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# **FAM-L with Python: Solutions and Code**

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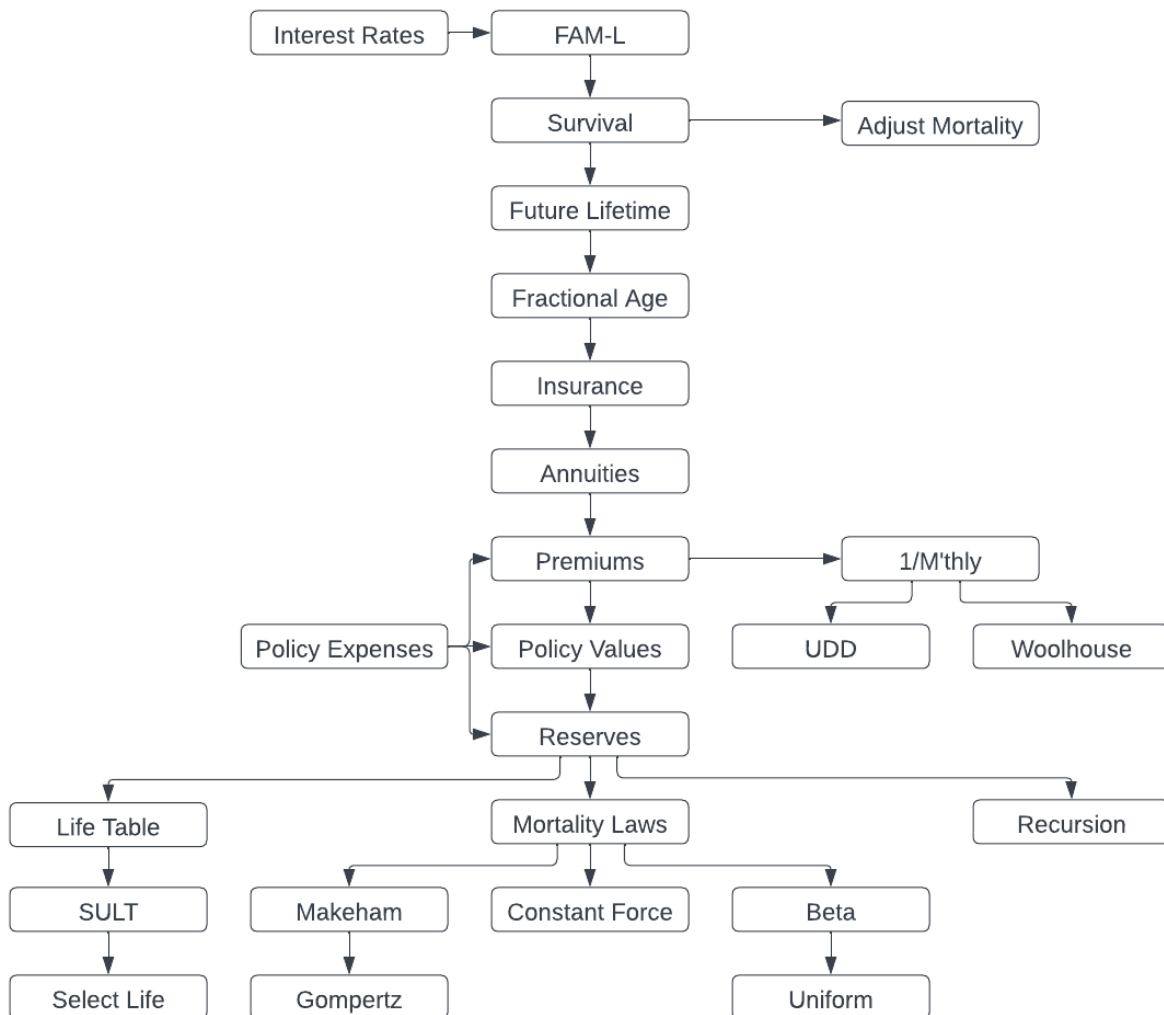
This package implements the general formulas, recursive relationships and shortcut equations for Fundamentals of Long Term Actuarial Mathematics, to solve the SOA sample exam questions and more.

- The concepts are developed hierarchically in object-oriented Python.
- Each module introduces the formulas (incrementally) used, with usage examples.
- The SOA sample questions released in August 2022 are solved in the last section of this Jupyter-Book, and in an [executable Google Colab notebook](#).

Enjoy!

Terence Lim

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## Sources:

- Documentation and formulas: [famL.pdf](#)
- Executable colab notebook: [mathlc.ipynb](#)

- Github repo: <https://github.com/terence-lim/mathlc.git>
- SOA FAM-L Sample Solutions: [copy](#) retrieved Aug 2022
- SOA FAM-L Sample Questions: [copy](#) retrieved Aug 2022

### Contact me

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## 1.1 Interest rates

$$d = \frac{i}{1+i}$$

$$v = \frac{1}{1+i}$$

$$\delta = \log(1+i)$$

$$(1+i)^t = (1-d)^{-t} = \left(1 + \frac{i^{(m)}}{m}\right)^{mt} = \left(1 - \frac{d^{(m)}}{m}\right)^{-mt} = e^{\delta t} = v^{-t}$$

**Doubling the force of interest:**

- $i' \leftarrow 2i + i^2$
- $d' \leftarrow 2d - d^2$
- $v' \leftarrow v^2$
- $\delta' \leftarrow 2\delta$

**Annuity certain:**

- Due:  $\ddot{a}_{\overline{n}|} = \frac{1-v^n}{d}$
- Immediate:  $a_{\overline{n}|} = \frac{1-v^n}{i} = \ddot{a}_{\overline{n+1}|} - 1$
- Continuous:  $\bar{a}_{\overline{n}|} = \frac{1-v^n}{\delta}$

## 1.2 Probability functions

$$\text{Var}(aX, bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) + 2 a b \text{Cov}(X, Y)$$

$$\text{Cov}(X, Y) = E[XY] - E[X] \cdot E[Y]$$

**Bernoulli**  $(p) : Y \in (a, b)$  w.p.  $(p, 1 - p) \Rightarrow$

- $E[Y] = a p + b (1 - p)$
- $\text{Var}[Y] = (a - b)^2 p (1 - p)$

**Binomial**  $(N, p) : Y$  is sum of  $N$  i.i.d. 0-1 Bernoulli( $p$ )  $\Rightarrow$

- $E[Y] = N p$
- $\text{Var}[Y] = N p (1 - p)$

**Mixture**  $(p, p_1, p_2) : Y$  is Binomial  $(p', N)$ , where  $p' \in (p_1, p_2)$  w.p.  $(p, 1 - p) \Rightarrow$

- $E[Y] = p N p_1 + (1 - p) N p_2$
- $\text{Var}[Y] = E[Y^2] - E[Y]^2 = E[\text{Var}(Y | p') + E(Y | p')^2] - E[Y]^2$

**Conditional Variance** shortcut:

- $\text{Var}[Y] = \text{Var}(E[Y | p'] + E[\text{Var}(Y | p')])$

**Portfolio Percentile**  $(p) : Y$  is sum of  $N$  i.i.d. r.v. each with mean  $\mu$  and variance  $\sigma^2 \Rightarrow$

- $Y \sim \text{Normal}$  with mean  $E[Y] = N\mu$  and variance  $\text{Var}[Y] = N\sigma^2$
- $Y_p = E[y] + z_p \sqrt{\text{Var}[Y]}$

```
"""FAM-L base class
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"""
from mathlib.faml import FAML
FAML.doc()
```

```
variance(**args):
    Compute variance of weighted sum of two r.v.

covariance(**args):
    Compute covariance of two r.v.

bernoulli(**args):
    Compute mean or variance of bernoulli r.v. with range (a, b)

binomial(**args):
    Compute mean or variance of binomial r.v.

mixture(**args):
    Mean and variance of mixture of two binomial r.v.

conditional_variance(**args):
    Conditional variance formula
```

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```

portfolio_percentile(**args):
    Percentile of a cumulative probability in the sum of N iid r.v.

solve(**args):
    Wrapper to solve root of equation

add_term(**args):
    Add two terms, allowing either term to be whole life

max_term(**args):
    Adjust term if adding term and deferral to (x) exceeds maxage

Interest(**args):
    Class for interest rate conversion and math

```

### 1.3 Examples

```

import math

print("SOA Question 2.2: (D) 400")
p1 = (1. - 0.02) * (1. - 0.01) # 2_p_x if vaccine given
p2 = (1. - 0.02) * (1. - 0.02) # 2_p_x if vaccine not given
print(math.sqrt(FAML.conditional_variance(p=.2, p1=p1, p2=p2, N=100000)))
print(math.sqrt(FAML.mixture(p=.2, p1=p1, p2=p2, N=100000, variance=True)))
print()

print("SOA Question 3.10: (C) 0.86")
interest = FAML.Interest(v=0.75)
L = 35 * interest.annuity(t=4, due=False) + 75 * interest.v_t(t=5)
interest = FAML.Interest(v=0.5)
R = 15 * interest.annuity(t=4, due=False) + 25 * interest.v_t(t=5)
print(L / (L + R))
print()

print("Example: double the force of interest i=0.05")
i = 0.05
i2 = FAML.Interest.double_force(i=i)
d2 = FAML.Interest.double_force(d=i/(1+i))
print('i:', round(i2, 6), round(FAML.Interest(d=d2).i, 6))
print('d:', round(d2, 6), round(FAML.Interest(i=i2).d, 6))
print()

print()
print("Values of z for selected values of Pr(Z<=z)")
print("-----")
print(FAML.frame().to_string(float_format=lambda x: f"{x:.3f}"))
FAML.frame()

```

```

SOA Question 2.2: (D) 400
396.5914603215815
396.5914603237804

```

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```
SOA Question 3.10:  (C) 0.86
0.8578442833761983
```

```
Example: double the force of interest i=0.05
i: 0.1025 0.1025
d: 0.092971 0.092971
```

```
Values of z for selected values of Pr(Z<=z)
```

```
-----
z          0.842  1.036  1.282  1.645  1.960  2.326  2.576
Pr(Z<=z)  0.800  0.850  0.900  0.950  0.975  0.990  0.995
```

```
z          0.842  1.036  1.282  1.645  1.960  2.326  2.576
Pr(Z<=z)   0.8    0.85   0.9    0.95   0.975   0.99   0.995
```

## SURVIVAL

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### 2.1 Survival and mortality functions

$T_x$  is time-to-death, or future lifetime, of  $(x)$

**Survival:**

$$S_x(t) = {}_t p_x = \text{Prob}(T_x > t) = \frac{S_0(x+t)}{S_0(x)} = 1 - F_x(t) = e^{-\int_0^t \mu_{x+s} ds} = \int_t^\infty f_x(s) ds = \frac{l_{x+t}}{l_x}$$

**Mortality:**

$$f_x(t) = {}_t p_x \mu_{x+t}$$

$${}_u | t q_x = \int_u^{u+t} {}_s p_x \mu_{x+s} ds = {}_u p_x - {}_{u+t} p_x = \frac{l_{x+u} - l_{x+u+t}}{l_x}$$

$${}_t p_x + {}_t q_x = 1$$

### 2.2 Force of mortality

$$\mu_{x+t} = \frac{f_x(t)}{S_x(t)} = \frac{-\frac{\partial}{\partial t} {}_t p_x}{{}_t p_x} = -\frac{\partial}{\partial t} \ln {}_t p_x$$

Note limits:  $S_x(0) = 1$ ,  $S_x(\infty) = 0$ ,  $\int_0^\infty \mu_{x+s} ds = \infty$

```
"""Survival and Mortality probability functions
```

```
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```

```
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```

```
"""
```

```
from math import exp, log
from survival import Survival
Survival.doc()
```

```
l_x(**args):
    Number of lives age ([x]+s): l_[x]+s

p_x(**args):
```

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```

    Probability that (x) lives t years: t_p_x

q_x(**args):
    Probability that (x) lives for u, but not for t+u: u|t_q_[x]+s

f_x(**args):
    probability density function of mortality

mu_x(**args):
    Force of mortality of (x+t): mu_x+t

survival_curve(**args):
    Construct curve of survival probabilities at integer ages

```

## 2.3 Examples

```

import math

print("SOA Question 2.3: (A) 0.0483")
B, c = 0.00027, 1.1
life = Survival(S=lambda x,s,t: (math.exp(-B * c**(x+s)
                                     * (c**t - 1)/math.log(c))))

print(life.f_x(x=50, t=10))
print()

print("# SOA Question 2.6: (C) 13.3")
life = Survival(l=lambda x,s: (1 - (x+s) / 60)**(1 / 3))
print(1000*life.mu_x(35))
print()

print("SOA Question 2.7: (B) 0.1477")
life = Survival(l=lambda x,s: (1 - ((x+s) / 250) if (x+s)<40
                                else 1 - ((x+s) / 100)**2))

print(life.q_x(30, t=20))
print()

print("CAS41-F99:12: k = 41")
fun = (lambda k:
        Survival(l=lambda x,s: 100*(k - .5*(x+s))**(2/3)).mu_x(50))
print(int(Survival.solve(fun, target=1/48, guess=50)))

```

```

SOA Question 2.3: (A) 0.0483
0.048327399045049846

```

```

# SOA Question 2.6: (C) 13.3
13.340451278922776

```

```

SOA Question 2.7: (B) 0.1477
0.1477272727272727

```

```

CAS41-F99:12: k = 41
41

```

## FUTURE LIFETIME

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### 3.1 Complete expectation of life

First moment:  $\overset{\circ}{e}_x = \int_0^\infty {}_t p_x \mu_{x+t} ds = \int_0^\infty {}_t p_x dt$

Second moment:  $E[T_x^2] = \int_0^\infty t^2 {}_t p_x \mu_{x+t} ds = \int_0^\infty 2t {}_t p_x dt$

- Variance:  $Var[T_x] = E[T_x^2] - (\overset{\circ}{e}_x)^2$

### 3.2 Curtate expectation of life

Curtate future lifetime:  $K_x$  is number of completed future years by (x) prior to death =  $\lfloor T_x \rfloor$

First moment:  $e_x = \sum_{k=0}^\infty k {}_k|q_x = \sum_{k=1}^\infty {}_k p_x dt$

Second moment:  $E[K_x^2] = \sum_{k=0}^\infty k^2 {}_k|q_x = \sum_{k=1}^\infty (2k-1) {}_k p_x dt$

- Variance:  $Var[K_x] = E[K_x^2] - (e_x)^2$

### 3.3 Temporary expectation of life

$\overset{\circ}{e}_{x:\overline{n}|} = \int_0^n {}_t p_x \mu_{x+t} ds + n {}_n p_x = \int_0^n {}_t p_x dt$

$e_{x:\overline{n}|} = \sum_{k=0}^{n-1} k {}_k|q_x + n {}_n p_x = \sum_{k=1}^n {}_k p_x$

```
"""Expected future lifetimes
```

```
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```

```
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```

```
"""
```

```
from mathlc.lifetime import Lifetime  
Lifetime.doc()
```

```
e_x(**args):  
    Compute moments of expected future lifetime
```

## 3.4 Examples

```
import math

print("SOA Question 2.1: (B) 2.5")
def fun(omega): # Solve first for omega, given mu_65 = 1/180
    life = Lifetime(l=lambda x,s: (1 - (x+s)/omega)**0.25)
    return life.mu_x(65)
omega = int(Lifetime.solve(fun, target=1/180, guess=100)) # solve for omega
life = Lifetime(l=lambda x,s: (1 - (x+s)/omega)**0.25, maxage=omega)
print(life.e_x(106))
print()

print("SOA Question 2.4: (E) 8.2")
life = Lifetime(l=lambda x,s: 0. if (x+s) >= 100 else 1 - ((x+s)**2)/10000.)
print(life.e_x(75, t=10, curtate=False))
print()

print("SOA Question 2.8: (C) 0.938")
def fun(mu): # Solve first for mu, given start and end proportions
    male = Lifetime(mu=lambda x,s: 1.5 * mu)
    female = Lifetime(mu=lambda x,s: mu)
    return (75 * female.p_x(0, t=20)) / (25 * male.p_x(0, t=20))
mu = Lifetime.solve(fun, target=85/15, guess=-math.log(0.94))
life = Lifetime(mu=lambda x,s: mu)
print(life.p_x(0, t=1))
print()
```

```
SOA Question 2.1: (B) 2.5
2.4786080555423604
```

```
SOA Question 2.4: (E) 8.2
8.20952380952381
```

```
SOA Question 2.8: (C) 0.938
0.9383813306903798
```



## FRACTIONAL AGES

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### 4.1 Uniform distribution of deaths

$l_{x+r} = (1-r) l_x + r l_{x+1}$  (i.e. linear interpolation)

$${}_r q_x = r q_x$$

$${}_r q_{x+s} = \frac{r q_x}{1-s q_x}$$

$$\mu_{x+r} = \frac{1}{1-r q_x}$$

$f_x(r) = q_x$  (i.e. constant within age)

$$\dot{e}_x = q_x \frac{1}{2} + p_x (1 + \dot{e}_{x+1})$$

$$\dot{e}_{x:\overline{1}|} = 1 - q_x \frac{1}{2} = q_x \frac{1}{2} + p_x$$

$$\dot{e}_x \approx e_x + 0.5$$

### 4.2 Constant force of mortality

$l_{x+r} = (l_x)^{1-r} \cdot (l_{x+1})^r$  (i.e. exponential interpolation)

$${}_r p_x = (p_x)^r$$

$\mu_{x+r} = -\ln p_x$  (i.e. constant within age)

$$f_x(r) = e^{-\mu t} \cdot \mu$$

```
"""Fractional ages

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"""
from mathlc.fractional import Fractional
Fractional.doc()
```

```
E_r(**args):  
    Pure endowment through fractional age:  $t_E[x]+s+r$   
  
l_r(**args):  
    Lives at fractional age:  $l[x]+s+r$   
  
p_r(**args):  
    Survival from and through fractional age:  $t_p[x]+s+r$   
  
q_r(**args):  
    Deferred mortality within fractional ages:  $u|t_q[x]+s+r$   
  
mu_r(**args):  
    Force of mortality at fractional age:  $\mu[x]+s+r$   
  
f_r(**args):  
    probability distribution function at fractional age:  $f[x]+s+r$  (t)  
  
e_r(**args):  
    Expectation of future lifetime through fractional age:  $e[x]+s:t$   
  
e_curtate(**args):  
    Convert curtate and complete lifetime assuming UDD within age
```

## 4.3 Examples

## INSURANCE

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## 5.1 Pure endowment

$${}_nE_x = A_{x:n|}^1 = v^n {}_np_x$$

## 5.2 Life insurance

**Whole life insurance:**

$$\bar{A}_x = \int_{t=0}^{\infty} v^t {}_tp_x \mu_{x+t} dt$$

$$A_x = \sum_{k=0}^{\infty} v^{k+1} {}_k|q_x$$

**Term insurance:**

$$\bar{A}_{x:\overline{t}|}^1 = \int_{s=0}^t v^s {}_sp_x \mu_{x+s} ds = \bar{A}_x - {}_tE_x \bar{A}_{x+t}$$

$$A_{x:\overline{t}|}^1 = \sum_{k=0}^{t-1} v^{k+1} {}_k|q_x = A_x - {}_tE_x A_{x+t}$$

**Endowment insurance:**

$$\bar{A}_{x:\overline{t}|} = \bar{A}_{x:\overline{t}|}^1 + {}_tE_x$$

$$A_{x:\overline{t}|} = A_{x:\overline{t}|}^1 + {}_tE_x$$

**Deferred insurance:**

$${}_u|\bar{A}_{x:\overline{t}|} = {}_uE_x \bar{A}_{x+u:\overline{t}|}$$

$${}_u|A_{x:\overline{t}|} = {}_uE_x A_{x+u:\overline{t}|}$$

## 5.3 Annuity Twin

**Whole life and Endowment Insurance ONLY:**

$$\bar{A}_x = 1 - \delta \bar{a}_x$$

$$A_x = 1 - d \ddot{a}_x$$

$$\bar{A}_{x:\overline{t}|} = 1 - \delta \bar{a}_{x:\overline{t}|}$$

$$A_{x:\overline{t}|} = 1 - d \ddot{a}_{x:\overline{t}|}$$

**Double the force of interest:**

$${}^2\bar{A}_x = 1 - (2\delta) {}^2\bar{a}_x$$

$${}^2A_x = 1 - (2d - d^2) {}^2\ddot{a}_x$$

$${}^2\bar{A}_{x:\overline{t}|} = 1 - (2\delta) {}^2\bar{a}_{x:\overline{t}|}$$

$${}^2A_{x:\overline{t}|} = 1 - (2d - d^2) {}^2\ddot{a}_{x:\overline{t}|}$$

## 5.4 Variances

Notationally:  ${}^2\bar{A}_x$  and  ${}^2A_x = \bar{A}_x$  and  $A_x$ , respectively, at double the force of interest.

$${}_t^2E_x = v^{2t} {}_tp_x = v_t {}_tE_x$$

**Pure Endowment:**

$$Var({}_tE_x) = v^{2t} {}_tp_x {}_tq_x = v^{2t} {}_tp_x - (v^t {}_tp_x)^2$$

**Whole life insurance:**

$$Var(\bar{A}_x) = {}^2\bar{A}_x - (\bar{A}_x)^2$$

$$Var(A_x) = {}^2A_x - (A_x)^2$$

**Term insurance:**

$$Var(\bar{A}_{x:\overline{t}|}^1) = {}^2\bar{A}_{x:\overline{t}|}^1 - (\bar{A}_{x:\overline{t}|}^1)^2$$

$$Var(A_{x:\overline{t}|}^1) = {}^2A_{x:\overline{t}|}^1 - (A_{x:\overline{t}|}^1)^2$$

**Deferred insurance:**

$$Var({}_u\bar{A}_{x:\overline{t}|}) = {}^2{}_u\bar{A}_{x:\overline{t}|} - ({}_u\bar{A}_{x:\overline{t}|})^2$$

$$Var({}_uA_{x:\overline{t}|}) = {}^2{}_uA_{x:\overline{t}|} - ({}_uA_{x:\overline{t}|})^2$$

**Endowment insurance:**

$$Var(\bar{A}_{x:\overline{t}|}) = {}^2\bar{A}_{x:\overline{t}|} - (\bar{A}_{x:\overline{t}|})^2$$

$$Var(A_{x:\overline{t}|}) = {}^2A_{x:\overline{t}|} - (A_{x:\overline{t}|})^2$$

## 5.5 Varying insurance

**Increasing insurance:**

$$(\overline{IA})_x = \int_{t=0}^{\infty} t v^t {}_t p_x \mu_{x+t} dt$$

$$(IA)_x = \sum_{k=0}^{\infty} (k+1) v^{k+1} {}_k|q_x$$

$$(\overline{IA})_{x:\overline{t}|} = \int_{s=0}^t s v^s {}_s p_x \mu_{x+s} ds$$

$$(IA)_{x:\overline{t}|} = \sum_{k=0}^{t-1} (k+1) v^{k+1} {}_k|q_x$$

**Decreasing insurance:**

$$(\overline{DA})_{x:\overline{t}|} = \int_{s=0}^t (t-s) v^s {}_s p_x \mu_{x+s} ds$$

$$(DA)_{x:\overline{t}|} = \sum_{k=0}^{t-1} (t-k) v^{k+1} {}_k|q_x$$

$$(\overline{DA})_{x:\overline{t}|} + (\overline{IA})_{x:\overline{t}|} = t \overline{A}_{x:\overline{t}|}^1$$

$$(DA)_{x:\overline{t}|} + (IA)_{x:\overline{t}|} = (t+1) A_{x:\overline{t}|}^1$$

## 5.6 Present value random variable $Z$

Expected present value of insurance benefits = EPV( $Z$ )

**Whole life insurance:**

$$Z = v^{K_x+1} \text{ (discrete) or } v^{T_x} \text{ (continuous)}$$

**Term insurance:**

$$Z = 0 \text{ when } K_x \geq t \text{ or } T_x > t, \text{ else whole life}$$

**Deferred whole life insurance:**

$$Z = 0 \text{ when } K_x < t \text{ or } T_x \leq t, \text{ else whole life}$$

**Endowment insurance:**

$$Z = v^t \text{ when } K_x \geq t \text{ or } T_x > t, \text{ else whole life}$$

```
"""Life insurance functions
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MIT License
"""
from mathlib.insurance import Insurance
Insurance.doc()
```

```
E_x(**args):
    Pure endowment: t_E_x

A_x(**args):
    Numerically compute APV of insurance from survival functions
```

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```

insurance_variance(**args):
    Compute variance of insurance given its two moments and benefit

insurance_twin(**args):
    Returns WL or Endowment Insurance twin from annuity

whole_life_insurance(**args):
    Whole life insurance:  $A_x$ 

term_insurance(**args):
    Term life insurance:  $A_x:t^1$ 

deferred_insurance(**args):
    Deferred insurance  $n|_A_x:t^1$  = discounted term or whole life

endowment_insurance(**args):
    Endowment insurance:  $A_x^1:t$  = term insurance + pure endowment

increasing_insurance(**args):
    Increasing life insurance:  $(IA)_x$ 

decreasing_insurance(**args):
    Decreasing life insurance:  $(DA)_x$ 

Z_t(**args):
     $T_x$  given percentile of the r.v.  $Z$ : PV of WL or Term insurance

Z_from_t(**args):
    PV of insurance payment  $Z(t)$ , given  $T_x$  (or  $K_x$  if discrete)

Z_to_t(**args):
     $T_x$  s.t. PV of insurance payment is  $Z$ 

Z_from_prob(**args):
    Percentile of insurance PV r.v.  $Z$ , given probability

Z_to_prob(**args):
    Cumulative density of insurance PV r.v.  $Z$ , given percentile value

Z_x(**args):
    APV of year  $t$  insurance death benefit

Z_plot(**args):
    Plot PV of insurance r.v.  $Z$  vs  $T$ 

```

## 5.7 Examples

```

import matplotlib.pyplot as plt
import numpy as np
import math

print("SOA Question 6.33: (B) 0.13")
life = Insurance(mu=lambda x,t: 0.02*t, interest=dict(i=0.03))
x = 0
print(life.p_x(x, t=15))
var = life.E_x(x, t=15, moment=life.VARIANCE, endowment=10000)
print(var)
p = 1- life.portfolio_cdf(mean=0, variance=var, value=50000, N=500)
print(p)
print()

print("SOA Question 4.18 (A) 81873 ")
life = Insurance(interest=dict(delta=0.05),
                 maxage=10,
                 f=lambda x,s,t: .1 if t < 2 else .4*t**(-2))
benefit = lambda x,t: 0 if t < 2 else 100000
prob = 0.9 - life.q_x(0, t=2)
x, y = life.survival_curve()
T = life.Z_t(0, prob=prob)
life.Z_plot(0, T=T, benefit=benefit, discrete=False, curve=(x,y))
print(life.Z_from_t(T) * benefit(0, T))
print()

print("SOA Question 4.10: (D)")
life = Insurance(interest=dict(i=0.01), S=lambda x,s,t: 1, maxage=40)
def fun(x, t):
    if 10 <= t <= 20: return life.interest.v_t(t)
    elif 20 < t <= 30: return 2 * life.interest.v_t(t)
    else: return 0
def A(x, t): # Z_x+k (t-k)
    return life.interest.v_t(t - x) * (t > x)
x = 0
benefits=[lambda x,t: (life.E_x(x, t=10) * A(x+10, t)
                    + life.E_x(x, t=20) * A(x+20, t)
                    - life.E_x(x, t=30) * A(x+30, t)),
          lambda x,t: (A(x, t)
                    + life.E_x(x, t=20) * A(x+20, t)
                    - 2 * life.E_x(x, t=30) * A(x+30, t)),
          lambda x,t: (life.E_x(x, t=10) * A(x, t)
                    + life.E_x(x, t=20) * A(x+20, t)
                    - 2 * life.E_x(x, t=30) * A(x+30, t)),
          lambda x,t: (life.E_x(x, t=10) * A(x+10, t)
                    + life.E_x(x, t=20) * A(x+20, t)
                    - 2 * life.E_x(x, t=30) * A(x+30, t)),
          lambda x,t: (life.E_x(x, t=10)
                    * (A(x+10, t)
                    + life.E_x(x+10, t=10) * A(x+20, t)
                    - life.E_x(x+20, t=10) * A(x+30, t)))]

fig, ax = plt.subplots(3, 2)
ax = ax.ravel()
for i, b in enumerate([fun] + benefits):

