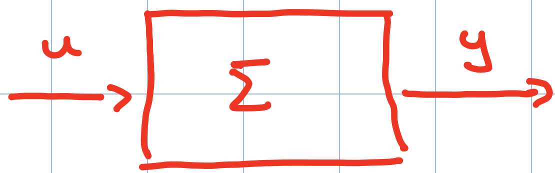
SISTEMA LTI-TC BIBO STABILE



$$Y(s) = G(s)\,U(s)$$

DETERMINARE LA RISPOSTA DEL SISTEMA A TRANSITORIO ESAURITO (REGIME) NEL CASO IN CUI L'INGRESSO SIA UNA ARMONICA ELEMENTARE NELLA FORMA

$$u(t) = A \sin(\omega t + \theta)$$

$A \leftarrow$ AMPIEZZA

$\theta \leftarrow$ SFASAMENTO INIZIALE

$\omega \leftarrow$ PULSAZIONE $\left[\frac{rad}{s}\right]$ $\qquad \omega = 2\pi f$

$$e^{j\omega t} = \cos(\omega t) + j\sin(\omega t)$$

$$Re[e^{j\omega t}] = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$Im[e^{j\omega t}] = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$U e^{j\omega t}$$

$U \in \mathbb{C}$ AMPIEZZA COMPLESSA

$$U = A \cdot e^{j\theta}$$

$A = |U| \qquad \theta = \measuredangle U$

1. DETERMINARE LA L-TRASF. DI

$$U e^{j\omega t} \stackrel{\mathcal{L}}{=} \frac{U}{s - j\omega}$$

$$Y(s) = G(s) \frac{U}{s - j\omega} =$$

$= \text{FRATTI\_SEMPLICI\_TRANSITORIA} + \dfrac{X}{s-j\omega}$

FORMULA DI HEAVISIDE PER X

$$X = \lim_{s \to j\omega} (s-j\omega)\, Y(s) =$$

$$= \lim_{s \to j\omega} (\cancel{s-j\omega})\, G(s) \frac{U}{\cancel{s-j\omega}} = G(j\omega)\, U$$

$$Y_{ss}(s) = \frac{G(j\omega)\, U}{s-j\omega}$$

$$y_{ss}(t) = G(j\omega)\, U\, e^{j\omega t}$$

$$u(t) = A \sin(\omega t + \theta) = \mathrm{Im}\left[U e^{j\omega t} \right] =$$

$$= \mathrm{Im}\left[A e^{j\theta} \cdot e^{j\omega t} \right] = \mathrm{Im}\left[A e^{j(\omega t + \theta)} \right]$$

$$= \mathrm{Im}\left[A\cos(\omega t+\theta) + jA\sin(\omega t+\theta)\right]$$

$$y_{ss}(t) = \mathrm{Im}\left[G(j\omega)\, U\, e^{j\omega t}\right]$$

$$G(j\omega) = |G(j\omega)| \cdot e^{j \measuredangle G(j\omega)}$$

$$y_{ss}(t) = \mathrm{Im}\left[|G(j\omega)| \cdot e^{j\measuredangle G(j\omega)} \cdot A \cdot e^{j\theta} \cdot e^{j\omega t}\right]$$

$$= \mathrm{Im}\left[A \cdot |G(j\omega)| \cdot e^{j(\omega t+\theta+\measuredangle G(j\omega))}\right] =$$

$$= \mathrm{Im}\left[A \cdot |G(j\omega)| \cdot \cos(\omega t+\theta+\measuredangle G(j\omega)) + \right.$$

$$\left. + jA\,|G(j\omega)|\sin(\omega t+\theta+\measuredangle G(j\omega))\right]$$

$$y_{ss}(t) = |G(j\omega)|\, A \sin(\omega t+\theta+\measuredangle G(j\omega))$$

$$y_{ss}(t) = Y \sin(\omega t + \phi)$$

$$u(t) = A \sin(\omega t + \theta)$$

$$\frac{Y}{A} = \frac{|G(j\omega)| \cdot \cancel{A}}{\cancel{A}} = |G(j\omega)|$$

$$\phi - \theta = \measuredangle G(j\omega) + \cancel{\theta} - \cancel{\theta} = \measuredangle G(j\omega)$$

$G(j\omega)$ SI OTTIENE VALUTANDO LA FdT DEL SISTEMA $G(s)$ SULL' ASSE IMMAGINARIO

$G(j\omega) = G(s)\big|_{s=j\omega}$ SI DEFINISCE RISPOSTA ARMONICA O RISPOSTA IN FREQUENZA

