Calcolo della risposta forzata per Sistemi LTI-TD

In[∘]:= $G[z\_] := \dfrac{z-2}{z^3 + \frac{3}{2} z^2 + \frac{3}{4} z + \frac{1}{8}}$

Calcolo i poli del sistema

In[∘]:= `Solve[Denominator[G[z]] == 0, z]`

Out[∘]=

$$\left\{\left\{z \to -\frac{1}{2}\right\}, \left\{z \to -\frac{1}{2}\right\}, \left\{z \to -\frac{1}{2}\right\}\right\}$$

Scrivo la risposta forzata in z

In[∘]:= $Y_f[z\_] := G[z] \left(\dfrac{z}{z-1}\right)$

In[∘]:= $Y_f[z]$

Out[∘]=

$$\frac{(-2+z)\, z}{(-1+z)\left(\frac{1}{8} + \frac{3z}{4} + \frac{3z^2}{2} + z^3\right)}$$

In[∘]:= $\text{Factor}\left[\dfrac{Y_f[z]}{z}\right]$

Out[∘]=

$$\frac{8\,(-2+z)}{(-1+z)\,(1+2z)^3}$$

In[∘]:= $C_1 \left(\dfrac{1}{z-1}\right) + C_{21} \left(\dfrac{1}{z+\frac{1}{2}}\right) + C_{22} \left(\dfrac{1}{\left(z+\frac{1}{2}\right)^2}\right) + C_{23} \left(\dfrac{1}{\left(z+\frac{1}{2}\right)^3}\right)$

Out[∘]=

$$\frac{C_1}{-1+z} + \frac{C_{21}}{\frac{1}{2}+z} + \frac{C_{22}}{\left(\frac{1}{2}+z\right)^2} + \frac{C_{23}}{\left(\frac{1}{2}+z\right)^3}$$

In[∘]:= $C_1 = \lim_{z\to 1} (z-1) \left(\dfrac{Y_f[z]}{z}\right)$

Out[∘]=

$$-\frac{8}{27}$$

In[∘]:= `G[1]`

Out[∘]=

$$-\frac{8}{27}$$

In[∘]:= $C_{23} = \lim_{z\to\frac{-1}{2}} \left(z+\frac{1}{2}\right)^3 \left(\frac{Y_f[z]}{z}\right)$

Out[∘]=

$$\frac{5}{3}$$

In[∘]:= $C_{22} = \lim_{z\to\frac{-1}{2}} D\left[\left(z+\frac{1}{2}\right)^3 \left(\frac{Y_f[z]}{z}\right), z\right]$

Out[∘]=

$$\frac{4}{9}$$

In[∘]:= $C_{21} = \left(\frac{1}{2}\right) \lim_{z\to\frac{-1}{2}} D\left[D\left[\left(z+\frac{1}{2}\right)^3 \left(\frac{Y_f[z]}{z}\right), z\right], z\right]$

Out[∘]=

$$\frac{8}{27}$$

In[∘]:= $C_1 \left(\frac{1}{z-1}\right) + C_{21}\left(\frac{1}{z+\frac{1}{2}}\right) + C_{22}\left(\frac{1}{\left(z+\frac{1}{2}\right)^2}\right) + C_{23}\left(\frac{1}{\left(z+\frac{1}{2}\right)^3}\right)$

Out[∘]=

$$-\frac{8}{27\,(-1+z)} + \frac{5}{3\left(\frac{1}{2}+z\right)^3} + \frac{4}{9\left(\frac{1}{2}+z\right)^2} + \frac{8}{27\left(\frac{1}{2}+z\right)}$$

Per “sistemare” i fratti semplici di Yf[z] bisogna moltiplicare per z ciascun fratto semplice

In[∘]:= $C_1 \left(\frac{z}{z-1}\right) + C_{21}\left(\frac{z}{z+\frac{1}{2}}\right) + C_{22}\left(\frac{z}{\left(z+\frac{1}{2}\right)^2}\right) + C_{23}\left(\frac{z}{\left(z+\frac{1}{2}\right)^3}\right)$

Out[∘]=

$$-\frac{8\,z}{27\,(-1+z)} + \frac{5\,z}{3\left(\frac{1}{2}+z\right)^3} + \frac{4\,z}{9\left(\frac{1}{2}+z\right)^2} + \frac{8\,z}{27\left(\frac{1}{2}+z\right)}$$

Scriviamo la risposta forzata nel dominio del Tempo

In[∘]:= $y_f[k\_] := C_1\,\text{UnitStep}[k] + C_{21}\left(-\frac{1}{2}\right)^k \text{UnitStep}[k] +$
$C_{22}\,\text{Binomial}[k, 1]\left(-\frac{1}{2}\right)^{k-1} \text{UnitStep}[k] + C_{23}\,\text{Binomial}[k, 2]\left(-\frac{1}{2}\right)^{k-2} \text{UnitStep}[k]$

