$$x(k+1) = \begin{bmatrix} 1/8 & 1/8 \\ -13/8 & 3/8 \end{bmatrix} x(k)$$

$$x_0 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$x(k) = A^k x_0$$

$$A \overset{\hat{T}}{\sim} \hat{\Lambda} \qquad \hat{T} = [\,Re(v) \quad Im(v)\,]$$

1. $z_0 = \hat{T}^{-1} x_0$

2. $A^k \overset{\hat{T}}{\sim} \hat{\Lambda}^k$

$$\hat{\Lambda} = \begin{bmatrix} \rho\cos(\theta) & \rho\sin(\theta) \\ -\rho\sin(\theta) & \rho\cos(\theta) \end{bmatrix}$$

$$\hat{\Lambda}^k = \begin{bmatrix} \rho^k\cos(\theta k) & \rho^k\sin(\theta k) \\ -\rho^k\sin(\theta k) & \rho^k\cos(\theta k) \end{bmatrix}$$

$$x(k+1) = A^k x_0$$

$$A^k \hat{T} = \hat{T}\hat{\Lambda}^k$$

$$z_0 = \hat{T}^{-1} x_0 \qquad x_0 = \hat{T} z_0$$

$$x(k+1) = A^k \hat{T} z_0 = \boxed{\hat{T}\hat{\Lambda}^k z_0} =$$

$$= \hat{T} \begin{bmatrix} \rho^k \cos(\theta k) & \rho^k \sin(\theta k) \\ -\rho^k \sin(\theta k) & \rho^k \cos(\theta k) \end{bmatrix} z_0$$

$$A \hat{T} = \hat{T} \hat{\Lambda}$$

$$\hat{\Lambda} = \hat{T}^{-1} A \hat{T}$$

SE HO UNA MATRICE 2x2 E VOGLIO VERIFICARE CHE QUESTA MATRICE È IN FORMA CANONICA DI ROTATION - SCALING

$$\hat{\Lambda} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \qquad \begin{matrix} a > 0 \\ b > 0 \end{matrix}$$

OPPURE

$$\hat{\Lambda} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$y(k) = \begin{bmatrix} 1 & -3 \end{bmatrix} x(k)$$

$$y(k) = C \hat{T} \hat{\Lambda}^{k} z_0$$

COSA RAPPRESENTANO DA UN PUNTO GEOMETRICO LE COLONNE DI $\hat{T}$?

$$\hat{T} = \begin{bmatrix} Re(v) & Im(v) \end{bmatrix}$$

$$x(k) = \hat{T}\, z(k)$$

$$\begin{cases} z(k+1) = \hat{\Lambda}\, z(k) \\ z(0) = z_0 \end{cases}$$

$z(k)$ SONO CO RAPPRESENTANO GLI STATI DEL SISTEMA IN QUEL PIANO CARTESIANO IL CUI ASSE DELLE ASCISSE È LA I^a COLONNA DI $\hat{T}$ E L'ASSE DELLE ORDINATE È LA II^a COLONNA DI $\hat{T}$.

$$z(k) = \hat{\Lambda}^k z_0 =$$

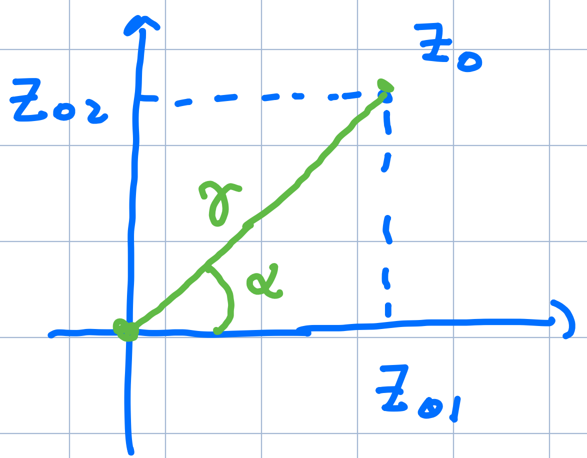
$$= \begin{bmatrix} \rho^k \cos(k\theta) & \rho^k \sin(k\theta) \\ -\rho^k \sin(k\theta) & \rho^k \cos(k\theta) \end{bmatrix} z_0$$

QUANTO VALE LA LUNGHEZZA DI $z(k)$?