```

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```

    life.Z_plot(0, benefit=b, ax=ax[i], verbose=False, color=f"C{i+1}")
    ax[i].legend(["(" + "abcde"[i-1] + ")" if i else "Z"])
z = [sum(abs(b(0, t) - fun(0, t))) for t in range(40)] for b in benefits]
print("ABCDE"[np.argmin(z)])
print()

print("SOA Question 4.12: (C) 167")
cov = Insurance.covariance(a=1.65, b=10.75, ab=0) # E[Z1 Z2] = 0 nonoverlapping
print(Insurance.variance(a=2, b=1, var_a=46.75, var_b=50.78, cov_ab=cov))
print()

print("SOA Question 4.11: (A) 143385")
A1 = 528/1000 # E[Z1] term insurance
C1 = 0.209 # E[pure_endowment]
C2 = 0.136 # E[pure_endowment^2]
def fun(A2):
    B1 = A1 + C1 # endowment = term + pure_endowment
    B2 = A2 + C2 # double force of interest
    return Insurance.insurance_variance(A2=B2, A1=B1)
A2 = Insurance.solve(fun, target=15000/(1000*1000), guess=[143400, 279300])
print(Insurance.insurance_variance(A2=A2, A1=A1, b=1000))
print()

print("SOA Question 4.15 (E) 0.0833 ")
life = Insurance(mu=lambda x: 0.04,
                 interest=dict(delta=0.06))
benefit = lambda x,t: math.exp(0.02*t)
A1 = life.A_x(0, benefit=benefit, discrete=False)
A2 = life.A_x(0, moment=2, benefit=benefit, discrete=False)
print(A2 - A1**2)
print()

print("SOA Question 4.4 (A) 0.036")
life = Insurance(f=lambda x: 0.025,
                 maxage=40+40,
                 interest=dict(v_t=lambda t: (1 + .2*t)**(-2)))
benefit = lambda x,t: 1 + .2 * t
A1 = life.A_x(40, benefit=benefit, discrete=False)
A2 = life.A_x(40, moment=2, benefit=benefit, discrete=False)
print(A2 - A1**2)
print()

# Example: plot Z vs T
life = Insurance(interest=dict(delta=0.06), mu=lambda x: 0.04)
prob = 0.8
x = 0
discrete = False
t = life.Z_t(0, prob, discrete=discrete)
Z = life.Z_from_prob(x, prob=prob, discrete=discrete)
print(t, life.Z_to_t(Z))
print(Z, life.Z_from_t(t, discrete=discrete))
print(prob, life.Z_to_prob(x, Z=Z))
life.Z_plot(0, T=t, discrete=discrete)

print("Other examples of usage")
life = Insurance(interest=dict(delta=0.06), mu=lambda x: 0.04)

```

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```

benefit = lambda x,t: math.exp(0.02 * t)
A1 = life.A_x(0, benefit=benefit)
A2 = life.A_x(0, moment=2, benefit=benefit)
print(A1, A2, A2 - A1**2)  # 0.0833

life = Insurance(interest=dict(delta=0.05), mu=lambda x,s: 0.03)
benefit = lambda x,t: math.exp(0.04 * t)
print(life.A_x(0, benefit=benefit))  #0.75
print(life.A_x(0, moment=2, benefit=benefit)) #0.60

life = Insurance(interest=dict(delta=0.08),
                 maxage=25,
                 S=lambda x,s,t: 1 - (0.02*t + 0.0008*(t**2)))
print(life.A_x(0)*10000)  #3647
print()

```

```

SOA Question 6.33:  (B) 0.13
0.10539922456186429
3884632.549746798
0.12828940905648634

SOA Question 4.18  (A) 81873
81873.07530779815

SOA Question 4.10:  (D)
D

SOA Question 4.12:  (C) 167
166.82999999999998

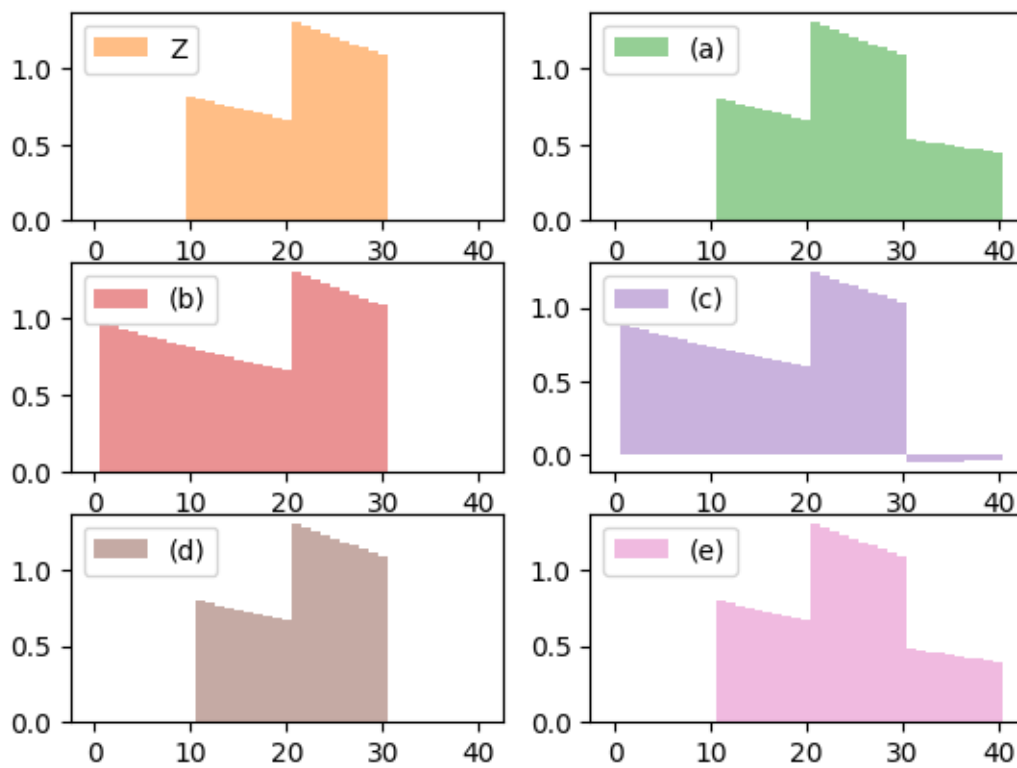
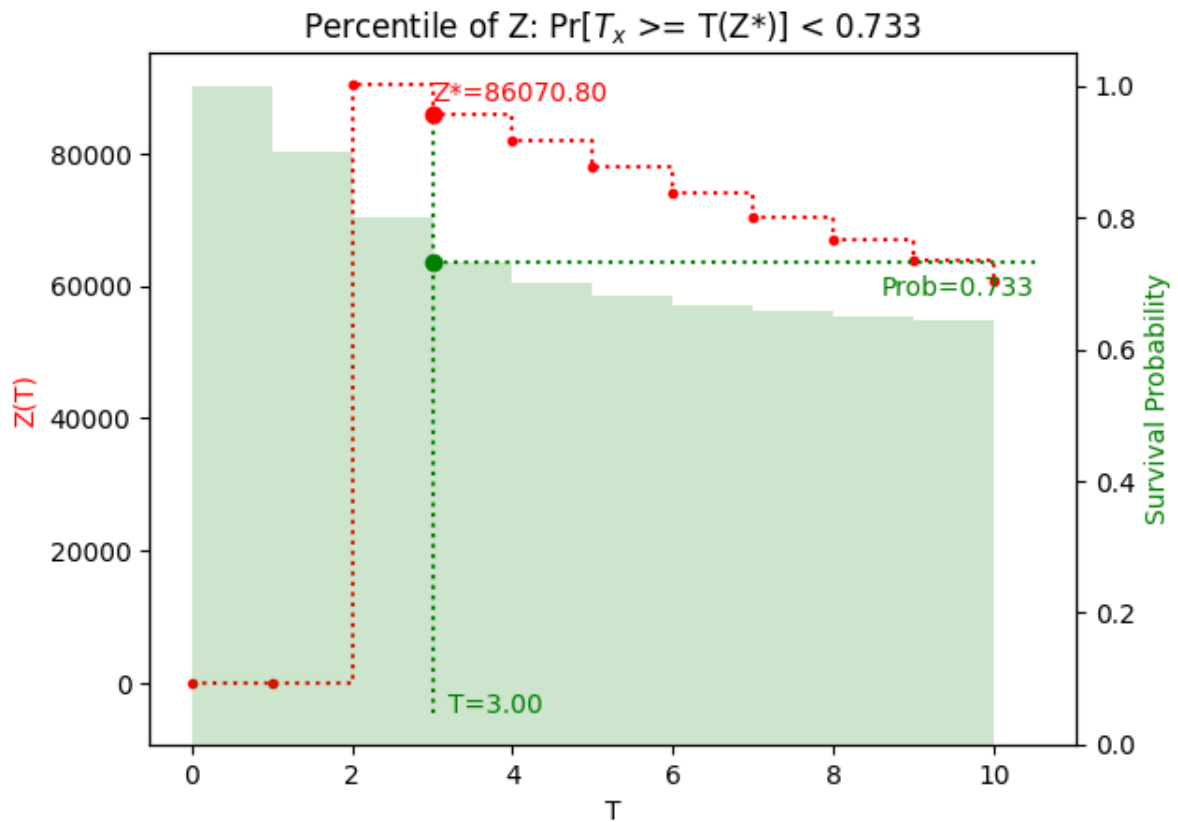
SOA Question 4.11:  (A) 143385
143384.99999999997

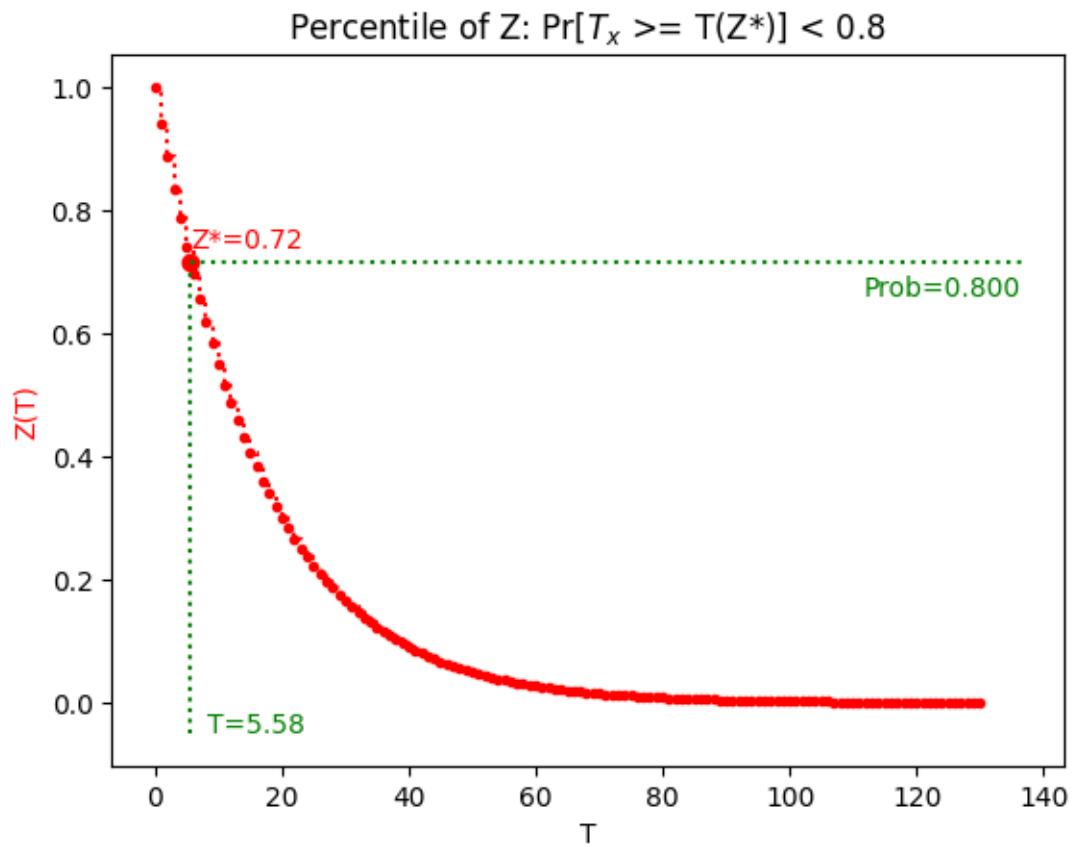
SOA Question 4.15  (E) 0.0833
0.08334849338238598

SOA Question 4.4   (A) 0.036
0.03567680106032681

5.578588782855243 5.000000000000001
0.7408182206817179 0.7155417527999328
0.8 0.8187307530779818
Other examples of usage
0.48998642116279045 0.32009235393201546 0.080005661008096
0.7421209265054034
0.5930972723997999
3503.833219537252

```







## ANNUITIES

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## 6.1 Life annuities

**Whole life annuity:**

$$\bar{a}_x = \int_{t=0}^{\infty} v^t {}_t p_x dt$$

$$\ddot{a}_x = \sum_{k=0}^{\infty} v^k {}_k|p_x$$

**Temporary annuity:**

$$\bar{a}_{x:\overline{t}|} = \int_{s=0}^t v^s {}_s p_x ds = \bar{A}_x - {}_t E_x \bar{a}_{x+t}$$

$$\ddot{a}_{x:\overline{t}|} = \sum_{k=0}^t v^k {}_k|p_x = \ddot{a}_x - {}_t E_x \ddot{a}_{x+t}$$

**Deferred whole life annuity:**

$${}_u|\bar{a}_x = \bar{a}_x - \bar{a}_{x+u}$$

$${}_u|\ddot{a}_x = \ddot{a}_x - \ddot{a}_{x+u}$$

**Certain and life annuity:**

$$\overline{\bar{a}}_{x:n|} = \bar{a}_{n|} + {}_n|\bar{a}_x$$

$$\overline{\ddot{a}}_{x:n|} = \ddot{a}_{n|} + {}_n|\ddot{a}_x$$

## 6.2 Insurance twin

**Whole life and Temporary Annuities ONLY:**

$$\bar{a}_x = \frac{1 - \bar{A}_x}{\delta}$$

$$\ddot{a}_x = \frac{1 - A_x}{d}$$

$$\bar{a}_{x:\overline{t}|} = \frac{1 - \bar{A}_{x:\overline{t}|}}{\delta} \text{ (continuous endowment insurance twin)}$$

$$\ddot{a}_{x:\overline{t}|} = \frac{1 - A_{x:\overline{t}|}}{d} \text{ (discrete endowment insurance twin)}$$

**Double the force of interest:**

$${}^2\bar{a}_x = \frac{1 - {}^2\bar{A}_x}{2\delta}$$

$${}^2\ddot{a}_x = \frac{1 - {}^2A_x}{2d - d^2}$$

$${}^2\bar{a}_{x:\overline{t}|} = \frac{1 - {}^2\bar{A}_{x:\overline{t}|}}{2\delta}$$

$${}^2\ddot{a}_{x:\overline{t}|} = \frac{1 - {}^2A_{x:\overline{t}|}}{2d - d^2}$$

## 6.3 Immediate annuities

$$a_x = \ddot{a}_x - 1$$

$$a_{x:\overline{t}|} = \ddot{a}_{x:\overline{t}|} - 1 + {}_tE_x$$

## 6.4 Variances

**Whole life annuity:**

$$Var(\bar{a}_x) = \frac{{}^2\bar{A}_x - (\bar{A}_x)^2}{d^2}$$

$$Var(\ddot{a}_x) = \frac{{}^2\ddot{A}_x - (A_x)^2}{\delta^2}$$

**Temporary annuity:**

$$Var(\bar{a}_{x:\overline{t}|}) = \frac{{}^2\bar{A}_{x:\overline{t}|} - (\bar{A}_{x:\overline{t}|})^2}{d^2}$$

$$Var(\ddot{a}_{x:\overline{t}|}) = \frac{{}^2A_{x:\overline{t}|} - (A_{x:\overline{t}|})^2}{\delta^2}$$

## 6.5 Varying annuities

**Increasing annuity:**

$$(\bar{Ia})_x = \int_{t=0}^{\infty} t v^t {}_t p_x dt$$

$$(I\ddot{a})_x = \sum_{k=0}^{\infty} (k+1) v^{k+1} {}_k p_x$$

$$(\bar{Ia})_{x:\overline{t}|} = \int_{s=0}^t s v^s {}_s p_x ds$$

$$(I\ddot{a})_{x:\overline{t}|} = \sum_{k=0}^{t-1} (k+1) v^{k+1} {}_k p_x$$

**Decreasing annuity:**

$$(\overline{Da})_{x:\overline{t}|} = \int_{s=0}^t (t-s) v^s {}_s p_x ds$$

$$(D\ddot{a})_{x:\overline{t}|} = \sum_{k=0}^{t-1} (t-k) v^{k+1} {}_k p_x$$

$$(\overline{Da})_{x:\overline{t}|} + (\bar{Ia})_{x:\overline{t}|} = t \bar{a}_{x:\overline{t}|}$$

$$(D\ddot{a})_{x:\overline{t}|} + (I\ddot{a})_{x:\overline{t}|} = (t+1) \ddot{a}_{x:\overline{t}|}$$

## 6.6 Present value random variable $Y$

Expected present value of a life annuity = EPV( $Y$ )

**Whole life annuity:**

$$Y = \ddot{a}_{\overline{K_x+1}|} \text{ (discrete) or } \bar{a}_{\overline{T_x}|} \text{ (continuous)}$$

**Temporary insurance:**

$$Y = \ddot{a}_{\overline{t}|} \text{ (discrete) or } \bar{a}_{\overline{t}|} \text{ (continuous) when } K_x \geq t \text{ or } T_x > t, \text{ else whole life}$$

**Certain and life annuity:**

$$Y = \ddot{a}_{\overline{n}|} \text{ (discrete) or } \bar{a}_{\overline{n}|} \text{ (continuous) when } K_x < n \text{ or } T_x \leq n, \text{ else whole life}$$

```
"""Life annuity functions
```

```
Copyright 2022, Terence Lim
```

```
MIT License
```

```
"""
```

```
from mathlib.annuity import Annuity
Annuity.doc()
```

```
a_x(**args):
    Numerically compute APV of annuities from survival functions

immediate_annuity(**args):
    Compute APV of immediate life annuity

annuity_twin(**args):
    Returns annuity from its WL or Endowment Insurance twin

whole_life_annuity(**args):
    Whole life annuity: a_x

temporary_annuity(**args):
    Temporary life annuity: a_x:t

deferred_annuity(**args):
    Deferred life annuity n|t_a_x = n+t_a_x - n_a_x

certain_life_annuity(**args):
    Certain and life annuity = certain + deferred

increasing_annuity(**args):
    Increasing annuity

decreasing_annuity(**args):
    Identity (Da)_x:n + (Ia)_x:n = (n+1) a_x:n temporary annuity

Y_t(**args):
    T_x given percentile of the r.v. Y = PV of WL or Temporary Annuity
```

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```

Y_from_t(**args):
    PV of insurance payment Y(t), given T_x (or K_x if discrete)

Y_from_prob(**args):
    Percentile of annuity PV r.v. Y, given probability

Y_to_prob(**args):
    Cumulative density of insurance PV r.v. Y, given percentile value

Y_x(**args):
    APV of t'th year's annuity benefit

Y_plot(**args):
    Plot PV of annuity r.v. Y vs T

```

## 6.7 Examples

```

if __name__ == "__main__":
    import matplotlib.pyplot as plt

    print("SOA Question 5.6: (D) 1200")
    life = Annuity(interest=dict(i=0.05))
    var = life.annuity_variance(A2=0.22, A1=0.45)
    mean = life.annuity_twin(A=0.45)
    print(life.portfolio_percentile(mean=mean, variance=var, prob=.95, N=100))
    print()

    print("Plot example")
    life = Annuity(interest=dict(delta=0.06), mu=lambda *x: 0.04)
    prob = 0.8
    x = 0
    discrete = True
    t = life.Y_t(0, prob, discrete=discrete)
    Y = life.Y_from_prob(x, prob=prob, discrete=discrete)
    print(t, life.Y_to_t(Y))
    print(Y, life.Y_from_t(t, discrete=discrete))
    print(prob, life.Y_to_prob(x, Y=Y))
    life.Y_plot(0, T=t, discrete=discrete)
    plt.show()

    print("Other usage")
    mu = 0.04
    delta = 0.06
    life = Annuity(interest=dict(delta=delta), mu=lambda *x: mu)
    print(life.temporary_annuity(50, t=20, b=10000, discrete=False))
    print(life.endowment_insurance(50, t=20, b=10000, discrete=False))
    print(life.E_x(50, t=20))
    print(life.whole_life_annuity(50, b=10000, discrete=False))
    print(life.whole_life_annuity(70, b=10000, discrete=False))

    mu = 0.07
    delta = 0.02
    life = Annuity(interest=dict(delta=delta), mu=lambda *x: mu)

```

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```

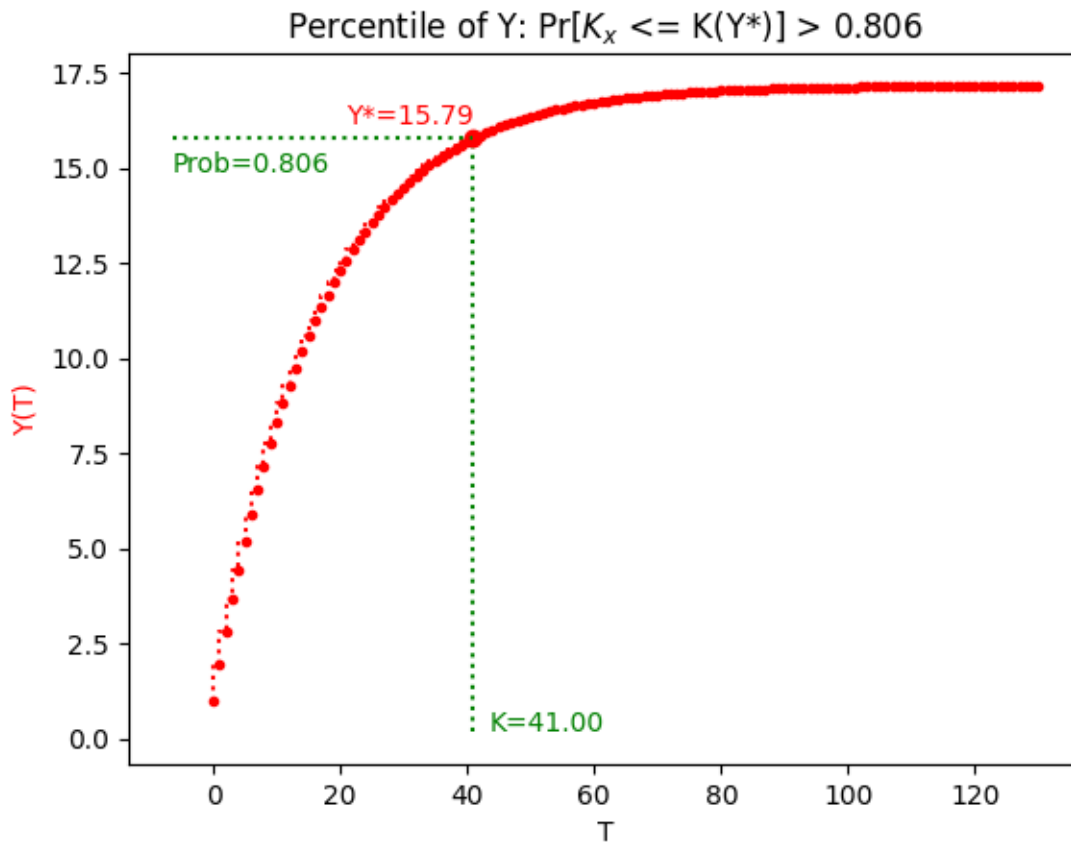
print(life.whole_life_annuity(0, discrete=False) * 30) # 333.33
print(life.temporary_annuity(0, t=10, discrete=False) * 30) # 197.81
print(life.interest_annuity(5, m=0)) # 4.7581
print(life.deferred_annuity(0, u=5, discrete=False)) # 7.0848
print(life.certain_life_annuity(0, u=5, discrete=False)) # 11.842

mu = 0.02
delta = 0.05
life = Annuity(interest=dict(delta=delta), mu=lambda x: mu)
print(life.decreasing_annuity(0, t=5, discrete=False)) # 6.94

```

SOA Question 5.6: (D) 1200  
1200.6946732201702

Plot example  
41 49.084762499581856  
15.790040843594133 15.790040843594133  
0.8 0.8596183508486661



Other usage  
86466.4716763387  
4812.011699419677  
0.13533528323661273  
100022.36417519346

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```
100165.250145111104  
333.3430094556871  
197.81011341979988  
4.758129098202016  
7.085079777653729  
11.843208875855744  
6.94209306102519
```

## PREMIUMS

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### 7.1 Equivalence Principle

Set premiums s.t. expected loss at issue equals zero:

$$E[_0L] = EPV_0(\text{future benefits}) - EPV_0(\text{future premiums}) = 0$$

- Fully continuous: both benefits and premiums are payable continuously
- Fully discrete: benefits are paid at the end of the year, premiums are paid at the beginning of the year
- Semi-continuous: benefits are paid at moment of death, premiums are paid at the beginning of the year

### 7.2 Net premium

Is the premium is determined excluding expenses under the equivalence principle

- Whole life insurance:  $P_x = \frac{A_x}{\ddot{a}_x}$  or  $\frac{\bar{A}_x}{\bar{a}_x}$
- Term life insurance:  $P_{x:\overline{t}|}^1 = \frac{A_{x:\overline{t}|}^1}{\ddot{a}_{x:\overline{t}|}}$  or  $\frac{\bar{A}_{x:\overline{t}|}^1}{\bar{a}_{x:\overline{t}|}}$
- Pure endowment:  $P_{x:\overline{t}|}^1 = \frac{{}_tE_x}{\ddot{a}_{x:\overline{t}|}}$  or  $\frac{{}_tE_x}{\bar{a}_{x:\overline{t}|}}$
- Endowment insurance:  $P_{x:\overline{t}|} = \frac{A_{x:\overline{t}|}}{\ddot{a}_{x:\overline{t}|}}$  or  $\frac{\bar{A}_{x:\overline{t}|}}{\bar{a}_{x:\overline{t}|}}$

**Shortcuts for whole life and endowment insurance only:**

$$P = b \left( \frac{1}{\ddot{a}_x} - d \right) = b \left( \frac{dA_x}{1 - A_x} \right) \text{ (fully discrete)}$$

$$P = b \left( \frac{1}{\bar{a}_x} - \delta \right) = b \left( \frac{d\bar{A}_x}{1 - \bar{A}_x} \right) \text{ (fully continuous)}$$

## 7.3 Gross premium

Accounts for expenses. If set under equivalence principle, then expected loss at issue equals zero:

$$E[L^g] = EPV_0(\text{future benefits}) + EPV_0(\text{future expenses}) - EPV_0(\text{future premiums}) = 0$$

Return of premiums paid without interest upon death:

$$\bullet \quad EPV_0 = \sum_{k=0}^{t-1} P(k+1) v^{k+1} {}_k|q_x = P \cdot (IA)_{x:\overline{t}|}^1$$

Expenses:

- per policy and/or percent of premium initial expenses in year 1:  
 $e_i = \text{initial\_per\_policy} + \text{initial\_per\_premium} \times \text{gross\_premium}$
- per policy and/or percent of premium renewal expenses in year 2+:  
 $e_r = \text{renewal\_per\_policy} + \text{renewal\_per\_premium} \times \text{gross\_premium}$
- settlement expense paid with death benefit:  $E$   
 $b + E = \text{death benefit} + \text{settlement expense} = \text{claim\_costs}$

