Problem set 1

04124880 Philosophical Logic

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Due: November 20, 2024 (in class)

Exercise 1. *(40 points) Use the truth table method to show that* $(\neg P \to (Q \vee R)) \to ((\neg P \to Q) \vee (\neg P \to R))$ *is a valid propositional formula (tautology). In other words, draw a truth table containing all possible values that the propositional letters may take, and check that the formula above remains true in every row of the table.*

Exercise 2. *(60 points) Give a Fitch-style natural deduction proof of the following:*

1. $(P \vee (Q \vee R)) \to ((P \vee Q) \vee R)$ *(20 points)*

2. $\neg(P \wedge Q) \to (\neg P \vee \neg Q)$ *(20 points)*

3. $\exists x \forall y (x = y) \vdash \forall x \forall y (x = y)$ *(20 points)*

Do not forget to number each proof line on the left of the proved formula and on its right side write the rule and the lines you have used. It is a good idea to check what the major logical connective of the formula you want to prove is and try to use its introduction rule. When this strategy seems to fail try to use DNE and prove its double-negated form instead. To use a formula you already have in a line, determine its major logical connective and use the corresponding elimination rule. (In order to simplify your solution to 2.3 you are allowed to directly obtain symmetry $\forall x \forall y (x = y \to y = x)$ *and transitivity* $\forall x \forall y \forall z (x = y \to y = z \to x = z)$ *anywhere in a subproof under the justifications SYM and TRANS respectively.)*