$$z_0 = \begin{bmatrix} z_{01} \\ z_{02} \end{bmatrix}$$

$$z(k) = \begin{bmatrix} \rho^k \cos(\theta k) & \rho^k \sin(\theta k) \\ -\rho^k \sin(\theta k) & \rho^k \cos(\theta k) \end{bmatrix} \begin{bmatrix} z_{01} \\ z_{02} \end{bmatrix}$$

$$= \begin{bmatrix} \rho^{k}\left(z_{01}\cos(\theta k) + z_{02}\sin(\theta k)\right) \\ \\ \rho^{k}\left(-z_{01}\sin(\theta k) + z_{02}\cos(\theta k)\right) \end{bmatrix}$$



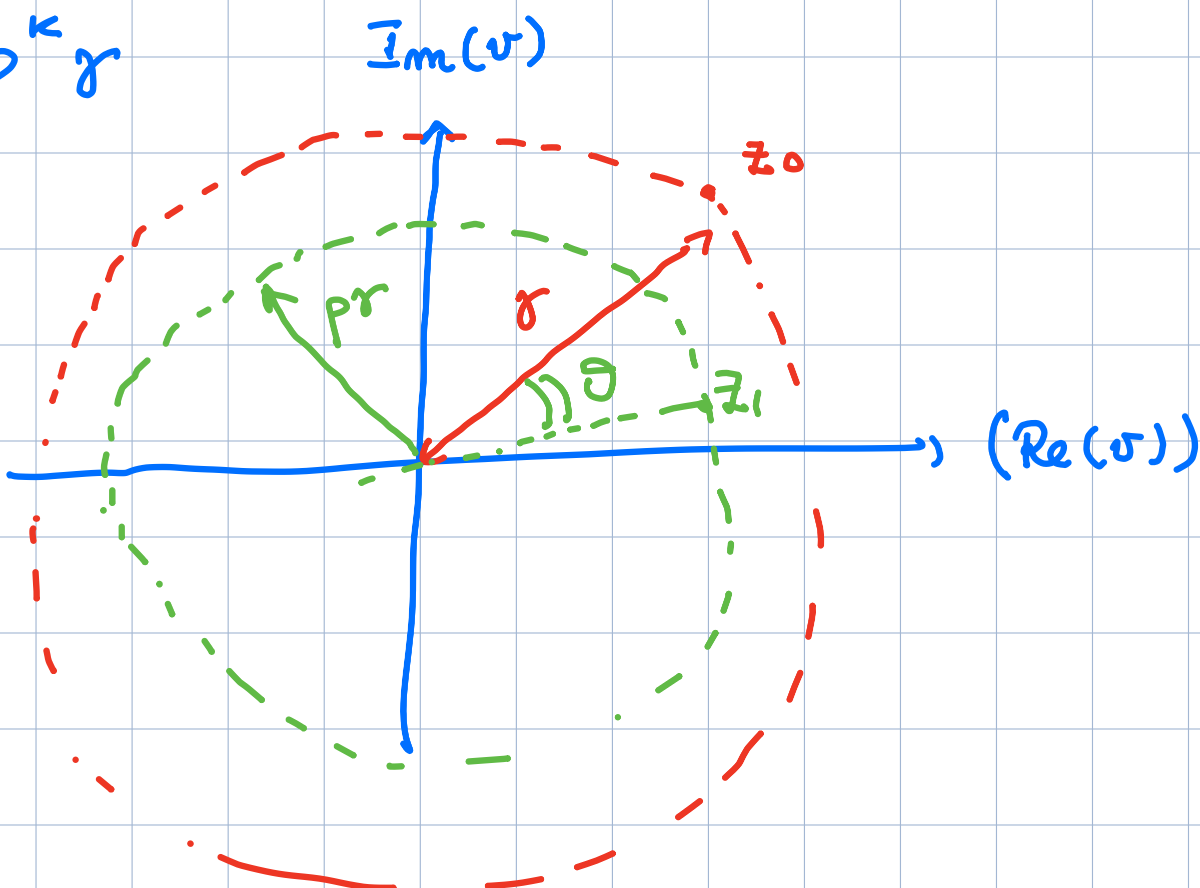
$$z_{01} = \gamma\cos(\alpha) \qquad z_{02} = \gamma\sin(\alpha)$$

