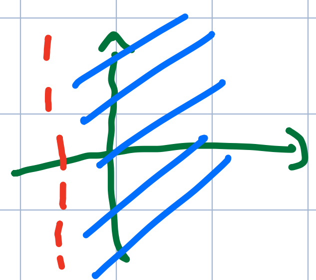
IMPULSO DI DIRAC

$$Y_f(s) = G(s)\,U(s)$$

$U(s) = 1$ CHI È LA "FUNZIONE"
LA CUI L-TRASFORMATA
È UNITARIA ?

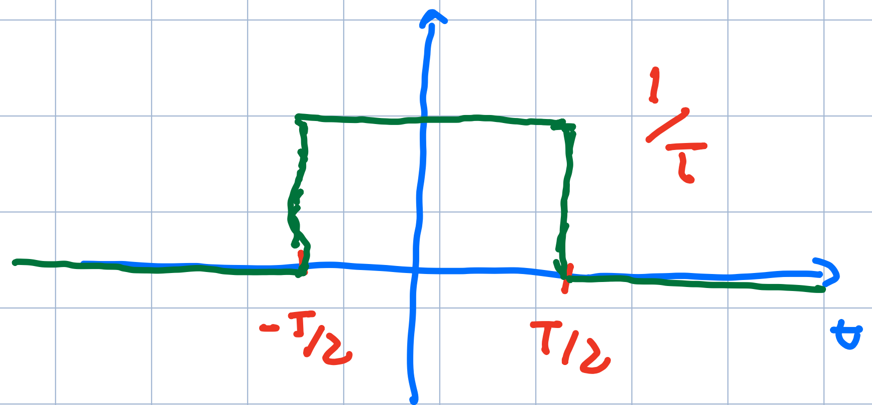
$$\frac{1}{s},\ \frac{1}{s^2},\ \frac{1}{s-a},\ \frac{1}{(s-a)^n},\ \frac{\omega}{s^2+\omega^2},\ \frac{s}{s^2+\omega^2}$$



PUNTO MATERIALE $\rightarrow$ PROPRIETÀ È
LA MASSA (FINITA)
"CONCENTRATA"
IN UN VOLUME
INFINITESIMO

$$m = \rho \cdot V$$

$$f_T(t) = \begin{cases} \frac{1}{T}, & |t| \leq \frac{T}{2} \quad \left(-\frac{T}{2} \leq t \leq \frac{T}{2}\right) \\ 0, & |t| > \frac{T}{2} \end{cases}$$

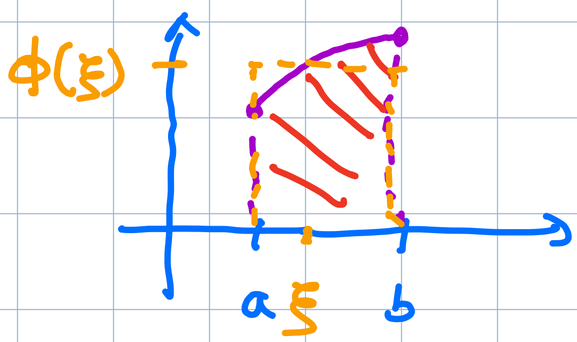


IMPULSO RETTANGOLARE DI DURATA PARI A T (BASE) E AMPIEZZA $1/T$ (ALTEZZA)

$\text{RECT}_T(t)$

$$\int_{-\infty}^{+\infty} f_T(t)\, \phi(t)\, dt = \int_{-T/2}^{T/2} \frac{1}{T} \phi(t)\, dt =$$

$$= \frac{1}{\cancel{T}} \cancel{T} \cdot \phi(\xi) \qquad -\frac{T}{2} \le \xi \le \frac{T}{2}$$



$$\int_{-\infty}^{+\infty} f_T(t)\, \phi(t)\, dt = \phi(\xi)$$

$$\lim_{T \to 0} \int_{-\infty}^{+\infty} f_T(t)\, \phi(t)\, dt = \lim_{T \to 0} \phi(\xi) = \phi(0)$$

$$\delta(t) \left(= \lim_{T \to 0} f_T(t) \right)$$

$$\int_{-\infty}^{+\infty} \delta(t)\, \phi(t)\, dt = \phi(0)$$

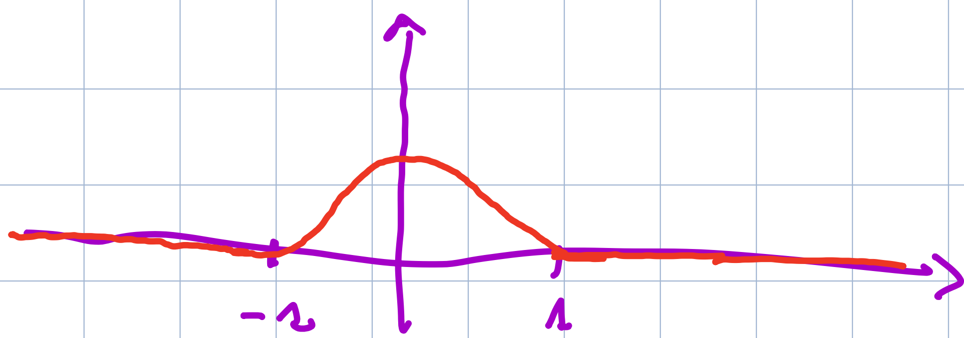
$$\int_{-\infty}^{+\infty} \delta(t)\, e^{-st}\, dt = e^{-st}\Big|_{t=0} = 1$$

$$1(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$\phi(t)$ FUNZIONE CONTINUA A SUPPORTO COMPATTO

