$$\begin{cases} \dot{x}_1(t) = -\alpha_{10} x_1(t) + \beta_{11} u_1(t) \\ x_1(0) = x_{10} \end{cases}$$

$$x_1(t) = e^{-\alpha_{10} t} \cdot x_{10} + \beta_{11} \int_0^t e^{-\alpha_{10}(t-\tau)} u_1(\tau)\, d\tau$$

$$\dot{x}_1(t) = -\alpha_{10} x_1(t) + \beta_{11} u(t)$$

$$\boxed{x_1(0) = x_{10}}$$

$$\underbrace{e^{-\alpha_{10} t} \cdot x_{10} \Big|_{t=0}}_{x_{10}} + \beta_{11} \int_0^t e^{-\alpha_{10}(t-\tau)} \cdot u(\tau)\, d\tau \Big|_{t=0}$$

$$x_{10} \qquad + \qquad 0 \qquad = x_{10}$$

$$x_1(t) = e^{-\alpha_{10} t} \cdot x_{10} + \beta_{11}\, \phi(t)$$

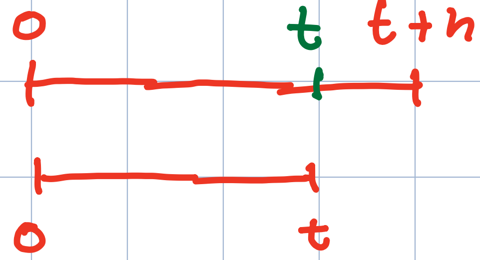
$$\phi(t) = \int_0^t e^{-\alpha_{10}(t-\tau)}\, u(\tau)\, d\tau$$

$$\frac{d}{dt} x_1(t) = -\alpha_{10}\, e^{-\alpha_{10} t}\, x_{10} + \beta_{11} \frac{d}{dt}\phi(t)$$

$$\frac{d}{dt}\phi(t) = \lim_{h \to 0} \frac{\phi(t+h) - \phi(t)}{h} \quad (h>0)$$

$$\frac{\phi(t+h) - \phi(t)}{h} = \frac{1}{h}\left(\int_0^{t+h} e^{-\alpha_{10}(t+h-\tau)}\, u(\tau)\, d\tau - \right.$$

$$\left. - \int_0^t e^{-\alpha_{10}(t-\tau)}\, u(\tau)\, d\tau \right)$$



$$\frac{\phi(t+h)-\phi(t)}{h} = \frac{1}{h}\left(\int_0^t e^{-\alpha_{10}(t+h-\tau)}\,u(\tau)\,d\tau + \right.$$

$$\left. + \int_t^{t+h} e^{-\alpha_{10}(t+h-\tau)}\,u(\tau)\,d\tau - \int_0^t e^{-\alpha_{10}(t-\tau)}\,u(\tau)\,d\tau\right)$$

$$= \frac{1}{h}\left(\int_0^t \left(e^{-\alpha_{10}(t+h-\tau)} - e^{-\alpha_{10}(t-\tau)}\right)u(\tau)\,d\tau + \right.$$

$$\left. + \int_t^{t+h} e^{-\alpha_{10}(t+h-\tau)}\,u(\tau)\,d\tau\right)$$

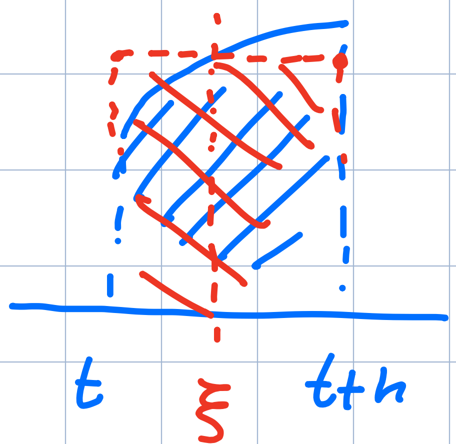
$$\int_0^t \lim_{h\to 0}\frac{1}{h}\left(e^{-\alpha_{10}(t+h-\tau)} - e^{-\alpha_{10}(t-\tau)}\right)\cdot u(\tau)\,d\tau$$

$$+\lim_{h\to 0}\frac{1}{h}\int_t^{t+h} e^{-\alpha_{10}(t+h-\tau)}\,u_1(\tau)\,d\tau =$$

