RLHF

Deep Reinforcement Learning from Human Preferences (OpenAI, 2017)

Environment giving reward? No.

$\Longrightarrow$ **human overseer** who can express preferences between trajectory segments

How to make the reward model $\hat{r}(o_t, a_t)$ for this?

If we have two trajectory with following **preference**,

$$((o_0^1, a_0^1), \ldots, (o_{k-1}^1, a_{k-1}^1)) \succ ((o_0^2, a_0^2), \ldots, (o_{k-1}^2, a_{k-1}^2))$$

It should mean that

$$r(o_0^1, a_0^1) + \ldots + r(o_{k-1}^1, a_{k-1}^1) > r(o_0^2, a_0^2) + \ldots + r(o_{k-1}^2, a_{k-1}^2)$$

Bradley-Terry model

For trajectory σ_1, σ_2 generated by policy π:

$$\hat{P}(\sigma_1, \sigma_2) = \frac{\exp\left(\sum \hat{r}(o_t^1, a_t^1)\right)}{\exp\left(\sum \hat{r}(o_t^1, a_t^1)\right) + \exp\left(\sum \hat{r}(o_t^2, a_t^2)\right)}$$

$$\text{loss}(\hat{r}) = -\sum_{(\sigma_1, \sigma_2) \in D} I(\sigma_1 \succ \sigma_2) \log \hat{P}(\sigma_1, \sigma_2) + I(\sigma_1 \prec \sigma_2) \log \hat{P}(\sigma_2, \sigma_1)$$

$\rightarrow$ Estimate $\hat{r}$ as adding data to D. Apply RL algorithms to update π. Can be done asynchronously.

ChatGPT

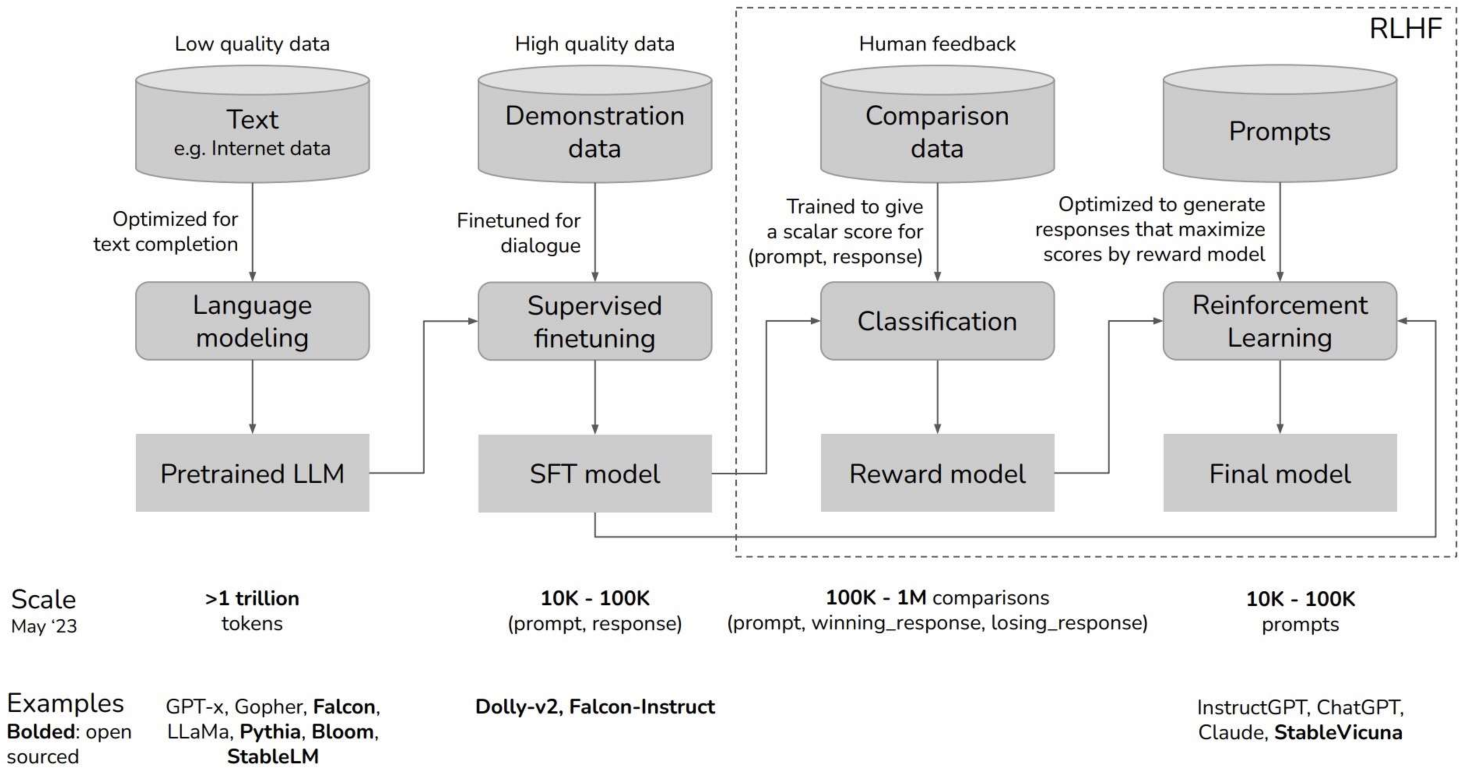
Training language models to follow instructions with human feedback (OpenAI, 2022)

