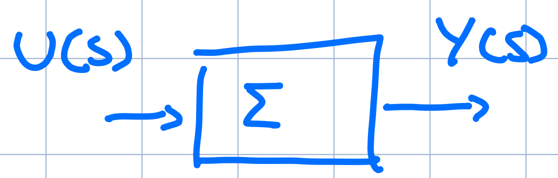
$$G(s) = \frac{s-1}{s^3+8s^2+25s+26}$$



$$u(t) = 1(t) \quad \doteq \quad U(s) = \frac{1}{s}$$

I POLI SONO $-2, -3 \pm 2j$

MODI NATURALI

$$-2 \Rightarrow e^{-2t} \, 1(t)$$

$$-3 \pm 2j = \sigma \pm j\omega \Rightarrow \begin{array}{l} e^{-3t} \sin(2t) \, 1(t) \\ e^{-3t} \cos(2t) \, 1(t) \end{array}$$

ABBIAMO UNO ZERO IN 1

$$C_i = \lim_{s \to p_i} (s-p_i) \cdot F(s)$$

$$C_1 = \lim_{s \to 0} s \cdot Y_F(s) = \lim_{s \to 0} \cancel{s} \cdot G(s) \cdot \frac{1}{\cancel{s}} =$$

$$= G(0)$$

$$y_F(t) = G(0) \cdot 1(t) + \dots$$

$$C_3 = -\frac{17}{130} - \frac{3}{65} j \qquad C_4 = \bar{C}_3$$

$$\frac{C_3}{s+3-2j} + \frac{\bar{C}_3}{s+3+2j}$$

$$C_3 \, e^{(-3+2j)t} + \bar{C}_3 \, e^{(-3-2j)t}$$

$$e^{-3t}\left(C_3 e^{2jt} + \bar{C}_3 e^{-2jt}\right) =$$

$$= 2e^{-3t} \operatorname{Re}\left(C_3 e^{2jt}\right) =$$

$$= 2e^{-3t} \operatorname{Re}\left(\left(\operatorname{Re}(C_3) + j\operatorname{Im}(C_3)\right)\left(\cos(2t) + \right.\right.$$

$$\left.\left. + j\sin(2t)\right)\right) =$$

$$= 2e^{-3t}\left(\operatorname{Re}(C_3)\cos(2t) - \operatorname{Im}(C_3)\sin(2t)\right)$$

MODELLO I/U NEL DOMINIO DEL TEMPO

$$G(s) = \frac{s-1}{s^3 + 8s^2 + 25s + 26}$$

$$\frac{Y_f(s)}{U(s)} = \frac{s-1}{s^3+8s^2+25s+26}$$

$$Y_f(s)\,(s^3+8s^2+25s+26) = U(s)(s-1)$$

$$s^3 Y_f(s) + 8s^2 Y_f(s) + 25\,s\,Y_f(s) + 26\,Y_f(s) =$$

$$= s\,U(s) - U(s)$$

$$\mathcal{L}\left[f^{(n)}(t)\right] = s^n F(s) - s^{n-1} f(0) - s^{n-2} \dot{f}(0) - \ldots - s f^{(n-2)}(0) - f^{(n-1)}(0)$$

$$\dddot{y}(t) + 8\ddot{y}(t) + 25\dot{y}(t) + 26\,y(t) =$$

$$= \dot{u}(t) - u(t)$$

ARMA

$$G(z) = \frac{z-3}{z^3 + \frac{z^2}{4} - \frac{z}{16} + \frac{1}{32}}$$

POLI

$$z = -\frac{1}{2} \Rightarrow \left(-\frac{1}{2}\right)^k 1(k)$$

$$z = \frac{1}{8} \pm j\frac{\sqrt{3}}{8} = \rho e^{\pm j\theta}, \quad \rho = \frac{1}{4}, \quad \theta = \frac{\pi}{3}$$

$$\Rightarrow \rho^k \cos(\theta k)\, 1(k) \quad , \quad \rho^k \sin(\theta k)\, 1(k)$$