$$= \int_0^t -\alpha_{10}\, e^{-\alpha_{10}(t-\tau)}\,u_1(\tau)\,d\tau \;+$$

$$+\lim_{h\to 0}\frac{1}{h}\int_t^{t+h} e^{-\alpha_{10}(t+h-\tau)}\,u_1(\tau)\,d\tau$$

$$t \le \tau \le t+h$$

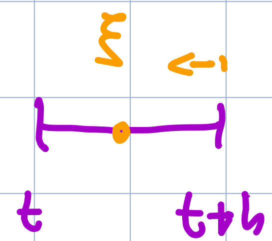


TEOREMA DELLA MEDIA SU INTEGRALE SECONDO ADDENDO

$$\lim_{h\to 0} \frac{\phi(t+h)-\phi(t)}{h} =$$

$$-\alpha_{10}\int_0^t e^{-\alpha_{10}(t-\tau)}\, u_1(\tau)\, d\tau \; +$$

$$+ \lim_{h\to 0} \frac{1}{\cancel{h}}\, \cancel{h}\cdot \left(e^{-\alpha_{10}(t+h-\xi)}\, u_1(\xi)\right)$$



QUANDO $h \to 0$ $\quad t+h \to t \quad$ E $\quad \xi \to t$

$$\lim_{h\to 0} \frac{\phi(t+h)-\phi(t)}{h} =$$

$$= -\alpha_{10}\int_0^t e^{-\alpha_{10}(t-\tau)}\, u_1(\tau)\, d\tau + u_1(t)$$

$$\dot{x}_1(t) = -\alpha_{10}\, e^{-\alpha_{10} t}\, x_{10} +$$

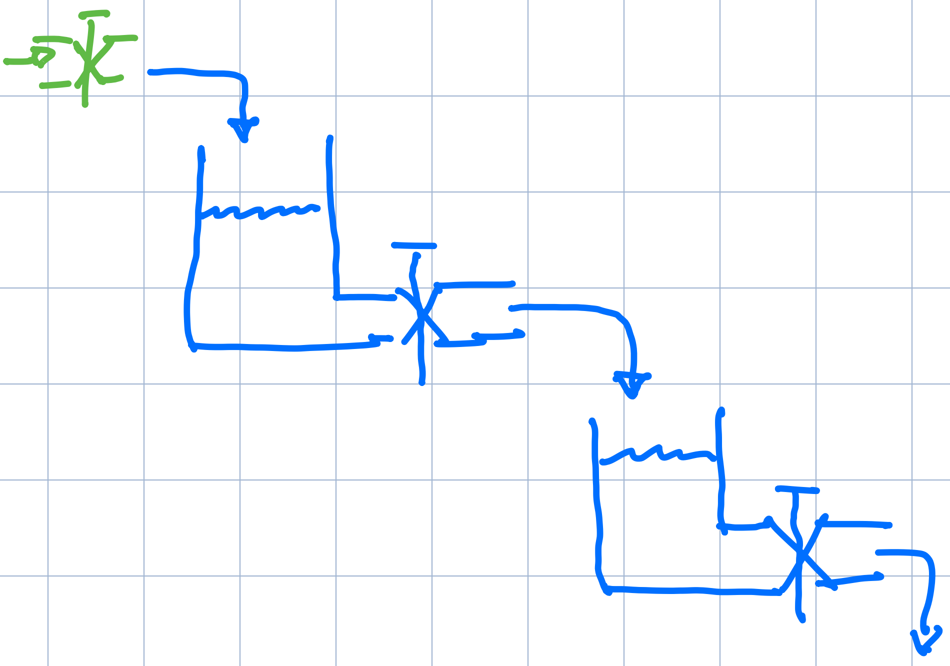
$$+\beta_{11}\left(-\alpha_{10}\int_0^t e^{-\alpha_{10}(t-\tau)}u_1(\tau)\,d\tau+u_1(t)\right)=$$

$$=-\alpha_{10}e^{-\alpha_{10}t}x_{10}-\alpha_{10}\beta_{11}\int_0^t e^{-\alpha_{10}(t-\tau)}u_1(\tau)+$$

$$+\beta_{11}u_1(t)=$$

$$=-\alpha_{10}\left(e^{-\alpha_{10}t}x_{10}+\beta_{11}\int_0^t e^{-\alpha_{10}(t-\tau)}u_1(\tau)d\tau\right)$$

