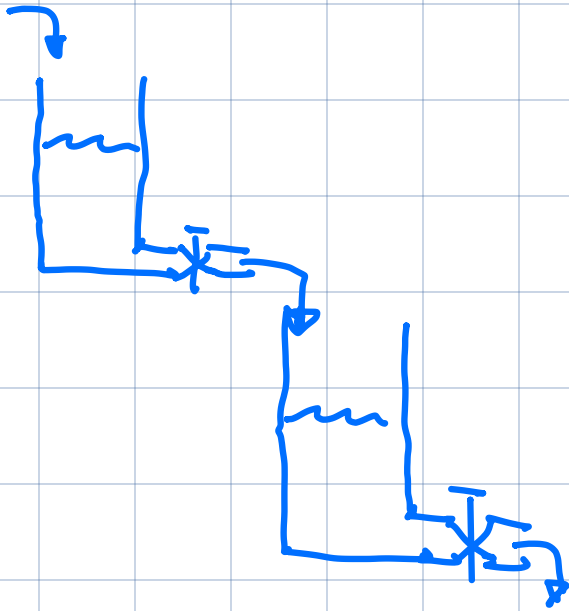


SISTEMI INTERCONNESSI



SOTTOSISTEMI

- SERIE (CASCATA)
- PARALLELO
- RETROAZIONE NEGATIVA (FEEDBACK NEGATIVO)

SERIE

SUPPONIAMO DI AVERE DUE
SOTTOSISTEMI (LTI-TC SISO)

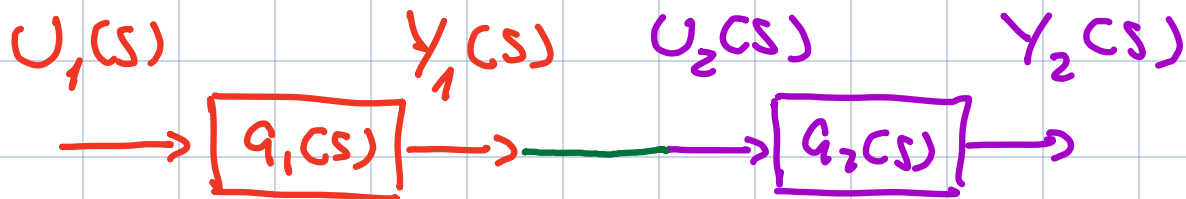


$$\Sigma_1: Y_1(s) = G_1(s) U_1(s)$$

$$\Sigma_2: Y_2(s) = G_2(s) U_2(s)$$

L'INTERCONNESSIONE IN SERIE IMPLICA

a) INDIVIDUARE CHI "VIENE" PRIMA
E CHI DOPO



b) L'INGRESSO DEL SOTTO SISTEMA
A VALLE (DOPO) È UGUALE
ALL'USCITA DEL SOTTO SISTEMA
A MONTE (PRIMA)

DETERMINANDO FDT DEL SISTEMA IN
SERIE

$$\begin{cases} Y_1(s) = G_1(s) U_1(s) \\ Y_2(s) = G_2(s) U_2(s) \end{cases}$$

$$U_2(s) = Y_1(s)$$

$$Y(s) = Y_2(s) = G_2(s) \cdot G_1(s) U_1(s) = \\ = G_2(s) G_1(s) U(s)$$

QUALI SONO I POLI DELLA SERIE?

