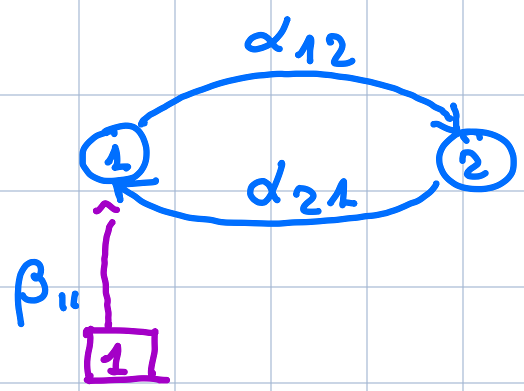
VARIABILI INDIPENDENTI



SISTEMA ISOLATO

IMMIGRAZIONE

$x_1(k) \leftarrow$ # ITALIANI IN ITALIA

$x_2(k) \leftarrow$ # ITALIANI ALL'ESTERO

$u_1(k) \leftarrow$ # IMMIGRATI

$\alpha_{12} \leftarrow$ FRAZIONE DI ITALIANI CHE EMIGRA (DA UN ANNO AL SUCCESSIVO)

$\alpha_{21} \leftarrow$ FRAZIONE DI EMIGRATI CHE DECIDE DI RIENTRARE

$\beta_{11} \leftarrow \;?\quad (1)$

$x_1(k+1) - x_1(k) = -\alpha_{12}\, x_1(k)$

$+\alpha_{21}\, x_2(k) + u_1(k)$

$x_2(k+1) - x_2(k) = \alpha_{12}\, x_1(k)$

$-\alpha_{21}\, x_2(k)$

$$\begin{cases} x_1(k+1) = x_1(k) - \alpha_{12}\, x_1(k) + \alpha_{21} x_2(k) + \\ \quad + u_1(k) \\ x_2(k+1) = \alpha_{12} x_1(k) + x_2(k) - \alpha_{21}\, x_2(k) \end{cases}$$

$$\begin{cases} x_1(k+1) = (1-\alpha_{12})\, x_1(k) + \alpha_{21} x_2(k) + u_1(k) \\ x_2(k+1) = \alpha_{12}\, x_1(k) + (1-\alpha_{21})\, x_2(k) \\ \qquad\qquad + 0 \end{cases}$$

$$\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \in \mathbb{N}^2 \qquad \begin{matrix} \alpha_{ij} \\ \beta_{ij} \end{matrix} \quad \underline{\underline{\text{ADIMENSIONALI}}}$$

$$X(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \quad (\text{VETTORE DI STATO})$$

$$(1-\alpha_{12}) \cdot x_1(k) + \alpha_{21} x_2(k)$$

$$\alpha_{12} x_1(k) + (1-\alpha_{21}) x_2(k)$$

$$\begin{bmatrix} (1-\alpha_{12}) & \alpha_{21} \\ \alpha_{12} & (1-\alpha_{21}) \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$\begin{matrix} u_1(k) \\ 0 \end{matrix} \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix} u_1(k)$$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} (1-\alpha_{12}) & \alpha_{21} \\ \alpha_{12} & (1-\alpha_{21}) \end{bmatrix}}_{A} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} +$$

$$\underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B} u_1(k)$$

$$X(k+1) = A\,X(k) + B\,u(k)$$

VARIABILI OSSERVABILI

MISURO ISTANTE PER ISTANTE
I CITTADINI ITALIANI E SEPARATAMENTE
GLI IMMIGRATI

$$\begin{cases} y_1(k) = x_1(k) + x_2(k) \\ y_2(k) = u_1(k) \end{cases}$$

VARIABILI OSSERVABILI , USCITE
SINONIMI

VARIABILI DIPENDENTI , STATO
SINONIMI

VARIABILI INDIPENDENTI , INGRESSI
SINONIMI

SCRITTURA IN FORMA VETTORIALE
DELLE VARIABILI OSSERVABILI

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} \in \mathbb{N}^2$$

$$\begin{matrix} x_1(k) + x_2(k) \\ 0 \end{matrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ u_1(k) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1(k)$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{D} u_1(k)$$

$$y(k) = C\,x(k) + D\,u(k)$$

$$\begin{cases} x(k+1) = A\,x(k) + B\,u(k) \\ y(k) = C\,x(k) + D\,u(k) \end{cases}$$

$n \leftarrow$ STATI $\qquad x(k) \in \mathbb{R}^n$

$m \leftarrow$ INGRESSI $\qquad u(k) \in \mathbb{R}^m$

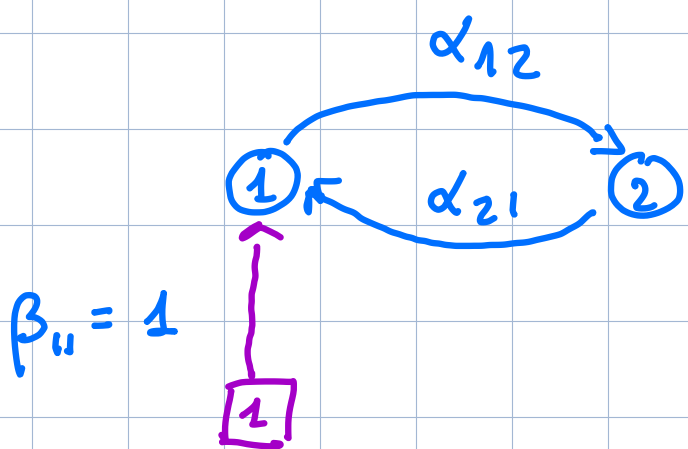
$p \leftarrow$ USCITE $\qquad y(k) \in \mathbb{R}^p$

$A \in \mathbb{R}^{n \times n}$ MATRICE DINAMICA

$B \in \mathbb{R}^{n \times m}$ MATRICE DEGLI INGRESSI

$C \in \mathbb{R}^{p \times n}$ MATRICE DI USCITA

$D \in \mathbb{R}^{p \times m}$ MATRICE INGRESSO - USCITA



$x_1(t)$

$x_2(t)$

$$\begin{cases} \dot{x}_1(t) = -\alpha_{12}\, x_1(t) + \alpha_{21}\, x_2(t) + u_1(t) \\ \dot{x}_2(t) = \alpha_{12}\, x_1(t) - \alpha_{21}\, x_2(t) \end{cases}$$

$[\alpha_{ij}] \rightarrow [sec^{-1}]$

COSTANTI DI TEMPO

$[\beta_{ij}] \rightarrow [sec^{-1}]$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \dot{X}(t)$$

$$\dot{X}(t) = A\, X(t) + B\, u(t)$$

$$A = \begin{bmatrix} -\alpha_{12} & \alpha_{21} \\ \alpha_{12} & -\alpha_{21} \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

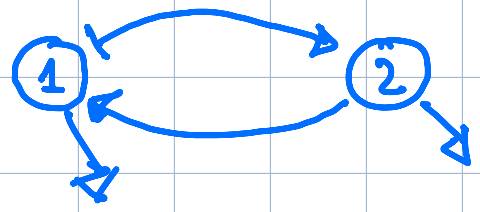
$$y_1(t) = x_1(t) + x_2(t)$$

$$y_2(t) = u_1(t)$$

$$Y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$$

$$y(t) = C\,x(t) + D\,u(t)$$

INDIVIDUI ADULTI E GIOVANI



ACCUMULO DI RISORSA

CAPITALE CHE MATURA INTERESSI