SISTEMA LTI - TD

$$x(k+1) = A\,x(k)$$

$$A = \begin{bmatrix} 9/10 & -3/10 & 1 \\ -7/5 & 4/5 & -2 \\ -7/5 & 3/10 & -3/2 \end{bmatrix} \qquad x_0 = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$$

$$x_e(k) = \sum_{i=1}^{n} v_i \lambda_i^k z_{0i}$$

autovalori di A $\left\{ -\frac{1}{2}, \frac{1}{2}, \frac{1}{5} \right\}$

modi naturali

$$-\frac{1}{2} \longrightarrow \left(-\frac{1}{2}\right)^k \qquad k \geq 0$$

$\frac{1}{2} \rightarrow \left(\frac{1}{2}\right)^k \qquad k \geq 0$

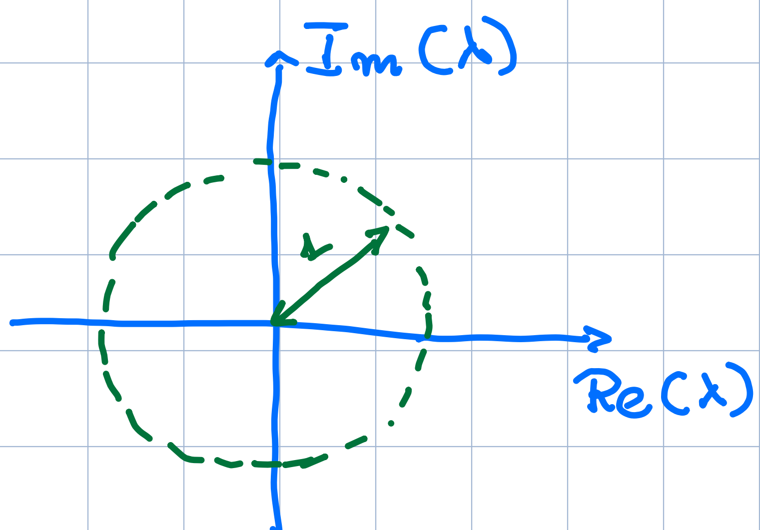
$\frac{1}{5} \rightarrow \left(\frac{1}{5}\right)^k \qquad k \geq 0$

LA CONVERGENZA ZERO RICHIEDE CHE

$$|\lambda_i| < 1$$

SONO NUMERICAMENTE PIÙ PESANTI QUEI MODI CHE HANNO MASSIMO MODULO SULLA BASE DELLA POTENZA. .

MODI (AUTOVALORI) DOMINANTI.



$$A \cdot v_1 = \lambda_1 v_1$$

$$x_\ell(k) = \begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix} \left(-\frac{1}{2}\right)^k \cdot 6 + \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \left(\frac{1}{2}\right)^k (-2) +$$

$$+ \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \left(\frac{1}{5}\right)^k \cdot (-4)$$

$$x_\ell(k) = A^k \cdot x_0$$

$$x_0 = \begin{bmatrix} 2 \\ -4 \\ -4 \end{bmatrix}$$

$$x_\ell(k) = \begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix} \left(-\frac{1}{2}\right)^k (-4)$$

SE $x_0 \in \ker(A - \lambda_i I)$

ALLORA $x_\ell(k) \in \ker(A - \lambda_i I)$

$$x_0 \in \ker(A - \lambda_i I)$$

$$(A - \lambda_i I)\, v_i = 0_n$$

$$A v_i = \lambda_i v_i$$

$\exists\, \gamma \in \mathbb{R}$ $\quad x_0 = \gamma v_i$

$$z_0 = \begin{bmatrix} 0 \\ \vdots \\ \gamma \\ 0 \\ \vdots \\ 0 \end{bmatrix} \leftarrow i$$

$$x_e(k) = \sum_{i=1}^{n} v_i \lambda_i^k z_{0i} = v_i \lambda_i^k \gamma$$

$$\begin{cases} \dot{x}(t) = A\,x(t) \\ x(0) = x_0 \end{cases}$$

$$x_e(t) = \underbrace{e^{At}}_{n \times n} \cdot \underbrace{x_0}_{n \times 1}$$

$$e^{at} = 1 + at + \frac{(at)^2}{2!} + \frac{(at)^3}{3!} + \ldots + \frac{(at)^k}{k!} + \ldots$$

$$e^{At} = I_n + A \cdot t + \frac{A^2 t^2}{2!} + \frac{A^3 t^3}{3!} + \ldots + \frac{A^k t^k}{k!} + \ldots$$