DEL SISTEMA_

$$G(s) = \frac{s+2}{s^4+3s^3+4s^2+3s+1}$$

$$u(t) = \sin(2t) \qquad \omega = 2 \frac{rad}{sec}$$

QUALE È LA RAPPRESENTAZIONE I/U NEL DOMINIO DEL TEMPO? (MODELLARMA)

$$G(s) = \frac{Y(s)}{U(s)} = \frac{s+2}{s^4+3s^3+4s^2+3s+1}$$

$$(s^4+3s^3+4s^2+3s+1)\,Y(s) = (s+2)\,U(s)$$

$$s^4 Y(s) + 3s^3 Y(s) + 4s^2 Y(s) + 3sY(s) + Y(s) =$$

$$= sU(s) + 2U(s)$$

$$\mathcal{L}[f^{(n)}(t)] = s^n F(s) - s^{n-1} f(0) - s^{n-2} \dot{f}(0) - \dots - s f^{(n-2)}(0) - f^{(n-1)}(0)$$

$$y^{IV}(t) + 3\dddot{y}(t) + 4\ddot{y}(t) + 3\dot{y}(t) + y(t) =$$

$$= \dot{u}(t) + 2u(t)$$ (C I/U IMPLICITO)

$$y(t) = \int_0^t g(t-\tau)\, u(\tau)\, d\tau$$

(C I/U ESPLICITO)

I MODI SONO

$-1, -1 \Rightarrow e^{-t}, \; t \cdot e^{-t}$

$-\frac{1}{2} \pm j\frac{\sqrt{3}}{2} \Rightarrow e^{-\frac{1}{2}t} \cdot \cos\left(\frac{\sqrt{3}}{2}t\right), \; e^{-\frac{1}{2}t} \sin\left(\frac{\sqrt{3}}{2}t\right)$

$$\sin(2t) \doteq \frac{2}{s^2+4}$$

$G(j\omega)$

$\omega \in \mathbb{R}$ $\quad\quad$ $G(j\omega) \in \mathbb{C}$

$(\omega, |G(j\omega)|)$

$(\omega, \measuredangle G(j\omega))$

SE $G(s)$ È UNA FUNZIONE DI VARIABILE COMPLESSA RAPPORTO DI POLINOMI IN S A COEFFICIENTI REALI (REAL-RAZIONALE)

