COME "INTERPRETARE" LA MATRICE DI CAMBIAMENTO DI BASE T NELLA FORMA DI JORDAN

$$A_1 = \left[\begin{array}{c|c|cc|c} -4 & & & & \\ \hline & -3 & & & \\ \hline & & -1 & 1 & \\ & & 0 & -1 & \\ \hline & & & & -1 \end{array}\right]$$

$$T_1 = [\, v_1 \mid v_2 \mid v_{31} \; v_{32} \mid v_4 \,]$$

$$(A_1 - (-4)I)\, v_1 = 0$$

$$(A_1 - (-3)I)\, v_2 = 0$$

$$(A_1 - (-1)I)\, v_4 = 0$$

$$(A_1 - (-1)I)\, v_{31} = 0$$

$$(A_1 - (-1)\,I)\, v_{32} = v_{31}$$

v_{32} È UN AUTOVETTORE GENERALIZZATO DI -1.

v_{31}, v_{32} SONO UNA CATENA DI AUTOVETTORI GENERALIZZATI DI LUNGHEZZA 2

$$(A_1 - (-1)\,I)^2\, v_{32} = 0$$

CASO A_2

$$A_2 = \left[\begin{array}{c|c|ccc} -4 & & & & \\ \hline & -3 & & & \\ \hline & & -1 & 1 & 0 \\ & & 0 & -1 & 1 \\ & & 0 & 0 & -1 \end{array}\right]$$

$$T_2 = [\, v_1 \;\vdots\; v_2 \;\vdots\; v_{31} \;\; v_{32} \;\; v_{33} \,]$$

$$(A_2 - (-4) I) v_1 = 0$$

$$(A_2 - (-3) I) v_2 = 0$$

$$(A_2 - (-1) I) v_{31} = 0$$

$$(A_2 - (-1) I) v_{32} = v_{31} ; \quad (A_2 - (-1) I)^2 v_{32} = 0$$

$$(A_2 - (-1) I) v_{33} = v_{32} ; \quad (A_2 - (-1) I)^3 v_{33} = 0$$

$$\begin{cases} \dot{x}(t) = A x(t) \\ y(t) = C x(t) \end{cases}$$

$$A = \begin{bmatrix} -4 & -2 & 2 & -4 \\ -1 & -3 & 1 & -4 \\ -1 & -1 & -1 & -2 \\ 1 & 1 & -1 & 1 \end{bmatrix} \qquad x_0 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C = [\,0 \quad 1 \quad 0 \quad 0\,]$$

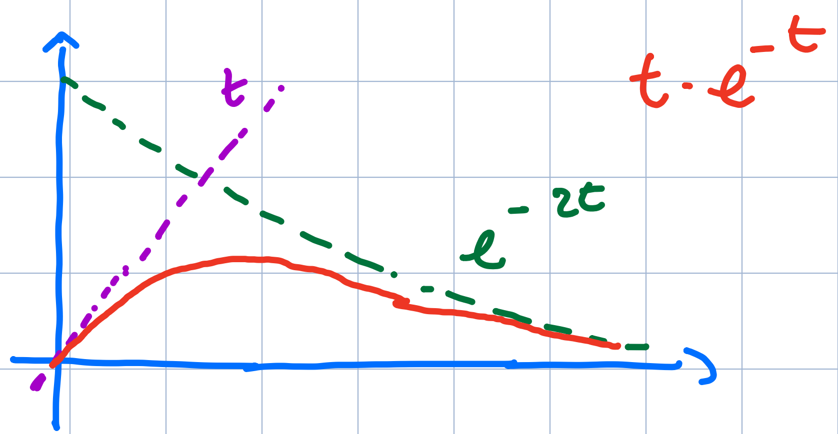
$\lambda_1 = -2$ m.a. $= 3$

$\lambda_2 = -1$ m.a. $= 1$

$$m(\lambda) = (\lambda+1)(\lambda+2)^2$$

$$\lambda + 1 \Rightarrow e^{-t}$$

$$(\lambda+2)^2 \Rightarrow e^{-2t}, \quad t\, e^{-2t}$$



$$x(t) = T \cdot e^{\Lambda t} z_0 \qquad z_0 = T^{-1} x_0$$

$$y(t) = C T e^{\Lambda t} z_0$$

$$\Lambda = \left[\begin{array}{c|cc|c} -2 & & & \\ \hline & -2 & 1 & \\ & 0 & -2 & \\ \hline & & & -1 \end{array}\right]$$

