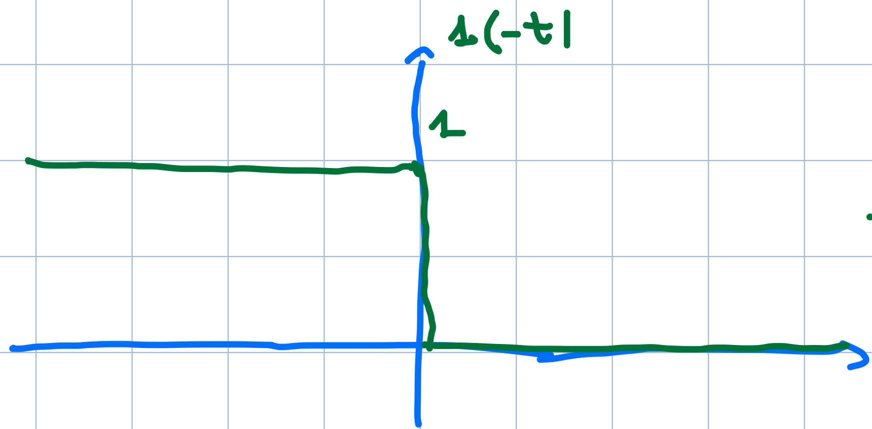
PROBLEMA DELLA DETERMINAZIONE DELLA RISPOSTA DI UN SISTEMA LTI-TC ALL'INGRESSO $1(-t)$



$y(t)$ PER $t<0 \rightarrow y_{ss}(t) = G(0)$

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases}$$

$$y(0) = Cx_0$$

$$\dot{y}(t) = C\dot{x}(t) + D\dot{u}(t) =$$
$$= C(Ax(t) + Bu(t)) + D\dot{u}(t)$$

$$\dot{y}(0) = CAx_0$$

$$\ddot{y}(t) = \dots$$

$$\ddot{y}(0) = CA^2 x_0$$

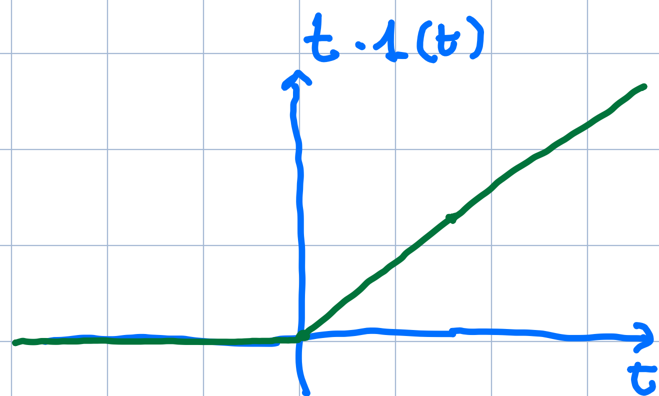
$$\begin{bmatrix} y(0) \\ \dot{y}(0) \\ \ddot{y}(0) \\ \dddot{y}(0) \end{bmatrix} = \underbrace{\begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix}}_{\Theta} x_0$$

$$x_0 = \Theta^{-1} \begin{bmatrix} y(0) \\ \dot{y}(0) \\ \ddot{y}(0) \\ \dddot{y}(0) \end{bmatrix}$$

$y(t)$, $t>0$ $\rightarrow$ $\mathcal{L}^{-1}\left[C\,(sI_n - A)^{-1}\, x_0\right]$

$$\frac{C_{11}}{s+1} + \frac{C_{12}}{(s+1)^2} + \frac{C_{21}}{(s+2)} + \frac{C_{22}}{(s+2)^2}$$

RAMPA



$$\mathcal{L}\left[t \cdot 1(t)\right] = \frac{1}{s^2}$$

TEOREMA DELLA MOLTIPLICAZIONE PER t

$$f(t) \doteq F(s)$$

$$t\, f(t) \doteq -\frac{dF}{ds}$$

$$\int_0^{+\infty} t \cdot f(t)\, e^{-st}\, dt = \int_0^{+\infty} -\frac{\partial}{\partial s}\left(f(t)\, e^{-st}\right) dt$$

$$f(t) \cdot e^{-st} \qquad\qquad t \cdot f(t) \cdot e^{-st} =$$

$$= -\frac{\partial}{\partial s}\left(f(t) \cdot e^{-st} \right)$$

$$= -\frac{d}{ds}\left(\int_0^{+\infty} f(t)\, e^{-st}\, dt \right) = -\frac{dF}{ds}$$

$$\mathcal{L}\left(t \cdot 1(t)\right) = -\frac{d}{ds}\left(\mathcal{L}\left(1(t)\right)\right) =$$

$$= -\frac{d}{ds}\left(\frac{1}{s}\right) = \frac{1}{s^2}$$

$$Y(s) = G(s) \cdot \frac{1}{s^2}$$

$$\frac{s-1}{s^2(s+1)^2(s+2)^2} = \frac{C_{11}}{s} + \frac{C_{12}}{s^2} + \frac{C_{21}}{s+1} +$$