$$+\beta_{11}u_1(t)=-\alpha_{10}x_1(t)+\beta_{11}u_1(t)$$

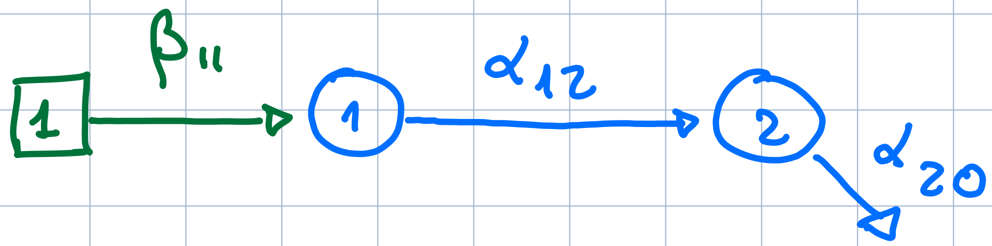


$x_1 \leftarrow$ VOLUME DI RISORSA A MONTE $[m^3]$

$x_2 \leftarrow$ VOLUME DI RISORSA A VALLE $[m^3]$

$u_1 \leftarrow$ AFFLUSSO DI RISORSA PROVENIENTE DA UN FORNITORE ESTERNO $\left[\frac{m^3}{s}\right]$

MISURO IL DEFLUSSO DEL SERBATOIO A VALLE $\left[\frac{m^3}{s}\right]$



$$
\begin{cases}
\underset{\left[\frac{m^3}{s}\right]}{\dot{x}_1(t)} = \beta_{11}\, \underset{\left[\frac{m^3}{s}\right]}{u_1(t)} - \underset{\left[\frac{1}{s}\right]}{\alpha_{12}}\, \underset{[m^3]}{x_1(t)} \\
\dot{x}_2(t) = \alpha_{12}\, x_1(t) - \alpha_{20}\, x_2(t) \\
y_1(t) = \alpha_{20}\, x_2(t)
\end{cases}
$$

$$
\begin{cases}
\dot{X}(t) = A\,X(t) + B\,u(t) \\
y(t) = C\,x(t) + D\,u(t)
\end{cases}
$$

$$
X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2 \qquad u = u_1 \in \mathbb{R}
$$

$Y = y_1 \in \mathbb{R}$ $\qquad n=2,\ m=1,\ p=1$

$A \in \mathbb{R}^{n\times n} = \mathbb{R}^{2\times 2}$

$B \in \mathbb{R}^{n\times m} = \mathbb{R}^{2\times 1}$

$C \in \mathbb{R}^{p\times n} = \mathbb{R}^{1\times 2}$

$D \in \mathbb{R}^{p\times m} = \mathbb{R}^{1\times 1}$

$$A = \begin{bmatrix} -\alpha_{12} & 0 \\ \alpha_{12} & -\alpha_{20} \end{bmatrix} \qquad B = \begin{bmatrix} \beta_{11} \\ 0 \end{bmatrix}$$

$$\dot{x}_1(t) = -\alpha_{12} x_1(t) + \beta_{11} u_1(t)$$
$$\dot{x}_2(t) = \alpha_{12} x_1(t) - \alpha_{20} x_2(t)$$

$$y_1(t) = \alpha_{20} x_2(t)$$

$$C = [\, 0 \quad \alpha_{20} \,] \qquad D = 0$$

$$\alpha_{12} = 0.5 \, \frac{1}{s} \qquad \alpha_{20} = 0.2 \, \frac{1}{s}$$

$$\beta_{11} = 1 \qquad x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$u_1(t) = 1 \, \frac{m^3}{s} \qquad \forall \; t \geq 0$$

$$\begin{cases} \dot{x}_1(t) = -\alpha_{12} x_1(t) + \beta_{11} u_1(t) \\ \dot{x}_2(t) = \alpha_{12} x_1(t) - \alpha_{20} x_2(t) \\ y_1(t) = \alpha_{20} x_2(t) \end{cases}$$

$$x_1(t) = x_{1e} \qquad x_2(t) = x_{2e}$$

$$u_1(t) = u_{1e}$$

$$\begin{cases} 0 = -\alpha_{12}\, x_{1e} + \beta_{11}\, u_{1e} \\ 0 = \alpha_{12}\, x_{1e} - \alpha_{20}\, x_{2e} \\ y_{1e} = \alpha_{20}\, x_{2e} \end{cases}$$

