Automatica 79 (2017) 214–217

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Technical communique

# A new discrete-time stabilizability condition for Linear Parameter-Varying systems☆

Amit Prakash Pandey, Maurício C. de Oliveira

*Department of Mechanical and Aerospace Engineering, UC San Diego, La Jolla, CA 92093, USA*

ARTICLE INFO

*Article history:*
Received 3 October 2016
Received in revised form
2 December 2016
Accepted 11 January 2017
Available online 2 March 2017

*keywords:*
Parameter-varying systems
Stabilizability
Time-varying systems
Linear matrix inequalities

ABSTRACT

We introduce a new condition for the stabilizability of discrete-time Linear Parameter-Varying (LPV) systems in the form of Linear Matrix Inequalities (LMIs). A distinctive feature of the proposed condition is the ability to handle variation in both the dynamics as well as in the input matrix without resorting to dynamic augmentation or iterative procedures. We show that this new condition contains the existing poly-quadratic stabilizability result as a particular case. A numerical example illustrates the results.



## 1. Introduction and motivation

Consider the class of time-varying discrete-time linear systems of the form

$$x(k+1) = A(\xi(k))x(k) + B(\xi(k))u(k), \tag{1}$$

where $x \in \mathbb{R}^n$ and the matrices $A(\xi(k))$ and $B(\xi(k))$ are assumed to depend affinely on the time-varying parameter $\xi(k)$, which takes values in the unit simplex

$$\Xi = \left\{ \xi \in \mathbb{R}_+^N : \sum_{i=1}^{N} \xi_i = 1 \right\}.$$

The affine assumption means that matrices $A(\xi(k))$ and $B(\xi(k))$ can be written as

$$A(\xi(k)) = \sum_{i=1}^{N} \xi_i(k) A_i, \qquad B(\xi(k)) = \sum_{i=1}^{N} \xi_i(k) B_i.$$

In this paper, we are concerned with *stabilizability* by a *gain-scheduled* controller of the form:

$$u(k) = K(\xi(k))\,x(k) \tag{2}$$

using Linear Matrix Inequalities (LMIs). There are various examples of practical applications of the above model, including spacecraft control (Calloni, Corti, Zanchettin, & Lovera, 2012; Corti, Dardanelli, & Lovera, 2012), active suspension systems (Do, da Silva, Sename, & Dugard, 2011; Do, Sename, & Dugard, 2010), and the many other applications in Mohammadpour and Scherer (2012) and related references.

To the best of our knowledge, the most general necessary and sufficient stabilizability conditions for this class of time-varying systems that can still be expressed as LMIs are the ones from Daafouz and Bernussou (2001), which we reproduce in the next lemma.

**Lemma 1** (*Daafouz & Bernussou, 2001*)**.** *Consider the time-varying discrete-time linear system of the form (1). Assume that $B_i = B$ for all $i = 1, \ldots, N$. The following statements are equivalent:*

(a) *System (1) is* poly − quadratically stabilizable*;*
(b) *There exist matrices $X_i$, $L_i$ and $Q_i \succ 0$, $i = 1, \ldots, N$, such that*

$$\begin{bmatrix} X_i + X_i^T - Q_i & X_i^T A_i^T + L_i^T B^T \\ A_i X_i + B L_i & Q_j \end{bmatrix} \succ 0, \tag{3}$$

*for all $i, j = 1, \ldots, N$.*

*Furthermore, if inequalities (3) are feasible the gain-scheduled state-feedback controller (2) with gains*

$$K(\xi(k)) = \sum_{i=1}^{N} \xi_i(k) K_i, \quad K_i = L_i X_i^{-1}, \tag{4}$$

*poly-quadratically stabilizes the system (1).*

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Tongwen Chen under the direction of Editor André L. Tits.

*E-mail addresses:* appandey@ucsd.edu (A.P. Pandey), mauricio@ucsd.edu (M.C. de Oliveira).

http://dx.doi.org/10.1016/j.automatica.2017.02.006
0005-1098/

The above lemma makes use of the notion of *poly-quadratic stability*, in which stability of the time-varying system (1) is proved by constructing an affine *parameter-dependent* Lyapunov function (Gahinet, Apkarian, & Chilali, 1996) of the form

$$\begin{aligned} V(x(k), \xi(k)) &= x(k)^T P(\xi(k)) x(k), \\ P(\xi(k)) &= \sum_{i=1}^{N} \xi_i(k) P_i \succ 0. \end{aligned} \tag{5}$$

In Lemma 1, if inequalities (3) are feasible, then $P_i = Q_i^{-1}$ provides such a Lyapunov function.