```
"""Premiums

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"""
from mathlc.premiums import Premiums
Premiums.doc()
```

```
net_premium(**args):
    Net level premium for n-pay, u-deferred t-year term insurance

gross_premium(**args):
    Gross premium by equivalence principle

insurance_equivalence(**args):
    Whole life or endowment insurance factor, given net premium

annuity_equivalence(**args):
    Whole life or temporary annuity factor, given net premium

premium_equivalence(**args):
    Premium given whole life or temporary/endowment annuity/insurance
```

## 7.4 Examples

```
import numpy as np

print("SOA Question 6.29 (B) 20.5")
life = Premiums(interest=dict(i=0.035))
def fun(a):
    return life.gross_premium(A=life.insurance_twin(a=a),
```

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```

        a=a, benefit=100000,
        initial_policy=200, initial_premium=.5,
        renewal_policy=50, renewal_premium=.1)
print(life.solve(fun, target=1770, guess=[20, 22]))
print()

print("SOA Question 6.2: (E) 3604")
life = Premiums()
A, IA, a = 0.17094, 0.96728, 6.8865
print(life.gross_premium(a=a, A=A, IA=IA, benefit=100000,
        initial_premium=0.5, renewal_premium=.05,
        renewal_policy=200, initial_policy=200))

print()

print("SOA Question 6.16: (A) 2408.6")
life = Premiums(interest=dict(d=0.05))
A = life.insurance_equivalence(premium=2143, b=100000)
a = life.annuity_equivalence(premium=2143, b=100000)
p = life.gross_premium(A=A, a=a, benefit=100000, settlement_policy=0,
        initial_policy=250, initial_premium=.04+.35,
        renewal_policy=50, renewal_premium=.04+.02)

print(A, a, p)
print()

print("SOA Question 6.20: (B) 459")
life = Premiums(interest=dict(i=0.04),
        l=lambda x,s: dict(zip([75, 76, 77, 78],
        np.cumprod([1,.9,.88,.85])))).get(x+s, 0))
a = life.temporary_annuity(75, t=3)
IA = life.increasing_insurance(75, t=2)
A = life.deferred_insurance(75, u=2, t=1)
print(life.solve(lambda P: P*IA + A*10000 - P*a, target=0, guess=100))
print()

print("Other usage")
life = Premiums(interest=dict(delta=0.06), mu=lambda x,s: 0.04)
print(life.net_premium(0))

```

SOA Question 6.29 (B) 20.5  
20.480268314431726

SOA Question 6.2: (E) 3604  
3604.229940320728

SOA Question 6.16: (A) 2408.6  
0.3000139997200056 13.999720005599887 2408.575206281868

SOA Question 6.20: (B) 459  
458.83181728297285

Other usage  
0.03692697915432344



## POLICY VALUES

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### 8.1 Net Policy Value

$${}_tV = EPV_t(\text{future benefits}) - EPV_t(\text{future premiums})$$

- ${}_0V = E[{}_0L] = 0$  (assumes equivalence principle)
- ${}_nV = E[{}_nL] = 0$  for a n-year term insurance
- ${}_nV = E[{}_nL] = \text{endowment benefit for a n-year endowment insurance}$

**Shortcuts for whole life and endowment insurance:**

$${}_tV = E[{}_tL] = b[1 - \frac{\ddot{a}_{x+t}}{\ddot{a}_x}] \text{ or } b[\frac{A_{x+t} - A_x}{1 - A_x}] \text{ (fully discrete)}$$

$${}_tV = E[{}_tL] = b[1 - \frac{\bar{a}_{x+t}}{\bar{a}_x}] \text{ or } b[\frac{\bar{A}_{x+t} - \bar{A}_x}{1 - \bar{A}_x}] \text{ (fully continuous)}$$

### 8.2 Gross Policy Value

$${}_tV^g = E[{}_tL^g] = EPV_t(\text{future benefits}) + EPV_t(\text{future expenses}) - EPV_t(\text{future premiums})$$

### 8.3 Variance of future loss

**Whole life and endowment insurance only:**

- Net future loss\*:

$$Var[{}_tL] = (b + \frac{P}{d})^2 [{}^2A_{x+t:n-t} - (A_{x+t:n-t})^2] \text{ (discrete)}$$

$$Var[{}_tL] = (b + \frac{P}{\delta})^2 [{}^2\bar{A}_{x+t:n-t} - (\bar{A}_{x+t:n-t})^2] \text{ (continuous)}$$

- Gross future loss\*:

$$Var[{}_tL] = (b + E + \frac{G - e_r}{d})^2 [{}^2A_{x+t:n-t} - (A_{x+t:n-t})^2] \text{ (discrete)}$$

**Shortcuts for variance of net future loss** (net premiums under equivalence principle)\*:

$$Var[_tL] = b^2 \left[ \frac{{}^2A_{x+t:\overline{n-t}|} - (\overline{A}_{x+t:\overline{n-t}|})^2}{(1 - \overline{A}_{x:\overline{n}|})^2} \right] \text{ (discrete)}$$

$$Var[_tL] = b^2 \left[ \frac{{}^2\overline{A}_{x+t:\overline{n-t}|} - (\overline{\overline{A}}_{x+t:\overline{n-t}|})^2}{(1 - \overline{\overline{A}}_{x:\overline{n}|})^2} \right] \text{ (continuous)}$$

\*For whole life insurance, remove the  $\overline{n}|$  and  $\overline{n-t}|$  notations.

## 8.4 Expense reserve

$${}_tV^e = {}_tV^g - {}_tV = EPV_t(\text{future expenses}) - EPV_t(\text{future expense loadings})$$

Generally:

- ${}_tV^e < 0$
- ${}_tV > {}_tV^g > 0 > {}_tV^e$

## 8.5 Present value of loss random variable $L$

\*\*Note: we have used the terms “expected future loss”, “policy value” and “reserves” interchangeably and loosely.

For full discrete whole life or endowment insurance:

- Net future loss

$${}_0L = b v^{K_x+1} - P \ddot{a}_{\overline{K_x+1}|} = \left(b + \frac{P}{d}\right) v^{K_x+1} - \frac{P}{d} \text{ (discrete)}$$

$${}_0L = b v^{T_x} - P \overline{a}_{\overline{T_x}|} = \left(b + \frac{P}{\delta}\right) v^{T_x} - \frac{P}{\delta} \text{ (continuous)}$$

- Gross future loss

$${}_0L = \left(b + E + \frac{G - e_r}{d}\right) v^{K_x+1} - \frac{G - e_r}{d} + (e_i - e_r)$$

```
"""Policy Values
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"""
from mathlc.policyvalues import PolicyValues
PolicyValues.doc()
```

```
net_future_loss(**args):
    Assume WL or Endowment Ins for shortcuts since P from equivalence

net_variance_loss(**args):
    Helper for variance of net loss shortcuts of WL or Endowment Ins loss

net_policy_variance(**args):
    Shortcuts for variance of future loss for WL or Endow Ins
```

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```

gross_future_loss(**args):
    Shortcut for WL or Endowment Ins gross future loss

gross_policy_variance(**args):
    Shortcut for gross policy value of WL and Endowment Insurance

gross_policy_value(**args):
    Gross policy values for insurance:  $t_V = E[L_t]$ 

L_from_t(**args):
    PV of Loss  $L(t)$ , given  $T_x$  (or  $K_x$  if discrete)

L_to_t(**args):
     $T_x$  s.t. PV of loss is  $Z$ 

L_from_prob(**args):
    Percentile of loss PV r.v.  $L$ , given probability

L_to_prob(**args):
    Cumulative density of loss PV r.v.  $L$ , given percentile value

L_plot(**args):
    Plot loss r.v.  $L$  vs  $T$ 

```

## 8.6 Examples

```

import matplotlib.pyplot as plt
from mathlc.sult import SULT

print("SOA Question 6.24: (E) 0.30")
life = PolicyValues(interest=dict(delta=0.07))
x, A1 = 0, 0.30 # Policy for first insurance
P = life.premium_equivalence(A=A1, discrete=False) # Need its premium
policy = life.Policy(premium=P, discrete=False)
def fun(A2): # Solve for A2, given Var(Loss)
    return life.gross_variance_loss(A1=A1, A2=A2, policy=policy)
A2 = life.solve(fun, target=0.18, guess=0.18)
print()

policy = life.Policy(premium=0.06, discrete=False) # Solve second insurance
variance = life.gross_variance_loss(A1=A1, A2=A2, policy=policy)
print(variance)
print()

print("SOA Question 6.30: (A) 900")
life = PolicyValues(interest=dict(i=0.04))
policy = life.Policy(premium=2.338, benefit=100, initial_premium=.1,
                    renewal_premium=0.05)
var = life.gross_variance_loss(A1=life.insurance_twin(16.50),
                              A2=0.17, policy=policy)
print(var)
print()

```

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```

print("SOA Question 7.32: (B) 1.4")
life = PolicyValues(interest=dict(i=0.06))
policy = life.Policy(benefit=1, premium=0.1)
def fun(A2):
    return life.gross_variance_loss(A1=0, A2=A2, policy=policy)
A2 = life.solve(fun, target=0.455, guess=0.455)
policy = life.Policy(benefit=2, premium=0.16)
var = life.gross_variance_loss(A1=0, A2=A2, policy=policy)
print(var)
print()

print("SOA Question 6.12: (E) 88900")
life = PolicyValues(interest=dict(i=0.06))
a = 12
A = life.insurance_twin(a)
policy = life.Policy(benefit=1000, settlement_policy=20,
                    initial_policy=10, initial_premium=0.75,
                    renewal_policy=2, renewal_premium=0.1)
policy.premium = life.gross_premium(A=A, a=a, **policy.premium_terms)
print(A, policy.premium)
L = life.gross_variance_loss(A1=A, A2=0.14, policy=policy)
print(L)
print()

print("Plot Example -- SOA Question 6.6: (B) 0.79")
life = SULT()
P = life.net_premium(62, b=10000)
policy = life.Policy(premium=1.03*P, renewal_policy=5,
                    initial_policy=5, initial_premium=0.05, benefit=10000)
L = life.gross_policy_value(62, policy=policy)
var = life.gross_policy_variance(62, policy=policy)
prob = life.portfolio_cdf(mean=L, variance=var, value=40000, N=600)
print(prob, 0.79)
life.L_plot(62, policy=policy)
print()

print("Plot Example -- SOA QQuestion 7.6: (E) -25.4")
life = SULT()
P = life.net_premium(45, b=2000)
policy = life.Policy(benefit=2000, initial_premium=.25, renewal_premium=.05,
                    initial_policy=2*1.5 + 30, renewal_policy=2*.5 + 10)
G = life.gross_premium(a=life.whole_life_annuity(45), **policy.premium_terms)
gross = life.gross_policy_value(45, t=10, policy=policy.set(premium=G))
net = life.net_policy_value(45, t=10, b=2000)
V = gross - net
print(V, -25.4)
T = life.L_to_t(G, policy=policy)
print(G)
life.L_plot(45, T=int(T), policy=policy)

plt.show()

```

SOA Question 6.24: (E) 0.30

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```
0.30419999999999975
```

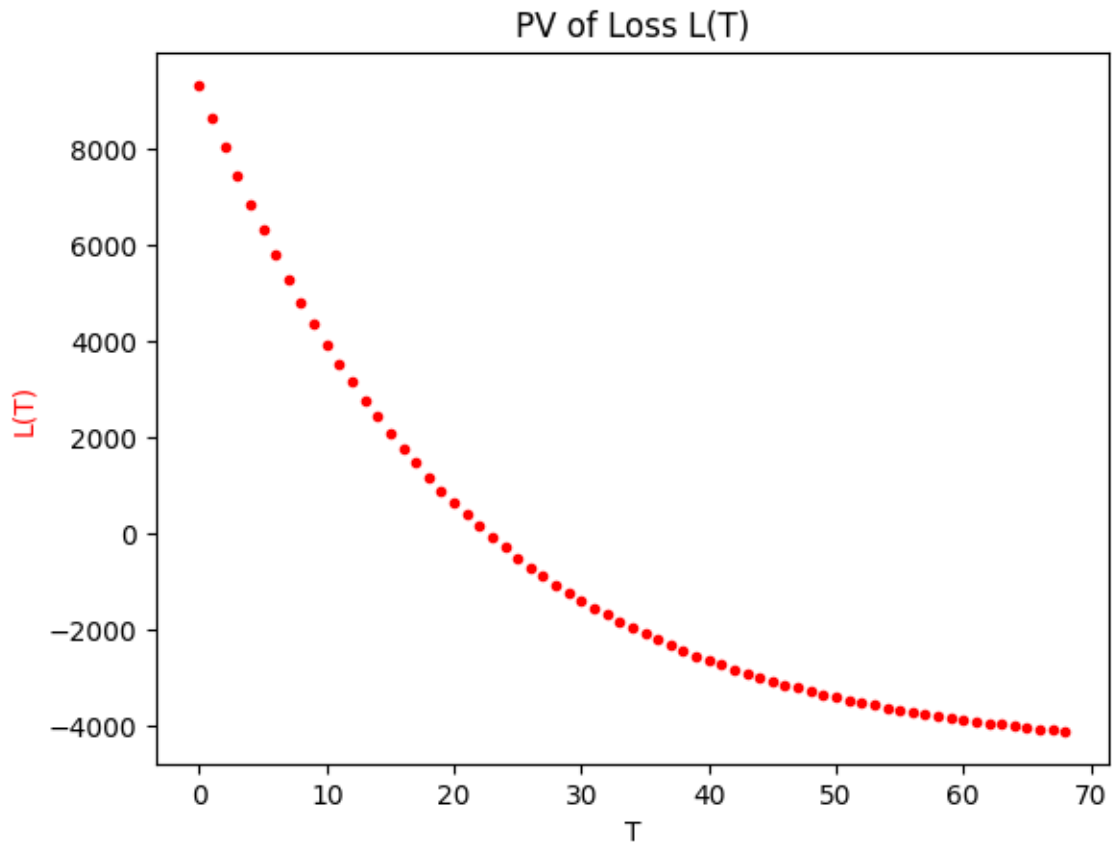
```
SOA Question 6.30: (A) 900
908.141412994607
```

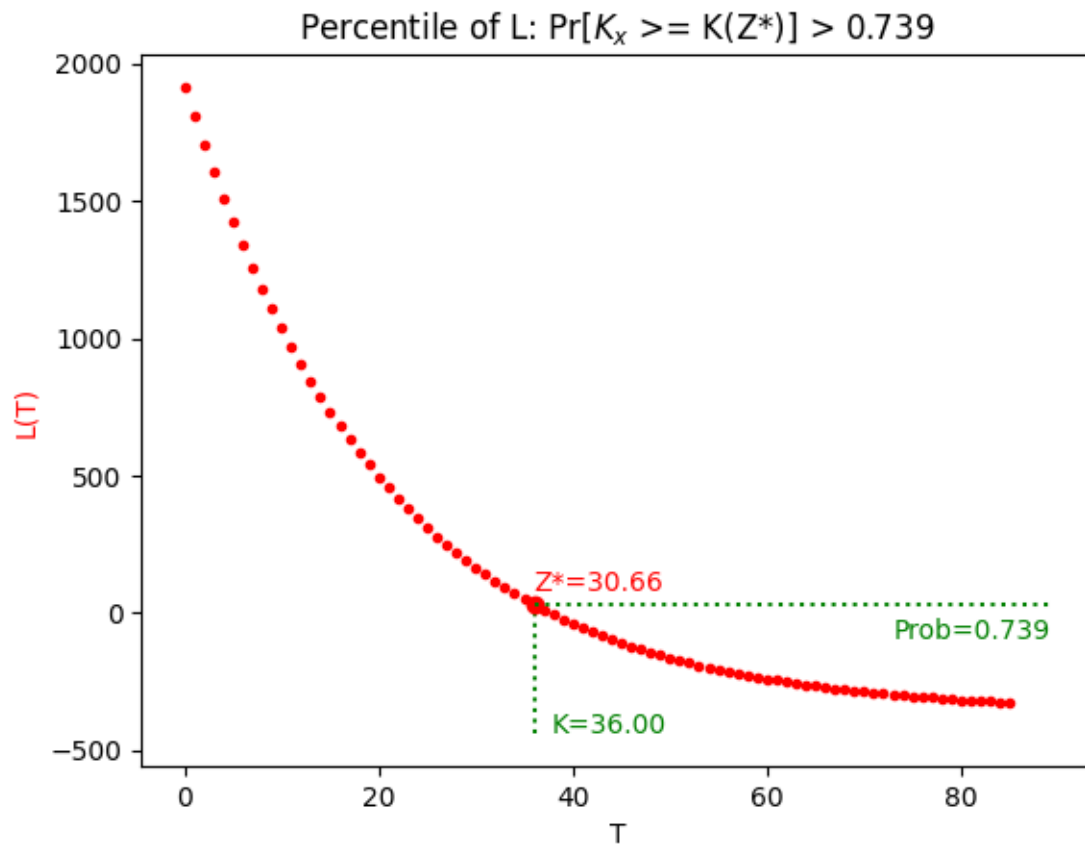
```
SOA Question 7.32: (B) 1.4
1.3848168384380901
```

```
SOA Question 6.12: (E) 88900
0.3207547169811321 35.38618830746352
88862.59592874818
```

```
Plot Example -- SOA Question 6.6: (B) 0.79
0.7914321142683509 0.79
```

```
Plot Example -- SOA QQuestion 7.6: (E) -25.4
-25.44920289521204 -25.4
31.161950196480408
```





## RESERVES

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## 9.1 Recursion

Gross reserves:  $({}_tV^g + G - e)(1 + i) = q_{x+t} (b + E) + p_{x+t \ t+1} V^g$

Net reserves:  $({}_tV + P)(1 + i) = q_{x+t} b + p_{x+t \ t+1} V$

Expense reserves:  $({}_tV^e + P^e - e)(1 + i) = q_{x+t} E + p_{x+t \ t+1} V^e$

## 9.2 Interim reserves

$({}_tV + P)(1 + i)^r = {}_r q_{x+t} b v^{1-r} + {}_r p_{x+t \ t+r} V$

${}_{t+r} V (1 + i)^{1-r} = {}_{1-r} q_{x+t+r} b + {}_{1-r} p_{x+t+r \ t+1} V$

## 9.3 Modified reserves

### Full Preliminary Term

- Initial premium:  $\alpha = A_{x:\overline{1}|}^1 = v q_x$

- Renewal premium:  $\beta = \frac{A_{x+1}}{\ddot{a}_{x+1}}$

${}_0V^{FPT} = {}_1V^{FPT} = 0$

${}_tV^{FPT} \text{ for } (x) = {}_{t-1}V^{FPT} \text{ for } (x+1)$

```
"""Recursive, Interim and Modified Reserves
```

```
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```

```
MIT License
```

```
"""
```

```
from mathlc.reserves import Reserves
```

```
Reserves.doc()
```

```

set_reserves(**args):
    None

fill_reserves(**args):
    None

V_plot(**args):
    None

t_V_forward(**args):
    Forward recursion (allows for optional reserve benefit)

t_V_backward(**args):
    Backward recursion (allows for optional reserve benefit)

t_V(**args):
    Try to solve time-t Reserve by forward or backward recursion

r_V_forward(**args):
    Forward recursion for interim reserves

r_V_backward(**args):
    Backward recursion for interim reserves

FPT_premium(**args):
    Initial or renewal Full Preliminary Term premiums

FPT_policy_value(**args):
    Compute Full Preliminary Term policy value at time t

```

## 9.4 Examples

```

import matplotlib.pyplot as plt
from mathlc.sult import SULT
from mathlc.policyvalues import PolicyValues

print("SOA Question 7.31: (E) 0.310")
x = 0
life = Reserves().set_reserves(T=3)
print(life._reserves)
G = 368.05
def fun(P): # solve net premium from expense reserve equation
    return life.t_V(x=x, t=2, premium=G-P, benefit=lambda t: 0,
                    per_policy=5 + .08*G)
P = life.solve(fun, target=-23.64, guess=[.29, .31]) / 1000
print(P)
print()

print("SOA Question 7.13: (A) 180")
life = SULT()
V = life.FPT_policy_value(40, t=10, n=30, endowment=1000, b=1000)
print(V)
print()

```

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```

print("Plot example: TODO from 6.12 -- this needs more work!!!")
life = PolicyValues(interest=dict(i=0.06))
a = 12
A = life.insurance_twin(a)
policy = life.Policy(benefit=1000, settlement_policy=20,
                    initial_policy=10, initial_premium=0.75,
                    renewal_policy=2, renewal_premium=0.1)
policy.premium = life.gross_premium(A=A, a=a, **policy.premium_terms)

life = Reserves(interest=dict(delta=0.06), mu=lambda x,s: 0.04)
life.set_reserves(T=100)
life.fill_reserves(x=0, policy=policy)
life.V_plot()

plt.show()

```

```

SOA Question 7.31: (E) 0.310
{'V': {0: 0, 3: 0}}
0.309966

```

```

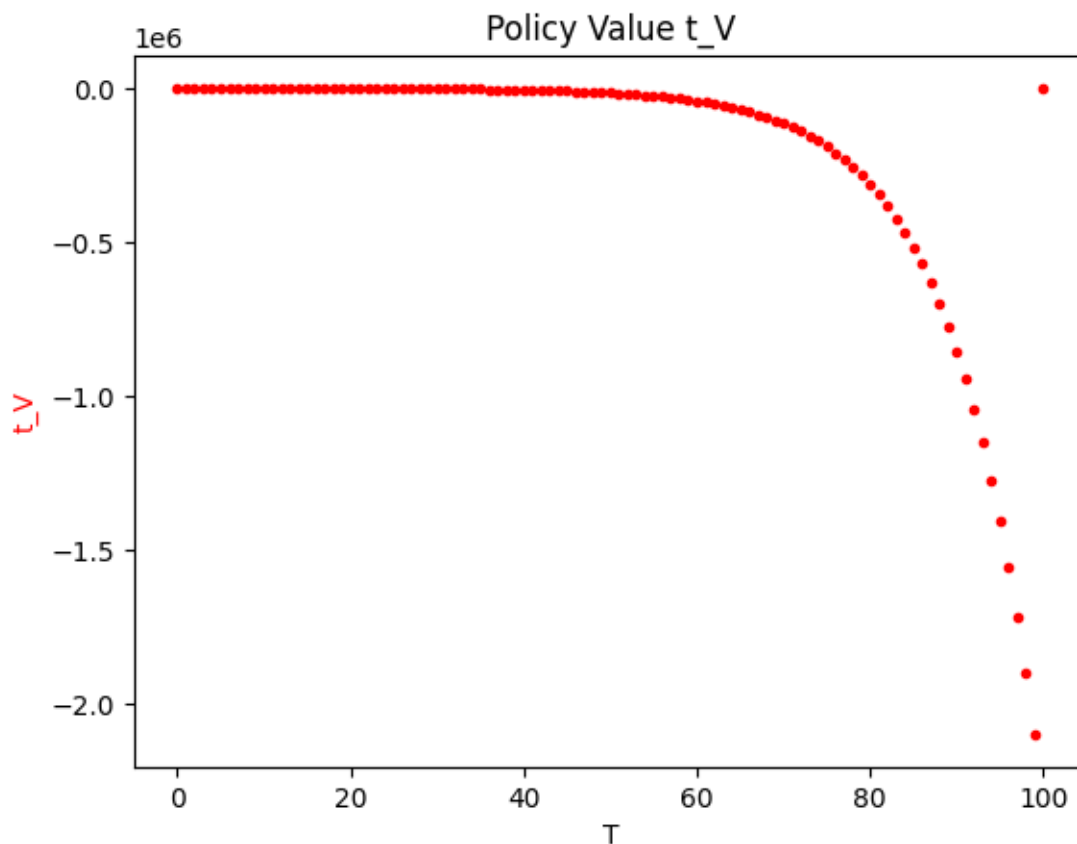
SOA Question 7.13: (A) 180
180.1071785904076

```

```

Plot example: TODO from 6.12 -- this needs more work!!!

```







## MORTALITY LAWS

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### 10.1 Uniform and Beta

**Beta**  $(\omega, \alpha)$ :

$$l_x \sim (\omega - x)^\alpha$$

$$\mu_x = \frac{\alpha}{\omega - x}$$

$${}_t p_x = \left( \frac{\omega - (x + t)}{\omega - x} \right)^\alpha$$

$${}^\circ e_x = \frac{\omega - x}{\alpha + 1}$$

**Uniform**  $(\omega)$ : Beta with  $\alpha = 1$

$$l_x \sim \omega - x$$

$$\mu_x = \frac{1}{\omega - x}$$

$${}_t p_x = \frac{\omega - (x + t)}{\omega - x}$$

$${}^\circ e_x = \frac{\omega - x}{2}$$

$${}^\circ e_{x:\overline{n}|} = {}_n p_x \cdot n + {}_n q_x \cdot \frac{n}{2}$$

$${}_n E_x = v^n \frac{\omega - (x + n)}{\omega - x}$$

$$\bar{A}_x = \frac{\bar{a}_{\omega-x|}}{\omega - x}$$

$$\bar{A}_{x:\overline{n}|}^1 = \frac{\bar{a}_{n|}}{\omega - x}$$

## 10.2 Gompertz and Makeham

**Makeham's Law:**  $c > 1$ ,  $B > 0$ ,  $A \geq -B$

$$\mu_x = A + Bc^x$$

$${}_tp_x = e^{\frac{Bc^x}{\ln c}(c^t - 1) - At}$$

**Gompertz's Law:** Makeham's Law with  $A = 0$

```
"""Mortality Laws: Uniform, Beta, Gompertz, Makeham
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"""
import math
from mathlib.mortalitylaws import MortalityLaws, Uniform, Beta, Makeham, Gompertz
MortalityLaws.doc()
```

```
l_r(**args):
    Fractional age lifes given continuous mortality law: l_[x]+s+r

p_r(**args):
    Fractional age survival given continuous mortality law

q_r(**args):
    Fractional age deferred mortality given continuous mortality law

mu_r(**args):
    Fractional age force of mortality given continuous mortality law

f_r(**args):
    fractional age mortality pdf given continuous mortality law

e_r(**args):
    Fractional age future lifetime given continuous mortality law
```

## 10.3 Examples

```
import math

print('Beta')
life = Beta(omega=100, alpha=0.5)
print(life.q_x(25, t=1, u=10))      # 0.0072
print(life.e_x(25))                  # 50
print(Beta(omega=60, alpha=1/3).mu_x(35) * 1000)
print()

print('Uniform')
uniform = Uniform(80, interest=dict(delta=0.04))
print(uniform.whole_life_annuity(20))      # 15.53
print(uniform.temporary_annuity(20, t=5))  # 4.35
print(Uniform(161).p_x(70, t=1))          # 0.98901
```

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```

print(Uniform(95).e_x(30, t=40, curtate=False)) # 27.692
print()

uniform = Uniform(omega=80, interest=dict(delta=0.04))
print(uniform.E_x(20, t=5)) # .7505
print(uniform.whole_life_insurance(20, discrete=False)) # .3789
print(uniform.term_insurance(20, t=5, discrete=False)) # .0755
print(uniform.endowment_insurance(20, t=5, discrete=False)) # .8260
print(uniform.deferred_insurance(20, u=5, discrete=False)) # .3033
print()

print('Gompertz/Makeham')
life = Gompertz(B=0.000005, c=1.10)
p = life.p_x(80, t=10) # 869.4
print(life.portfolio_percentile(N=1000, mean=p, variance=p*(1-p), prob=0.99))

print(Gompertz(B=0.00027, c=1.1).f_x(50, t=10)) # 0.04839
life = Makeham(A=0.00022, B=2.7e-6, c=1.124)
print(life.mu_x(60) * 0.9803) # 0.00316