Let's Apply RLHF in **Language Models**.

Alignment

"Follow the user's instructions helpfully and safely"

$\rightarrow$ fine-tune LM on the reward model representing human preference scores



ChatGPT

Simplify the problem, assume **Contextual Bandit**

O: prompt x

A: response y

For K responses, 1 vs 1 preference comparison $\rightarrow$ get $\binom{K}{2}$ tuples of (x, y_w, y_l)

Reward Modeling

$$\text{loss}(\theta) = -\frac{1}{\binom{K}{2}}\mathbb{E}_{(x,y_w,y_l)\sim D}\left[\log(\sigma(r_\theta(x, y_w) - r_\theta(x, y_l)))\right]$$

$$\left(\because \sigma(a-b) = \frac{1}{1+e^{-(a-b)}} = \frac{e^a}{e^a + e^b}\right)$$

RL(policy gradient), by sampling response y from π_ϕ^{RL}

$$\text{objective}(\phi) = \mathbb{E}_{(x,y)\sim D_{\pi_\phi^{\text{RL}}}}\left[r_\theta(x, y) - \beta \log \frac{\pi_\phi^{\text{RL}}(y|x)}{\pi^{\text{SFT}}(y|x)}\right]$$

* Response y is **tokenized** as $y_1, \ldots, y_n$

$$\pi(y|x) = \pi(y_1|x)\pi(y_2|x, y_1)\ldots\pi(y_n|x, y_1, \ldots, y_{n-1})$$

DPO

Direct Preference Optimization: Your Language Model is Secretly a Reward Model

(Stanford, 2023)

RLHF is unstable $\rightarrow$ Do we really need Reward Modeling + RL ?

Generalized form of RLHF using KL constraint:

Train the reward model with loss function:

$$\mathcal{L}_R(r_\phi, \mathcal{D}) = -\mathbb{E}_{(x,y_w,y_l)\sim\mathcal{D}}\left[\log(\sigma(r_\theta(x,y_w) - r_\theta(x,y_l)))\right] \tag{1}$$

And find optimal policy that maximizes:

$$\mathbb{E}_{(x\sim D, y\sim\pi_\theta(y|x))}\left[r_\phi(x,y)\right] - \beta\mathbb{D}_{\mathsf{KL}}\left[\pi_\theta(y|x)\,||\,\pi_{\mathsf{ref}}(y|x)\right] \tag{2}$$

