A presenta autovalori reali e distinti

$$e^{At} \overset{T}{\sim} e^{\Lambda t}$$

$$e^{At} = I_n + At + \frac{A^2 t^2}{2!} + \dots + \frac{A^k t^k}{k!} + \dots =$$

$$= \sum_{i=0}^{+\infty} \frac{A^i t^i}{i!}$$

$$e^{At} \quad T$$

$$T = [\, v_1 \;\; v_2 \;\; \dots \;\; v_n \,] \qquad A v_i = \lambda_i v_i$$

$$e^{At} T = \left(\sum_{i=0}^{+\infty} \frac{A^i t^i}{i!} \right) T =$$

$$= \sum_{i=0}^{+\infty} \frac{A^i T t^i}{i!} = \sum_{i=0}^{+\infty} \frac{T \Delta^i t^i}{i!}$$

$$A^i \overset{T}{\sim} \Delta^i$$

$$A^i T = T \Delta^i$$

$$= T \left(\sum_{i=0}^{+\infty} \frac{\Delta^i t^i}{i!} \right) = T e^{\Delta t}$$

$$e^{At} T = T e^{\Delta t}$$

$$x_e(t) = e^{At} \cdot x_0$$

$$e^{At} = T e^{\Lambda t} T^{-1}$$

$$x_e(t) = T e^{\Lambda t} T^{-1} x_0 = T e^{\Lambda t} z_0 =$$

$$= \begin{pmatrix} v_1 & v_2 & \cdots & v_n \end{pmatrix} \begin{pmatrix} e^{\lambda_1 t} & & & 0 \\ & e^{\lambda_2 t} & & \\ & & \ddots & \\ 0 & & & e^{\lambda_n t} \end{pmatrix} \begin{pmatrix} z_{01} \\ z_{02} \\ \vdots \\ z_{0n} \end{pmatrix}$$

$$= \begin{pmatrix} v_1 & v_2 & \cdots & v_n \end{pmatrix} \begin{pmatrix} e^{\lambda_1 t} z_{01} \\ e^{\lambda_2 t} z_{02} \\ \vdots \\ e^{\lambda_n t} z_{0n} \end{pmatrix} =$$

$$= v_1 e^{\lambda_1 t} z_{01} + v_2 e^{\lambda_2 t} z_{02} + \dots + v_n e^{\lambda_n t} z_{0n}$$

$$= \sum_{i=1}^{n} v_i e^{\lambda_i t} z_{0i} .$$

UN SISTEMA LTI-TC I CUI AUTOVALORI

$$\lambda_1 , \lambda_2 , \dots , \lambda_n$$

SONO REALI E DISTINTI PRESENTA

n MODI NATURALI ESPONENZIALI

$$e^{\lambda_1 t} , e^{\lambda_2 t} , \dots , e^{\lambda_n t}$$

SE GLI AUTOVALORI SONO TUTTI STRETTAMENTE NEGATIVI

$$\lambda_i < 0 \quad \forall i$$

ALLORA

$$\lim_{t \to +\infty} x_e(t) = 0_n$$

$$\dot{x}(t) = \begin{bmatrix} -8 & 3/4 & -2 \\ 0 & -2 & 0 \\ 0 & -3/2 & -4 \end{bmatrix} x(t)$$

$$x_0 = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$$

$$\lambda_1 = -8 \Rightarrow e^{\lambda_1 t} = e^{-8t}$$

$$\lambda_2 = -4 \Rightarrow e^{\lambda_2 t} = e^{-4t}$$

$$\lambda_3 = -2 \Rightarrow e^{\lambda_3 t} = e^{-2t}$$

COSTANTE DI TEMPO

$$e^{-\alpha t} = e^{-\frac{t}{T}}$$

α È IL VALORE ASSOLUTO DELL'ARGOMENTO DELL'ESPONENZIALE.