```

```

Beta
0.007188905547861446
50.0
13.333333333333332

Uniform
16.03290804858584
4.47503070125663
0.989010989010989
27.692307692307693

0.7505031903214833
0.378867519462745
0.07552885288417432
0.8260320432056576
0.30333866657857067

Gompertz/Makeham
869.3908338193208
0.048389180223511644
0.0031580641631654026

```



## CONSTANT FORCE

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### 11.1 Constant force of mortality

$${}_t p_x = e^{-\mu t}$$

**Future lifetime:**

$${}^{\circ}e_x = \frac{1}{\mu}$$

$${}^{\circ}e_{x:n|} = \frac{1}{\mu}(1 - e^{-\mu n})$$

**Pure endowment:**

$${}_n Ex = e^{-(\mu+\delta)n}$$

**Insurance:**

$$\bar{A}_x = \frac{\mu}{\mu + \delta}$$

$$\bar{A}_{x:\overline{t}|} = \frac{\mu}{\mu + \delta}(1 - e^{-\mu t})$$

**Annuities:**

$$\bar{a}_x = \frac{1}{\mu + \delta}$$

$$\bar{a}_{x:\overline{t}|} = \frac{1}{\mu + \delta}(1 - e^{-\mu t})$$

```
"""Constant Force mortality law shortcuts
```

```
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```

```
MIT License
```

```
"""
```

```
from mathlc.constantforce import ConstantForce
```

```
ConstantForce.doc()
```

```
e_x(**args):
```

```
    Expected lifetime E[T_x] is memoryless: does not depend on (x)
```

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```

E_x(**args):
    Shortcut for APV of whole life: does not depend on age x

whole_life_insurance(**args):
    Shortcut for APV of whole life: does not depend on age x

temporary_annuity(**args):
    Shortcut for temporary life annuity: does not depend on age x

term_insurance(**args):
    Shortcut for APV of term life: does not depend on age x

Z_t(**args):
    Shortcut for T_x (or K_x) given survival probability for insurance

Y_t(**args):
    Shortcut for T_x (or K_x) given survival probability for annuity

```

## 11.2 Examples

```

from scipy.stats import norm
import math

print("SOA Question 6.36: (B) 500")
life = ConstantForce(mu=0.04, interest=dict(delta=0.08))
a = life.temporary_annuity(50, t=20, discrete=False)
A = life.term_insurance(50, t=20, discrete=False)
print(a, A)
def fun(R):
    return life.gross_premium(a=a, A=A, initial_premium=R/4500,
                              renewal_premium=R/4500, benefit=100000)
R = life.solve(fun, target=4500, guess=[400, 800])
print(R)
print()

print("SOA Question 6.31: (D) 1330")
life = ConstantForce(mu=0.01, interest=dict(delta=0.05))
A = life.term_insurance(35, t=35) + life.E_x(35, t=35) * 0.51791 # A_35
A = (life.term_insurance(35, t=35, discrete=False)
     + life.E_x(35, t=35) * 0.51791) # A_35
P = life.premium_equivalence(A=A, b=100000, discrete=False)
print(P)
print()

print("SOA Question 6.27: (D) 10310")
life = ConstantForce(mu=0.03, interest=dict(delta=0.06))
x = 0
payments = (3 * life.temporary_annuity(x, t=20, discrete=False)
            + life.deferred_annuity(x, u=20, discrete=False))
benefits = (1000000 * life.term_insurance(x, t=20, discrete=False)
            + 500000 * life.deferred_insurance(x, u=20, discrete=False))
print(benefits, payments)
print(life.term_insurance(x, t=20), life.deferred_insurance(x, u=20))

```

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```

P = benefits / payments
print(P)
print()

print("SOA Question 5.4: (A) 213.7")
life = ConstantForce(mu=0.02, interest=dict(delta=0.01))
P = 10000 / life.certain_life_annuity(40, u=life.e_x(40, curtate=False),
                                     discrete=False)

print()

print("SOA Question 5.1: (A) 0.705")
life = ConstantForce(mu=0.01, interest=dict(delta=0.06))
EY = life.certain_life_annuity(0, u=10, discrete=False)
print(life.p_x(0, t=life.Y_to_t(EY))) # 0.705
print()

print("Other examples")
life = ConstantForce(mu=0.03, interest=dict(delta=0.04))
print(life.whole_life_annuity(20, discrete=False)) # 14.286
print(life.temporary_annuity(20, t=5, discrete=False)) # 4.219

life = ConstantForce(mu=0.04, interest=dict(delta=0.07))
#print(life.T_p(30, 0.7), 1000*life.certain.Z_t(30)) # 8.9169 535.7, , 0.7 ???

life = ConstantForce(mu=.03, interest=dict(delta=.04))
print(life.E_x(20, t=5)) # .7047
print(life.whole_life_insurance(20, discrete=False)) # .4286
print(life.term_insurance(20, t=5, discrete=False)) # .1266
print(life.endowment_insurance(20, t=5, discrete=False)) # .8313
print(life.deferred_insurance(20, u=5, discrete=False)) # .3020

life1 = ConstantForce(mu=.04, interest=dict(delta=.02))
life2 = ConstantForce(mu=.05, interest=dict(delta=.02))
life3 = ConstantForce(mu=.05, interest=dict(delta=.03))

A1 = life1.term_insurance(0, t=5, discrete=False)
E1 = life1.E_x(0, t=5)
A2 = life2.term_insurance(5, t=7, discrete=False)
E2 = life2.E_x(5, t=7)
A3 = life3.whole_life_insurance(12, discrete=False)
print(A1, E1, A2, E2, A1 + E1 * (A2 + E2 * A3))

life = ConstantForce(mu=.04, interest=dict(delta=.06))
A1 = 10 * life.deferred_insurance(0, u=5, discrete=False)
A2 = 10 * 10 * life.deferred_insurance(0, u=5, moment=2, discrete=False)
E = 100 * A1
V = 100 * (A2 - A1**2)
print(A1, A2, E, V) # 2.426, 11.233, 242.6, 534.8
print(E + norm.ppf(0.95) * math.sqrt(V)) # 281

```

SOA Question 6.36: (B) 500

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```
7.577350389254893 0.3030940155701957
500.0
```

```
SOA Question 6.31: (D) 1330
1326.5406293909457
```

```
SOA Question 6.27: (D) 10310
305783.51862973545 29.66002470618696
0.26992967028309356 0.053452469414929524
10309.617799001708
```

```
SOA Question 5.4: (A) 213.7
```

```
SOA Question 5.1: (A) 0.705
0.7053680433746505
```

```
Other examples
```

```
14.28571428571429
4.21874157544695
0.7046880897187136
0.42857142857142866
0.1265622472634085
0.831250336982122
0.3020091813080202
0.17278785287885476 0.740818220681718 0.27669543272541713 0.6126263941844162 0.
↪6614218680727285
2.426122638850533 11.233224102930535 242.61226388505327 534.7153044187462
280.64771434478587
```



## LIFE TABLE

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### 12.1 Pure endowment

$${}_tE_x = v^t \frac{l_{x+t}}{l_x}$$

$${}_t^2E_x = v^{2t} \frac{l_{x+t}}{l_x} = v^t {}_tE_x$$

```
"""Life Tables

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"""

from mathlc.lifetable import LifeTable

LifeTable.doc()
```

```
fill(**args):
    Fill in missing lives and mortality. Does not check consistency

l_x(**args):
    Lookup l_x from life table

d_x(**args):
    Compute from lifetable lives at x_t divided by lives at x

p_x(**args):
    t_p_x = lives beginning year x+t divided lives beginning year x

q_x(**args):
    Deferred mortality: u|t_q_x = (l[x+u] - l[x+u+t]) / l[x]

f_x(**args):
    probability density function of mortality

mu_x(**args):
    Compute mu_x from p_x in life table
```

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```
e_x(**args):
    Expected curtate lifetime from sum of lives in table

E_x(**args):
    Pure Endowment from life table and interest rate
```

## 12.2 Examples

```
import math
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd

print("SOA Question 6.53: (D) 720")
x = 0
life = LifeTable(interest=dict(i=0.08), q={x:.1, x+1:.1, x+2:.1}).fill()
A = life.term_insurance(x, t=3)
P = life.gross_premium(a=1, A=A, benefit=2000, initial_premium=0.35)
print(A, P)
print(life.frame())
print()

print("SOA Question 6.41: (B) 1417")
x = 0
life = LifeTable(interest=dict(i=0.05), q={x:.01, x+1:.02}).fill()
P = 1416.93
a = 1 + life.E_x(x, t=1) * 1.01
A = (life.deferred_insurance(x, u=0, t=1)
     + 1.01 * life.deferred_insurance(x, u=1, t=1))
print(a, A)
P = 100000 * A / a
print(P)
print(life.frame())
print()

print("SOA Question 3.11: (B) 0.03")
life = LifeTable(q={50//2: .02, 52//2: .04}, udd=True).fill()
print(life.q_r(50//2, t=2.5/2))
print(life.frame())
print()

print("SOA Question 3.5: (E) 106")
l = [99999, 88888, 77777, 66666, 55555, 44444, 33333, 22222]
a = LifeTable(l={age:l for age,l in zip(range(60, 68), l)}, udd=True)\
    .q_r(60, u=3.4, t=2.5)
b = LifeTable(l={age:l for age,l in zip(range(60, 68), l)}, udd=False)\
    .q_r(60, u=3.4, t=2.5)
print(100000 * (a - b))
print()
```

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```

print("SOA Question 3.14: (C) 0.345")
life = LifeTable(l={90: 1000, 93: 825},
                 d={97: 72},
                 p={96: .2},
                 q={95: .4, 97: 1}, udd=True).fill()
print(life.q_r(90, u=93-90, t=95.5-93))
print(life.frame())
print()

print("Other usage examples")
l = [110, 100, 92, 74, 58, 38, 24, 10, 0]
table = LifeTable(l={age:l for age,l in zip(range(79, 88), l)},
                 interest=dict(i=0.06), maxage=87)
print(table.mu_x(80))
# print(table.temporary_annuity(80, t=4, m=4, due=True)) # 2.7457
# print(table.whole_life_annuity(80, m=4, due=True, woolhouse=True)) # 3.1778
# print(table.whole_life_annuity(80, m=4, due=False, woolhouse=True)) # 2.9278
print(table.temporary_annuity(80, t=4)) # 2.7457
print(table.whole_life_annuity(80)) # 3.1778
print('*', table.whole_life_annuity(80, discrete=False)) # 2.9278

l = [100, 90, 70, 50, 40, 20, 0]
table = LifeTable(l={age:l for age,l in zip(range(70, 77), l)},
                 interest=dict(i=0.08), maxage=76)
print(table.A_x(70),
      table.A_x(70, moment=2)) # .75848, .58486
print(table.endowment_insurance(70, t=3)) # .81974
print('*', table.endowment_insurance(70, t=3, discrete=False)) # .81974

print(table.E_x(70, t=3)) # .39692
print('*', table.term_insurance(70, t=3, discrete=False)) # .43953
print('*', table.endowment_insurance(70, t=3, discrete=False)) # .83644
print(table.E_x(70, t=3, moment=2)) # .31503
print('*', table.term_insurance(70, t=3, moment=2, discrete=False)) # .38786
print('*', table.endowment_insurance(70, t=3, moment=2, discrete=False)) # .70294

l = [1000, 990, 975, 955, 925, 890, 840]
table = LifeTable(l={age:l for age,l in zip(range(70, 77), l)},
                 interest=dict(i=0.08), maxage=76)
print(table.increasing_annuity(70, t=4, discrete=True))
print(table.decreasing_annuity(71, t=5, discrete=False))

print('*', table.endowment_insurance(70, t=3, discrete=False)) # .7976

l = [100, 90, 70, 50, 40, 20, 0]
table = LifeTable(l={age:l for age,l in zip(range(70, 77), l)},
                 interest=dict(i=0.08), maxage=76)
print(1e6*table.whole_life_annuity(70, variance=True)) #1743784

```

```

SOA Question 6.53: (D) 720
0.23405349794238678 720.1646090534978
      l      d      q      p
0 100000.0 10000.0 0.1 0.9

```

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```

1  90000.0  9000.0  0.1  0.9
2  81000.0  8100.0  0.1  0.9
3  72900.0      NaN  NaN  NaN

SOA Question 6.41:  (B) 1417
1.9522857142857144 0.027662585034013608
1416.9332301924137
      l      d      q      p
0  100000.0  1000.0  0.01  0.99
1   99000.0  1980.0  0.02  0.98
2   97020.0      NaN  NaN  NaN

SOA Question 3.11:  (B) 0.03
0.0298
      l      d      q      p
25  100000.0  2000.0  0.02  0.98
26   98000.0  3920.0  0.04  0.96
27   94080.0      NaN  NaN  NaN

SOA Question 3.5:   (E) 106
106.16575827938624

SOA Question 3.14:  (C) 0.345
0.345
      l      d      q      p
90   1000.0      NaN  NaN  NaN
93    825.0      NaN  NaN  NaN
95    600.0  240.0  0.4  0.6
96    360.0  288.0  0.8  0.2
97     72.0   72.0  1.0  0.0
98      0.0      NaN  NaN  NaN

Other usage examples
0.08338160893905101
3.013501078071164
3.564334685633434
* 3.0554886512477943
0.7584803549199998 0.5848621157624223
0.8197429253670677
* 0.8364390776999064
0.3969161205100847
* 0.4395229571898218
* 0.8364390776999064
0.31508481344155215
* 0.3878582966930021
* 0.7029431101345542
8.373488543413096
6.7673201739834585
* 0.7976061232001961
1744071.8039800397

```

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## 13.1 Standard ultimate life table

From SOA's "Excel Workbook for FAM-L Tables":

- interest rate  $i = 0.05$
- 100000 initial lives aged 20
- Makeham's Law with  $A = 0.00022$ ,  $B = 0.0000027$ ,  $c = 1.124$

## 13.2 Temporary Annuity

$$A_{x:\overline{t}|}^1 = A_x - {}_tE_x A_{x+t} = A_{x:\overline{t}|} - {}_tE_x$$

$${}^2A_{x:\overline{t}|}^1 = {}^2A_x - {}_tE_x {}^2A_{x+t} = {}^2A_x - v^t {}_tE_x {}^2A_{x+t}$$

```
"""Standard Ultimate Life Table

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"""

from math import exp, log
from math import dict, float, int
import pandas as pd

class SULT(LifeTable):
    """Standard Ultimate Life Table"""

    def __init__(self, interest = dict(i=0.05), lifes=LifeTable.LIFES,
                 A: float = 0.00022, B: float = 0.0000027, c: float = 1.124,
                 minage: int = 20, maxage = 130, **kwargs):
        """SULT from SOA's "Excel Workbook for FAM-L Tables"""
        t_p_x = lambda x,t: math.exp(-A*t - (B*c**x*(c**t - 1))/math.log(c))
        l = {t+minage: lifes*t_p_x(minage, t) for t in range(1+maxage-minage)}
        super().__init__(interest=interest, l=l, minage=minage, maxage=maxage,
                        **kwargs)
```

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```

def frame(self):
    """Displays tables used for FAM-L exam"""
    tab = pd.DataFrame(dict(l_x=self._table['l']).sort_index())
    t = {'q_x': self['q'],
         'a_x': self['a'],
         'A_x': self['A'],
         '2A_x': self['2A'],
         'a_x:10': self['a10'],
         'A_x:10': self['A10'],
         'a_x:20': self['a20'],
         'A_x:20': self['A20'],
         '5_E_x': self['5E'],
         '10_E_x': self['10E'],
         '20_E_x': self['20E']}
    tab = tab.join(pd.DataFrame.from_dict(t).set_index(tab.index[:-1]))
    for digits, col in zip([1,6,4,5,5,4,5,4,5,5,5,5], tab.columns):
        tab[col] = tab[col].map(f"{{:.{digits}f}}".format)
    return tab.loc[20:100]

```

## 13.3 Examples

```

print("SOA Question 6.52: (D) 50.80")
sult = SULT()
a = sult.temporary_annuity(45, t=10)
other_cost = 10 * sult.deferred_annuity(45, u=10)
P = sult.gross_premium(a=a, A=0, benefit=0,
                       initial_premium=1.05, renewal_premium=0.05,
                       initial_policy=100 + other_cost, renewal_policy=20)

print(a, P)
print()

print("SOA Question 6.47: (D) 66400")
sult = SULT()
a = sult.temporary_annuity(70, t=10)
A = sult.deferred_annuity(70, u=10)
P = sult.gross_premium(a=a, A=A, benefit=100000, initial_premium=0.75,
                       renewal_premium=0.05)

print(P)
print()

print("SOA Question 6.43: (C) 170")
sult = SULT()
a = sult.temporary_annuity(30, t=5)
A = sult.term_insurance(30, t=10)
other_expenses = 4 * sult.deferred_annuity(30, u=5, t=5)
P = sult.gross_premium(a=a, A=A, benefit=200000, initial_premium=0.35,
                       initial_policy=8 + other_expenses, renewal_policy=4,
                       renewal_premium=0.15)

print(P)
print()

```

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```

print("SOA Question 6.39: (A) 29")
sult = SULT()
P40 = sult.premium_equivalence(sult.whole_life_insurance(40), b=1000)
P80 = sult.premium_equivalence(sult.whole_life_insurance(80), b=1000)
p40 = sult.p_x(40, t=10)
p80 = sult.p_x(80, t=10)
P = (P40 * p40 + P80 * p80) / (p80 + p40)
print(P)
print()

print("SOA Question 6.37: (D) 820")
sult = SULT()
benefits = sult.whole_life_insurance(35, b=50000 + 100)
expenses = sult.immediate_annuity(35, b=100)
a = sult.temporary_annuity(35, t=10)
print(benefits, expenses, a)
print((benefits + expenses) / a)
print()

print("SOA Question 6.35: (D) 530")
sult = SULT()
A = sult.whole_life_insurance(35, b=100000)
a = sult.whole_life_annuity(35)
print(sult.gross_premium(a=a, A=A, initial_premium=.19, renewal_premium=.04))
print()

print("SOA Question 5.8: (C) 0.92118")
sult = SULT()
a = sult.certain_life_annuity(55, u=5)
print(sult.p_x(55, t=math.floor(a)))
print()

print("SOA Question 5.3: (C) 0.6239")
sult = SULT()
t = 10.5
print(t * sult.E_r(40, t=t))
print()

print("SOA Question 4.17: (A) 1126.7")
sult = SULT()
median = sult.Z_t(48, prob=0.5, discrete=False)
benefit = lambda x,t: 5000 if t < median else 10000
print(sult.A_x(48, benefit=benefit))
print()

print("SOA Question 4.14: (E) 390000 ")
sult = SULT()
p = sult.p_x(60, t=85-60)

```

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```

mean = sult.bernoulli(p)
var = sult.bernoulli(p, variance=True)
F = sult.portfolio_percentile(mean=mean, variance=var, prob=.86, N=400)
print(F * 5000 * sult.interest.v_t(85-60))
print()

print("SOA Question 4.5: (C) 35200")
sult = SULT()
print(100000 * sult.Interest(delta=0.05).v_t(sult.Z_t(45, prob=.95)))
print()

print("SOA Question 3.9: (E) 3850")
sult = SULT()
p1 = sult.p_x(20, t=25)
p2 = sult.p_x(45, t=25)
mean = sult.bernoulli(p1) * 2000 + sult.bernoulli(p2) * 2000
var = (sult.bernoulli(p1, variance=True) * 2000
      + sult.bernoulli(p2, variance=True) * 2000)
print(sult.portfolio_percentile(mean=mean, variance=var, prob=.99))
print()

print("SOA Question 3.8: (B) 1505")
sult = SULT()
p1 = sult.p_x(35, t=40)
p2 = sult.p_x(45, t=40)
mean = sult.bernoulli(p1) * 1000 + sult.bernoulli(p2) * 1000
var = (sult.bernoulli(p1, variance=True) * 1000
      + sult.bernoulli(p2, variance=True) * 1000)
print(sult.portfolio_percentile(mean=mean, variance=var, prob=.95))
print()

print("SOA Question 3.4: (B) 815")
sult = SULT()
mean = sult.p_x(25, t=95-25)
var = sult.bernoulli(mean, variance=True)
print(sult.portfolio_percentile(N=4000, mean=mean, variance=var, prob=.1))
print()

print("Other usage examples")
print(sult.temporary_annuity(80, t=10)*20000) # ~130770

E = sult.E_x(60, t=5)
print(E, E*sult.a_x(65, benefit=(lambda x,t: 1000 + .05*t)))

print(sult.whole_life_annuity(60)) # 14.9041
print(sult.whole_life_annuity(60, discrete=False)) # ~13.9041
print(sult.deferred_annuity(60, u=10)) # 6.9485
print(sult.temporary_annuity(60, t=10)) # 7.9555
print(sult.temporary_annuity(60, t=15)) # 10.5282
print(sult.temporary_annuity(60, t=10))
print(sult.E_x(60, t=10)) #
print(sult.temporary_annuity(60, t=10, discrete=False)) # ~7.5341

```

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```

print(sult.certain_life_annuity(60, u=10)) # 15.0563
print(sult.whole_life_annuity(60, variance=True, discrete=False)) # ~10.6182
print(sult.endowment_insurance(60, t=10)) # .62116
print(sult.whole_life_insurance(60, moment=2)) # .10834
print(sult.whole_life_insurance(70, moment=2)) # .21467
print(sult.endowment_insurance(60, t=10, moment=2)) # .38732
print(sult.temporary_annuity(60, t=10, variance=True)) #.6513

print(sult.p_x(70)) # 0.989587
print(math.log(sult.p_x(70, t=2)) / -2) # 0.011103

A1 = sult.whole_life_insurance(20, discrete=False)
#print(A1, sult.whole_life(20))
A2 = sult.whole_life_insurance(50, discrete=False)
#print(A2, sult.whole_life(50))
A3 = sult.whole_life_insurance(70, discrete=False)
E2 = sult.E_x(20, t=30)
E3 = sult.E_x(20, t=50)
print(A1, E2, A2, E3, A3, 125*A1 + E2*175*A2 - E3*250*A3) # 5,335

print(sult.whole_life_insurance(50)) # .18931
print(sult.term_insurance(50, t=20)) # .0402
print(sult.endowment_insurance(50, t=20)) # ~.31471
print(sult.E_x(50, t=20),
      sult.whole_life_insurance(70),
      sult.deferred_insurance(50, u=20)) # .14911

print(sult.whole_life_insurance(50, discrete=False)) # .19400
print(sult.term_insurance(50, t=20, discrete=False)) # .0412
print(sult.endowment_insurance(50, t=20, discrete=False)) #.38944
print(sult.E_x(50, t=20),
      sult.whole_life_insurance(70, discrete=False),
      sult.deferred_insurance(50, u=20, discrete=False)) # .15281
print()

print("Standard Ultimate Life Table at i=0.05")
sult.frame()

```

SOA Question 6.52: (D) 50.80  
8.0750937741422 50.80135534704229

SOA Question 6.47: (D) 66400  
66384.13293704337

SOA Question 6.43: (C) 170  
171.22371939459944

SOA Question 6.39: (A) 29  
29.033866427845496

SOA Question 6.37: (D) 820  
4836.382819496279 1797.2773668474615 8.092602358383987  
819.7190338249138

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```

SOA Question 6.35: (D) 530
534.4072234303344

SOA Question 5.8: (C) 0.92118
0.9211799771029529

SOA Question 5.3: (C) 0.6239
6.23871918627528

SOA Question 4.17: (A) 1126.7
1126.774772894844

SOA Question 4.14: (E) 390000
389322.86778416135

SOA Question 4.5: (C) 35200
36787.94411714415

SOA Question 3.9: (E) 3850
3850.144345130047

SOA Question 3.8: (B) 1505
1504.8328375406456

SOA Question 3.4: (B) 815
815.0943255167722

Other usage examples
135770.41601330126
0.7668687235541889 10395.824343647037
14.904074300627297
14.39854504493635
6.9485261567485095
7.955548143878787
10.52811141378872
7.955548143878787
0.5786434508971754
7.74293075434579
15.05634783239256
10.621710987844606
0.6211643741010102
0.10834081779190481
0.21466683367433795
0.38732012436971175
0.6504506203786906
0.9895866730368528
0.01110329636560811
0.05042882180359127 0.22808348858314714 0.1940084167264656 0.0794272723450692 0.
↵4388106987422976 5.333988893971686
0.18930786030072838
0.04020082061028696
0.3884385332061035
0.34823771259581654 0.4281760254481774 0.14910703969044142
0.1940084167264656
0.041197982733875926
0.38943569532969247

```

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```
0.34823771259581654 0.4388106987422976 0.15281043399258967
```

```
Standard Ultimate Life Table at i=0.05
```

	<code>l_x</code>	<code>q_x</code>	<code>a_x</code>	<code>A_x</code>	<code>...</code>	<code>A_x:20</code>	<code>5_E_x</code>	<code>10_E_x</code>	<code>20_E_x</code>
20	100000.0	0.000250	19.9664	0.04922	...	0.37829	0.78252	0.61224	0.37440
21	99975.0	0.000253	19.9197	0.05144	...	0.37833	0.78250	0.61220	0.37429
22	99949.7	0.000257	19.8707	0.05378	...	0.37837	0.78248	0.61215	0.37417
23	99924.0	0.000262	19.8193	0.05622	...	0.37842	0.78245	0.61210	0.37404
24	99897.8	0.000267	19.7655	0.05879	...	0.37848	0.78243	0.61205	0.37390
..	...	...	...	...	...	...	...	...	...
96	17501.8	0.192887	3.5597	0.83049	...	0.83049	0.19872	0.01330	0.00000
97	14125.9	0.214030	3.3300	0.84143	...	0.84143	0.16765	0.00827	0.00000
98	11102.5	0.237134	3.1127	0.85177	...	0.85177	0.13850	0.00485	0.00000
99	8469.7	0.262294	2.9079	0.86153	...	0.86153	0.11173	0.00266	0.00000
100	6248.2	0.289584	2.7156	0.87068	...	0.87068	0.08777	0.00136	0.00000

```
[81 rows x 12 columns]
```



## SELECT LIFE TABLE

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### 14.1 Select and ultimate life table

```
"""Select and Ultimate Life Table

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"""
from mathlc.selectlife import Select
Select.doc()
```

```
fill(**args):
    Fills in missing mortality values. Does not check for consistency

l_x(**args):
    Returns number of lifes computed from select table

p_x(**args):
    t_p_[x]+s by chain rule: prod(1_p_[x]+s+y) for y in range(t)

q_x(**args):
    t|u_q_[x]+s = [x]+s survives u years, does not survive next t

e_x(**args):
    Returns curtate expected life time computed from select table

A_x(**args):
    Returns insurance value computed from select table

a_x(**args):
    Returns annuity value computed from select table
```

## 14.2 Examples