$$\text{POLI_SERIE} \subseteq \text{POLI_}\Sigma_1 \cup \text{POLI_}\Sigma_2$$

$$G_1(s) = \frac{s+1}{(s+2)(s+5)}$$

$$G_2(s) = \frac{1}{(s+1)(s+6)}$$

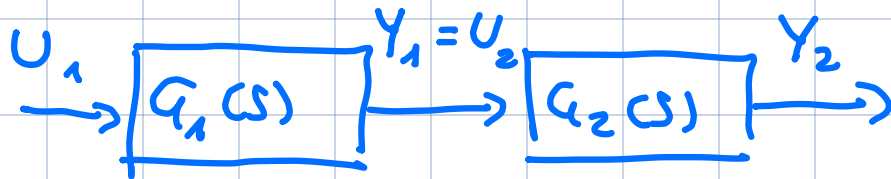
$$G_2(s) \cdot G_1(s) = \frac{1}{\cancel{(s+1)}(s+6)} \cdot \frac{\cancel{s+1}}{(s+2)(s+5)}$$

Σ_1 BIBO STABILE

Σ_2 BIBO STABILE

L'INTERCONNESSIONE SERIE È

BIBO STABILE?



SE UNO DEI DUE (O TUTTI E DUE)
NON È BIBO STABILE?

$$G_1(s) = \frac{s-1}{(s+1)(s+4)}$$

$$G_2(s) = \frac{1}{(s-1)(s+6)}$$

A PARTIRE DAI MODI DI Σ_1 E
 Σ_2 SE NE GENERANO DI NUOVI?

