Determinare le CI che compensano ESATTAMENTE la risposta transitoria nel caso di ingresso a gradino unitario per un Sistema LTI-TD

In[◦]:=
```
Clear["Global`*"]
```

In[◦]:=
```
A = {{11 / 12, -1, 31 / 12}, {1, -1, 1}, {1 / 12, 0, -7 / 12}}
```

Out[◦]=

$$\left\{\left\{\frac{11}{12}, -1, \frac{31}{12}\right\}, \{1, -1, 1\}, \left\{\frac{1}{12}, 0, -\frac{7}{12}\right\}\right\}$$

In[◦]:=
```
B = {{1}, {0}, {-1}}
```

Out[◦]=

$$\{\{1\}, \{0\}, \{-1\}\}$$

In[◦]:=
```
C1 = {{-1, 3 / 2, -1}}
```

Out[◦]=

$$\left\{\left\{-1, \frac{3}{2}, -1\right\}\right\}$$

In[◦]:=
```
Eigenvalues[A]
```

Out[◦]=

$$\left\{-\frac{1}{2}, -\frac{1}{2}, \frac{1}{3}\right\}$$

In[◦]:=
```
JordanDecomposition[A]〚2〛 // MatrixForm
```

Out[◦]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{2} & 1 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

I modi saranno (1/3)^k, (-1/2)^k, Binom(k,1) (-1/2)^(k-1).

In[◦]:=
```
x0 = {{x1}, {x2}, {x3}}
```

Out[◦]=

$$\{\{x_1\}, \{x_2\}, \{x_3\}\}$$

Associo alla variabile libera la risposta libera del sistema in z

In[∘]:= `libera = Simplify[z C1.Inverse[z IdentityMatrix[3] - A].x0][[1]][[1]]`

Out[∘]=

$$-\left(\left(z\left(4\left(-2+z+6z^2\right)x_1+\left(11-12z-36z^2\right)x_2+4\left(-5+7z+6z^2\right)x_3\right)\right)/\left(2(1+2z)^2(-1+3z)\right)\right)$$

In[∘]:= `Σ = StateSpaceModel[{A, B, C1}, SamplingPeriod → 1]`

Out[∘]=

$$\left(\begin{array}{ccc|c} \frac{11}{12} & -1 & \frac{31}{12} & 1 \\ 1 & -1 & 1 & 0 \\ \frac{1}{12} & 0 & -\frac{7}{12} & -1 \\ \hline -1 & \frac{3}{2} & -1 & 0 \end{array}\right)_1^{\mathcal{S}}$$

In[∘]:= `G[z_] := Simplify[C1.Inverse[z IdentityMatrix[3] - A].B][[1]][[1]]`

In[∘]:= `G[z]`

Out[∘]=

$$\frac{6(-1+2z)}{(1+2z)^2(-1+3z)}$$

Utilizzo la variabile regime per la risposta a regime al gradino unitario (tempo discreto)

In[∘]:= `regime = G[1]` $\left(\frac{z}{z-1}\right)$

Out[∘]=

$$\frac{z}{3(-1+z)}$$

Utilizzo la variabile forzata per la risposta al gradino unitario (tempo discreto)

In[∘]:= `forzata = Simplify[G[z]` $\left(\frac{z}{z-1}\right)$`]`

Out[∘]=

$$\frac{6z(-1+2z)}{(-1+z)(1+2z)^2(-1+3z)}$$

Calcolo in z la risposta transitoria

In[∘]:= `transitoria = Factor[forzata - regime]`

Out[∘]=

$$-\frac{z\left(-17+20z+12z^2\right)}{3(1+2z)^2(-1+3z)}$$

In[∘]:= `Numerator[Simplify[Expand[libera + transitoria]]]`

Out[∘]=

$$-z\left(-34+40z+24z^2+12\left(-2+z+6z^2\right)x_1-3\left(-11+12z+36z^2\right)x_2-60x_3+84zx_3+72z^2x_3\right)$$

In[∘]:= `CoefficientList[Numerator[Simplify[Expand[libera + transitoria]]], z]`

Out[∘]=

$$\{0,\ 34+24x_1-33x_2+60x_3,\ -40-12x_1+36x_2-84x_3,\ -24-72x_1+108x_2-72x_3\}$$

In[ ]:=
```
Solve[CoefficientList[Numerator[Simplify[Expand[libera + transitoria]]], z] ==
  {0, 0, 0, 0}, {x1, x2, x3}]
```

Out[ ]=

$$\left\{\left\{x_1 \to -\frac{2}{3},\ x_2 \to -\frac{2}{3},\ x_3 \to -\frac{2}{3}\right\}\right\}$$

In[ ]:=
```
FullSimplify[OutputResponse[{Σ, {-2 / 3, -2 / 3, -2 / 3}}, 1, k]]
```

Out[ ]=

$$\left\{\frac{1}{3}\right\}$$