```

from mathlc.sult import SULT

print("SOA Question 3.2: (D) 14.7")
e_curtate = Select.e_curtate(e=15)
life = Select(l={65: [1000, None],
                    66: [955, None]},
              e={65: [e_curtate, None]},
              d={65: [40, None],
                66: [45, None]}, udd=True).fill()
print(life.e_r(66))
print(life.frame('e'))
print()

print("SOA Question 4.16: (D) .1116")
q = [.045, .050, .055, .060]
q_ = {50+x: [0.7 * q[x] if x < 4 else None,
            0.8 * q[x+1] if x+1 < 4 else None,
            q[x+2] if x+2 < 4 else None]
      for x in range(4)}
life = Select(q=q_, interest=dict(i=.04)).fill()
print(life.term_insurance(50, t=3))
print()

print("SOA Question 4.13: (C) 350 ")
life = Select(q={65: [.08, .10, .12, .14],
                    66: [.09, .11, .13, .15],
                    67: [.10, .12, .14, .16],
                    68: [.11, .13, .15, .17],
                    69: [.12, .14, .16, .18]}, interest=dict(i=.04)).fill()
print(life.deferred_insurance(65, t=2, u=2, b=2000))
print()

print("SOA Question 3.13: (B) 1.6")
life = Select(l={55: [10000, 9493, 8533, 7664],
                    56: [8547, 8028, 6889, 5630],
                    57: [7011, 6443, 5395, 3904],
                    58: [5853, 4846, 3548, 2210]},
              e={57: [None, None, None, 1]}).fill()
print(life.e_r(58, s=2))
print()

print("SOA Question 3.12: (C) 0.055 ")
life = Select(l={60: [10000, 9600, 8640, 7771],
                    61: [8654, 8135, 6996, 5737],
                    62: [7119, 6549, 5501, 4016],
                    63: [5760, 4954, 3765, 2410]}, udd=False).fill()
print(life.q_r(60, s=1, t=3.5) - life.q_r(61, s=0, t=3.5))

print()

```

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```

print("SOA Question 3.7: (b) 16.4")
life = Select(q={50: [.0050, .0063, .0080],
                    51: [.0060, .0073, .0090],
                    52: [.0070, .0083, .0100],
                    53: [.0080, .0093, .0110]}) .fill()
print(1000*life.q_r(50, s=0, r=0.4, t=2.5))
print()

print("SOA Question 3.6: (D) 15.85")
life = Select(q={60: [.09, .11, .13, .15],
                    61: [.1, .12, .14, .16],
                    62: [.11, .13, .15, .17],
                    63: [.12, .14, .16, .18],
                    64: [.13, .15, .17, .19]},
              e={61: [None, None, None, 5.1]}) .fill()
print(life.e_x(61))
print()

print("SOA Question 3.3: (E) 1074")
life = Select(l={50: [99, 96, 93],
                 51: [97, 93, 89],
                 52: [93, 88, 83],
                 53: [90, 84, 78]})
print(10000*life.q_r(51, s=0, r=0.5, t=2.2))

print()

print("SOA Question 3.1: (B) 117")
life = Select(l={60: [80000, 79000, 77000, 74000],
                 61: [78000, 76000, 73000, 70000],
                 62: [75000, 72000, 69000, 67000],
                 63: [71000, 68000, 66000, 65000]})
print(1000*life.q_r(60, s=0, r=0.75, t=3, u=2))
print()

print("Other usage examples")
life = Select(minage=20, maxage=30, n=3)
life.set_select(column=3, select_age=False, q=SULT()['q']) .fill()
print(life._select)
print(life.frame('l'))
print('-----')
print(life.frame('q'))
print('=====')

life = Select(q={21: [0.00120, 0.00150, 0.00170, 0.00180],
                  22: [0.00125, 0.00155, 0.00175, 0.00185],
                  23: [0.00130, 0.00160, 0.00180, 0.00195]}) .fill()
print(life.frame('l'))
print('-----')
print(life.frame('q'))

```

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```
print('=====')
print(life.p_x(21, 1, 4)) #0.99317
```

SOA Question 3.2: (D) 14.7

14.67801047120419

0 1

e

65 14.50000 14.104167

66 14.17801 13.879121

SOA Question 4.16: (D) .1116

0.1115661982248521

SOA Question 4.13: (C) 350

351.0578236056159

SOA Question 3.13: (B) 1.6

1.6003382187147688

SOA Question 3.12: (C) 0.055

0.05465655938591829

SOA Question 3.7: (b) 16.4

16.420207214428586

SOA Question 3.6: (D) 15.85

5.846832

SOA Question 3.3: (E) 1074

1073.684210526316

SOA Question 3.1: (B) 117

116.7192429022082

Other usage examples

```
{'A': {20: {}, 21: {}, 22: {}, 23: {}, 24: {}, 25: {}, 26: {}, 27: {}, 28: {}, 29:
↵ {}, 30: {}}, 'a': {20: {}, 21: {}, 22: {}, 23: {}, 24: {}, 25: {}, 26: {}, 27: {}
↵, 28: {}, 29: {}, 30: {}}, 'q': {20: {3: 0.0002620983481647125}, 21: {3: 0.
↵00026732142024017664}, 22: {3: 0.0002731921206805849}, 23: {3: 0.
↵0002797907468233207}, 24: {3: 0.0002872075506188112}, 25: {3: 0.
↵0002955439724024386}, 26: {3: 0.0003049140275073012}, 27: {3: 0.
↵0003154458646110006}, 28: {3: 0.0003272835170716096}, 29: {3: 0.
↵00034058887111311693}, 30: {3: 0.00035554387766526267}}, 'd': {20: {}, 21: {},
↵22: {}, 23: {}, 24: {}, 25: {}, 26: {}, 27: {}, 28: {}, 29: {}, 30: {}}, 'l':
↵{20: {0: 100000}, 21: {}, 22: {}, 23: {}, 24: {}, 25: {}, 26: {}, 27: {}, 28: {}},
↵29: {}, 30: {}}, 'e': {20: {}, 21: {}, 22: {}, 23: {}, 24: {}, 25: {}, 26: {}},
↵27: {}, 28: {}, 29: {}, 30: {}}}
```

0

1

20 100000

-----

3

q

20 0.000262

21 0.000267

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```

22  0.000273
23  0.000280
24  0.000287
25  0.000296
26  0.000305
27  0.000315
28  0.000327
29  0.000341
30  0.000356
=====
                                0                1                2                3
1
21  100000.000000  99880.000000  99730.180000  99560.638694
22   99834.996495  99710.202749  99555.651935  99381.429544
23   99665.273502  99535.708646  99376.451512  99197.573900
-----
                                0                1                2                3
q
21  0.00120  0.00150  0.00170  0.00180
22  0.00125  0.00155  0.00175  0.00185
23  0.00130  0.00160  0.00180  0.00195
=====
0.9931675400449915

```



## RECURSION

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### 15.1 Chain rule

$${}_{t+n}p_x = {}_np_x \cdot {}_tp_{x+n}$$

$${}_{t+n}E_x = {}_nE_x \cdot {}_tE_{x+n}$$

### 15.2 Future lifetime

**Complete expectation of life:**

$$\dot{e}_x = \dot{e}_{x:\overline{m}|} + {}_mp_x \dot{e}_{x+m}$$

- One-year recursion:  $\dot{e}_x = \dot{e}_{x:\overline{1}|} + p_x \dot{e}_{x+1}$
- Temporary expectation:  $\dot{e}_{x:\overline{m+n}|} = \dot{e}_{x:\overline{m}|} + {}_mp_x \dot{e}_{x+m:\overline{n}|}$

**Curtate expectation of life:**

$$e_x = e_{x:\overline{m}|} + {}_mp_x e_{x+m}$$

- One-year recursion:  $e_x = p_x(1 + e_{x+1})$
- Temporary expectation:  $e_{x:\overline{m+n}|} = e_{x:\overline{m}|} + {}_mp_x e_{x:\overline{n}|}$

### 15.3 Insurance

$$A_x = v q_x + v p_x A_{x+1} \Rightarrow A_{x+1} = \frac{A_x - v q_x}{v p_x}$$

$$A_{x:\overline{t}|}^1 = v q_x + v p_x A_{x+1:\overline{t-1}|}^1$$

$$A_{x:\overline{0}|} = b$$

$$A_{x:\overline{1}|} = q_x v b + p_x v b = v b$$

$$IA_x = v q_x + v p_x A_{x+1}$$

$$IA_{x:\overline{t}|}^1 = v q_x + v p_x (A_{x+1} + IA_{x+1:\overline{t-1}|}^1)$$

$$DA_{x:\overline{t}|}^1 = t v q_x + v p_x (DA_{x+1:\overline{t-1}|}^1)$$

## 15.4 Annuities

$$\ddot{a}_x = 1 + v p_x \ddot{a}_{x+1} \Rightarrow \ddot{a}_{x+1} = \frac{\ddot{a}_x - 1}{v p_x}$$

$$\ddot{a}_{x:\overline{t}|} = 1 + v p_x \ddot{a}_{x+1:\overline{t-1}|}$$

```
"""Recursion Formulas and Identity Relationships
```

```
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```

```
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```

```
"""
```

```
from mathlc.recursion import Recursion
Recursion.doc()
```

```
set_q(**args):
    Set in key-value store

set_p(**args):
    Set in key-value store

set_e(**args):
    Set in key-value store

set_E(**args):
    Set in key-value store

set_A(**args):
    Set in key-value store

set_IA(**args):
    Set in key-value store

set_DA(**args):
    Set in key-value store

set_a(**args):
    Set in key-value store
```

## 15.5 Examples

```
from mathlc.constantforce import ConstantForce

print("SOA Question 6.48: (A) 3195")
life = Recursion(interest=dict(i=0.06), depth=5)
x = 0
life.set_p(0.95, x=x, t=5)
life.set_q(0.02, x=x+5)
```

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```

life.set_q(0.03, x=x+6)
life.set_q(0.04, x=x+7)
a = 1 + life.E_x(x, t=5)
A = life.deferred_insurance(x, u=5, t=3)
P = life.gross_premium(A=A, a=a, benefit=100000)
print(P)
print()

print("SOA Question 6.40: (C) 116 ")
# - standard formula discounts/accumulates by too much (i should be smaller)
x = 0
life = Recursion(interest=dict(i=0.06)).set_a(7, x=x+1).set_q(0.05, x=x)
a = life.whole_life_annuity(x)
A = 110 * a / 1000
print(a, A)
life = Recursion(interest=dict(i=0.06)).set_A(A, x=x).set_q(0.05, x=x)
A1 = life.whole_life_insurance(x+1)
P = life.gross_premium(A=A1 / 1.03, a=7) * 1000
print(P)
print()

print("SOA Question 6.17: (A) -30000")
x = 0
life = ConstantForce(mu=0.1, interest=dict(i=0.08))
A = life.endowment_insurance(x, t=2, b=100000, endowment=30000)
a = life.temporary_annuity(x, t=2)
P = life.gross_premium(a=a, A=A)
print(A, a, P)

life1 = Recursion(interest=dict(i=0.08))\
    .set_q(life.q_x(x, t=1) * 1.5, x=x, t=1)\
    .set_q(life.q_x(x+1, t=1) * 1.5, x=x+1, t=1)
policy = life1.Policy(premium=P * 2, benefit=100000, endowment=30000)
L = life1.gross_policy_value(x, t=0, n=2, policy=policy)
print(L)
print()

```

```

SOA Question 6.48: (A) 3195
[ Pure Endowment: 5_E_0 ]
    pure endowment 5_E_0 = 5_p_0 * v^5
[ Pure Endowment: 5_E_0 ]
    pure endowment 5_E_0 = 5_p_0 * v^5
[ Term Insurance: A_5^1:3 ]
    forward: A_5 = qv + pvA_6
        forward: A_6 = qv + pvA_7
            endowment insurance - pure endowment = A_7^1:1
                pure endowment 1_E_7 = 1_p_7 * v^1
[ Term Insurance: A_5^1:3 ]
    pure endowment 1_E_7 = 1_p_7 * v^1
        endowment insurance - pure endowment = A_7^1:1
            forward: A_6 = qv + pvA_7
                forward: A_5 = qv + pvA_6
3195.1189176587473

SOA Question 6.40: (C) 116

```

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```

[ Whole Life Annuity: a_0 ]
    forward: a_0 = 1 + E_0 a_1
    pure endowment 1_E_0 = 1_p_0 * v^1
7.2735849056603765 0.8000943396226414
[ Whole Life Insurance: A_1 ]
    backward: A_1 = (A_0/v - q) / p
    backward: A_1 = (A_0/v - q) / p
    backward: A_1 = (A_0/v - q) / p
    backward: A_1 = (A_0/v - q) / p
116.51945397474269

SOA Question 6.17: (A) -30000
37251.49857703497 1.8378124241073728 20269.478042694187
[ Term Insurance: A_0^1:2 ]
    forward: A_0 = qv + pvA_1
    endowment insurance - pure endowment = A_1^1:1
    pure endowment 1_E_1 = 1_p_1 * v^1
[ Temporary Annuity: a_0:2 ]
    forward: a_0:2 = 1 + E_0 a_1:1
    pure endowment 1_E_0 = 1_p_0 * v^1
    1-year discrete annuity: a_x:1 = 1
[ Pure Endowment: 2_E_0 ]
    chain Rule: 2_E_0 = E_0 * 1_E_1
    pure endowment 1_E_1 = 1_p_1 * v^1
    pure endowment 1_E_0 = 1_p_0 * v^1
-30107.42633581125

```

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## 16.1 1/mthly insurance

$$K_x^{(m)} = \frac{1}{m} \lfloor mT_x \rfloor$$

$$A_x^{(m)} = \sum_{k=0}^{\infty} v^{\frac{k+1}{m}} \frac{k}{m} \lfloor \frac{1}{m} \rfloor q_x$$

$$A_{x:\overline{t}|}^{(m)} = \sum_{k=0}^{mt-1} v^{\frac{k+1}{m}} \frac{k}{m} \lfloor \frac{1}{m} \rfloor q_x$$

## 16.2 Annuity twin

$$A_x^{(m)} = 1 - d^{(m)} \ddot{a}_x^{(m)} \iff \ddot{a}_x^{(m)} = \frac{1 - A_x^{(m)}}{d^{(m)}}$$

$$A_{x:\overline{t}|}^{(m)} = 1 - d^{(m)} \ddot{a}_{x:\overline{t}|}^{(m)} \iff \ddot{a}_{x:\overline{t}|}^{(m)} = \frac{1 - A_{x:\overline{t}|}^{(m)}}{d^{(m)}}$$

## 16.3 Immediate annuity

$$a_x^{(m)} = \ddot{a}_x^{(m)} - \frac{1}{m}$$

$$a_{x:\overline{t}|}^{(m)} = \ddot{a}_{x:\overline{t}|}^{(m)} - \frac{1}{m} (1 - {}_tE_x)$$

```
"""Mthly-paid insurance and annuities
```

```
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```

```
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```

```
"""
```

```
from mathlc.mthly import Mthly
```

```
Mthly.doc()
```

```
v_m(**args):
    Return discount rate after k mthly periods

p_m(**args):
    Return survival rate over k mthly periods

q_m(**args):
    Return mortality rate over k mthly periods

Z_m(**args):
    Return PV of insurance r.v. Z and probability by mthly period

E_x(**args):
    Return pure endowment factor

A_x(**args):
    Compute insurance factor with mthly benefits

whole_life_insurance(**args):
    Whole life insurance: A_x

term_insurance(**args):
    Term life insurance: A_x:t^1

deferred_insurance(**args):
    Deferred insurance n|_A_x:t^1 = discounted whole life

endowment_insurance(**args):
    Endowment insurance: A_x:t = term insurance + pure endowment

immediate_annuity(**args):
    Immediate mthly annuity

annuity_twin(**args):
    Return annuity twin of mthly insurance

annuity_variance(**args):
    Variance of mthly annuity from mthly insurance moments

whole_life_annuity(**args):
    Whole life mthly annuity: a_x

temporary_annuity(**args):
    Temporary mthly life annuity: a_x:t

deferred_annuity(**args):
    Deferred mthly life annuity n|t_a_x = n+t_a_x - n_a_x
```



## 16.4 Examples

```
import math
import pandas as pd
from mathlc.premiums import Premiums
from mathlc.lifetable import LifeTable

print("SOA Question 6.4: (E) 1893.9")
mthly = Mthly(m=12, life=Premiums(interest=dict(i=0.06)))
A1, A2 = 0.4075, 0.2105
mean = mthly.annuity_twin(A1)*15*12
var = mthly.annuity_variance(A1=A1, A2=A2, b=15 * 12)
S = Premiums.portfolio_percentile(mean=mean, variance=var, prob=.9, N=200)
print(S / 200)
print()

print("SOA Question 4.2: (D) 0.18")
life = LifeTable(q={0: .16, 1: .23}, interest=dict(i_m=.18, m=2),
                 udd=False).fill()
mthly = Mthly(m=2, life=life)
Z = mthly.Z_m(0, t=2, benefit=lambda x,t: 300000 + t*30000*2)
print(Z)
print(Z[Z['Z'] >= 277000].iloc[:, -1].sum())
print()
```

```
SOA Question 6.4: (E) 1893.9
1893.912859650868
```

```
SOA Question 4.2: (D) 0.18
              Z          p
m
1  275229.357798  0.083485
2  277754.397778  0.076515
3  277986.052822  0.102903
4  276285.832315  0.090297
0.17941813045022975
```



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## 17.1 Annuities

$$\alpha(m) = \frac{id}{i^{(m)} d^{(m)}}$$

$$\beta(m) = \frac{i - i^{(m)}}{i^{(m)} d^{(m)}}$$

$$\ddot{a}_x^{(m)} = \alpha(m) \ddot{a}_x - \beta(m)$$

$$\ddot{a}_{x:\overline{n}|}^{(m)} = \alpha(m) \ddot{a}_{x:\overline{n}|} - \beta(m)(1 - {}_tE_x)$$

$${}_u|\ddot{a}_x^{(m)} = \alpha(m) {}_u|\ddot{a}_x - \beta(m) {}_uE_x$$

## 17.2 Insurance

**Discrete insurance:**

$$\text{Whole life: } A_x^{(m)} = \frac{i}{i^{(m)}} A_x$$

$$\text{Temporary: } A_{x:\overline{t}|}^{1(m)} = \frac{i}{i^{(m)}} A_{x:\overline{t}|}^1$$

$$\text{Endowment: } A_{x:\overline{t}|}^{(m)} = \frac{i}{i^{(m)}} A_{x:\overline{t}|}^1 + {}_tE_x$$

$$\text{Deferred: } {}_u|A_x^{(m)} = {}_uE_x \frac{i}{i^{(m)}} A_{x+u}$$

**Continuous insurance:**

$$\text{Whole life: } \bar{A}_x = \frac{i}{\delta} A_x$$

$$\text{Temporary: } \bar{A}_{x:\overline{t}|}^1 = \frac{i}{\delta} A_{x:\overline{t}|}^1$$

$$\text{Endowment: } \bar{A}_{x:\overline{t}|} = \frac{i}{\delta} A_{x:\overline{t}|}^1 + {}_tE_x$$

$$\text{Deferred: } {}_u|\bar{A}_x = {}_uE_x \frac{i}{\delta} A_{x+u}$$

Double the force of interest:

$${}^2\overline{A}_x = \frac{i^2 - 2i}{2\delta} {}^2A_x$$

$${}^2A_x^{(m)} = \frac{i^2 - 2i}{(i^{(m)})^2 - 2i^{(m)}} {}^2A_x$$

```
"""Fractional ages
```

```
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```

```
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```

```
"""
```

```
from mathlc.udd import UDD
```

```
UDD.doc()
```

```
alpha(**args):
    1/Mthly UDD interest rate beta function

beta(**args):
    1/Mthly UDD interest rate alpha function

whole_life_insurance(**args):
    1/Mthly UDD Whole life insurance: A_x

term_insurance(**args):
    1/Mthly UDD Term insurance: A_x:t

deferred_insurance(**args):
    Deferred insurance n|_A_x:t^1 = discounted whole life

whole_life_annuity(**args):
    1/Mthly UDD Whole life annuity: a_x

temporary_annuity(**args):
    1/Mthly UDD Temporary life annuity: a_x:t

deferred_annuity(**args):
    1/Mthly UDD Deferred life annuity n|t_a_x = n+t_a_x - n_a_x

frame(**args):
    Construct 1/mthly UDD interest function values
```

## 17.3 Examples

```
from mathlc.sult import SULT
from mathlc.recursion import Recursion

print("SOA Question 7.9: (A) 38100")
sult = SULT(udd=True)
x, n, t = 45, 20, 10
a = UDD(m=12, life=sult).temporary_annuity(x+10, t=n-10)
print(a)
A = UDD(m=0, life=sult).endowment_insurance(x+10, t=n-10)
```

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```

print(A)
print(A*100000 - a*12*253)
policy = sult.Policy(premium=253*12, endowment=100000, benefit=100000)
print(sult.gross_future_loss(A=A, a=a, policy=policy))
print()

print("SOA Question 6.49: (C) 86")
sult = SULT(udd=True)
a = UDD(m=12, life=sult).temporary_annuity(40, t=20)
A = sult.whole_life_insurance(40, discrete=False)
P = sult.gross_premium(a=a, A=A, benefit=100000, initial_policy=200,
                      renewal_premium=0.04, initial_premium=0.04)

print(P/12)
print()

print("SOA Question 6.38: (B) 11.3")
x, n = 0, 10
life = Recursion(interest=dict(i=0.05))
life.set_A(0.192, x=x, t=n, endowment=1, discrete=False)
life.set_E(0.172, x=x, t=n)
a = life.temporary_annuity(x, t=n, discrete=False)
print(a)

def fun(a):      # solve for discrete annuity, given continuous
    life = Recursion(interest=dict(i=0.05))
    life.set_a(a, x=x, t=n).set_E(0.172, x=x, t=n)
    return UDD(m=0, life=life).temporary_annuity(x, t=n)
a = life.solve(fun, target=a, guess=a) # discrete annuity
P = life.gross_premium(a=a, A=0.192, benefit=1000)
print(P)
print()

print("SOA Question 6.32: (C) 550")
x = 0
life = Recursion(interest=dict(i=0.05)).set_a(9.19, x=x)
benefits = UDD(m=0, life=life).whole_life_insurance(x)
payments = UDD(m=12, life=life).whole_life_annuity(x)
print(benefits, payments)
print(life.gross_premium(a=payments, A=benefits, benefit=100000)/12)
print()

print("SOA Question 6.22: (C) 102")
life = SULT(udd=True)
a = UDD(m=12, life=life).temporary_annuity(45, t=20)
A = UDD(m=0, life=life).whole_life_insurance(45)
print(life.gross_premium(A=A, a=a, benefit=100000)/12)
print()

print("Interest Functiona at i=0.05")
print("-----")
print(UDD.frame())
print()
UDD.frame()

```

SOA Question 7.9: (A) 38100

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```

7.831075686716718
0.6187476755196442
38099.62176709247
38099.62176709246

```

```

SOA Question 6.49: (C) 86
85.99177833261696

```

```

SOA Question 6.38: (B) 11.3
[ Temporary Annuity: a_0:10 ]
    Annuity twin: a = (1 - A) / d
16.560714925944584
11.308644185253657

```

```

SOA Question 6.32: (C) 550
0.5763261529803323 8.72530251348809
550.4356936711871

```

```

SOA Question 6.22: (C) 102
102.40668704849178

```

```

Interest Functiona at i=0.05
-----

```

	i (m)	d (m)	i/i (m)	d/d (m)	alpha (m)	beta (m)
1	0.05000	0.04762	1.00000	1.00000	1.00000	0.00000
2	0.04939	0.04820	1.01235	0.98795	1.00015	0.25617
4	0.04909	0.04849	1.01856	0.98196	1.00019	0.38272
12	0.04889	0.04869	1.02271	0.97798	1.00020	0.46651
0	0.04879	0.04879	1.02480	0.97600	1.00020	0.50823

	i (m)	d (m)	i/i (m)	d/d (m)	alpha (m)	beta (m)
1	0.05000	0.04762	1.00000	1.00000	1.00000	0.00000
2	0.04939	0.04820	1.01235	0.98795	1.00015	0.25617
4	0.04909	0.04849	1.01856	0.98196	1.00019	0.38272
12	0.04889	0.04869	1.02271	0.97798	1.00020	0.46651
0	0.04879	0.04879	1.02480	0.97600	1.00020	0.50823

## WOOLHOUSE MTHLY

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### 18.1 Annuities

**Whole life annuity:**

$$\ddot{a}_x^{(m)} = \ddot{a}_x - \frac{m-1}{2m} - \frac{m^2-1}{12m^2}(\mu_x + \delta)$$

**Temporary annuity:**

$$\ddot{a}_{x:t|}^{(m)} = \ddot{a}_x^{(m)} - {}_tE_x \ddot{a}_{x+t}^{(m)}$$

- Approximate  $\mu_x \approx -\frac{1}{2}(\ln p_{x-1} + \ln p_x)$

```
"""Mthly with Woolhouse approximation
```

```
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```

```
MIT License
```

```
"""
```

```
from mathlc.woolhouse import Woolhouse
```

```
Woolhouse.doc()
```

```
v_m(**args):
    Return discount rate after k mthly periods

p_m(**args):
    Return survival rate over k mthly periods

q_m(**args):
    Return mortality rate over k mthly periods

Z_m(**args):
    Return PV of insurance r.v. Z and probability by mthly period

E_x(**args):
    Return pure endowment factor

A_x(**args):
    Compute insurance factor with mthly benefits
```

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```

whole_life_insurance(**args):
    1/Mthly Woolhouse Whole life insurance: A_x

term_insurance(**args):
    1/Mthly Woolhouse Term insurance: A_x:t

deferred_insurance(**args):
    1/Mthly Woolhouse Deferred insurance = discounted term or whole life

endowment_insurance(**args):
    1/Mthly Woolhouse Term insurance: A_x:t

immediate_annuity(**args):
    Immediate mthly annuity

annuity_twin(**args):
    Return annuity twin of mthly insurance

annuity_variance(**args):
    Variance of mthly annuity from mthly insurance moments

whole_life_annuity(**args):
    1/Mthly Woolhouse Whole life annuity: a_x

temporary_annuity(**args):
    1/Mthly Woolhouse Temporary life annuity: a_x

deferred_annuity(**args):
    1/Mthly Woolhouse Temporary life annuity: a_x

```

## 18.2 Examples

```

from mathlc.sult import SULT
from mathlc.recursion import Recursion
from mathlc.udd import UDD

print("SOA Question 7.7: (D) 1110")
x = 0
life = Recursion(interest=dict(i=0.05)).set_A(0.4, x=x+10)
a = Woolhouse(m=12, life=life).whole_life_annuity(x+10)
print(a)
policy = life.Policy(premium=0, benefit=10000, renewal_policy=100)
V = life.gross_future_loss(A=0.4, policy=policy.future)
print(V)
policy = life.Policy(premium=30*12, renewal_premium=0.05)
V1 = life.gross_future_loss(a=a, policy=policy.future)
print(V, V1, V+V1)
print()

print("SOA Question 6.25: (C) 12330")
life = SULT()

```

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```

woolhouse = Woolhouse(m=12, life=life)
benefits = woolhouse.deferred_annuity(55, u=10, b=1000 * 12)
expenses = life.whole_life_annuity(55, b=300)
payments = life.temporary_annuity(55, t=10)
print(benefits + expenses, payments)
def fun(P):
    return life.gross_future_loss(A=benefits + expenses, a=payments,
                                   policy=life.Policy(premium=P))
P = life.solve(fun, target=-800, guess=[12110, 12550])
print(P)
print()

print("SOA Question 6.15: (B) 1.002")
life = Recursion(interest=dict(i=0.05)).set_a(3.4611, x=0)
A = life.insurance_twin(3.4611)
udd = UDD(m=4, life=life)
a1 = udd.whole_life_annuity(x=x)
woolhouse = Woolhouse(m=4, life=life)
a2 = woolhouse.whole_life_annuity(x=x)
print(life.gross_premium(a=a1, A=A)/life.gross_premium(a=a2, A=A))
print()

print("SOA Question 5.7: (C) 17376.7")
life = Recursion(interest=dict(i=0.04))
life.set_A(0.188, x=35)
life.set_A(0.498, x=65)
life.set_p(0.883, x=35, t=30)
mthly = Woolhouse(m=2, life=life, three_term=False)
print(mthly.temporary_annuity(35, t=30))
print(1000 * mthly.temporary_annuity(35, t=30))
print()

```

```

SOA Question 7.7: (D) 1110
12.141666666666666
5260.0
5260.0 -4152.028174603174 1107.9718253968258

```

```

SOA Question 6.25: (C) 12330
98042.52569470297 8.019169307712845
12325.781125438532

```