$$z(k) = \begin{bmatrix} \rho^{k}\left(\gamma\cos(\alpha)\cos(\theta k) + \gamma\sin(\alpha)\sin(\theta k)\right) \\ \\ \rho^{k}\left(-\gamma\cos(\alpha)\sin(\theta k) + \gamma\sin(\alpha)\cos(\theta k)\right) \end{bmatrix}$$

$$= \begin{bmatrix} \rho^k r \cdot \cos(\alpha - \theta k) \\ \\ \rho^k r \cdot \sin(\alpha - \theta k) \end{bmatrix}$$

$$|z(k)| = \sqrt{\rho^{2k} r^2 \cos^2(\alpha - \theta k) + \rho^{2k} r^2 \sin^2(\alpha - \theta k)} =$$

$$= \rho^k r$$



Z(K) SI "SPOSTA" LUNGO CIRCONFE-
RENZE CONCENTRICHE VARIANDO

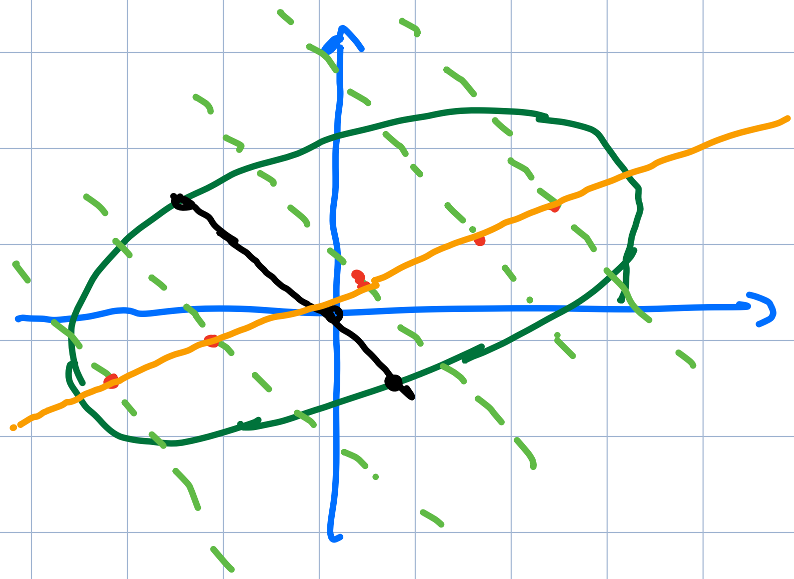
LA FASE IN SENSO ORARIO DI θ RADIANTI DA UN PASSO AL SUCCESSIVO E SCALANDO IL MODULO DI ρ

$$x(k) = \hat{T}\, z(k) =$$

$$= \begin{bmatrix} Re(v) & Im(v) \end{bmatrix} \begin{bmatrix} z_1(k) \\ \\ z_2(k) \end{bmatrix}$$

$$x(k) = Re(v)\, z_1(k) + Im(v)\, z_2(k)$$

$Re(v)$ e $Im(v)$ SONO I DIAMETRI CONIUGATI DI UN ELLISSI



$$x = v_1 \cos(\xi) + v_2 \sin(\xi)$$

N.B. I DIAMETRI CONIUGATI NON SONO IN GENERALE ORTOGONALI FRA LORO.

$$x_0 = T z_0$$

CASO TC

$$\begin{cases} \dot{x}(t) = A x(t) \\ x(0) = x_0 \\ y(t) = C x(t) \end{cases}$$

$A \in \mathbb{R}^{2\times 2}$ E PRESENTA UNA COPPIA DI AUTOVALORI C.C.

$$\lambda, \bar{\lambda} \in \mathbb{C}$$

$T = [v \quad \bar{v}]$

$\hat{T} = [Re(v) \quad Im(v)]$

$$\hat{\Lambda} = \begin{bmatrix} Re(\lambda) & Im(\lambda) \\ -Im(\lambda) & Re(\lambda) \end{bmatrix}$$