$$\left(\frac{1}{4}\right)^k \cos\left(\frac{\pi}{3} k\right) 1(k), \quad \left(\frac{1}{4}\right)^k \sin\left(\frac{\pi}{3} k\right) 1(k)$$

$$u(k) = 1(k) \div U(z) = \frac{z}{z-1}$$

$$Y_F(z) = G(z)\,\frac{z}{z-1}$$

$$D_1 = \lim_{z\to 1}\,(z-1)\,\frac{Y_F(z)}{z} =$$

$$= \lim_{z\to 1}\,\cancel{(z-1)}\,G(z)\,\frac{\cancel{z}}{\cancel{z-1}}\cdot\frac{1}{\cancel{z}} = G(1)$$

$$y(k) = G(1)\cdot 1(k) + \dots$$

$$y_F(k) = -\frac{64}{39}\,1(k) + \frac{16}{3}\left(-\frac{1}{2}\right)^k 1(k)$$

$$\frac{D_3}{z-\left(\frac{1}{4}\right)e^{j\frac{\pi}{3}}}+\frac{\overline{D}_3}{z-\left(\frac{1}{4}\right)e^{-j\frac{\pi}{3}}}$$

$$D_3\left(\frac{1}{4}\right)^k e^{j\frac{\pi}{3}k}+\overline{D}_3\left(\frac{1}{4}\right)^k e^{-j\frac{\pi}{3}k}$$

$$\left(\frac{1}{4}\right)^k\left[D_3\, e^{j\frac{\pi}{3}k}+\overline{D}_3\, e^{-j\frac{\pi}{3}k}\right]$$

$$2\left(\frac{1}{4}\right)^k \mathrm{Re}\left(D_3\, e^{j\frac{\pi}{3}k}\right) =$$

$$2\left(\frac{1}{4}\right)^k\left(\mathrm{Re}(D_3)\cos\left(\frac{\pi}{3}k\right)-\mathrm{Im}(D_3)\sin\left(\frac{\pi}{3}k\right)\right)$$

MODELLO ARMA TD

$$G(z) = \frac{z-3}{z^3 + \frac{z^2}{4} - \frac{z}{16} + \frac{1}{32}}$$

$$Y_f(z)\left(z^3 + \frac{z^2}{4} - \frac{z}{16} + \frac{1}{32}\right) = U(z)(z-3)$$

$$z^3 Y_f(z) + \frac{z^2}{4} Y_f(z) - \frac{z}{16} Y_f(z) + \frac{1}{32} Y_f(z) =$$

$$= z\,U(z) - 3\,U(z)$$

$$\mathcal{Z}[f(k+n)] = z^n F(z) - z^n f(0) - z^{n-1} f(1) -$$

$$- \dots - z^2 f(n-2) - z\, f(n-1)$$

$$y(k+3) + \frac{1}{4} y(k+2) - \frac{1}{16} y(k+1) + \frac{1}{32} y(k) =$$

$$= u(k+1) - 3u(k)$$

$$y(k') + \frac{1}{4} y(k'-1) - \frac{1}{16} y(k'-2) + \frac{1}{32} y(k'-3) =$$

$$= u(k'-2) - 3u(k'-3)$$

$$F(s) = \frac{n_F(s)}{(s-p_1)^{\nu_1}(s-p_2)^{\nu_2} \cdots (s-p_r)^{\nu_r}} =$$

$$= \frac{C_{11}}{(s-p_1)} + \frac{C_{12}}{(s-p_1)^2} + \dots + \frac{C_{1\nu_1}}{(s-p_1)^{\nu_1}} +$$

$$+ \frac{C_{21}}{(s-p_2)} + \frac{C_{22}}{(s-p_2)^2} + \dots + \frac{C_{2\nu_2}}{(s-p_2)^{\nu_2}}$$

$\dots$

$$+ \frac{C_{r1}}{s-p_r} + \frac{C_{r2}}{(s-p_r)^2} + \dots + \frac{C_{r\nu_r}}{(s-p_r)^{\nu_r}}$$