The main deficiency of the condition in Lemma 1 is the fact that the system cannot have time-variation in the input matrix $B$, hence the assumption $B(\xi(k)) = B$. Note that one cannot simply let the $B$'s in inequalities (3) vary with $i$. In fact, a key element proved in Daafouz and Bernussou (2001) is the fact that inequalities (3) and $P_i = Q_i^{-1}$ imply that

$$\begin{bmatrix} P_i & (A_i + BK_i)^T P_j \\ P_j(A_i + BK_i) & P_j \end{bmatrix} \succ 0 \tag{6}$$

for all $i, j = 1, \dots, N$ from which

$$\begin{bmatrix} P(\xi(k)) & (A(\xi(k)) + BK(\xi(k)))^T P(\xi(k+1)) \\ \star & P(\xi(k+1)) \end{bmatrix} \succ 0 \tag{7}$$

for all $\xi(k), \xi(k+1) \in \Xi$ after a convex combination, where the $\star$ notation stand for symmetric blocks omitted for brevity. In fact, inequality (7) can be taken as a definition of poly-quadratic stabilizability. If $B$ and $K$ vary with $i$ then (6) no longer implies (7).

The main contribution of the present paper is to introduce the following design condition for the poly-quadratic stabilizability of system (1) which does not require the assumption that $B(\xi(k)) = B$ and recovers Lemma 1 as a particular case.

**Theorem 2.** *Consider the time-varying discrete-time linear system of the form (1). If there exist $X_i, L_i, Y_i, Z_i$ and $Q_i \succ 0$, $i = 1, \dots, N$ such that*

$$\begin{bmatrix} X_i + X_i^T - Q_i & X_i^T A_i^T & -L_i^T \\ A_i X_i & Q_j - R_{i,j} & B_i Z_j - Y_j^T \\ -L_i & Z_j^T B_i^T - Y_j & Z_j + Z_j^T \end{bmatrix} \succ 0, \tag{8}$$

*where*

$$R_{i,j} = B_i Y_j + Y_j^T B_i^T, \tag{9}$$

*for all $i, j = 1, \dots, N$, then the gain-scheduled state-feedback controller (2) with gain $K(\xi(k))$ as in (4) poly-quadratic stabilizes the system (1). Furthermore, if $B_i = B$ for all $i = 1, \dots, N$, then the converse also holds.*

As we will show in detail later in Section 4, it is guaranteed to hold whenever the one in Lemma 1 holds.

## 2. Comparison with existing results

It is important to point out that there have been other attempts to overcome the limitations of Daafouz and Bernussou (2001), e.g. Borges, Oliveira, Abdallah, and Peres (2008, 2010), Mao (2003) and Montagner, Oliveira, Leite, and Peres (2005). Refs. Borges et al. (2008, 2010) are BMIs (Bilinear Matrix Inequalities), hence not computationally attractive. The conditions presented in Mao (2003) and Montagner et al. (2005) are LMI based. Ref. Mao (2003) is concerned with poly-quadratically stabilizing robust controllers only. Ref. Montagner et al. (2005) deal with gain-scheduling but does not seem to recover the stabilizability results (Daafouz & Bernussou, 2001) in the case $B_i = B_j = B$ as possible with Theorem 2. In the case $B_i \neq B_j$ both Theorem 2 and Montagner et al. (2005) are sufficient and a direct comparison is not straightforward. However, Montagner et al. (2005) require checking $O(N^3)$ inequalities whereas Theorem 2 involves $O(N^2)$ inequalities. Furthermore, the gain scheduled controller (4) is linear in the time-varying parameter $\xi(k)$ whereas the one from Montagner et al. (2005) is rational.

