$$\ell(\theta, \hat{\theta}) = a\theta^2 + b\theta\hat{\theta} + c\hat{\theta}^2$$

$$p(\theta | x) = N(\theta \mid M, L^{-1}).$$

min $\rho(\theta, \hat{\theta})$ wrt $\hat{\theta}$.

Find the Bayes rule.

$$\rho(\theta, \hat{\theta}) = E[L(\theta, \hat{\theta}) \mid x]$$

$$= E_\theta[a\theta^2 + b\theta\hat{\theta} + c\hat{\theta}^2 \mid x]$$

$$= aE_\theta[\theta^2 \mid x] + b\hat{\theta}E[\theta \mid x] + c\hat{\theta}^2$$

To find the Bayes rule

$$\frac{\partial \rho(\theta, \hat{\theta})}{\partial \hat{\theta}} \left\{ \qquad \right\}$$

$$= 0 + bE[\theta \mid x] + c2\hat{\theta} \overset{\text{set}}{=} 0$$

$$\hat{\theta} = -\frac{b}{c2}E[\theta \mid x]$$

$$\boxed{\hat{\theta} = -\frac{bM}{c2}}$$

show that $\hat{\theta}$ is unique.

$$\frac{\partial^2 \ell(\theta, \hat{\theta})}{\partial \hat{\theta}} = 2c > 0 \quad \text{by defn.}$$