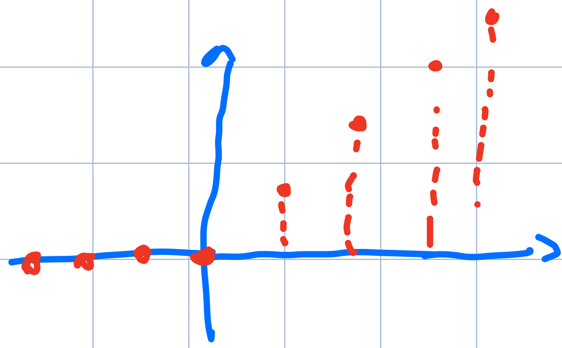
$$+ \frac{C_{22}}{(s+1)^2} + \frac{C_{31}}{(s+2)} + \frac{C_{32}}{(s+2)^2}$$

$$y(t) = C_{11}\, 1(t) + C_{12}\, t \cdot 1(t) + C_{21}\, e^{-t}\, 1(t) +$$

$$+ C_{22}\, t \cdot e^{-t}\, 1(t) + C_{31}\, e^{-2t}\, 1(t) + C_{32}\, t \cdot e^{-2t}\, 1(t).$$

RAMPA DISCRETA

$$K \cdot 1(K) = \begin{cases} K & K \geq 0 \\ 0 & K < 0 \end{cases}$$



TEOREMA DELLA MOLTIPLICAZIONE

PER k

$$f(k) \doteq F(z)$$

$$k\,f(k) \doteq -z\,\frac{dF}{dz}$$

$$1(k) \doteq \frac{z}{z-1}$$

$$k \cdot 1(k) \doteq -z\,\frac{d}{dz}\left(\frac{z}{z-1}\right) =$$

$$= -z\,\frac{(\cancel{z}-1)-\cancel{z}}{(z-1)^2} = \frac{z}{(z-1)^2}$$

$$Y(z) = G(z) \cdot \frac{z}{(z-1)^2}$$

TEOREMI ASINTOTICI

- VALORE INIZIALE
- VALORE FINALE

CT $\lim_{t \to 0^+} f(t)$ VAL. INIZ.

DT $f(0)$

CT $\lim_{t \to +\infty} f(t)$ VAL. FINALE

DT $\lim_{k \to +\infty} f(k)$

TEOREMA DEL VALORE INIZIALE

SIA $f(t)$ DI CLASSE $\mathcal{L}$

$$\lim_{t \to 0^+} f(t) = \lim_{s \to \infty} s F(s)$$

SIA $f(k)$ Z-TRASFORMABILE

$$f(0) = \lim_{z \to \infty} F(z)$$

$$F(z) = f(0) + \frac{f(1)}{z} + \frac{f(2)}{z^2} + \dots$$

TEOREMA DEL VALORE FINALE

SIA $f(t)$ CONTINUA E DI CLASSE $\mathcal{L}$

E

$$f_\infty = \lim_{t \to +\infty} f(t)$$

ESISTA. ALLORA

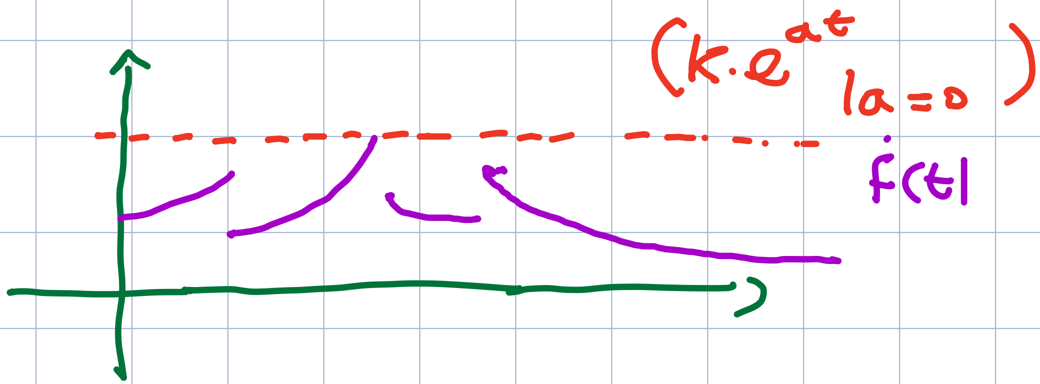
$$f_\infty = \lim_{s \to 0} s F(s)$$

SE $f(t)$ È CONTINUA E DI CLASSE $\mathcal{L}$ ALLORA $\dot{f}(t)$ È DI CLASSE $\mathcal{L}$