$$G_1(s) = \frac{1}{(s+1)(s+2)}$$

e^{-t}, e^{-2t}

$$G_2(s) = \frac{1}{(s+2)}$$

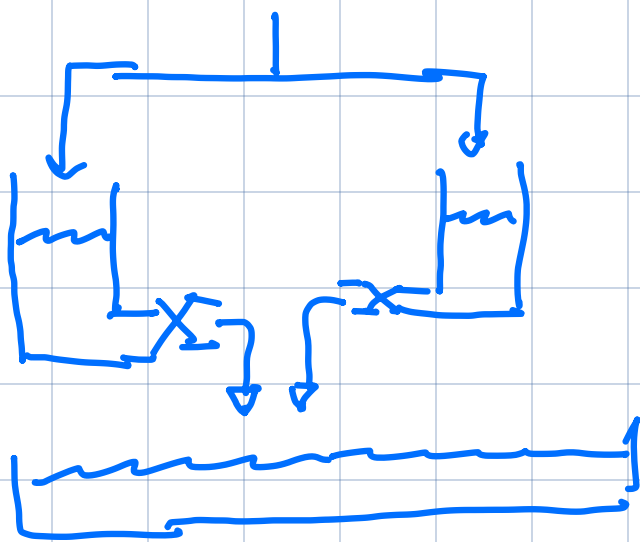
e^{-2t}

$$G_2(s)G_1(s) = \frac{1}{(s+1)(s+2)^2}$$

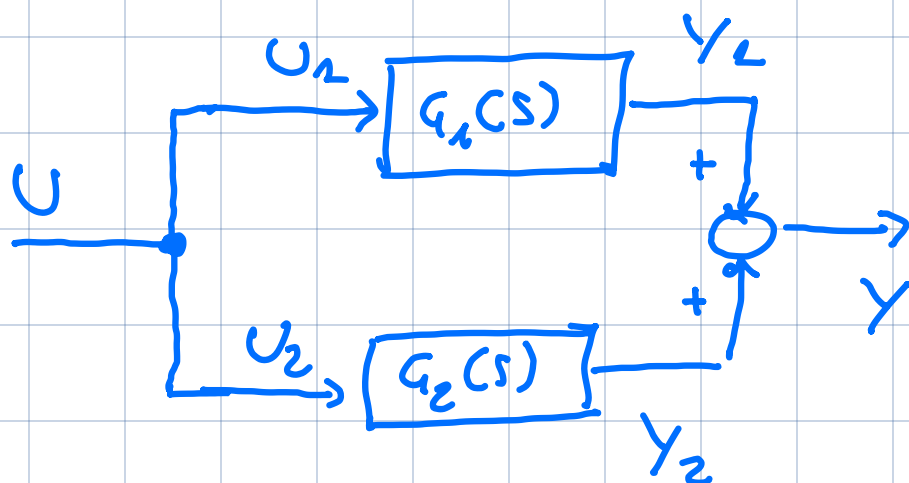
$e^{-t}, e^{-2t}, t \cdot e^{-2t}$



$$G_n(s) G_{n-1}(s) G_{n-2}(s) \dots G_2(s) G_1(s)$$



PARALLELO



$$\left\{ \begin{array}{l} Y_1(s) = G_1(s) U_1(s) \\ Y_2(s) = G_2(s) U_2(s) \\ Y(s) = Y_1(s) + Y_2(s) \\ U_1(s) = U(s), \quad U_2(s) = U(s) \end{array} \right.$$

$$Y_1(s) = G_1(s) U(s)$$

$$Y_2(s) = G_2(s) U(s)$$

$$\begin{aligned} Y(s) &= Y_1(s) + Y_2(s) = G_1(s) U(s) + G_2(s) U(s) \\ &= (G_1(s) + G_2(s)) U(s) \end{aligned}$$

$$\text{POLY_PAR} \subseteq \text{mcm}(\text{POLY_}\Sigma_1, \text{POLY_}\Sigma_2)$$

$$G_1(s) = \frac{1}{(s+4)(s-1)}$$

$$G_2(s) = - \frac{1}{(2s+3)(s-1)}$$

$$G_1(s) + G_2(s) = \frac{1}{(s+4)(s-1)} - \frac{1}{(2s+3)(s-1)}$$

$$= \frac{2s+3 - s-4}{(s+4)(2s+3)(s-1)} = \frac{\cancel{s}-1}{(s+4)(2s+3)\cancel{(s-1)}}$$

Σ_1 BIBO STABILE

Σ_2 BIBO STABILE

PARALLELO BIBO STABILE

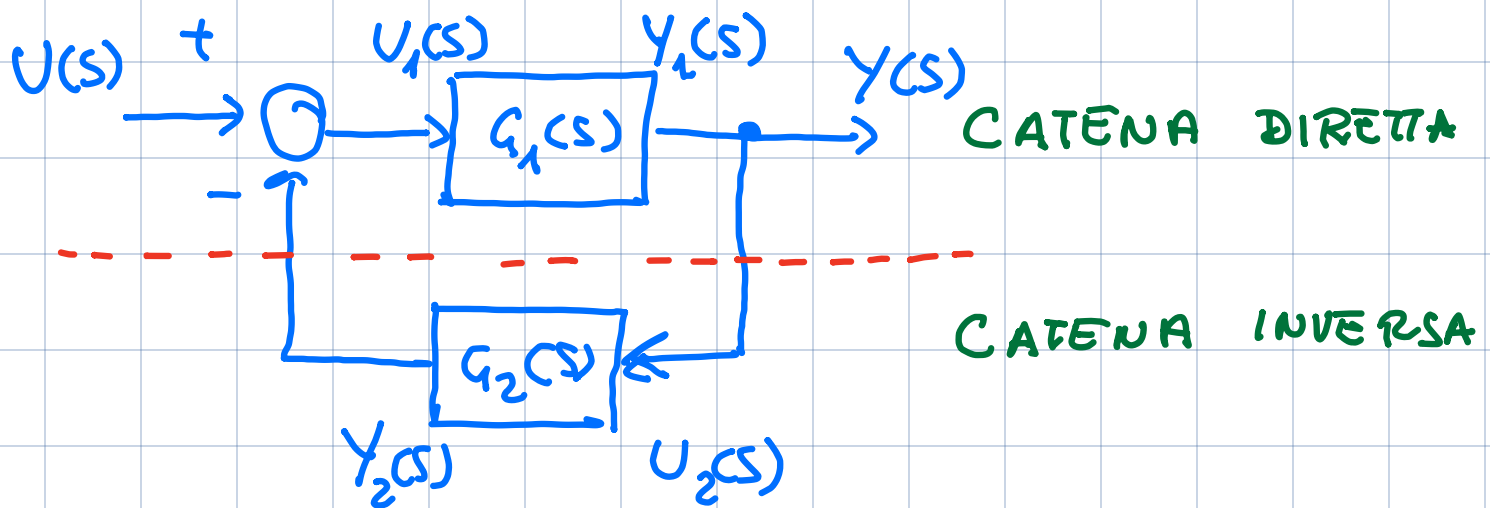
SE UNO DEI DUE (OR ESCLUSIVO)

