CALCOLO DELLA RISPOSTA FORZATA

$$Y(s) = G(s)\, U(s)$$

$$G(s) = \frac{1+2s}{(s+1)(s+3)(s+7)}$$

$$u(t) = \sin(t) \cdot 1(t) \qquad U(s) = \frac{1}{s^2+1}$$

$$Y(s) = \frac{1+2s}{(s+1)(s+3)(s+7)} \cdot \frac{1}{s^2+1} =$$

$Y(s)$ È UNA FUNZIONE REAL - RAZIONALE

COEFF. POLINOMI NUMERI REALI

$$Y(s) = \frac{1+2s}{(s+1)(s+3)(s+7)(s-j)(s+j)} =$$

$$= \frac{C_1}{s-j} + \frac{C_2}{s+j} + \frac{C_3}{s+1} +$$

$$+ \frac{C_4}{s+3} + \frac{C_5}{s+7}$$

FORMULA ELEMENTARE DI HEAVISIDE

$$F(s) = \frac{n_F(s)}{(s-p_1)(s-p_2)\cdots(s-p_n)} =$$

$$= \frac{C_1}{s-p_1} + \frac{C_2}{s-p_2} + \dots + \frac{C_n}{s-p_n}$$

$$C_i = \lim_{s \to p_i} (s - p_i) F(s)$$

$$y(t) = y_{ss}(t) - \frac{1}{24} e^{-t} 1(t) + \frac{1}{16} e^{-3t} 1(t) - \frac{13}{1200} e^{-7t} 1(t)$$

$$C_1 = -\frac{1}{200} - \frac{7}{200} j \qquad C_2 = \overline{C}_1$$

$$y_{ss}(t) = \mathcal{L}^{-1}\left[C_1 \frac{1}{s-j} + \overline{C}_1 \frac{1}{s+j} \right] =$$

$$= C_1 \mathcal{L}^{-1}\left[\frac{1}{s-j}\right] + \overline{C}_1 \mathcal{L}^{-1}\left[\frac{1}{s+j}\right]$$

$$e^{at} 1(t) \;\dot{=}\; \frac{1}{s-a}$$

$$y_{ss}(t) = C_1 e^{jt} 1(t) + \overline{C}_1 e^{-jt} 1(t) =$$

$$= 2\,\mathrm{Re}\left(C_1 e^{jt}\right) 1(t)$$

LA RISPOSTA A REGIME AL SEGNALE PERIODICO ELEMENTARE

$$u(t) = \sin(t)\,1(t)$$

È

$$y_{ss}(t) = -\frac{1}{100}\cos(t) + \frac{7}{100}\sin(t) =$$

$$= X \sin(t + \theta)$$

$$G(s) = \frac{1+2s}{s^3 + 11s^2 + 31s + 21} = \frac{Y(s)}{U(s)}$$

$$\frac{1+2s}{s^3 + 11s^2 + 31s + 21} = \frac{Y(s)}{U(s)}$$

$$(s^3 + 11 s^2 + 31 s + 21)\, Y(s) =$$

$$(1 + 2s)\, U(s)$$

$$s^3 Y(s) + 11 s^2 Y(s) + 31 s Y(s) + 21 Y(s) =$$

$$= 2s\, U(s) + U(s)$$

$f(t)$ CLASSE $\mathcal{L}$, CONTINUA

$$F(s) \doteq f(t)$$

$$\mathcal{L}(\dot{f}(t)) = s F(s) - f(0)$$

$f(t)$ CLASSE $\mathcal{L}$, CONTINUA FINO ALL'ORDINE $n-1$

$F(s) \doteq f(t)$

$$\mathcal{L}(f^{(n)}(t)) = s^n F(s) - s^{n-1} f(0) - s^{n-2} \dot{f}(0) - \dots - s f^{(n-2)}(0) - f^{(n-1)}(0)$$

$$s^3 Y(s) + 11 s^2 Y(s) + 31 s Y(s) + 21 Y(s) =$$

$$= 2 s U(s) + U(s)$$

$$\mathcal{L}^{-1}[s^3 Y(s) + 11 s^2 Y(s) + 31 s Y(s) + 21 Y(s)] =$$

$$\mathcal{L}^{-1}[2 s U(s) + U(s)]$$

$$y(t) = \mathcal{L}^{-1}[Y(s)]$$

$$u(t) = \mathcal{L}^{-1}[U(s)]$$

$$\dddot{y}(t) + 11\ddot{y}(t) + 31\dot{y}(t) + 21\, y(t) =$$

$$= 2\dot{u}(t) + u(t) \qquad I/U$$

$$\dot{y}(t) = \mathcal{L}^{-1}[s\,Y(s)]$$

$$\dot{u}(t) = \mathcal{L}^{-1}[s\,U(s)]$$

$$\ddot{y}(t) = \mathcal{L}^{-1}[s^2 Y(s)]$$

$$\dddot{y}(t) = \mathcal{L}^{-1}[s^3 Y(s)]$$

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -21 & -31 & -11 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) \\ y(t) = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} x(t) \end{cases}$$

