Calcolo della risposta al gradino per un sistema LTI-TD (caso poli complessi e coniugati)

Immetto la FdT

In[∘]:= $G[z\_] := \frac{z-3}{z^3 + \frac{z^2}{4} - \frac{z}{16} + \frac{1}{32}}$

Calcolo i poli della FdT

In[∘]:= `Solve[Denominator[G[z]] == 0, z]`

Out[∘]=

$\left\{\left\{z \to -\frac{1}{2}\right\}, \left\{z \to \frac{1}{8}\left(1 - i\sqrt{3}\right)\right\}, \left\{z \to \frac{1}{8}\left(1 + i\sqrt{3}\right)\right\}\right\}$

Calcolo modulo, argomento dei poli complessi e coniugati

In[∘]:= $\text{Abs}\left[\frac{1}{8}\left(1 + i\sqrt{3}\right)\right]$

Out[∘]=

$\frac{1}{4}$

In[∘]:= $\text{Arg}\left[\frac{1}{8}\left(1 + i\sqrt{3}\right)\right]$

Out[∘]=

$\frac{\pi}{3}$

Calcolo della Risposta Forzata in z

In[∘]:= $Y_f[z\_] := G[z]\left(\frac{z}{z-1}\right)$

In[∘]:= $Y_f[z]$

Out[∘]=

$\frac{(-3+z)\, z}{(-1+z)\left(\frac{1}{32} - \frac{z}{16} + \frac{z^2}{4} + z^3\right)}$

Divido Yf[z] per z e successivamente scrivo l’espansione in fratti semplici nello stile Trasformata di Laplace

In[∘]:= $\frac{Y_f[z]}{z}$

Out[∘]=

$$\frac{-3+z}{(-1+z)\left(\frac{1}{32}-\frac{z}{16}+\frac{z^2}{4}+z^3\right)}$$

In[∘]:= $\text{Apart}\left[\frac{Y_f[z]}{z}\right]$

Out[∘]=

$$-\frac{64}{39\,(-1+z)}+\frac{32}{3\,(1+2\,z)}-\frac{64\,(-17+12\,z)}{13\left(1-4\,z+16\,z^2\right)}$$

Scrivo in forma simbolica i fratti semplici di Yf[z]/z

In[∘]:= $D_1\left(\frac{1}{z-1}\right)+D_2\left(\frac{1}{z+\frac{1}{2}}\right)+D_3\left(\frac{1}{z-\frac{1}{4}\,\text{Exp}\left[I\,\left(\frac{Pi}{3}\right)\right]}\right)+D_4\left(\frac{1}{z-\frac{1}{4}\,\text{Exp}\left[-I\,\left(\frac{Pi}{3}\right)\right]}\right)$

Out[∘]=

$$\frac{D_1}{-1+z}+\frac{D_2}{\frac{1}{2}+z}+\frac{D_3}{-\frac{1}{4}\,e^{\frac{i\pi}{3}}+z}+\frac{D_4}{-\frac{1}{4}\,e^{-\frac{i\pi}{3}}+z}$$

In[∘]:= $D_1=\lim_{z\to 1}(z-1)\left(\frac{Y_f[z]}{z}\right)$

Out[∘]=

$$-\frac{64}{39}$$

In[∘]:= $D_2=\lim_{z\to -\frac{1}{2}}\left(z+\frac{1}{2}\right)\left(\frac{Y_f[z]}{z}\right)$

Out[∘]=

$$\frac{16}{3}$$

In[∘]:= $D_3=\text{Expand}\left[\lim_{z\to\left(\frac{1}{4}\right)\text{Exp}\left[\frac{I\,Pi}{3}\right]}\left(z-\left(\frac{1}{4}\right)\text{Exp}\left[\frac{I\,Pi}{3}\right]\right)\left(\frac{Y_f[z]}{z}\right)\right]$

Out[∘]=

$$-\frac{24}{13}-\frac{248\,i}{13\,\sqrt{3}}$$

In[∘]:= $D_4=\text{Conjugate}[D_3]$

Out[∘]=

$$-\frac{24}{13}+\frac{248\,i}{13\,\sqrt{3}}$$

Una volta calcolati i coefficienti Di scrivo la Yf[z]

In[∘]:=
$$\frac{z\,D_1}{-1+z}+\frac{z\,D_2}{\frac{1}{2}+z}+\frac{z\,D_3}{-\frac{1}{4}\,e^{\frac{i\pi}{3}}+z}+\frac{z\,D_4}{-\frac{1}{4}\,e^{-\frac{i\pi}{3}}+z}$$

Out[∘]=
$$-\frac{64\,z}{39\,(-1+z)}+\frac{16\,z}{3\left(\frac{1}{2}+z\right)}+\frac{\left(-\frac{24}{13}+\frac{248\,i}{13\,\sqrt{3}}\right)z}{-\frac{1}{4}\,e^{-\frac{i\pi}{3}}+z}+\frac{\left(-\frac{24}{13}-\frac{248\,i}{13\,\sqrt{3}}\right)z}{-\frac{1}{4}\,e^{\frac{i\pi}{3}}+z}$$

In[∘]:= **F[Z_, γ_, t_] := ComplexExpand[Re[Z Exp[I γ t]]]**

In[∘]:=
$$y_f[k\_] := D_1\,\text{UnitStep}[k]+D_2\left(-\frac{1}{2}\right)^k\text{UnitStep}[k]+2\left(\frac{1}{4}\right)^k F\left[D_3,\frac{\pi}{3},k\right]\text{UnitStep}[k]$$

In[∘]:= $y_f[k]$

Out[∘]=
$$-\frac{64\,\text{UnitStep}[k]}{39}+\frac{1}{3}\,(-1)^k\,2^{4-k}\,\text{UnitStep}[k]+$$
$$2^{1-2k}\left(-\frac{24}{13}\,\text{Cos}\left[\frac{k\pi}{3}\right]+\frac{248\,\text{Sin}\left[\frac{k\pi}{3}\right]}{13\,\sqrt{3}}\right)\text{UnitStep}[k]$$

In[∘]:= **DiscretePlot[y_f[k], {k, 0, 10}, PlotRange → All]**

Out[∘]=