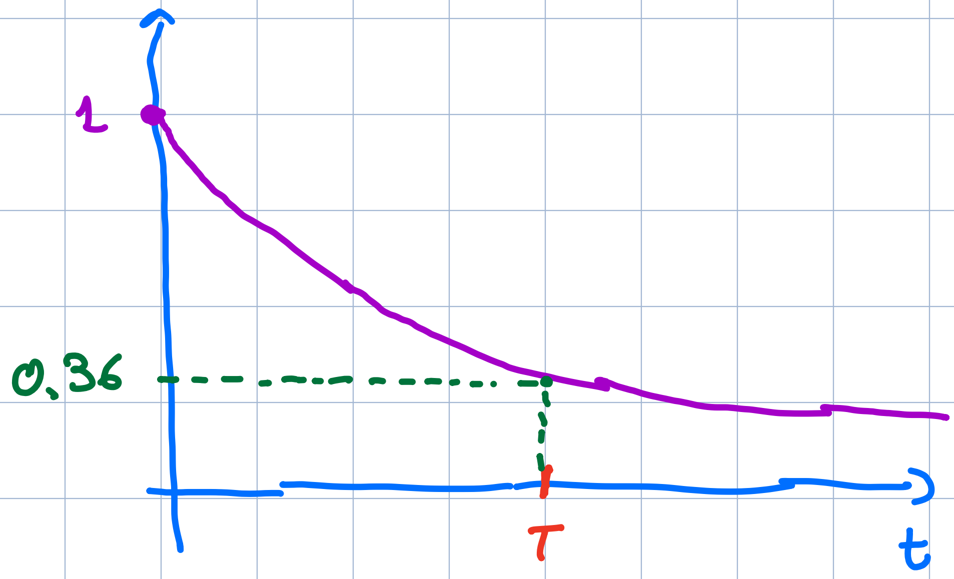
$$\alpha = |-\alpha|$$

$$T = \frac{1}{\alpha} = \alpha^{-1}$$

$T \rightarrow$ COSTANTE DI TEMPO DELLA FUNZIONE ESPONENZIALE.

$[T] = [sec]$ $\quad [\alpha] = [sec^{-1}]$

$$e^{-\frac{t}{T}}\Big|_{t=T} = e^{-1} = \frac{1}{e} \simeq 0.36$$



ESPONENZIALE "LENTO" $\Rightarrow$ T "ELEVATA"

" "VELOCE" $\Rightarrow$ T "PICCOLA"

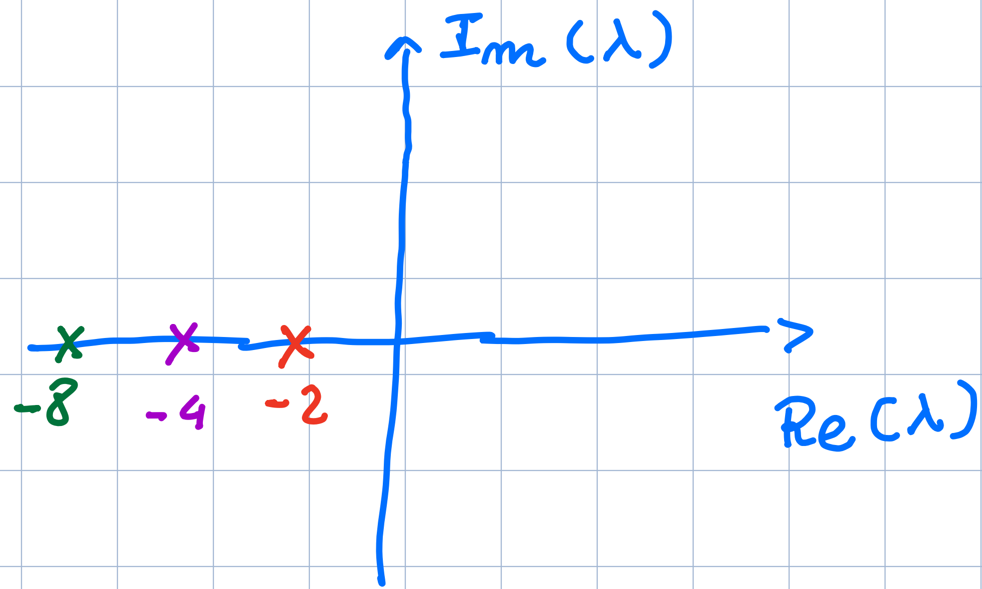
ESPONENZIALE "LENTO" $\Rightarrow$ α "PICCOLA"

" "VELOCE" $\Rightarrow$ α "ELEVATA"

$\lambda_1 = -8 \Rightarrow \alpha_1 = 8 \frac{1}{sec}$, $T_1 = 0.125\ sec$

$\lambda_2 = -4 \Rightarrow \alpha_2 = 4 \frac{1}{sec}$, $T_2 = 0.25\ sec$

$\lambda_3 = -2 \Rightarrow \alpha_3 = 2 \frac{1}{sec}$, $T_3 = 0.5\ sec$



MODO DOMINANTE ⇒ QUELLO ASSOCIATO ALL'AUTOVALORE PIÙ VICINO ALL'ASSE IMMAGINARIO

$$e^{-2t}$$

$$x_e(t) = \sum_{i=1}^{3} v_i \, e^{\lambda_i t} z_{0i} =$$

$$= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-8t} (3) + \begin{pmatrix} -1/2 \\ 0 \\ 1 \end{pmatrix} e^{-4t} \left(-\frac{3}{2}\right) +$$

$$+ \begin{pmatrix} -1/2 \\ -4/3 \\ 1 \end{pmatrix} e^{-2t} \left(\frac{3}{2} \right)$$

$$x_0 = \alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad x_e(t) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} e^{-8t} (\alpha)$$