1. DERIVABILE CON CONTINUITÀ (SMOOTH)
2. DIVERSA DA ZERO SU UN COMPATTO DI $\mathbb{R}$

$$\phi(t) = \begin{cases} 0 & |t| \geq 1 \\ e^{-\frac{1}{1-t^2}} & |t| < 1 \end{cases}$$



$\mathcal{D}$

$$\int_{-\infty}^{+\infty} 1(t)\,\phi(t)\,dt = \int_{0}^{+\infty} \phi(t)\,dt$$

$$\int_{-\infty}^{+\infty} 1'(t)\,\phi(t)\,dt = \overbrace{\cancel{1(t)\,\phi(t)\Big|_{-\infty}^{+\infty}}}^{0} -$$

$$- \int_{-\infty}^{+\infty} 1(t)\,\phi'(t)\,dt =$$

$$= - \int_{-\infty}^{+\infty} 1(t)\, \phi'(t)\, dt = - \int_{0}^{+\infty} \phi'(t)\, dt =$$

$$= - \left(\phi(+\infty) - \phi(0) \right) = \phi(0)$$

L'IMPULSO UNITARIO È LA DERIVATA DEBOLE DEL GRADINO UNITARIO.

$$G(z) = \frac{z-2}{z^3 + \frac{3}{2} z^2 + \frac{3}{4} z + \frac{1}{8}}$$

1. RISPOSTA GRADINO UNITARIO DISCRETO

$$U(z) = \frac{z}{z-1}$$

$$G(z) = \frac{z-2}{\left(z+\frac{1}{2}\right)^3}$$

$$\left(z+\frac{1}{2}\right)^3 \rightarrow \left(-\frac{1}{2}\right)^k 1(k) \quad \binom{k}{1}\left(-\frac{1}{2}\right)^{k-1} 1(k) \quad \binom{k}{2}\left(-\frac{1}{2}\right)^{k-2} 1(k)$$

$$\binom{k}{n} a^{k-n} 1(k) \doteq \frac{z}{(z-a)^{n+1}}$$

$$Y(z) = -\frac{8}{27}\frac{z}{z-1} + \frac{8}{27}\frac{z}{z+1/2} + \frac{4}{9}\frac{z}{\left(z+1/2\right)^2} + \frac{5}{3}\frac{z}{\left(z+1/2\right)^3}$$

$$y(k) = -\frac{8}{27} 1(k) + \frac{8}{27}\left(-\frac{1}{2}\right)^{k} 1(k) + \frac{4}{9}\binom{k}{1}\left(-\frac{1}{2}\right)^{k-1} 1(k)$$

$$+ \frac{5}{3}\binom{k}{2}\left(-\frac{1}{2}\right)^{k-2} 1(k)$$

RISPOSTA ALL'IMPULSO

$$Y_f(z) = G(z) \cdot U(z) =$$

$$F(z) = f(0) + f(1)\, z^{-1} + f(2)\, z^{-2} + \ldots$$

$$\delta(k) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

$$\mathcal{Z}[\delta(k)] = 1 + 0 \cdot z^{-1} + 0 \cdot z^{-2} + \ldots = 1$$

LA RISPOSTA ALL'IMPULSO È L'ANTI-TRASFORMATA ZETA DELLA FdT

$$G(z) = \frac{z-2}{\left(z+\frac{1}{2}\right)^3}$$

$$\frac{G(z)}{z} = \frac{z-2}{z\left(z+\frac{1}{2}\right)^3} = \frac{D_1}{z} + \frac{D_{21}}{z+\frac{1}{2}} + \frac{D_{22}}{\left(z+\frac{1}{2}\right)^2} +$$

$$+ \frac{D_{23}}{\left(z+\frac{1}{2}\right)^3}$$

$$G(z) = -16 + 16\,\frac{z}{z+\frac{1}{2}} + 8\,\frac{z}{\left(z+\frac{1}{2}\right)^2} + 5\,\frac{z}{\left(z+\frac{1}{2}\right)^3}$$

$$g(k) = -16\,\delta(k) + 16\left(-\frac{1}{2}\right)^k 1(k) + 8\binom{k}{1}\left(-\frac{1}{2}\right)^{k-1} 1(k)$$

$$+ 8\binom{k}{2}\left(-\frac{1}{2}\right)^{k-2} 1(k)$$

$$g(k) = y(k+1) - y(k)$$