SISTEMA LTI-TD

$$\begin{cases} x(k+1) = A x(k) + B u(k) \\ y(k) = C x(k) + D u(k) \end{cases}$$

$$G(z) = C(zI - A)^{-1} B + D = \frac{n_G(z)}{d_G(z)}$$

$d_G(z)$ CONTIENE ALCUNI (SE NON TUTTI) FATTORI DEL POLINOMIO CARATTERISTICO DI A

$$\partial(n_G) \leq \partial(d_G)$$

$\partial(n_G) = \partial(d_G) \iff D \neq 0$ <-- IMPROPRIO

$\partial(n_G) < \partial(d_G) \iff D = 0$ <-- PROPRIO

$$A = \begin{bmatrix} 0 & 1 \\ 1/12 & 1/12 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

ORDINE DI UN SISTEMA LTI

IN BASE ALLA RAPPRESENTAZIONE

$\Rightarrow$ I/S/U $\rightarrow$ DIMENSIONE SPAZIO DI STATO

$\Rightarrow$ I/U (FdT) $\rightarrow$ GRADO DEL DENOMINATORE DI FdT

$$G(z) = \frac{z}{z^2 - \frac{1}{12} z - \frac{1}{12}}$$

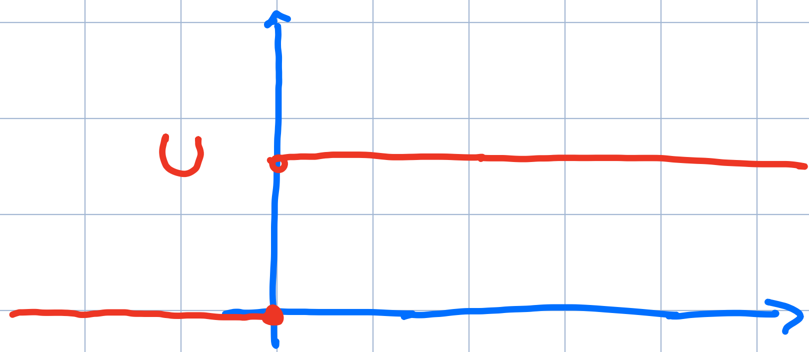
$$Y(s) = G(s) \cdot U(s) \qquad TC$$

$$Y(z) = G(z) \cdot U(z) \qquad TD$$

INGRESSI O SEGNALI CANONICI

POLINOMIALI | PERIODICI

GRADINO



$$f(t) = \begin{cases} U & t > 0 \\ ? & t = 0 \quad \leftarrow \quad U \quad t = 0 \\ 0 & t < 0 \end{cases}$$

$$F(s) = \int_{0}^{+\infty} f(t)\, e^{-st}\, dt$$

INTEGRALE DI RIEMANN

$$|U| \leq \underline{\underline{K e^{at}}} \qquad (K = ?\,,\ a = ?)$$

$a = 0 \quad , \quad k = |U|$

$$e^{At} \doteq (sI - A)^{-1}$$

$$e^{at} \doteq \frac{1}{s-a}$$

$$f(t) = \begin{cases} U \cdot e^{at} \big|_{a=0} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$U \cdot e^{at} \big|_{a=0} \doteq U \cdot \frac{1}{s-a} \Big|_{a=0} = U \frac{1}{s}$$

$U \cdot 1(t)$

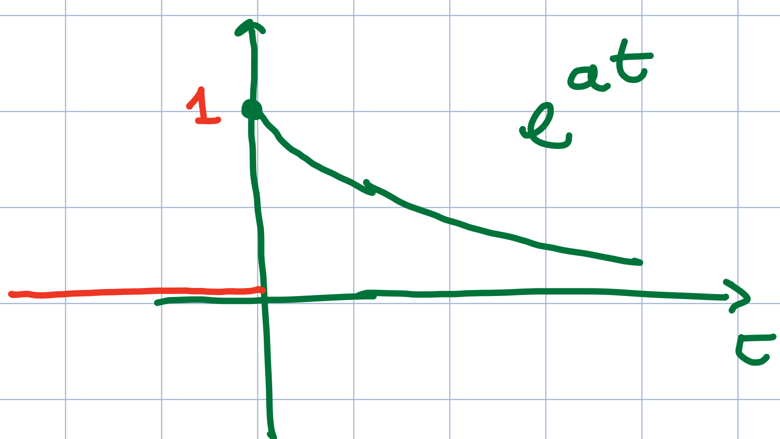
GRADINO UNITARIO

$$1(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$1(t) \doteq \frac{1}{s}$$

