Caso “ibrido” di un sistema dinamico che presenta autovalori reali e compliessi e coniugati (TC)

In[]:=
```
A = {{-9/2, 3, 1/2}, {-1, -1, 0}, {-5/2, 5, -3/2}}
```
Out[]=

$$\left\{\left\{-\frac{9}{2}, 3, \frac{1}{2}\right\}, \{-1, -1, 0\}, \left\{-\frac{5}{2}, 5, -\frac{3}{2}\right\}\right\}$$

In[]:=
```
C1 = {1, 0, 0}
```
Out[]=

$\{1, 0, 0\}$

Calcolo gli autovalori di A

In[]:=
```
λ = Eigenvalues[A]
```
Out[]=

$\{-3, -2 + i, -2 - i\}$

In[]:=
```
CharacteristicPolynomial[A, x]
```
Out[]=

$-15 - 17\,x - 7\,x^2 - x^3$

In[]:=
```
Simplify[A.A.A + 7 A.A + 17 A + 15 IdentityMatrix[3]]
```
Out[]=

$\{\{0, 0, 0\}, \{0, 0, 0\}, \{0, 0, 0\}\}$

Costruisco la matrice di cambiamento di base, inizio dalla funzione Eigenvectors di Mathematica

In[]:=
```
T0 = Transpose[Eigenvectors[A]]
```
Out[]=

$$\left\{\left\{2, \frac{2}{5} + \frac{i}{5}, \frac{2}{5} - \frac{i}{5}\right\}, \left\{1, \frac{1}{10} + \frac{3\,i}{10}, \frac{1}{10} - \frac{3\,i}{10}\right\}, \{0, 1, 1\}\right\}$$

In[]:=
```
T0 // MatrixForm
```
Out[]//MatrixForm=

$$\begin{pmatrix} 2 & \frac{2}{5} + \frac{i}{5} & \frac{2}{5} - \frac{i}{5} \\ 1 & \frac{1}{10} + \frac{3\,i}{10} & \frac{1}{10} - \frac{3\,i}{10} \\ 0 & 1 & 1 \end{pmatrix}$$

In[]:=
```
T = Transpose[{T0[[All, 1]], Re[T0[[All, 2]]], Im[T0[[All, 2]]]}]
```
Out[]=

$$\left\{\left\{2, \frac{2}{5}, \frac{1}{5}\right\}, \left\{1, \frac{1}{10}, \frac{3}{10}\right\}, \{0, 1, 0\}\right\}$$

In[]:=
```
T // MatrixForm
```
Out[]//MatrixForm=

$$\begin{pmatrix} 2 & \frac{2}{5} & \frac{1}{5} \\ 1 & \frac{1}{10} & \frac{3}{10} \\ 0 & 1 & 0 \end{pmatrix}$$

Verifichiamo che la forma canonica e’ diagonale a blocchi

In[∘]:=
```
Λ = Simplify[Inverse[T].A.T]
```
Out[∘]=
```
{{-3, 0, 0}, {0, -2, 1}, {0, -1, -2}}
```

