Analisi della Risposta libera, caso TC, per un sistema “planare” che presenta una coppia di autovalori complessi e coniugati

```
In[◦]:= A = {{-1 / 2, 1 / 2}, {-5 / 2, -3 / 2}}
```

Out[◦]=

$$\left\{\left\{-\frac{1}{2}, \frac{1}{2}\right\}, \left\{-\frac{5}{2}, -\frac{3}{2}\right\}\right\}$$

```
In[◦]:= C1 = {2, -1}
```

Out[◦]=

{2, -1}

Calcolo il polinomio caratteristico di A

```
In[◦]:= CharacteristicPolynomial[A, x]
```

Out[◦]=

$2 + 2x + x^2$

```
In[◦]:= Simplify[A.A + 2 A + 2 IdentityMatrix[2]]
```

Out[◦]=

{{0, 0}, {0, 0}}

Calcolo gli autovalori di A

```
In[◦]:= λ = Eigenvalues[A]
```

Out[◦]=

{-1 + ⅈ, -1 - ⅈ}

```
In[◦]:= σ = Re[λ[[1]]]
```

Out[◦]=

-1

```
In[◦]:= ω = Im[λ[[1]]]
```

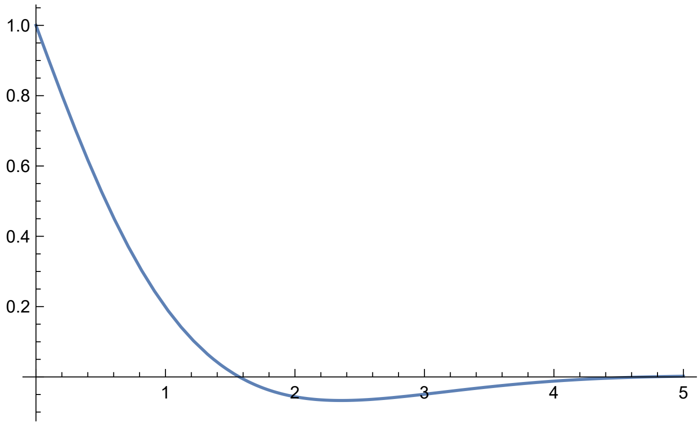
Out[◦]=

1

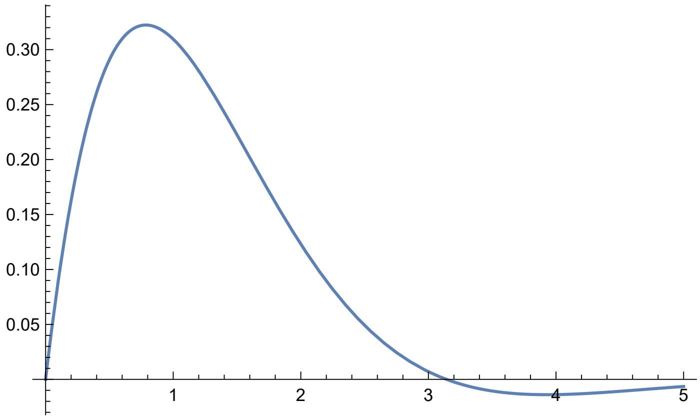
Grafico dei modi naturali

```
In[◦]:= Plot[Exp[σ t] Cos[ω t], {t, 0, 5}, PlotRange → All]
```

Out[◦]=



In[∘]:=
```
Plot[Exp[σ t] Sin[ω t], {t, 0, 5}, PlotRange → All]
```
Out[∘]=



Calcolo la risposta libera a partire dalla matrice di cambiamento di base che genera la forma Rotation-Scaling Tempo Continuo

In[∘]:=
```
T = Simplify[Transpose[Eigenvectors[A]]]
```
Out[∘]=

$$\left\{\left\{-\frac{1}{5}-\frac{2\,i}{5},\,-\frac{1}{5}+\frac{2\,i}{5}\right\},\,\{1,\,1\}\right\}$$

In[∘]:=
```
T // MatrixForm
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{5}-\frac{2\,i}{5} & -\frac{1}{5}+\frac{2\,i}{5} \\ 1 & 1 \end{pmatrix}$$

In[∘]:=
```
λ
```
Out[∘]=

$$\{-1+i,\,-1-i\}$$

In[∘]:=
```
T̂ = Simplify[Transpose[{Re[T[[All, 1]]], Im[T[[All, 1]]]}]]
```
Out[∘]=

$$\left\{\left\{-\frac{1}{5},\,-\frac{2}{5}\right\},\,\{1,\,0\}\right\}$$

In[∘]:=
```
T̂ // MatrixForm
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{5} & -\frac{2}{5} \\ 1 & 0 \end{pmatrix}$$

In[∘]:=
```
T // MatrixForm
```
Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{5}-\frac{2\,i}{5} & -\frac{1}{5}+\frac{2\,i}{5} \\ 1 & 1 \end{pmatrix}$$

In[∘]:=
```
Λ̂ = Simplify[Inverse[T̂].A.T̂]
```
Out[∘]=

$$\{\{-1,\,1\},\,\{-1,\,-1\}\}$$

In[∘]:=
```
MatrixForm[Λ̂]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}$$

In[∘]:=
```
x0 = {{1}, {0}}
```

Out[∘]=

{{1}, {0}}

In[∘]:=
```
z0 = Inverse[T̂].x0
```

Out[∘]=

$$\left\{\{0\}, \left\{-\frac{5}{2}\right\}\right\}$$

In[∘]:=

$$x[t\_] := \text{Simplify}\left[\hat{T}.\begin{pmatrix} \text{Exp}[\sigma\, t]\ \text{Cos}[\omega\, t] & \text{Exp}[\sigma\, t]\ \text{Sin}[\omega\, t] \\ -\text{Exp}[\sigma\, t]\ \text{Sin}[\omega\, t] & \text{Exp}[\sigma\, t]\ \text{Cos}[\omega\, t] \end{pmatrix}.z_0\right]$$

In[∘]:=
```
x[t]
```

Out[∘]=

$$\left\{\left\{\frac{1}{2}\, e^{-t}\, (2\, \text{Cos}[t] + \text{Sin}[t])\right\}, \left\{-\frac{5}{2}\, e^{-t}\, \text{Sin}[t]\right\}\right\}$$

In[∘]:=
```
y[t_] := Simplify[C1.x[t]]
```

In[∘]:=
```
y[t]
```

Out[∘]=

$$\left\{\frac{1}{2}\, e^{-t}\, (4\, \text{Cos}[t] + 7\, \text{Sin}[t])\right\}$$

In[∘]:=
```
Plot[y[t], {t, 0, 5}, PlotRange → All]
```

Out[∘]=