UN MODO DI CALCOLARE L'EQUILIBRIO CONSISTE NELLA "IMPOSIZIONE" DI u_{1e}

$$\begin{cases} 0 = -\alpha_{12}\, x_{1e} + \beta_{11}\, u_{1e} \\ 0 = \alpha_{12}\, x_{1e} - \alpha_{20}\, x_{2e} \end{cases}$$

$$x_{1e} = \frac{\beta_{11}}{\alpha_{12}}\, u_{1e}$$

$$x_{2e} = \frac{\alpha_{12}}{\alpha_{20}}\, x_{1e} = \frac{\cancel{\alpha_{12}}}{\alpha_{20}} \frac{\beta_{11}}{\cancel{\alpha_{12}}}\, u_{1e} =$$

$$= \frac{\beta_{11}}{\alpha_{20}}\, u_{1e}$$

$$y_{1e} = \alpha_{20}\, x_{2e} = \cancel{\alpha_{20}} \frac{\beta_{11}}{\cancel{\alpha_{20}}} u_{1e} = \beta_{11} u_{1e}$$

PARCO MACCHINE

VOGLIO DESCRIVERE UN MODELLO DI FLUSSO CHE DESCRIVE L'ANDAMENTO TEMPORALE DI UNA AGENZIA DI NOLEGGIO AUTOMOBILI

1. COMPARTIMENTI

LE MACCHINE SONO DIVISE IN TRE CATEGORIE IN BASE AI Km PERCORSI

I FINO A 10^4 Km
II DA 10^4 Km A $2\cdot 10^4$ Km
III OLTRE $2\cdot 10^4$ Km

LA MACCHINA PUÒ TROVARSI NELLE SEGUENTI CONFIGURAZIONI

DISPONIBILE, IN MANUTENZIONE.

$$\left.\begin{array}{ll} \mathrm{I} & x_1 \\ \mathrm{II} & x_2 \\ \mathrm{III} & x_3 \end{array}\right\}$$ DISPONIBILI PER CATEGORIA

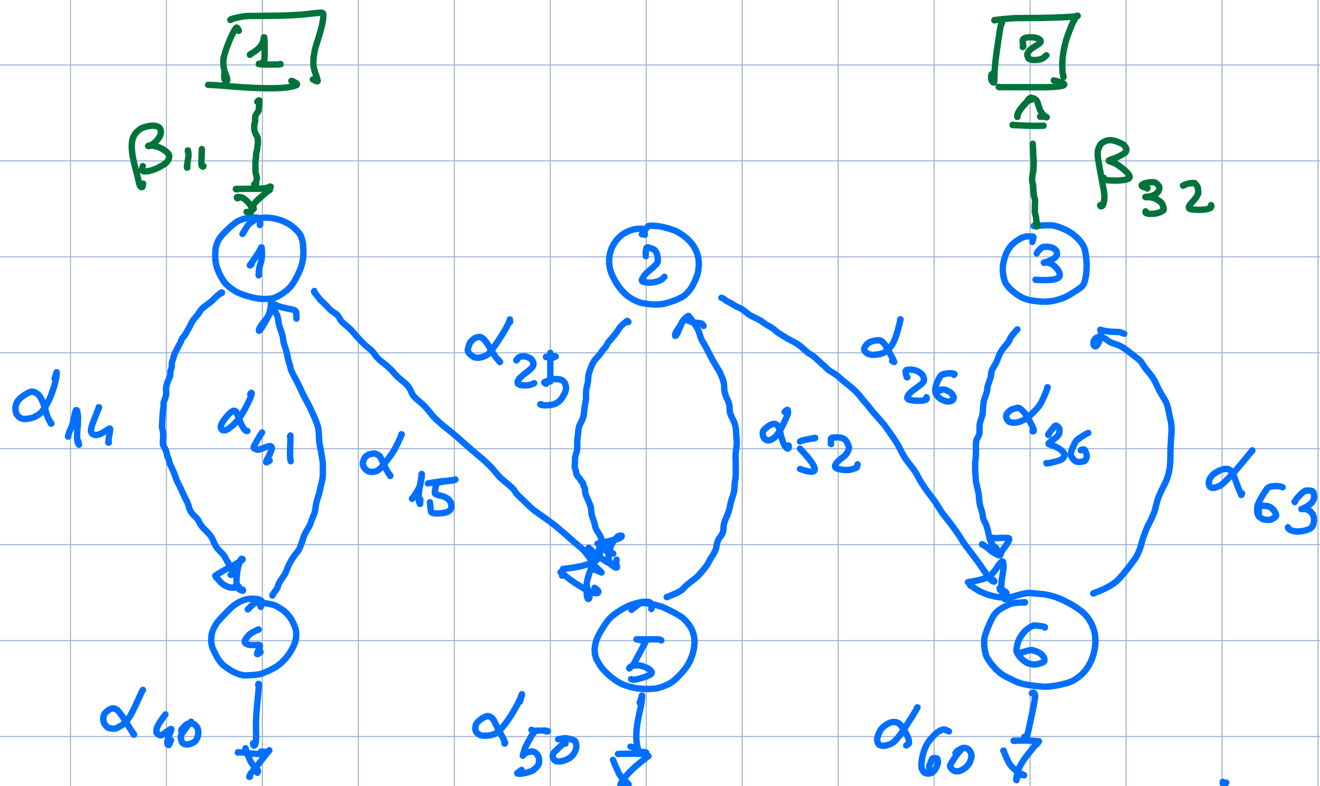
$$\left.\begin{array}{ll} \mathrm{I} & x_4 \\ \mathrm{II} & x_5 \\ \mathrm{III} & x_6 \end{array}\right\}$$ IN MANUTENZIONE PER CATEGORIA