```

SOA Question 6.15: (B) 1.002
1.0022973504113772

```

```

SOA Question 5.7: (C) 17376.7
[ Pure Endowment: 30_E_35 ]
    pure endowment 30_E_35 = 30_p_35 * v^30
17.37671459632958
[ Pure Endowment: 30_E_35 ]
    pure endowment 30_E_35 = 30_p_35 * v^30
17376.71459632958

```



## ADJUST MORTALITY

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### 19.1 Extra mortality risk

1. Add constant to force of mortality:  $\mu_{x+t} + k \Rightarrow {}_t p_x^* = {}_t p_x e^{-kt}$
2. Multiply force of mortality by constant:  $\mu_{x+t} \cdot k \Rightarrow {}_t p_x^* = ({}_t p_x)^k$
3. Multiply mortality rate by a constant:  $q_x \rightarrow q_x \cdot k$
4. Age rating – add years to age:  $(x) \rightarrow (x + k)$

```
"""Adjust Mortality
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"""
from mathlc.adjustmortality import Adjust
Adjust.doc()
```

```
q_x(**args):
    Add constant to mortality rate or age rating

p_x(**args):
    Adjust force of mortality by adding or multiplying a constant
```

### 19.2 Examples

```
from mathlc.selectlife import Select
from mathlc.sult import SULT

print("SOA Question 5.5: (A) 1699.6")
life = SULT()
adjust = Adjust(life=life)
q = adjust(extra=0.05, adjust=Adjust.ADD_FORCE) ['q']
select = Select(n=1) \
    .set_select(column=0, select_age=True, q=q) \
```

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```

        .set_select(column=1, select_age=False, a=life['a']).fill()
print(100*select['a'][45][0])
print()

print("SOA Question 4.19:  (B) 59050")
life = SULT()
adjust = Adjust(life=life)
q = adjust(extra=0.8, adjust=Adjust.MULTIPLY_RATE)['q']
select = Select(n=1)\
        .set_select(column=0, select_age=True, q=q)\
        .set_select(column=1, select_age=False, q=life['q']).fill()
print(100000*select.whole_life_insurance(80, s=0))
print()

print("Other usage examples")
life = SULT()
adjust = Adjust(life=life)(extra=0.05, adjust=Adjust.ADD_FORCE)
print(life.p_x(45), adjust.p_x(45))

```

```

SOA Question 5.5: (A) 1699.6
1699.6076593190103

SOA Question 4.19:  (B) 59050
59050.59973285648

Other usage examples
0.9992288829941123 0.9504959153149807

```

## SOLUTIONS

```
"""Solutions to SOA sample questions
```

```
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```

```
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```

```
"""
```

```
import math
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np
from typing import Union, Optional
from mathlc.faml import FAML
from mathlc.survival import Survival
from mathlc.lifetime import Lifetime
from mathlc.insurance import Insurance
from mathlc.annuity import Annuity
from mathlc.premiums import Premiums
from mathlc.policyvalues import PolicyValues
from mathlc.reserves import Reserves
from mathlc.recursion import Recursion
from mathlc.lifetable import LifeTable
from mathlc.sult import SULT
from mathlc.selectlife import Select
from mathlc.constantforce import ConstantForce
from mathlc.adjustmortality import Adjust
from mathlc.mthly import Mthly
from mathlc.udd import UDD
from mathlc.woolhouse import Woolhouse
```

```
class SOA:
```

```
    """To keep score of solutions correct"""
```

```
    def __init__(self, terminate: bool = False):
        self.score = {}
        self.terminate = terminate
```

```
    def __call__(self, solution: Union[float, str], answer: Union[float, str],
                 msg: str, rel_tol: float = 0.01):
        """Grade this question, and accumulate score"""
        section, question = str(msg).split('.')
        msg = "SOA Question " + str(msg) + ":"
        if isinstance(solution, str) or isinstance(answer, str):
            correct = (solution == answer)
```

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```

else:
    correct = math.isclose(solution, answer, rel_tol=rel_tol)
    print(msg, '[', solution, ']', answer, '*' * 10 * (1-correct))
    if section not in self.score:
        self.score[section] = {}
    self.score[section][question] = correct
    if self.terminate:
        assert correct, msg

def summary(self):
    """Return final score and by section"""
    score = {int(k): [len(v), sum(v.values())] for k, v in self.score.items()}
    out = pd.DataFrame.from_dict(score, orient='index',
                                columns=['num', 'correct'])
    out.loc[0] = [sum(out['num']), sum(out['correct'])]
    return out.sort_index()

soa = SOA(terminate=False)

```

## 20.1 Tables

```

print("Interest Functiona at i=0.05")
UDD.frame()

```

Interest Functiona at i=0.05

	i (m)	d(m)	i/i (m)	d/d(m)	alpha (m)	beta (m)
1	0.05000	0.04762	1.00000	1.00000	1.00000	0.00000
2	0.04939	0.04820	1.01235	0.98795	1.00015	0.25617
4	0.04909	0.04849	1.01856	0.98196	1.00019	0.38272
12	0.04889	0.04869	1.02271	0.97798	1.00020	0.46651
0	0.04879	0.04879	1.02480	0.97600	1.00020	0.50823

```

print("Values of z for selected values of Pr(Z<=z)")
print(FAML.frame().to_string(float_format=lambda x: f"{x:.3f}"))

```

Values of z for selected values of Pr(Z<=z)

z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z<=z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995

```

print("Standard Ultimate Life Table at i=0.05")
SULT().frame()

```

Standard Ultimate Life Table at i=0.05

	l_x	q_x	a_x	A_x	2A_x	a_x:10	A_x:10	a_x:20	\
20	100000.0	0.000250	19.9664	0.04922	0.00580	8.0991	0.61433	13.0559	
21	99975.0	0.000253	19.9197	0.05144	0.00614	8.0990	0.61433	13.0551	

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```

22    99949.7  0.000257  19.8707  0.05378  0.00652  8.0988  0.61434  13.0541
23    99924.0  0.000262  19.8193  0.05622  0.00694  8.0986  0.61435  13.0531
24    99897.8  0.000267  19.7655  0.05879  0.00739  8.0983  0.61437  13.0519
..      ...      ...      ...      ...      ...      ...      ...
96    17501.8  0.192887  3.5597  0.83049  0.69991  3.5356  0.83164  3.5597
97    14125.9  0.214030  3.3300  0.84143  0.71708  3.3159  0.84210  3.3300
98    11102.5  0.237134  3.1127  0.85177  0.73356  3.1050  0.85214  3.1127
99     8469.7  0.262294  2.9079  0.86153  0.74930  2.9039  0.86172  2.9079
100    6248.2  0.289584  2.7156  0.87068  0.76427  2.7137  0.87078  2.7156

      A_x:20    5_E_x    10_E_x    20_E_x
20    0.37829  0.78252  0.61224  0.37440
21    0.37833  0.78250  0.61220  0.37429
22    0.37837  0.78248  0.61215  0.37417
23    0.37842  0.78245  0.61210  0.37404
24    0.37848  0.78243  0.61205  0.37390
..      ...      ...      ...      ...
96    0.83049  0.19872  0.01330  0.00000
97    0.84143  0.16765  0.00827  0.00000
98    0.85177  0.13850  0.00485  0.00000
99    0.86153  0.11173  0.00266  0.00000
100    0.87068  0.08777  0.00136  0.00000

[81 rows x 12 columns]

```

## 20.2 2 Survival models

SOA Question 2.1: (B) 2.5

```

def fun(omega): # Solve first for omega, given mu_65 = 1/180
    return Lifetime(l=lambdax,s:(1 - (x+s)/omega)**0.25).mu_x(65)
omega = int(Lifetime.solve(fun, target=1/180, guess=(106, 126)))
life = Lifetime(l=lambdax,s:(1 - (x+s)/omega)**0.25, maxage=omega)
soa(2.5, life.e_x(106, curtate=True), 2.1)

```

SOA Question 2.1: [ 2.5 ] 2.4786080555423604

SOA Question 2.2: (D) 400

```

p1 = (1. - 0.02) * (1. - 0.01) # 2_p_x if vaccine given
p2 = (1. - 0.02) * (1. - 0.02) # 2_p_x if vaccine not given
v = math.sqrt(FAML.conditional_variance(p=.2, p1=p1, p2=p2, N=100000))
soa(400, v, 2.2)

```

SOA Question 2.2: [ 400 ] 396.5914603215815

SOA Question 2.3: (A) 0.0483

```

B, c = 0.00027, 1.1
life = Survival(S=lambdax,s,t: (math.exp(-B * c**(x+s)

```

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```

                * (c**t - 1)/math.log(c)))
soa(0.0483, life.f_x(x=50, t=10), 2.3)

```

SOA Question 2.3: [ 0.0483 ] 0.048327399045049846

## SOA Question 2.4: (E) 8.2

```

life = Lifetime(l=lambda x,s: 0. if (x+s) >= 100 else 1 - ((x+s)**2)/10000.)
soa(8.2, life.e_x(75, t=10, curtate=False), 2.4)

```

SOA Question 2.4: [ 8.2 ] 8.20952380952381

## SOA Question 2.5: (B) 37.1

```

life = Recursion().set_e(25, x=60, curtate=True)
life.set_q(0.2, x=40, t=20).set_q(0.003, x=40)
def fun(e): # solve e_40 from math1c.e_40:20 = e_40 - 20_p_40 e_60
    return life.set_e(e, x=40, curtate=True).e_x(x=40, t=20, curtate=True)
life.set_e(life.solve(fun, target=18, guess=[36, 41]), x=40, curtate=True)
soa(37.1, life.e_x(41, curtate=True), 2.5)

```

```

[ Lifetime: e_41 ]
    backward e_41 = e_41:1 + p_41 e_42
    shortcut 1-year curtate e_40:1
SOA Question 2.5: [ 37.1 ] 37.11434302908726

```

## SOA Question 2.6: (C) 13.3

```

life = Survival(l=lambda x,s: (1 - (x+s)/60)**(1/3))
soa(13.3, 1000*life.mu_x(35), 2.6)

```

SOA Question 2.6: [ 13.3 ] 13.340451278922776

## SOA Question 2.7: (B) 0.1477

```

life = Survival(l=lambda x,s:
                (1-((x+s)/250) if (x+s)<40 else 1-((x+s)/100)**2))
soa(0.1477, life.q_x(30, t=20), 2.7)

```

SOA Question 2.7: [ 0.1477 ] 0.1477272727272727

## SOA Question 2.8: (C) 0.94

```

def fun(p): # Solve first for mu, given start and end proportions
    mu = -math.log(p)
    male = Lifetime(mu=lambda x,s: 1.5 * mu)
    female = Lifetime(mu=lambda x,s: mu)
    return (75 * female.p_x(0, t=20)) / (25 * male.p_x(0, t=20))
soa(0.94, Lifetime.solve(fun, target=85/15, guess=[0.89, 0.99]), 2.8)

```



SOA Question 2.8: [ 0.94 ] 0.9383813306903798

## 20.3 3 Life tables and selection

SOA Question 3.1: (B) 117

```
life = Select(l={60: [80000, 79000, 77000, 74000],
                  61: [78000, 76000, 73000, 70000],
                  62: [75000, 72000, 69000, 67000],
                  63: [71000, 68000, 66000, 65000]})
soa(117, 1000*life.q_r(60, s=0, r=0.75, t=3, u=2), 3.1)
```

SOA Question 3.1: [ 117 ] 116.7192429022082

SOA Question 3.2: (D) 14.7

```
e_curtate = Select.e_curtate(e=15)
life = Select(l={65: [1000, None],
                  66: [955, None]},
              e={65: [e_curtate, None]},
              d={65: [40, None],
                  66: [45, None]}, udd=True).fill()
soa(14.7, life.e_r(66), 3.2)
```

SOA Question 3.2: [ 14.7 ] 14.67801047120419

SOA Question 3.3: (E) 1074

```
life = Select(l={50: [99, 96, 93],
                  51: [97, 93, 89],
                  52: [93, 88, 83],
                  53: [90, 84, 78]})
soa(1074, 10000*life.q_r(51, s=0, r=0.5, t=2.2), 3.3)
```

SOA Question 3.3: [ 1074 ] 1073.684210526316

SOA Question 3.4: (B) 815

```
sult = SULT()
mean = sult.p_x(25, t=95-25)
var = sult.bernoulli(mean, variance=True)
p = sult.portfolio_percentile(N=4000, mean=mean, variance=var, prob=0.1)
soa(815, p, 3.4)
```

SOA Question 3.4: [ 815 ] 815.0943255167722

SOA Question 3.5: (E) 106

```

l = [99999, 88888, 77777, 66666, 55555, 44444, 33333, 22222]
a = LifeTable(l={age:l for age,l in zip(range(60, 68), l)}, udd=True)\
.q_r(60, u=3.4, t=2.5)
b = LifeTable(l={age:l for age,l in zip(range(60, 68), l)}, udd=False)\
.q_r(60, u=3.4, t=2.5)
soa(106, 100000 * (a - b), 3.5)

```

SOA Question 3.5: [ 106 ] 106.16575827938624

#### SOA Question 3.6: (D) 15.85

```

life = Select(q={60: [.09, .11, .13, .15],
                    61: [.1, .12, .14, .16],
                    62: [.11, .13, .15, .17],
                    63: [.12, .14, .16, .18],
                    64: [.13, .15, .17, .19]},
              e={61: [None, None, None, 5.1]}) .fill()
soa(5.85, life.e_x(61), 3.6)

```

SOA Question 3.6: [ 5.85 ] 5.846832

#### SOA Question 3.7: (b) 16.4

```

life = Select(q={50: [.0050, .0063, .0080],
                    51: [.0060, .0073, .0090],
                    52: [.0070, .0083, .0100],
                    53: [.0080, .0093, .0110]}) .fill()
soa(16.4, 1000*life.q_r(50, s=0, r=0.4, t=2.5), 3.7)

```

SOA Question 3.7: [ 16.4 ] 16.420207214428586

#### SOA Question 3.8: (B) 1505

```

sult = SULT()
p1 = sult.p_x(35, t=40)
p2 = sult.p_x(45, t=40)
mean = sult.bernoulli(p1) * 1000 + sult.bernoulli(p2) * 1000
var = (sult.bernoulli(p1, variance=True) * 1000
       + sult.bernoulli(p2, variance=True) * 1000)
soa(1505, sult.portfolio_percentile(mean=mean, variance=var, prob=.95), 3.8)

```

SOA Question 3.8: [ 1505 ] 1504.8328375406456

#### SOA Question 3.9: (E) 3850

```

sult = SULT()
p1 = sult.p_x(20, t=25)
p2 = sult.p_x(45, t=25)
mean = sult.bernoulli(p1) * 2000 + sult.bernoulli(p2) * 2000
var = (sult.bernoulli(p1, variance=True) * 2000
       + sult.bernoulli(p2, variance=True) * 2000)
soa(3850, sult.portfolio_percentile(mean=mean, variance=var, prob=.99), 3.9)

```

SOA Question 3.9: [ 3850 ] 3850.144345130047

SOA Question 3.10: (C) 0.86

```
interest = FAML.Interest(v=0.75)
L = 35*interest.annuity(t=4, due=False) + 75*interest.v_t(t=5)
interest = FAML.Interest(v=0.5)
R = 15*interest.annuity(t=4, due=False) + 25*interest.v_t(t=5)
soa(0.86, L / (L + R), "3.10")
```

SOA Question 3.10: [ 0.86 ] 0.8578442833761983

SOA Question 3.11: (B) 0.03

```
life = LifeTable(q={50//2: .02, 52//2: .04}, udd=True).fill()
soa(0.03, life.q_r(50//2, t=2.5/2), 3.11)
```

SOA Question 3.11: [ 0.03 ] 0.0298

SOA Question 3.12: (C) 0.055

```
life = Select(l={60: [10000, 9600, 8640, 7771],
                  61: [8654, 8135, 6996, 5737],
                  62: [7119, 6549, 5501, 4016],
                  63: [5760, 4954, 3765, 2410]}, udd=False).fill()
soa(0.055, life.q_r(60, s=1, t=3.5)-life.q_r(61, s=0, t=3.5), 3.12)
```

SOA Question 3.12: [ 0.055 ] 0.05465655938591829

SOA Question 3.13: (B) 1.6

```
life = Select(l={55: [10000, 9493, 8533, 7664],
                  56: [8547, 8028, 6889, 5630],
                  57: [7011, 6443, 5395, 3904],
                  58: [5853, 4846, 3548, 2210]},
              e={57: [None, None, None, 1]}).fill()
soa(1.6, life.e_r(58, s=2), 3.13)
```

SOA Question 3.13: [ 1.6 ] 1.6003382187147688

SOA Question 3.14: (C) 0.345

```
life = LifeTable(l={90: 1000, 93: 825},
                 d={97: 72},
                 p={96: .2},
                 q={95: .4, 97: 1}, udd=True).fill()
soa(0.345, life.q_r(90, u=93-90, t=95.5 - 93), 3.14)
```

SOA Question 3.14: [ 0.345 ] 0.345

## 20.4 4 Insurance benefits

SOA Question 4.1: (A) 0.27212

```
life = Recursion(interest=dict(i=0.03))
life.set_A(0.36987, x=40).set_A(0.62567, x=60)
life.set_E(0.51276, x=40, t=20).set_E(0.17878, x=60, t=20)
Z2 = 0.24954
A = (2 * life.term_insurance(40, t=20)
      + life.deferred_insurance(40, u=20))
soa(0.27212, math.sqrt(life.insurance_variance(A2=Z2, A1=A)), 4.1)
```

SOA Question 4.1: [ 0.27212 ] 0.2721117749374753

SOA Question 4.2: (D) 0.18

```
life = LifeTable(q={0: .16, 1: .23},
                 interest=dict(i_m=.18, m=2),
                 udd=False).fill()
mthly = Mthly(m=2, life=life)
Z = mthly.Z_m(0, t=2, benefit=lambda x,t: 300000 + t*30000*2)
soa(0.18, Z[Z['Z'] >= 277000].iloc[:, -1].sum(), 4.2)
```

SOA Question 4.2: [ 0.18 ] 0.17941813045022975

SOA Question 4.3: (D) 0.878 – multi recursion on endowment insurance

```
life = Recursion(interest=dict(i=0.05)).set_q(0.01, x=60)
def fun(q): # solve for q_61
    return life.set_q(q, x=61).endowment_insurance(60, t=3)
q = life.solve(fun, target=0.86545, guess=0.01)
life.set_q(q, x=61).set_interest(i=0.045)
A = life.endowment_insurance(60, t=3)
soa(0.878, A, "4.3")
```

```
[ Endowment Insurance: A_60:3 ]
    forward: A_60 = qv + pvA_61
    forward: A_61 = qv + pvA_62
SOA Question 4.3: [ 0.878 ] 0.8777667236003878
```

SOA Question 4.4 (A) 0.036

```
life = Insurance(f=lambda *x: 0.025,
                 maxage=40+40,
                 interest=dict(v_t=lambda t: (1 + .2*t)**(-2)))
benefit = lambda x,t: 1 + .2 * t
A1 = life.A_x(40, benefit=benefit, discrete=False)
A2 = life.A_x(40, moment=2, benefit=benefit, discrete=False)
soa(0.036, life.insurance_variance(A2=A2, A1=A1), 4.4)
```

SOA Question 4.4: [ 0.036 ] 0.03567680106032681

SOA Question 4.5: (C) 35200

```
sult = SULT(interest=dict(delta=0.05))
Z = 100000 * sult.Z_from_prob(45, 0.95)
soa(35200, Z, 4.5)
```

SOA Question 4.5: [ 35200 ] 34993.774911115455

#### SOA Question 4.6: (B) 29.85

```
sult = SULT()
life = LifeTable(interest=dict(i=0.05),
                 q={70+k: .95**k * sult.q_x(70+k) for k in range(3)}).fill()
A = life.term_insurance(70, t=3, b=1000)
soa(29.85, A, 4.6)
```

SOA Question 4.6: [ 29.85 ] 29.84835110355902

#### SOA Question 4.7: (B) 0.06

```
def fun(i):
    life = Recursion(interest=dict(i=i), verbose=False).set_p(0.57, x=0, t=25)
    return 0.1*life.E_x(0, t=25) - life.E_x(0, t=25, moment=life.VARIANCE)
soa(0.06, Recursion.solve(fun, target=0, guess=[0.058, 0.066]), 4.7)
```

SOA Question 4.7: [ 0.06 ] 0.06008023738770262

#### SOA Question 4.8 (C) 191

```
v_t = lambda t: 1.04**(-t) if t < 1 else 1.04**(-1) * 1.05**(-t+1)
life = SULT(interest=dict(v_t=v_t))
soa(191, life.whole_life_insurance(50, b=1000), 4.8)
```

SOA Question 4.8: [ 191 ] 191.1281281882354

#### SOA Question 4.9: (D) 0.5

```
life = Recursion().set_A(0.39, x=35, t=15, endowment=1)\
               .set_A(0.25, x=35, t=15)
E = life.E_x(35, t=15)
life = Recursion().set_A(0.32, x=35)\
               .set_E(E, x=35, t=15)
def fun(A):
    return life.set_A(A, x=50).term_insurance(35, t=15)
A = life.solve(fun, target=0.25, guess=[0.35, 0.55])
soa(0.5, A, 4.9)
```

```
[ Pure Endowment: 15_E_35 ]
    endowment - term insurance = 15_E_35
SOA Question 4.9: [ 0.5 ] 0.5
```

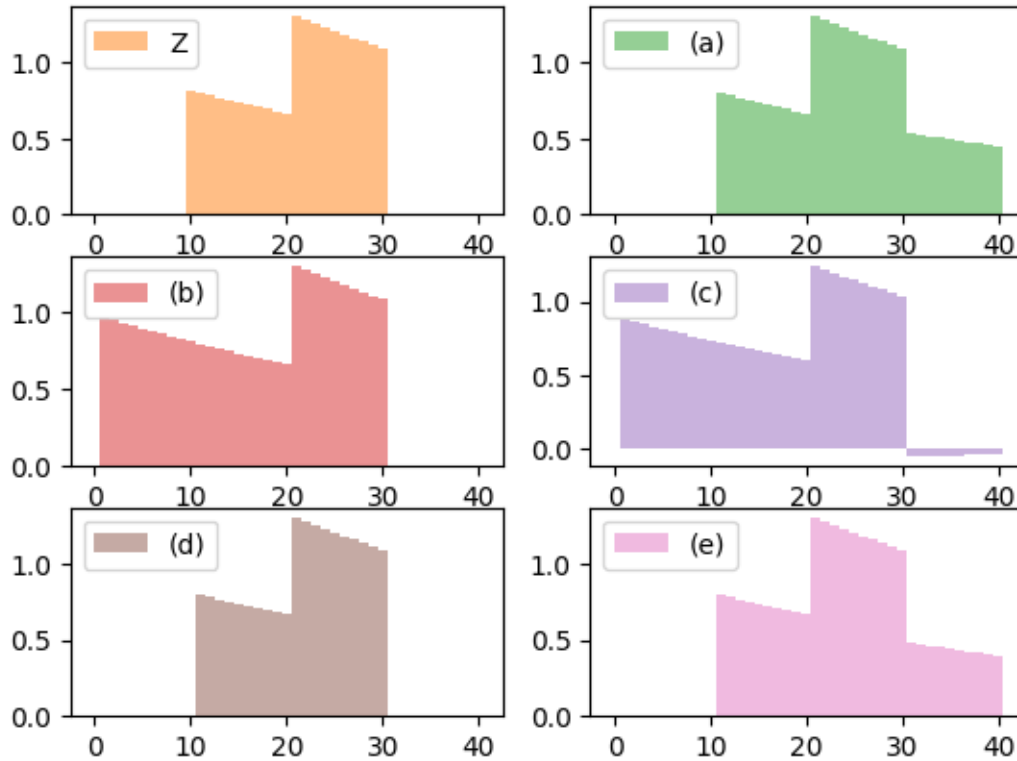
#### SOA Question 4.10: (D)

```

life = Insurance(interest=dict(i=0.01), S=lambda x,s,t: 1, maxage=40)
def fun(x, t):
    if 10 <= t <= 20: return life.interest.v_t(t)
    elif 20 < t <= 30: return 2 * life.interest.v_t(t)
    else: return 0
def A(x, t): #  $Z_{x+k}(t-k)$ 
    return life.interest.v_t(t - x) * (t > x)
x = 0
benefits=[lambda x,t: (life.E_x(x, t=10) * A(x+10, t)
                    + life.E_x(x, t=20) * A(x+20, t)
                    - life.E_x(x, t=30) * A(x+30, t)),
          lambda x,t: (A(x, t)
                    + life.E_x(x, t=20) * A(x+20, t)
                    - 2 * life.E_x(x, t=30) * A(x+30, t)),
          lambda x,t: (life.E_x(x, t=10) * A(x, t)
                    + life.E_x(x, t=20) * A(x+20, t)
                    - 2 * life.E_x(x, t=30) * A(x+30, t)),
          lambda x,t: (life.E_x(x, t=10) * A(x+10, t)
                    + life.E_x(x, t=20) * A(x+20, t)
                    - 2 * life.E_x(x, t=30) * A(x+30, t)),
          lambda x,t: (life.E_x(x, t=10)
                    * (A(x+10, t)
                    + life.E_x(x+10, t=10) * A(x+20, t)
                    - life.E_x(x+20, t=10) * A(x+30, t)))]
fig, ax = plt.subplots(3, 2)
ax = ax.ravel()
for i, b in enumerate([fun] + benefits):
    life.Z_plot(0, benefit=b, ax=ax[i], verbose=False, color=f"C{i+1}")
    ax[i].legend(["(" + "abcde"[i-1] + ")" if i else "Z"])
z = [sum(abs(b(0, t) - fun(0, t)) for t in range(40)) for b in benefits]
soa('D', "ABCDE"[np.argmin(z)], '4.10')

```

SOA Question 4.10: [ D ] D



SOA Question 4.11: (A) 143385

```
A1 = 528/1000 # E[Z1] term insurance
C1 = 0.209    # E[pure_endowment]
C2 = 0.136    # E[pure_endowment^2]
B1 = A1 + C1  # endowment = term + pure_endowment
def fun(A2):
    B2 = A2 + C2 # double force of interest
    return Insurance.insurance_variance(A2=B2, A1=B1)
A2 = Insurance.solve(fun, target=15000/(1000*1000), guess=[143400, 279300])
soa(143385, Insurance.insurance_variance(A2=A2, A1=A1, b=1000), 4.11)
```

SOA Question 4.11: [ 143385 ] 143384.99999999997

SOA Question 4.12: (C) 167

```
cov = FAML.covariance(a=1.65, b=10.75, ab=0) # E[Z1 Z2] = 0 nonoverlapping
soa(167, FAML.variance(a=2, b=1, var_a=46.75, var_b=50.78, cov_ab=cov), 4.12)
```

SOA Question 4.12: [ 167 ] 166.82999999999998

SOA Question 4.13: (C) 350

```
life = Select(q={65: [.08, .10, .12, .14],
                    66: [.09, .11, .13, .15],
                    67: [.10, .12, .14, .16],
                    68: [.11, .13, .15, .17],
```

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```

        69: [.12, .14, .16, .18]}, interest=dict(i=.04)).fill()
soa(350, life.deferred_insurance(65, t=2, u=2, b=2000), 4.13)

```

SOA Question 4.13: [ 350 ] 351.0578236056159

SOA Question 4.14: (E) 390000

```

sult = SULT()
p = sult.p_x(60, t=85-60)
mean = sult.bernoulli(p)
var = sult.bernoulli(p, variance=True)
F = sult.portfolio_percentile(mean=mean, variance=var, prob=.86, N=400)
soa(390000, F * 5000 * sult.interest.v_t(85-60), 4.14)

```

SOA Question 4.14: [ 390000 ] 389322.86778416135

SOA Question 4.15 (E) 0.0833

```

life = Insurance(mu=lambda x: 0.04, interest=dict(delta=0.06))
benefit = lambda x,t: math.exp(0.02*t)
A1 = life.A_x(0, benefit=benefit, discrete=False)
A2 = life.A_x(0, moment=2, benefit=benefit, discrete=False)
soa(0.0833, life.insurance_variance(A2=A2, A1=A1), 4.15)

