

```
In[*]:= A = 
$$\begin{pmatrix} -6 & -7 & -1 & -3 & -3 \\ 4 & 5 & 1 & 3 & 3 \\ 4/3 & 2/3 & -10/3 & -4/3 & 0 \\ -4/3 & -5/3 & 1/3 & -8/3 & 0 \\ -10/3 & -17/3 & -2/3 & -5/3 & -5 \end{pmatrix}$$

```

```
Out[*]= 
$$\left\{ \{-6, -7, -1, -3, -3\}, \{4, 5, 1, 3, 3\}, \left\{ \frac{4}{3}, \frac{2}{3}, -\frac{10}{3}, -\frac{4}{3}, 0 \right\}, \left\{ -\frac{4}{3}, -\frac{5}{3}, \frac{1}{3}, -\frac{8}{3}, 0 \right\}, \left\{ -\frac{10}{3}, -\frac{17}{3}, -\frac{2}{3}, -\frac{5}{3}, -5 \right\} \right\}$$

```

```
In[*]:= Eigenvalues[A]
```

```
Out[*]= 
$$\{-3, -3, -2, -2, -2\}$$

```

```
In[*]:= NullSpace[A + 3 IdentityMatrix[5]]
```

```
Out[*]= 
$$\{\{1, -1, -2, 1, 1\}\}$$

```

```
NullSpace[A + 2 IdentityMatrix[5]]
```

```
Out[*]= 
$$\left\{ \{2, -2, 0, 1, 1\}, \left\{ \frac{3}{2}, -1, 1, 0, 0 \right\} \right\}$$

```

```
In[*]:= CharacteristicPolynomial[A, x]
```

```
Out[*]= 
$$-72 - 156 x - 134 x^2 - 57 x^3 - 12 x^4 - x^5$$

```

```
In[*]:= MatrixMinimalPolynomial[a_List?MatrixQ, x_] :=  
Module[{i, n = 1, qu = {}, mnm = {Flatten[IdentityMatrix[Length[a]]]}},  
While[Length[qu] == 0, AppendTo[mnm, Flatten[MatrixPower[a, n]]];  
qu = NullSpace[Transpose[mnm]];  
n++];  
First[qu].Table[x^i, {i, 0, n - 1}]]
```

```
In[*]:= MatrixMinimalPolynomial[A, x]
```

```
Out[*]= 
$$36 + 60 x + 37 x^2 + 10 x^3 + x^4$$

```

```
In[*]:= Simplify[A.A.A.A.A + 12 A.A.A.A + 57 A.A.A + 134 A.A + 156 A + 72 IdentityMatrix[5]]
```

```
Out[*]= 
$$\{\{0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0\}\}$$

```

```
In[*]:= Simplify[A.A.A.A + 10 A.A.A + 37 A.A + 60 A + 36 IdentityMatrix[5]]
```

```
Out[*]= 
$$\{\{0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0\}\}$$

```

```
In[*]:= Solve[MatrixMinimalPolynomial[A, x] == 0, x]
```

```
Out[*]= 
$$\{\{x \rightarrow -3\}, \{x \rightarrow -3\}, \{x \rightarrow -2\}, \{x \rightarrow -2\}\}$$

```

```
In[*]:= Eigenvalues[A]
```

```
Out[*]= 
$$\{-3, -3, -2, -2, -2\}$$

```