In[∘]:=
```
Λ // MatrixForm
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} -3 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & -1 & -2 \end{pmatrix}$$

Calcolo la risposta libera nello stato a partire dallo stato iniziale

In[∘]:= x_0 = {{1}, {0}, {0}}

Out[∘]=
```
{{1}, {0}, {0}}
```

Proietto lo stato iniziale lungo le colonne della matrice “ibrida” T

In[∘]:= z_0 = Inverse[T].x_0

Out[∘]=

$$\left\{\left\{\frac{3}{4}\right\}, \{0\}, \left\{-\frac{5}{2}\right\}\right\}$$

In[∘]:= σ = Re[λ[[2]]]

Out[∘]=
```
-2
```

In[∘]:= ω = Im[λ[[2]]]

Out[∘]=
```
1
```

Calcolo la risposta libera sfruttando la decomposizione modale

In[∘]:=

$$x_1[t\_] := \text{Simplify}\left[T.\begin{pmatrix} \text{Exp}[\lambda[[1]]\,t] & 0 & 0 \\ 0 & \text{Exp}[\sigma t]\,\text{Cos}[\omega t] & \text{Exp}[\sigma t]\,\text{Sin}[\omega t] \\ 0 & \text{Exp}[\sigma t]\,\text{Sin}[\omega t] & \text{Exp}[\sigma t]\,\text{Cos}[\omega t] \end{pmatrix}.z_0\right]$$

In[∘]:= x_1[t] // MatrixForm

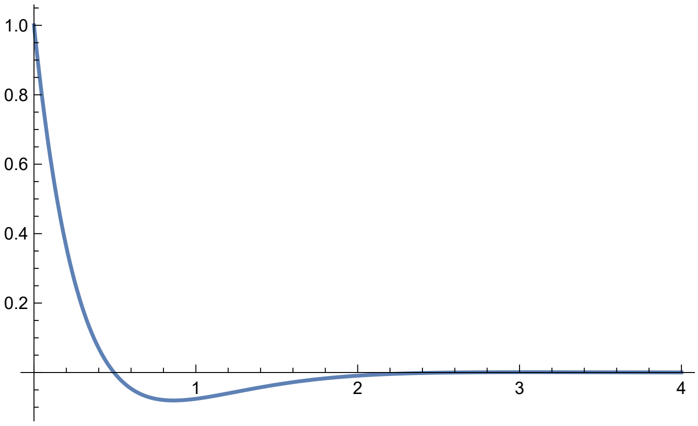
Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{2}\,e^{-3t}\left(-3 + e^{t}\,\text{Cos}[t] + 2\,e^{t}\,\text{Sin}[t]\right) \\ -\frac{1}{4}\,e^{-3t}\left(-3 + 3\,e^{t}\,\text{Cos}[t] + e^{t}\,\text{Sin}[t]\right) \\ -\frac{5}{2}\,e^{-2t}\,\text{Sin}[t] \end{pmatrix}$$

In[∘]:= y_1[t_] := C1.x_1[t]

In[ ]:= `Plot[y1[t], {t, 0, 4}, PlotRange → All]`

Out[ ]=



In[ ]:= `T // MatrixForm`

Out[ ]//MatrixForm=

$$\begin{pmatrix} 2 & \frac{2}{5} & \frac{1}{5} \\ 1 & \frac{1}{10} & \frac{3}{10} \\ 0 & 1 & 0 \end{pmatrix}$$

Analizziamo cosa accade quando “cambia” lo stato iniziale

In[ ]:= $x_0 = \{\{-4\}, \{-2\}, \{0\}\}$

Out[ ]=

$\{\{-4\}, \{-2\}, \{0\}\}$

In[ ]:= z_0 = Inverse[T].x_0

Out[ ]=

$\{\{-2\}, \{0\}, \{0\}\}$

In[ ]:= x_1[t_] := Simplify[T.MatrixExp[Λ t].z_0]

In[ ]:= x_1[t]

Out[ ]=

$\{\{-4\, e^{-3t}\}, \{-2\, e^{-3t}\}, \{0\}\}$

In[ ]:= `T // MatrixForm`

Out[ ]//MatrixForm=

$$\begin{pmatrix} 2 & \frac{2}{5} & \frac{1}{5} \\ 1 & \frac{1}{10} & \frac{3}{10} \\ 0 & 1 & 0 \end{pmatrix}$$

In[ ]:= $x_0 = \left\{\left\{\frac{1}{5}\right\}, \left\{-\frac{1}{5}\right\}, \{1\}\right\}$

Out[ ]=

$\left\{\left\{\frac{1}{5}\right\}, \left\{-\frac{1}{5}\right\}, \{1\}\right\}$

In[ ]:= z_0 = Inverse[T].x_0

Out[ ]=

$\{\{0\}, \{1\}, \{-1\}\}$

In[◦]:=
```
x1[t_] := Simplify[T.MatrixExp[Λ t].z0]
```

In[◦]:=
```
x1[t]
```

Out[◦]=

$$\left\{\left\{\frac{1}{5}\, e^{-2t}\, (\text{Cos}[t] - 3\,\text{Sin}[t])\right\},\ \left\{-\frac{1}{5}\, e^{-2t}\, (\text{Cos}[t] + 2\,\text{Sin}[t])\right\},\ \left\{e^{-2t}\, (\text{Cos}[t] - \text{Sin}[t])\right\}\right\}$$

In[◦]:=
```
g2 = Graphics3D[InfinitePlane[{0, 0, 0}, {T[[All, 2]], T[[All, 3]]}]];
```

In[◦]:=
```
g1 = ParametricPlot3D[x1[t], {t, 0, 5}, PlotRange → All, PlotStyle → Red];
```

In[◦]:=
```
Show[g1, g2]
```

Out[◦]=