Before proceeding, let us briefly discuss another alternative for incorporating time-variation in the input matrix in the discrete-time case. A standard way to handle time-variation in the input matrix $B$ is to work with an augmented system, for example:

$$\tilde{x}(k+1) = \tilde{A}(\xi(k))\, \tilde{x}(k) + \tilde{B}\, \tilde{u}(k), \quad \tilde{x} = \begin{pmatrix} x \\ u \end{pmatrix}, \tag{10}$$

where $A(\xi)$ and $B$ have as vertices the matrices

$$\tilde{A}_i = \begin{bmatrix} A_i & B_i \\ 0 & 0 \end{bmatrix}, \qquad \tilde{B}_i = \tilde{B} = \begin{bmatrix} 0 \\ I \end{bmatrix}. \tag{11}$$

This approach remains popular, as attested by recent applications in spacecraft control (Calloni et al., 2012; Corti et al., 2012) and the design of active suspension systems (Do et al., 2011, 2010). However, we have shown in Pandey, Sehr, and de Oliveira (2016) that for quadratic continuous-time stabilizability and in Sehr, Pandey, and de Oliveira (submitted for publication) for quadratic continuous-time performance, that such augmentation is counter-productive, in the sense that quadratic stabilizability or performance of the augmented system by a gain-scheduled controller of the form (2) in fact implies existence of a *robust controller*, that is one in which $K(\xi(k)) = K$ is independent of the time-varying parameter, $\xi(k)$, with same stability and performance guarantees. A similar property also holds for discrete-time quadratic stabilizability but not for poly-quadratic stabilizability (Pandey et al., 2016). This is surprising since in discrete-time augmentation necessarily comes with an additional cost. Indeed, a controller of the form

$$\tilde{u}(k) = \tilde{K}(\xi(k))\, \tilde{x}(k) \tag{12}$$

that stabilizes the augmented system (10) corresponds to the dynamic and strictly proper controller with realization:

$$\begin{aligned} z(k+1) &= K_u(\xi(k))\, z(k) + K_x(\xi(k))\, x(k), \\ u(k) &= z(k), \end{aligned} \tag{13}$$

where $K_u$ and $K_x$ are obtained from the augmented gain

$$\tilde{K}(\xi(k)) = \begin{bmatrix} K_x(\xi(k)) & K_u(\xi(k)) \end{bmatrix}. \tag{14}$$

Because it is strictly proper, it necessarily introduces an additional delay in the feedback loop. For this reason, we expect that a procedure that can directly handle variation in the input matrix $B$ will lead to even better closed-loop performance as compared with controllers obtained through augmentation. Indeed, this is the case with the condition we propose in Theorem 2 as illustrated by the following comparative numerical example.

## 3. Comparative numerical example

Consider the following time-varying linear discrete-time system from de Oliveira, Bernussou, and Geromel (1999) with:

$$A(\alpha) = \begin{bmatrix} 0.8 & -0.25 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0.8\alpha & -0.5\alpha & 0.2 & 0.03 + \alpha \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad B(\beta) = \begin{bmatrix} \beta \\ 0 \\ 1 - \beta \\ 0 \end{bmatrix}.$$

Our goal is to determine the largest $\gamma > 0$ such that the above discrete-time system can be stabilized for all $0 \leq \beta \leq 1$ and $|\alpha| \leq \gamma$. This system can be put in the form (1) with 4 vertices. The maximum possible values of $\gamma$ using different conditions in the literature are summarized in Table 1. We have compared:

**Table 1**
Maximum $\gamma$ for different control approaches.

| Quadratic | | Poly-quadratic | | | |
|---|---|---|---|---|---|
| SR Gahinet and Apkarian (1994) | DGS Gahinet and Apkarian (1994) | SR Mao (2003) | DGS Daafouz and Bernussou (2001) | SGS Montagner et al. (2005) | SGS Theorem 2 |
| 0.53 | 0.53 | 0.56 | 0.59 | 0.59 | **0.64** |

- Static Robust (SR) state-feedback controllers of the form (2) with $K(\xi(k)) = K$ using a quadratic condition (Gahinet & Apkarian, 1994) and a poly-quadratic condition (Mao, 2003);
- Dynamic Gain-Scheduled (DGS) state-feedback controllers of the form (12) using the quadratic condition from Gahinet and Apkarian (1994) and the poly-quadratic conditions from Daafouz and Bernussou (2001);
- Static Gain-Scheduled (SGS) state-feedback controllers of the form (2) using the poly-quadratic conditions of Montagner et al. (2005) and Theorem 2.

