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Exercise 6

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Question 1: Random Walk in biology

You observed two spherical objects of radius $R_1 = 1\mu m$, $R_2 = 100\mu m$ moving randomly by about $1\mu m$ and $1mm$ every second correspondingly. Use Stokes-Einstein equation for diffusion in low Reynolds numbers to give an an educated guess about which object might be living. Assume a reasonable value for the temperature ($\sim$ 300 K).

Question 2: 1D-Diffusion problem

Consider the following problem:

$$\frac{\partial u(x,t)}{\partial t} = \nu \cdot \frac{\partial^2 u(x,t)}{\partial x^2}$$

$$\begin{array}{ll} u(x,t=0) = xe^{-x^2} & x \in [0,\infty) \\ u(x=0,t) = 0 & t \in [0,\infty) \end{array}$$

To solve the diffusion equation you have been introduced to Random Walk (RW). The objective of this exercise is to implement the two methods and to validate your code. The exact (analytic) solution of the problem is

$$u^{\text{ex}}(x,t) = \frac{x}{(1+4\nu t)^{3/2}} \cdot e^{-x^2/(1+4\nu t)}.$$

Random Walk for diffusion in space

Please implement the Random Walk particle method in 1D. Test your code with the problem stated above! At $x = 0$ we have the Dirichlet boundary condition $u(x = 0, t) = 0$. By computing the solution on $x \in [-X, X]$ this boundary condition is implicitly satisfied for all t due to the point symmetric nature of the initial concentration. At $x = X$ we have an open boundary. Discuss the problems and effects of the boundary conditions!

Parameters:

particle number:	$N = \{50, 100, 200, 400, 800\}$
domain:	$x \in [-X, X]\,,\ X = 4$
interparticle spacing:	$h = 2X/(2N-1)$
diffusion constant:	$\nu = 0.0001$
time step:	$\Delta t = 0.1$
integration time:	$T = t_{\max} = 10$
kernel size:	$\epsilon = h$