$$\overline{G(j\omega)} = G(-j\omega)$$

$$G(j\omega) \quad\quad G(-j\omega) = \overline{G(j\omega)} =$$

$$= |G(j\omega)| \cdot e^{-j \measuredangle G(j\omega)}$$

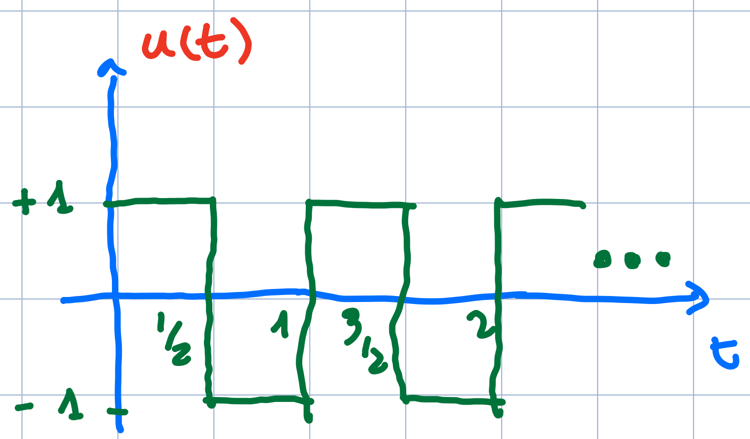
$|G(j\omega)| > 1$ AMPLIFICAZIONE

$|G(j\omega)| < 1$ ATTENUAZIONE

$\measuredangle G(j\omega) > 0$ ANTICIPO

$\measuredangle G(j\omega) < 0$ RITARDO

$$y^{IV} + 3y''' + 4y'' + 3y' + y = u' + 2u$$



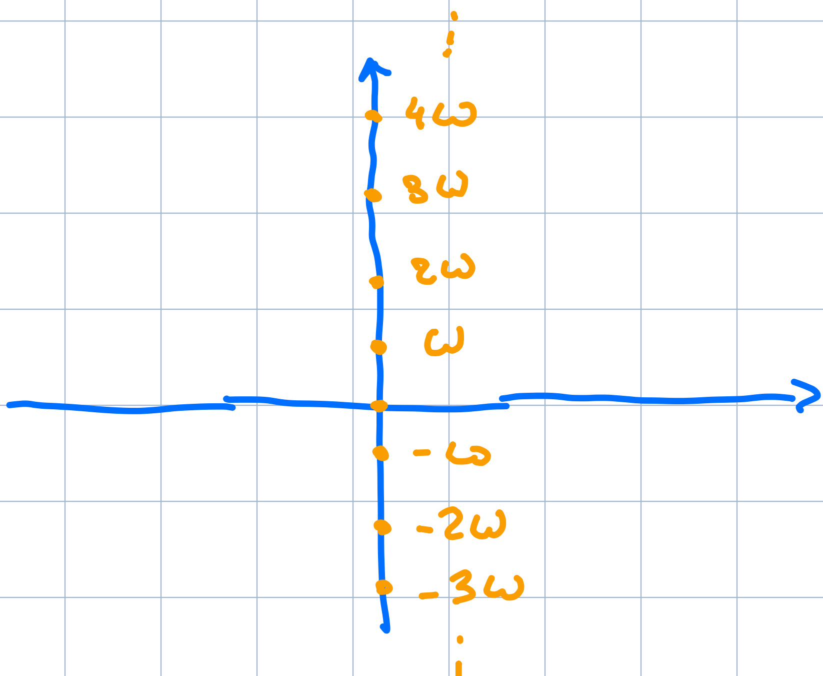
PERIODO , DUTY CYCLE

ANALISI DI FOURIER.

$$u(t) = \sum_{\nu=-\infty}^{+\infty} c_\nu \, e^{j \overbrace{\frac{2\pi}{T}}^{\omega} \nu \cdot t} \qquad e^{j\omega t}$$

$$c_\nu = \frac{1}{T} \int_0^T u(t) \, e^{-j \frac{2\pi}{T} \nu \cdot t}$$

$$y_{ss}(t) = \sum_{\nu=-\infty}^{+\infty} c_\nu \, G\left(j \frac{2\pi}{T} \cdot \nu\right) e^{j \frac{2\pi}{T} \nu t}$$



$$G(s) = \frac{1}{s+1}$$

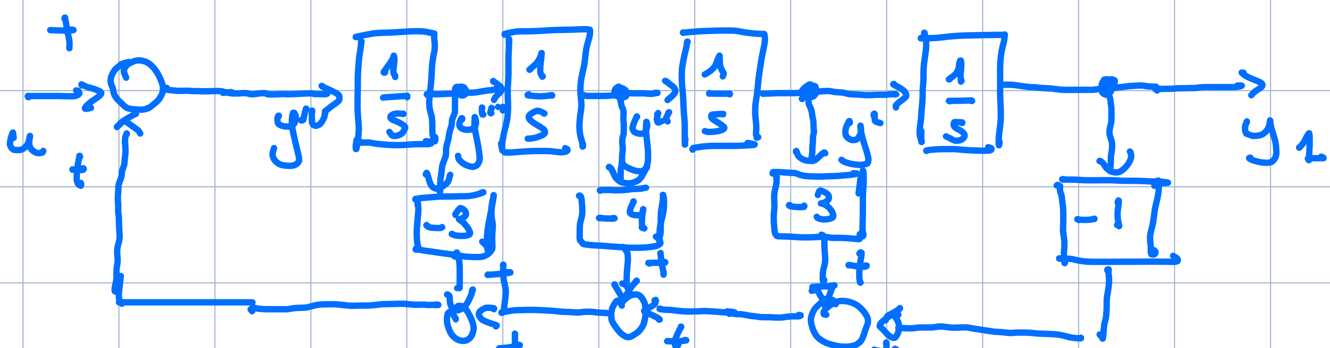
$$f = 2^{\frac{n}{12}} \cdot f_0$$

$$y^{IV} + 3y''' + 4y'' + 3y' + y = u' + 2u$$

4 "registri"

$$y_1^{IV} + 3y_1''' + 4y_1'' + 3y_1' + y_1 = u$$

$$y_1^{IV} = -3y_1''' - 4y_1'' - 3y_1' - y_1 + u$$



$$y_1 \longleftrightarrow u$$

$$\dot{y}_1 + 2y_1 \longleftrightarrow \dot{u} + 2u$$