x_0 noto $\qquad I/S/U$

$$\dddot{y}(t) + 11\ddot{y}(t) + 31\dot{y}(t) + 21y(t) = 2\dot{u}(t) + u(t)$$

I/U

$$y(0) = Cx_0$$

$$\dot{y}(0) = CAx_0$$

$$\ddot{y}(0) = CA^2x_0$$

$$y(t) = Cx(t)$$

$$\dot{y}(t) = C\dot{x}(t) = C(Ax(t) + Bu(t))$$

$$\dot{y}(0) = C(Ax(t) + Bu(t))\Big|_{t=0} = CAx_0$$

$$\ddot{y}(t) = C\ddot{x}(t) = C\frac{d}{dt}(Ax(t) + Bu(t)) =$$

$$= CA\left(\frac{d}{dt}x(t) + B\frac{d}{dt}u(t)\right) =$$

$$= CA\left(Ax(t) + Bu(t) + B\frac{d}{dt}u(t)\right)$$

$\ddot{y}(0) = CA^2 x_0$

$\dddot{y}(t) + 11\ddot{y}(t) + 31\dot{y}(t) + 21 y(t) = 2\dot{u}(t) + u(t)$

I/U

$y(0) = C x_0$

$\dot{y}(0) = CA x_0$

$\ddot{y}(0) = CA^2 x_0$

MODELLO ARMA TC

AUTO-REGRESSIVE-MOVING-AVERAGE

$$A = \begin{bmatrix} \frac{\ }{9} & -\frac{125}{72} & -\frac{89}{144} \\ \frac{10}{9} & -\frac{197}{72} & -\frac{233}{144} \\ -1 & 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

$$C = [\,5 \quad -5 \quad -1\,] \qquad T_1$$

$$G(z) = \frac{12(z+1)}{(2z-1)(2z+1)(3z+1)}$$

POLI →

$2z-1=0 \qquad z = 1/2$

$2z+1=0 \qquad z = -1/2$

$3z+1=0 \qquad z = -1/3$

$$a^k \, 1(k) \doteq \frac{z}{z-a}$$

$$1(k) \doteq \frac{z}{z-1}$$

$$Y(z) = G(z) \cdot \frac{z}{z-1} =$$

$$= \frac{12(z+1)}{(2z-1)(2z+1)(3z+1)} \cdot \frac{z}{z-1}$$

$$\frac{z}{z-p_i} \qquad \cdot$$

$$\frac{Y(z)}{z} = \frac{C_1}{z-p_1} + \frac{C_2}{z-p_2} + \dots + \frac{C_n}{z-p_n}$$

$$C_i = \lim_{z \to p_i} (z - p_i) \left(\frac{Y(z)}{z} \right)$$

$$Y(z) = C_1 \frac{z}{z - p_1} + C_2 \frac{z}{z - p_2} + \dots + C_n \frac{z}{z - p_n}$$

$$C_1 = \lim_{z \to 1} (z - 1) \frac{Y(z)}{z}$$

$$\frac{Y(z)}{z} = 2 \frac{1}{z-1} - \frac{18}{5} \frac{1}{z - 1/2} - 2 \frac{1}{z + 1/2} + \frac{18}{5} \frac{1}{z + 1/3}$$

$$Y(z) = 2 \frac{z}{z-1} - \frac{18}{5} \frac{z}{z - 1/2} - 2 \frac{z}{z + 1/2} + \frac{18}{5} \frac{z}{z + 1/3}$$

$$y(k) = 2 \cdot 1(k) - \frac{18}{5} \left(\frac{1}{2} \right)^k 1(k) - 2 \left(-\frac{1}{2} \right)^k 1(k) + \frac{18}{5} \left(-\frac{1}{3} \right)^k 1(k)$$

$$G(z) = \frac{z+1}{z^3 + \frac{1}{3}z^2 - \frac{1}{4}z - \frac{1}{12}} = \frac{Y(z)}{U(z)}$$

$$\left(z^3 + \frac{1}{3}z^2 - \frac{1}{4}z - \frac{1}{12}\right)Y(z) = (z+1)\,U(z)$$

$$z^3 Y(z) + \frac{1}{3}z^2 Y(z) - \frac{1}{4}z\,Y(z) - \frac{1}{12}Y(z) =$$

$$= z\,U(z) + U(z)$$

$$\mathcal{Z}[f(k+1)] = z\,F(z) - z f(0)$$

$$\mathcal{Z}[f(k+n)] = z^n F(z) - z^n f(0) - z^{n-1} f(1) - \dots - z f(n-1)$$

$$\mathcal{Z}[f(k-n)] = z^{-n} F(z)$$

$$z^3 Y(z) + \frac{1}{3} z^2 Y(z) - \frac{1}{4} z Y(z) - \frac{1}{12} Y(z) =$$

$$= z\,U(z) + U(z)$$

$$u(k) \doteq U(z)$$

$$y(k) \doteq Y(z)$$

$$y(\underbrace{k+3}) + \frac{1}{3} y(\underbrace{k+2}) - \frac{1}{4} y(\underbrace{k+1}) - \frac{1}{12} y(k) = u(k+1) + u(k)$$

$$y(0) = C x_0$$

$$y(1) = CA x_0$$

$$y(2) = CA^2 x_0$$

MODELLO ARMA

$$k' = k+3, \quad k'-1 = k+2, \quad k'-2 = k+1, \quad k'-3 = k$$

$$y(k') + \frac{1}{3} y(k'-1) - \frac{1}{4} y(k'-2) - \frac{1}{12} y(k'-3) =$$

$$= u(k'-2) + u(k'-3)$$

$$y(k') = \overbrace{-\frac{1}{3}y(k'-1) + \frac{1}{4}y(k'-2) + \frac{1}{12}y(k'-3)}^{AR}$$

$$\underbrace{+ u(k'-2) + u(k'-3)}_{\Pi A}$$