Lab3

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October 2018

Part II

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- **a) What is the risk of melt-down in the power plant during a day if no observations have been made?**
  0.02578
- **What if there is icy weather?**
  0.03472
- **b) Suppose that both warning sensors indicate failure. What is the risk of a meltdown in that case?**
  $P(Meltdown|PumpFailureWarning, WaterLeakWarning) = 0.14535$
- **Compare this result with the risk of a melt-down when there is an actual pump failure and water leak. What is the difference?**
  $P(Meltdown|PumpFailure, WaterLeak) = 0.20000$
- **c) The conditional probabilities for the stochastic variables are often estimated by repeated experiments or observations. Why is it sometimes very difficult to get accurate numbers for these? What conditional probabilites in the model of the plant do you think are difficult or impossible to estimate?**
  Especially IcyWeather and Meltdown are very difficult to induce. This forces us to model their probabilities, which introduces inaccuracies in our model as a whole.
- **d) Assume that the "IcyWeather" variable is changed to a more accurate "Temperature" variable instead. What are the different alternatives for the domain of this variable?**
  The domain could be either continuous or discrete values in some temperature scale.
- **What will happen with the probability distribution of** $P(WaterLeak|Temperature)$ **in each alternative?**
  The lower the temperature, the higher chance for a water leak.

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- **a) What does a probability table in a Bayesian network represent?**
  Probability tables for top nodes show prior probabilities whilst probability tables for child nodes show conditional probabilities.

- **b) What is a joint probability distribution?**
  It is the probability that several conditions are fulfilled. Written as e.g. $P(x, y)$

- **Using the chain rule on the structure of the Bayesian network to rewrite the joint distribution as a product of $P(child|parent)$ expressions, calculate manually the particular entry in the joint distribution of P(Meltdown=F, PumpFailureWarning=F, PumpFailure=F, WaterLeakWaring=F, WaterLeak=F, IcyWeather=F).**

$$
\begin{aligned}
&P(Meltdown = F, PumpFailureWarning = F, PumpFailure = F,\\
&\quad WaterLeakWarning = F, WaterLeak = F, IcyWeather = F) =\\
&\quad P(Meltdown = F|PumpFailure = F, WaterLeak = F)\times\\
&\quad P(PumpFailureWarning = F|PumpFailure = F)\times\\
&\quad P(WaterLeakWarning = F|WaterLeak = F)\times\\
&\quad P(WaterLeak = F|IcyWeather = F)\times\\
&\quad P(PumpFailure = F)\times\\
&\quad P(IcyWeather = F) =\\
&\quad 0.999 \times 0.95 \times 0.95 \times 0.9 \times 0.9 \times 0.95 =\\
&\quad 0.69
\end{aligned}
$$

- **Is this a common state for the nuclear plant to be in?**
  Yes.

- **c) What is the probability of a meltdown if you know that there is both a water leak and a pump failure?**
  0.20.

- **Would knowing the state of any other variable matter? Explain your reasoning!**

  No. Water leak and Pump failure are the only parents to Meltdown, and their states are set.

- **d) Calculate manually the probability of a meltdown when you happen to know that PumpFailureWarning=F, WaterLeak=F, WaterLeakWarning=F and IcyWeather=F but you are not really sure about a pump failure.**

$$
\begin{aligned}
&X = \{Meltdown\} \\
&e = \{PumpFailureWarning = F, WaterLeakWarning = F, \\
&\quad WaterLeak = F, IcyWeather = F\} \\
&y = \{PumpFailure\}
\end{aligned}
$$

$$
\begin{aligned}
P(X|e) &= \alpha \times \sum_{y} P(X, e, y) = \\
&= \alpha \times (P(X, e, PumpFailure = F) + P(X, e, PumpFailure = T)) = \\
&= \alpha \times \{0.0019, 0.7007\} = \{0.0027, 0.9973\}
\end{aligned}
$$

