarrow$ BIBO STABILITÀ

TEST SU AUTOVALORI DI A — TEST SUI POLI DI G

QUANTI SONO GLI EQUILIBRI PER UN SISTEMA LTI-TC ?

$$(x_{eq}, u_{eq})$$

$$A x_{eq} + B u_{eq} = 0_x$$

1. $u_{eq} = 0_u$

1a SICURAMENTE $(0_x, 0_u)$

$$A x_{eq} = 0_x$$

1b SE $\det(A) \neq 0$ NON VE NE SONO ALTRI

SE $\det(A) = 0$ VE NE SONO INFINITI

$$A x_{eq} = 0_x$$

$$x_{eq} \in \ker(A)$$

2. $u_{eq} \neq 0$

$$A x_{eq} + B u_{eq} = 0_x$$

$$A x_{eq} = - B u_{eq}$$

2a) $DET(A) \neq 0$

$$x_{eq} = -A^{-1} B u_{eq}$$

$DET(A) = 0$

$$A x_{eq} = -B u_{eq}$$

SE $B u_{eq} \in Im(A)$ $(B \subseteq Im(A))$

ALLORA ESISTONO INFINITI

EQUILIBRI x_{eq}

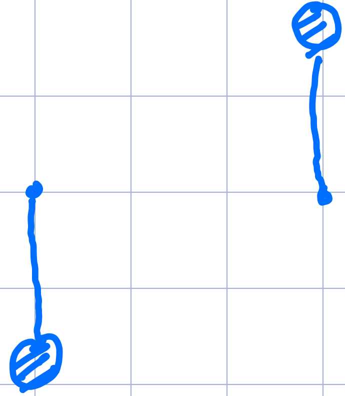
SE $B u_{eq} \notin Im(A)$ $(B \not\subseteq Im(A))$

NO EQUILIBRI.

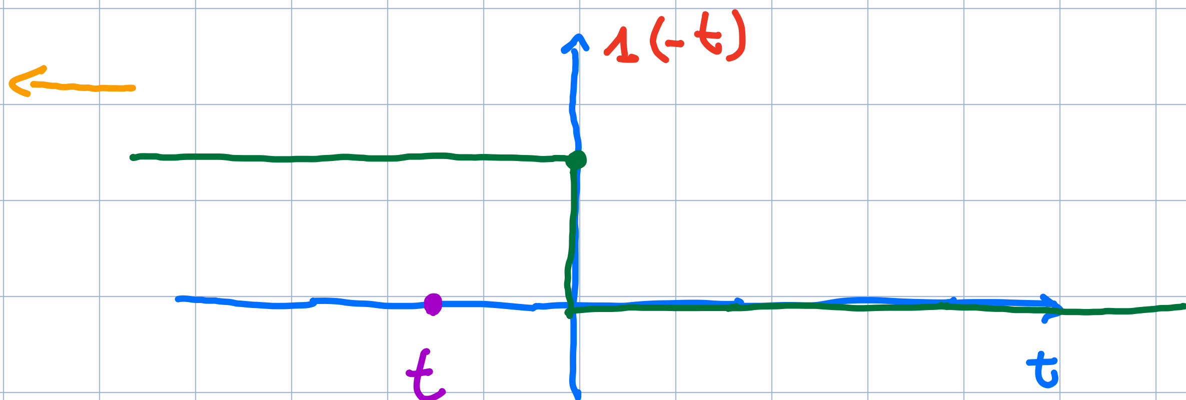
— • —

$$A x_{eq} + B u_{eq} = x_{eq}$$

$$(A - I) x_{eq} + B u_{eq} = 0_x$$



DETERMINARE LA RISPOSTA DEL SISTEMA ALL' INGRESSO $1(-t)$



$$y(t) = \begin{cases} y_{ss}(t) & t < 0 \\ y_{\text{"libera"}}(t) & t \geq 0 \end{cases}$$

$$y_{ss}(0^-) = y_{\text{"libera"}}(0^+)$$

$$\dot{y}_{ss}(0^-) = \dot{y}_{\text{"libera"}}(0^+)$$

$$\vdots$$

$$y_{ss}^{(n-1)}(0^-) = y_{\text{"libera"}}^{(n-1)}(0^+)$$

$$y_e(0) = G(0) = C x_0$$

$$\dot{y}_e(0) = 0 = CA x_0$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$y^{(n-1)}(0) = 0 = CA^{n-1} x_0$$

$$\Theta = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} \qquad x_0 = \Theta^{-1} \cdot \begin{bmatrix} G(0) \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$y_{\text{"libera"}}(t) = \mathcal{L}^{-1}\left[C(sI - A)^{-1} x_0\right]$$