Assignment 2

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Exercise 1

1. Give a formula, using quantifiers and identity, that is true in every model with a domain of one object and false in some model with a domain of two objects.

Solution:

$$\forall x \forall y (x = y)$$

2. Give a formula, not using quantifiers or identity, that has this property.

Solution:

$$F_x \leftrightarrow F_y$$

Exercise 2

Solution:

$$I\,\imath\,x\,(M_{xs})$$

Exercise 3

Solution:

The expression with negation:

$$\neg S\,\imath\,x\,(M_{xs})$$

For the wide-scope negation, the expression is :

$$\neg \exists x (M_{xs} \wedge \forall y (M_{ys} \rightarrow y = x) \wedge S_x)$$

For the narrow-scope negation, the expression is:

$$\exists x (M_{xs} \wedge \forall y (M_{ys} \rightarrow y = x) \wedge \neg S_x)$$

	True Model	**False Model**
Wide-Scope Negation	No one is Smith's murderer (M_{xs} = False for all x).	a is Smith's unique murderer (M_{as} = True) and is sane (S_a = True).
Narrow-Scope Negation	a is Smith's unique murderer (M_{as} = True) and is not sane (S_a = False).	a is Smith's unique murderer (M_{as} = True) and is sane (S_a = True).

Exercise 4

1. "There are few hard exercises in this homework"

Solution:

E - "is an exercise in this homework"

$$\forall x (\frac{|E_x|}{2} > |E_x \wedge H_x|)$$

2. "The number of happy people divided by the number of logicians that read the works of Gödel but not Frege is less than the number of planets"

Solution:

$$\forall x (|H_x| \div |L_x \wedge R_g \wedge \neg R_f| < |P_x|)$$

3. "Over 75% of students who passed this course live happily ever after"

Solution:

$$\forall x (0.75 < |S_x \wedge P_x \wedge H_x| \div |S_x \wedge P_x|)$$