$$\mathrm{Re}(\lambda) = \sigma \qquad \mathrm{Im}(\lambda) = \omega$$

$$\hat{\Lambda} = \begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix}$$

$$x(t) = e^{At} x_0$$

$$y(t) = C\,x(t) = C\,e^{At} x_0$$

$$e^{At} \overset{T}{\sim} e^{\Lambda t}$$

$$e^{At} \overset{\hat{T}}{\sim} e^{\hat{\Lambda} t}$$

$$x(t) = e^{At} x_0 = e^{At} \hat{T} z_0 =$$

$$= \hat{T} e^{\hat{\Lambda} t} z_0$$

$$y(t) = C\,x(t) = C\,\hat{T}\,e^{\hat{\Lambda} t}\,z_0$$

$$e^{\begin{bmatrix} \sigma & \omega \\ -\omega & \sigma \end{bmatrix} t} =$$

$$= \begin{bmatrix} e^{\sigma t}\cos(\omega t) & e^{\sigma t}\sin(\omega t) \\ -e^{\sigma t}\sin(\omega t) & e^{\sigma t}\cos(\omega t) \end{bmatrix}$$

I MODI NATURALI SONO

$$e^{\sigma t}\cos(\omega t) \qquad t \geq 0$$

$$e^{\sigma t}\sin(\omega t) \qquad t \geq 0$$

$\sigma = Re(\lambda) \qquad \omega = Im(\lambda)$

FUNZIONI PSEUDO - PERIODICHE

$T = \frac{2\pi}{\omega}$ $\quad \omega \leftarrow$ PULSAZIONE $\left[\frac{rad}{s}\right]$

SE $\sigma < 0$

$$\lim_{t \to +\infty} e^{\sigma t} \cos(\omega t) = 0$$

$$\lim_{t \to +\infty} e^{\sigma t} \sin(\omega t) = 0$$

SE $\sigma > 0$ I MODI SONO "DIVERGENTI"

SE $\sigma = 0$ I MODI SONO LIMITATI MA NON CONVERGENTI (A ZERO)

$$\dot{x}(t) = \begin{bmatrix} -1/2 & 1/2 \\ -5/2 & -3/2 \end{bmatrix} x(t)$$

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$y(t) = \begin{bmatrix} 2 & -1 \end{bmatrix} x(t)$

$\lambda^2 + 2\lambda + 2$

$A^2 + 2A + 2I_2$

TEOREMA DI CAYLEY - HAMILTON

OGNI MATRICE $A \in \mathbb{R}^{n \times n}$ È UNO ZERO DEL SUO POLINOMIO CARATTERISTICO.

$$p_A(\lambda) = \det(\lambda I - A)$$

$$p_A(A) = 0_{n \times n}$$

$$\lambda = -1 \pm j$$

$\sigma = -1 \quad \omega = 1$

$e^{-t} \cos(t)$

$e^{-t} \sin(t)$

GLI ZERI DELLE FUNZIONI PSEUDO - PERIODICHE

$$e^{\sigma t} \cos(\omega t)$$

$$e^{\sigma t} \sin(\omega t)$$

SONO DATE DAGLI ZERI DELLA COMPONENTE PERIODICA

$$e^{\sigma t} \cos(\omega t) = 0$$

$$\omega t_1 = \frac{\pi}{2}, \quad \omega t_2 = \frac{3}{2}\pi, \quad \omega t_3 = \frac{5}{2}\pi, \ \dots$$

$$t_1 = \frac{\pi}{2\omega}, \quad t_2 = \frac{3\pi}{2\omega}, \quad t_3 = \frac{5\pi}{2\omega} \ \dots$$

$$\frac{(2n+1)\pi}{2\omega}$$

$$e^{\sigma t}\sin(\omega t) = 0$$

$$\omega t_1 = 0 \qquad \omega t_2 = \pi \qquad \omega t_3 = 2\pi \; \text{---}$$

$$t_1 = 0 \qquad t_2 = \frac{\pi}{\omega} \qquad t_3 = \frac{2\pi}{\omega} \; \text{---}$$

$$\frac{n\pi}{\omega}$$

1. $z_0 = \hat{T}^{-1} x_0$

2. $x(t) = e^{At} \cdot x_0 = \hat{T} e^{\hat{\Lambda} t} z_0 =$

$$= \hat{T} \begin{bmatrix} e^{\sigma t}\cos(\omega t) & e^{\sigma t}\sin(\omega t) \\ -e^{\sigma t}\sin(\omega t) & e^{\sigma t}\cos(\omega t) \end{bmatrix} z_0$$