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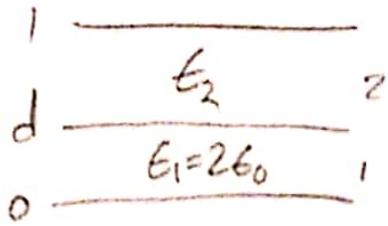
ECE 329 - HW5
DUE 06/28/19

1f) $C = \frac{Q}{V}$

$Q = \rho_s A$



$V = ED$

$V = E_1 d + E_2(1-d)$

$\rho_s = \epsilon_1 E_1 = \epsilon_2 E_2$

$E_2 = \frac{\epsilon_1 E_1}{\epsilon_2}$

$$\frac{A(E_2)(\epsilon_2)}{E_1 d + E_2(1-d)} = \frac{A(\epsilon_2)}{\frac{E_1}{E_2} d + (1-d)}$$

$$= \frac{A\epsilon_2}{\frac{\epsilon_2 d}{\epsilon_1} + (1-d)} \left(\frac{\epsilon_1}{\epsilon_1}\right) \boxed{= \frac{A\epsilon_2\epsilon_1}{\epsilon_2 d + \epsilon_1(1-d)} = C}$$

2a) $C = \frac{\varepsilon A}{d} = \frac{\pi(8.85\,pF/m)(0.02m)^2}{0.005m} \boxed{= 2.224\,pF}$

2b) $G = \frac{\sigma A}{d} = \frac{(10^{-3}\frac{1}{\Omega \cdot m})(0.02m)^2}{0.005m} \boxed{= 8 \times 10^{-5}\,S}$

$R = \frac{1}{G} = 12{,}500\,\Omega \boxed{= 12.5\,k\Omega}$

2c)

$\frac{dV}{dt} + \frac{1}{RC}V = 0$

general solution $V(t) = V(0)e^{-\frac{t}{\tau}}$

$V(t) = \frac{(1C)}{2.224\times10^{-12}} e^{-\frac{t}{RC}}$

$V(t) = (4.496\times10^{11})\, e^{-\frac{t}{2.78\times10^{-8}}}$

$V(10ns) = 1.528\times10^{11}\,V$

$Q = CV = (1.528\times10^{11})(2.224\times10^{-12})$

$\boxed{Q = 0.3398\,C}$

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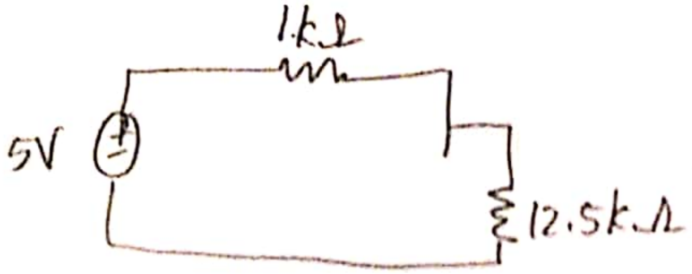
2d) 10s is significantly larger than τ so a long time has passed in which the capacitor behaves as an open circuit



$I = \frac{V}{R} = \frac{5}{(1+12.5)(1000)} = 3.7037\times10^{-4} A \quad (12.5)(1000)$

By voltage div, $\boxed{V_C = 4.63V \text{ 10s later}}$

3a)

$E_r = \frac{Q_s}{\varepsilon 4\pi r^2}$

$V = \int_a^b E \cdot dl = \frac{Q_s}{4\varepsilon\pi}\int_a^b r^{-2}\,dr$

$V = \frac{Q_s}{4\varepsilon\pi}\left(r^{-1}\Big|_a^b\right) = \frac{Q_s}{4\varepsilon\pi}\left(\frac{1}{b}-\frac{1}{a}\right)$

$\frac{a}{ab} - \frac{b}{ab}$

$V = \frac{Q_s}{4\varepsilon\pi}\,\frac{a-b}{ab}$

$Q_s = \left(4\varepsilon\pi\frac{ab}{a-b}\right)V$

$Q_s = CV$ ✓

3b) $C = 4\pi\epsilon \frac{ab}{b-a}$ when $a=1$ $\lim b \to \infty$

$C = 4\pi\epsilon = 4\pi(2\epsilon_0)$

$\boxed{C = 8\pi\epsilon_0 F}$

3c) $G = 4\pi\sigma = 25.13\ \mu S$

3d)

$V_R = -V_c$

$\frac{dQ}{dt}R = -\frac{Q}{C}$

$\frac{dQ}{dt}R + \frac{Q}{C} = 0$

3e) $\int \frac{1}{Q}\,dQ = -\int \frac{1}{RC}\,dt$

$\ln Q = \frac{-t}{RC} + C$

$Ce^{-\frac{t}{RC}} = Q$

$Qe^{-\frac{t}{RC}} = Q(t)$

$\boxed{Q(t) = 1e^{-\frac{t}{RC}}}$

4) - Silicon
- damp wood

$\sim 10^{-3} \frac{1}{\Omega \cdot m}$

- castor oil
- pyrex glass

$\sim \varepsilon_r \sim 4.7$

- formamide $\varepsilon_r \sim 84.0$ @ $20°C$
- sulfuric acid $\sim \varepsilon_r \sim 84.0$ @ $20°C$

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5a) $R = \frac{d}{A\sigma} = \frac{200m}{\pi(0.001m)^2(5.8\times10^7 \frac{1}{\Omega\cdot m})} \boxed{= 1.0976\,\Omega}$

$\frac{\cancel{m}\ \Omega\cdot \cancel{m}}{\cancel{m^2}}$

5b) $I = \sigma E A = (5.8\times10^7 \frac{1}{\Omega\cdot m})(E)\underbrace{(\pi(0.001m)^2)}_{3.14\times10^{-6}} = 1A$

$\boxed{E = 0.00549\ V/m}$

5c) $\sigma E = N_e q v$

$|v| = \frac{\sigma E}{N_e q} = \frac{(5.8\times10^7 \frac{1}{\Omega\cdot m})(0.00549\ V/m)}{(8.45\times10^{28} m^{-3})(1.6\times10^{-19}C)}$

$\boxed{|v| = 2.355\times10^{-5}\ m/s}$

5d) $vt = d$

$t = \frac{200m}{2.355\times10^{-5} m/s}$

$\boxed{t = 8.49\times10^6 s.}$

$\approx 141,542.8$ min

~ 2.359 hours

~ 98.3 days?

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