$$T = [v_1 \; v_{21} \; v_{22} \; v_3]$$

$$(A - (-2) I) v_1 = 0$$

$$(A - (-1) I) v_3 = 0$$

$$(A - (-2) I) v_{21} = 0$$

$$(A - (-2) I) v_{22} = v_{21}$$

$$(A - (-2) I)^2 v_{22} = 0$$

$$x_0 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ 0 \end{pmatrix} \qquad z_0 = T^{-1} \cdot x_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & -4 & -4 \\ 22 & 7 & -25 & -28 \\ 12 & 5 & -15 & -14 \\ -1 & -1 & 1 & -1 \end{bmatrix} \qquad X_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = [\, 2 \quad 0 \quad 1 \quad 0 \,]$$

$$m(\lambda) = p(\lambda) = (\lambda+1)(\lambda+2)^3$$

$$(\lambda+1) \Rightarrow e^{-t}$$

$$(\lambda+2)^3 \Rightarrow e^{-2t},\ t e^{-2t},\ \frac{t^2}{2} e^{-2t}$$

$$T = [\, v_{11} \quad v_{12} \quad v_{13} \;\vdots\; v_2 \,]$$

$$(A-(-1)I)\, v_2 = 0$$

$$(A-(-2)I)\, v_{11} = 0$$

$$(A-(-2)I)\, v_{12} = v_{11} \qquad (A-(-2)I)^2\, v_{12} = 0$$

$$(A-(-2)I)\, v_{13} = v_{12} \qquad (A-(-2)I)^3\, v_{13} = 0$$

$$(A-(-2)I)^2\, v_{13} = v_1$$

ESEMPIO TD

$$x(k+1) = A\,x(k)$$

$$y(k) = C\,x(k)$$

$$A = \begin{bmatrix} -\frac{16}{45} & \frac{38}{45} & \frac{7}{45} & \frac{7}{45} \\ \frac{8}{45} & -\frac{1}{45} & -\frac{8}{45} & -\frac{8}{45} \\ 1 & -1 & -\frac{1}{5} & 0 \\ -\frac{68}{45} & \frac{67}{45} & \frac{23}{45} & \frac{14}{45} \end{bmatrix} \qquad x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

$$m(\lambda) = p(\lambda) = \left(\lambda - \frac{1}{3}\right)\left(\lambda + \frac{1}{5}\right)^3$$

$$\lambda - \frac{1}{3} \Rightarrow \left(\frac{1}{3}\right)^k$$

$$\left(\lambda+\frac{1}{5}\right)^3 \Rightarrow \left(-\frac{1}{5}\right)^k, \binom{k}{1}\left(-\frac{1}{5}\right)^{k-1}, \binom{k}{2}\left(-\frac{1}{5}\right)^{k-2}$$

$$k\left(-\frac{1}{5}\right)^{k-1}, \quad \frac{k(k-1)}{2}\left(-\frac{1}{5}\right)^{k-2}$$

$$\Lambda^k = \begin{pmatrix} \left(-\frac{1}{5}\right)^k & \binom{k}{1}\left(-\frac{1}{5}\right)^{k-1} & \binom{k}{2}\left(-\frac{1}{5}\right)^{k-2} & 0 \\ 0 & \left(-\frac{1}{5}\right)^k & \binom{k}{1}\left(-\frac{1}{5}\right)^{k-1} & 0 \\ 0 & 0 & \left(-\frac{1}{5}\right)^k & 0 \\ 0 & 0 & 0 & \left(\frac{1}{3}\right)^k \end{pmatrix}$$