In the case of quadratic stabilizability, we observe that the maximum possible value of $\gamma$ is identical for both the static robust controller and the dynamic gain-scheduled controller (see Pandey et al., 2016). In the case of poly-quadratic stabilizability, Theorem 2 reaches a maximum value of $\gamma$ that is superior to all competing designs.

## 4. Proof of Theorem 2

The main goal of this section is to prove Theorem 2 and discuss some of its implications. We will make use of the following technical result.

**Lemma 3.** *If $X + X^T \succ Y \succ 0$ then $X$ is nonsingular,*

$$X^T Y^{-1} X \succeq X + X^T - Y$$

*and*

$$Y^{-1} \succeq X^{-1} + X^{-T} - X^{-T} Y X^{-1}.$$

*Furthermore, equality always holds for $X = Y$.*

Let us start by proving that feasibility of the inequalities (8) imply poly-quadratic stabilizability by the gain-scheduled controller (2) with gains (4). With that in mind, assume that (8) holds. Because $X_i + X_i^T \succ Q_i \succ 0$ then $X_i$ is nonsingular. Calculate $K_i = L_i X_i^{-1}$ and substitute to obtain

$$\begin{bmatrix} X_i + X_i^T - Q_i & X_i^T A_i^T & -X_i^T K_i^T \\ A_i X_i & Q_j - R_{i,j} & B_i Z_j - Y_j^T \\ -K_i X_i & Z_j^T B_i^T - Y_j & Z_j + Z_j^T \end{bmatrix} \succ 0,$$

where $R_{i,j}$ is as in (9) for all $i, j = 1, \dots, N$. Now use Lemma 3 with $X = X_i$ and $Y = Q_i$ to show that

$$X_i^T Q_i^{-1} X_i \succeq X_i + X_i^T - Q_i,$$

which implies

$$\begin{bmatrix} X_i^T Q_i^{-1} X_i & X_i^T A_i^T & -X_i^T K_i^T \\ A_i X_i & Q_j - R_{i,j} & B_i Z_j - Y_j^T \\ -K_i X_i & Z_j^T B_i^T - Y_j & Z_j + Z_j^T \end{bmatrix} \succ 0. \tag{15}$$

Multiplying inequalities (15) by

$$S_i = \begin{bmatrix} X_i^{-T} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix},$$

on the left and its transpose on the right gives,

$$\begin{bmatrix} Q_i^{-1} & A_i^T & -K_i^T \\ A_i & Q_j - R_{i,j} & B_i Z_j - Y_j^T \\ -K_i & Z_j^T B_i^T - Y_j & Z_j + Z_j^T \end{bmatrix} \succ 0, \tag{16}$$

for all $i, j = 1, \dots, N$. Since $Q_i$ and $Z_i$ are nonsingular, the definitions

$$P_i = Q_i^{-1}, \qquad H_i = Z_i^{-T}, \qquad F_i = P_i Y_i^T H_i,$$

for all $i = 1, \dots, N$, allows one to rewrite (16) in the form

$$\begin{bmatrix} P_i & \star & \star \\ A_i & P_j^{-1} + M_{i,j} & \star \\ -K_i & H_j^{-1} B_i^T - H_j^{-T} F_j^T P_j^{-1} & H_j^{-T} + H_j^{-1} \end{bmatrix} \succ 0, \tag{17}$$

where

$$M_{i,j} = -B_i H_j^{-T} F_j^T P_j^{-1} - P_j^{-1} F_j H_j^{-1} B_i^T,$$

for all $i, j = 1, \dots, N$ and the $\star$ stands for symmetric blocks omitted for brevity. Another congruence transformation multiplying (17) by

$$S_j = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & H_j \\ 0 & P_j & F_j \end{bmatrix},$$

on the left and by its transpose on the right produces

$$\begin{bmatrix} P_i & -K_i^T H_j^T & A_i^T P_j^T - K_i^T F_j^T \\ -H_j K_i & H_j + H_j^T & B_i^T P_j^T + F_j^T \\ P_j A_i - F_j K_i & P_j B_i + F_j & P_j \end{bmatrix} \succ 0, \tag{18}$$

for all $i, j = 1, \dots, N$. Taking convex combinations of (18) over $i$ and $j$ gives (19) which is given in Box I. The functions $P(\xi(k))$, $H(\xi(k))$, $F(\xi(k))$ and $K(\xi(k))$ are given by