Meltdown=T

$$
\begin{aligned}
&P(Meltdown = T, PumpFailureWarning = F, PumpFailure = F, \\
&\quad WaterLeakWarning = F, WaterLeak = F, IcyWeather = F)+ \\
&P(Meltdown = T, PumpFailureWarning = F, PumpFailure = T, \\
&\quad WaterLeakWarning = F, WaterLeak = F, IcyWeather = F) = \\
&\qquad P(WaterLeakWarning = F|WaterLeak = F)\times \\
&\qquad P(WaterLeak = F|IcyWeather = F)\times \\
&\qquad P(IcyWeahter = F)\times \\
&\qquad (P(Meltdown = T|PumpFailure = F, WaterLeak = F)\times \\
&\qquad P(PumpFailureWarning = F|PumpFailure = F)\times \\
&\qquad P(PumpFailure = F)+ \\
&\qquad P(Meltdown = T|PumpFailure = T, WaterLeak = F)\times \\
&\qquad P(PumpFailureWarning = F|PumpFailure = T)\times \\
&\qquad P(PumpFailure = T)) = \\
&\qquad 0.95 \times 0.9 \times 0.95 \times (0.001 \times 0.95 \times 0.9 + 0.15 \times 0.1 \times 0.1) = \\
&\qquad 0.0019
\end{aligned}
$$

Meltdown=F

$$
\begin{aligned}
&P(Meltdown = F, PumpFailureWarning = F, PumpFailure = F,\\
&\quad WaterLeakWarning = F, WaterLeak = F, IcyWeather = F)+\\
&P(Meltdown = F, PumpFailureWarning = F, PumpFailure = T,\\
&\quad WaterLeakWarning = F, WaterLeak = F, IcyWeather = F) =\\
&\qquad P(WaterLeakWarning = F|WaterLeak = F)\times\\
&\qquad P(WaterLeak = F|IcyWeather = F)\times\\
&\qquad P(IcyWeahter = F)\times\\
&\qquad (P(Meltdown = F|PumpFailure = F, WaterLeak = F)\times\\
&\qquad P(PumpFailureWarning = F|PumpFailure = F)\times\\
&\qquad P(PumpFailure = F)+\\
&\qquad P(Meltdown = F|PumpFailure = T, WaterLeak = F)\times\\
&\qquad P(PumpFailureWarning = F|PumpFailure = T)\times\\
&\qquad P(PumpFailure = T)) =\\
&0.95 \times 0.9 \times 0.95 \times (0.999 \times 0.95 \times 0.9 + 0.85 \times 0.1 \times 0.1) =\\
&0.7007
\end{aligned}
$$

$$
\alpha = \frac{1}{0.0019 + 0.7007} \approx 1.4233
$$

$$
\alpha\{0.0019, 0.7007\} = \{0.0027, 0.9973\}
$$

Part III

- **During the lunch break, the owner tries to show off for his employees by demonstrating the many features of his car stereo. To everyone's disappointment, it doesn't work. How did the owner's chances of surviving the day change after this observation?**
  Before: 0.99001
  After: 0.98116
- **How does the bicycle change the owner's chances of survival?**
  Before: 0.99505
  After: 0.99116
- **It is possible to model any function in propositional logic with Bayesian Networks. What does this fact say about the complexity of exact inference in Bayesian Networks?**
  The complexity of exact inference in Bayesian Networks is, in the worst case, as high as in propositional logic.

- **What alternatives are there to exact inference?**
  One alternative is *the naive Bayes' model*. This can be more efficient under the assumption that some of the stochastic variables are conditionally independent.

Part IV

- **The owner had an idea that instead of employing a safety person, to replace the pump with a better one. Is it possible, in your model, to compensate for the lack of Mr H.S.'s expertise with a better pump?**

  Yes.

- **Mr H.S. fell asleep on one of the plant's couches. When he wakes up he hears someone scream: "There is one or more warning signals beeping in your control room!". Mr H.S. realizes that he does not have time to fix the error before it is to late (we can assume that he wasn't in the control room at all). What is the chance of survival for Mr H.S. if he has a car with the same properties as the owner?**

  0.98

- **What unrealistic assumptions do you make when creating a Bayesian Network model of a person?**

  That a complex being like a person can be modeled with only four stochastic variables.

- **Describe how you would model a more dynamic world where for example the "IcyWeather" is more likely to be true the next day if it was true the day before. You only have to consider a limited sequence of days.**

  One could construct a chain of nodes, representing the different days, where the probability for the first node is the initial probability for icyWeather and the following nodes have some higher probability if the parent node is true. These nodes could then be tied together in a single node that essentially "ORs" the values of the nodes in the chain.