$$\int_0^{+\infty} \dot{f}(t)\, e^{-st}\, dt = s F(s) - f(0)$$

$$\lim_{s \to 0} \int_0^{+\infty} \dot{f}(t)\, e^{-st}\, dt = \lim_{s \to 0} s F(s) - f(0)$$

$$\lim_{t \to +\infty} \dot{f}(t) = 0$$



$$\int_0^{+\infty} \dot{f}(t) \lim_{s \to 0} e^{-st} \, dt = \lim_{s \to 0} sF(s) - f(0)$$

$$\int_0^{+\infty} \dot{f}(t) \, dt = \lim_{s \to 0} sF(s) - f(0)$$

$$f(t) \Big|_0^{+\infty} = \lim_{s \to 0} sF(s) - f(0)$$

$$f_\infty - \cancel{f(0)} = \lim_{s \to 0} sF(s) - \cancel{f(0)}$$

SIA $f(k)$ Z-TRASFORMABILE E

SUPPONIAMO CHE

$$f_\infty = \lim_{k \to \infty} f(k)$$

ESISTA

$$f_\infty = \lim_{z \to 1} (z-1)\, F(z)$$

— o —

PREMESSA : SISTEMA BIBO STABILE

OPERAZIONE : CALCOLO DELLA

RISPOSTA FORZATA AD UN GRADINO DI

AMPIEZZA U

$$U(s) = \frac{U}{s}$$

$$Y(s) = G(s)\frac{U}{s}$$

$$y_\infty = \lim_{t \to +\infty} y(t) = Y = \lim_{s \to 0} sY(s) =$$

$$= \lim_{s \to 0} \cancel{s} \cdot G(s) \cdot \frac{U}{\cancel{s}} = G(0)\, U$$

$$Y = G(0)\, U$$

$G(0) \rightarrow$ GUADAGNO STATICO

GUADAGNO IN CONTINUA

$$G(0) = \frac{Y}{U}$$

$- \circ -$

$$U(z) = U \frac{z}{z-1}$$

$$Y(z) = G(z)\, U \frac{z}{z-1}$$

$$y_\infty = \lim_{k \to \infty} y(k) = Y = \lim_{z \to 1} (z-1)\, Y(z) =$$

$$= \lim_{z \to 1} \cancel{(z-1)}\, G(z) \cdot U \frac{z}{\cancel{z-1}} =$$

$$= G(1) \cdot U$$

$$G(1) = \frac{Y}{U}$$

CHE TIPO DI SISTEMA DINAMICO È

$$G(s) = \frac{1}{s}$$

→ BIBO STABILE ?

DEVO TROVARE SE ESISTE (OVVIAMENTE SI) UN INGR. LIMITATO T.C. L'USCITA FORZATA NON È LIMITATA

$$Y(s) = \frac{1}{s} \cdot \frac{1}{s} = \frac{1}{s^2}$$

RAMPA NON È LIMITATA! (NO BIBO STABILE)

→ CHE TIPO DI OPERAZIONE VIENE EFFETTUATA NEL DOMINIO DEL TEMPO SU UN INGRESSO GENERICO.

$$Y(s) = \frac{1}{s} \cdot U(s)$$

$$Y(s) = G(s)\, U(s)$$

$$y(t) = \int_0^t g(\tau)\, u(t-\tau)\, d\tau \cdot$$

$$y(t) = \int_0^t 1(\tau)\, u(t-\tau)\, d\tau =$$

$$= \int_0^t u(t-\tau)\, d\tau \quad .$$

$$\zeta = t - \tau \qquad d\zeta = -d\tau$$

$$\zeta_i = t - \tau_i = t - 0 = t$$

$$\xi_s = t - \tau_s = t - t = 0$$

$$y(t) = -\int_t^0 u(\xi)\,d\xi = \int_0^t u(\xi)\,d\xi$$

INTEGRATORE ELEMENTARE

$$\rightarrow \boxed{\frac{1}{s}} \rightarrow$$

$$\rightarrow \boxed{\int} \rightarrow$$

$$G(z) = \frac{z}{z-1} = \frac{Y(z)}{U(z)}$$

$$Y(z)(z-1) = U(z)\,z$$

$$z\,Y(z) - Y(z) = z\,U(z)$$

$$y(k+1) - y(k) = u(k+1)$$

$$y(k') - y(k'-1) = u(k')$$

$$y(k') = y(k'-1) + u(k')$$

$$y\_corr = y\_prec + u\_corr$$

$$y(k') = \sum_{l=0}^{k'} u(l)$$