Caso “ibrido” di un sistema dinamico che presenta autovalori reali e compliessi e coniugati (TD)

In[•]:= $A = \left\{\left\{-\frac{1}{2}, 0, \frac{-7}{3}\right\}, \left\{\frac{-13}{60}, \frac{-1}{5}, \frac{-9}{5}\right\}, \left\{\frac{1}{12}, 0, \frac{1}{3}\right\}\right\}$

Out[•]=

$\left\{\left\{-\frac{1}{2}, 0, -\frac{7}{3}\right\}, \left\{-\frac{13}{60}, -\frac{1}{5}, -\frac{9}{5}\right\}, \left\{\frac{1}{12}, 0, \frac{1}{3}\right\}\right\}$

In[•]:= **C1 = {1, 0, 1}**

Out[•]=

{1, 0, 1}

Calcolo gli Autovalori di A

In[•]:= **λ = Eigenvalues[A]**

Out[•]=

$\left\{-\frac{1}{5}, \frac{1}{12}\left(-1 + i\sqrt{3}\right), \frac{1}{12}\left(-1 - i\sqrt{3}\right)\right\}$

In[•]:= **ρ = Abs[λ⟦2⟧]**

Out[•]=

$\frac{1}{6}$

In[•]:= **θ = Arg[λ⟦2⟧]**

Out[•]=

$\frac{2\pi}{3}$

In[•]:= **CharacteristicPolynomial[A, x]**

Out[•]=

$-\frac{1}{180} - \frac{11x}{180} - \frac{11x^2}{30} - x^3$

Inserisco lo stato iniziale e lo proietto lungo T

In[•]:= **T0 = FullSimplify[Transpose[Eigenvectors[A]]]**

Out[•]=

$\left\{\left\{0, -5 + i\sqrt{3}, -5 - i\sqrt{3}\right\}, \left\{1, -4 + i\sqrt{3}, -4 - i\sqrt{3}\right\}, \{0, 1, 1\}\right\}$

In[•]:= **T0 // MatrixForm**

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & -5 + i\sqrt{3} & -5 - i\sqrt{3} \\ 1 & -4 + i\sqrt{3} & -4 - i\sqrt{3} \\ 0 & 1 & 1 \end{pmatrix}$$

In[•]:= **T = Transpose[{T0⟦All, 1⟧, Re[T0⟦All, 2⟧], Im[T0⟦All, 2⟧]}]**

Out[•]=

$\left\{\left\{0, -5, \sqrt{3}\right\}, \left\{1, -4, \sqrt{3}\right\}, \{0, 1, 0\}\right\}$

In[•]:= T // MatrixForm

Out[•]//MatrixForm=

$$\begin{pmatrix} 0 & -5 & \sqrt{3} \\ 1 & -4 & \sqrt{3} \\ 0 & 1 & 0 \end{pmatrix}$$

In[•]:= z_0 = Inverse[T].x_0

Out[•]=

$$\left\{\left\{-\frac{7}{5}\right\}, \{1\}, \left\{\frac{26}{5\sqrt{3}}\right\}\right\}$$

Determino la forma canonica a blocchi

In[•]:= Λ = Inverse[T].A.T

Out[•]=

$$\left\{\left\{-\frac{1}{5}, 0, 0\right\}, \left\{0, -\frac{1}{12}, \frac{1}{4\sqrt{3}}\right\}, \left\{0, -\frac{1}{4\sqrt{3}}, -\frac{1}{12}\right\}\right\}$$

In[•]:= Λ // MatrixForm

Out[•]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{5} & 0 & 0 \\ 0 & -\frac{1}{12} & \frac{1}{4\sqrt{3}} \\ 0 & -\frac{1}{4\sqrt{3}} & -\frac{1}{12} \end{pmatrix}$$

Calcolo la risposta libera utilizzando la decomposizione modale

In[•]:= $$x_1[k\_] := \text{FullSimplify}\left[T.\begin{pmatrix} \lambda[[1]]^k & 0 & 0 \\ 0 & \rho^k \text{Cos}[\theta\, k] & \rho^k \text{Sin}[\theta\, k] \\ 0 & -\rho^k \text{Sin}[\theta\, k] & \rho^k \text{Cos}[\theta\, k] \end{pmatrix}.z_0\right]$$