COME SCRIVO IN FORMA "COMPATTA"

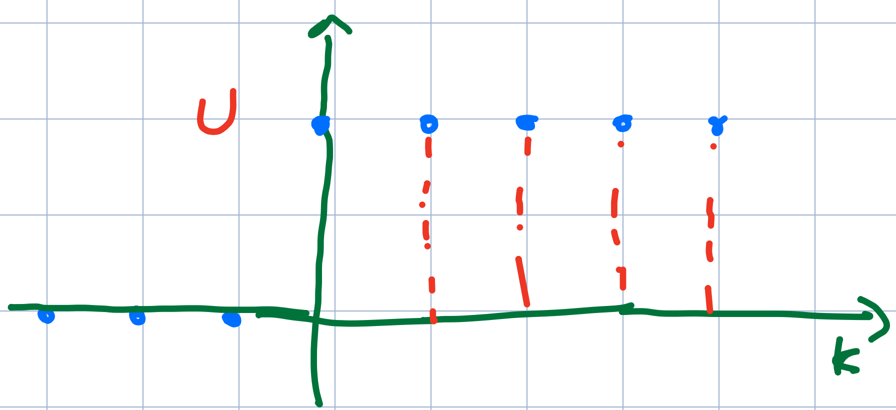
I SEGNALI RIGHT-SIDED?



$$F(t) = \begin{cases} e^{at} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$e^{at}\,1(t)$$

GRADINO CASO TD



$$F(k) = \begin{cases} U & k \geq 0 \\ 0 & k < 0 \end{cases}$$

$$|U| \leq M\alpha^k \quad (M = ?,\ \alpha = ?)$$

$$\alpha = 1 \quad M = |U|$$

$$A^k \doteq z(zI - A)^{-1}$$

$$\alpha^k \doteq \frac{z}{z-\alpha}$$

$$U \cdot \alpha^k \Big|_{\alpha=1} \Rightarrow$$ gradino di ampiezza U

$$U \cdot \alpha^k \Big|_{\alpha=1} \doteq U \frac{z}{z-\alpha}\Big|_{\alpha=1} = U \frac{z}{z-1}$$

GRADINO UNITARIO DISCRETO

$$1(k) = \begin{cases} 1 & k \geq 0 \\ 0 & k < 0 \end{cases}$$

$$1(k) \doteq \frac{z}{z-1}$$

$\alpha^{k} 1(k)$

CALCOLO DELLA RISPOSTA AL GRADINO

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -21 & -31 & -11 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}$$

$$Y(s) = G(s) \cdot U(s) = G(s) \cdot \frac{1}{s}$$

$$U(s) = \mathcal{L}\left[1(t)\right] = \frac{1}{s}$$

$$G(s) = \frac{2s+1}{(s+3)(s+7)(s+1)}$$

$$Y(s) = \frac{2s+1}{(s+3)(s+7)(s+1)} \cdot \frac{1}{s}$$

ANTITRASFORMATA DI LAPLACE

$$y(t) = \mathcal{L}^{-1}[Y(s)]$$

$$\frac{2s+1}{s(s+3)(s+7)(s+1)} = \frac{C_1}{s} + \frac{C_2}{s+3} + \frac{C_3}{s+7} + \frac{C_4}{s+1}$$

$$\frac{2s+1}{(s+3)(s+7)(s+1)} = C_1 + \frac{s\,C_2}{s+3} + \frac{s\,C_3}{s+7} + \frac{s\,C_4}{s+1}$$

$$C_1 = \lim_{s \to 0} s\, Y(s)$$

$$C_2 = \lim_{s \to -3} (s+3)\, Y(s)$$

$$C_3 = \lim_{s \to -7} (s+7)\, Y(s)$$

$$C_4 = \lim_{s \to -1} (s+1)\, Y(s)$$

$$Y(s) = \frac{1}{21} \cdot \frac{1}{s} - \frac{5}{24} \frac{1}{s+3} + \frac{13}{168} \frac{1}{s+7} + \frac{1}{12} \frac{1}{s+1}$$

$$y(t) = \underbrace{\frac{1}{21} \cdot 1(t)}_{\text{RISPOSTA A REGIME}} \underbrace{- \frac{5}{24} e^{-3t} 1(t) + \frac{13}{168} e^{-7t} 1(t) + \frac{1}{12} e^{-t} 1(t)}_{\text{RISPOSTA TRANSITORIA}}$$