NON È BIBO STABILE ALLORA

IL PARALLELO NON È BIBO STABILE

NON SI GENERANO MODI NUOVI.

RETROAZIONE NEGATIVA



$U_1(s) \leftarrow$ "SEGNALE DI ERRORE"

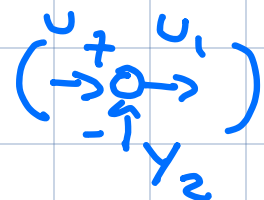
$$Y_1(s) = G_1(s) U_1(s)$$

$$Y_2(s) = G_2(s) U_2(s)$$

$$U_1(s) = U(s) - Y_2(s)$$

$$Y(s) = Y_1(s)$$

$$Y(s) = U_2(s)$$



$$Y_1(s) = G_1(s) (U(s) - Y_2(s)) =$$

$$= G_1(s) (U(s) - G_2(s) U_2(s))$$

$$Y(s) = G_1(s) (U(s) - G_2(s) Y(s))$$

$$Y(s) + G_1(s) G_2(s) Y(s) = G_1(s) U(s)$$

$$(1 + G_1(s) G_2(s)) Y(s) = G_1(s) U(s)$$

$$Y(s) = \frac{G_1(s)}{1 + G_1(s) G_2(s)} U(s)$$

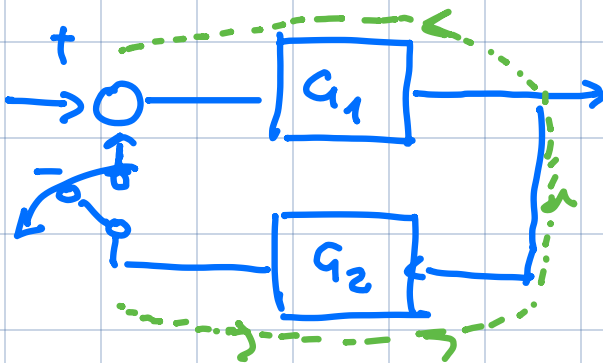
$$G_1 = \frac{n_1}{d_1} \quad G_2 = \frac{n_2}{d_2}$$

$$\frac{G_1(s)}{1 + G_1(s) G_2(s)} = \frac{\frac{n_1}{d_1}}{1 + \frac{n_1}{d_1} \frac{n_2}{d_2}} =$$