$i = 1, \dots, r \qquad j = 1, \dots, \nu_i$

$$C_{ij} = \frac{1}{(\nu_i - j)!} \cdot \lim_{s \to p_i} \frac{d^{\nu_i - j}}{ds^{\nu_i - j}} \left((s-p_i)^{\nu_i} F(s) \right)$$

$$\frac{1}{(s-a)^n} \doteq \frac{t^{(n-1)}}{(n-1)!} e^{at} 1(t)$$

$n \geq 1$

$$G(s) = \frac{s-3}{s^3 + 6s^2 + 9s + 4}$$

-4, $m.a. = 1$, $e^{-4t} 1(t)$

-1, $m.a = 2$, $e^{-t} 1(t)$, $t \cdot e^{-t} 1(t)$

$$C_{32} = \frac{1}{(2-2)!} \lim_{s \to -1} \frac{d^{(2-2)}}{ds^{(2-2)}} ((s+1)^2 Y_f(s))$$

$$= \lim_{s \to -1} (s+1)^2 Y_f(s)$$

$$C_{31} = \frac{1}{(2-1)!} \lim_{s \to -1} \frac{d^{2-1}}{ds^{2-1}} ((s+1)^2 Y_f(s))$$

$$= \frac{1}{1!} \lim_{s \to -1} \frac{d}{ds} ((s+1)^2 Y_f(s))$$

$$Y_F(s) = -\frac{3}{4} \cdot \frac{1}{s} + \frac{7}{36} \frac{1}{s+4} + \frac{4}{3} \frac{1}{(s+1)^2} +$$

$$+ \frac{5}{9} \frac{1}{(s+1)}$$

$$y_F(t) = -\frac{3}{4} 1(t) + \frac{7}{36} e^{-4t} 1(t) + \frac{5}{9} e^{-t} 1(t) +$$

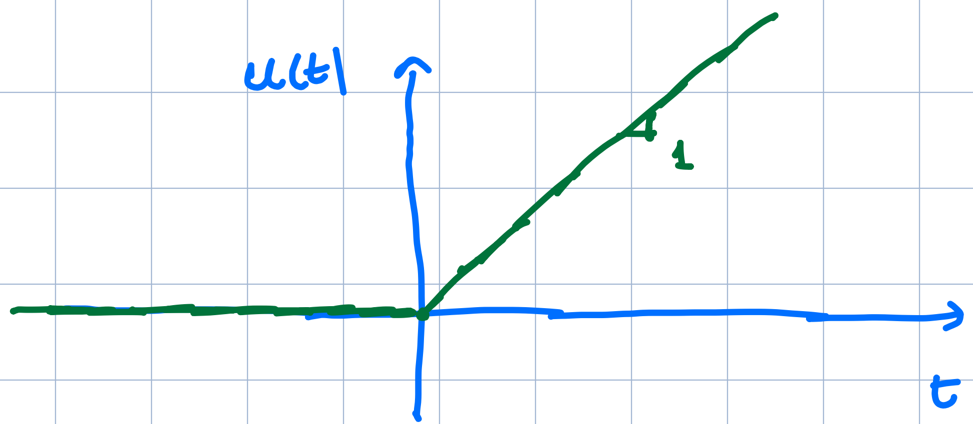
$$+ \frac{4}{3} t \cdot e^{-t} 1(t)$$

$$\frac{1}{(s-a)^n} \overset{\cdot}{\underset{\cdot}{=}} \frac{t^{n-1}}{(n-1)!} e^{at} 1(t)$$

TIPOLOGIE DI INGRESSI CANONICI

POLINOMIALI	PERIODICI
$1(t)$	$\sin(\omega t)\, 1(t)$, $\cos(\omega t)\, 1(t)$
$t \cdot 1(t)$	

