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Exercise 1

- Write the relation R as $(x,y)\in R$

(a) The relation R on {1, 2, 3, 4} defined by $(x,y)\in R$ if $x^2 \geq y$.

(b) The relation R on {1,2,3,4,5} defined by $(x,y)\in R$ if 3 divides x-y.

(a) $R = \{(1,1), (2,1), (3,1), (4,1), (2,2), (3,2), (4,2), (2,3), (3,3), (4,3), (2,4), (3,4), (4,4)\}$

(b) $x-y=0$ @ $x-y=\pm 3$

$R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (4,1), (5,2), (1,4), (2,5)\}$

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Exercise 2

- Find range and domain for:

(a) The relation R on {1, 2, 3, 4} defined by $(x,y)\in R$ if $x^2 \geq y$.

(b) The relation R = { (1,2), (2,1), (3,3), (1,1), (2,2)} on X = {1, 2, 3}

(a) $R = \{(1,1), (2,1), (3,1), (4,1), (2,2), (3,2), (4,2), (2,3), (3,3), (4,3), (2,4), (3,4), (4,4)\}$

Range : $\{1, 2, 3, 4\}$

Domain : $\{1, 2, 3, 4\}$

(b) Range : $\{1, 2, 3\}$

Domain : $\{1, 2, 3\}$

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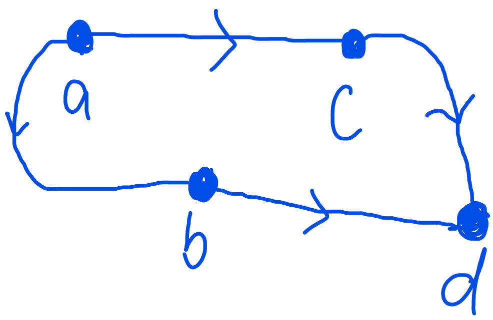
Exercise 3

Draw the diagraph of the relation:

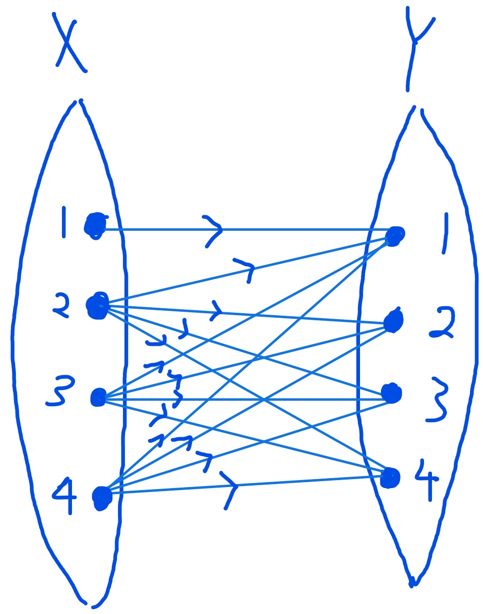
(a) $R = \{ (a,c), (b, d), (a,b), (c,d) \}$

(b) The relation R on $\{1, 2, 3, 4\}$ defined by $(x,y) \in R$ if $x^2 \geq y$

(a)

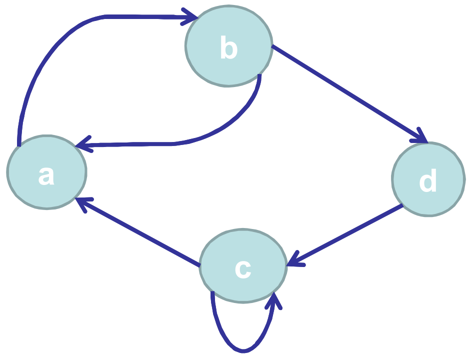


(b)



Exercise 4

Write the relation as a set of ordered pair.



$R = \{(a,b), (b,a), (c,a), (c,c), (b,d), (d,c)\}$

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Exercise 5

- Let $A=\{1, 2, 3, 4\}$ and R be a relation on A.
  $R = \{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}$

(a) What is R (represent)?

