Macro Theory: Assignment 3

Ricardo Mayer

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1 Revenge of the Sith, I

Submit a correct answer to Question 4 of Assignment 1 (a.k.a Problem Set 1).
For you convenience I'll repeat the question here:

1. Write down Bayes rule for $f_{X|Y}(x|y)$. Note that this notation refers to *densities*! Please employ the same notation in your answer.
2. Write down Bayes rule for $f_{X|Y,Z}(x|y,z)$, maintaining the conditioning on $Z = z$.
3. Show how you can obtain $f(x, y)$ using only $f(x, y, z)$ or using together $f_{Y,X|Z}(x, y|z)$ and $f_Z(z)$.
4. Suppose that x and y have a joint distribution that is **uniform**. The support of the joint distribution is given by $\{(x, y) : x \in (0, y), y \in (0, 1)\}$.
   (a) Draw a diagram showing the support of $f(x, y)$.
   (b) You already know the joint is uniform (thus, a constant). Find out what is proper constant height of the joint.
   (c) Find the marginal density $f_X(x)$.
   (d) Find the conditional density $f_{Y|X}(y|x)$

2 Revenge of the Sith, II

Submit a correct answer to the entire Assignment 2 (a.k.a Computational Problem Set 1).

Even if you are the *one* case that submitted a correct answer, this time please emulate the instruction's .Rmd file and hide unimportant messages or results when loading packages or calling functions . . . that applies to everyone, BTW.

3 Past shocks, adaptive expectations and Muth 1961

Consider equation (6) in Chapter 3 of Benassy's book. Starting from there,

1. Write p_t as a function of $p_{t-2}, \epsilon_t, \epsilon_{t-1}$.

2. Write p_t as a function of $p_{t-3}, \epsilon_t, \epsilon_{t-1}, \epsilon_{t-2}$.

3. Write p_t as a function of $p_{t-4}, \epsilon_t, \epsilon_{t-1}, \epsilon_{t-4}, \epsilon_{t-3}$. In this case, use summation symbols to simplify your formula

4. Starting from the previous point, write p_t as a function of past shocks into an infinite past. Take limits where you must, in order to get the same expression you'll find in equation (7) of the same chapter.

4 Sargent and Wallace's increase in quantity of money

Consider the Sargent and Wallace's version of the Cagan model as depicted in section (3.5.2) of Benassy's book. Read the mental experiment there concerning an increment in the quantity of money. Prove that the equations describing the evolution of p_t are correct. Looking at Figure 2 may aid your intuition. Assume, throughout your answer, that at time $t_0 + \theta$ money becomes 25% higher than before.

5 Economic model and pseudocode

1. Get familiar with the concept and examples of Pseudocode and algorithm. They are close cousins of Flow Charts, but they are not the same. You may start drawing a flow chart if that helps you start thinking on your problem. A complete pseudocode may contain the data gathering steps of your work, the actual computations you want to carry and what output to display and when. Algorithms are usually understood to refer to specific parts of your work. Here are some links so you may enlighten yourself:

    - Wikipedia reference for Pseudocode. Note the difference between the pseudocode that has a particular programming language in mind and the more numerical-math oriented, that tends to use more traditional math notation and borrows a few control flow structures from computer science.
    - Here are two examples of numerical algorithms, commonly used in numerical mathematics : the Jacobi method and the Conjugate-Gradient method.
    - I'd rather have you to write your pseudocode by hand and not in LaTeX(at least for this assignment) but this page have additional examples of how pieces of pseudocode look like.

2. This is the actual question: **(hand)write a pseudocode that, in principle, would allow you to obtain time series of simulated data for consumption, investment, capital and labor, *generated* by the model presented in section 8.**. This will force you to visualize the general aspects of the problem and to distinguish specific ways to accomplish some of the steps. There is always a trade-off between generality and usefulness of the pseudocode as a guide for action.

6 Practising log-linearization

Take a look at Harald Uhlig's toolkit. For this assignment you may stop before the paper starts solving the system of equation by the method of undetermined coefficients. The model featured in section 8 is very similar to the one in that paper. This question will be easy if you first do and understand Uhlig's example of steady state and log-linearizations first.

1. Find the *system of equations* that determine the steady states values of the relevant variables, for the Social Planner's problem (SPP) in the economy of section 8.

2. Log-linearize the resource constraint for the SPP in the economy of section 8.

7 Log-linearization as a special case of change of variables

Suppose that you have these equations:

$$c_t(k_{t-1}, z_t) = \bar{c} + \alpha_{ck} k_{t-1} + \alpha_{cz} z_t \tag{1}$$
$$l_t(k_{t-1}, z_t) = \bar{l} + \alpha_{lk} k_{t-1} + \alpha_{lz} z_t \tag{2}$$

and

$$C_t(K_{t-1}, z_t) = \bar{C} + \beta_{ck} K_{t-1} + \beta_{cz} z_t \tag{3}$$
$$L_t(K_{t-1}, z_t) = \bar{L} + \beta_{lk} K_{t-1} + \beta_{lz} z_t \tag{4}$$

where $x_t := \ln X_t$. Now suppose that you already have the β's, that is, the coefficients used when consumption, labor and capital are measured in levels. Show that if you want to use the variables in *logs*, you don't have to redo all your hard work, because it's sufficient to transform the β's to find the appropriate α's. Tip: this is just an application of the Chain Rule and of the fact that coefficients are just partial derivatives.

8 A two-real-productivities model

At date zero, a social planner seeks to maximize the following criteria by choosing a sequence of capital stock, labor fraction of available time and consumption level:

$$\max_{\{K_t, C_t, Lt\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\ln C_t + \Psi \ln(1 - L_t) \right) \right]$$

Output, in turn, has two uses: consumption and investment,

$$C_t + I_t \leq Y_t \tag{5}$$

Capital stock grows by investing, I_t, and decreases due to depreciation of existing capital, δK_{t-1}.

$$K_t = V_t I_t + (1 - \delta) K_{t-1} \tag{6}$$

Combining the resource constraint 5 and the capital accumulation 6 results in the following constraint,

$$A_t K_{t-1}^{\alpha} L_t^{1-\alpha} + (1-\delta)\frac{K_{t-1}}{V_t} - C_t - \frac{K_t}{V_t}$$

There is also a transversality condition to be met:

$$\lim_{T\to\infty} E_0 \beta^T \frac{1}{C_T}\frac{1}{V_T} K_T = 0$$

Then, a Lagrangian function for the SPP is given by

$$\max_{\{K_t, C_t, Lt\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\ln C_t + \Psi \ln(1-L_t) + \lambda_t \left[A_t K_{t-1}^{\alpha} L_t^{1-\alpha} + (1-\delta)\frac{K_{t-1}}{V_t} - C_t - \frac{K_t}{V_t}\right]\right)\right] \tag{7}$$

Finally, we temporally assume that A and V are stationary :

$$\ln A_t = (1-\rho_a)\ln \bar{A} + \rho_a \ln A_{t-1} + \epsilon_{a,t}, \quad \epsilon_a \sim \text{ i.i.d. } N(0, \sigma_a^2) \tag{8}$$

$$\ln V_t = (1-\rho_v)\ln \bar{V} + \rho_v \ln V_{t-1} + \epsilon_{v,t}, \quad \epsilon_v \sim \text{ i.i.d. } N(0, \sigma_v^2) \tag{9}$$