ESPONENZIALE DI MATRICE .

$$x_e(0) = x_0$$

$$x_e(t) = e^{At} x_0$$

$$x_e(0) = e^{At}\Big|_{t=0} x_0 = I_n x_0 = x_0$$

$$e^{At}\Big|_{t=0} = \left(I_n + At + \frac{A^2 t^2}{2!} + \dots + \frac{A^k t^k}{k!} + \dots \right)\Big|_{t=0} =$$

$$= I_n$$

$$x_e(t) = e^{At} x_0$$

$$\frac{dx_e}{dt} = A \cdot x_e$$

$$\frac{d}{dt}\left(e^{At} x_0\right) = \frac{d}{dt}\left(e^{At}\right) x_0$$

$$\frac{d}{dt}\left(e^{At}\right) = \frac{d}{dt}\left(I_n + At + \frac{A^2t^2}{2!} + \dots + \frac{A^kt^k}{k!} + \dots\right) =$$

$$= A + A^2t + \frac{A^3t^2}{2!} + \dots + \frac{A^kt^{k-1}}{(k-1)!} + \dots =$$

$$= A\left(I_n + At + \frac{A^2t^2}{2!} + \dots + \frac{A^{k-1}t^{k-1}}{(k-1)!} + \dots\right) =$$

$$= \left(I_n + At + \frac{A^2t^2}{2!} + \dots + \frac{A^{k-1}t^{k-1}}{(k-1)!} + \dots\right)A$$

$$\frac{1}{k!}\frac{d}{dt}\left(t^k\right) = \frac{k \cdot t^{k-1}}{k!} = \frac{t^{k-1}}{(k-1)!}$$

$$\frac{d}{dt}\left(e^{At}\right) = A \cdot e^{At} = e^{At} \cdot A$$

$$\frac{d}{dt} x_e(t) = \frac{d}{dt}\left(e^{At} x_0\right) = \frac{d}{dt}\left(e^{At}\right) x_0 =$$

$$= A \underbrace{e^{At} x_0}_{x_e(t)} = A x_e(t)$$

1. $e^{At}\big|_{t=0} = I_n$

2. $\frac{d}{dt}\left(e^{At}\right) = A \cdot e^{At} = e^{At} \cdot A$

$$A = (a_{ij}) \quad 1 \leq i, j \leq n$$

$$e^{At} \neq \left(e^{a_{ij} t}\right) \quad 1 \leq i, j \leq n$$

$$A = \begin{pmatrix} a_1 & & & 0 \\ & a_2 & & \\ & & \ddots & \\ 0 & & & a_n \end{pmatrix}$$

$$A^k = \begin{pmatrix} a_1^k & & & 0 \\ & a_2^k & & \\ & & \ddots & \\ 0 & & & a_n^k \end{pmatrix}$$

$$e^{At} = I_n + At + \frac{A^2 t^2}{2!} + \dots + \frac{A^k t^k}{k!} + \dots$$

$A = \mathrm{diag}(a_i) \qquad 1 \leq i \leq n$

$A^k = \mathrm{diag}(a_i^k) \qquad 1 \leq i \leq n$

$$e^{At} = I_n + \operatorname{diag}(a_i t) + \operatorname{diag}\left(\frac{a_i^2 t^2}{2!}\right) + \dots +$$

$$+ \operatorname{diag}\left(\frac{a_i^k t^k}{k!}\right) + \dots =$$

$$= \operatorname{diag}\left(\sum_{j=0}^{+\infty} \frac{a_i^j t^j}{j!}\right) = \operatorname{diag}\left(e^{a_i t}\right)$$

$$A \overset{T}{\sim} \Lambda$$

$$e^{At} \overset{T}{\sim} ?$$

$$A T = T \Lambda$$

$$e^{At} T = ?$$

$$A \overset{T}{\sim} \Lambda \Rightarrow A^k \overset{T}{\sim} \Lambda^k$$

VERA $k=1 \rightarrow$ OK PER HYP.

$$A^k \overset{T}{\sim} \Lambda^k \Rightarrow A^{k+1} \overset{T}{\sim} \Lambda^{k+1}$$

VERA

$$A^k T = T \Lambda^k$$

$$A(A^k T) = A(T \Lambda^k)$$

$$A^{k+1} T = A T \Lambda^k = T \Lambda \Lambda^k = T \Lambda^{k+1}$$