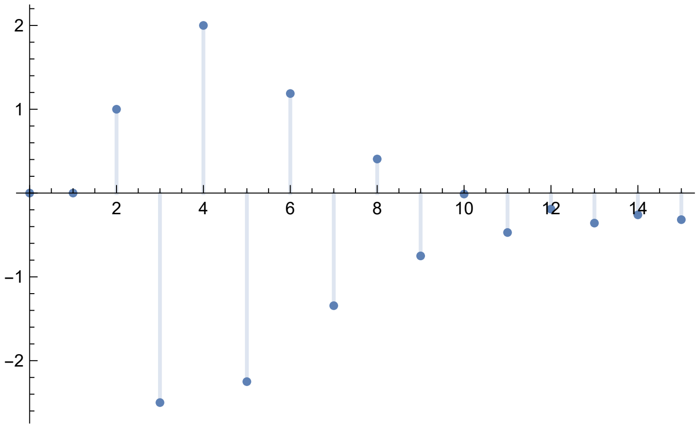
In[∘]:= $y_f[k]$

Out[∘]=

$$-\frac{8\,\text{UnitStep}[k]}{27} + \frac{1}{27}(-1)^k\,2^{3-k}\,\text{UnitStep}[k] +$$
$$\frac{1}{9}(-1)^{-1+k}\,2^{3-k}\,k\,\text{UnitStep}[k] + \frac{5}{3}(-1)^{-2+k}\,2^{1-k}\,(-1+k)\,k\,\text{UnitStep}[k]$$

In[∘]:= **DiscretePlot[y_f[k], {k, 0, 15}, PlotRange → All]**

Out[∘]=



Calcolo della risposta all'impulso che e' per definizione l'antitrasformata zeta della FdT del sistema

In[∘]:= **G[z]**

Out[∘]=

$$\frac{-2+z}{\frac{1}{8}+\frac{3z}{4}+\frac{3z^2}{2}+z^3}$$

In[∘]:= $\mathbf{Factor}\left[\frac{G[z]}{z}\right]$

Out[∘]=

$$\frac{8\,(-2+z)}{z\,(1+2z)^3}$$

In[∘]:= $\mathbf{Apart}\left[\frac{G[z]}{z}\right]$

Out[∘]=

$$-\frac{16}{z}+\frac{40}{(1+2z)^3}+\frac{32}{(1+2z)^2}+\frac{32}{1+2z}$$

In[∘]:= $D_1\left(\frac{1}{z}\right)+D_{21}\left(\frac{1}{z+\frac{1}{2}}\right)+D_{22}\left(\frac{1}{\left(z+\frac{1}{2}\right)^2}\right)+D_{23}\left(\frac{1}{\left(z+\frac{1}{2}\right)^3}\right)$

Out[∘]=

$$\frac{D_1}{z}+\frac{D_{21}}{\frac{1}{2}+z}+\frac{D_{22}}{\left(\frac{1}{2}+z\right)^2}+\frac{D_{23}}{\left(\frac{1}{2}+z\right)^3}$$

In[∘]:= $D_1 = \lim_{z\to 0} z\left(\frac{G[z]}{z}\right)$

Out[∘]=

-16

In[∘]:= $D_{23} = \lim_{z\to \frac{-1}{2}} \left(z+\frac{1}{2}\right)^3\left(\frac{G[z]}{z}\right)$

Out[∘]=

5

In[∘]:= $D_{22} = \lim_{z \to \frac{-1}{2}} D\left[\left(z + \frac{1}{2}\right)^3 \left(\frac{G[z]}{z}\right), z\right]$

Out[∘]=

8

In[∘]:= $D_{21} = \left(\frac{1}{2}\right) \lim_{z \to \frac{-1}{2}} D\left[D\left[\left(z + \frac{1}{2}\right)^3 \left(\frac{G[z]}{z}\right), z\right], z\right]$

Out[∘]=

16

Moltiplico tutto per z al fine di sistema l'antitrasformata ed identificare le successioni elementari

In[∘]:= $D_1 + D_{21}\left(\frac{z}{z + \frac{1}{2}}\right) + D_{22}\left(\frac{z}{\left(z + \frac{1}{2}\right)^2}\right) + D_{23}\left(\frac{z}{\left(z + \frac{1}{2}\right)^3}\right)$

Out[∘]=

$$-16 + \frac{5\,z}{\left(\frac{1}{2} + z\right)^3} + \frac{8\,z}{\left(\frac{1}{2} + z\right)^2} + \frac{16\,z}{\frac{1}{2} + z}$$

In[∘]:= $g[k\_] := D_1\,\text{KroneckerDelta}[k] + D_{21}\left(-\frac{1}{2}\right)^k \text{UnitStep}[k] +$
$D_{22}\,\text{Binomial}[k, 1]\left(-\frac{1}{2}\right)^{k-1} \text{UnitStep}[k] + D_{23}\,\text{Binomial}[k, 2]\left(-\frac{1}{2}\right)^{k-2} \text{UnitStep}[k]$

In[∘]:= g[k]

Out[∘]=

$$-16\,\delta_k + (-1)^k\, 2^{4-k}\,\text{UnitStep}[k] + (-1)^{-1+k}\, 2^{4-k}\, k\,\text{UnitStep}[k] + 5\,(-1)^{-2+k}\, 2^{1-k}\,(-1+k)\, k\,\text{UnitStep}[k]$$

In[∘]:= DiscretePlot[g[k], {k, 0, 20}, PlotRange → All]

Out[∘]=