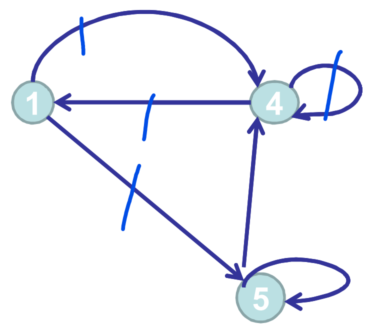
(b) What is matrix representation of R?

(a)

(b) $$M_R = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Exercise 6

- Let $A = \{1, 4, 5\}$ and let R be given by the digraph shown below.
- Find M_R and R



$$R = \{(1,4), (1,5), (4,1), (4,4), (5,4), (5,5)\}$$

$$M_R = \begin{array}{c c} & \begin{array}{ccc} 1 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

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Exercise 7

- An airline services the five cities c_1, c_2, c_3, c_4 and c_5.
- Table below gives the cost (in dollars) of going from c_i to c_j. Thus the cost of going from c_1 to c_3 is \$100, while the cost of going from c_4 to c_2 is \$200

To / from	c_1	c_2	c_3	c_4	c_5
c_1					200
c_2	190		200		220
c_3				190	250
c_4	190	200			
c_5	200		200		

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- If the relation R on the set of cities $A = \{c_1, c_2, c_3, c_4, c_5\}$: $c_i \, R \, c_j$ if and only if the cost of going from c_i to c_j is defined and less than or equal to \$180.

Find:

i) *R.*

ii)Matrices of relations for *R.*

(i) $R = \{(c_1, c_2), (c_1, c_3), (c_1, c_4), (c_2, c_4), (c_3, c_1), (c_3, c_2), (c_4, c_3), (c_4, c_5), (c_5, c_2), (c_5, c_4)\}$

(ii)

$$M_R = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{array} \begin{array}{c} \begin{array}{ccccc} c_1 & c_2 & c_3 & c_4 & c_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

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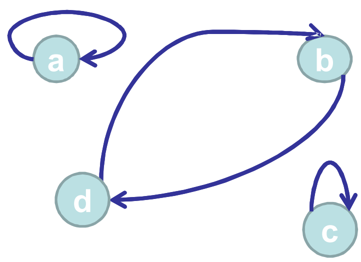
Exercise 8

(i) The relation R on $X=\{a,b,c,d\}$ given by the below diagraph. Is R a reflexive relation?



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(i) $R = \{(a,a), (b,d), (d,b), (c,c)\}$

The relation R is not reflexive as $b \in X$ but $(b,b) \notin R$ and $d \in X$ but $(d,d) \notin R$

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Exercise 9

Let A = { 1,2,3,4). Construct the matrix of relation of *R*. Then, determine whether the relation is reflexive, not reflexive or irreflexive.

(i) *R* = { (1,1), (1,2), (2,1), (2,2), (3,3), (3,4), (4,3), (4,4))
(ii) *R* = { (1,3), (1,1), (3,1), (1,2), (3,3), (4,4)}
(iii) *R* ={ (1,2), (1,3), (3,1), (1,1), (3,3), (3,2), (1,4), (4,2), (3,4)}
(iv) *R* ={ (1,2), (1,3), (3,2), (1,4), (4,2), (3,4)}

(i) $M_R = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$ $\therefore$ Reflexive

(ii) $M_R = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$ $\therefore$ Not reflexive

(iii) $M_R = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$ $\therefore$ Not reflexive.

(iv) $M_R = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$ $\therefore$ Irreflexive

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Exercise 10

Let A = { 1,2,3,4). Construct the matrix of relation of *R*. Then, determine whether the relation is symmetric, asymmetric, not symmetric or antisymmetric.

