3A) $\hat{\lambda} = \bar{X} = \frac{1}{n}\sum_{1}^{n} X_i = 24.9, \; n = 1$

$\delta = 0.5, 1, 1.5, 2, 2.5$

$\mathbb{E}[X_i] = Var[X_i] = \lambda_0$

CLT:

$$\sqrt{n}\,\frac{(\bar{X} - \lambda_0)}{\sqrt{\lambda_0}} \xrightarrow{n \to \infty} N(0,1)$$

$$\sqrt{\frac{\lambda_0}{n}} \approx \sqrt{\frac{\hat{\lambda}}{n}} = \sqrt{\frac{24.9}{23}} \boxed{\approx 1.04}$$

$$\begin{aligned}
P(|\lambda_0 - \hat{\lambda}| > \delta) &= P(|\lambda_0 - \bar{X}| > \delta) \\
&= P\left(\sqrt{n}\,\frac{|\lambda_0 - \bar{X}|}{\sqrt{\lambda_0}} > \sqrt{n}\,\frac{\delta}{\sqrt{\lambda_0}}\right) \\
&\approx P\left(|N(0,1)| > \sqrt{n}\,\frac{\delta}{\sqrt{\hat{\lambda}}}\right) \\
&= P\left(|N(0,1)| > \frac{\sqrt{23}}{\sqrt{24.9}}\,\delta\right) \\
&= 2\left(1 - \Phi\left(\sqrt{\frac{23}{24.9}}\,\delta\right)\right)
\end{aligned}$$

$\approx 1.315, 1.168, 1.075, 1.027, 1.008$

3B)

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{\hat{\lambda}^{x_i} e^{-\hat{\lambda}}}{x_i!}(300) = \hat{n}_i$$

MLE of λ, $\hat{\lambda} = \bar{X}$

$$\hat{\lambda} = \frac{292}{75} \approx 3.89\overline{3}$$

estimate $; 3.89\dot{3}$

The difference in the observed $(3.89\overline{3})$ and the expected (3.893) is quite close to one another, which shows this is a good fit

3 c)

$\{X_1, X_2, \dots X_n)$, $\hat{\mu}_k = \frac{1}{n}\sum_{i=1}^{n} X_i^k$

$E(\hat{\mu}_k) = \mu_k$

Kth moment $\mu_k = E(X^k)$

$$\mathbb{E}(\hat{\mu}_k) = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i^k\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n} \mathbb{E}(X_i^k)$$

$$= \frac{1}{n}\sum_{i=1}^{n} \mu_k$$

$$= \frac{n\mu_k}{n}$$

$$\boxed{= \mu_k}$$

3D) $\hat{p}_{MM}$ $\{X_1, X_2, \dots, X_n\} \sim \text{Geo}(p)$

$k = 1$ $\mathbb{E}(X) = \frac{1}{p}$

$$\frac{1}{\hat{p}_{MM}} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

$$= \bar{X}_n$$

$$\boxed{\hat{p}_{MM} = \frac{1}{\bar{X}_n}}$$

3E)

a) $\Gamma(r+1) = r\Gamma(r)$ $\qquad e^{-t} = \frac{d}{dt}(-e^{-t})$

$$\Gamma(r+1) = \int_0^\infty t^r e^{-t} dt$$

$$= \left[t^r(-e^{-t})\right]_0^\infty - \int_0^\infty r t^{r-1}(-e^{-t}) dt$$

$$= 0 + r\boxed{\int_0^\infty t^{r-1}(-e^{-t})dt} = \Gamma(r)$$

$$= 0 + r\Gamma(r)$$

$$\boxed{\Gamma(r+1) = r\Gamma(r)}$$

b) $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

$$\Gamma(1/2) = \int_{t=0}^{+\infty} t^{\frac{1}{2}-1} e^{-t} dt$$

$$= \int_{t=0}^{+\infty} \frac{e^{-t}}{t^{1/2}} dt$$

$y = \sqrt{t}$, $dy = \frac{dt}{2\sqrt{t}}$

$$= 2\int_{y=0}^{+\infty} e^{-y^2} dy$$

$$= \int_{y=-\infty}^{+\infty} e^{-y^2} dy \boxed{= \sqrt{\pi}}$$