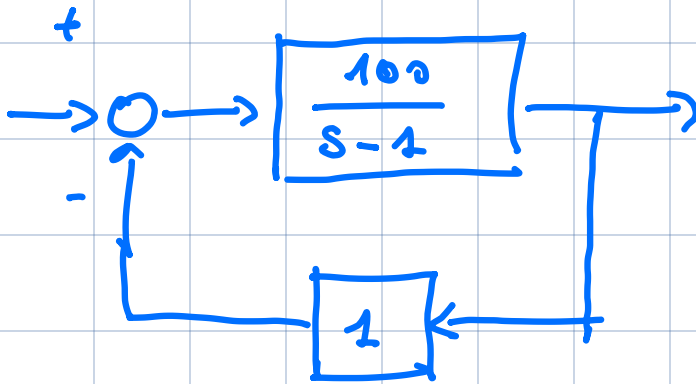
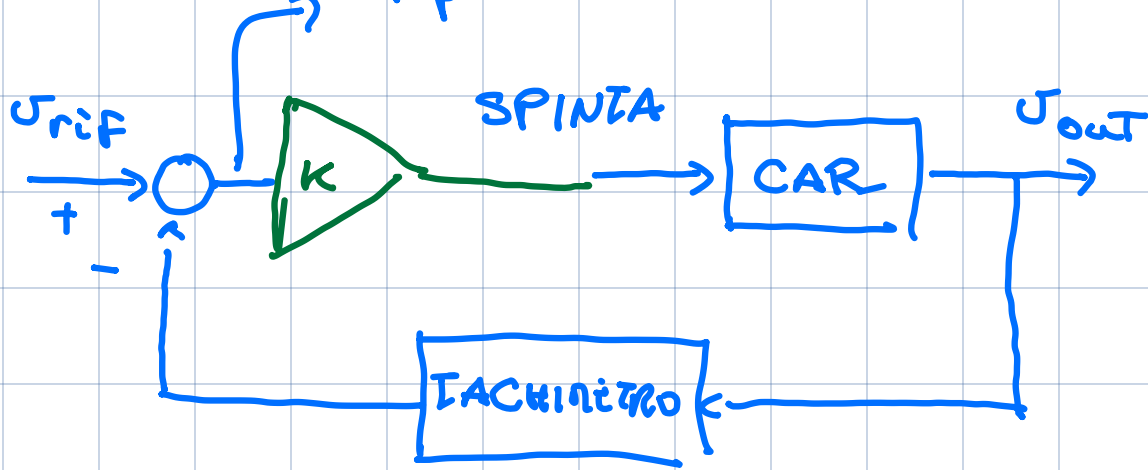
$$= \frac{\frac{n_1}{d_1}}{\frac{d_1 d_2 + n_1 n_2}{d_1 d_2}} = \frac{n_1 d_2}{n_1 n_2 + d_1 d_2}$$

$$G_1(s) G_2(s)$$

LOOP GAIN



$$5_{in} - 5_{out, m}$$

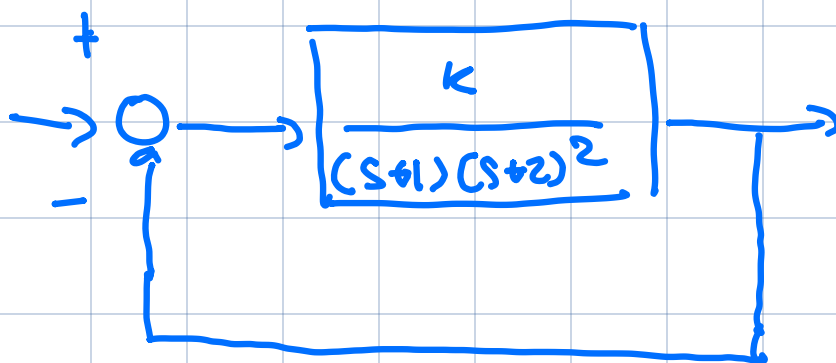


$$G_1(s) = \frac{100}{s-1}$$

$$G_2(s) = 1$$

$$\frac{G_1(s)}{1 + G_1(s) G_2(s)} = \frac{\frac{100}{s-1}}{1 + \frac{100}{s-1} \cdot 1} =$$

$$= \frac{\frac{100}{\cancel{s-1}}}{\frac{s-1+100}{\cancel{s-1}}} = \frac{100}{s+99}$$



$$G_1(s) = \frac{k}{(s+1)(s+2)^2}$$

$$G_2(s) = 1$$

$$\frac{G_1(s)}{1 + G_1(s)G_2(s)} = \frac{\frac{k}{(s+1)(s+2)^2}}{1 + \frac{k}{(s+1)(s+2)^2} \cdot 1} =$$

k

$$= \frac{\frac{(s+1)(s+2)^2}{(s+1)(s+2)^2 + k}}{(s+1)(s+2)^2} = \frac{k}{(s+1)(s+2)^2 + k}$$