(i) *R* = { (1,1), (1,2), (2,1), (2,2), (3,3), (3,4), (4,3), (4,4))

(ii) *R* = { (1,3), (1,1), (3,1), (1,2), (3,3), (4,4)}

(iii) *R* ={ (1,2), (1,3), (1,1), (3,3), (3,2), (1,4), (4,2), (3,4)}

(i) $$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$\therefore$ Symmetric

(ii) $$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$\therefore$ Not symmetric

(iii) $$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$\therefore$ Antisymmetric

Exercise 11

Let A = { 1,2,3,4). Construct the matrix of relation of *R*. Then, determine whether the relation is symmetric, asymmetric, not symmetric or antisymmetric.

(iii) R ={ (1,2), (1,3), (1,1), (3,3), (3,2), (1,4), (4,2), (3,4)}

(iii)

$$M_R = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$\because \forall x, y \in A,\ (x,y) \in R,\ (y,x) \notin R$

when $x \neq y$

$\therefore R$ is antisymmetric

Exercise 12

The relation *R* on the set {1,2,3,4,5} defined by the rule $(x,y) \in R$ if $x+y \leq 6$

(i) List the elements of R
(ii) Find the domain of R
(iii) Find the range of R
(iv) Is the relation of R reflexive, symmetric, assymmetric, antisymmetric, transitive, and/or equivalence relation or partial order?

(i) $R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)\}$

(ii) Domain $= \{1,2,3,4,5\}$

(iii) Range $= \{1,2,3,4,5\}$

(iv) $M_R = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$

→ Not reflexive as there is no $(4,4)$ and $(5,5)$

→ Symmetric as $\forall x,y \in \{1,2,3,4,5\}$, $(x,y) \in R \rightarrow (y,x) \in R$

→ Not asymmetric as $\exists\, m_{11} = 1$

→ Not antisymmetric as $(2,1) \in R$, $(1,2) \in R$, but $1 \neq 2$

→ Not transitive. $(2,1)$ and $(1,5) \in R$, but $(2,5) \notin R$

→ Not an equivalence relation, as R is not reflexive, and not transitive.

→ Not a partial order, as R is not reflexive, not antisymmetric and not transitive.

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Exercise 13

The relation *R* on the set {1,2,3,4,5} defined by the rule $(x,y) \in R$ if 3 divides *x-y*

(i) List the elements of R
(ii) Find the domain of R
(iii) Find the range of R
(iv) Is the relation of R refelxive, symmetric, assymmetric, antisymmetric, transitive, and/or equivalence relation or partial order?

(i) $R = \{(1,1),(1,4),(2,2),(2,5),(3,3),(4,1),(4,4),(5,2),(5,5)\}$

(ii) Domain $= \{1,2,3,4,5\}$

(iii) Range $= \{1,2,3,4,5\}$

(iv)
$$M_R = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

→ Reflexive. $\forall x \in \{1,2,3,4,5\}, (x,x) \in R$

→ Symmetric $\forall x,y \in \{1,2,3,4,5\}, (x,y) \in R \rightarrow (y,x) \in R$

→ Not assymetric. $\exists m_{ii} = 1$

→ Not antisymetric $(1,4) \in R, (4,1) \in R$, but $1 \neq 4$

→ Transitive

→ Equivalence relation since R is reflexive, symmetric and transitive.

→ Not partial order relation since R is not antisymmetric

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$\forall i \forall j$ when $(n_{ij} = 1)$ then $(m_{ij} = 1)$

$n_{41} = 1, \; m_{41} = 1$

$(4,4)$ and $(4,1) \in R$, $(4,1) \in R$

$\therefore$ R is transitive.

Exercise 14

The relation *R* on the set {1,2,3,4,5} defined by the rule $(x,y) \in R$ if $x=y-1$

(i) List the elements of R

(ii) Find the domain of R

(iii) Find the range of R

(iv) Is the relation of R reflexive, symmetric, assymmetric, antisymmetric, transitive, and/or equivalence relation or partial order?