$$x_0 = \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix}$$

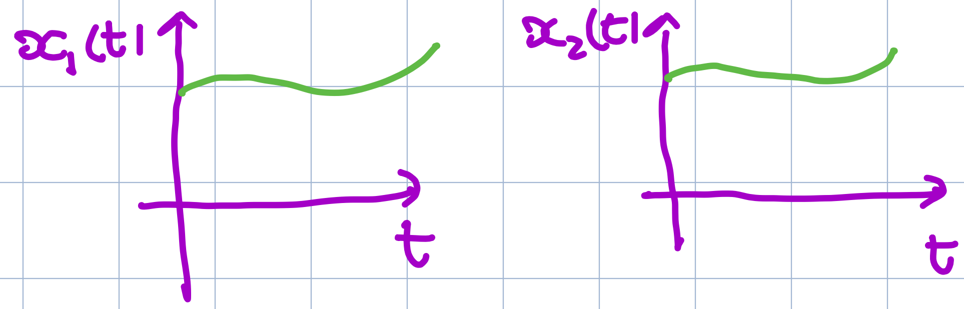
$$x_0 = \alpha \begin{pmatrix} -1/2 \\ -4/3 \\ 1 \end{pmatrix} \qquad \alpha \in \mathbb{R}$$

$$x_0 = \begin{pmatrix} -1 \\ -8/3 \\ 2 \end{pmatrix}$$

$$\dot{x}(t) = \begin{bmatrix} -10/7 & 6/7 \\ 9/7 & -25/7 \end{bmatrix} x(t)$$

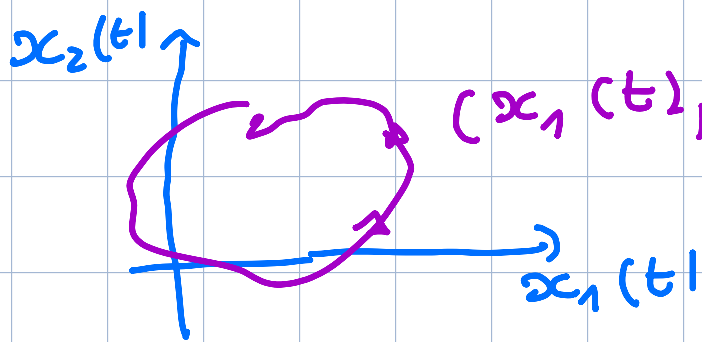
DEF.

MOVIMENTO $\Rightarrow$ $(t, x(t))$



TRAIETTORIA $\Rightarrow$ PROIEZIONE SULLO SPAZIO DI STATO DEL MOVIMENTO

$$(x_1(t), x_2(t))$$



$$e^{(a+b)t} = e^{at} \cdot e^{bt}$$

$$e^{(A+B)t} \neq e^{At} \cdot e^{Bt}$$

SE A e B COMMUTANO

$$A \cdot B = B \cdot A$$

ALLORA

$$e^{(A+B)t} = e^{At} \cdot e^{Bt}$$

$$e^{(A+B)t}\Big|_{t=0} = e^{At}\Big|_{t=0} \cdot e^{Bt}\Big|_{t=0}$$

$$I_n = I_n \cdot I_n$$

$$(A+B) \cdot e^{(A+B)t}\Big|_{t=0} = (A+B)$$

$$\frac{d}{dt}\left(e^{At} \cdot e^{Bt}\right) =$$

$$\frac{d}{dt}\left(e^{At}\right) \cdot e^{Bt} + e^{At} \cdot \frac{d}{dt}\left(e^{Bt}\right) =$$

$$= A \cdot e^{At} \cdot e^{Bt} + e^{At} \cdot B\, e^{Bt}$$

VALUTATA PER $t=0$

$$A+B$$

$$\frac{d^2}{dt^2}\left(e^{(A+B)t}\right) = \frac{d}{dt}\left((A+B)\, e^{(A+B)t}\right) =$$

$$= (A+B)\frac{d}{dt}\left(e^{(A+B)t}\right) = (A+B) \cdot (A+B)\, e^{(A+B)t}$$

$$(A+B)(A+B) = A^2 + AB + BA + B^2$$

SOLO SE A E B COMMUTANO

$$(A+B)(A+B) = A^2 + 2AB + B^2$$

$$\frac{d}{dt}\left(A e^{At} e^{Bt} + e^{At} B e^{Bt}\right) =$$

$$= A \cdot A e^{At} \cdot e^{Bt} + A e^{At} B e^{Bt} +$$

$$+ A e^{At} B e^{Bt} + e^{At} B \cdot B e^{Bt}$$

$$A^2 + AB + AB + B^2 = A^2 + 2AB + B^2$$

L'ESPONENZIALE DI MATRICE

$$e^{At}$$

È NONSINGOLARE $(\forall A \in \mathbb{R}^{n \times n}, \forall t \in \mathbb{R})$

DEVO INDIVIDUARE $X \in \mathbb{R}^{n \times n}$ T.C.

$$e^{At} X = I_n$$

$$X = e^{-At}$$

$$e^{At} e^{-At} = e^{(A-A)t} =$$

$$= e^{A(I_n - I_n)t} = e^{A(t-t)} = I_n$$

UN SISTEMA DINAMICO LTI-TC È REVERSIBILE.

$$x_e(t) = e^{At} x_0$$

$$x_e(-t) = e^{-At} x_0$$