COME UNA MACCHINA PASSA DI CATEGORIA? SE SFORO I KM PASSO IN MANUTENZIONE NELLA CATEGORIA SUPERIORE

POTREBBE ACCADERE CHE, INDIPENDENTEMENTE DALLA CATEGORIA LA MACCHINA VA ROTTAMATA

LE MACCHINE VENGONO ACQUISTATE NUOVE

ACCADE CHE MACCHINE "ANZIANE" MA EFFICIENTI SONO VENDUTE A PRIVATI



$$\dot{x}_1(t) = \beta_{11} u_1(t) + \alpha_{41} x_4(t) - \alpha_{14} x_1(t) - \alpha_{15} x_1(t)$$

$$\dot{x}_2(t) = \alpha_{52} x_5(t) - \alpha_{25} x_2(t) - \alpha_{26} x_2(t)$$

$$\dot{x}_3(t) = \alpha_{63} x_6(t) - \beta_{32} u_2(t) - \alpha_{36} x_3(t)$$

$$\dot{x}_4(t) = \alpha_{14} x_1(t) - \alpha_{40} x_4(t) - \alpha_{41} x_4(t)$$

$$\dot{x}_5(t) = \alpha_{15} x_1(t) + \alpha_{25} x_2(t) - \alpha_{50} x_5(t) - \alpha_{52} x_5(t)$$

$$\dot{x}_6(t) = \alpha_{26} x_2(t) + \alpha_{36} x_3(t) -$$

$$- \alpha_{60} x_6(t) - \alpha_{63} x_6(t)$$

$x(t) \in \mathbb{R}^6$ $\quad$ $u(t) \in \mathbb{R}^2$

$y(t) \in \mathbb{R}^6$

$n = 6, \quad m = 2, \quad p = 6$

$A \in \mathbb{R}^{6\times 6}$ $\quad$ $B \in \mathbb{R}^{6\times 2}$

$C \in \mathbb{R}^{6\times 6}$ $\quad$ $D \in \mathbb{R}^{6\times 2}$

$$A = \begin{bmatrix} -(\alpha_{14}+\alpha_{15}) & 0 & 0 & \alpha_{41} & 0 & 0 \\ 0 & -(\alpha_{25}+\alpha_{26}) & 0 & 0 & \alpha_{52} & 0 \\ 0 & 0 & -\alpha_{36} & 0 & 0 & \alpha_{63} \\ \alpha_{14} & 0 & 0 & -(\alpha_{41}+\alpha_{40}) & 0 & 0 \\ \alpha_{15} & \alpha_{25} & 0 & 0 & -(\alpha_{52}+\alpha_{50}) & 0 \\ 0 & \alpha_{26} & \alpha_{36} & 0 & 0 & -(\alpha_{63}+\alpha_{60}) \end{bmatrix}$$

$$B = \begin{bmatrix} \beta_{11} & 0 \\ 0 & 0 \\ 0 & -\beta_{32} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = I_6 \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = O_{6\times 2}$$