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Exercise 1

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Question 1: Dimensional Analysis

In the lecture we performed a dimensional analysis of the Couette Flow based on Taylor's method. This exercise is dedicated to another phenomenon, viscous flow past a sphere as depicted in Figure 1.

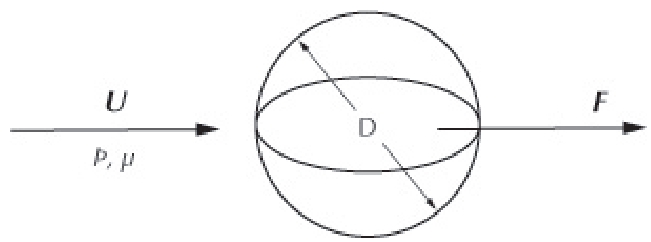

Figure 1: Viscous flow past a sphere

The objective is to correlate the force that develops on the sphere, F, in terms of the dimensionless parameters in the problem. Assume the sphere is not heated or cooled and that gravitational effects (or any other external forces) are of no consequence.

a) Find the dimensional quantities that describe the problem. Use Figure 1 as guidance.

b) Set up the matrix according to Taylor's method and find the dimensionless groupings. How many do you expect?

c) Based on your groupings explain the connection between the force F and the Reynolds number $Re = \frac{\rho U D}{\mu}$

Question 2: Dynamic similitude

In the lecture you have encountered the so-called Navier-Stokes equation. You shortly analyzed the dimensions of each term in the equation. In this exercise we want to derive a dimensionless form of this equation (in one spatial dimension).
The Navier-Stokes equation describes the motion of a viscous fluid. In one spatial dimension, the equation reads

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} \tag{1}$$

where t is the time, x the spatial dimension, ρ is the density, u the velocity, p the pressure of the fluid and μ its constant viscosity (assuming the temperature to be constant).

Suppose now a small one-dimensional sound source of size L moving at a constant velocity U emits a tone with frequency ω. For simplicity, we assume the motion can be represented by Equation (1). We denote the density of the fluid far away from the source with ρ_∞.

a) Based on the given nomenclature define all dimensionless variables and set up the dimensionless form of the Navier-Stokes equation.

b) In the lecture you have seen some important dimensionless groupings, e.g. the Reynolds number. Recast your dimensionless grouping such that the Navier-Stokes equation contains only the dimensionless quantities, the Reynolds number $Re = \frac{\rho_\infty U L}{\mu}$ and the Strouhal number $St = \frac{\omega L}{U}$.

c) In the lecture you have been introduced to the concept of "dynamic similitude". Explain this concept in short! Assume another otherwise identical sound source moves with increased velocity $\hat{U} = 10U$. What other physical quantities have to be changed to ensure dynamic similitude of both systems?

Question 3: Continuum assumption (Optional)

To get a feeling for the continuum assumption let's do a simple toy problem in the computer. Take a box and randomly seed a large number of particles homogeneously inside. Then take averaging volumes of increasing size, centered at the box center and count the number of particles inside.

a) Plot the density vs. volume size. Assume that all particles have the same mass $m = 1$.

b) Find the mean free path length λ and the physical length scale L in function of the particle seed density. Check that you recover the density that you used to create the particles.

c) Let's do now an inhomogeneous seeding of the particle. What changes do you see in L?