The optimal solution of (2) is:

$$\pi_r(y|x) = \tfrac{1}{Z(x)}\pi_{\mathsf{ref}}(y|x)\exp\left(\tfrac{1}{\beta}r(x,y)\right) \tag{3}$$

with partition function

$$Z(x) = \sum_y \pi_{\mathsf{ref}}(y|x)\exp\left(\frac{1}{\beta}r(x,y)\right)$$

DPO

Proof of (3)

$$
\begin{aligned}
&\underset{\pi}{\operatorname{argmax}}\, \mathbb{E}_{x\sim\mathcal{D},y\sim\pi}\left[r(x,y)\right] - \beta\mathbb{D}_{\mathsf{KL}}\left[\pi(y|x)||\pi_{\mathsf{ref}}(y|x)\right] \\
&= \underset{\pi}{\operatorname{argmax}}\, \mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi(y|x)}\left[r(x,y) - \beta\log\frac{\pi(y|x)}{\pi_{\mathsf{ref}}(y|x)}\right] \\
&= \underset{\pi}{\operatorname{argmin}}\, \mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi(y|x)}\left[\log\frac{\pi(y|x)}{\pi_{\mathsf{ref}}(y|x)} - \frac{1}{\beta}r(x,y)\right] \\
&= \underset{\pi}{\operatorname{argmin}}\, \mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi(y|x)}\left[\log\frac{\pi(y|x)}{\pi_{\mathsf{ref}}(y|x)} - \frac{1}{\beta}r(x,y) + \log Z(x) - \log Z(x)\right] \\
&= \underset{\pi}{\operatorname{argmin}}\, \mathbb{E}_{x\sim\mathcal{D}}\mathbb{E}_{y\sim\pi(y|x)}\left[\log\frac{\pi(y|x)}{\frac{1}{Z(x)}\pi_{\mathsf{ref}}(y|x)\exp(\frac{1}{\beta}r(x,y))} - \log Z(x)\right]
\end{aligned}
$$

where $Z(x) = \sum_y \pi_{\mathsf{ref}}(y|x)\exp\left(\frac{1}{\beta}r(x,y)\right)$. Let

$$
\pi^*(y|x) = \frac{1}{Z(x)}\pi_{\mathsf{ref}}(y|x)\exp\left(\frac{1}{\beta}r(x,y)\right)
$$

$\pi^*(y|x) \geq 0$ and $\sum_y \pi^*(y|x) = 1 \implies \pi^*$ is valid policy.

Also, $Z(x)$ is independent to π

DPO

Continue. Substituting π^*,

Proof.

$$
\begin{aligned}
\ldots &= \underset{\pi}{\operatorname{argmin}}\, \mathbb{E}_{x\sim D}\left[\mathbb{E}_{y\sim\pi(y|x)}\left[\log\frac{\pi(y|x)}{\pi^*(y|x)}\right] - \log Z(x)\right] \\
&= \underset{\pi}{\operatorname{argmin}}\, \mathbb{E}_{x\sim D}\left[\mathbb{D}_{\mathrm{KL}}(\pi(y|x)||\pi^*(y|x)) - \log Z(x)\right] \\
&= \underset{\pi}{\operatorname{argmin}}\, \mathbb{E}_{x\sim D}\left[\mathbb{D}_{\mathrm{KL}}(\pi(y|x)||\pi^*(y|x))\right] \\
&= \pi^*(y|x)
\end{aligned}
$$

□

From (3), we get:

$$
r(x, y) = \beta \log\frac{\pi_r(y|x)}{\pi_{\mathsf{ref}}(y|x)} + \beta \log Z(x) \tag{4}
$$

Apply (4) to (1), and $Z(x)$ cancels:

$$
\therefore \mathcal{L}_{\mathsf{DPO}}(\pi_\theta; \pi_{\mathsf{ref}}) = -\mathbb{E}_{(x,y_w,y_l)\sim\mathcal{D}}\left[\log\sigma(\beta\log\frac{\pi_\theta(y_w|x)}{\pi_{\mathsf{ref}}(y_w|x)} - \beta\log\frac{\pi_\theta(y_l|x)}{\pi_{\mathsf{ref}}(y_l|x)})\right] \tag{5}
$$

$\rightarrow$ No longer a RL problem. No need of reward model. **Direct optimization**

$+$ Enhanced Stability