In[•]:= x_1[k]

Out[•]=

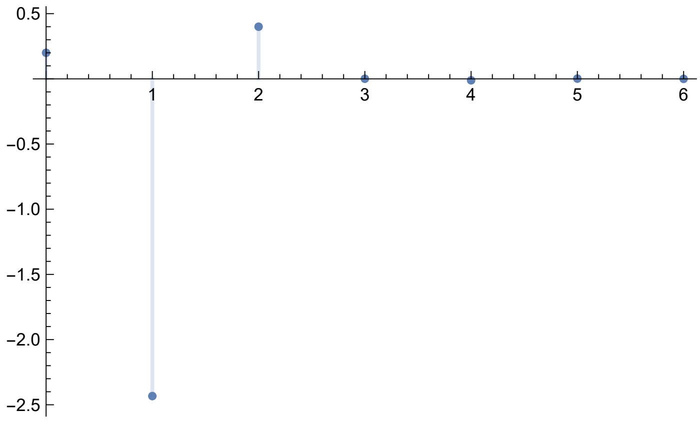
$$\left\{\left\{\frac{1}{5}\times 6^{-k}\left(\text{Cos}\left[\frac{2\,k\,\pi}{3}\right]-\frac{145\,\text{Sin}\left[\frac{2\,k\,\pi}{3}\right]}{\sqrt{3}}\right)\right\},\right.$$

$$\left\{\frac{1}{5}\left(-7\left(-\frac{1}{5}\right)^k+6^{1-k}\,\text{Cos}\left[\frac{2\,k\,\pi}{3}\right]-119\times 2^{-k}\times 3^{-\frac{1}{2}-k}\,\text{Sin}\left[\frac{2\,k\,\pi}{3}\right]\right)\right\},$$

$$\left.\left\{\frac{1}{5}\times 6^{-k}\left(5\,\text{Cos}\left[\frac{2\,k\,\pi}{3}\right]+\frac{26\,\text{Sin}\left[\frac{2\,k\,\pi}{3}\right]}{\sqrt{3}}\right)\right\}\right\}$$

In[ ]:= `DiscretePlot[x1[k][[1]], {k, 0, 6}, PlotRange → All]`

Out[ ]=



In[ ]:= `y1[k_] := FullSimplify[C1.x1[k]]`

In[ ]:= `y1[k]`

Out[ ]=

$$\left\{\frac{1}{5}\times 2^{-k}\times 3^{-1-k}\left(18\,\mathrm{Cos}\left[\frac{2\,k\,\pi}{3}\right]-119\,\sqrt{3}\,\mathrm{Sin}\left[\frac{2\,k\,\pi}{3}\right]\right)\right\}$$

In[ ]:= `x1[k]`

Out[ ]=

$$\left\{\left\{\frac{1}{5}\times 6^{-k}\left(\mathrm{Cos}\left[\frac{2\,k\,\pi}{3}\right]-\frac{145\,\mathrm{Sin}\left[\frac{2\,k\,\pi}{3}\right]}{\sqrt{3}}\right)\right\},\right.$$
$$\left\{\frac{1}{5}\left(-7\left(-\frac{1}{5}\right)^{k}+6^{1-k}\,\mathrm{Cos}\left[\frac{2\,k\,\pi}{3}\right]-119\times 2^{-k}\times 3^{-\frac{1}{2}-k}\,\mathrm{Sin}\left[\frac{2\,k\,\pi}{3}\right]\right)\right\},$$
$$\left.\left\{\frac{1}{5}\times 6^{-k}\left(5\,\mathrm{Cos}\left[\frac{2\,k\,\pi}{3}\right]+\frac{26\,\mathrm{Sin}\left[\frac{2\,k\,\pi}{3}\right]}{\sqrt{3}}\right)\right\}\right\}$$