$$\begin{bmatrix} P(\xi(k)) & F(\xi(k)) \\ K(\xi(k)) & H(\xi(k)) \end{bmatrix} = \sum_{i=1}^{N} \xi_i(k) \begin{bmatrix} P_i & F_i \\ K_i & H_i \end{bmatrix},$$

where $K(\xi(k))$ is as in (4). Finally, multiplying (19) by

$$S(\xi(k)) = \begin{bmatrix} I & 0 \\ K(\xi(k)) & 0 \\ 0 & I \end{bmatrix},$$

on the right and by its transpose on the left yields

$$\begin{bmatrix} P(\xi(k)) & \star \\ P(\xi(k+1))\mathcal{A}_{cl}(\xi(k)) & P(\xi(k+1)) \end{bmatrix} \succ 0,$$

where $\mathcal{A}_{cl}(\xi(k)) = A(\xi(k)) + B(\xi(k))K(\xi(k))$, for all $\xi(k), \xi(k+1) \in \Xi$. As discussed in the introduction, this is poly-quadratic stabilizability.

The above argument proves the sufficiency part of Theorem 2. Under the additional assumption that $B_i = B$ for all $i = 1, \dots, N$, it is possible to prove the converse implication, that is, that poly-quadratic stabilizability imply feasibility of the inequalities (8).

$$\begin{bmatrix} P(\xi(k)) & \star & \star \\ -H(\xi(k+1))K(\xi(k)) & H(\xi(k+1)) + H(\xi(k+1))^T & \star \\ P(\xi(k+1))A(\xi(k)) - F(\xi(k+1))K(\xi(k)) & P(\xi(k+1))B(\xi(k)) + F(\xi(k+1)) & P(\xi(k+1)) \end{bmatrix} \succ 0,$$

for all $\xi(k), \xi(k+1) \in \Xi$. (19)

**Box I.**

**Lemma 4.** *Consider the time-varying discrete-time linear system of the form (1). Assume that $B_i = B$ for all $i = 1, \dots, m$. The following statements are equivalent:*

(a) *System (1) is* poly − quadratically stabilizable*;*
(b) *There exist $X_i$, $L_i$ and $Q_i \succ 0$, $i = 1, \dots, N$, such that*

$$\begin{bmatrix} X_i + X_i^T - Q_i & X_i^T A_i^T + L_i^T B^T \\ A_i X_i + BL_i & Q_j \end{bmatrix} \succ 0, \tag{20}$$

*for all $i, j = 1, \dots, N$.*
(c) *There exist $Y$, $Z$ and $X_i$, $L_i$, $Q_i \succ 0$, $i = 1, \dots, N$, such that*

$$\begin{bmatrix} X_i + X_i^T - Q_i & X_i^T A_i^T & -L_i^T \\ A_i X_i & Q_j - R & BZ - Y^T \\ -L_i & Z^T B^T - Y & Z + Z^T \end{bmatrix} \succ 0, \tag{21}$$

*where*

$$R = BY + Y^T B^T, \tag{22}$$

*for all $i, j = 1, \dots, N$.*

*Furthermore, in case inequalities (20) or (21) are feasible, the gain-scheduled controller of the form (2) with gain (4) poly-quadratically stabilizes the system (1).*

**Proof.** That (a) is equivalent to (b) is Lemma 1 which is proved in Daafouz and Bernussou (2001). That (c) implies (a) has been proved before. So it remains to prove that (b) implies (c).

Assume that (20) holds for some $X_i$, $L_i$ and $Q_i$. Now let $\rho$ be sufficiently large so that

$$\begin{bmatrix} X_i + X_i^T - Q_i & X_i^T A_i^T + L_i^T B^T & -L_i^T \\ A_i X_i + BL_i & Q_j & 0 \\ -L_i & 0 & \rho I \end{bmatrix} \succ 0, \tag{23}$$

for all $i, j = 1, \dots, N$. Now define

$$Z = \frac{\rho}{2} I, \qquad Y = -ZB^T, \tag{24}$$

such that (23) becomes

$$\begin{bmatrix} X_i + X_i^T - Q_i & X_i^T A_i^T + L_i^T B^T & -L_i^T \\ A_i X_i + BL_i & Q_j & -BZ^T - Y^T \\ -L_i & -ZB^T - Y & Z + Z^T \end{bmatrix} \succ 0,$$

which multiplied by

$$S = \begin{bmatrix} I & 0 & 0 \\ 0 & I & B \\ 0 & 0 & I \end{bmatrix},$$

on the left and its transpose on the right gives (21) for all $i, j = 1, \dots, N$. □

That Lemma 4 implies the converse implication in Theorem 2 holds when $B_i = B$ follows from the fact that in this case $Z_j = Z$, $Y_j = Y$, and $R_{ij} = R$ for all $i, j = 1, \dots, N$, in (9) and (22).