STEADY STATE RESPONSE

$$e^{at}\,1(t) \doteq \frac{1}{s-a}$$

$\sin(\omega t)\,1(t)$

$\cos(\omega t)\,1(t)$

$$\sin(\varphi) = \frac{e^{j\varphi} - e^{-j\varphi}}{2j}$$

$$\cos(\varphi) = \frac{e^{j\varphi} + e^{-j\varphi}}{2}$$

$$\sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j} =$$

$$= \frac{1}{2j} e^{j\omega t} - \frac{1}{2j} e^{-j\omega t}$$

$$\mathcal{L}[\sin(\omega t)] = \frac{1}{2j}\mathcal{L}[e^{j\omega t}] - \frac{1}{2j}\mathcal{L}[e^{-j\omega t}]$$

$$e^{at} \doteq \frac{1}{s-a}$$

$$\mathcal{L}[\sin(\omega t)] = \frac{1}{2j} \frac{1}{s-j\omega} - \frac{1}{2j} \frac{1}{s+j\omega} =$$

$$= \frac{1}{2j} \left(\frac{1}{s-j\omega} - \frac{1}{s+j\omega} \right) =$$

$$= \frac{1}{2j} \left(\frac{s+j\omega - (s-j\omega)}{(s-j\omega)(s+j\omega)} \right) =$$

$$= \frac{1}{\cancel{2j}} \frac{\cancel{2j}\omega}{(s-j\omega)(s+j\omega)} = \frac{\omega}{s^2 + \cancel{j\omega s} - \cancel{j\omega s} + \omega^2} =$$

$$= \frac{\omega}{s^2+\omega^2}$$

$$\cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2} =$$

$$= \frac{1}{2} e^{j\omega t} + \frac{1}{2} e^{-j\omega t}$$

$$\mathcal{L}[\cos(\omega t)] = \frac{1}{2} \frac{1}{s - j\omega} + \frac{1}{2} \frac{1}{s + j\omega} =$$

$$= \frac{1}{2} \left(\frac{s - \cancel{j\omega} + s + \cancel{j\omega}}{(s-j\omega)(s+j\omega)} \right) =$$

$$= \frac{1}{\cancel{2}} \frac{\cancel{2}s}{s^2+\omega^2} = \frac{s}{s^2+\omega^2}$$

$$|\sin(\omega t)| \leq k e^{at} \qquad k > 0, \quad a \in \mathbb{R}$$

$$k = 1 \qquad a = 0$$

$$u(t) = \sin(t)\,1(t)$$

$$Y(s) = \frac{2s+1}{(s+1)(s+3)(s+7)(s^2+1)} =$$

$$\frac{D_1}{s+j} + \frac{D_2}{s-j} + \frac{D_3}{s+1} + \frac{D_4}{s+3} + \frac{D_5}{s+7}$$

$$D_1 = \lim_{s \to -j} (s+j)\, Y(s)$$

$$D_2 = \lim_{s \to j} (s-j)\, Y(s)$$

$$D_3 = \lim_{s \to -1} (s+1)\, Y(s)$$

$$D_4 = \lim_{s \to -3} (s+3)\, Y(s)$$

$$D_5 = \lim_{s \to -7} (s+7)\, Y(s)$$

$$\frac{D}{s-j} + \frac{\bar{D}}{s+j}$$

$$D \cdot e^{jt} + \bar{D}\, e^{-jt} = 2\,\mathrm{Re}\left(D e^{jt}\right)$$

$$z(t) + \bar{z}(t) = 2\,\mathrm{Re}\left(z(t)\right)$$

$$\frac{7}{100} \sin(t) - \frac{1}{100} \cos(t) = X \sin(t + \theta)$$