Calcolo della risposta al gradino per un sistema LTI-TC (caso poli complessi e coniugati)

Fdt del sistema

In[∘]:= $G[s\_] := \frac{(s-1)}{s^3 + 8 s^2 + 25 s + 26}$

Poli del sistema (ci danno una informazione in merito ai modi naturali “visibili” lato uscita)

In[∘]:= `Solve[Denominator[G[s]] == 0, s]`

Out[∘]=

$\{\{s \to -3 - 2 i\}, \{s \to -3 + 2 i\}, \{s \to -2\}\}$

Calcolo la risposta forzata in s al gradino unitario

In[∘]:= $Y_f[s\_] := G[s] \left(\frac{1}{s}\right)$

In[∘]:= $Y_f[s]$

Out[∘]=

$$\frac{-1 + s}{s\,(26 + 25 s + 8 s^2 + s^3)}$$

In[∘]:= `Apart[`$Y_f[s]$`]`

Out[∘]=

$$-\frac{1}{26 s} + \frac{3}{10\,(2 + s)} + \frac{-63 - 17 s}{65\,(13 + 6 s + s^2)}$$

Scrivo in maniera “verbosa” la Yf[s] mettendo in evidenza i quattro fratti semplici

In[∘]:= $C_1 \left(\frac{1}{s}\right) + C_2 \left(\frac{1}{s+2}\right) + C_3 \left(\frac{1}{s + 3 - 2 i}\right) + C_4 \left(\frac{1}{s + 3 + 2 i}\right)$

Out[∘]=

$$\frac{C_1}{s} + \frac{C_2}{2 + s} + \frac{C_3}{(3 - 2 i) + s} + \frac{C_4}{(3 + 2 i) + s}$$

Formula di Heaviside semplificata per il calcolo dei coefficienti Ci

In[∘]:= $C_1 = \lim_{s \to 0} s\, Y_f[s]$

Out[∘]=

$$-\frac{1}{26}$$

In[∘]:= $C_2 = \lim_{s\to -2} (s+2)\, Y_f[s]$

Out[∘]=

$$\frac{3}{10}$$

In[∘]:= $C_3 = \lim_{s\to -3+2\,i} (s+3-2\,i)\, Y_f[s]$

Out[∘]=

$$-\frac{17}{130} + \frac{3\,i}{65}$$

In[∘]:= C_4 = Conjugate[C_3]

Out[∘]=

$$-\frac{17}{130} - \frac{3\,i}{65}$$

In[∘]:= $C_1 \left(\frac{1}{s}\right) + C_2 \left(\frac{1}{s+2}\right) + C_3 \left(\frac{1}{s+3-2\,i}\right) + C_4 \left(\frac{1}{s+3+2\,i}\right)$

Out[∘]=

$$-\frac{1}{26\,s} + \frac{3}{10\,(2+s)} - \frac{\frac{17}{130} - \frac{3\,i}{65}}{(3-2\,i)+s} - \frac{\frac{17}{130} + \frac{3\,i}{65}}{(3+2\,i)+s}$$

In[∘]:= F[Z_, γ_, t_] := ComplexExpand[Re[Z Exp[I γ t]]]

In[∘]:= y_f[t_] :=
C_1 UnitStep[t] + C_2 Exp[-2 t] × UnitStep[t] + 2 Exp[-3 t] × F[C_3, 2, t] × UnitStep[t]

In[∘]:= y_f[t]

Out[∘]=

$$-\frac{\text{UnitStep}[t]}{26} + \frac{3}{10} e^{-2t}\,\text{UnitStep}[t] + 2\,e^{-3t} \left(-\frac{17}{130}\,\text{Cos}[2\,t] - \frac{3}{65}\,\text{Sin}[2\,t]\right) \text{UnitStep}[t]$$

In[∘]:= Simplify[ComplexExpand[InverseLaplaceTransform[Y_f[s], s, t]]]

Out[∘]=

$$-\frac{1}{130}\, e^{-3t} \left(-39\, e^{t} + 5\, e^{3t} + 34\,\text{Cos}[2\,t] + 12\,\text{Sin}[2\,t]\right)$$

In[∘]:= Plot[y_f[t], {t, 0, 10}, PlotRange → All]

Out[∘]=