That the choice $X_i = X_i^T = Q_i$ can be made without loss of generality comes from the fact that (15) implies (8) using Lemma 3 with $X = X_i = Y = Q_i$.

## 5. Discussion

We have introduced a new LMI condition for the stabilizability of time-varying discrete-time linear systems. Contrary to some similar conditions existing in the literature, our condition allows for time-variation in the input matrix as well as in the dynamic matrix. We have shown that it includes the poly-quadratic stabilizability condition from Daafouz and Bernussou (2001) as a particular case.

## References


Borges, R. A., Oliveira, R. C., Abdallah, C. T., & Peres, P. L. (2008). $H_\infty$ gain scheduling for discrete-time systems with control delays and time-varying parameters: a BMI approach. In *American control conference, 2008* (pp. 3088–3093). IEEE.

Borges, R. A., Oliveira, R. C., Abdallah, C. T., & Peres, P. L. (2010). A BMI approach for $H_\infty$ gain scheduling of discrete time-varying systems. *International Journal of Robust and Nonlinear Control*, *20*(11), 1255–1268.

Calloni, A., Corti, A., Zanchettin, A. M., & Lovera, M. (2012). Robust attitude control of spacecraft with magnetic actuators. In *2012 American control conference*, (ACC). (pp. 750–755). IEEE.

Corti, A., Dardanelli, A., & Lovera, M. (2012). LPV methods for spacecraft control: An overview and two case studies. In *2012 American control conference*, (ACC). (pp. 1555–1560). IEEE.

Daafouz, J., & Bernussou, J. (2001). Parameter dependent Lyapunov functions for discrete time systems with time varying parametric uncertainties. *Systems & Control Letters*, *43*(5), 355–359.

de Oliveira, M. C., Bernussou, J., & Geromel, J. C. (1999). A new discrete-time robust stability condition. *Systems & Control Letters*, *37*(4), 261–265.

Do, A. L., da Silva, J. G., Sename, O., & Dugard, L. (2011). Control design for LPV systems with input saturation and state constraints: an application to a semi-active suspension. In *2011 50th IEEE conference on decision and control and European control conference* (pp. 3416–3421). IEEE.

Do, A.-L., Sename, O., & Dugard, L. (2010). An LPV control approach for semi-active suspension control with actuator constraints. In *Proceedings of the 2010 American control conference* (pp. 4653–4658). IEEE.

Gahinet, P., & Apkarian, P. (1994). A linear matrix inequality approach to $H_\infty$ control. *International Journal of Robust and Nonlinear Control*, *4*(4), 421–448.

Gahinet, P., Apkarian, P., & Chilali, M. (1996). Affine parameter-dependent Lyapunov functions and real parametric uncertainty. *IEEE Transactions on Automatic Control*, *41*(3), 436–442.

Mao, W.-J. (2003). Robust stabilization of uncertain time-varying discrete systems and comments on an improved approach for constrained robust model predictive control. *Automatica*, *39*(6), 1109–1112.

Mohammadpour, J., & Scherer, C. W. (Eds.) (2012). *Control of linear parameter varying systems with applications*. Springer.

Montagner, V. F., Oliveira, R. C., Leite, V. J., & Peres, P. L. (2005). Gain scheduled state feedback control of discrete-time systems with time-varying uncertainties: an LMI approach. In *44th IEEE conference on decision and control, 2005 and 2005 European control conference*, CDC-ECC'05. (pp. 4305–4310). IEEE.

Pandey, A., Sehr, M., & de Oliveira, M. (2016). Pre-filtering in gain-scheduled and robust control. In *American control conference* (pp. 3698-3703). Boston, MA, USA.

Sehr, M., Pandey, A., & de Oliveira, M. (2015). Pre-filtering in continuous-time quadratic gain-scheduled and robust control (submitted for publication).