$$u(t) = \begin{cases} t \,, & t \geq 0 \\ 0 \,, & t < 0 \end{cases}$$



RAMPA DI PENDENZA UNITARIA

$$\frac{1}{(s-a)^n} \doteq \frac{t^{n-1}}{(n-1)!} e^{at} \, 1(t)$$

$$\frac{1}{s^2} \doteq t \cdot 1(t)$$

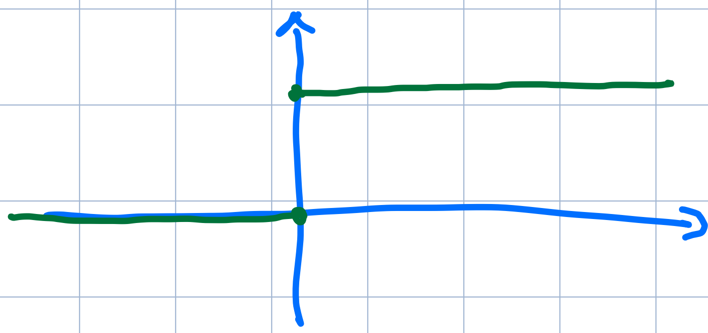
$$u(t) = \begin{cases} \frac{t^2}{2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$\frac{1}{s^3} \doteq \frac{t^2}{2} \cdot 1(t)$$

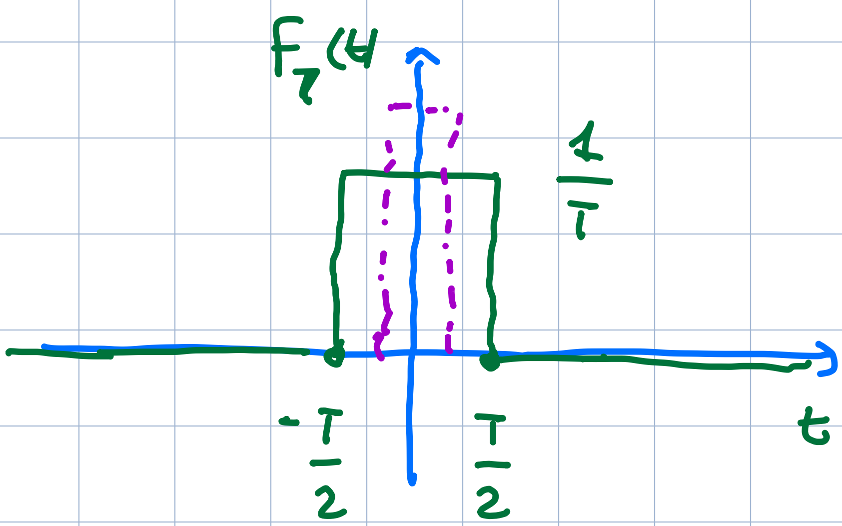
GRADINO $\overset{?}{\longleftrightarrow}$ RAMPA

$1(t)$ $\qquad$ $t \cdot 1(t)$

IL GRADINO È LA DERIVATA ORDINARIA DELLA RAMPA.



$$F_T(t) = \begin{cases} \dfrac{1}{T}, & |t| \le T/2 \\ 0, & |t| > T/2 \end{cases}$$



$$\lim_{T \to 0} F_T(t) \, .$$

$\phi(t)$ CONTINUA SU $\mathbb{R}$

È INTEGRABILE

$$\int_{-\infty}^{+\infty} f_T(t)\,\phi(t)\,dt =$$

$$= \int_{-T/2}^{T/2} \frac{1}{T}\,\phi(t)\,dt =$$

$$= \frac{1}{T}\int_{-T/2}^{+T/2} \phi(t)\,dt = \frac{1}{\cancel{T}}\left(\cancel{T}\cdot\phi(\xi)\right)$$

$$-\frac{T}{2} < \xi < \frac{T}{2}$$

$$= \phi(\xi)$$

$$\lim_{T\to 0}\int_{-\infty}^{+\infty} f_T(t)\,\phi(t)\,dt = \lim_{T\to 0}\phi(\xi) = \phi(0)$$

IMPULSO DI DIRAC

IMPULSO ELEMENTARE

$$f_T(t) = \begin{cases} \frac{1}{T} & |t| \le T/2 \\ 0 & |t| > T/2 \end{cases}$$

$$\phi(t) = e^{-st} 1(t) \qquad s \in \mathbb{C}$$

$$\lim_{T \to 0} \int_{-\infty}^{+\infty} f_T(t)\, e^{-st} 1(t)\, dt = e^{-st} 1(t) \Big|_{t=0} =$$

$$= 1$$