```

SOA Question 4.15: [ 0.0833 ] 0.08334849338238598

SOA Question 4.16: (D) 0.11

```

q = [.045, .050, .055, .060]
q_ = {50+x: [0.7 * q[x] if x < len(q) else None,
            0.8 * q[x+1] if x+1 < len(q) else None,
            q[x+2] if x+2 < len(q) else None]
      for x in range(4)}
life = Select(q=q_, interest=dict(i=.04)).fill()
soa(0.1116, life.term_insurance(50, t=3), 4.16)

```

SOA Question 4.16: [ 0.1116 ] 0.1115661982248521

SOA Question 4.17: (A) 1126.7

```

sult = SULT()
median = sult.Z_t(48, prob=0.5, discrete=False)
benefit = lambda x,t: 5000 if t < median else 10000
A = sult.A_x(48, benefit=benefit)
soa(1130, A, 4.17)

```

SOA Question 4.17: [ 1130 ] 1126.774772894844

SOA Question 4.18 (A) 81873

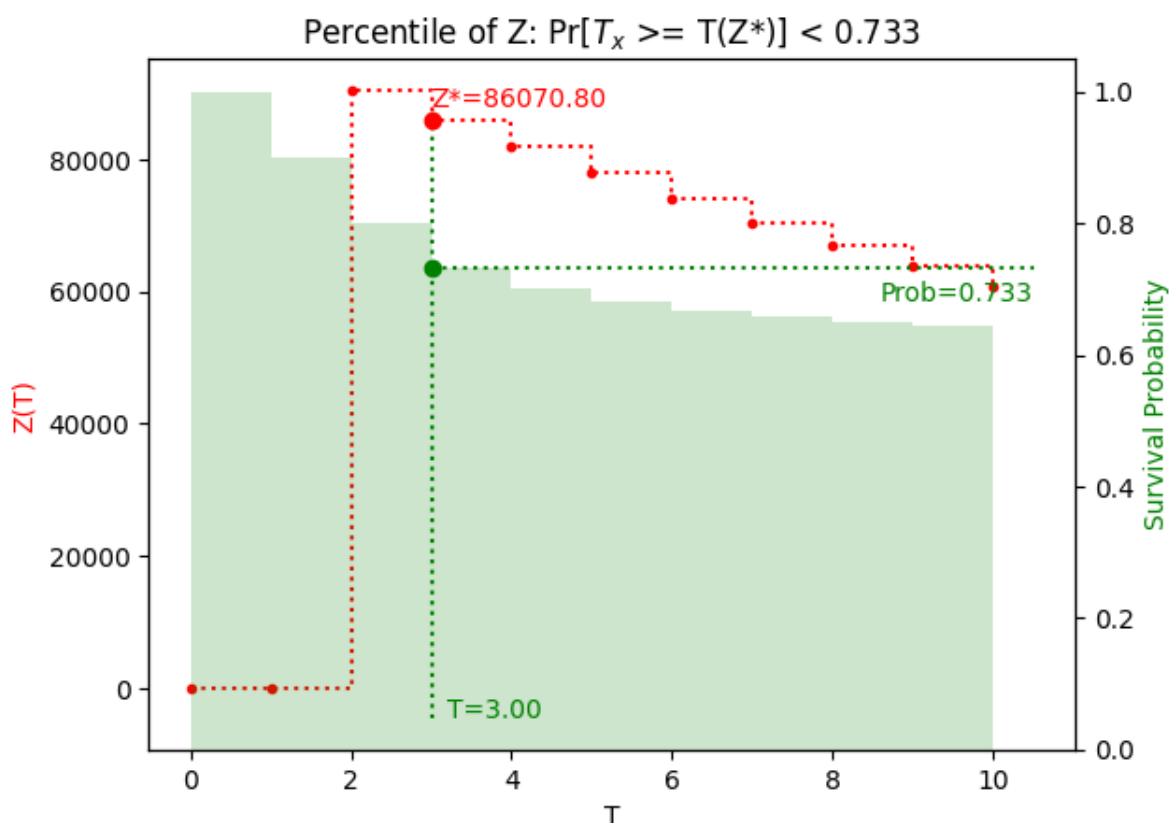


```

life = Insurance(interest=dict(delta=0.05),
                 maxage=10,
                 f=lambda x,s,t: .1 if t < 2 else .4*t**(-2))
benefit = lambda x,t: 0 if t < 2 else 100000
prob = 0.9 - life.q_x(0, t=2)
x, y = life.survival_curve()
T = life.Z_t(0, prob=prob)
life.Z_plot(0, T=T, benefit=benefit, discrete=False, curve=(x,y))
Z = life.Z_from_t(T) * benefit(0, T)
plt.show()
soa(81873, Z, 4.18)

```

SOA Question 4.18: [ 81873 ] 81873.07530779815



SOA Question 4.19: (B) 59050

```

life = SULT()
adjust = Adjust(life=life)
q = adjust(extra=0.8, adjust=Adjust.MULTIPLY_RATE)['q']
select = Select(n=1)\
    .set_select(column=0, select_age=True, q=q)\
    .set_select(column=1, select_age=False, q=life['q']).fill()
soa(59050, 100000*select.whole_life_insurance(80, s=0), 4.19)

```

SOA Question 4.19: [ 59050 ] 59050.59973285648

## 20.5 5 Annuities

SOA Question 5.1: (A) 0.705

```
life = ConstantForce(mu=0.01, interest=dict(delta=0.06))
EY = life.certain_life_annuity(0, u=10, discrete=False)
a = life.p_x(0, t=life.Y_to_t(EY))
soa(0.705, a, 5.1) # 0.705
```

SOA Question 5.1: [ 0.705 ] 0.7053680433746505

SOA Question 5.2: (B) 9.64

```
x, n = 0, 10
life = Recursion(interest=dict(i=0.05))
life.set_A(0.3, x).set_A(0.4, x+n).set_E(0.35, x, t=n)
a = life.immediate_annuity(x, t=n)
soa(9.64, a, 5.2)
```

SOA Question 5.2: [ 9.64 ] 9.639999999999999

SOA Question 5.3: (C) 6.239

```
sult = SULT()
t = 10.5
soa(6.239, t * sult.E_r(40, t=t), 5.3)
```

SOA Question 5.3: [ 6.239 ] 6.23871918627528

SOA Question 5.4: (A) 213.7

```
life = ConstantForce(mu=0.02, interest=dict(delta=0.01))
P = 10000 / life.certain_life_annuity(40, u=life.e_x(40, curtate=False),
                                     discrete=False)
soa(213.7, P, 5.4) # 213.7
```

SOA Question 5.4: [ 213.7 ] 213.74552118275955

SOA Question 5.5: (A) 1699.6

```
life = SULT()
adjust = Adjust(life=life)
q = adjust(extra=0.05, adjust=Adjust.ADD_FORCE)['q']
select = Select(n=1)\
    .set_select(column=0, select_age=True, q=q)\
    .set_select(column=1, select_age=False, a=life['a']).fill()
soa(1700, 100*select['a'][45][0], 5.5)
```

SOA Question 5.5: [ 1700 ] 1699.6076593190103

SOA Question 5.6: (D) 1200

```
life = Annuity(interest=dict(i=0.05))
var = life.annuity_variance(A2=0.22, A1=0.45)
mean = life.annuity_twin(A=0.45)
soa(1200, life.portfolio_percentile(mean, var, prob=.95, N=100), 5.6)
```

SOA Question 5.6: [ 1200 ] 1200.6946732201702

#### SOA Question 5.7: (C)

```
life = Recursion(interest=dict(i=0.04))
life.set_A(0.188, x=35).set_A(0.498, x=65).set_p(0.883, x=35, t=30)
mthly = Woolhouse(m=2, life=life, three_term=False)
soa(17376.7, 1000 * mthly.temporary_annuity(35, t=30), 5.7)
```

```
[ Pure Endowment: 30_E_35 ]
    pure endowment 30_E_35 = 30_p_35 * v^30
SOA Question 5.7: [ 17376.7 ] 17376.71459632958
```

#### SOA Question 5.8: (C) 0.92118

```
sult = SULT()
a = sult.certain_life_annuity(55, u=5)
soa(0.92118, sult.p_x(55, t=math.floor(a)), 5.8)
```

SOA Question 5.8: [ 0.92118 ] 0.9211799771029529

#### SOA Question 5.9: (C) 0.015

```
x, p = 0, 0.9 # set arbitrary p_x = 0.9
life1 = Recursion().set_a(21.854, x=x).set_p(p, x=x)
life2 = Recursion().set_a(22.167, x=x)
def fun(k):
    life2.set_p((1 + k) * p, x=x)
    return life1.whole_life_annuity(x+1) - life2.whole_life_annuity(x+1)
soa(0.015, life2.solve(fun, target=0, guess=[0.005, 0.025]), 5.9)
```

SOA Question 5.9: [ 0.015 ] 0.015009110961925157

## 20.6 6 Premium Calculation

#### SOA Question 6.1: (D) 35.36

```
life = SULT(interest=dict(i=0.03))
soa(35.36, life.net_premium(80, t=2, b=1000, return_premium=True), 6.1)
```

SOA Question 6.1: [ 35.36 ] 35.35922286190033

#### SOA Question 6.2: (E) 3604

```

life = Premiums()
A, IA, a = 0.17094, 0.96728, 6.8865
P = life.gross_premium(a=a, A=A, IA=IA, benefit=100000,
                      initial_premium=0.5, renewal_premium=.05,
                      renewal_policy=200, initial_policy=200)
soa(3604, P, 6.2)

```

SOA Question 6.2: [ 3604 ] 3604.229940320728

### SOA Question 6.3: (C) 0.390

```

life = SULT()
P = life.net_premium(45, u=20, annuity=True)
t = life.Y_to_t(life.whole_life_annuity(65))
p = 1 - life.p_x(65, t=math.floor(t) - 1)
soa(0.39, p, 6.3)

```

SOA Question 6.3: [ 0.39 ] 0.39039071872030084

### SOA Question 6.4: (E) 1890

```

mthly = Mthly(m=12, life=Reserves(interest=dict(i=0.06)))
A1, A2 = 0.4075, 0.2105
mean = mthly.annuity_twin(A1)*15*12
var = mthly.annuity_variance(A1=A1, A2=A2, b=15 * 12)
S = Reserves.portfolio_percentile(mean=mean, variance=var, prob=.9, N=200)
soa(1890, S / 200, 6.4)

```

SOA Question 6.4: [ 1890 ] 1893.912859650868

### SOA Question 6.5: (D) 33

```

life = SULT()
P = life.net_premium(30, b=1000)
def fun(k):
    return (life.Y_x(30, t=k) * P
            - life.Z_x(30, t=k) * 1000)
soa(33, min([k for k in range(20, 40) if fun(k) < 0]), 6.5)

```

SOA Question 6.5: [ 33 ] 33

### SOA Question 6.6: (B) 0.79

```

life = SULT()
P = life.net_premium(62, b=10000)
policy = life.Policy(premium=1.03*P, renewal_policy=5,
                    initial_policy=5, initial_premium=0.05, benefit=10000)
L = life.gross_policy_value(62, policy=policy)
var = life.gross_policy_variance(62, policy=policy)
prob = life.portfolio_cdf(mean=L, variance=var, value=40000, N=600)
soa(.79, prob, 6.6)

```

SOA Question 6.6: [ 0.79 ] 0.7914321142683509

SOA Question 6.7: (C) 2880

```
life=SULT()
a = life.temporary_annuity(40, t=20)
A = life.E_x(40, t=20)
IA = a - life.interest_annuity(t=20) * life.p_x(40, t=20)
soa(2880, life.gross_premium(a=a, A=A, IA=IA, benefit=100000), 6.7)
```

SOA Question 6.7: [ 2880 ] 2880.2463991134578

SOA Question 6.8: (B) 9.5

```
life = SULT()
initial_cost = (50 + 10 * life.deferred_annuity(60, u=1, t=9)
               + 5 * life.deferred_annuity(60, u=10, t=10))
soa(9.5, life.net_premium(60, initial_cost=initial_cost), 6.8)
```

SOA Question 6.8: [ 9.5 ] 9.526003201821927

SOA Question 6.9: (D) 647

```
life = SULT()
a = life.temporary_annuity(50, t=10)
A = life.term_insurance(50, t=20)
initial_cost = 25 * life.deferred_annuity(50, u=10, t=10)
P = life.gross_premium(a=a, A=A, benefit=100000,
                      initial_premium=0.42, renewal_premium=0.12,
                      initial_policy=75 + initial_cost, renewal_policy=25)
soa(647, P, 6.9)
```

SOA Question 6.9: [ 647 ] 646.8608151974504

SOA Question 6.10: (D) 0.91

```
x = 0
life = Recursion(interest=dict(i=0.06)).set_p(0.975, x=x)
a = 152.85/56.05 # solve a_x:3, given net premium and benefit APV

def fun(p): # solve p_x+2, given a_x:3
    return life.set_p(p, x=x+1).temporary_annuity(x, t=3)
life.set_p(life.solve(fun, target=a, guess=0.975), x=x+1)

def fun(p): # finally solve p_x+3, given A_x:3
    return life.set_p(p, x=x+2).term_insurance(x=x, t=3, b=1000)
p = life.solve(fun, target=152.85, guess=0.975)
soa(0.91, p, "6.10")
```

SOA Question 6.10: [ 0.91 ] 0.90973829505257

SOA Question 6.11: (C) 0.041

```

life = Recursion(interest=dict(i=0.04))
life.set_A(0.39788, 51)
life.set_q(0.0048, 50)
A = life.whole_life_insurance(50)
P = life.gross_premium(A=A, a=life.annuity_twin(A=A))
life.set_q(0.048, 50)
A = life.whole_life_insurance(50)
soa(0.041, A - life.annuity_twin(A) * P, 6.11)

```

```

[ Whole Life Insurance: A_50 ]
  forward: A_50 = qv + pvA_51
[ Whole Life Insurance: A_50 ]
  forward: A_50 = qv + pvA_51
SOA Question 6.11: [ 0.041 ] 0.04069206883563675

```

## SOA Question 6.12: (E) 88900

```

life = PolicyValues(interest=dict(i=0.06))
a = 12
A = life.insurance_twin(a)
policy = life.Policy(benefit=1000, settlement_policy=20,
                    initial_policy=10, initial_premium=0.75,
                    renewal_policy=2, renewal_premium=0.1)
policy.premium = life.gross_premium(A=A, a=a, **policy.premium_terms)
L = life.gross_variance_loss(A1=A, A2=0.14, policy=policy)
soa(88900, L, 6.12)

```

```
SOA Question 6.12: [ 88900 ] 88862.59592874818
```

## SOA Question 6.13: (D) -400

```

life = SULT(interest=dict(i=0.05))
A = life.whole_life_insurance(45)
policy = life.Policy(benefit=10000, initial_premium=.8, renewal_premium=.1)
def fun(P): # Solve for premium, given Loss(t=0) = 4953
    return life.L_from_t(t=10.5, policy=policy.set(premium=P))
policy.premium = life.solve(fun, target=4953, guess=100)
L = life.gross_policy_value(45, policy=policy)
soa(-400, L, 6.13)

```

```
SOA Question 6.13: [ -400 ] -400.94447599879277
```

## SOA Question 6.14 (D) 1150

```

life = SULT(interest=dict(i=0.05))
a = (life.temporary_annuity(40, t=10)
     + 0.5 * life.deferred_annuity(40, u=10, t=10))
A = life.whole_life_insurance(40)
P = life.gross_premium(a=a, A=A, benefit=100000)
soa(1150, P, 6.14)

```

```
SOA Question 6.14: [ 1150 ] 1148.5800555155263
```

## SOA Question 6.15: (B) 1.002

```

life = Recursion(interest=dict(i=0.05)).set_a(3.4611, x=0)
A = life.insurance_twin(3.4611)
udd = UDD(m=4, life=life)
a1 = udd.whole_life_annuity(x=x)
woolhouse = Woolhouse(m=4, life=life)
a2 = woolhouse.whole_life_annuity(x=x)
P = life.gross_premium(a=a1, A=A)/life.gross_premium(a=a2, A=A)
soa(1.002, P, 6.15)

```

SOA Question 6.15: [ 1.002 ] 1.0022973504113772

SOA Question 6.16: (A) 2408.6

```

life = Premiums(interest=dict(d=0.05))
A = life.insurance_equivalence(premium=2143, b=100000)
a = life.annuity_equivalence(premium=2143, b=100000)
p = life.gross_premium(A=A, a=a, benefit=100000, settlement_policy=0,
                       initial_policy=250, initial_premium=0.04 + 0.35,
                       renewal_policy=50, renewal_premium=0.04 + 0.02)
soa(2410, p, 6.16)

```

SOA Question 6.16: [ 2410 ] 2408.575206281868

SOA Question 6.17: (A) -30000

```

x = 0
life = ConstantForce(mu=0.1, interest=dict(i=0.08))
A = life.endowment_insurance(x, t=2, b=100000, endowment=30000)
a = life.temporary_annuity(x, t=2)
P = life.gross_premium(a=a, A=A)
life1 = Recursion(interest=dict(i=0.08))
life1.set_q(life.q_x(x, t=1) * 1.5, x=x, t=1)
life1.set_q(life.q_x(x+1, t=1) * 1.5, x=x+1, t=1)
policy = life1.Policy(premium=P*2, benefit=100000, endowment=30000)
L = life1.gross_policy_value(x, t=0, n=2, policy=policy)
soa(-30000, L, 6.17)

```

```

[ Term Insurance: A_0^1:2 ]
  forward: A_0 = qv + pvA_1
  endowment insurance - pure endowment = A_1^1:1
  pure endowment 1_E_1 = 1_p_1 * v^1
[ Temporary Annuity: a_0:2 ]
  forward: a_0:2 = 1 + E_0 a_1:1
  pure endowment 1_E_0 = 1_p_0 * v^1
  1-year discrete annuity: a_x:1 = 1
[ Pure Endowment: 2_E_0 ]
  chain Rule: 2_E_0 = E_0 * 1_E_1
  pure endowment 1_E_1 = 1_p_1 * v^1
  pure endowment 1_E_0 = 1_p_0 * v^1
SOA Question 6.17: [ -30000 ] -30107.42633581125

```

SOA Question 6.18: (D) 166400

```

life = SULT(interest=dict(i=0.05))
def fun(P):
    A = (life.term_insurance(40, t=20, b=P)
         + life.deferred_annuity(40, u=20, b=30000))
    return life.gross_premium(a=1, A=A) - P
P = life.solve(fun, target=0, guess=[162000, 168800])
soa(166400, P, 6.18)

```

SOA Question 6.18: [ 166400 ] 166362.83871487685

SOA Question 6.19: (B) 0.033

```

life = SULT()
policy = life.Policy(initial_policy=.2, renewal_policy=.01)
a = life.whole_life_annuity(50)
A = life.whole_life_insurance(50)
policy.premium = life.gross_premium(A=A, a=a, **policy.premium_terms)
L = life.gross_policy_variance(50, policy=policy)
soa(0.033, L, 6.19)

```

SOA Question 6.19: [ 0.033 ] 0.03283273381910885

SOA Question 6.20: (B) 459

```

life = LifeTable(interest=dict(i=0.04),
                 p={75: 0.9, 76: 0.88, 77: 0.85}).fill()
a = life.temporary_annuity(75, t=3)
IA = life.increasing_insurance(75, t=2)
A = life.deferred_insurance(75, u=2, t=1)
def fun(P):
    return life.gross_premium(a=a, A=P*IA + A*10000) - P
soa(459, life.solve(fun, target=0, guess=[449, 489]), "6.20")

```

SOA Question 6.20: [ 459 ] 458.83181728297353

SOA Question 6.21: (C) 100

```

life = Recursion(interest=dict(d=0.04))
life.set_A(0.7, x=75, t=15, endowment=1)
life.set_E(0.11, x=75, t=15)
def fun(P):
    P = float(P)
    return (P * life.temporary_annuity(75, t=15)
            - life.endowment_insurance(75, t=15, b=1000, endowment=15*P))
P = life.solve(fun, target=0, guess=(80, 120))
soa(100, P, 6.21)

```

SOA Question 6.21: [ 100 ] 100.85470085470084

SOA Question 6.22: (C) 102



```

life=SULT(udd=True)
a = UDD(m=12, life=life).temporary_annuity(45, t=20)
A = UDD(m=0, life=life).whole_life_insurance(45)
P = life.gross_premium(A=A, a=a, benefit=100000) / 12
soa(102, P, 6.22)

```

SOA Question 6.22: [ 102 ] 102.40668704849178

#### SOA Question 6.23: (D) 44.7

```

x = 0
life = Recursion().set_a(15.3926, x=x)\
    .set_a(10.1329, x=x, t=15)\
    .set_a(14.0145, x=x, t=30)

def fun(P):
    per_policy = 30 + (30 * life.whole_life_annuity(x))
    per_premium = (0.6 + 0.1 * life.temporary_annuity(x, t=15)
        + 0.1 * life.temporary_annuity(x, t=30))
    a = life.temporary_annuity(x, t=30)
    return (P * a) - (per_policy + per_premium * P)
P = life.solve(fun, target=0, guess=[30.3, 49.5])
soa(44.7, P, 6.23)

```

SOA Question 6.23: [ 44.7 ] 44.70806635781144

#### SOA Question 6.24: (E) 0.30

```

life = PolicyValues(interest=dict(delta=0.07))
x, A1 = 0, 0.30 # Policy for first insurance
P = life.premium_equivalence(A=A1, discrete=False) # Need its premium
policy = life.Policy(premium=P, discrete=False)
def fun(A2): # Solve for A2, given Var(Loss)
    return life.gross_variance_loss(A1=A1, A2=A2, policy=policy)
A2 = life.solve(fun, target=0.18, guess=0.18)

policy = life.Policy(premium=0.06, discrete=False) # Solve second insurance
variance = life.gross_variance_loss(A1=A1, A2=A2, policy=policy)
soa(0.304, variance, 6.24)

```

SOA Question 6.24: [ 0.304 ] 0.30419999999999975

#### SOA Question 6.25: (C) 12330

```

life = SULT()
woolhouse = Woolhouse(m=12, life=life)
benefits = woolhouse.deferred_annuity(55, u=10, b=1000 * 12)
expenses = life.whole_life_annuity(55, b=300)
payments = life.temporary_annuity(55, t=10)
def fun(P):
    return life.gross_future_loss(A=benefits + expenses, a=payments,
        policy=life.Policy(premium=P))
P = life.solve(fun, target=-800, guess=[12110, 12550])
soa(12330, P, 6.25)

```

```
SOA Question 6.25: [ 12330 ] 12325.781125438532
```

SOA Question 6.26 (D) 180

```
life = SULT(interest=dict(i=0.05))
def fun(P):
    return P - life.net_premium(90, b=1000, initial_cost=P)
P = life.solve(fun, target=0, guess=[150, 190])
soa(180, P, 6.26)
```

```
SOA Question 6.26: [ 180 ] 180.03164891315885
```

SOA Question 6.27: (D) 10310

```
life = ConstantForce(mu=0.03, interest=dict(delta=0.06))
x = 0
payments = (3 * life.temporary_annuity(x, t=20, discrete=False)
            + life.deferred_annuity(x, u=20, discrete=False))
benefits = (1000000 * life.term_insurance(x, t=20, discrete=False)
            + 500000 * life.deferred_insurance(x, u=20, discrete=False))
P = benefits / payments
soa(10310, P, 6.27)
```

```
SOA Question 6.27: [ 10310 ] 10309.617799001708
```

SOA Question 6.28 (B) 36

```
life = SULT(interest=dict(i=0.05))
a = life.temporary_annuity(40, t=5)
A = life.whole_life_insurance(40)
P = life.gross_premium(a=a, A=A, benefit=1000,
                      initial_policy=10, renewal_premium=.05,
                      renewal_policy=5, initial_premium=.2)
soa(36, P, 6.28)
```

```
SOA Question 6.28: [ 36 ] 35.72634219391481
```

SOA Question 6.29 (B) 20.5

```
life = Premiums(interest=dict(i=0.035))
def fun(a):
    return life.gross_premium(A=life.insurance_twin(a=a), a=a,
                              initial_policy=200, initial_premium=.5,
                              renewal_policy=50, renewal_premium=.1,
                              benefit=100000)
a = life.solve(fun, target=1770, guess=[20, 22])
soa(20.5, a, 6.29)
```

```
SOA Question 6.29: [ 20.5 ] 20.480268314431726
```

SOA Question 6.30: (A) 900

```

life = PolicyValues(interest=dict(i=0.04))
policy = life.Policy(premium=2.338, benefit=100, initial_premium=.1,
                    renewal_premium=0.05)
var = life.gross_variance_loss(A1=life.insurance_twin(16.50),
                              A2=0.17, policy=policy)
soa(900, var, "6.30")

```

SOA Question 6.30: [ 900 ] 908.141412994607

#### SOA Question 6.31: (D) 1330

```

life = ConstantForce(mu=0.01, interest=dict(delta=0.05))
A = (life.term_insurance(35, t=35, discrete=False)
     + life.E_x(35, t=35) * 0.51791) # A_35
P = life.premium_equivalence(A=A, b=100000, discrete=False)
soa(1330, P, 6.31)

```

SOA Question 6.31: [ 1330 ] 1326.5406293909457

#### SOA Question 6.32: (C) 550

```

x = 0
life = Recursion(interest=dict(i=0.05)).set_a(9.19, x=x)
benefits = UDD(m=0, life=life).whole_life_insurance(x)
payments = UDD(m=12, life=life).whole_life_annuity(x)
P = life.gross_premium(a=payments, A=benefits, benefit=100000)/12
soa(550, P, 6.32)

```

SOA Question 6.32: [ 550 ] 550.4356936711871

#### SOA Question 6.33: (B) 0.13

```

life = Insurance(mu=lambda x,t: 0.02*t, interest=dict(i=0.03))
x = 0
var = life.E_x(x, t=15, moment=life.VARIANCE, endowment=10000)
p = 1- life.portfolio_cdf(mean=0, variance=var, value=50000, N=500)
soa(0.13, p, 6.33, rel_tol=0.02)

```

SOA Question 6.33: [ 0.13 ] 0.12828940905648634

#### SOA Question 6.34: (A) 23300

```

life = SULT()
def fun(benefit):
    A = life.whole_life_insurance(61)
    a = life.whole_life_annuity(61)
    return life.gross_premium(A=A, a=a, benefit=benefit,
                              initial_premium=0.15, renewal_premium=0.03)
b = life.solve(fun, target=500, guess=[23300, 23700])
soa(23300, b, 6.34)

```

```
SOA Question 6.34: [ 23300 ] 23294.288659265632
```

SOA Question 6.35: (D) 530

```
sult = SULT()
A = sult.whole_life_insurance(35, b=100000)
a = sult.whole_life_annuity(35)
P = sult.gross_premium(a=a, A=A, initial_premium=.19, renewal_premium=.04)
soa(530, P, 6.35)
```

```
SOA Question 6.35: [ 530 ] 534.4072234303344
```

SOA Question 6.36: (B) 500

```
life = ConstantForce(mu=0.04, interest=dict(delta=0.08))
a = life.temporary_annuity(50, t=20, discrete=False)
A = life.term_insurance(50, t=20, discrete=False)
def fun(R):
    return life.gross_premium(a=a, A=A, initial_premium=R/4500,
                             renewal_premium=R/4500, benefit=100000)
R = life.solve(fun, target=4500, guess=[400, 800])
soa(500, R, 6.36)
```

```
SOA Question 6.36: [ 500 ] 500.0
```

SOA Question 6.37: (D) 820

```
sult = SULT()
benefits = sult.whole_life_insurance(35, b=50000 + 100)
expenses = sult.immediate_annuity(35, b=100)
a = sult.temporary_annuity(35, t=10)
P = (benefits + expenses) / a
soa(820, P, 6.37)
```

```
SOA Question 6.37: [ 820 ] 819.7190338249138
```

SOA Question 6.38: (B) 11.3

```
x, n = 0, 10
life = Recursion(interest=dict(i=0.05))
life.set_A(0.192, x=x, t=n, endowment=1, discrete=False)
life.set_E(0.172, x=x, t=n)
a = life.temporary_annuity(x, t=n, discrete=False)

def fun(a): # solve for discrete annuity, given continuous
    life = Recursion(interest=dict(i=0.05), verbose=False)
    life.set_a(a, x=x, t=n).set_E(0.172, x=x, t=n)
    return UDD(m=0, life=life).temporary_annuity(x, t=n)
a = life.solve(fun, target=a, guess=a) # discrete annuity
P = life.gross_premium(a=a, A=0.192, benefit=1000)
soa(11.3, P, 6.38)
```

```
[ Temporary Annuity: a_0:10 ]
    Annuity twin: a = (1 - A) / d
SOA Question 6.38: [ 11.3 ] 11.308644185253657
```

SOA Question 6.39: (A) 29

```
sult = SULT()
P40 = sult.premium_equivalence(sult.whole_life_insurance(40), b=1000)
P80 = sult.premium_equivalence(sult.whole_life_insurance(80), b=1000)
p40 = sult.p_x(40, t=10)
p80 = sult.p_x(80, t=10)
P = (P40 * p40 + P80 * p80) / (p80 + p40)
soa(29, P, 6.39)
```

```
SOA Question 6.39: [ 29 ] 29.033866427845496
```

SOA Question 6.40: (C) 116

```
# - standard formula discounts/accumulates by too much (i should be smaller)
x = 0
life = Recursion(interest=dict(i=0.06)).set_a(7, x=x+1).set_q(0.05, x=x)
a = life.whole_life_annuity(x)
A = 110 * a / 1000
life = Recursion(interest=dict(i=0.06)).set_A(A, x=x).set_q(0.05, x=x)
A1 = life.whole_life_insurance(x+1)
P = life.gross_premium(A=A1 / 1.03, a=7) * 1000
soa(116, P, "6.40")
```

```
[ Whole Life Annuity: a_0 ]
    forward: a_0 = 1 + E_0 a_1
    pure endowment 1_E_0 = 1_p_0 * v^1
[ Whole Life Insurance: A_1 ]
    backward: A_1 = (A_0/v - q) / p
    backward: A_1 = (A_0/v - q) / p
    backward: A_1 = (A_0/v - q) / p
    backward: A_1 = (A_0/v - q) / p
SOA Question 6.40: [ 116 ] 116.51945397474269
```

SOA Question 6.41: (B) 1417

```
x = 0
life = LifeTable(interest=dict(i=0.05), q={x:.01, x+1:.02}).fill()
a = 1 + life.E_x(x, t=1) * 1.01
A = (life.deferred_insurance(x, u=0, t=1)
     + 1.01 * life.deferred_insurance(x, u=1, t=1))
P = 100000 * A / a
soa(1417, P, 6.41)
```

```
SOA Question 6.41: [ 1417 ] 1416.9332301924137
```

SOA Question 6.42: (D) 0.113

```

x = 0
life = ConstantForce(interest=dict(delta=0.06), mu=0.06)
policy = life.Policy(discrete=True, premium=315.8,
                    T=3, endowment=1000, benefit=1000)
L = [life.L_from_t(t, policy=policy) for t in range(3)] # L(t)
Q = [life.q_x(x, u=u, t=1) for u in range(3)] # prob(die in year t)
Q[-1] = 1 - sum(Q[:-1]) # follows SOA Question Solution incorrect treat endowment!
p = sum([q for (q, l) in zip(Q, L) if l > 0])
soa(0.113, p, 6.42)

```

SOA Question 6.42: [ 0.113 ] 0.11307956328284252

#### SOA Question 6.43: (C) 170

```

sult = SULT()
a = sult.temporary_annuity(30, t=5)
A = sult.term_insurance(30, t=10)
other_expenses = 4 * sult.deferred_annuity(30, u=5, t=5)
P = sult.gross_premium(a=a, A=A, benefit=200000, initial_premium=0.35,
                    initial_policy=8 + other_expenses, renewal_policy=4,
                    renewal_premium=0.15)
soa(170, P, 6.43)

```

SOA Question 6.43: [ 170 ] 171.22371939459944

#### SOA Question 6.44: (D) 2.18

```

life = Recursion(interest=dict(i=0.05)).set_IA(0.15, x=50, t=10)
life.set_a(17, x=50).set_a(15, x=60).set_E(0.6, x=50, t=10)
A = life.deferred_insurance(50, u=10)
IA = life.increasing_insurance(50, t=10)
a = life.temporary_annuity(50, t=10)
P = life.gross_premium(a=a, A=A, IA=IA, benefit=100)
soa(2.2, P, 6.44)

```

SOA Question 6.44: [ 2.2 ] 2.183803457688809

#### SOA Question 6.45: (E) 690

```

life = SULT(udd=True)
policy = life.Policy(benefit=100000, premium=560, discrete=False)
p = life.L_from_prob(35, prob=0.75, policy=policy)
soa(690, p, 6.45)

```

SOA Question 6.45: [ 690 ] 689.2659416264196

#### SOA Question 6.46: (E) 208

```

life = Recursion(interest=dict(i=0.05)).set_IA(0.51213, x=55, t=10)
life.set_a(12.2758, x=55).set_a(7.4575, x=55, t=10)
A = life.deferred_annuity(55, u=10)
IA = life.increasing_insurance(55, t=10)

```

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```
a = life.temporary_annuity(55, t=10)
P = life.gross_premium(a=a, A=A, IA=IA, benefit=300)
soa(208, P, 6.46)
```

SOA Question 6.46: [ 208 ] 208.12282139036515

#### SOA Question 6.47: (D) 66400

```
sult = SULT()
a = sult.temporary_annuity(70, t=10)
A = sult.deferred_annuity(70, u=10)
P = sult.gross_premium(a=a, A=A, benefit=100000, initial_premium=0.75,
                      renewal_premium=0.05)
soa(66400, P, 6.47)
```

SOA Question 6.47: [ 66400 ] 66384.13293704337

#### SOA Question 6.48: (A) 3195 – example of deep insurance recursion

```
x = 0
life = Recursion(interest=dict(i=0.06), depth=5).set_p(.95, x=x, t=5)
life.set_q(.02, x=x+5).set_q(.03, x=x+6).set_q(.04, x=x+7)
a = 1 + life.E_x(x, t=5)
A = life.deferred_insurance(x, u=5, t=3)
P = life.gross_premium(A=A, a=a, benefit=100000)
soa(3195, P, 6.48)
```

```
[ Pure Endowment: 5_E_0 ]
  pure endowment 5_E_0 = 5_p_0 * v^5
[ Pure Endowment: 5_E_0 ]
  pure endowment 5_E_0 = 5_p_0 * v^5
[ Term Insurance: A_5^1:3 ]
  forward: A_5 = qv + pvA_6
  forward: A_6 = qv + pvA_7
  endowment insurance - pure endowment = A_7^1:1
  pure endowment 1_E_7 = 1_p_7 * v^1
[ Term Insurance: A_5^1:3 ]
  pure endowment 1_E_7 = 1_p_7 * v^1
  endowment insurance - pure endowment = A_7^1:1
  forward: A_6 = qv + pvA_7
  forward: A_5 = qv + pvA_6
SOA Question 6.48: [ 3195 ] 3195.1189176587473
```

#### SOA Question 6.49: (C) 86

```
sult = SULT(udd=True)
a = UDD(m=12, life=sult).temporary_annuity(40, t=20)
A = sult.whole_life_insurance(40, discrete=False)
P = sult.gross_premium(a=a, A=A, benefit=100000, initial_policy=200,
                      renewal_premium=0.04, initial_premium=0.04) / 12
soa(86, P, 6.49)
```

SOA Question 6.49: [ 86 ] 85.99177833261696

SOA Question 6.50: (A) -47000

```
life = SULT()
P = life.premium_equivalence(a=life.whole_life_annuity(35), b=1000)
a = life.deferred_annuity(35, u=1, t=1)
A = life.term_insurance(35, t=1, b=1000)
cash = (A - a * P) * 10000 / life.interest.v
soa(-47000, cash, "6.50")
```

SOA Question 6.50: [ -47000 ] -46948.2187697819

SOA Question 6.51: (D) 34700

```
life = Recursion()
life.set_DA(0.4891, x=62, t=10)
life.set_A(0.0910, x=62, t=10)
life.set_a(12.2758, x=62)
life.set_a(7.4574, x=62, t=10)
IA = life.increasing_insurance(62, t=10)
A = life.deferred_annuity(62, u=10)
a = life.temporary_annuity(62, t=10)
P = life.gross_premium(a=a, A=A, IA=IA, benefit=50000)
soa(34700, P, 6.51)
```

```
[ Increasing Insurance: IA_62:10 ]
    identity IA_62:10: (11)A - DA
SOA Question 6.51: [ 34700 ] 34687.207544453246
```

SOA Question 6.52: (D) 50.80 – hint: set face value benefits to 0

```
sult = SULT()
a = sult.temporary_annuity(45, t=10)
other_cost = 10 * sult.deferred_annuity(45, u=10)
P = sult.gross_premium(a=a, A=0, benefit=0, # set face value H = 0
                      initial_premium=1.05, renewal_premium=0.05,
                      initial_policy=100 + other_cost, renewal_policy=20)
soa(50.8, P, 6.52)
```

SOA Question 6.52: [ 50.8 ] 50.80135534704229

SOA Question 6.53: (D) 720

```
x = 0
life = LifeTable(interest=dict(i=0.08), q={x:.1, x+1:.1, x+2:.1}).fill()
A = life.term_insurance(x, t=3)
P = life.gross_premium(a=1, A=A, benefit=2000, initial_premium=0.35)
soa(720, P, 6.53)
```

SOA Question 6.53: [ 720 ] 720.1646090534978

SOA Question 6.54: (A) 25440



```
life = SULT()
s = math.sqrt(life.net_policy_variance(45, b=200000))
soa(25440, s, 6.54)
```

SOA Question 6.54: [ 25440 ] 25441.694847703857

## 20.7 7 Policy Values

SOA Question 7.1: (C) 11150

```
life = SULT()
x, n, t = 40, 20, 10
A = (life.whole_life_insurance(x+t, b=50000)
     + life.deferred_insurance(x+t, u=n-t, b=50000))
a = life.temporary_annuity(x+t, t=n-t, b=875)
L = life.gross_future_loss(A=A, a=a)
soa(11150, L, 7.1)
```

SOA Question 7.1: [ 11150 ] 11152.108749338717

SOA Question 7.2: (C) 1152

```
x = 0
life = Recursion(interest=dict(i=.1)).set_q(0.15, x=x).set_q(0.165, x=x+1)
life.set_reserves(T=2, endowment=2000)

def fun(P): # solve P s.t. V is equal backwards and forwards
    policy = dict(t=1, premium=P,
                  benefit=lambda t: 2000, reserve_benefit=True)
    return life.t_V_backward(x, **policy) - life.t_V_forward(x, **policy)
P = life.solve(fun, target=0, guess=[1070, 1230])
soa(1152, P, 7.2)
```

SOA Question 7.2: [ 1152 ] 1151.5151515151515

SOA Question 7.3: (E) 730

```
x = 0 # x=0 is (90) and interpret every 3 months as t=1 year
life = LifeTable(interest=dict(i=0.08/4),
                  l={0:1000, 1:898, 2:800, 3:706}).fill()
life.set_reserves(T=8, V={3: 753.72})
life.set_reserves(V={2: life.t_V_forward(x=0, t=2, premium=60*0.9,
                                          benefit=lambda t: 1000)})
V = life.t_V_forward(x=0, t=1, premium=0, benefit=lambda t: 1000)
soa(730, V, 7.3)
```

SOA Question 7.3: [ 730 ] 729.998398765594

SOA Question 7.4: (B) -74 – split benefits into two policies

```

life = SULT()
P = life.gross_premium(a=life.whole_life_annuity(40),
                      A=life.whole_life_insurance(40),
                      initial_policy=100, renewal_policy=10,
                      benefit=1000)
P += life.gross_premium(a=life.whole_life_annuity(40),
                      A=life.deferred_insurance(40, u=11),
                      benefit=4000) # for deferred portion
policy = life.Policy(benefit=1000, premium=1.02*P,
                    renewal_policy=10, initial_policy=100)
V = life.gross_policy_value(x=40, t=1, policy=policy)
policy = life.Policy(benefit=4000, premium=0)
A = life.deferred_insurance(41, u=10)
V += life.gross_future_loss(A=A, a=0, policy=policy) # for deferred portion
soa(-74, V, 7.4)

```

SOA Question 7.4: [ -74 ] -73.942155695248

#### SOA Question 7.5: (E) 1900

```

x = 0
life = Recursion(interest=dict(i=0.03), udd=True).set_q(0.04561, x=x+4)
life.set_reserves(T=3, V={4: 1405.08})
V = life.r_V_backward(x, s=4, r=0.5, benefit=10000, premium=647.46)
soa(1900, V, 7.5)

```

SOA Question 7.5: [ 1900 ] 1901.766021537228

#### Answer 7.6: (E) -25.4

```

life = SULT()
P = life.net_premium(45, b=2000)
policy = life.Policy(benefit=2000, initial_premium=.25, renewal_premium=.05,
                    initial_policy=2*1.5 + 30, renewal_policy=2*.5 + 10)
G = life.gross_premium(a=life.whole_life_annuity(45), **policy.premium_terms)
gross = life.gross_policy_value(45, t=10, policy=policy.set(premium=G))
net = life.net_policy_value(45, t=10, b=2000)
V = gross - net

soa(-25.4, V, 7.6)

```

SOA Question 7.6: [ -25.4 ] -25.44920289521204

#### SOA Question 7.7: (D) 1110

```

x = 0
life = Recursion(interest=dict(i=0.05)).set_A(0.4, x=x+10)
a = Woolhouse(m=12, life=life).whole_life_annuity(x+10)
policy = life.Policy(premium=0, benefit=10000, renewal_policy=100)
V = life.gross_future_loss(A=0.4, policy=policy.future)
policy = life.Policy(premium=30*12, renewal_premium=0.05)
V += life.gross_future_loss(a=a, policy=policy.future)
soa(1110, V, 7.7)

```

SOA Question 7.7: [ 1110 ] 1107.9718253968258

SOA Question 7.8: (C) 29.85

```
sult = SULT()
x = 70
q = {x: [sult.q_x(x+k)*(.7 + .1*k) for k in range(3)] + [sult.q_x(x+3)]}
life = Recursion(interest=dict(i=.05)).set_q(sult.q_x(70)*.7, x=x)\
    .set_reserves(T=3)
V = life.t_V(x=70, t=1, premium=35.168, benefit=lambda t: 1000)
soa(29.85, V, 7.8)
```

SOA Question 7.8: [ 29.85 ] 29.85469179271202

SOA Question 7.9: (A) 38100

```
sult = SULT(udd=True)
x, n, t = 45, 20, 10
a = UDD(m=12, life=sult).temporary_annuity(x+10, t=n-10)
A = UDD(m=0, life=sult).endowment_insurance(x+10, t=n-10)
policy = sult.Policy(premium=253*12, endowment=100000, benefit=100000)
V = sult.gross_future_loss(A=A, a=a, policy=policy)
soa(38100, V, 7.9)
```

SOA Question 7.9: [ 38100 ] 38099.62176709246

SOA Question 7.10: (C) -970

```
life = SULT()
G = 977.6
P = life.net_premium(45, b=100000)
policy = life.Policy(benefit=0, premium=G-P, renewal_policy=.02*G + 50)
V = life.gross_policy_value(45, t=5, policy=policy)
soa(-970, V, "7.10")
```

SOA Question 7.10: [ -970 ] -971.8909301877826

SOA Question 7.11: (B) 1460

```
life=Recursion(interest=dict(i=0.05)).set_a(13.4205, x=55)
policy=life.Policy(benefit=10000)
def fun(P):
    return life.L_from_t(t=10, policy=policy.set(premium=P))
P = life.solve(fun, target=4450, guess=400)
V = life.gross_policy_value(45, t=10, policy=policy.set(premium=P))
soa(1460, V, 7.11)
```

SOA Question 7.11: [ 1460 ] 1459.9818035330218

SOA Question 7.12: (E) 4.09

```
benefit = lambda k: 26 - k
x = 44
life = Recursion(interest=dict(i=0.04)).set_q(0.15, x=55)
life.set_reserves(T=25, endowment=1, V={11: 5.})
def fun(P): # solve for net premium, from mathlc.mathlc.mathlc.mathlc.mathlc.
    ↪mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.
    ↪mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.
    ↪recursion
    return life.t_V(x=x, t=24, premium=P, benefit=benefit)
P = life.solve(fun, target=0.6, guess=0.5) # solved net premium
V = life.t_V(x, t=12, premium=P, benefit=benefit) # recursion formula
soa(4.09, V, 7.12)
```

SOA Question 7.12: [ 4.09 ] 4.089411764705883

Answer 7.13: (A) 180

```
life = SULT()
V = life.FPT_policy_value(40, t=10, n=30, endowment=1000, b=1000)
soa(180, V, 7.13)
```

SOA Question 7.13: [ 180 ] 180.1071785904076

SOA Question 7.14: (A) 2200

```
x = 45
life = Recursion(interest=dict(i=0.05)).set_q(0.009, x=50)
life.set_reserves(T=10, V={5: 5500})
def fun(P): # solve for net premium, from mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.
    mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.
    mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.year 6 reserve
    return life.t_V(x=x, t=6, premium=P*0.96 - 50,
                    benefit=lambda t: 100000 + 200)
P = life.solve(fun, target=7100, guess=[2200, 2400])
soa(2200, P, 7.14)
```

SOA Question 7.14: [ 2200 ] 2197.8174603174602

SOA Question 7.15: (E) 50.91

```
x = 0
life = Recursion(udd=True, interest=dict(i=0.05)).set_q(0.1, x=x+15)
life.set_reserves(T=3, V={16: 49.78})
V = life.r_V_forward(x, s=15, r=0.6, benefit=100)
soa(50.91, V, 7.15)
```

SOA Question 7.15: [ 50.91 ] 50.91362826922369

SOA Question 7.16: (D) 380

```
life = Select(interest=dict(v=.95), A={86: [683/1000]},
              q={80+k: [.01*(k+1)] for k in range(6)}).fill()
```

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```
x, t, n = 80, 3, 5
A = life.whole_life_insurance(x+t)
a = life.temporary_annuity(x+t, t=n-t)
V = life.gross_future_loss(A=A, a=a,
                           policy=life.Policy(benefit=1000, premium=130))
soa(380, V, 7.16)
```

SOA Question 7.16: [ 380 ] 381.6876905200001

SOA Question 7.17: (D) 1.018

```
x = 0
life = Recursion(interest=dict(v=math.sqrt(0.90703)))
life.set_q(0.02067, x=x+10)
life.set_A(0.52536, x=x+11)
life.set_A(0.30783, x=x+11, moment=2)
A1 = life.whole_life_insurance(x+10)
A2 = life.whole_life_insurance(x+10, moment=2)
ratio = (life.insurance_variance(A2=A2, A1=A1)
         / life.insurance_variance(A2=0.30783, A1=0.52536))
soa(1.018, ratio, 7.17)
```

```
[ Whole Life Insurance: A_10 ]
  forward: A_10 = qv + pvA_11
[ Whole Life Insurance: A_10 ]
  forward: A_10 = qv + pvA_11
SOA Question 7.17: [ 1.018 ] 1.0182465434445054
```

SOA Question 7.18: (A) 17.1

```
x = 10
life = Recursion(interest=dict(i=0.04)).set_q(0.009, x=x)
def fun(a):
    return life.set_a(a, x=x).net_policy_value(x, t=1)
a = life.solve(fun, target=0.012, guess=[17.1, 19.1])
soa(17.1, a, 7.18)
```

SOA Question 7.18: [ 17.1 ] 17.07941929974385

SOA Question 7.19: (D) 720

```
life = SULT()
policy = life.Policy(benefit=100000, initial_policy=300, initial_premium=.5,
                    renewal_premium=.1)
P = life.gross_premium(A=life.whole_life_insurance(40),
                      **policy.premium_terms)
A = life.whole_life_insurance(41)
a = life.immediate_annuity(41) # after premium and expenses are paid
V = life.gross_future_loss(A=A, a=a, policy=policy.set(premium=P).future)
soa(720, V, 7.19)
```

SOA Question 7.19: [ 720 ] 722.7510208759086

## SOA Question 7.20: (E) -277.23

```

life = SULT()
S = life.FPT_policy_value(35, t=1, b=1000) # is 0 for FPT at t=0,1
policy = life.Policy(benefit=1000, initial_premium=.3, initial_policy=300,
                    renewal_premium=.04, renewal_policy=30)
P = life.gross_premium(A=life.whole_life_insurance(35),
                      **policy.premium_terms)
R = life.gross_policy_value(35, t=1, policy=policy.set(premium=P))
soa(-277.23, R - S, "7.20")

```

SOA Question 7.20: [ -277.23 ] -277.19303323929216

## SOA Question 7.21: (D) 11866

```

life = SULT()
x, t, u = 55, 9, 10
P = life.gross_premium(IA=0.14743, a=life.temporary_annuity(x, t=u),
                      A=life.deferred_annuity(x, u=u), benefit=1000)
policy = life.Policy(initial_policy=life.term_insurance(x+t, t=1, b=10*P),
                    premium=P, benefit=1000)
a = life.temporary_annuity(x+t, t=u-t)
A = life.deferred_annuity(x+t, u=u-t)
V = life.gross_future_loss(A=A, a=a, policy=policy)
soa(11866, V, 7.21)

```

SOA Question 7.21: [ 11866 ] 11866.30158100453

## SOA Question 7.22: (C) 46.24

```

life = PolicyValues(interest=dict(i=0.06))
policy = life.Policy(benefit=8, premium=1.250)
def fun(A2):
    return life.gross_variance_loss(A1=0, A2=A2, policy=policy)
A2 = life.solve(fun, target=20.55, guess=20.55/8**2)
policy = life.Policy(benefit=12, premium=1.875)
var = life.gross_variance_loss(A1=0, A2=A2, policy=policy)
soa(46.2, var, 7.22)

```

SOA Question 7.22: [ 46.2 ] 46.2375

## SOA Question 7.23: (D) 233

```

life = Recursion(interest=dict(i=0.04)).set_p(0.995, x=25)
A = life.term_insurance(25, t=1, b=10000)
def fun(beta): # value of premiums in first 20 years must be equal
    return beta * 11.087 + (A - beta)
beta = life.solve(fun, target=216 * 11.087, guess=[140, 260])
soa(233, beta, 7.23)

```

```

[ Term Insurance: A_25^1:1 ]
    endowment insurance - pure endowment = A_25^1:1
    pure endowment 1_E_25 = 1_p_25 * v^1
SOA Question 7.23: [ 233 ] 232.64747466274176

```

## SOA Question 7.24: (C) 680

```
life = SULT()
P = life.premium_equivalence(A=life.whole_life_insurance(50), b=1000000)
soa(680, 11800 - P, 7.24)
```

SOA Question 7.24: [ 680 ] 680.291823645397

## SOA Question 7.25: (B) 3947.37

```
life = Select(interest=dict(i=.04), A={55: [.23, .24, .25],
                                         56: [.25, .26, .27],
                                         57: [.27, .28, .29],
                                         58: [.20, .30, .31]})
V = life.FPT_policy_value(55, t=3, b=100000)
soa(3950, V, 7.25)
```

SOA Question 7.25: [ 3950 ] 3947.3684210526353

## SOA Question 7.26: (D) 28540 – backward-forward reserve recursion

```
x = 0
life = Recursion(interest=dict(i=.05)).set_p(0.85, x=x).set_p(0.85, x=x+1)
life.set_reserves(T=2, endowment=50000)
benefit = lambda k: k*25000
def fun(P): # solve P s.t. V is equal backwards and forwards
    policy = dict(t=1, premium=P, benefit=benefit, reserve_benefit=True)
    return life.t_V_backward(x, **policy) - life.t_V_forward(x, **policy)
P = life.solve(fun, target=0, guess=[27650, 28730])
soa(28540, P, 7.26)
```

SOA Question 7.26: [ 28540 ] 28542.392566782808

## SOA Question 7.27: (B) 213

```
x = 0
life = Recursion(interest=dict(i=0.03)).set_q(0.008, x=x)
life.set_reserves(V={0: 0})
def fun(G): # Solve gross premium from mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.
    ↪mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.
    ↪mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.expense_
    ↪reserves equation
    return life.t_V(x=x, t=1, premium=G-187, benefit=lambda t: 0,
                    per_policy=10 + 0.25*G)
G = life.solve(fun, target=-38.70, guess=[200, 252])
soa(213, G, 7.27)
```

SOA Question 7.27: [ 213 ] 212.970355987055

## SOA Question 7.28: (D) 24.3

```
life = SULT()
PW = life.net_premium(65, b=1000) # 20_V=0 => P+W is net premium for A_65
```

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```
P = life.net_premium(45, t=20, b=1000) # => P is net premium for A_45:20
soa(24.3, PW - P, 7.28)
```

SOA Question 7.28: [ 24.3 ] 24.334725400123975

## SOA Question 7.29: (E) 2270

```
x = 0
life = Recursion(interest=dict(i=0.04)).set_a(14.8, x=x)\
    .set_a(11.4, x=x+10)
def fun(B): # Solve for benefit B given net 10_V = 2290
    return life.net_policy_value(x, t=10, b=B)
B = life.solve(fun, target=2290, guess=2290*10)
policy = life.Policy(initial_policy=30, renewal_policy=5, benefit=B)
G = life.gross_premium(a=life.whole_life_annuity(x), **policy.premium_terms)
V = life.gross_policy_value(x, t=10, policy=policy.set(premium=G))
soa(2270, V, 7.29)
```

SOA Question 7.29: [ 2270 ] 2270.743243243244

## SOA Question 7.30: (E) 9035

```
policy = SULT.Policy(premium=0, benefit=10000) # premiums=0 after t=10
L = SULT().gross_policy_value(35, policy=policy)
V = SULT(interest=dict(i=0)).gross_policy_value(35, policy=policy) # 10000
soa(9035, V-L, "7.30")
```

SOA Question 7.30: [ 9035 ] 9034.654127845053

## SOA Question 7.31: (E) 0.310

```
x = 0
life = Reserves().set_reserves(T=3)
G = 368.05
def fun(P): # solve net premium from mathlc.mathlc.mathlc.mathlc.mathlc.
    ↪mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.
    ↪mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.mathlc.
    ↪reserve equation
    return life.t_V(x=x, t=2, premium=G-P, benefit=lambda t: 0,
        per_policy=5 + .08*G)
P = life.solve(fun, target=-23.64, guess=[.29, .31]) / 1000
soa(0.310, P, 7.31)
```

SOA Question 7.31: [ 0.31 ] 0.309966

## SOA Question 7.32: (B) 1.4

```
life = PolicyValues(interest=dict(i=0.06))
policy = life.Policy(benefit=1, premium=0.1)
def fun(A2):
    return life.gross_variance_loss(A1=0, A2=A2, policy=policy)
A2 = life.solve(fun, target=0.455, guess=0.455)
```

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```
policy = life.Policy(benefit=2, premium=0.16)
variance = life.gross_variance_loss(A1=0, A2=A2, policy=policy)
soa(1.39, variance, 7.32)
```

```
SOA Question 7.32: [ 1.39 ] 1.3848168384380901
```

```
from datetime import datetime
print(datetime.now())
soa.summary()
```

```
2022-11-05 09:52:09.278624
```

	num	correct
0	136	136
2	8	8
3	14	14
4	19	19
5	9	9
6	54	54
7	32	32